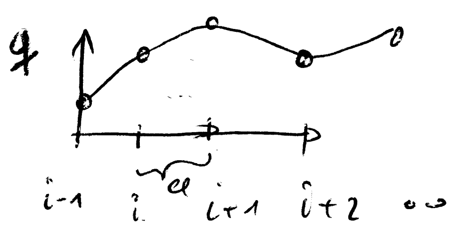
Quantum Field Theory

1 Introduction

Quantum Field Theory describes the fundamental interactions of matter.

$$
\begin{array}{ccc}
x, p & & \phi(\vec{x}),\ \pi(\vec{x}) \\
\text{classical mechanics} & \longrightarrow & \text{classical field theory} \\
[\hat{x},\hat{p}] = i\hbar \ \Big\downarrow & & [\hat{\phi}(\vec{x}), \hat{\pi}(\vec{x})] = i\hbar \cdot \Big\downarrow\ \delta(\vec{x}-\vec{y}) \\
\text{quantum mechanics} & \longrightarrow & \text{quantum field theory} \\
\text{operators } \hat{x}, \hat{p} & & \text{operators } \hat{\phi}(\vec{x}), \hat{\pi}(\vec{x})
\end{array}
$$

Example I: oscillating string; mass



$$\xrightarrow{a \to 0} \quad \phi(x,t)$$

$$\partial_t^2 q_i = c^2\left([q_{i+1} - q_i] - [q_i - q_{i-1}]\right)/a^2 \qquad (1.1)$$

$$\downarrow$$

$$\partial_t^2 \phi(t,x) = c^2\, \partial_x^2 \phi(t,x) \longleftarrow \left[\frac{q_i - q_{i-1}}{a} \to \partial_x \phi\right]$$

$$S[q] = \int dt \ \frac{1}{2} \sum_i a \left\{ (\partial_t q_i)^2 - c^2 (q_{i+1} - q_i)^2 / a^2 \right\}$$

$$\Big\downarrow \ a \to 0 \qquad (1.2)$$

$$S[\phi] = \int dt \int dx \left\{ (\partial_t \phi)^2 - c^2 (\partial_x \phi)^2 \right\}$$

in general dimensions: (with $c=1$)

$$S[\phi] = \int d^d x \left\{ \frac{1}{2} (\partial_t \phi)^2 - (\vec{\nabla}\phi)^2 - V(\phi) \right\} \qquad (1.3)$$

(i) 'simply a bunch of (coupled) harmonic oscillators'

(ii) $S[\phi]$ has Poincaré invariance

→ later

Example II: Electrodynamics

action: $S[A_\mu] = \int d^4x \, \mathcal{L}(A_\mu(x), \partial_\mu A_\nu(x))$ (1.4)

$\mu = 0, \underbrace{1,2,3}_{i}$, $x^0 = t$, $(x^i) = \vec{x}$

$i = 1,2,3$

Lagrangian: $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ (1.5)

$F^{\mu\nu} = \eta^{\mu\rho}\eta^{\nu\sigma} F_{\rho\sigma}$

with flat metric $\eta^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}^{\mu\nu}$ (1.6)

$\eta_\mu{}^\nu = \delta_\mu{}^\nu \quad (= \eta^{\nu\rho}\eta_{\mu\rho})$

Remarks: (i) $\eta^{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}^{\mu\nu}$ also used (mostly in GR, QG)

(ii) in general $g^{\nu\rho} g_{\mu\rho} = \delta_\mu{}^\nu$ (1.7)

inverse metric metric

see p. 5a,b

(iii) Content of lecture course

see cover sheet

In both examples

$$\begin{array}{lll} q \to \phi,\ A & \xrightarrow{\text{Quantisation}} & \hat{\phi},\ \hat{A} \\ p \to \dot{\phi},\ \dot{A} & & \hat{\pi}_\phi,\ \hat{\pi}_A \\ \phantom{p \to {}} \overset{\shortparallel}{\pi_\phi}\ \ \overset{\shortparallel}{\pi_A} & & \end{array}$$

describes annihilation and creation of particles

(1) Hilbert space construction, related to operators, e.g., $\hat{\phi}$, $\hat{\pi}_\phi$.

vacuum $|\Omega\rangle$
1 particle $|1\rangle$
$\vdots$

annihilation, creation

(2) Modern particle physics described by (renormalisable) quantum field theories:
scalar fields (Higgs)
fermion fields: (leptons, quarks)
vector fields: (photons; $W^\pm, Z$; gluons)
graviton (spin 2)
(perturbatively) non-renormalisable