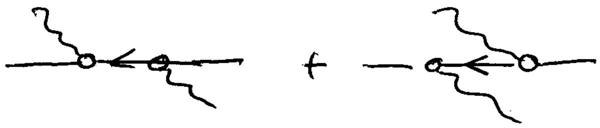
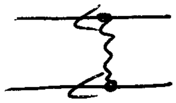
6.2 Elementary processes

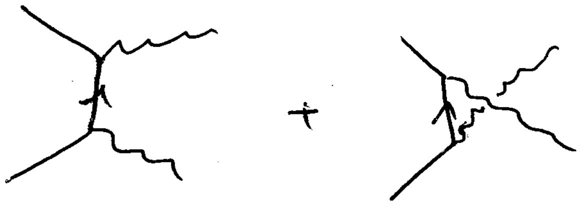
(i) Compton scattering: $e^- \gamma \to e^- \gamma$



(ii) Elastic $e^- e^-$-scattering



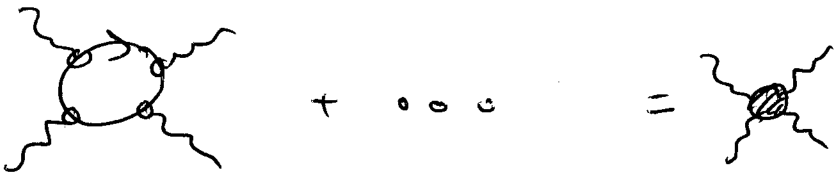
(iii) pair-annihilation/creation: $e^+ e^- \to \gamma\gamma$



(iv) Bhaba-scattering: $e^+ e^- \to e^+ e^-$

(i) – (iv) tree level processes

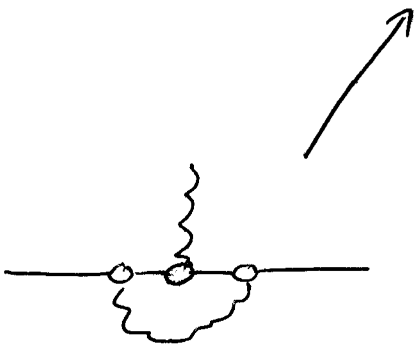
(v) light-by-light scattering
(non-linear electrodynamics)



 is an effective four photon vertex

(vi) Landé factor (gyromagnetic ratio)

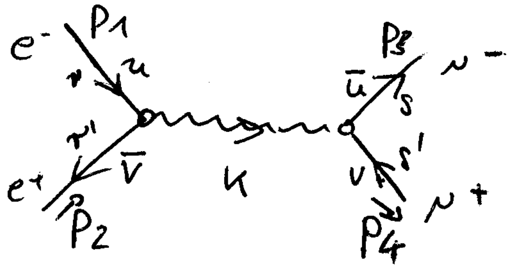
$$i\not{\partial} - m_e \to i\not{D} - m_e + \frac{\Delta g}{2}\,\frac{e}{4 m_e}\,\sigma_{\mu\nu}F^{\mu\nu} \qquad (6.17)$$



$$\Delta g = \alpha/\pi \qquad (6.18)$$

(v), (vi) loop effects

Tree level example: $e^+e^- \to \mu^+\mu^-$



only <u>one</u> diagram

2 – 2 scattering, eq. (3.93) in highly rel. case

$$d\sigma = \int \frac{1}{2s} \, |\mathcal{M}|^2 \, d\Pi_2 \qquad (6.19)$$

with
$$\int d\Pi_2 = \frac{1}{2}\frac{1}{(2\pi)^2} \frac{1}{\underbrace{4 p_3^0 p_4^0}_{s}} \, d\Omega \, s/4 \qquad (6.20)$$

where $s = (p_1 + p_2)^2$

$$|\mathcal{M}|^2 = \underbrace{\frac{1}{2}\sum_{r} \frac{1}{2}\sum_{r'}}_{\text{average}} \underbrace{\sum_{s,s'}}_{\text{sum}} |\mathcal{M}(r,r',s,s')|^2 \qquad (6.21)$$

The scattering amplitude is read-off from the Feynman rules:

$$i\mathcal{M} = \bar{u}_{\mu_s}(p_3)(ie\gamma_\rho) v_{\mu_{s'}}(p_4) \left[\frac{-\eta^{\rho\sigma}}{s}\right] \bar{v}_{e_{r'}}(p_2)(ie\gamma_\sigma) \cdot u_{e_r}(p_1)$$

(6.22)

It follows that

$$|\mathcal{M}|^2 = \frac{e^4}{4s^2} \cdot T_{\mu\,\alpha\beta}\; T_e^{\;\alpha\beta} \qquad (6.23)$$

with

$$T_{\mu\,\alpha\beta} = \sum_{s,s'} \bar{u}_{\mu s}(p_3)(ie\gamma_\alpha)v_{\mu s'}(p_4) \cdot \left(\bar{u}_{\mu s}(p_3)\,(ie\gamma_\beta)\,v_{\mu s'}(p_4)\right)^*$$

$$T_e^{\;\alpha\beta} = \sum_{r,r'} \bar{v}_{e r'}(p_2)\,(ie\gamma^\alpha)\,u_{e r}(p_1)\;(\ldots\gamma^\beta\ldots)^* \qquad (6.24)$$

We use that (p. 25 - 33 in e^+e^- to $\mu^+\mu^-$)

$$\sum_s u_{\mu s}(p_3)\,\bar{u}_{\mu s}(p_3) = \not{p}_3 + m_\mu \quad \leftarrow \text{eq. } (4.67)$$

$$\sum_{s,s'} \bar{u}_s(p_3)\,\gamma_\alpha\,\left[v_{\mu s'}(p_4)\,\bar{v}_{\mu s'}(p_4)\right]\gamma_\beta\, u_s(p_3)$$

$$\sum_{s'} v_{\mu s'}(p_4)\,\bar{v}_{\mu s'}(p_4) = \not{p}_4 - m_\mu \quad \leftarrow \text{eq. } (4.67)$$

$$= \mathrm{Tr}\,(\not{p}_3 + m_\mu)\,\gamma_\alpha\,(\not{p}_4 - m_\mu)\,\gamma_\beta \qquad (6.25)$$

with

$$\left[\bar{u}_s(p)\,\gamma_\alpha\,v_{s'}(q)\right]^* = \underbrace{v_{s'}^\dagger(q)\gamma^0}\,\underbrace{\gamma^0\gamma_\alpha^\dagger\gamma^0}\,\underbrace{\gamma^0\,\bar{u}_s^\dagger(p)}$$

$$= \bar{v}_{s'}(q)\;\gamma_\alpha\;u_s(p) \qquad (6.26)$$

Highly relativistic limit: drop m_ν, m_e

$$\Rightarrow T_{\nu\,\alpha\beta} = \mathrm{Tr}\,(\not{p}_3 + m_\nu)\,\gamma_\alpha\,(\not{p}_4 - m_\nu)\,\gamma_\beta$$

$\mathrm{Tr}\,\gamma^{2n+1} = 0 \longrightarrow$
$$= \mathrm{Tr}\,\not{p}_3\,\gamma_\alpha\,\not{p}_4\,\gamma_\beta + \mathrm{Tr}\,\gamma_\alpha\gamma_\beta\, m_\nu^2 \tag{6.27}$$

We use: $\mathrm{Tr}\,\gamma^{2n+1} = \mathrm{Tr}\,\underbrace{\gamma_5^2}_{1}\,\gamma^{2n+1} \overset{\{\gamma_5,\gamma\}=0}{=} -\mathrm{Tr}\,\gamma_5\,\gamma^{2n+1}\,\gamma_5 = -\mathrm{Tr}\,\gamma^{2n+1}$ cyclicity of trace

and

(a) $$\mathrm{Tr}\,\gamma^\rho\gamma^\sigma = \frac{1}{2}\,\mathrm{Tr}\,\underbrace{\{\gamma^\rho,\gamma^\sigma\}}_{2\eta^{\rho\sigma}} = 4\eta^{\rho\sigma}$$

(b) $$\mathrm{Tr}\,\gamma^\rho\gamma^\sigma\gamma^\alpha\gamma^\beta = \underbrace{2\eta^{\rho\sigma}\,\mathrm{Tr}\,\gamma^\alpha\gamma^\beta}_{8\eta^{\rho\sigma}\eta^{\alpha\beta}} - \mathrm{Tr}\,\gamma^\sigma\gamma^\rho\gamma^\alpha\gamma^\beta$$

$$= \ldots$$

$$\Rightarrow \mathrm{Tr}\,\gamma^\rho\gamma^\sigma\gamma^\alpha\gamma^\beta = 4\,(\eta^{\rho\sigma}\eta^{\alpha\beta} - \eta^{\rho\alpha}\eta^{\beta\sigma} + \eta^{\rho\beta}\eta^{\alpha\sigma}) \tag{6.28}$$

and hence:

$$T_{\nu\,\alpha\beta} = 4\,(p_{3\alpha}\,p_{4\beta} + p_{3\beta}\,p_{4\alpha} - \eta_{\alpha\beta}\,p_3\cdot p_4) - 4\,\eta_{\alpha\beta}\,m_\nu^2 \tag{6.29}$$

$$s \gg m_\nu^2 \longrightarrow \simeq 0$$

We similarily compute

$$T_e^{\alpha\beta} \simeq 4\left(p_1^{\alpha} p_2^{\beta} + p_1^{\beta} p_2^{\alpha} - \eta^{\alpha\beta} p_1\cdot p_2 \right) \tag{6.30}$$

and arrive at

$$|\mathcal{M}|^2 = \frac{e^4}{4s^2}\, 2\cdot 16 \left[(p_1\cdot p_4)(p_2\cdot p_3) + (p_1\cdot p_3)(p_2\cdot p_4) \right]$$

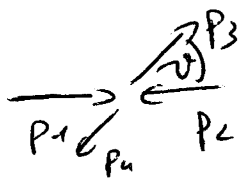
$$= \frac{8e^4}{s^2}\left[\qquad \right] \tag{6.31}$$

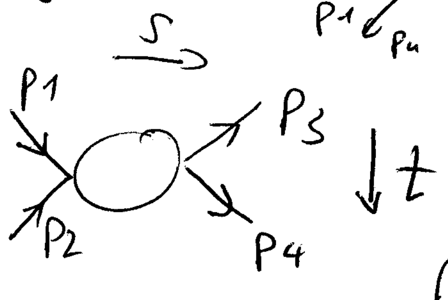
and in summary, after inserting (6.31) in (6.19)

$$\frac{d\sigma}{d\Omega} = \frac{2\alpha^2}{p_3^0\, p_4^0\, s^2}\left[(p_1\cdot p_4)(p_2\cdot p_3) + (p_1\cdot p_3)(p_2\cdot p_4) \right] \tag{6.32}$$

with $\alpha = e^2/4\pi$

Mandelstam variables:

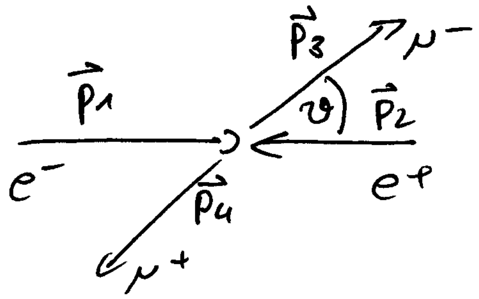




$$s = (p_1+p_2)^2$$

$$t = (p_1-p_3)^2 \tag{6.33}$$

$$u = (p_1-p_4)^2 \, :$$



scattering angle ϑ:

$$\cos\vartheta = \frac{\vec{p}_1 \cdot \vec{p}_3}{|\vec{p}_1||\vec{p}_3|}$$

highly relativistic limit

$$p_1 \cdot p_3 = \underbrace{p_1^0 p_3^0}_{\simeq \frac{1}{4}s} - \underbrace{\vec{p}_1 \cdot \vec{p}_3}_{\simeq \frac{1}{4}s \cos\vartheta} \simeq 1/4\, s(1-\cos\vartheta)$$

$$= p_2 \cdot p_4$$

$$p_1 \cdot p_4 = p_2 \cdot p_3 = \frac{1}{4} s(1+\cos\vartheta)$$

$$\Rightarrow (p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)$$

$$= \frac{1}{16} s(2 + 2\cos^2\vartheta) = \frac{1}{8} s(1+\cos^2\vartheta)$$

The final result for $|\mathcal{M}|^2$ is

$$\boxed{|\mathcal{M}|^2 = e^4(1+\cos^2\vartheta) = 16\pi^2\alpha^2(1+\cos^2\vartheta)}$$

with $\alpha = e^2/4\pi$. (6.34)

Eq. (6.34) has to be compared with that for the scalar 2-2 scattering, eq. (3.23), p. 45, $|\mathcal{M}|^2 = \lambda^2$.

We insert eq. (6.34) in eq. (6.19) for the cross section, $(4p_3^0p_4^0 \simeq s)$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1+\cos^2\vartheta)} \qquad (6.35)$$

Remarks:

(i) The QED $e^+e^- \to \mu^+\mu^-$ cross section

$\frac{d\sigma}{d\Omega} = \alpha^2/4\pi\,(1+\cos^2\vartheta)$ should be compared

with the scalar cross section $\phi\phi \to \phi\phi$,

$$\frac{d\sigma}{d\Omega} = \frac{1}{(4\pi)}\,\frac{\lambda^2}{4\pi}\ ,$$ eq. (3.93), p. 80:

(ii) The intermediate virtual photon was

chosen in Feynman gauge, $\xi = 1$. We have

have shown on p. 151, eq. (5.58) that

any choice of ξ leads to the same

result, in particular $\xi = 0$: $\underset{\nu\;k}{\sim\!\sim\!\sim}{}_{\mu}\ k^{\nu} = 0$.

(iii) In the high energy limit also

$$(p_1 - p_2)_\nu\, \bar{v}(p_2)\gamma^\nu u(p_1) \overset{\sim m_e}{\approx} 0\ ,$$ only

the physical polarisations $\varepsilon_1, \varepsilon_2$ play

a rôle, ε_3 drops out. See eq. (5.34), (5.35).

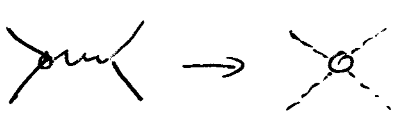
(iv) The argument done in (iii) also applies to $\bar{u}(p_3)\gamma^\nu v(p_4)$. In summary we have

$$\bar{u}(p_3)\gamma^\nu v(p_4)\, p_{3/4\,\nu} \approx 0$$

$$\left(\bar{v}(p_2)\gamma^\nu u(p_1)\, p_{1/2\,\nu} \approx 0\right)$$

So if $p_{3,4}$ are orthogonal to the beam axis, defined by $p_{1/2}$, the related polarisation ε_1 or ε_2 also 'drops out of the game'. In this case, $\vartheta = \frac{\pi}{2}$, only one polarisation contributes to the scattering, for $\vartheta = 0$, both.

(v) In the highly-relativistic case and

$\vartheta = \pi/2$: "  "