8 The Standard model

Electro-weak theory $\quad U(1) \times SU(2)$

& <u>Qu</u>antum <u>C</u>hromo dynamics $\quad SU(3)$

8.1 Non-Abelian gauge theories

Consider fermions with $(\partial_\nu U = 0)$

$$\psi(x) \to U\,\psi(x) \quad \text{with } U \in SU(N) \tag{8.1}$$

that is $\psi = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_N \end{pmatrix}$, ψ_i Dirac fermion (with four compo.) (8.2)

ψ is in the fundamental representation

Free Dirac action: invariant under (8.1)

$$S_0[\psi] = \int d^4x\, \underbrace{\bar\psi\,(i\partial\!\!\!/ - m)\,\psi}_{\bar\psi_i\,(i\partial\!\!\!/ - m)\,\psi_i} \tag{8.3}$$

Global invariance: $\bar{\Psi} \xrightarrow{U} \bar{\Psi}\, U^{+}$, U commutes with γ^0

$$S_D[U\Psi] = \int d^4x\, \bar{\Psi}\, U^{+} (i\partial\!\!\!/ - m)\, U \Psi$$

$$= \int d^4x\, \bar{\Psi} (i\partial\!\!\!/ - m)\, \underbrace{U^{+} U}_{\mathbb{1}_{SU(N)}}\, \Psi \qquad (8.4)$$

Gauge invariance: invariance of S under

$$\psi(x) \to U(x)\; \bar{\Psi}(x) \qquad (8.5)$$

Analogously to QED ($U(1)$)

we introduce the covariant derivative

gauge coupling, gauge field

$$D_\mu = \partial_\mu - i g \underset{\substack{\cap \\ [\text{Lie-algebra } su(N)]}}{A_\mu} \qquad (8.6)$$

(relat. minus sign to eq. (5.9))

and

$$S_D[\Psi, A] = \int d^4x\, \bar{\Psi} (i D\!\!\!/ - m)\, \Psi \qquad (8.7)$$

Invariance of $S_D[\psi, A]$ under the gauge transformation (8.5) enforces

$$D_\nu \xrightarrow{\mathcal{U}} \mathcal{U} D_\nu \mathcal{U}^+ \tag{8.8}$$

or (with $\mathcal{U} D_\nu \mathcal{U}^+ = \partial_\nu - ig A_\nu^{\mathcal{U}}$)

$$A_\nu(x) \rightarrow \mathcal{U}(x) A_\nu(x) \mathcal{U}^+(x) + \frac{i}{g} \mathcal{U}(x) \partial_\nu \mathcal{U}^+(x) \tag{8.9}$$

A_ν has to live in the algebra of $SU(N)$

We write $\underset{\text{Group}}{\mathcal{U}} = e^{i\omega}$ algebra (8.10)

and infinitesimally

$$A_\nu \rightarrow A_\nu - \frac{i}{g}[\omega, A_\nu] + \frac{i}{g}\partial_\nu \omega + O(\omega^2)$$

$$= A_\nu + \frac{i}{g} D_\nu \omega + O(\omega^2) \tag{8.11}$$

adjoint repr.

Gauge field action: (see p. 135 for $U(1)$)

Fieldstrength:

$$F_{\mu\nu} = \frac{i}{g}[D_\mu, D_\nu]$$

$su(N)$

Matrix

$$= \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \qquad (8.12)$$

$$F_{\mu\nu} = F^a_{\mu\nu}\, t^a \qquad (8.13)$$

generators of $SU(N)$

They satisfy $[t^a, t^b] = i f^{abc} t^c$

structure constants

e.g. $f^{abc} = \varepsilon^{abc}$ in $SU(2)$ (8.14)

Gauge transformations: $D_\mu \to U D_\mu U^\dagger$

$$F_{\mu\nu} \to U F_{\mu\nu} U^\dagger \qquad (8.15)$$

$\Rightarrow$ Gauge invariant action: $\operatorname{tr} t^a t^b = \frac{1}{2}\delta^{ab}$

$$S_{YM}[A] = -\frac{1}{2g^2}\int d^4x \operatorname{tr} F_{\mu\nu}F^{\mu\nu} \qquad (8.16)$$

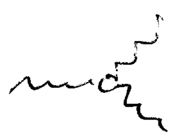
Yang-Mills

In summary we have

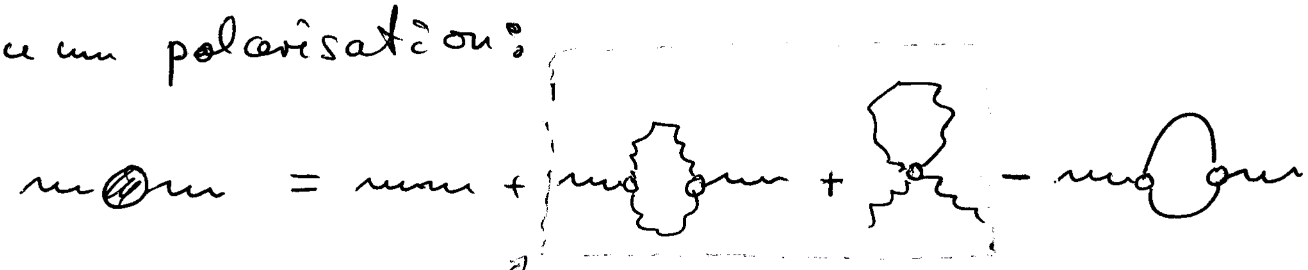
$$S[A,\psi] = S_D[A,\psi] + S_{YM}[A] + S_{gf}[A] + \underbrace{\dots}_{\text{ghosts}} \tag{8.17}$$

Difference to $U(1)$: self interaction of gauge fields!

$$\frac{1}{2}\,\mathrm{tr}\,F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\Big(\partial_\mu A_\nu^a\,\partial^\mu A^{\nu a} - \partial_\mu A_\nu^a\,\partial^\nu A^{\mu a}\Big)$$
$$+ g f^{abc} A_\mu^a A_\nu^b\,\partial^\mu A^{\nu c}$$
$$- \frac{1}{4} f^{abc} f^{ade} A_\mu^b A_\nu^c A_\mu^d A_\nu^e \tag{8.18}$$



Vacuum polarisation:



dominate: $\beta_g < 0$ $\to$ asympt. freedom

see p. 204