Quantum Field Theory 1 – Problem set 5

Suggested reading before solving these problems: Chapters 3.1 to 3.2 in the script and/or Chapters 4.2 to 4.3 of *Peskin & Schroeder*.

Problem 1: Time and normal ordering 1

Explain in a few words and formula

a) What is a time ordered product and why do we need it?

b) What is a normal ordered product and why is it useful?

c) How can we transform a time ordered expression to a normal ordered one?

Problem 2: Time and normal ordering 2

a) Show that the time ordered product $T(\phi(x_1)\phi(x_2))$ and the normal ordered product $:\phi(x_1)\phi(x_2):$ are both symmetric under the interchange of x_1 and x_2. Deduce that the Feynman propagator $D_F(x_1 - x_2)$ has the same property.

b) Check Wick's theorem for the case of three real scalar fields

$$
\begin{aligned}
T(\phi(x_1)\phi(x_2)\phi(x_3)) &= :\phi(x_1)\phi(x_2)\phi(x_3): +\phi(x_1)D_F(x_2 - x_3) \\
&\quad +\phi(x_2)D_F(x_1 - x_3) + \phi(x_3)D_F(x_1 - x_2).
\end{aligned}
$$

Problem 3: Three-point correlation function

Consider the following Lagrangian, involving two real scalar fields Φ and ϕ:

$$
\mathcal{L} = \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}M^2\Phi^2 + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - h\Phi\phi\phi.
$$

The last term is an interaction term that allows a Φ particle to decay into two ϕ's, provided that $M > 2m$. Assume that this is the case and calculate the $\mathcal{O}(h)$ contribution to the correlation function

$$
\langle 0| T \; \Phi(x)\,\phi(y)\,\phi(z)\; e^{-i\int dt\, H_{\text{int}}(t)}|0\rangle.
$$