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# Quantum Field Theory 1 – Tutorial 4

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## Problem 1: Relation between operators in different pictures

Consider the decomposition of the Hamiltonian operator  $H$  in terms of free and interaction parts,  $H = H_0 + H_{\text{int}}$ . You have learned that, in the Heisenberg picture, operators evolve in time with the *full* Hamiltonian  $H$ , while in the interaction picture they evolve with the *free* Hamiltonian  $H_0$ .

In the lecture we have derived a relation between time-ordered expectation values of products of fields in the Heisenberg picture and the interaction picture. To that end we have used the relation between the scalar field in the Heisenberg picture  $\phi_H$  and in the interaction picture  $\phi_I$ :

$$\phi_H(t, \mathbf{x}) = U^\dagger(t, 0) \phi_I(t, \mathbf{x}) U(t, 0). \quad (1)$$

In the following, we will derive this relation in two steps.

(a) First, consider the time evolution operator  $U(t, 0)$ , which is a unitary operator. Show that  $U(t, 0)$  can be written in terms of  $H_0$  and  $H$  as

$$U(t, 0) = e^{iH_0 t} e^{-iH t}, \quad (2)$$

by showing that the expression in Eq. (2) satisfies the defining equation,

$$i\partial_t U(t, 0) = H_I(t) U(t, 0),$$

where  $H_I(t) = H_{\text{int}}(t) = e^{iH_0 t} H_{\text{int}} e^{-iH_0 t}$ .

(b) Second, use Eq. (2) to show the relation given in Eq. (1).