Quantum Field Theory 1 – Tutorial 5

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tutorial date: 16 November 2016

Problem 1: Feynman propagator

(a) Calculate the integral

$$\int_{-\infty}^{\infty} \frac{e^{itx}}{x^2+1} dx$$

for $t > 0$ using the residue theorem.

(b) Consider the Feynman propagator for a scalar field

$$D_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\epsilon} e^{-ip(x-y)}$$

Assuming $x_0 - y_0 > 0$ and using the residue theorem, show that this can be written as

$$D_F(x-y) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} e^{-iE_p(x_0-y_0)+i\boldsymbol{p}(\boldsymbol{x}-\boldsymbol{y})}.$$

How does this change for $x_0 - y_0 < 0$?

Residue theorem: The contour integral of a function $f(z)$ around a closed, counter-clockwise path encircling a domain where $f(z)$ has a finite number of isolated singularities (poles at $z = z_i, i = 1, 2, ..., n$) is

$$\oint dz f(z) = 2\pi i \sum_{i=1}^{n} \mathrm{Res}(f, z_i)$$

where the residue of $f(z)$ at a simple pole z_i is $\mathrm{Res}(f, z_i) = \lim_{z\to z_i}(z - z_i)f(z)$.