Quantum Field Theory 1 – Tutorial 6

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Problem 1: Cross section and Lorentz boosts

Under Lorentz boosts coordinate differences transform according to

$$\begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix} \to \begin{pmatrix} dt' \\ dx' \\ dy' \\ dz' \end{pmatrix} = \Lambda \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}$$

with a 4×4 matrix Λ that leaves the metric tensor $\eta = \mathrm{diag}(1, -1, -1, -1)$ invariant

$$\eta = \Lambda^T \eta \Lambda.$$

We choose coordinates such that σ is a cross-section area in the $x - y$ plane:

$$\sigma = \int dx dy P(x, y)$$

with an appropriate dimensionless function P.

a) How does σ transform under Lorentz boosts?

b) Show that boosts along the z -axes leave σ invariant.

c) In the lecture it was shown that one has in the center of mass frame for ultrarelativistic particles

$$d\sigma = \frac{1}{2s} |\mathcal{M}_{fi}|^2 d\Pi_n \tag{1}$$

where $d\Pi_n$ is the relativistically invariant n-body phase space element and $s = (p_1 + p_2)^2$. Why can Eq. (1) hold only in frames where the initial momenta are either parallel or antiparallel?