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# Quantum Field Theory 1 – Tutorial 7

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## Problem 1: Lie groups and Lie algebras

The finite elements of a connected Lie group can be obtained from an infinite product of transformations infinitesimal close to  $\mathbb{1}$ . A well known example is the group  $SO(3)$  of rotations in  $\mathbb{R}^3$ .

a) Show that for a matrix  $M$

$$\lim_{N \rightarrow \infty} \left( \mathbb{1} + \frac{\alpha}{N} M \right)^N = \exp(\alpha M) \quad (1)$$

by comparing the Taylor series in  $\alpha$ . The expression in Eq. (1) is an element of a matrix Lie group while the matrix  $M$  is part of the corresponding Lie algebra.

b) Explicitly calculate  $\exp(\alpha M)$  for

$$M_1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad M_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$