Quantum Field Theory 1 – Tutorial 9

Lectures: Jan Pawlowski pawlowski@thphys.uni-heidelberg.de
Tutorials: Michael Scherer scherer@thphys.uni-heidelberg.de
Institut für Theoretische Physik, Uni Heidelberg tutorial date: 14 December 2016

Problem 1: Representations of Clifford algebra

Examine the following representation of the Clifford algebra

$$\begin{aligned} \Gamma^0 &= \gamma^0\gamma^2 \qquad & \Gamma^1 &= i\gamma^0\gamma^1 \\ \Gamma^2 &= i\gamma^0 \qquad & \Gamma^3 &= i\gamma^0\gamma^3 \end{aligned}$$

where the γ^μ are the Dirac matrices in the chiral representation, i.e.

$$\gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}, \quad \text{where} \quad \bar{\sigma}^\mu = (\mathbb{1}_2, -\boldsymbol{\sigma})$$

and $\boldsymbol{\sigma}$ is the vector of Pauli matrices.

Prove that the Γ^μ are anti-hermitian, and that they are a representation of the Clifford algebra,

$$\{\Gamma^\mu, \Gamma^\nu\} = 2\,\eta^{\mu\nu}\,.$$

Express the matrix $\Gamma^5 = i\,\Gamma^0\Gamma^1\Gamma^2\Gamma^3$ in terms of Dirac matrices γ^μ. Is this matrix hermitian or anti-hermitian? Show that it commutes with the Γ^μ.