Quantum Field Theory 1 – Tutorial 1

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Problem 1: Natural units

In particle physics one often chooses units such that $\hbar = c = 1$ and measures energies in MeV. Complete the following translation table (The charge of a proton is $e = 1.6 \cdot 10^{-19}\ C$):

	SI units	Natural units
c	$3 \cdot 10^{8}$ m/s	1
$\hbar$	$1.05 \cdot 10^{-34}$ Js	1
m_e	$9.1 \cdot 10^{-31}$ kg	
m_p		938.3 MeV
$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}$		
$a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2}$	$0.53 \cdot 10^{-10}$ m	
G	$6.67 \cdot 10^{-11}$ Nm^2/(kg^2)	

Problem 2: Upper and lower indices

In special relativity one distinguishes between upper or contra-variant indices (as e.g. x^μ) and lower or co-variant indices (e.g. x_μ). The metric tensor

$$(\eta_{\mu\nu}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

is used to raise or lower indices:

$$x_\mu = \eta_{\mu\nu} x^\nu, \qquad x^\mu = \eta^{\mu\nu} x_\nu.$$

For $x^\mu = (x^0, \boldsymbol{x})$ and $p^\mu = (p^0, \boldsymbol{p})$ calculate

$$x_\mu, \qquad\qquad p \cdot x = p_\mu x^\mu$$

and show that

$$\eta^\mu{}_\nu = \delta^\mu{}_\nu, \qquad\qquad (\eta^{\mu\nu}) = (\eta_{\mu\nu}).$$

Problem 3: Poincaré group

Under a Poincaré transformation (Λ, a) a coordinate vector x^μ transforms as

$$x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu + a^\mu.$$

Poincaré transformations leave the scalar product of differences of coordinate vectors, $(x-y)\cdot(x-y)$, invariant. Hence the matrix Λ satisfies

$$\Lambda^\rho{}_\mu \, \Lambda^\sigma{}_\nu \, \eta^{\mu\nu} = \eta^{\rho\sigma}. \tag{1}$$

a) Show that
$$(\Lambda^{-1})^\mu{}_\nu = \Lambda_\nu{}^\mu.$$

b) Consider the product of two Poincaré transformations
$$(\Lambda, a) = (\Lambda_1, a_1)(\Lambda_2, a_2).$$
Determine (Λ, a) and show that Λ satisfies Eq. (1).

c) Determine the inverse transformation
$$(\Lambda, a)^{-1}.$$

Remark: The properties shown here together with associativity

$$\Big[(\Lambda_1, a_1)(\Lambda_2, a_2)\Big](\Lambda_3, a_3) = (\Lambda_1, a_1)\Big[(\Lambda_2, a_2)(\Lambda_3, a_3)\Big]$$

and the existence of a unit element
$$(\mathbb{1}, 0)$$
imply that the set of Poincaré transformations constitutes a *group*.