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# Quantum Field Theory 1 – Tutorial 3

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## Problem 1: Lorentz invariant quantities

In the lecture we have used the following important identity for the three-momentum integral

$$\int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega_{\mathbf{p}}} f(p) = \int \frac{d^4p}{(2\pi)^4} 2\pi \delta(p^2 - m^2) \Theta(p_0) f(p). \quad (1)$$

Use the delta function identity

$$\delta(g(x) - g(a)) = \frac{1}{|g'(a)|} \delta(x - a).$$

to derive Eq. (1). Further, convince yourself that this implies that the three-momentum integral is invariant under Lorentz transformations  $\Lambda$  if  $f(\Lambda p) = f(p)$ .

*Remark:* We consider proper, orthochronous Lorentz transformations  $\Lambda \in SO(1, 3)$ .

*Optional:* In the lecture we have also introduced conveniently normalized one-particle states with momentum  $\mathbf{p}$  as  $|\mathbf{p}\rangle = \sqrt{2\omega_{\mathbf{p}}} a_{\mathbf{p}}^\dagger |0\rangle$  with  $\omega_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}$  (see Eq. (2.65) in the script). Consider a Lorentz boost in the  $p_3$  direction:

$$p'_3 = \gamma(p_3 + \beta E), \quad \text{and} \quad E' = \gamma(E + \beta p_3).$$

Use the delta function identity from above to show that the boost gives

$$\delta^{(3)}(\mathbf{p} - \mathbf{q}) = \delta^{(3)}(\mathbf{p}' - \mathbf{q}') \frac{E'}{E}.$$

This non-trivial transformation property is related to the fact that volumes are not invariant under boosts due to Lorentz contraction ( $V \rightarrow V/\gamma$ ). Instead, the quantity  $E \delta^{(3)}(\mathbf{p} - \mathbf{q})$  is invariant under boosts. More generally,  $E \delta^{(3)}\delta(\mathbf{p} - \mathbf{q})$  is Lorentz invariant which also makes  $\langle \mathbf{p} | \mathbf{q} \rangle = 2\omega_{\mathbf{p}} (2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{q})$  a Lorentz invariant quantity.