Quantum Field Theory 1 – Tutorial 6

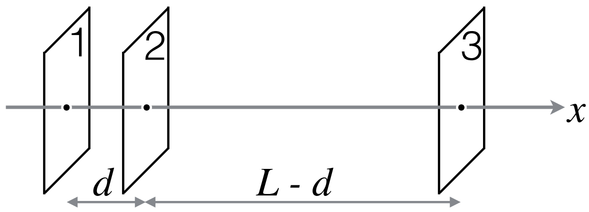
Lectures: Jörg Jäckel J.Jaeckel@thphys.uni-heidelberg.de
Jan Pawlowski J.Pawlowski@thphys.uni-heidelberg.de
Tutorials: Malo Tarpin M.Tarpin@thphys.uni-heidelberg.de
Institut für Theoretische Physik, Uni Heidelberg

tutorial date: 18 November 2019

This tutorial is a preparation for problem set 6 and corresponds to exercise 6.3.a)

Problem 1: Casimir effect (preparation)

Take the electromagnetic field with its divergent vacuum energy density $\epsilon = 2\int \frac{d^3p}{(2\pi)^3}\frac{1}{2}\hbar\omega_{\boldsymbol{p}}$ and $\omega_{\boldsymbol{p}} = c|\boldsymbol{p}|$. Now, introduce three parallel and perfectly conducting plates with distances d and $L-d$ into the electromagnetic vacuum, see figure. The plates of infinite size and zero thickness introduce boundary conditions for the electromagnetic field which comes with two possible polarizations.



To simplify the analysis, we won't do the full calculation for the electromagnetic field which is, e.g., complicated by considerations about polarization. Instead, we consider a massless scalar field in one-dimensional space. Therefore, the 'plates' define the positions x where the field vanishes and introduce discrete field modes. Show that the energy between 'plate' 1 and 'plate' 2 is given as

$$E(d) = \frac{\pi\hbar c}{2d}\sum_{n=1}^{\infty} n\,.$$

Calculate he energy between plates 2 and 3? What is the full energy $E_f(d)$ between plates 1 and 3?