Quantum Field Theory 1 – Tutorial 10

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Problem 1: QED Lagrangian and local $U(1)$ rotations

Quantum Electrodynamics (QED) is the quantum field theory for a spin-1/2 field interacting with the electromagnetic field represented by the Lagrangian

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\left(i\not{D} - m\right)\psi\,, \quad \text{where} \quad D_\mu = \partial_\mu - ieA_\mu\,.$$

Its predictions describe electromagnetic phenomena from macroscopic scales down to distances several hundred times smaller than the proton. QED is one of the most stringently tested and most accurate physical theories. For example, the QED prediction for the electron anomalous magnetic dipole moment agrees with the experimental value to more than 10 significant figures, see for example Phys. Rev. Lett. 109, 111807 (2012).

QED is an abelian gauge theory with symmetry group $U(1)$. The spinor field ψ and the gauge field A_μ transform under local $U(1)$ rotations as follows

$$\psi(x) \to e^{ie\alpha(x)}\psi(x)\,, \quad \text{and} \quad A_\mu(x) \to A_\mu(x) + \partial_\mu\alpha(x)\,.$$

a) Show that these transformations leave the QED Lagrangian invariant.

b) Show that a adding a mass term for the gauge field $\frac{m_A^2}{2}A_\mu A^\mu$ would break the invariance under gauge transformations.