Quantum Field Theory 1 – Tutorial 11

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Problem 1: QED Lagrangian and gauge fixing

The pure gauge field Lagrangian $\mathcal{L}_A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ gives an equation of motion of the form $Q^{\nu\sigma}A_\sigma = 0$ with operator $Q^{\nu\sigma} = \partial_\mu\partial^\mu\eta^{\nu\sigma} - \partial^\nu\partial^\sigma$.

a) Show that $Q^{\nu\sigma}$ is not invertible as it annihilates any function of the form $\partial_\sigma\alpha$.

This prevents us from obtaining a sensible photon propagator directly from $\mathcal{L}_A$ and is related to the redundancy of the gauge field, $A_\mu \to A_\mu + e\partial_\mu\alpha$. To remove the redundancy, we introduce a *gauge fixing* (cf. script Eq. (5.21), p. 138)

$$\partial_\mu A^\mu = 0\,. \tag{1}$$

b) Show that for any given field A'_μ we can always pick a representative configuration, which satisfies Eq. (1). Specify the required gauge transformation.

c) Show that Lorenz gauge leaves a subspace of gauge tranformations which satisfy Eq. (1). Which equation has to be satisfied by further gauge transformations?

A complete the gauge fixing can be achieved by adding appropriate boundary conditions. With the gauge fixing constraint, we can then write the gauge field Lagrangian as

$$\mathcal{L}_A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}\left(\partial_\mu A^\mu\right)^2\,, \tag{2}$$

and obtain the invertible operator $\partial_\mu\partial^\mu\eta^{\nu\sigma} - \left(1-\frac{1}{\xi}\right)\partial^\nu\partial^\sigma$, cf. script Eq. (5.22), p. 139.