
Quantum Field Theory 1 – Tutorial 2

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Problem 1: Fourier transform and Dirac distribution

Consider a three-dimensional cube $x_i \in (-L/2, L/2), i = 1, 2, 3$ with volume $V = L^3$ and periodic boundary conditions. A (complex) field configuration $\phi(\mathbf{x})$ can be written as

$$\phi(\mathbf{x}) = \frac{1}{V} \sum_{l,m,n} e^{i\mathbf{p}_{lmn}\mathbf{x}} \tilde{\phi}_{lmn} \quad (1)$$

with $\mathbf{p}_{lmn} = \left(\frac{2\pi l}{L}, \frac{2\pi m}{L}, \frac{2\pi n}{L}\right)$.

a) What is the range of the indices l, m, n ?

b) Show that

$$\int_V d^3x e^{i(\mathbf{p}_{lmn} - \mathbf{p}_{l'm'n'})\mathbf{x}} = V \delta_{ll'} \delta_{mm'} \delta_{nn'} \quad (2)$$

and hence

$$\tilde{\phi}_{lmn} = \int_V d^3x e^{-i\mathbf{p}_{lmn}\mathbf{x}} \phi(\mathbf{x}).$$

c) Consider now the limit $L \rightarrow \infty$. Show that Eq. (1) becomes

$$\phi(\mathbf{x}) = \frac{1}{(2\pi)^3} \int d^3p e^{i\mathbf{p}\mathbf{x}} \tilde{\phi}(\mathbf{p}).$$

d) By comparing Eq. (2) to its infinite volume limit

$$\int d^3x e^{i(\mathbf{p} - \mathbf{p}')\mathbf{x}} = (2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{p}')$$

convince yourself that

$$(2\pi)^3 \delta^{(3)}(\mathbf{q}) \big|_{\mathbf{q}=0} = \lim_{L \rightarrow \infty} V.$$