Quantum Field Theory 1 – Tutorial 4

Lectures: Jan Pawlowski J.Pawlowski@thphys.uni-heidelberg.de
Tutorials: Felipe Attanasio F.Attanasio@thphys.uni-heidelberg.de
Institut für Theoretische Physik, Uni Heidelberg tutorial date: week of 7.11.2022

Problem 1: Relation between operators in different pictures

Consider the decomposition of the Hamiltonian operator H in terms of free and interaction parts, $H = H_0 + H_{\text{int}}$. You have learned that in the Heisenberg picture operators evolve in time with the *full* Hamiltonian H, while in the interaction picture they evolve with the *free* Hamiltonian H_0.

In the lecture we have derived a relation between time-ordered expectation values of products of fields in the Heisenberg picture and the interaction picture. To that end we have used the relation between the scalar field in the Heisenberg picture ϕ_H and in the interaction picture ϕ_I,

$$\phi_H(t, \boldsymbol{x}) = U^\dagger(t,0)\, \phi_I(t, \boldsymbol{x})\, U(t,0)\,. \tag{1}$$

In the following, we will derive this relation in two steps.

(a) First, consider the time evolution operator $U(t,0)$, which is an unitary operator. Show that $U(t,0)$ can be written in terms of H_0 and H as

$$U(t,0) = e^{iH_0 t}\, e^{-iHt}\,, \tag{2}$$

by showing that the expression in Eq. (2) satisfies the defining equation,

$$i\partial_t U(t,0) = H_I(t) U(t,0)\,,$$

where $H_I(t) = H_{\text{int}}(t) = e^{iH_0 t} H_{\text{int}} e^{-iH_0 t}$.

(b) Second, use Eq. (2) to show the relation given in Eq. (1).