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# Quantum Field Theory 1 – Tutorial 6

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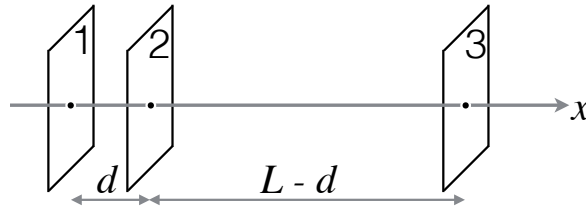
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*This tutorial is a preparation for problem set 6 and corresponds to exercise 6.3.a)*

## Problem 1: Casimir effect (preparation)

Take the electromagnetic field with its divergent vacuum energy density  $\epsilon = 2 \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2} \hbar \omega_{\mathbf{p}}$  and  $\omega_{\mathbf{p}} = c|\mathbf{p}|$ . Now, introduce three parallel and perfectly conducting plates with distances  $d$  and  $L - d$  into the electromagnetic vacuum, see figure. The plates of infinite size and zero thickness introduce boundary conditions for the electromagnetic field which comes with two possible polarisations.



To simplify the analysis, we will not do the full calculation for the electromagnetic field which is, e.g., complicated by considerations about polarisation. Instead, we consider a massless scalar field in one-dimensional space. Therefore, the 'plates' define the positions  $x$  where the field vanishes and introduce discrete field modes. Show that the energy between 'plate' 1 and 'plate' 2 is given as

$$E(d) = \frac{\pi \hbar c}{2d} \sum_{n=1}^{\infty} n.$$

Calculate the energy between plates 2 and 3. What is the full energy  $E_f(d)$  between plates 1 and 3?