
Quantum Field Theory 1 – Tutorial 7

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Problem 1: Lie groups and Lie algebras

The finite elements of a connected Lie group can be obtained from an infinite product of transformations infinitesimal close to $\mathbb{1}$. A well known example is the group $SO(3)$ of rotations in $\mathbb{R}^3$.

a) Show that for a matrix M

$$\lim_{N \rightarrow \infty} \left(\mathbb{1} + \frac{\alpha}{N} M \right)^N = \exp(\alpha M) \quad (1)$$

by comparing the Taylor series in α . The expression in Eq. (1) is an element of a matrix Lie group while the matrix M is part of the corresponding Lie algebra.

b) Explicitly calculate $\exp(\alpha M)$ for

$$M_1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad M_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$