Non-equilibrium dynamics of gauge theories and transport coefficients

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Outline

- **Gauge dynamics far from equilibrium**

- **Spectral functions and transport coefficients**

- **Summary and outlook**

Gauge dynamics far from equilibrium

Gasenzer, McLerran, JMP, Sexty '13

Gauge dynamics far from equilibrium

Abelian Higgs model in 2+1 dim

Gasenzer, McLerran, JMP, Sexty '13

Classical action:

$$S[A_\mu, \phi] = -\int_x \left[\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^* D^\mu \phi + V(\phi) \right]$$

ϕ **Higgs**

phase $\dfrac{\phi}{|\phi|} = e^{i\,\varphi}$

Gauge dynamics far from equilibrium

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Classical action of Yang-Mills theory in diagonalisation gauges:

$$S_{\rm YM} \simeq \frac{1}{2}\int_x {\rm tr} F^2_{\bar\mu\bar\nu} + \frac{1}{2}\int_x {\rm tr}\,(D_{\bar\mu} A_2)^2$$

$A_2 = A_2^c(x_0, x_1)$

Wilson loop

$$\mathcal{W}_2 = \mathcal{P}\exp\left\{i\int_0^{L_2} dx_2\, A_2(x)\right\} = \exp\{i\,\phi\}$$

Vortex winding

$$n(\mathcal{S}) = \frac{1}{16\pi\, i}\oint_{\mathcal{S}} d^2x\, \epsilon_{ij}{\rm tr}\,\hat\phi\,\partial_i\hat\phi\,\partial_j\hat\phi$$

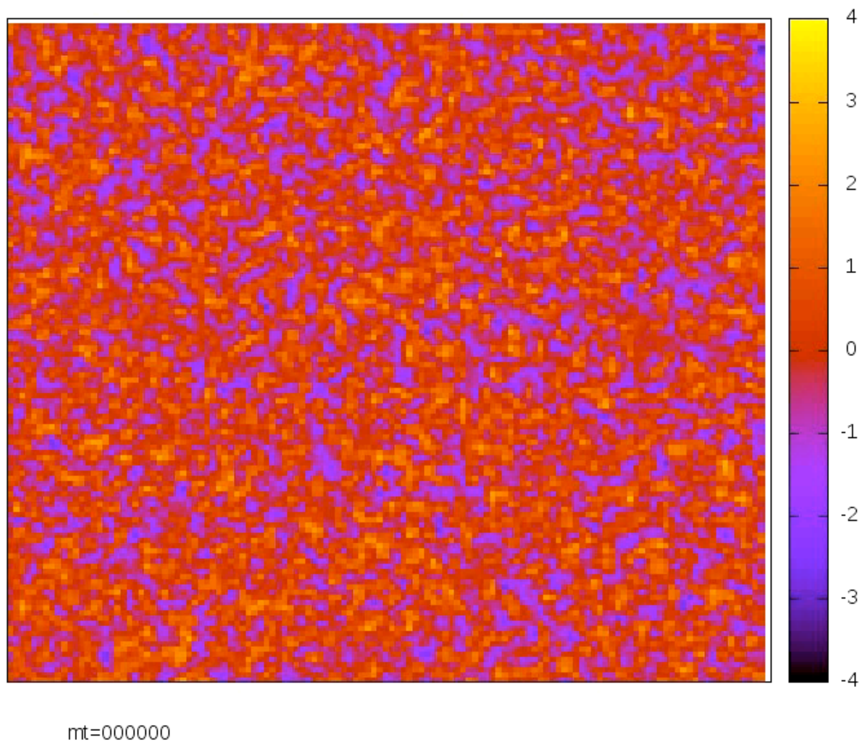
phase $\hat\phi = \frac{\phi}{\|\phi\|}$

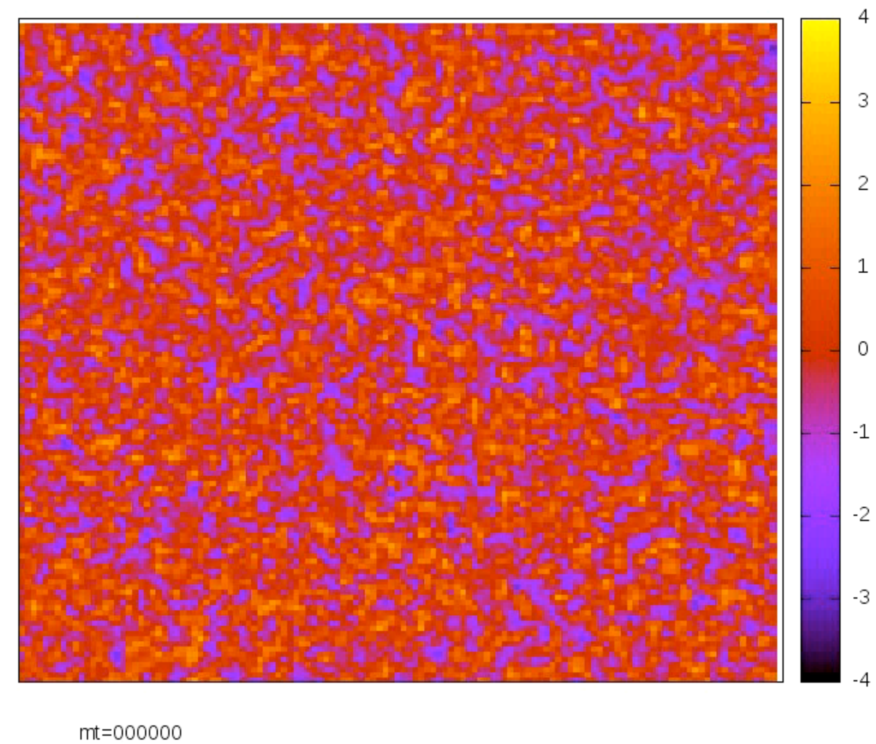
Quiz

Complex scalar vs Abelian Higgs

Gasenzer, McLerran, JMP, Sexty '13

phase φ of scalar field





'tachyonic' initial conditions

classical statistical lattice simulations

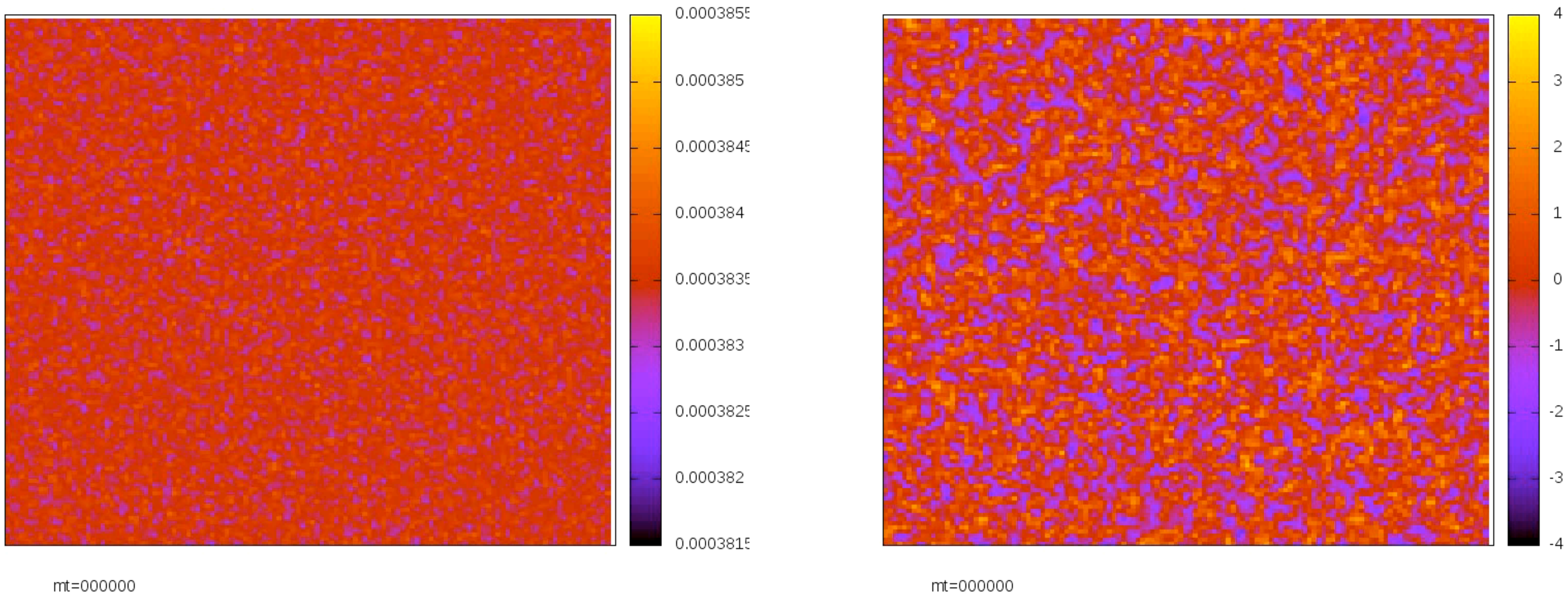
Which is which?

Gauge dynamics far from equilibrium

Abelian Higgs model in 2+1 dim

Gasenzer, McLerran, JMP, Sexty '13

magnetic field **phase of Higgs**



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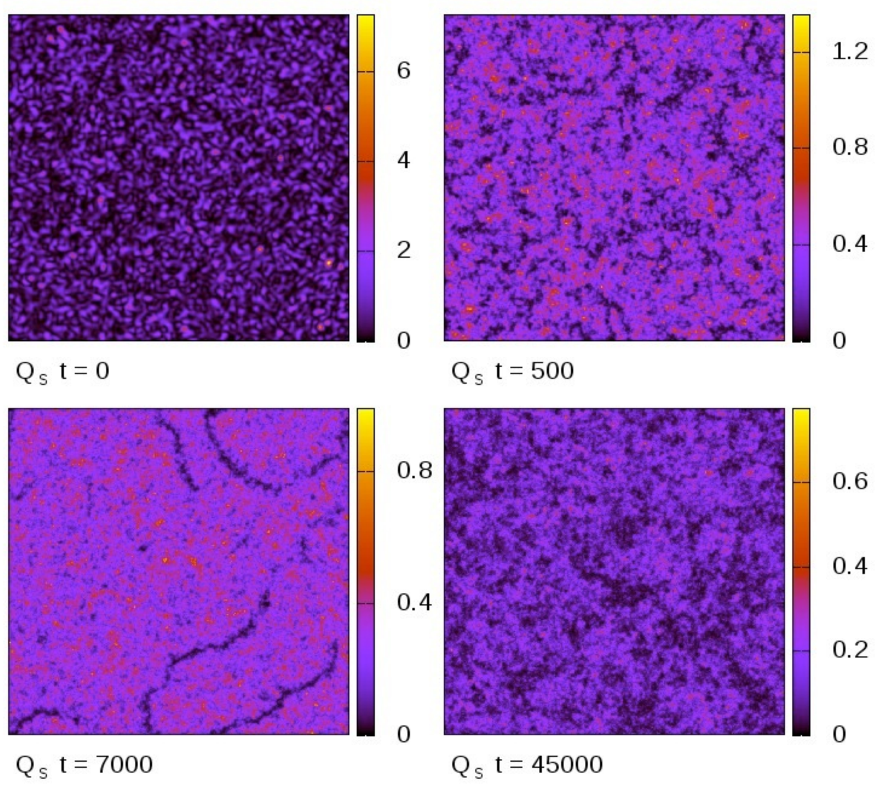
Gauge dynamics far from equilibrium

Abelian Higgs model in 2+1 dim

'overpopulation' initial conditions

modulus of Higgs

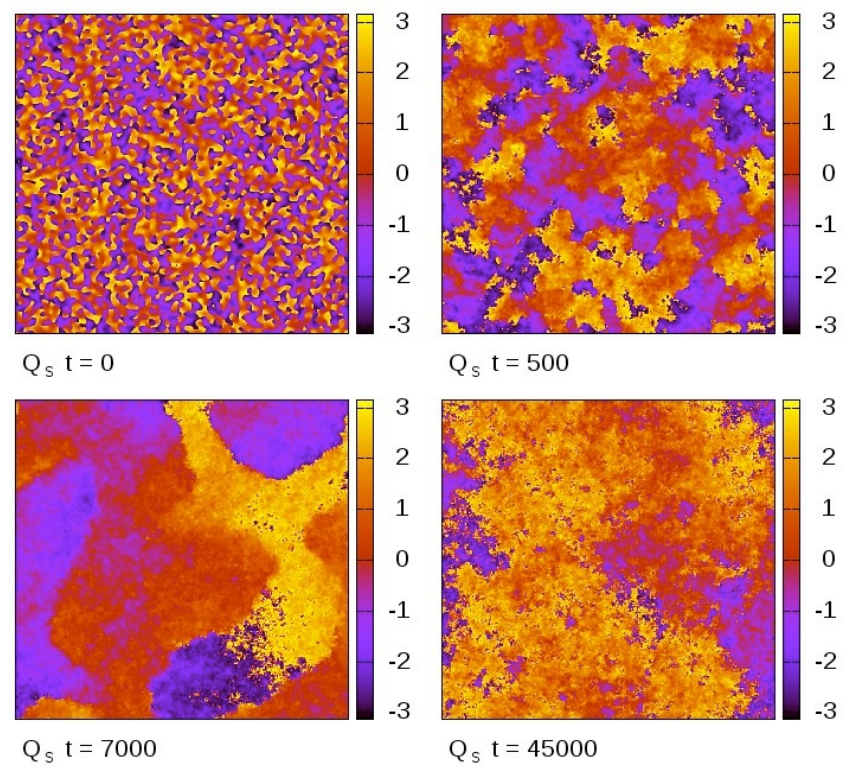


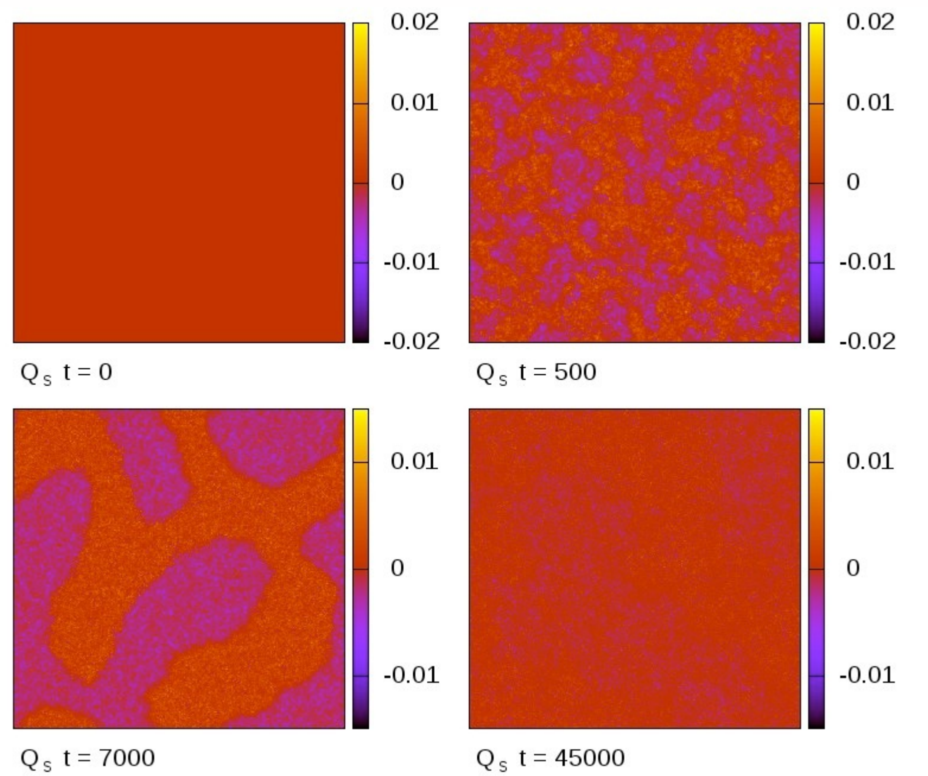
$$\xi = 0.025$$

coupling

$$\xi = \frac{6e^2}{\lambda}$$

relative phase





charge

$$\varphi^U(\vec{x},t) = \arg(G^U(\vec{0},\vec{x},t))$$

relative phase

$$G^U(\vec{x},\vec{y},t) = \langle \phi(\vec{x},t)U(\vec{x},\vec{y},t)\phi(\vec{y},t)^* \rangle_{\mathrm{cl}}$$

parallel transport U

Gasenzer, McLerran, JMP, Sexty '13

Gauge dynamics far from equilibrium

Abelian Higgs model in 2+1 dim

'overpopulation' initial conditions

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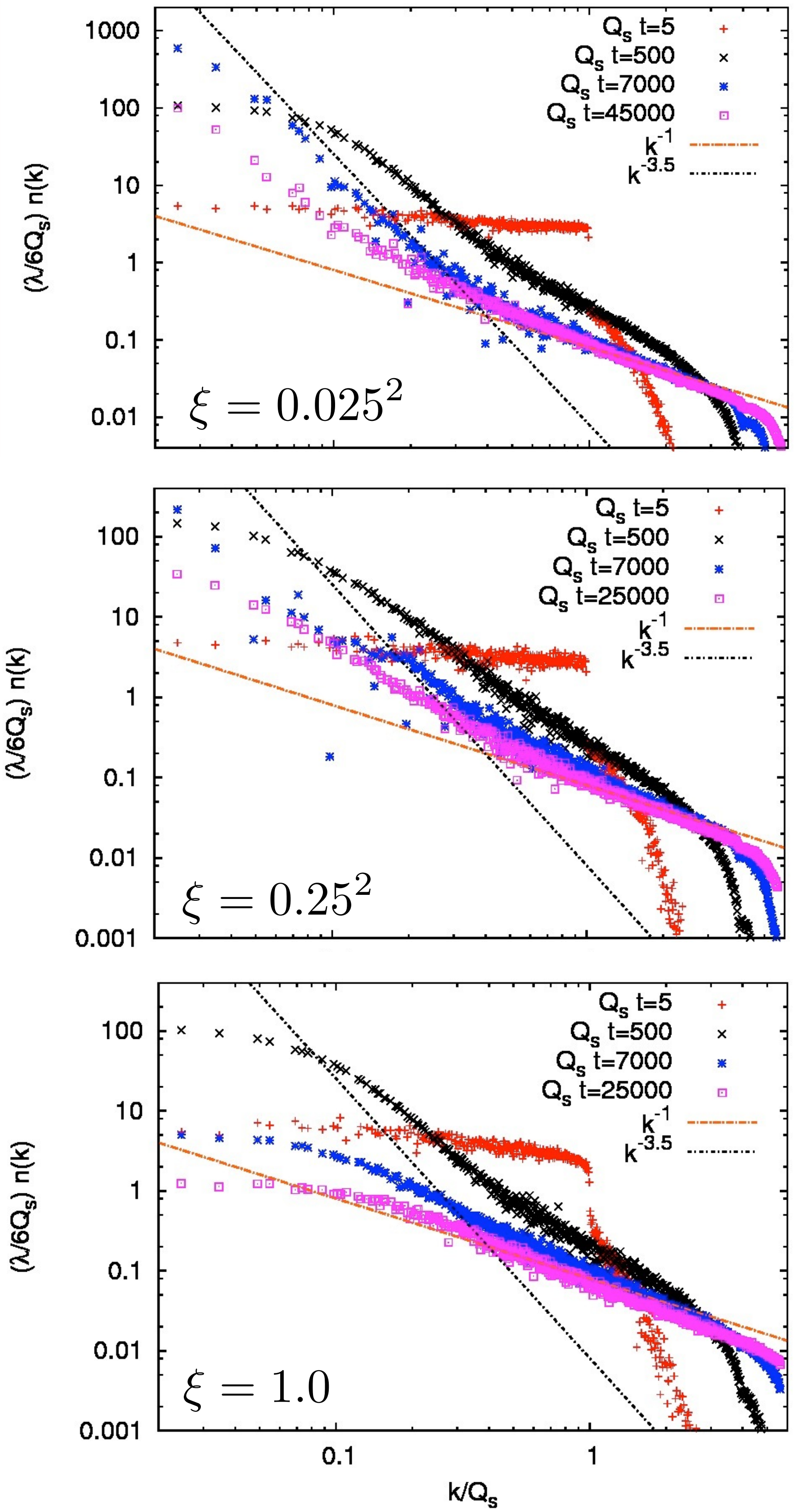
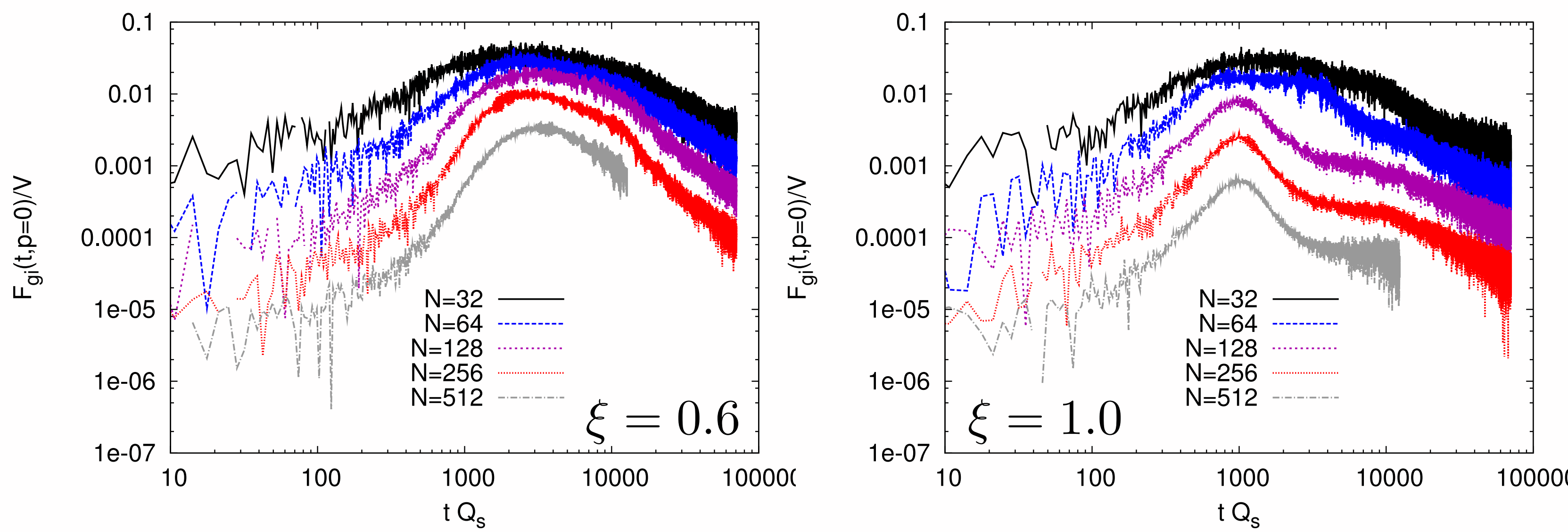


$$\frac{F_{\mathrm{gi}}(p=0)}{V} = \frac{1}{V^2}\int dx dy \phi^*(x) U(x,y) \phi(y)$$



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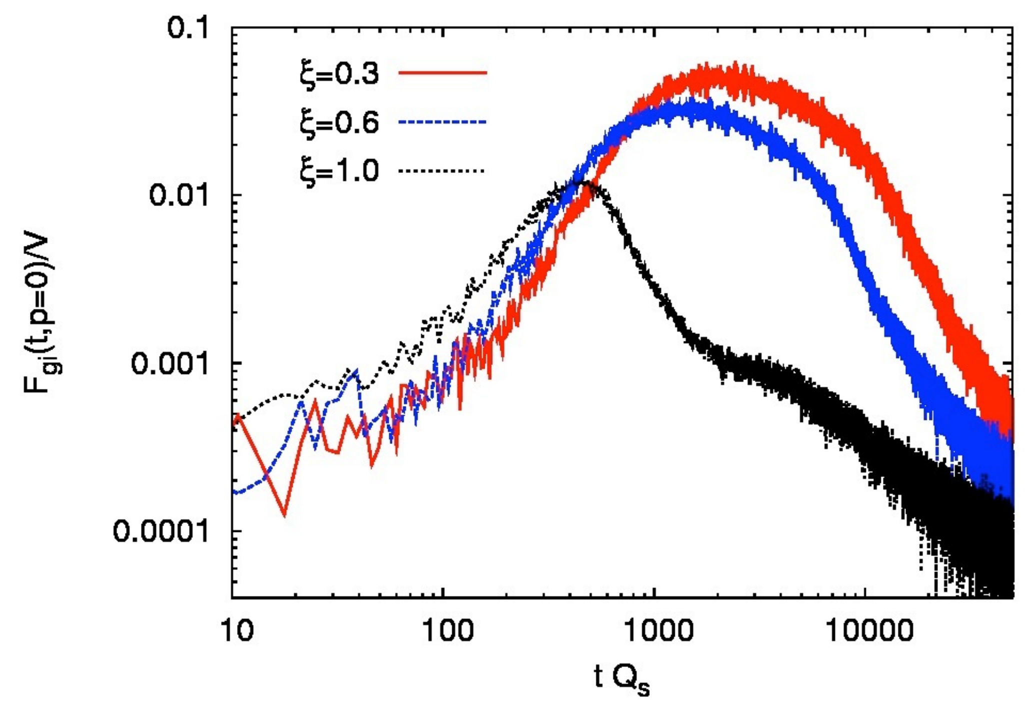


Spectral functions & transport coefficients

M. Haas, Fister, JMP ´13

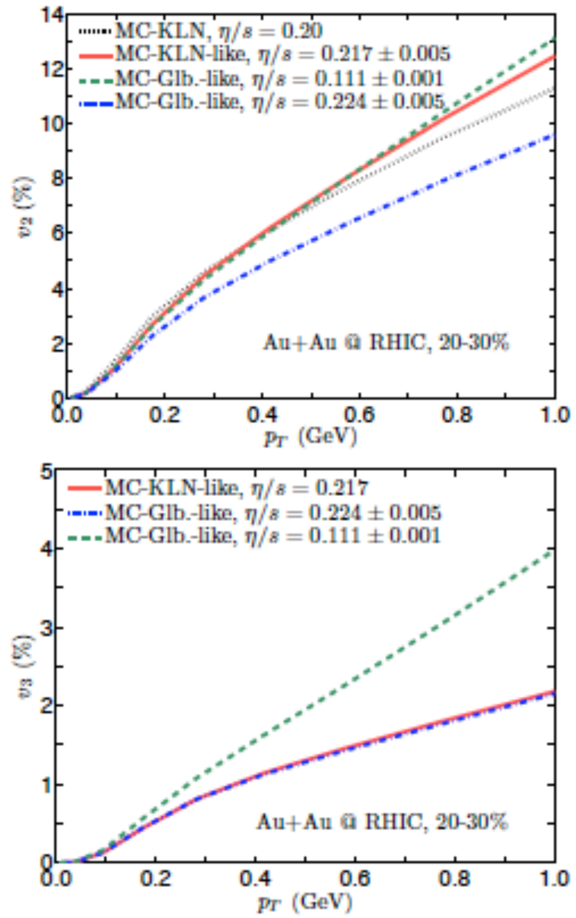
Christiansen, M. Haas, JMP, Strodthoff, in prep.

Helmboldt, JMP, Strodthoff, in prep.

Heavy ion collisions

Shooting the elephant

Zhi Qiu and U. Heinz, to be published



Proof of principle calculation:

- Take ensemble of sum of deformed Gaussian profiles, $s(\boldsymbol{r}_\perp) = s_2(\boldsymbol{r}_\perp; \bar{\varepsilon}_2, \psi_2) + s_3(\boldsymbol{r}_\perp; \bar{\varepsilon}_3, \psi_3)$, with
  1. equal Gaussian radii $R_2^2 = R_3^2 = 8\,\mathrm{fm}^2$ to reproduce $\langle r_\perp^2 \rangle$ of MC-KLN source for 20-30% AuAu
  2. $\bar{\varepsilon}_2$ and $\bar{\varepsilon}_3$ adjusted such that
     - $\bar{\varepsilon}_{2,3} = \langle \varepsilon_{2,3} \rangle_{\mathrm{KLN}}^{20-30\%}$ ("MC-KLN-like")
     - $\bar{\varepsilon}_{2,3} = \langle \varepsilon_{2,3} \rangle_{\mathrm{Gl}}^{20-30\%}$ ("MC-Glauber-like")
  3. $\psi_2 = 0$, ψ_3 (direction of triangularity) distributed randomly
- Use $v_2^\pi(p_T)$ from VISH2+1 for $\eta/s = 0.20$ with MC-KLN initial conditions for 20-30% AuAu as "mock data"
- Fit mock $v_2^\pi(p_T)$ data with VISH2+1 for "MC-Glauber-like" or "MC-KLN-like" Gaussian initial conditions with both elliptic and triangular deformations by adjusting η/s
  $\Longrightarrow$ $(\eta/s)_{\mathrm{KLN}} = 0.217 \pm 0.005$ for "MC-KLN-like",
  $(\eta/s)_{\mathrm{Gl}} = 0.111 \pm 0.001$ for "MC-Glauber-like"
- Compute $v_3^\pi(p_T)$ for "MC-KLN-like" fit with $(\eta/s)_{\mathrm{Gl}}$=0.217 and reproduce it with "MC-Glauber-like" initial condition by readjusting η/s
  $\Longrightarrow$ $(\eta/s)_{\mathrm{Gl}}^{v_3} = 0.224 \pm 0.005$ for "MC-Glauber-like"
- Compute $v_2^\pi(p_T)$ for "MC-Glauber-like" initial profiles with readjusted $(\eta/s)_{\mathrm{Gl}}^{v_3} = 0.224$ and compare with "MC-Glauber-like" fit to original mock data $\Longrightarrow$ clearly visible (and measurable) difference!

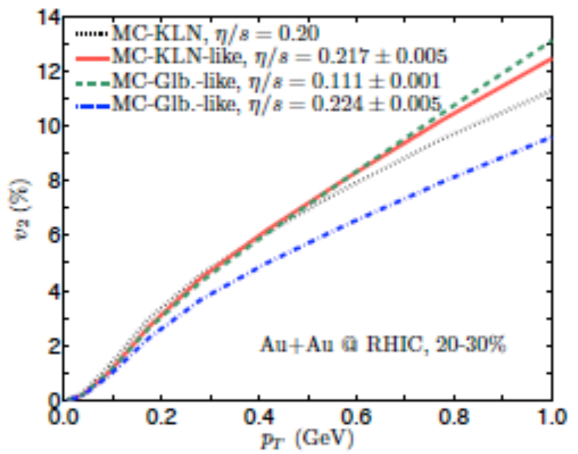
This exercise proves: (i) Fitting $v_3(p_T)$ data with MC-Glauber and MC-KLN initial conditions yields the same η/s (within narrow error band); (ii) The corresponding $v_2(p_T)$ fits are quite different, and only one (more precisely: at most one!) of the models will fit the corresponding $v_2(p_T)$ data.

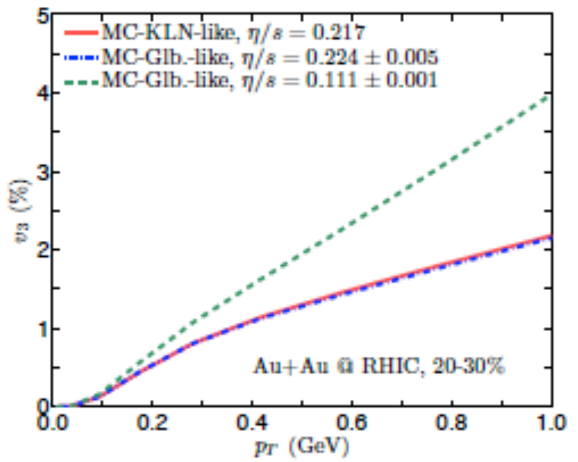
U. Heinz, talk at RETUNE '12

Heavy ion collisions

Computing the elephant

Zhi Qiu and U. Heinz, to be published





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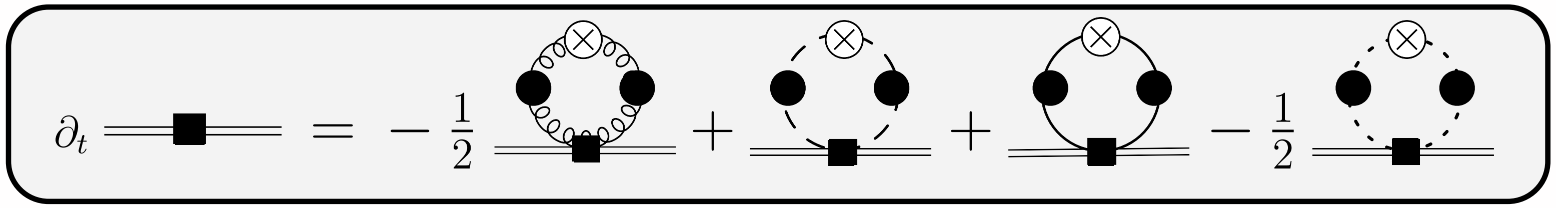
Transport in QCD

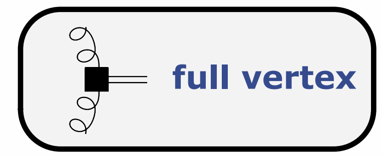
correlations of energy-momentum tensor

M. Haas, Fister, JMP ´13

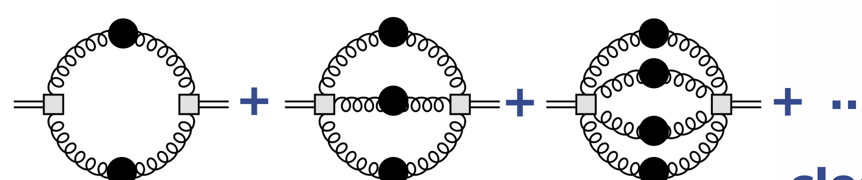
Flow



$\rho_{\pi\pi}$



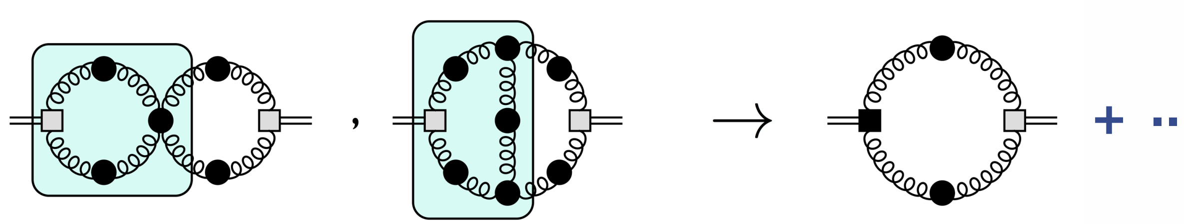
Diagrammatic representation

$\rho_{\pi\pi} =$ 

closed form

full computation Christiansen, Haas, JMP, Strodthoff, in prep.

Vertex corrections



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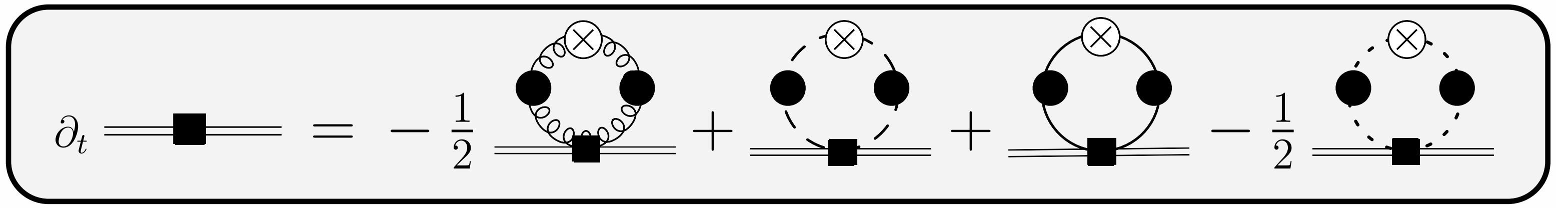

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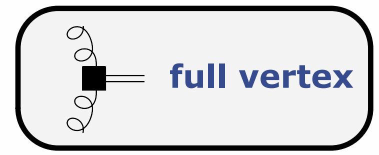

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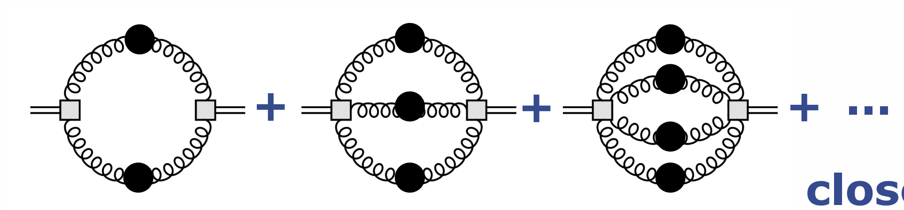

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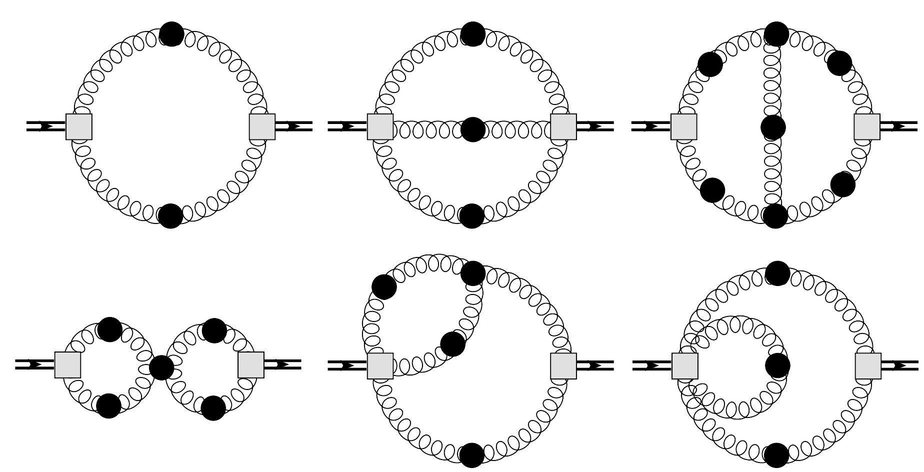
Complete 2-loop corrections



Transport in QCD

correlations of energy-momentum tensor

M. Haas, Fister, JMP ´13

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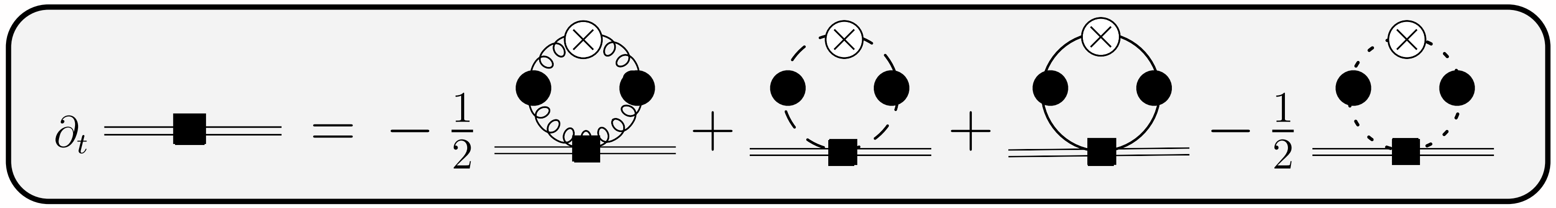
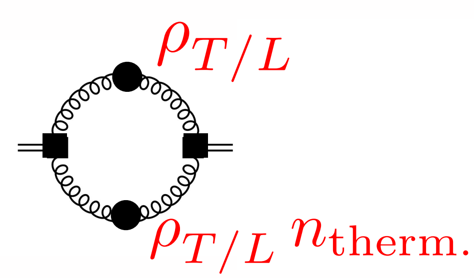
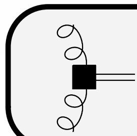


$\rho_{\pi\pi}$

Current approximation

$\rho_{\pi\pi} =$





with optimised RG-scheme from Fister, JMP ’13

$\rho_{T/L}$ with MEM

$$\rho_{\pi\pi}(p) = \frac{2d_A}{3}\int \frac{d^4k}{(2\pi)^4}\left[n(k^0) - n(k^0+p_0)\right]\left(V_{TT}\rho_T(k)\rho_T(k+p) + V_{TL}\rho_T(k)\rho_L(k+p) + V_{LL}\rho_L(k)\rho_L(k+p)\right)$$

Transport in QCD

correlations of energy-momentum tensor

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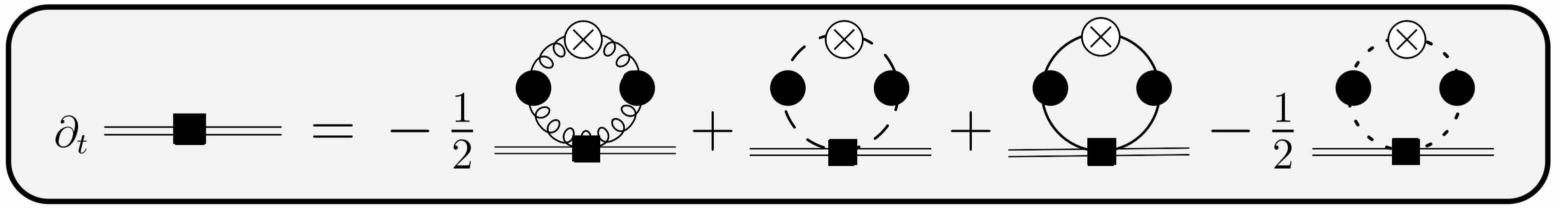
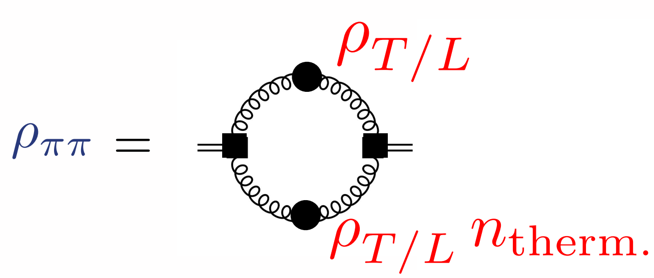
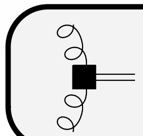


$\rho_{\pi\pi}$

Current approximation





full vertex

with optimised RG-scheme from Fister, JMP '13

$\rho_{T/L}$ with MEM

'Those are my methods (principles),
and if you don't like them...well, I have others'

direct computation **Groucho Marx**

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Transport in QCD

correlations of energy-momentum tensor

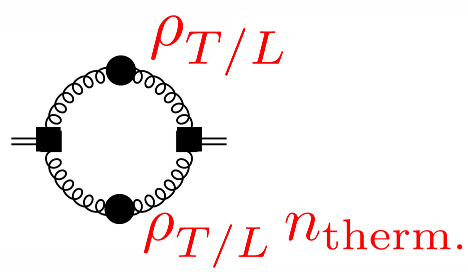
M. Haas, Fister, JMP ´13

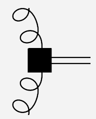
Shear viscosity

$$\eta = \frac{1}{20}\,\frac{d}{d\omega}\bigg|_{\omega=0} \rho_{\pi\pi}(\omega, 0)$$ **Kubo relation**

Current approximation

$\rho_{\pi\pi} =$





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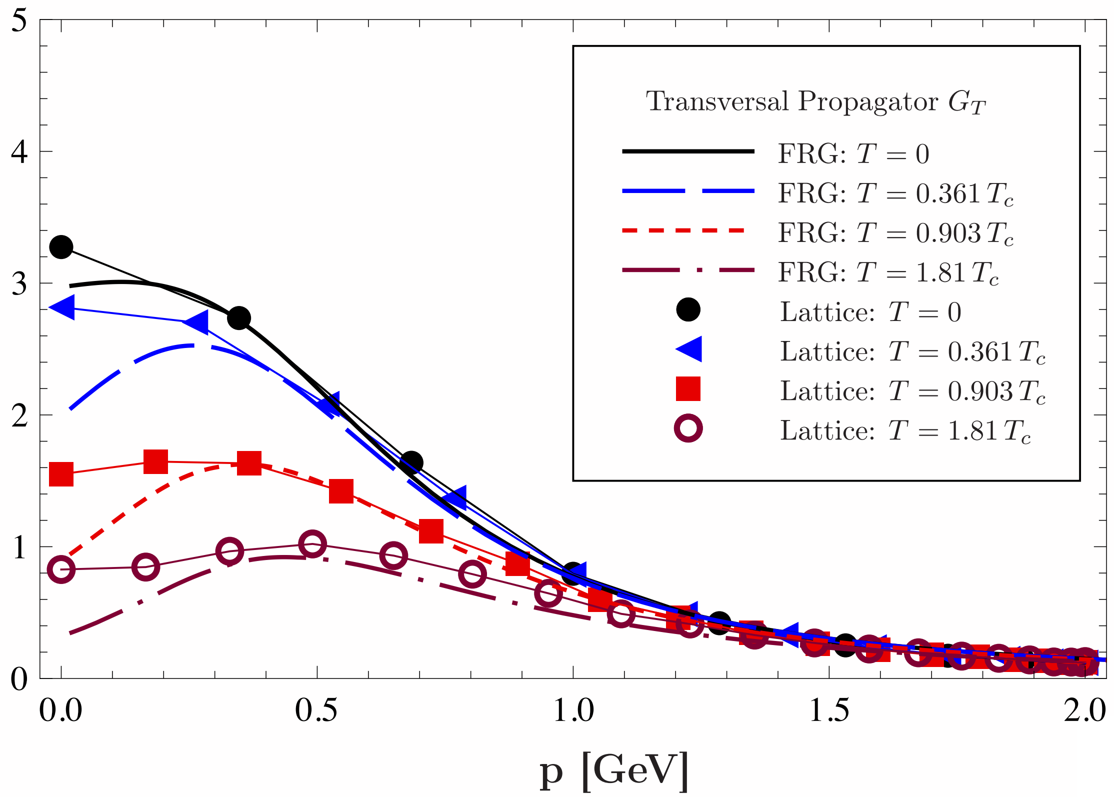
Viscosity in pure glue

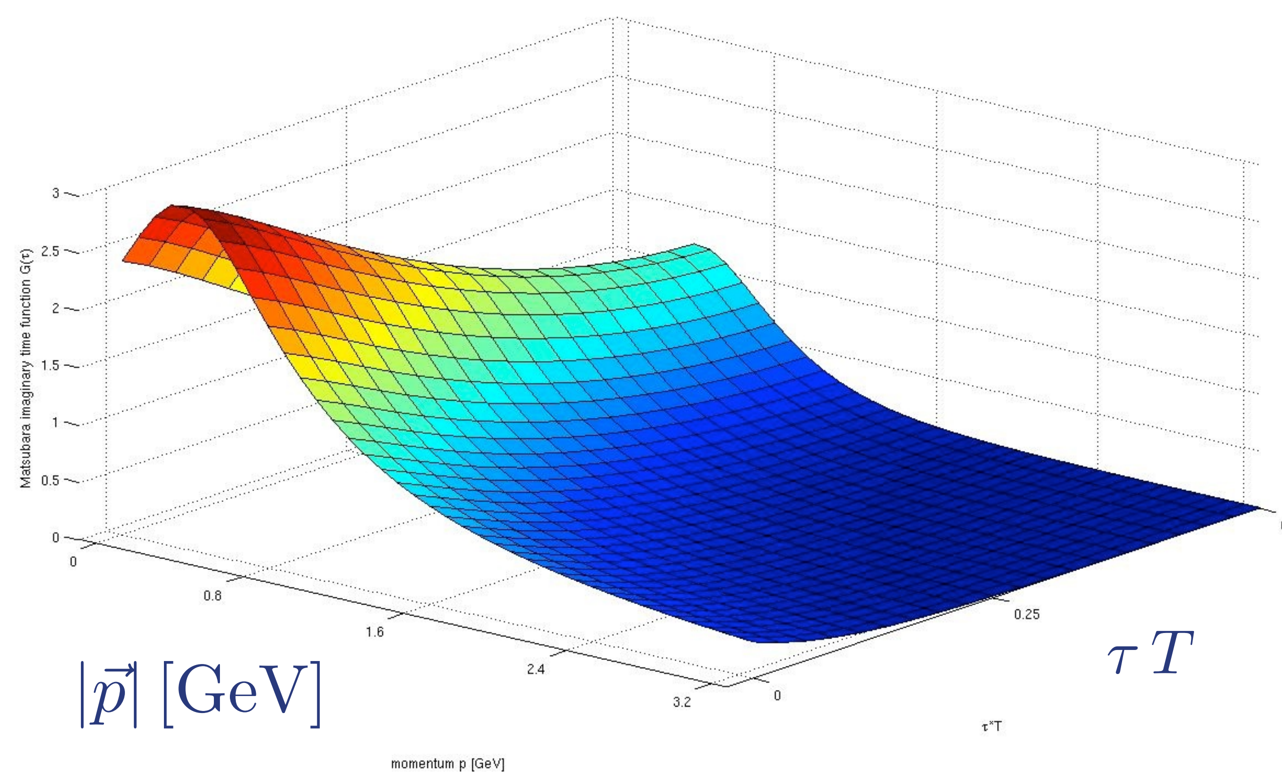
imaginary time correlations

M. Haas, Fister, JMP ´13



transversal gluon propagator

$G_T(\tau, \vec{p})$

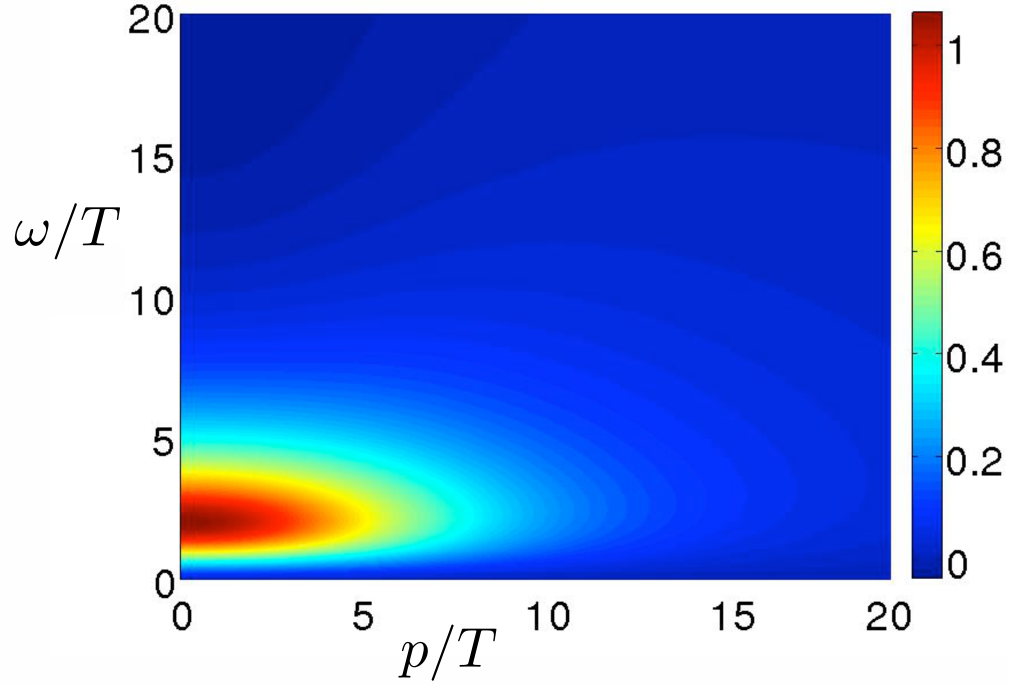


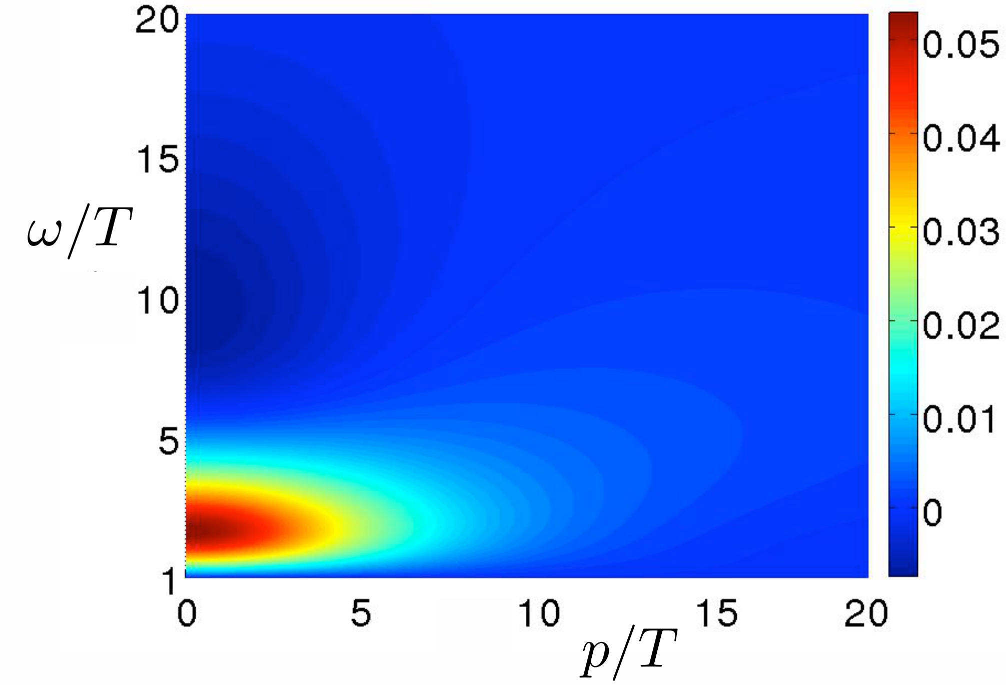
Viscosity in pure glue

gluon spectral functions

M. Haas, Fister, JMP ´13

transversal

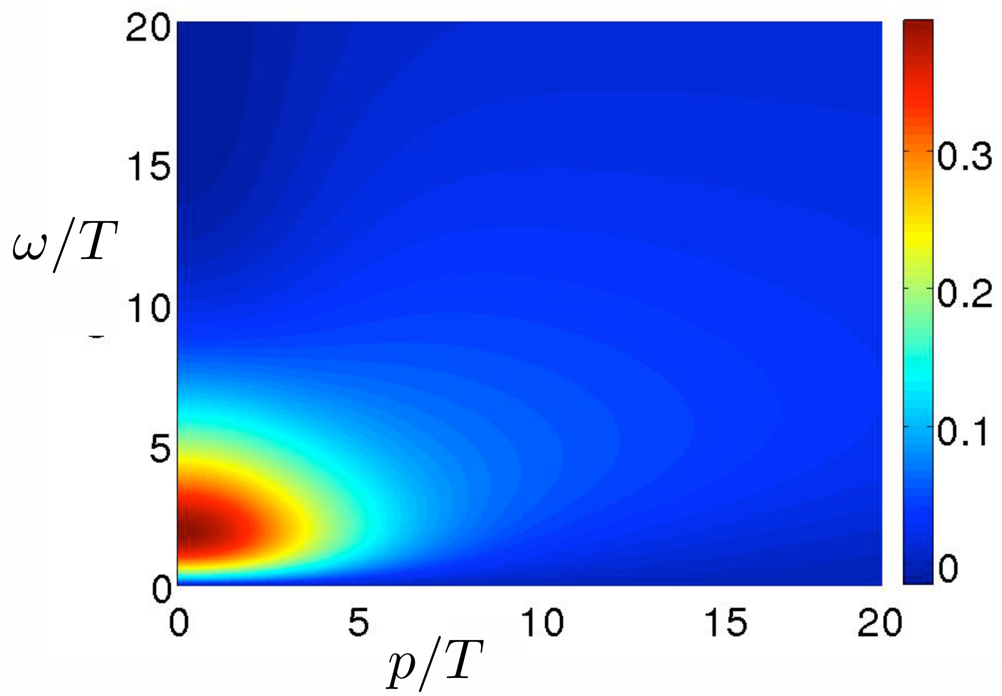


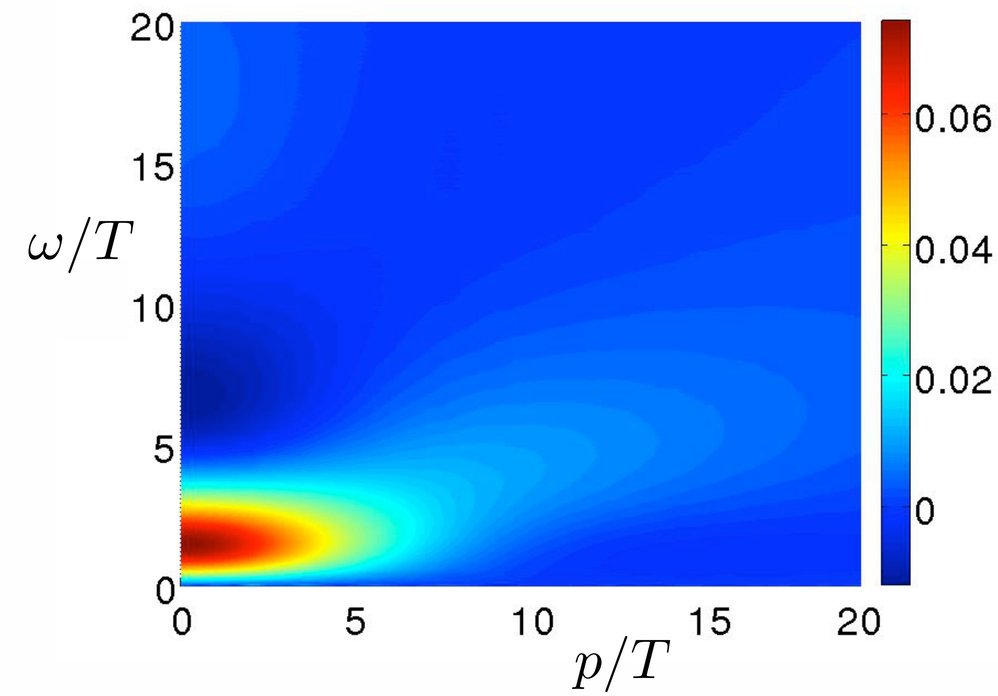


$\mathbf{T = 0.36 T_c}$

longitudinal

$\mathbf{T = 1.8 T_c}$

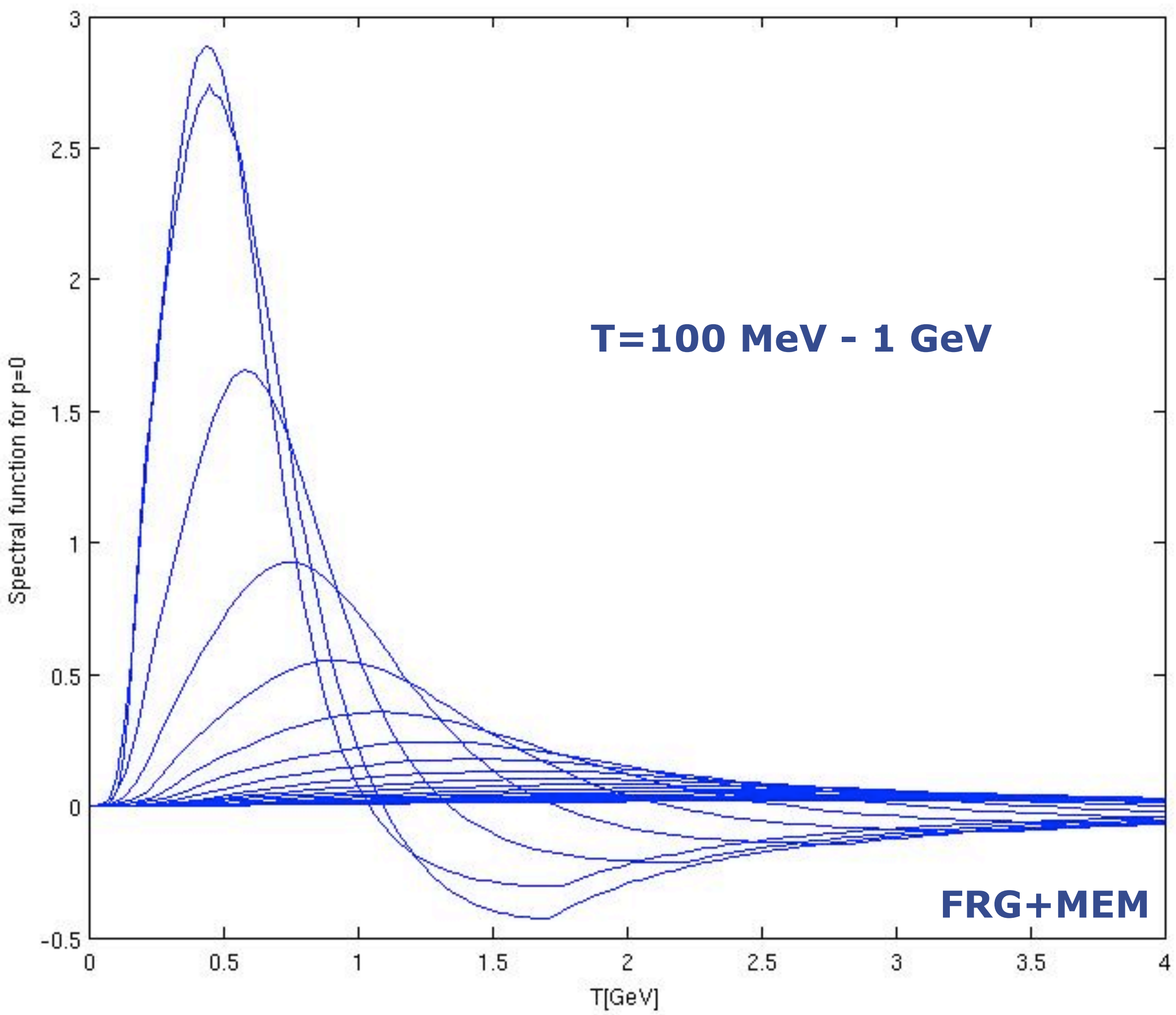




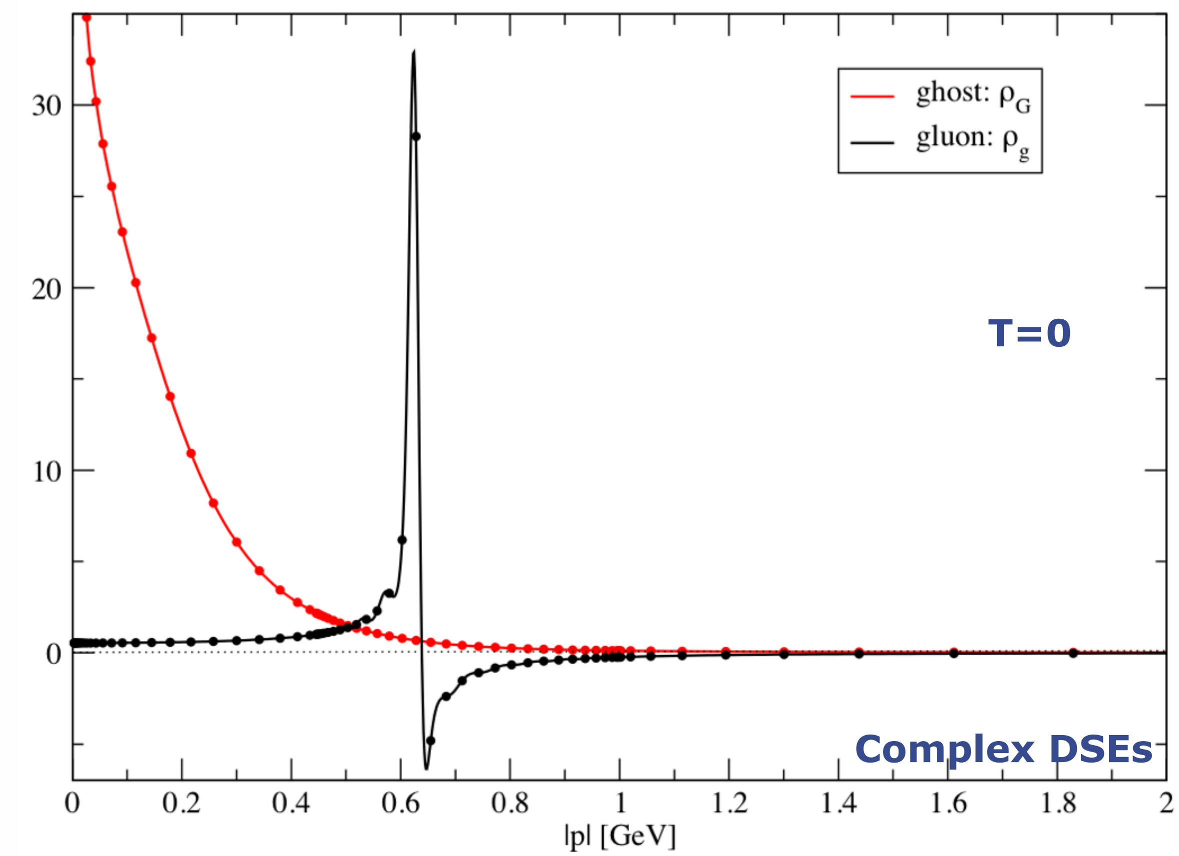
Viscosity in pure glue

spectral functions

transversal spectral function

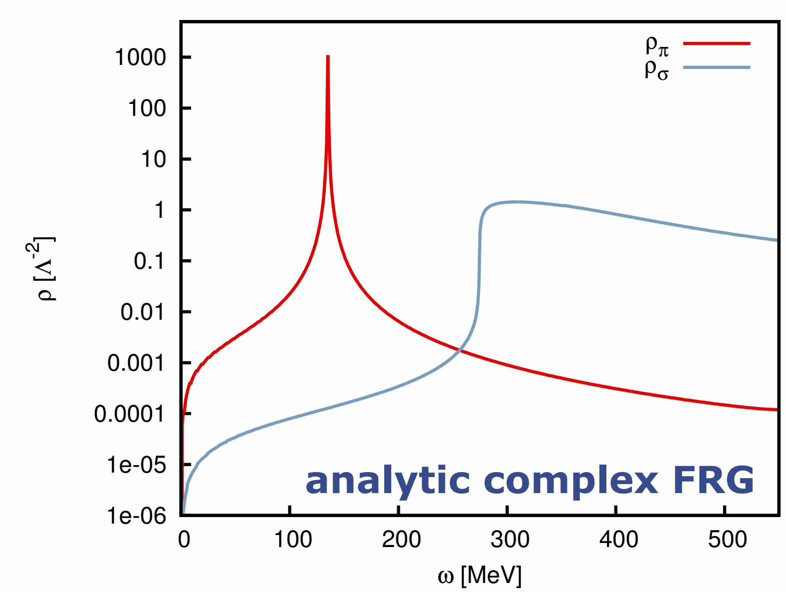


M. Haas, Fister, JMP ´13



Strauss, Fischer, Kellermann ’12

pion and sigma spectral functions

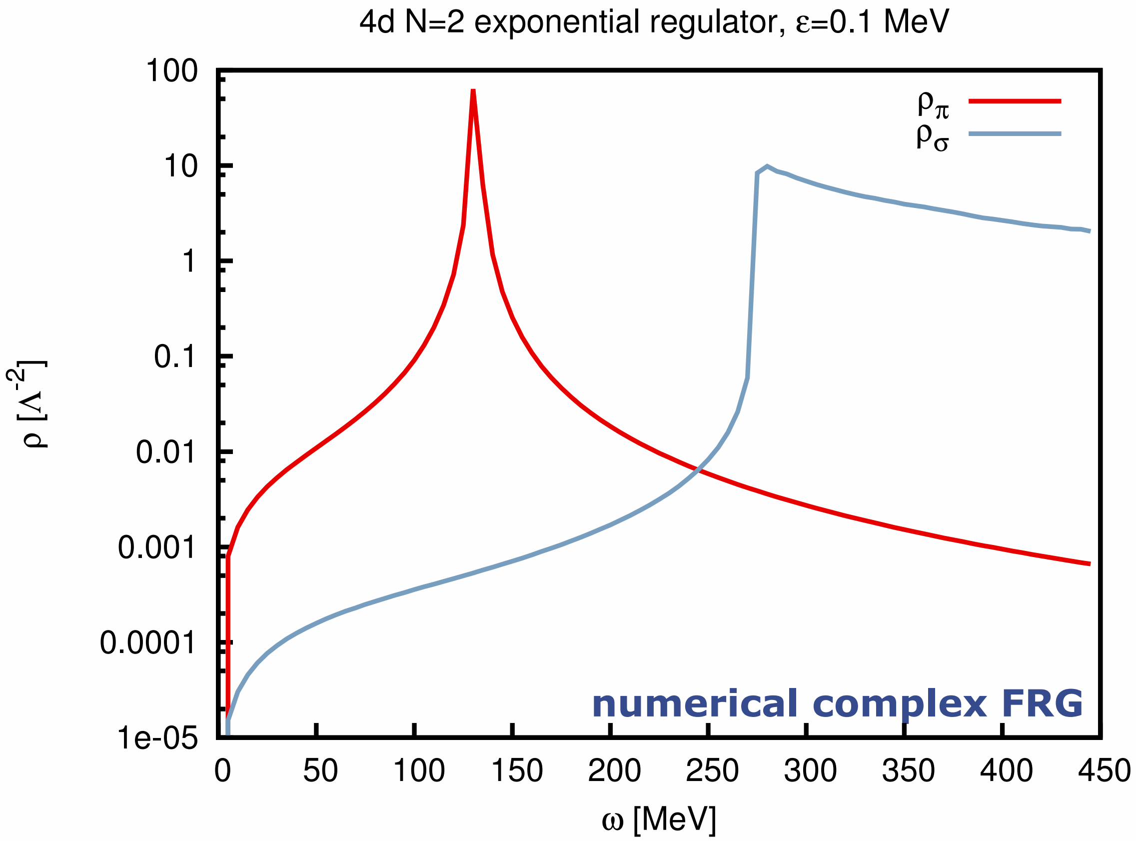


Kamikado, Strodthoff, von Smekal, Wambach ’13

Viscosity in pure glue

spectral functions

pion and sigma spectral functions



JMP, Strodthoff, in preparation

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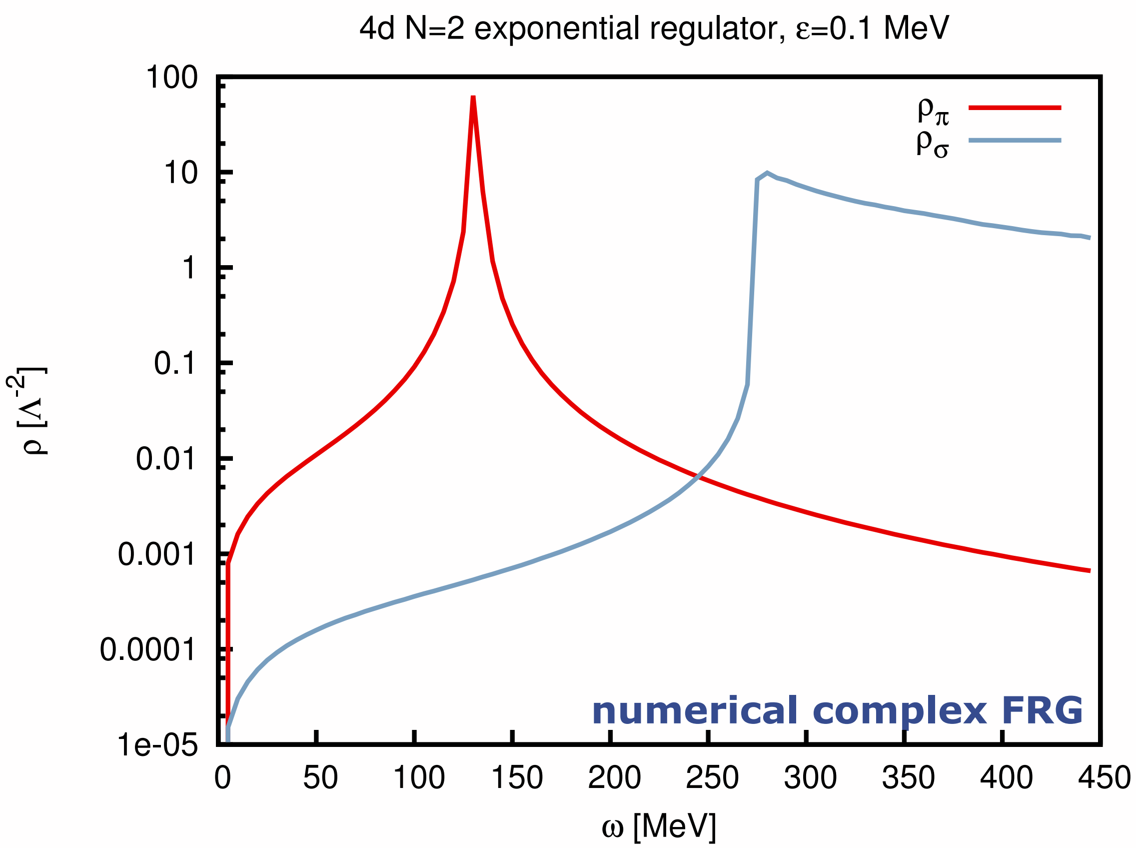
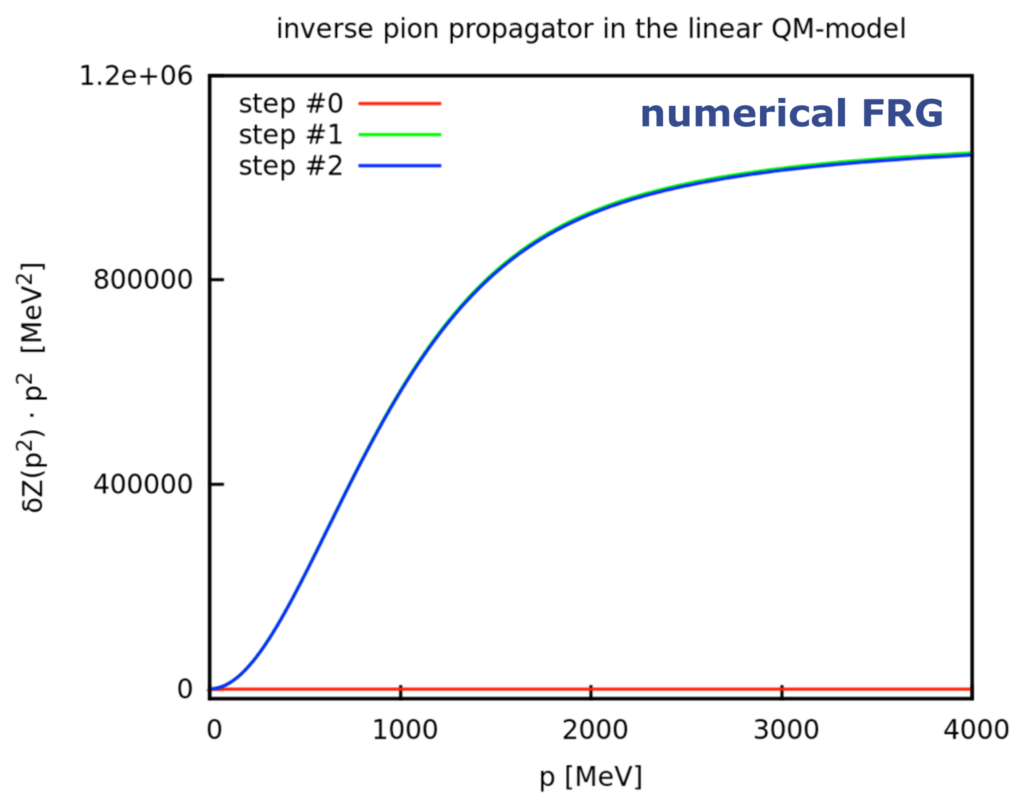


JMP, Strodthoff, in preparation

QM-model

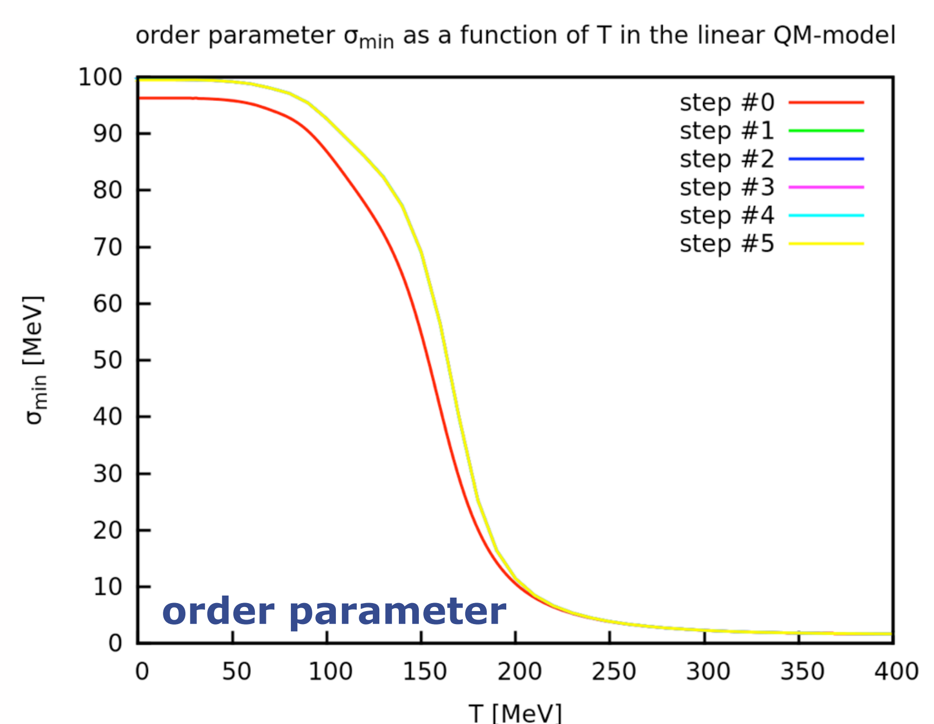


O(N)-model

iteration step	σ_0 [MeV]	δ_ρ [%]	m_{pole} [MeV]	m_{screen} [MeV]	δ_m [%]
0	93.55	0.0043	130.3113	136.7593	4.9
1	100.05	0.0028	126.6390	126.4590	0.14
5	99.38	0.0043	127.0347	127.0110	0.019

iteration step	σ_0 [MeV]	δ_ρ [%]	m_{pole} [MeV]	m_{screen} [MeV]	δ_m [%]
0	96.25	0.0052	91.4911	134.8281	47
1	99.56	0.0044	90.8841	91.1611	0.30
5	99.56	0.0073	90.9244	91.1551	0.25

QM-model



Helmboldt, JMP, Strodthoff, in preparation

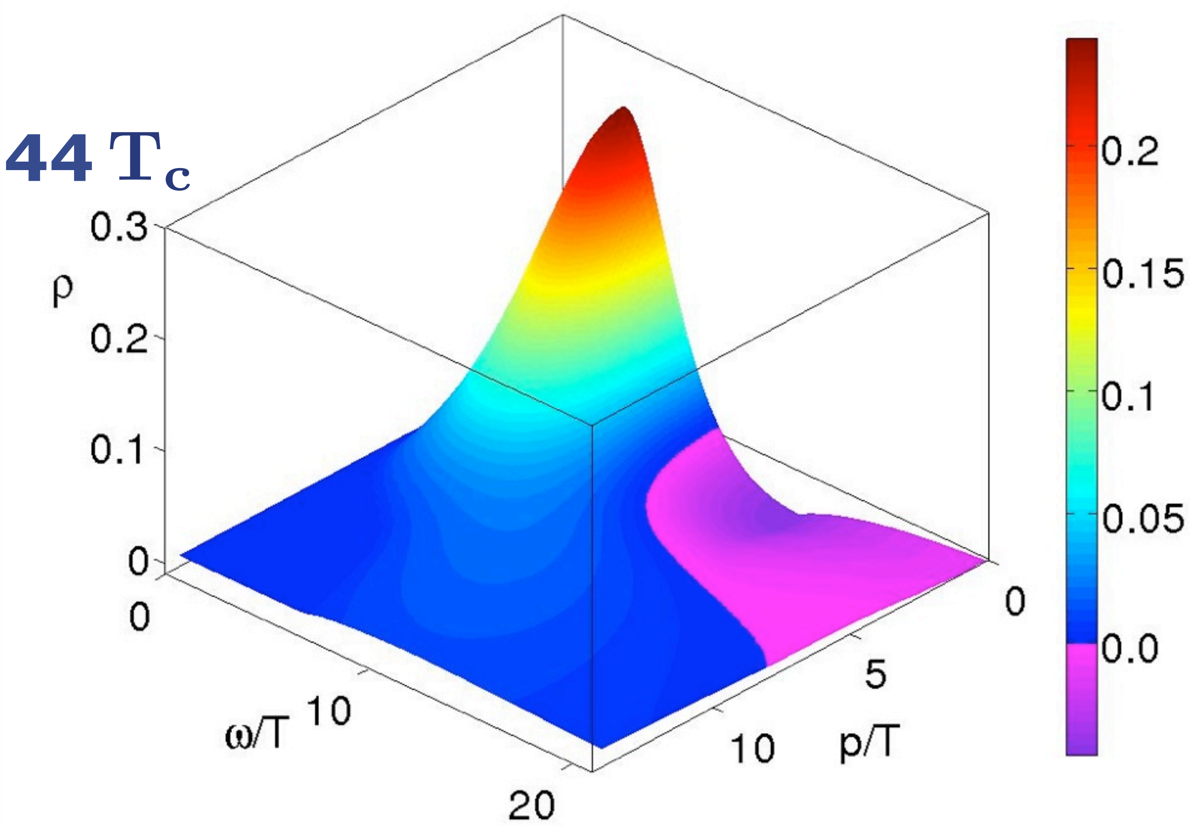
Viscosity in pure glue

spectral functions

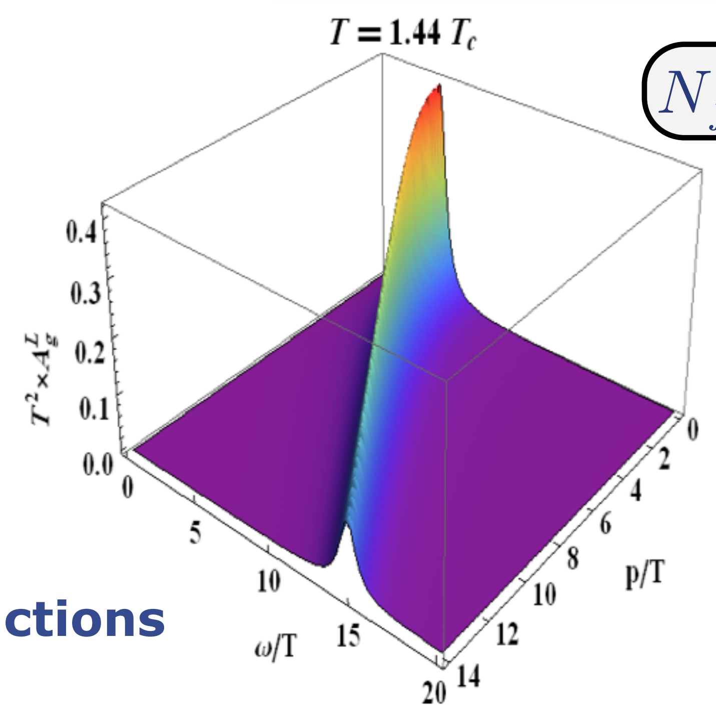
M. Haas, Fister, JMP ´13

transversal

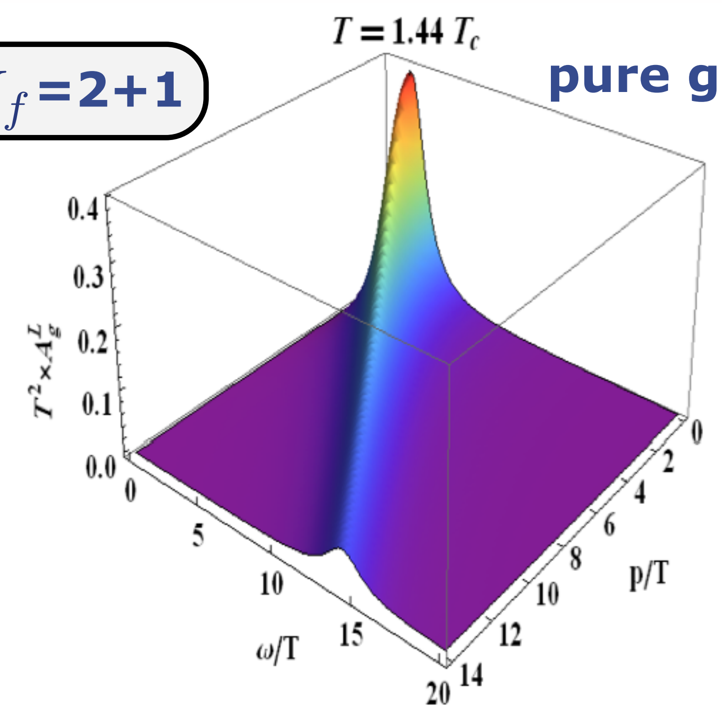
T=1.44 T_c



T=1.44 T_c



N_f=2+1



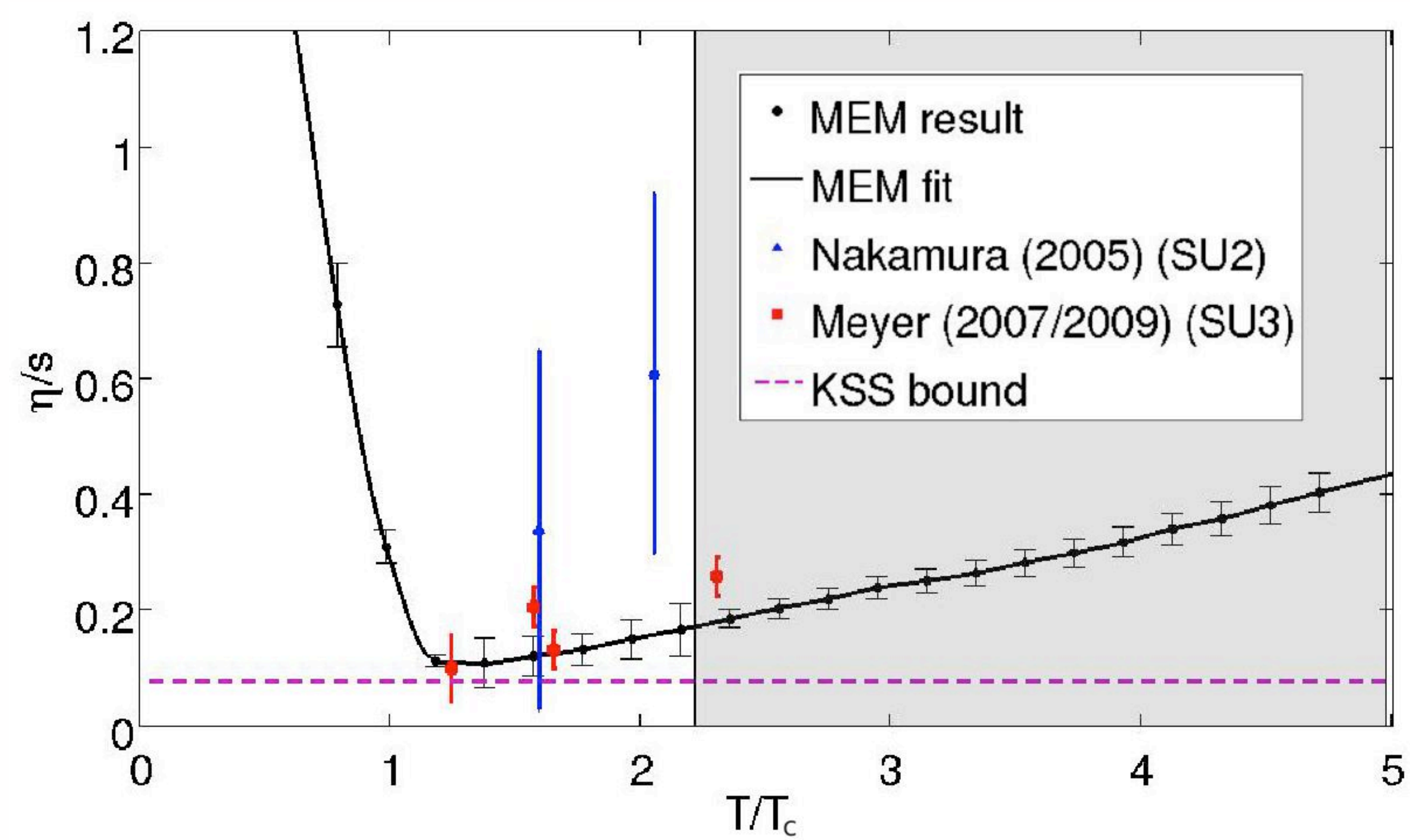
pure glue

PHSD spectral functions

Viscosity in pure glue

shear viscosity

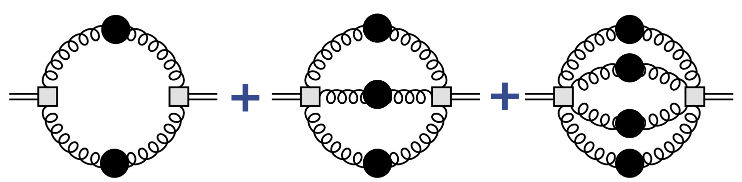
M. Haas, Fister, JMP ´13



Kubo relation

$$\eta = \frac{1}{20}\frac{d}{d\omega}\bigg|_{\omega=0}\rho_{\pi\pi}(\omega,0)$$

Diagrammatic representation

$\rho_{\pi\pi} =$ 

+ ··· closed form

$T \lesssim 2T_c$ **: MEM+optimised RG-scheme systematic error estimates**

Shaded area: MEM error estimates

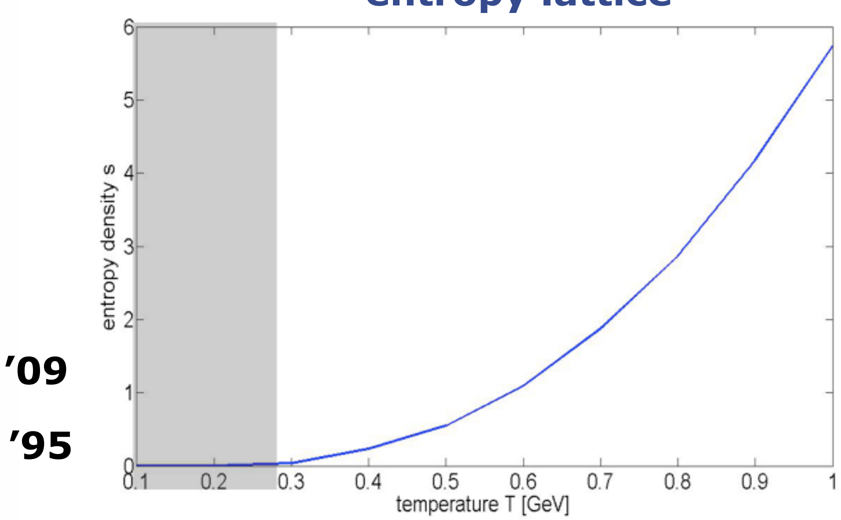
minimum at T = 1.25$\mathrm{T_c}$: $$\frac{\eta}{\mathrm{s}} = 1.45\frac{1}{4\pi}$$

scale matching with QCD: $$\frac{\eta}{\mathrm{s}} = 2.27\frac{1}{4\pi}$$

H. Meyer '09

Boyd, Engels, Karsch '95

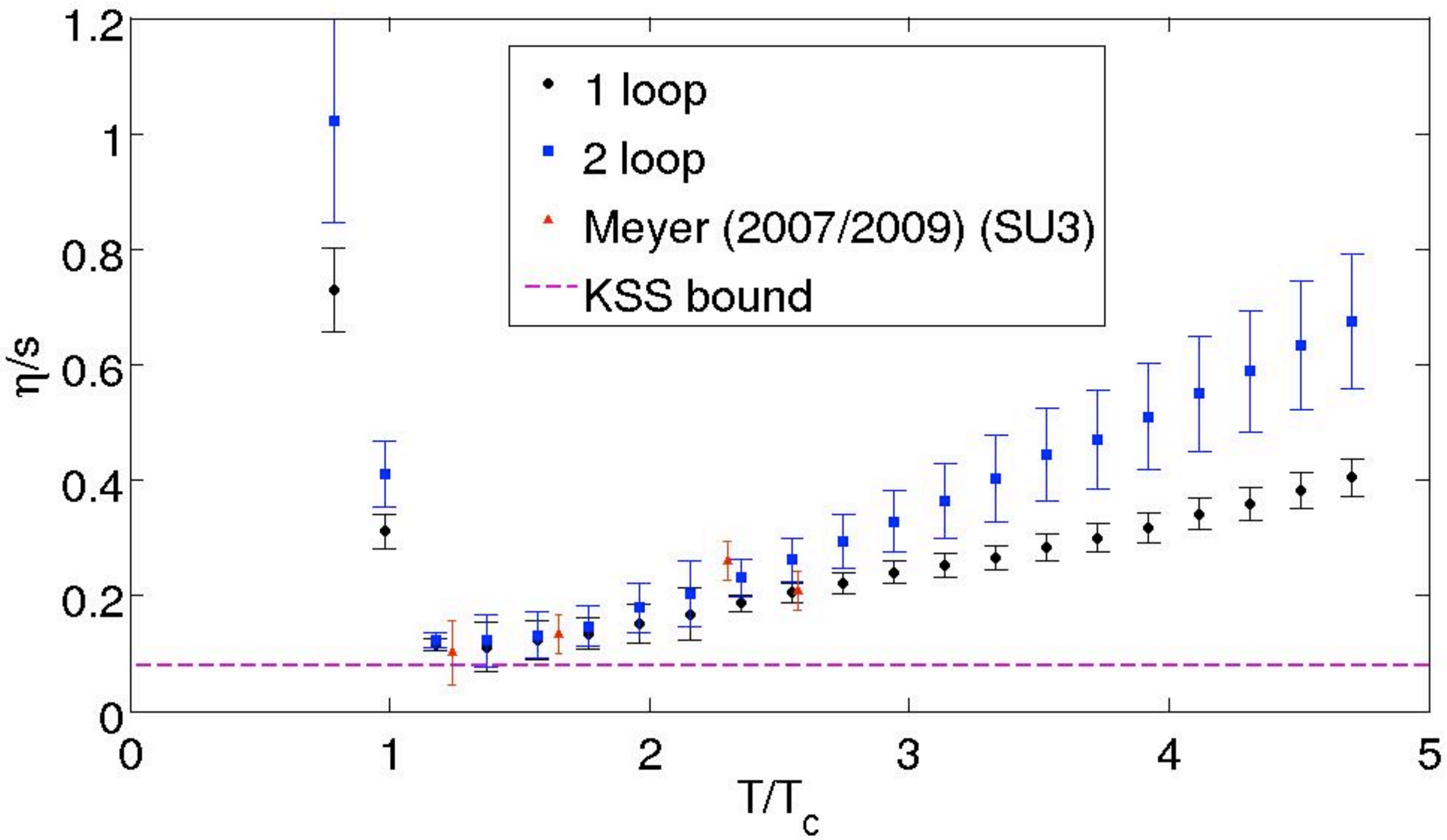
entropy lattice



Viscosity in pure glue

shear viscosity

Christiansen, M. Haas, JMP, Strodthoff, in prep.



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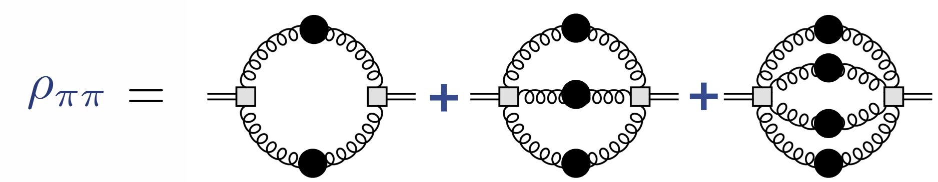
Diagrammatic representation



+ ⋯ closed form

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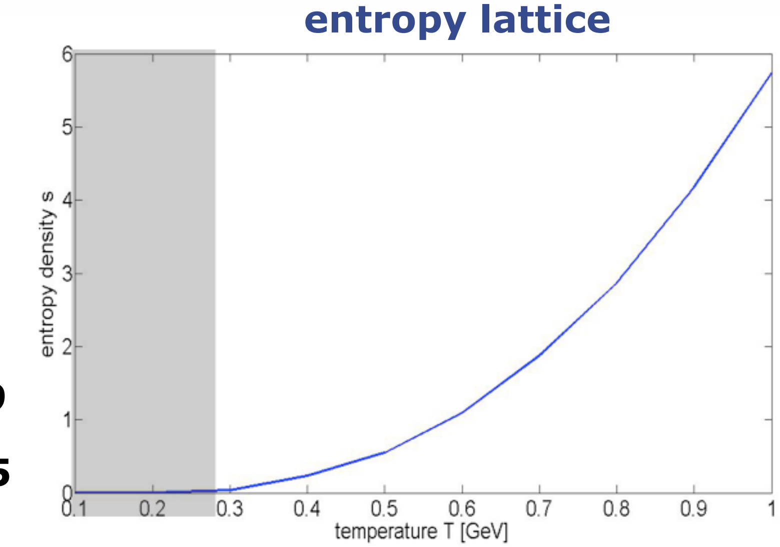


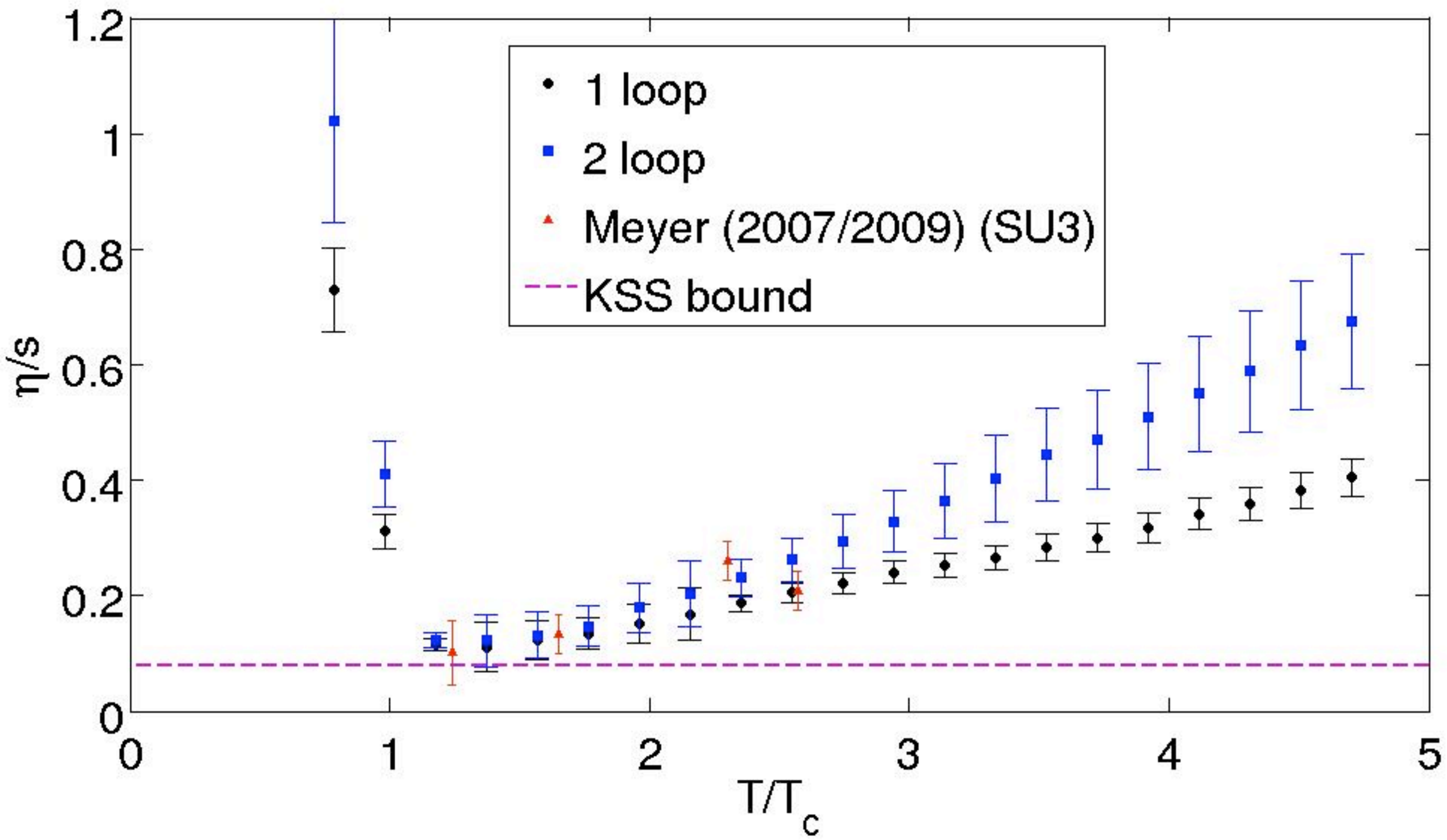
H. Meyer '09

Boyd, Engels, Karsch '95

Viscosity in pure glue

shear viscosity

Christiansen, M. Haas, JMP, Strodthoff, in prep.



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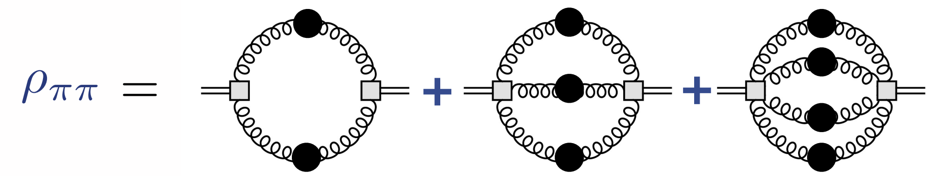
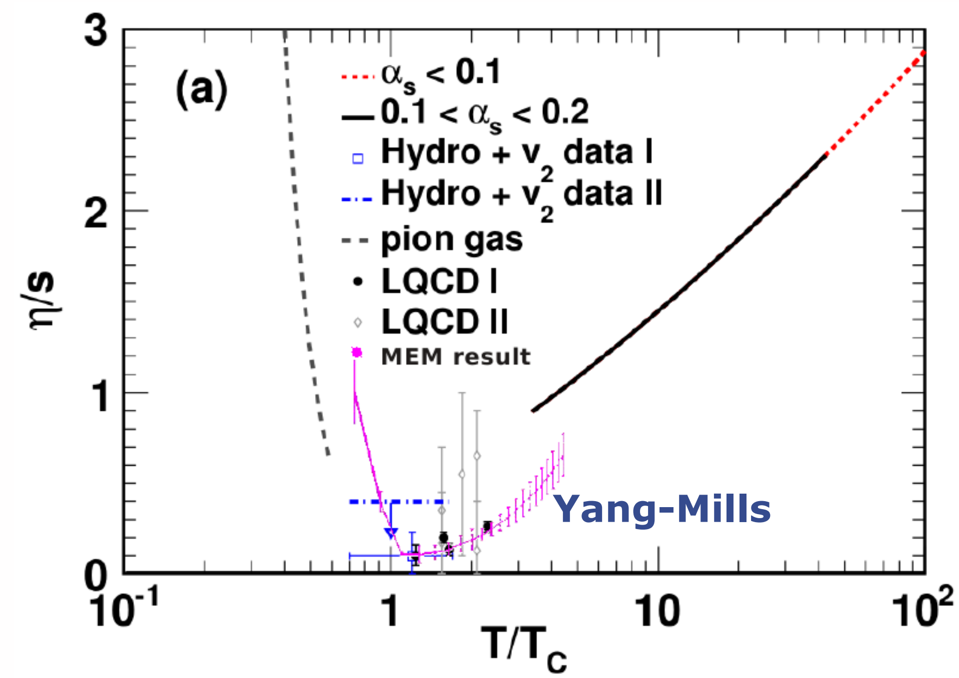
Diagrammatic representation



\+ ··· closed form



Chen, Deng, Dong, Wang ’11

minimum at T = 1.25T_c : $\frac{\eta}{s} = 1.45 \frac{1}{4\pi}$

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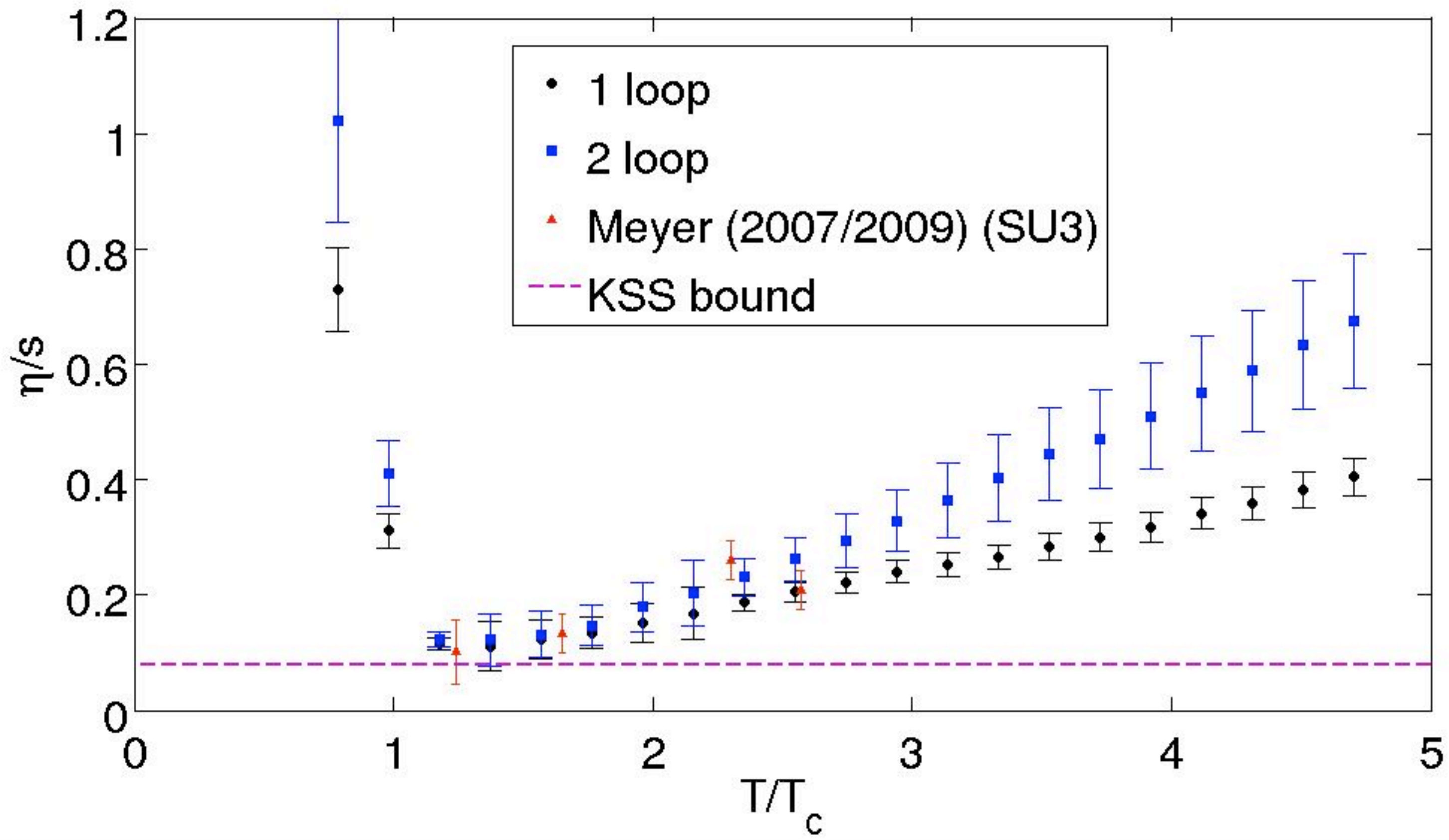
Viscosity in pure glue

shear viscosity

Christiansen, M. Haas, JMP, Strodthoff, in prep.



Kubo relation

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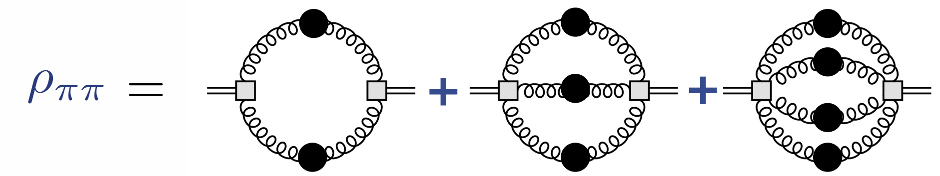
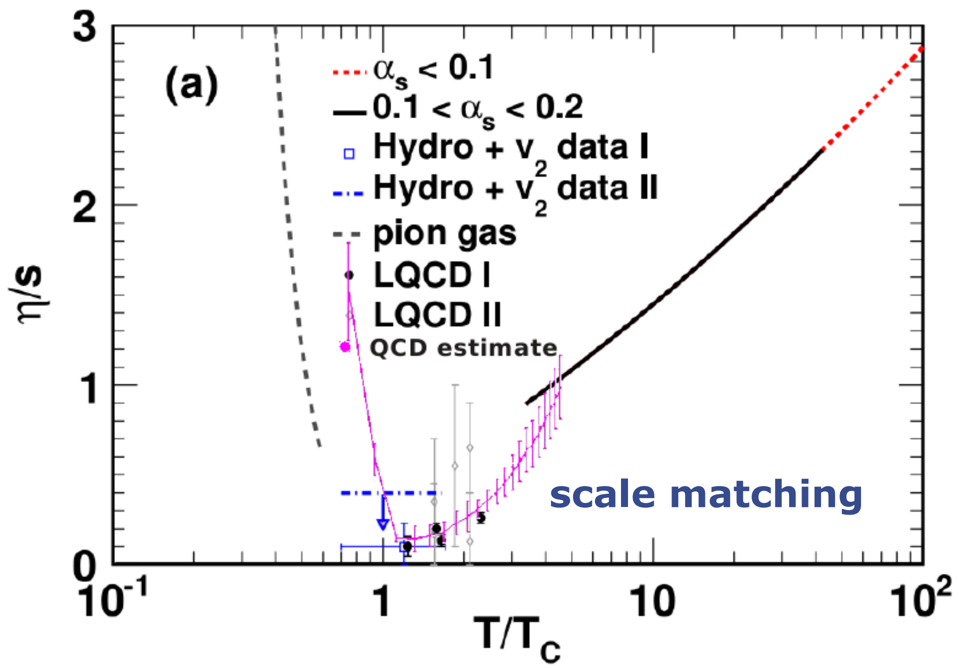
Diagrammatic representation



+ ··· closed form



Chen, Deng, Dong, Wang '11

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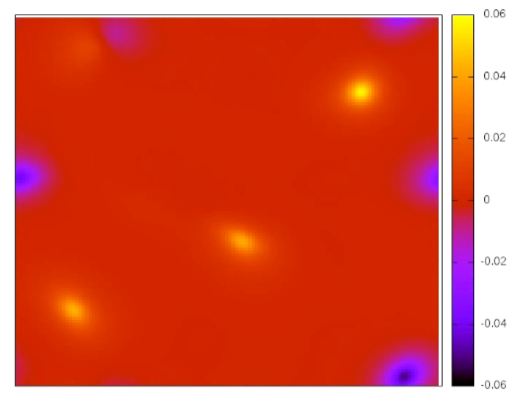
Summary & Outlook

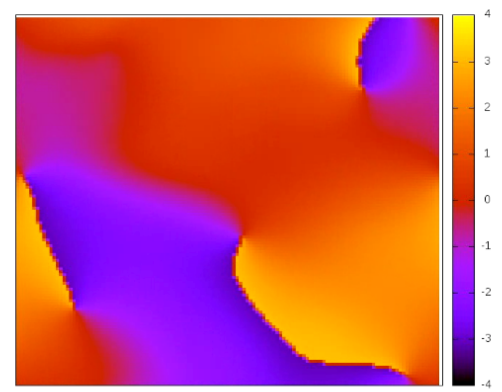
Summary & outlook

▪ Gauge dynamics far from equilibrium

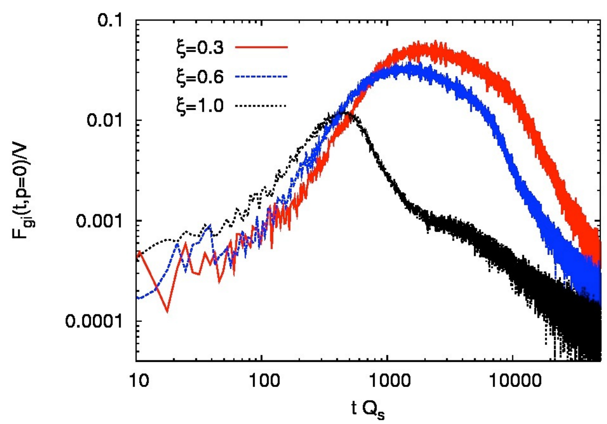
Abelian Higgs



magnetic field

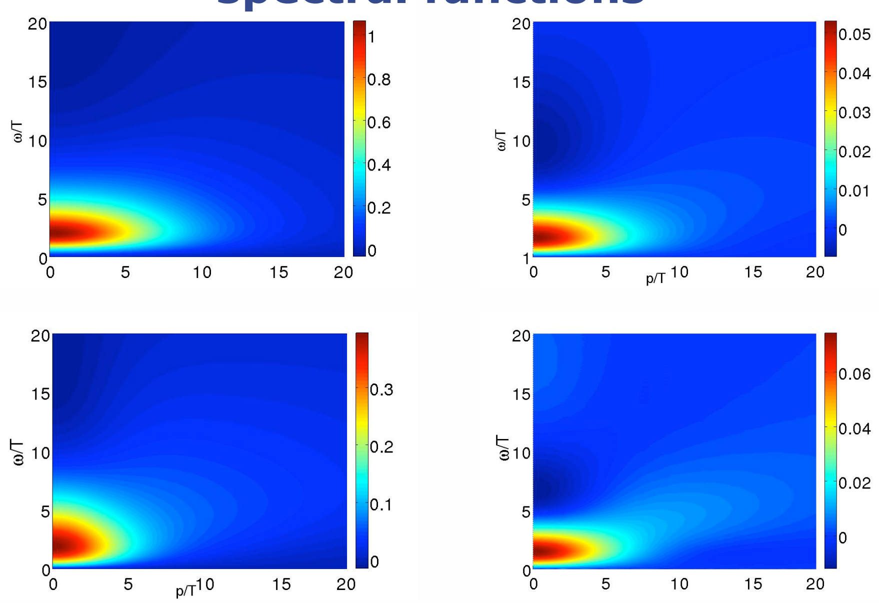


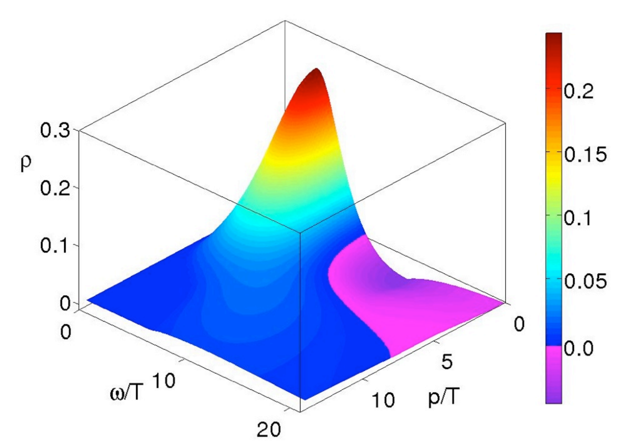
phase of Higgs



▪ Spectral functions and transport coefficients

spectral functions





viscosity over entropy ratio