Equation of state and phase structure of ultracold quantum gases in 2 & 3 dimensions

Jan M. Pawlowski

Universität Heidelberg & ExtreMe Matter Institute

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Boettcher, JMP, Wetterich, in preparation

Boettcher, JMP, Wetterich, arXiv:1312.0505 [cond-mat.quant-gas]

Boettcher, Diehl, JMP, Wetterich, Phys.Rev. A87, 023606 (2013)

Outline

- **BEC - BCS cross-over & the functional RG**

- **2d & 3d phase structure**

- **Summary and outlook**

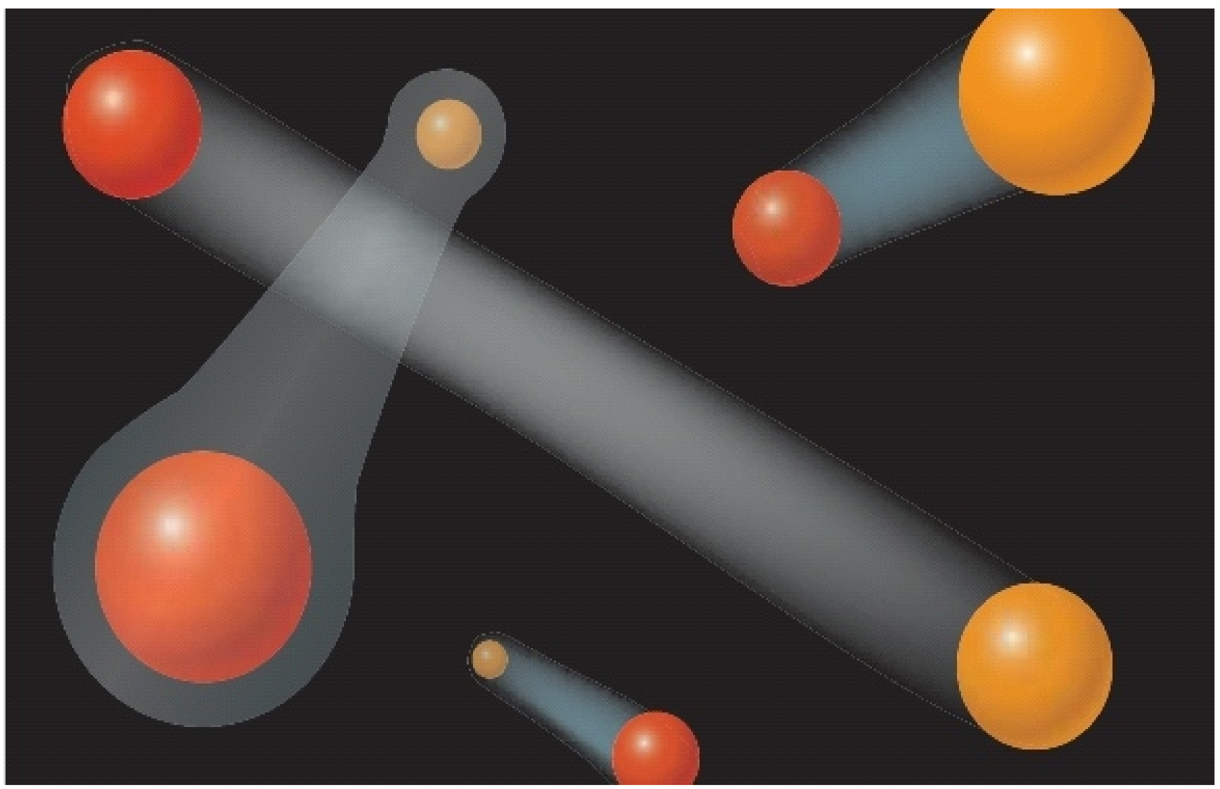
Phase diagram of cold quantum gases

BEC-BCS cross-over

Eagles '69, Leggett '80

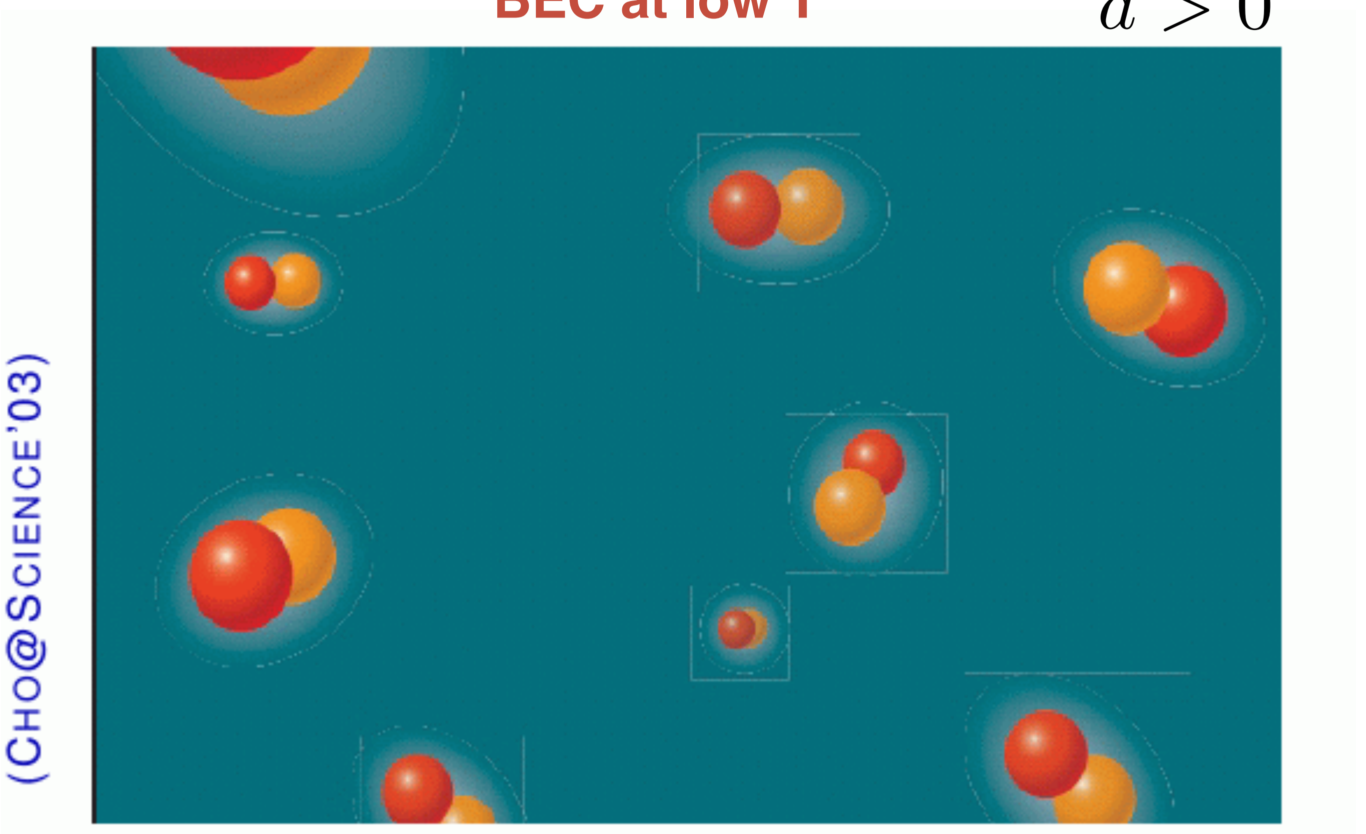
Fermions with attractive interactions

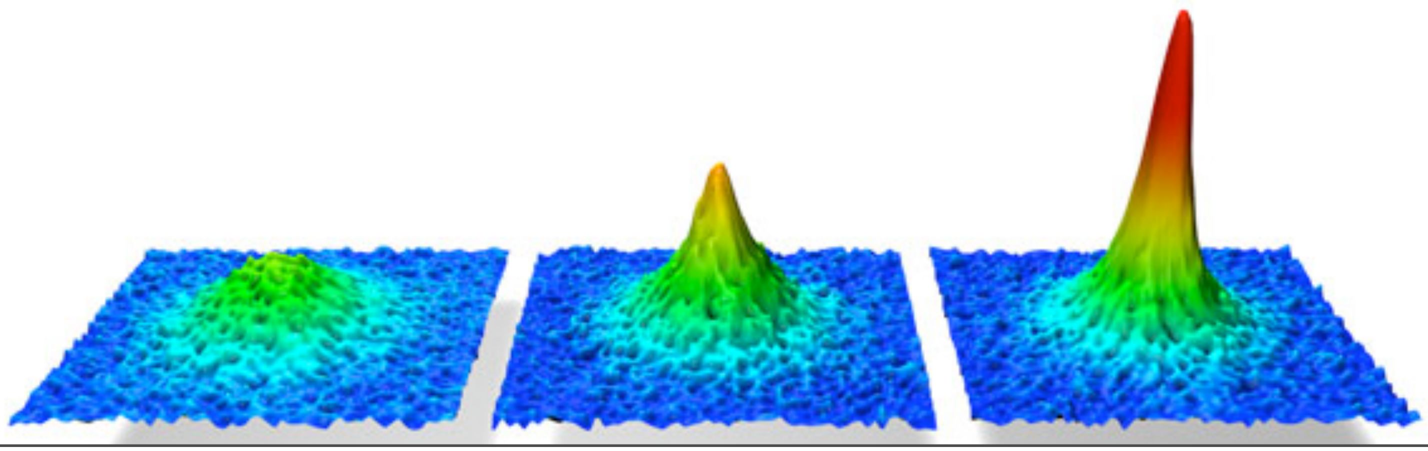
BCS superfluidity at low T $a < 0$



Bound molecules of two atoms on microscopic scale

BEC at low T $a > 0$

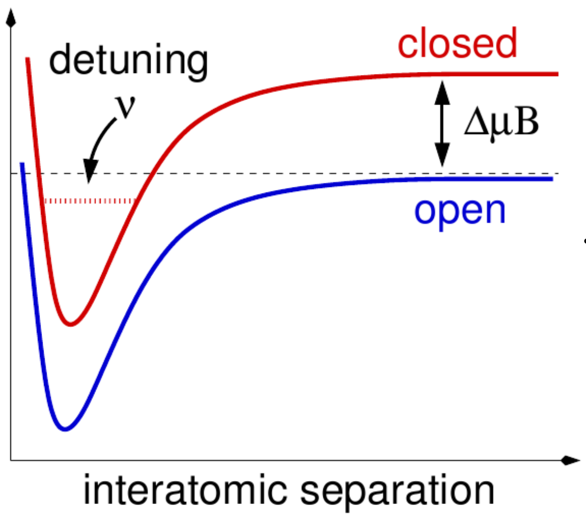


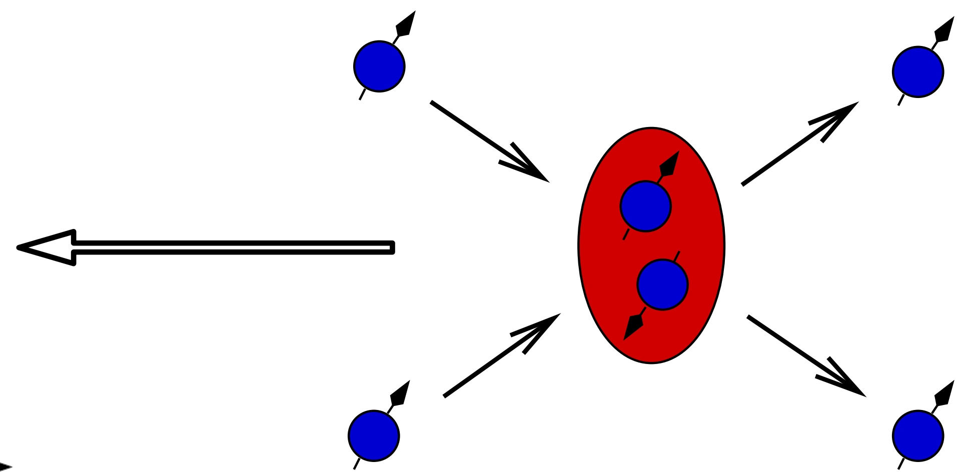


Regal et al '04

BEC-BCS cross-over

2-atom scattering





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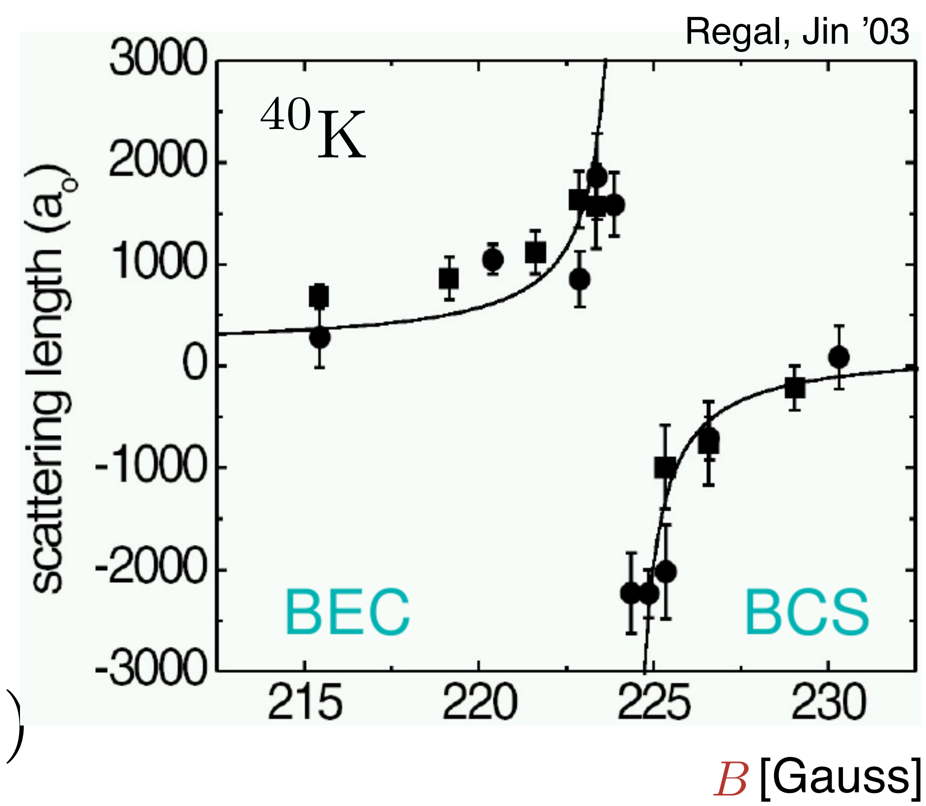
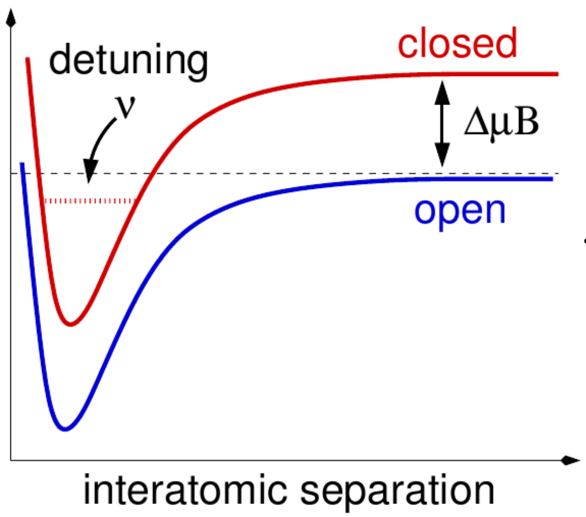


hyperfine interaction between interaction channels ⇨ **s-wave scattering length** $a(B)$

BEC-BCS cross-over



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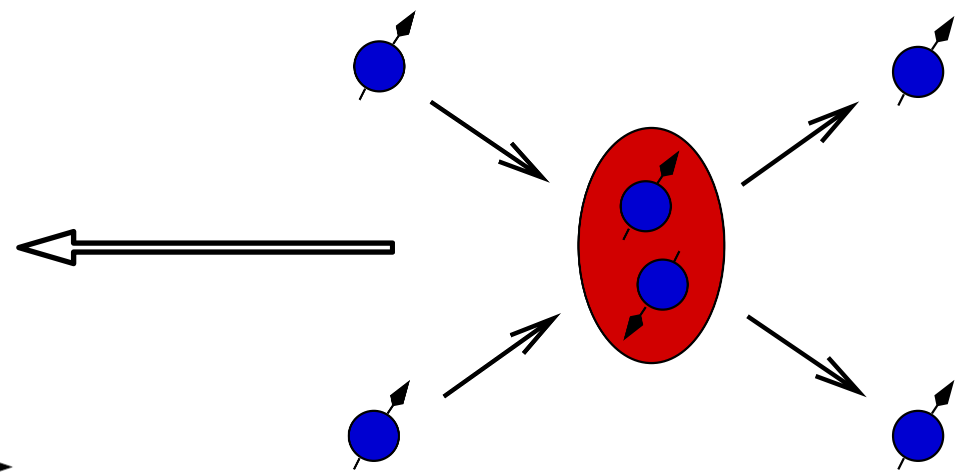

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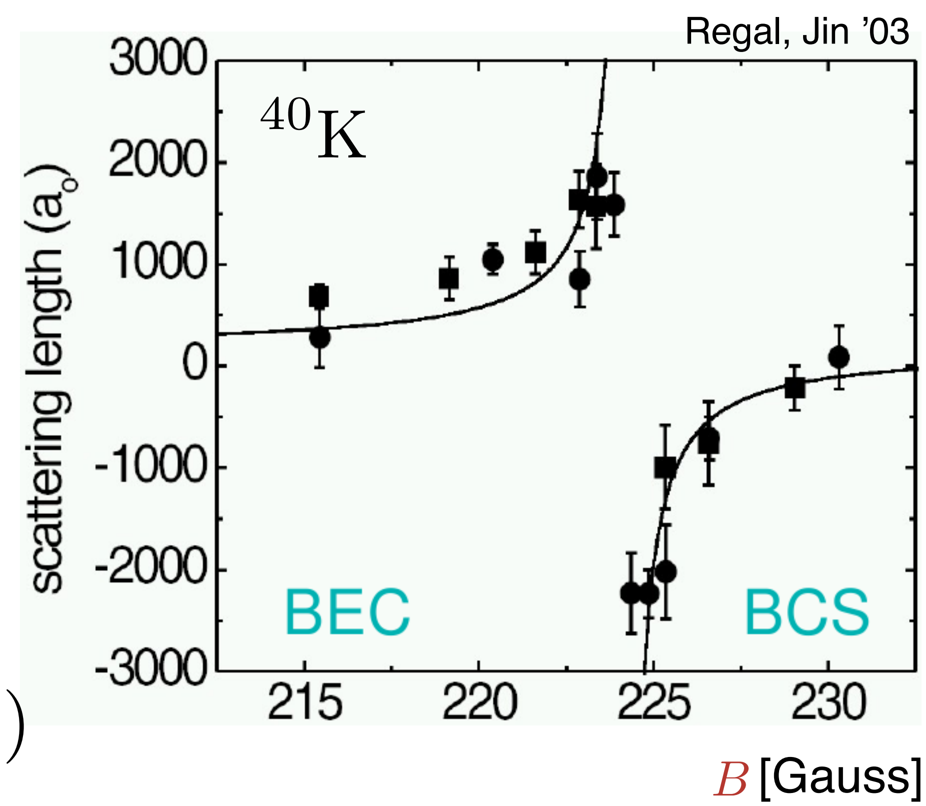
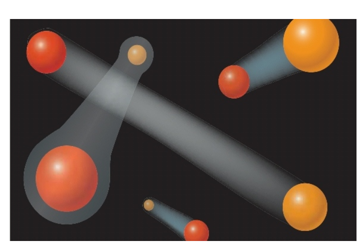


hyperfine interaction between interaction channels ⇨ **s-wave scattering length** $a(B)$

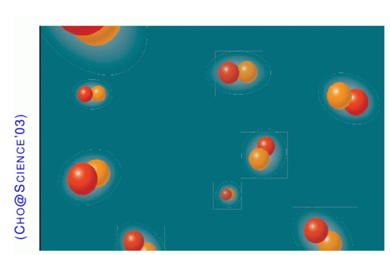
Strongly-correlated set-up

Relevant degrees of freedom

stable fermionic atom field ψ

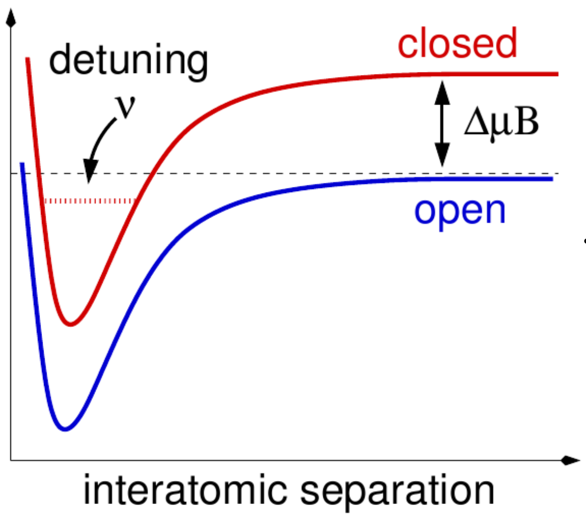


bosonic molecule field /Cooper pair ϕ



$$\hbar = k_B = 2M = 1$$

BEC-BCS cross-over



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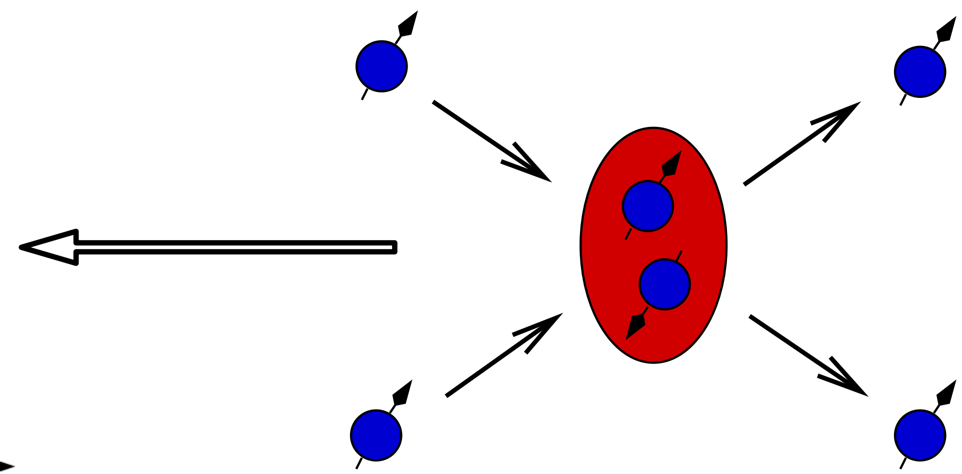

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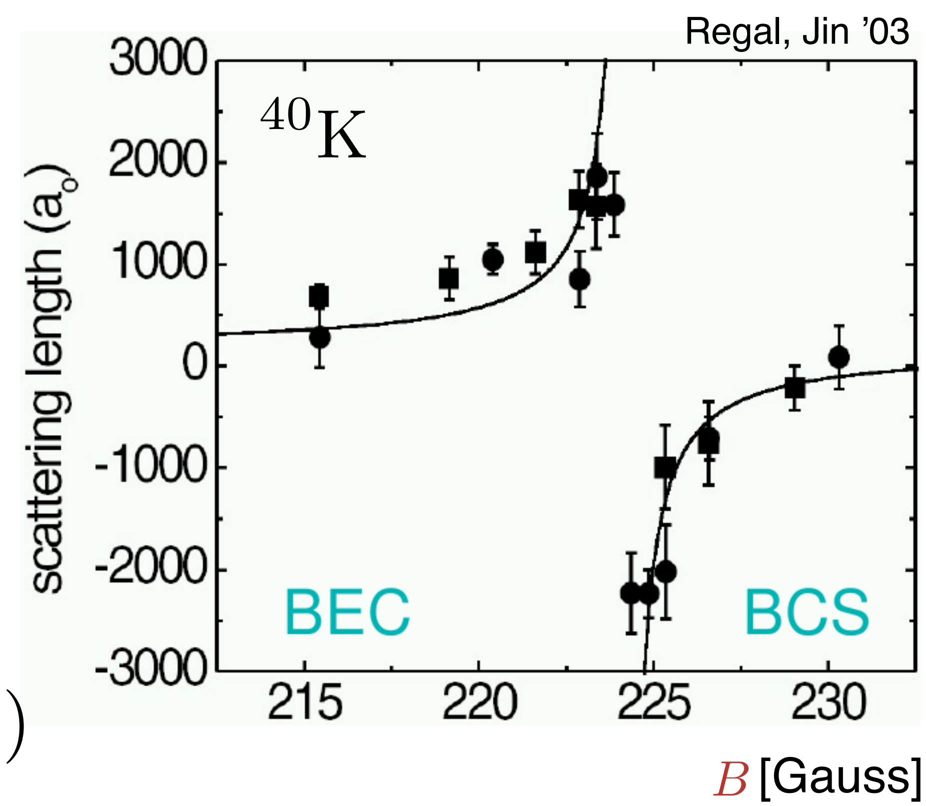

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B [Gauss]

Strongly-correlated set-up

Relevant degrees of freedom

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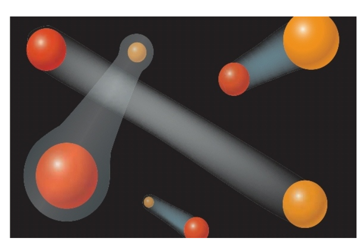

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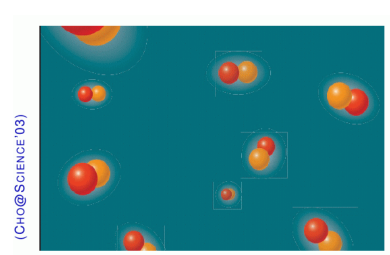


Effective action

$$\Gamma[\psi,\phi] = \int_{\tau,\vec{x}} \Big\{ \psi^\dagger \left(Z_\psi \partial_\tau - A_\psi \nabla^2 - \mu \right) \psi$$
$$+ \phi^* \left(Z_\phi \partial_\tau - A_\phi \frac{\nabla^2}{2} \right) \phi$$
$$+ \lambda_\psi (\psi^\dagger \psi)^2 + U(\phi)$$
$$- \frac{h_\phi}{2} (\phi^* \psi^{\,\mathrm{T}} \epsilon \psi - \phi \psi^\dagger \epsilon \psi^*) + \dots \Big\}$$

BEC-BCS cross-over

Effective action

$$\cdots + m_\phi^2 \phi^* \phi + \lambda_\psi (\psi^\dagger \psi)^2 - \frac{h_\phi}{2}(\phi^* \psi^{\mathrm{T}} \epsilon \psi - \phi \psi^\dagger \epsilon \psi^*) + \ldots$$

⇑ **relevant terms**

$$\lambda_\psi (\psi^\dagger \psi)^2 = \frac{1}{2} \lambda_\psi (\psi^\dagger \epsilon \psi^*)(\psi^{\mathrm{T}} \epsilon \psi)$$

Fierz transformation

BEC-BCS cross-over

Effective action

$$\cdots + m_\phi^2 \phi^* \phi + \lambda_\psi (\psi^\dagger \psi)^2 - \frac{h_\phi}{2}(\phi^* \psi^T \epsilon \psi - \phi \psi^\dagger \epsilon \psi^*) + \ldots$$

relevant terms

Relation to microphysics via Hubbard-Stratonovich

$$m_\phi^2 = \bar{\mu}(B - B_0) - 2\mu$$

chemical potential of molecule

$$\lambda_\psi = \frac{4\pi a_{bg}}{M}$$

$$\lambda_{\psi,\text{eff}} = \lambda_\psi - \frac{h^2}{m_\phi^2} \Longrightarrow$$

$$a(B) = \frac{M}{4\pi}\left(\lambda_\psi - \frac{h^2}{m_\phi^2}\right)$$

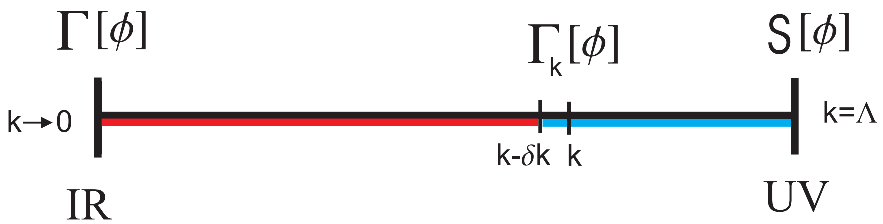
$$h_\phi^2 = \Delta B$$

Functional Methods for ultracold atoms

Functional RG

Boettcher, JMP, Diehl, Nucl.Phys.Proc.Suppl. 228 (2012) 63-135

free energy at momentum scale k



RG-scale k: $t = \ln k$

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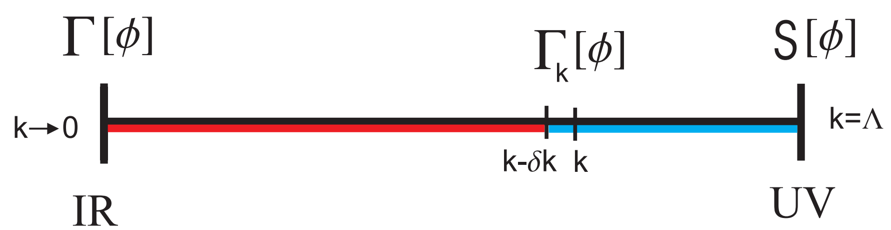
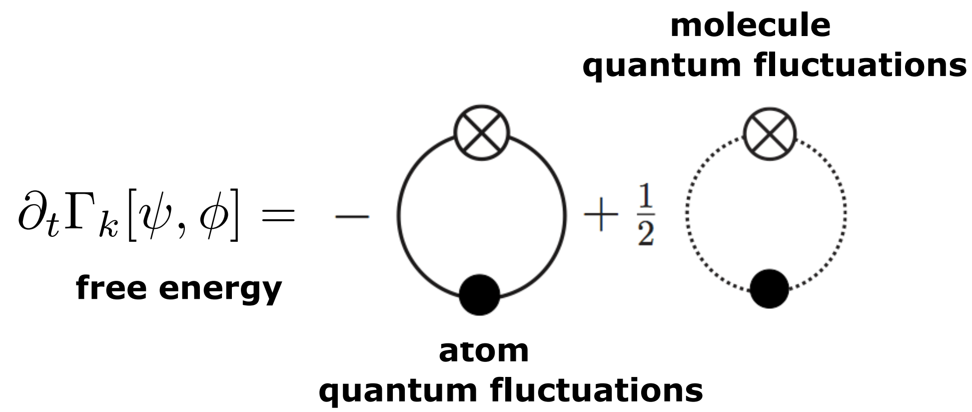


RG-scale k: $t = \ln k$



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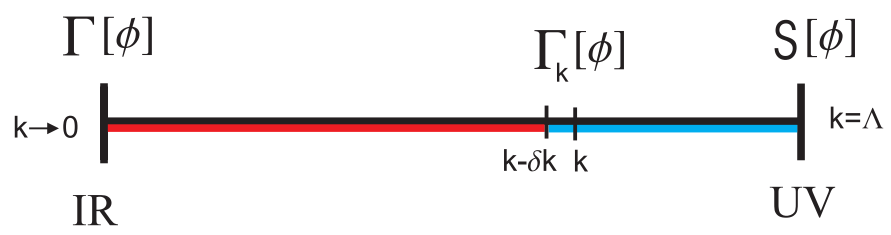
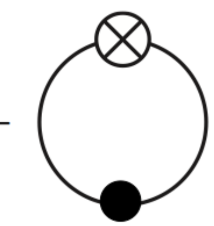
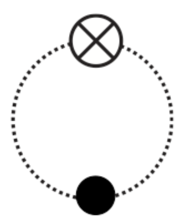


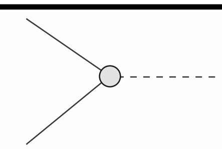
RG-scale k: $t = \ln k$

molecule
quantum fluctuations

$\partial_t \Gamma_k[\psi, \phi] = -$  $+ \frac{1}{2}$ 

free energy

atom
quantum fluctuations

Dynamical bosonisation  dynamical

Gies, Wetterich '01
JMP '05
Flörchinger, Wetterich '09

Functional Methods for ultracold atoms

Functional RG

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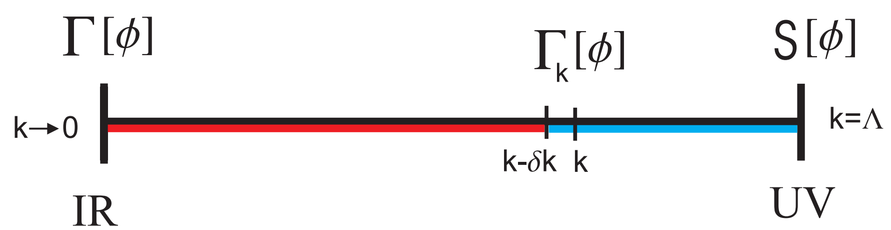
ultracold atoms

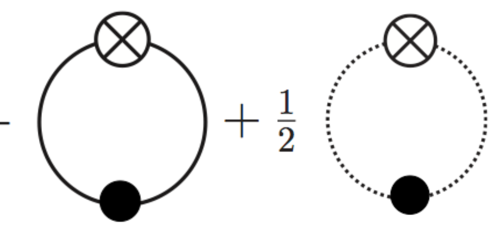
free energy at momentum scale k



RG-scale k: $t = \ln k$

molecule quantum fluctuations

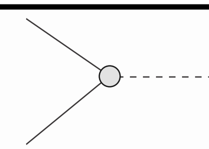
$$\partial_t \Gamma_k[\psi, \phi] = -\ \bigcirc + \frac{1}{2}\ \bigcirc$$



free energy

atom quantum fluctuations

Dynamical bosonisation



dynamical

Gies, Wetterich '01
JMP '05
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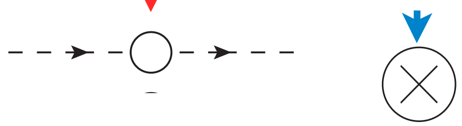
Bosons

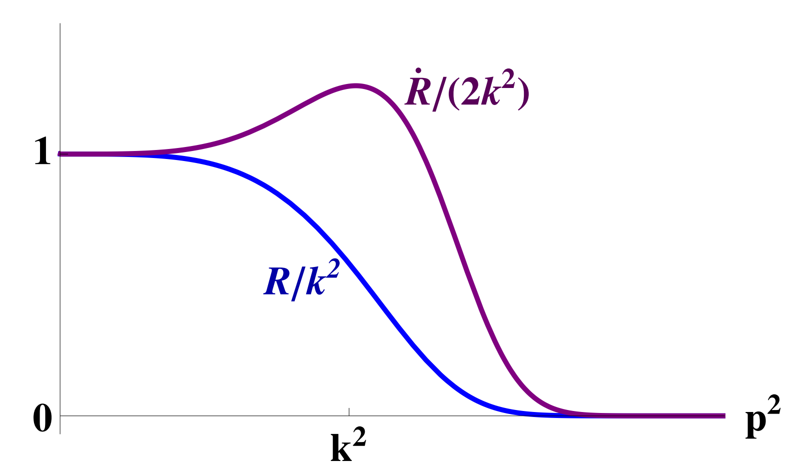
$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \left\{ \frac{1}{\Gamma^{(2)}[\phi] + R_k} \partial_t R_k \right\} - \partial_t C_k$$

$\partial_t = k\, \partial_k$

full propagator

regulator





BEC-BCS cross-over

Effective action

$$\Gamma[\psi,\phi] = \int_{\tau,\vec{x}} \Big\{ \psi^\dagger \left(Z_\psi \partial_\tau - A_\psi \nabla^2 - \mu \right) \psi + \phi^* \left(Z_\phi \partial_\tau - A_\phi \frac{\nabla^2}{2} \right) \phi \;\; + \lambda_\psi (\psi^\dagger \psi)^2 + U(\phi) - \frac{h_\phi}{2} (\phi^* \psi^{\,\mathrm{T}} \epsilon \psi - \phi \psi^\dagger \epsilon \psi^*) \Big\} \; + \; \dots$$

Approximation

Fermion propagator

$$A_{\psi,k}(\omega_0, \vec{q}^2) \qquad Z_{\psi,k}(\omega_0, \vec{q}^2)$$

Molecule propagator

$$A_{\phi,k} + B_{\phi,k} \frac{p_0^2}{\vec{p}^2} \qquad Z_{\phi,k}$$

Molecule effective potential

$$U_k(\phi)$$

Molecule-atom coupling

$$h_{\phi,k}$$

BEC-BCS cross-over

Effective action

$$\Gamma[\psi,\phi]=\int_{\tau,\vec{x}}\Big\{\psi^\dagger\left(Z_\psi\partial_\tau-A_\psi\nabla^2-\mu\right)\psi+\phi^*\left(Z_\phi\partial_\tau-A_\phi\frac{\nabla^2}{2}\right)\phi\;+\lambda_\psi(\psi^\dagger\psi)^2+U(\phi)-\frac{h_\phi}{2}(\phi^*\psi^{\mathrm{T}}\epsilon\psi-\phi\psi^\dagger\epsilon\psi^*)\Big\}+\dots$$

Approximation

Fermion propagator

$A_{\psi,k}(\omega_0,\vec{q}^2)$

$Z_{\psi,k}(\omega_0,\vec{q}^2)$

Molecule propagator

$A_{\phi,k}+B_{\phi,k}\frac{p_0^2}{\vec{p}^2}\qquad Z_{\phi,k}$

Molecule effective potential

$U_k(\phi)$

Molecule-atom coupling

$h_{\phi,k}$

Why?

BEC-BCS cross-over

Effective action

$$\Gamma[\psi,\phi] = \int_{\tau,\vec{x}} \Big\{ \psi^\dagger \left(Z_\psi \partial_\tau - A_\psi \nabla^2 - \mu \right) \psi + \phi^* \left(Z_\phi \partial_\tau - A_\phi \frac{\nabla^2}{2} \right) \phi \quad + \lambda_\psi (\psi^\dagger \psi)^2 + U(\phi) - \frac{h_\phi}{2} (\phi^* \psi^{\,\mathrm{T}} \epsilon \psi - \phi \psi^\dagger \epsilon \psi^*) \Big\} \textbf{ + ...}$$

Approximation

Fermion propagator

$$A_{\psi,k}(\omega_0, \vec{q}^2) \qquad Z_{\psi,k}(\omega_0, \vec{q}^2)$$

Molecule propagator

$$A_{\phi,k} + B_{\phi,k} \frac{p_0^2}{\vec{p}^2} \qquad Z_{\phi,k}$$

Molecule effective potential

$$U_k(\phi)$$

Molecule-atom coupling

$$h_{\phi,k}$$

Why? density!

$$n(\mu, T) = 2 \int_{\vec{p}} \left(\frac{1}{2} - \int_{p_0} G_{\psi^* \psi}(P) \right)$$

BEC-BCS cross-over

Effective action

$$\Gamma[\psi,\phi] = \int_{\tau,\vec{x}} \Big\{ \psi^\dagger \left(Z_\psi \partial_\tau - A_\psi \nabla^2 - \mu \right) \psi + \phi^* \left(Z_\phi \partial_\tau - A_\phi \frac{\nabla^2}{2} \right) \phi \;\; + \lambda_\psi (\psi^\dagger \psi)^2 + U(\phi) - \frac{h_\phi}{2} (\phi^* \psi^{\,\mathrm{T}} \epsilon \psi - \phi \psi^\dagger \epsilon \psi^*) \Big\} + \ldots$$

Approximation

Fermion propagator

$A_{\psi,k}(\omega_0, \vec{q}^2)$

$Z_{\psi,k}(\omega_0, \vec{q}^2)$

Molecule propagator

$A_{\phi,k} + B_{\phi,k} \dfrac{p_0^2}{\vec{p}^{\,2}} \qquad Z_{\phi,k}$

Molecule effective potential

$U_k(\phi)$

Molecule-atom coupling

$h_{\phi,k}$

Why? density!

$$n(\mu,T) = 2 \int_{\vec{p}} \left(\frac{1}{2} - \int_{p_0} \frac{[i\, p_0 + \vec{p}^2 - \mu + \Sigma_{\psi^*\psi}(P)]^*}{|i\, p_0 + \vec{p}^2 - \mu + \Sigma_{\psi^*\psi}(P)|^2 + |\Sigma_{\psi^T\psi}|^2} \right)$$

BEC-BCS cross-over

Effective action

$$\Gamma[\psi,\phi]=\int_{\tau,\vec{x}}\Big\{\psi^\dagger\left(Z_\psi\partial_\tau-A_\psi\nabla^2-\mu\right)\psi+\phi^*\left(Z_\phi\partial_\tau-A_\phi\frac{\nabla^2}{2}\right)\phi\quad+\lambda_\psi(\psi^\dagger\psi)^2+U(\phi)-\frac{h_\phi}{2}(\phi^*\psi^{\,\mathrm{T}}\epsilon\psi-\phi\psi^\dagger\epsilon\psi^*)\Big\}$$ **+ ...**

Approximation

Fermion propagator	Molecule propagator	Molecule effective potential	Molecule-atom coupling
$A_{\psi,k}(\omega_0,\vec{q}^2)$ $Z_{\psi,k}(\omega_0,\vec{q}^2)$	$A_{\phi,k}+B_{\phi,k}\frac{p_0^2}{\vec{p}^2}$ $\quad Z_{\phi,k}$	$U_k(\phi)$	$h_{\phi,k}$

Why? **density!**

$$n(\mu,T)=2\int_{\vec{p}}\left(\frac{1}{2}-\int_{p_0}\frac{[i\,p_0+\vec{p}^2-\mu+\Sigma_{\psi^*\psi}(P)]^*}{|i\,p_0+\vec{p}^2-\mu+\Sigma_{\psi^*\psi}(P)|^2+|\Sigma_{\psi^T\psi}|^2}\right)$$

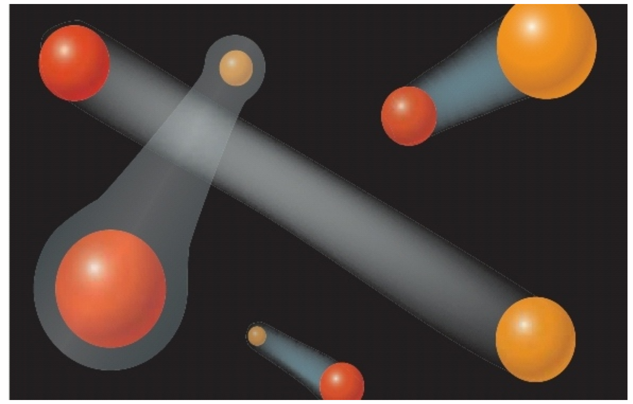
Contact

$$\Sigma_{\psi^*\psi}(P)]\simeq\frac{4C}{-ip_0+\vec{p}^{\,2}-\mu}-\delta\mu$$

for $\vec{p}^{\,2}\to\infty$

Bound molecules of two atoms on microscopic scale

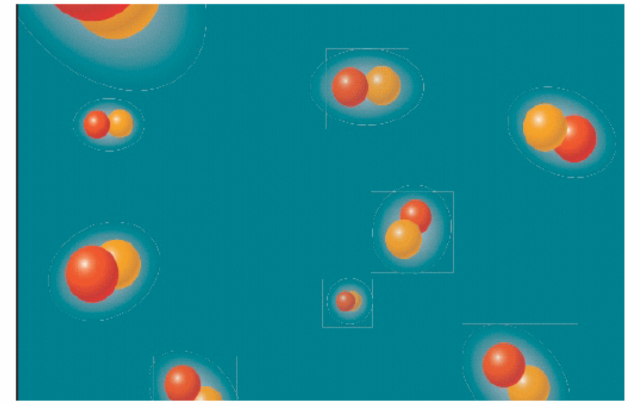
BEC at low T $a > 0$

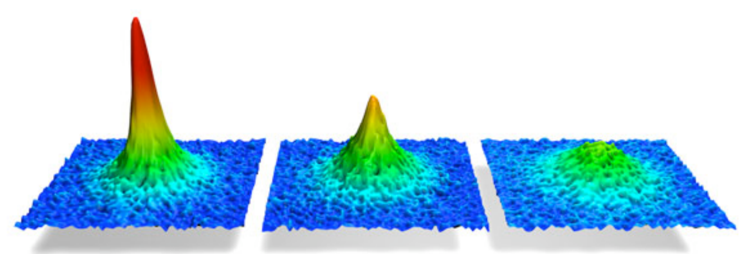


Fermions with attractive interactions

BCS superfluidity at low T $a < 0$

(Cho@Science'03)



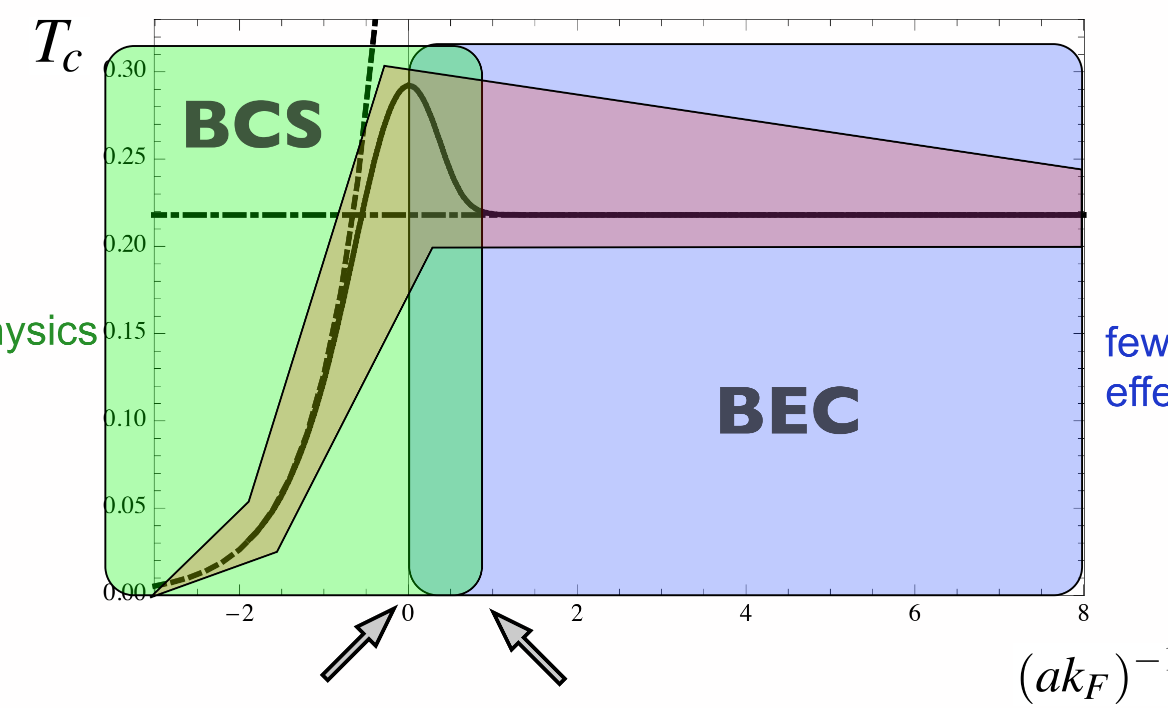


Regal et al '04

Phase diagram of cold quantum gases

critical behavior: long distance scales

Many-Body fermion physics



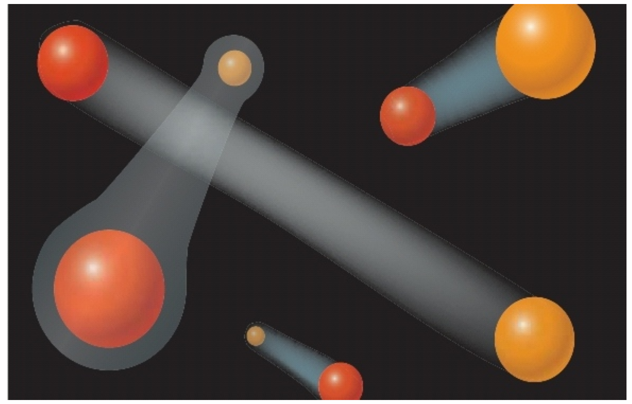
few-body physics of effective dimers

two-body bound state

zero crossing of fermion chemical potential

Bound molecules of two atoms on microscopic scale

BEC at low T $a > 0$



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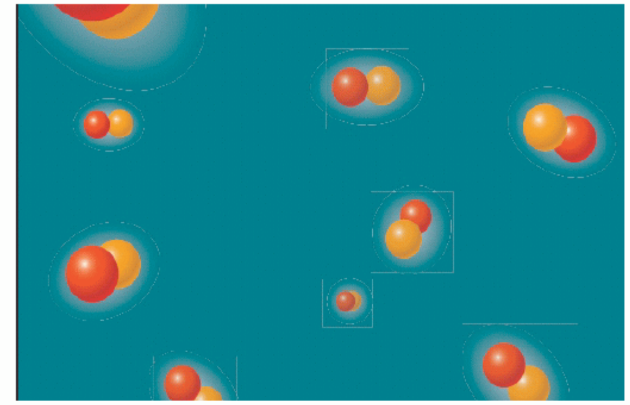

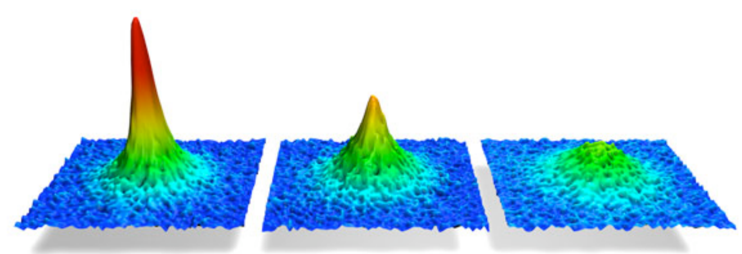

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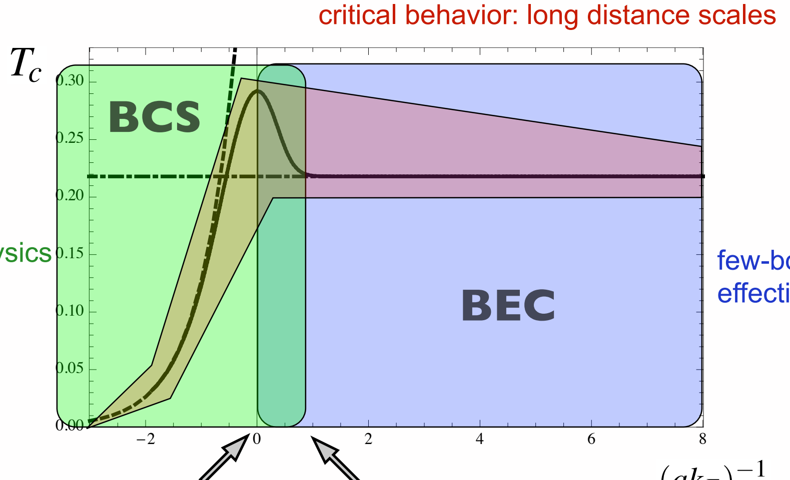

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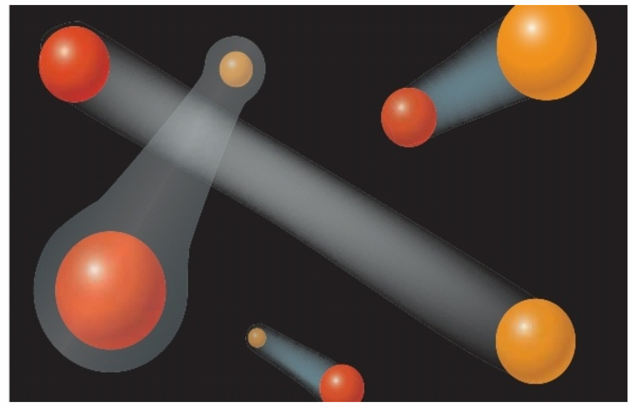

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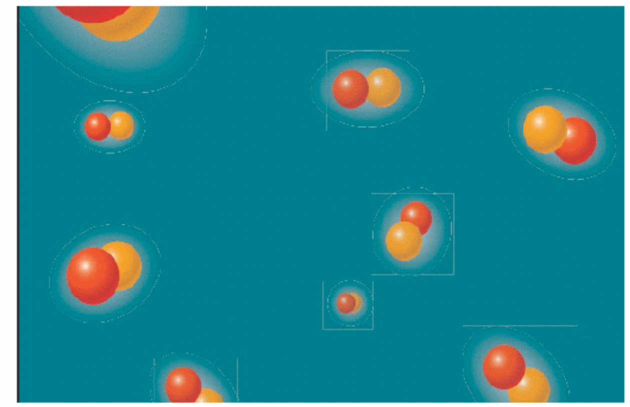

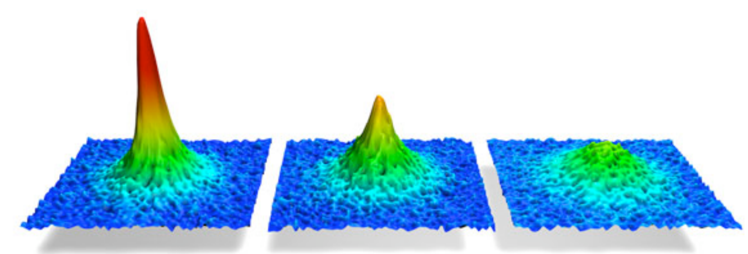

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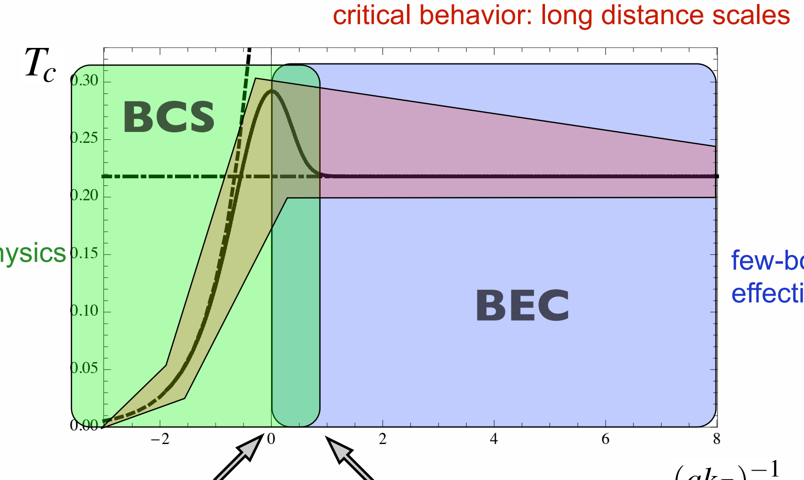
Phase diagram of QCD

critical behavior: long distance scales

Many-Body fermion physics

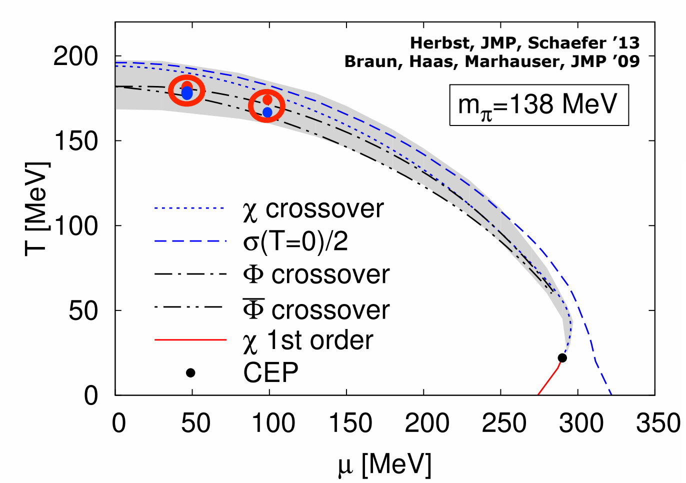


few-body physics of effective dimers

two-body bound state

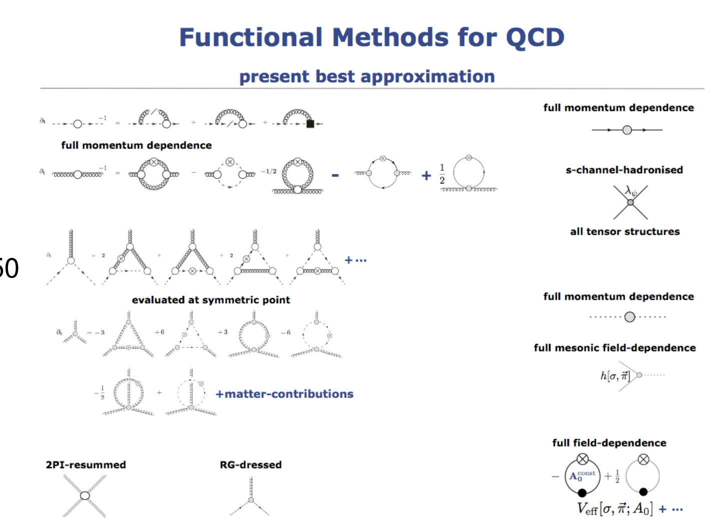
zero crossing of fermion chemical potential

FRG-UCG



FRG-QCD

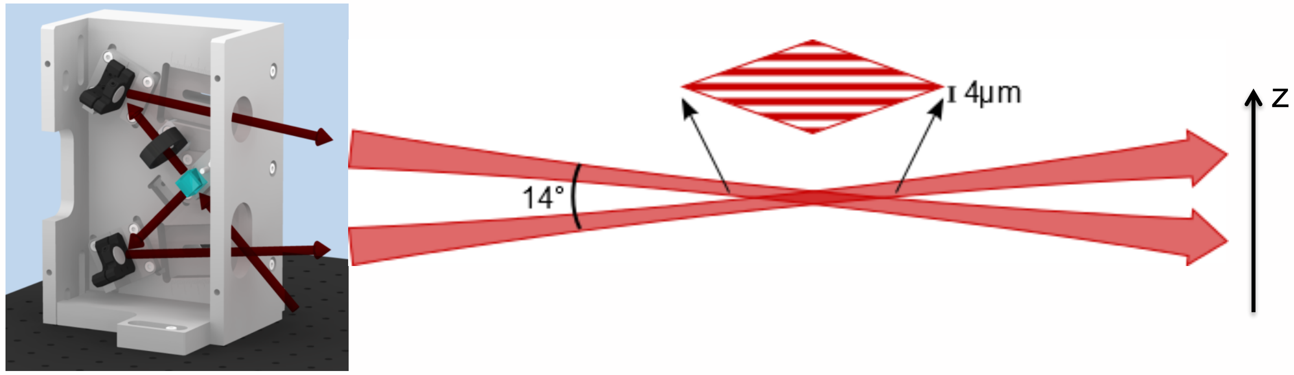
Fister, Herbst, Mitter, Rennecke, Strodthoff, JMP



ultracold quantum gases in 2 dimensions

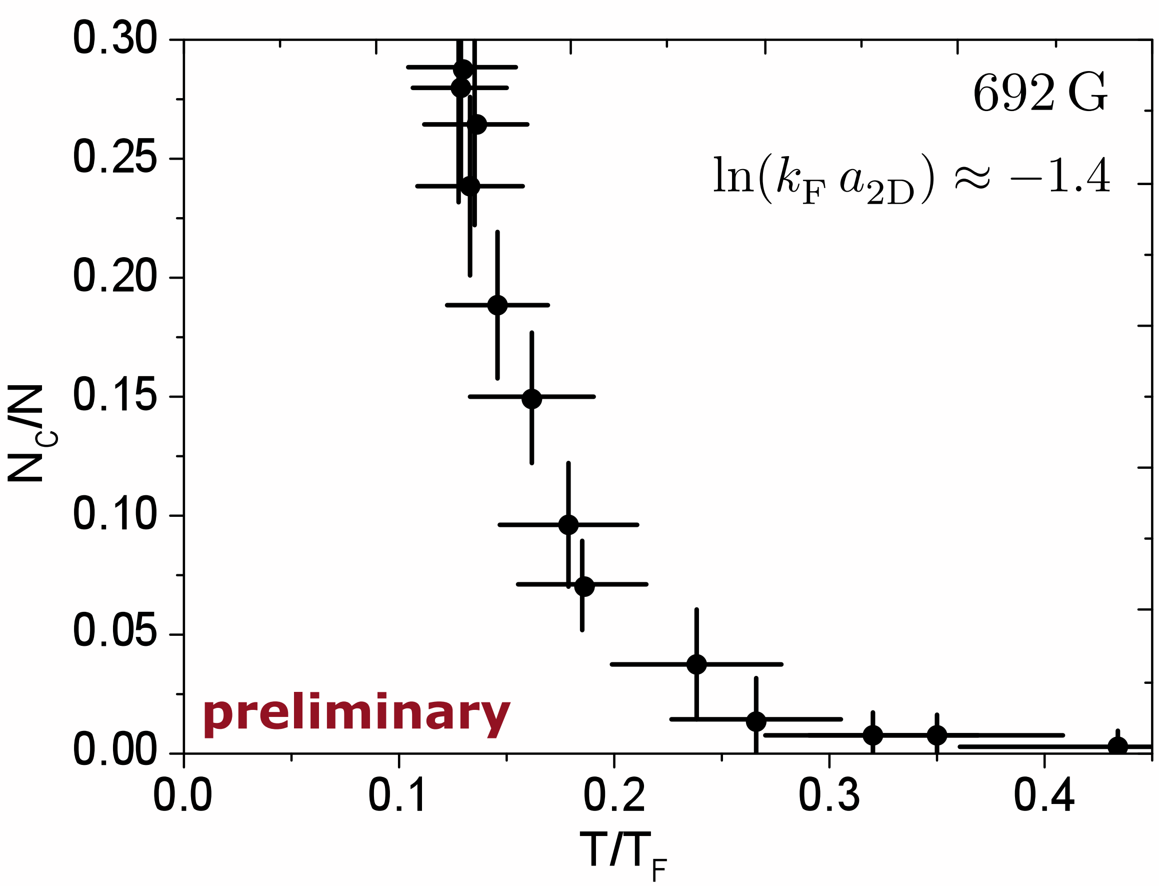
Experimental realisation

Jochim group in Heidelberg



ω_r = 2π x 19Hz
ω_z = 2π x 5900Hz

ω_r / ω_z ≈ 1:310 ⇨ ~ 50 000 radial states in transversal ground state



ultracold quantum gases in 2 dimensions

Experimental realisation

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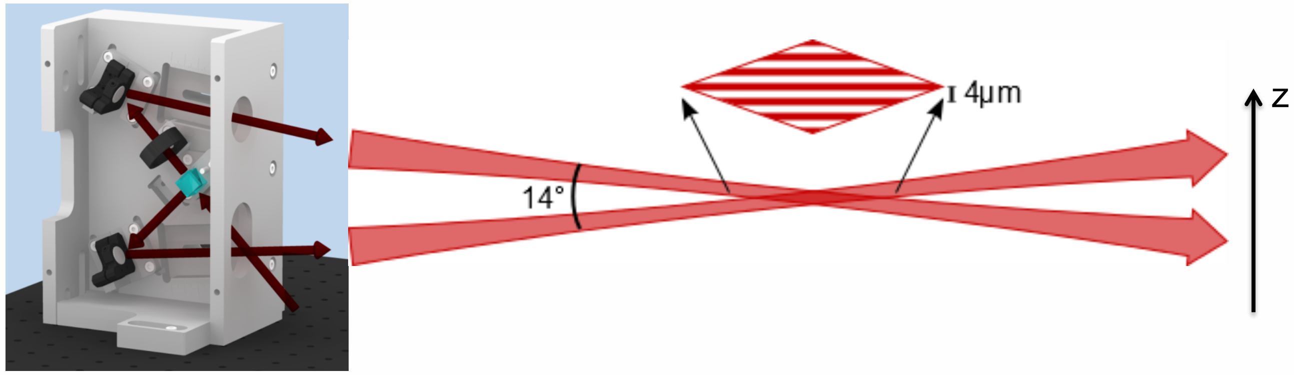
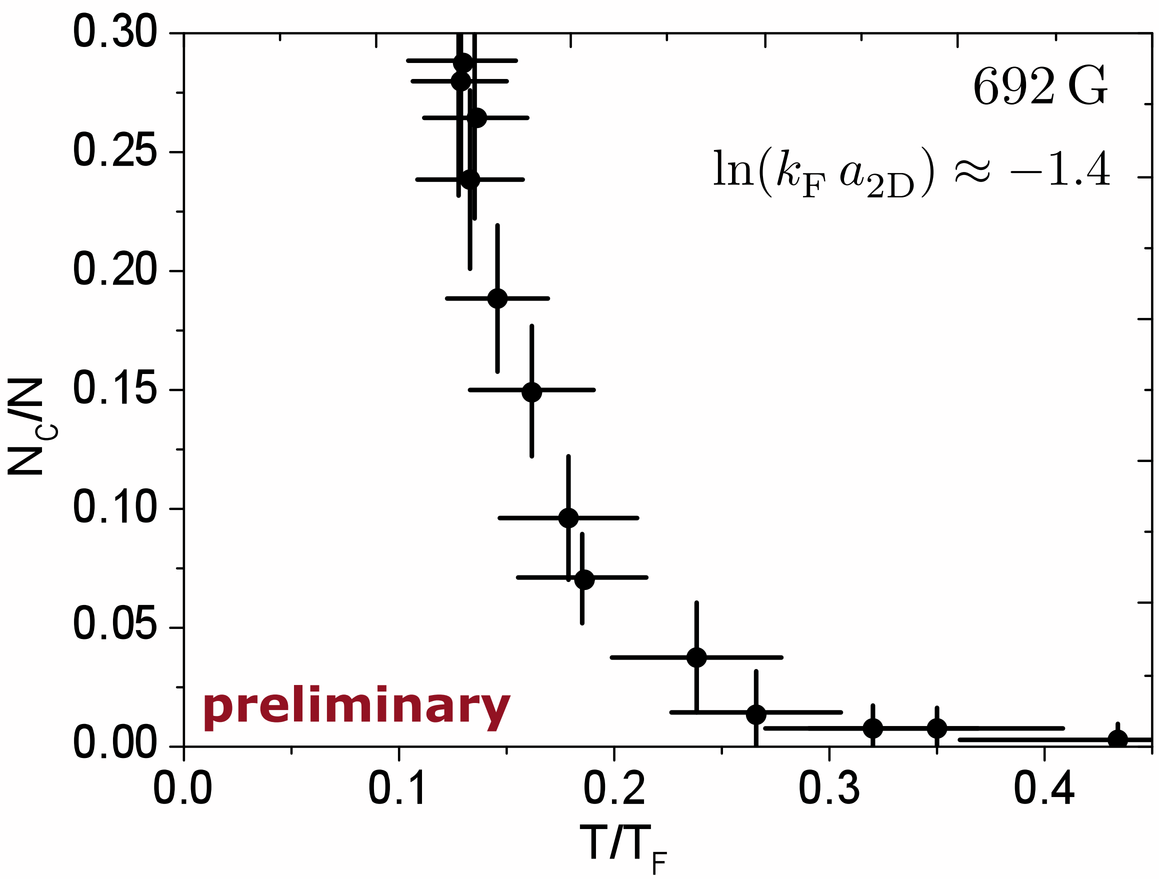


for details ask Thomas Lompe

$\omega_r = 2\pi$ x 19Hz
$\omega_z = 2\pi$ x 5900Hz

$\omega_r / \omega_z \approx$ 1:310 ⇨ ~ 50 000 radial states in transversal ground state

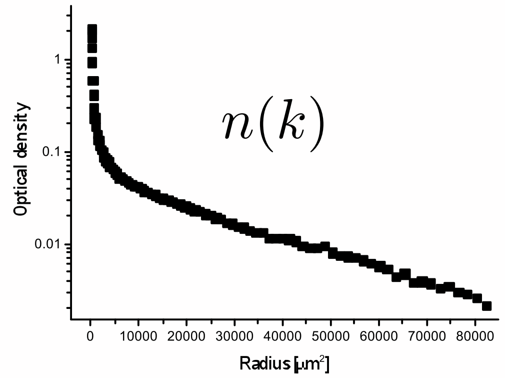


ultracold quantum gases in 2 dimensions

Measurements

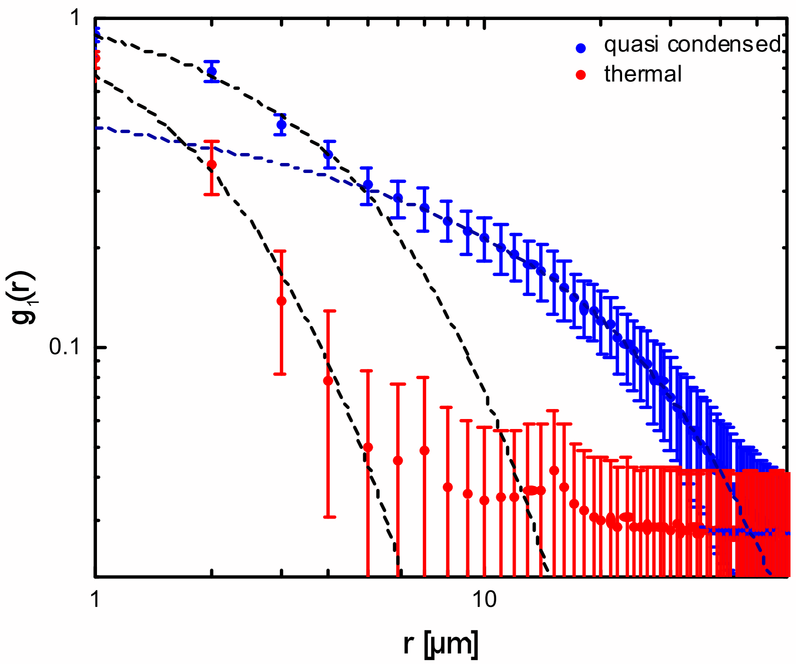
Jochim group in Heidelberg

$$\langle \hat{\psi}^\dagger(r_0)\hat{\psi}(r_0+r)\rangle \propto r^{-\eta}$$



Fourier transform ⟹

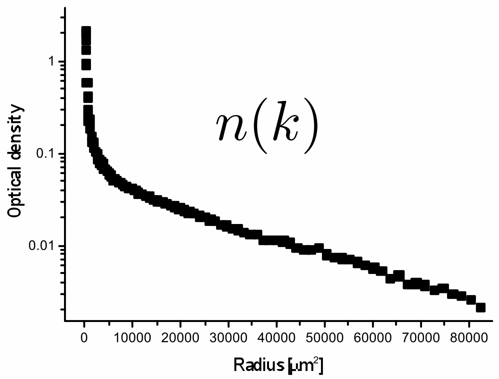
$$g_1(r) \sim \langle \hat{\psi}^\dagger(r_0)\hat{\psi}(r_0+r)\rangle$$



ultracold quantum gases in 2 dimensions

Measurements

Jochim group in Heidelberg



Fourier transform

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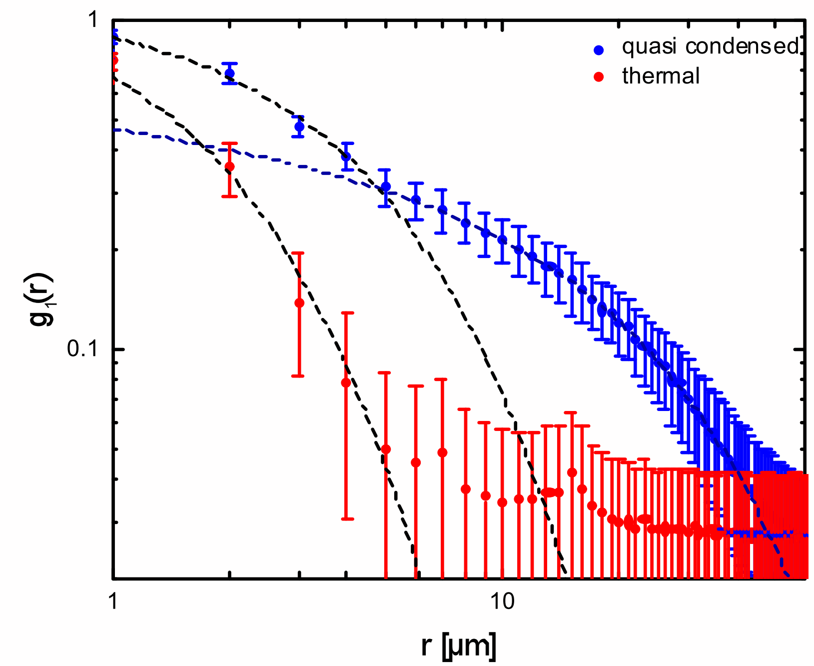
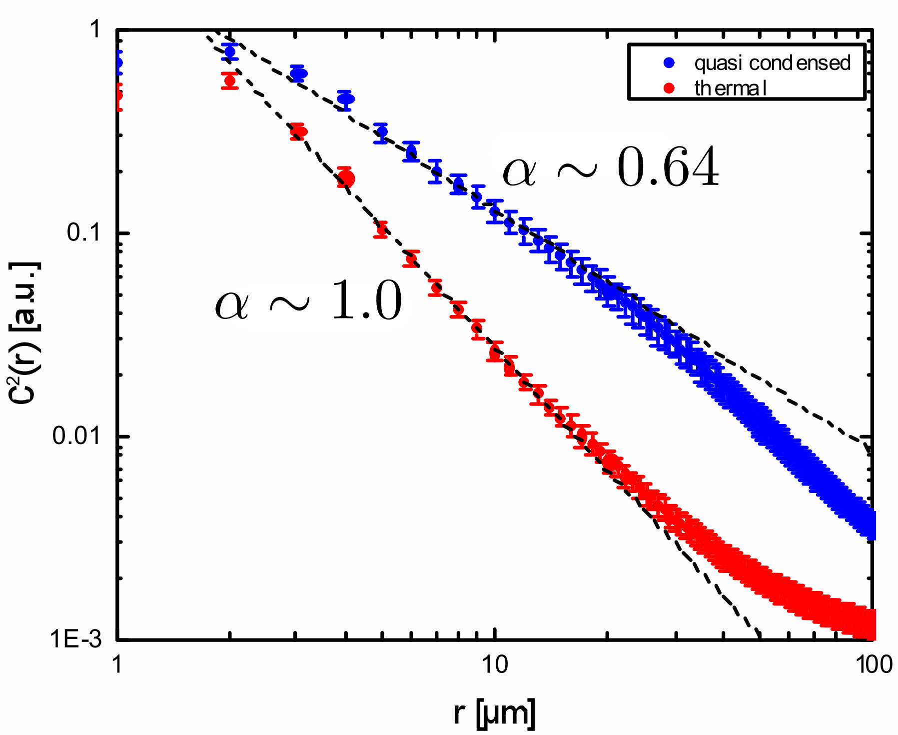


$$g_1(r) \sim r^{-\eta}$$

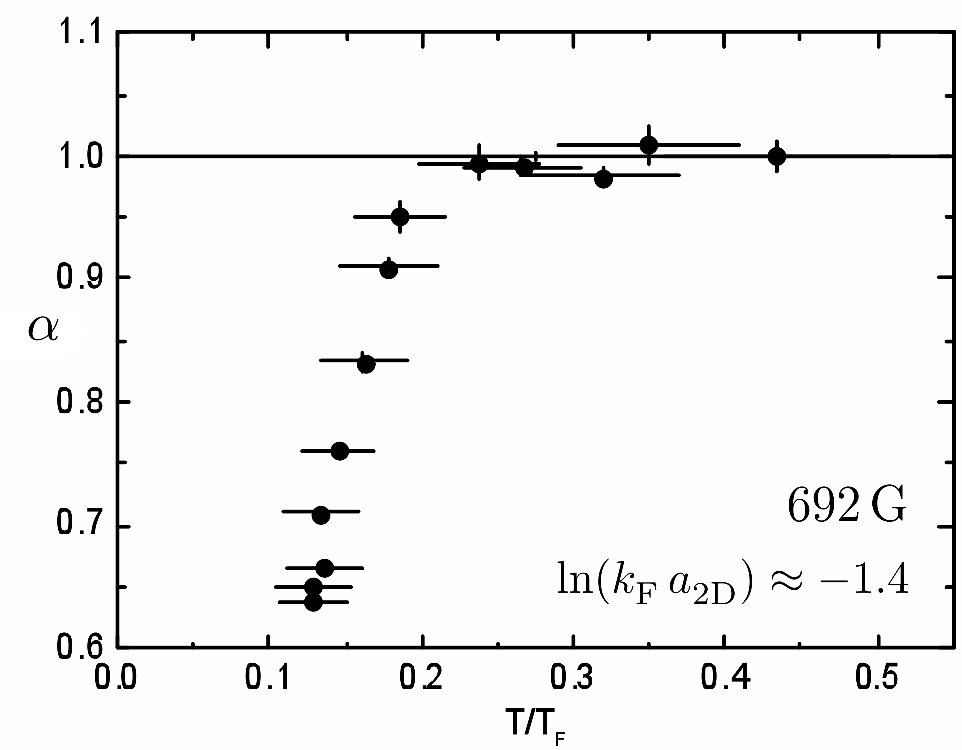
$$C^2(r) = \frac{1}{\pi r^2}\int_0^r d^2r' \, |g_1(r')|^2 \propto r^{-2\alpha}$$



thermal: $\alpha = 1$

hom. BEC: $\alpha = 0$

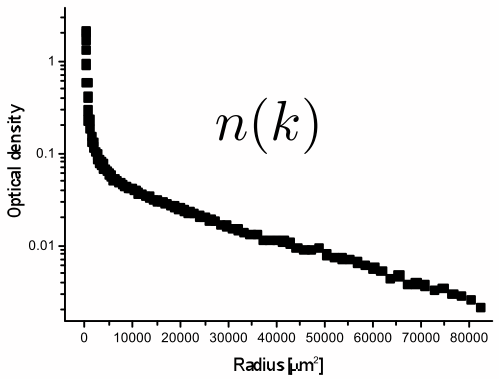
hom. BKT: $\alpha = 0.25$



ultracold quantum gases in 2 dimensions

Measurements

Jochim group in Heidelberg



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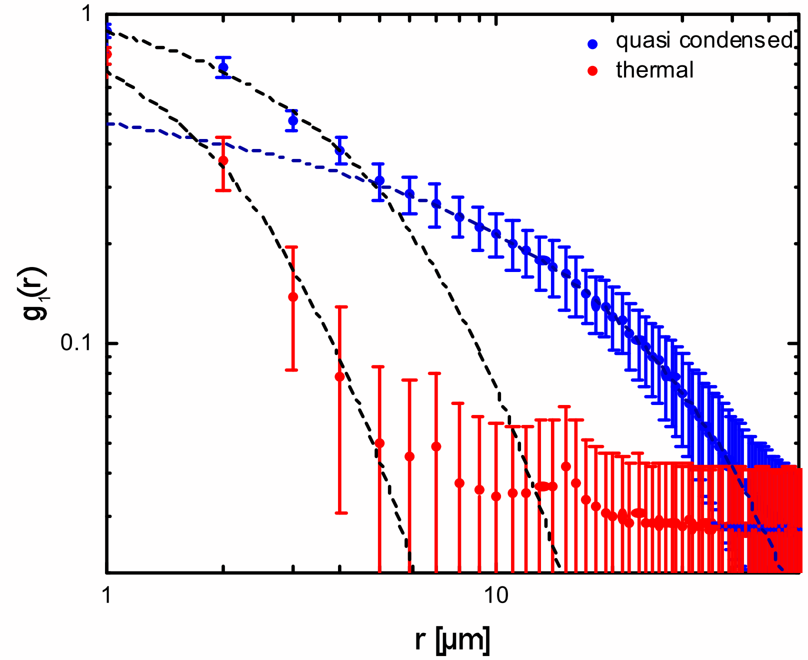

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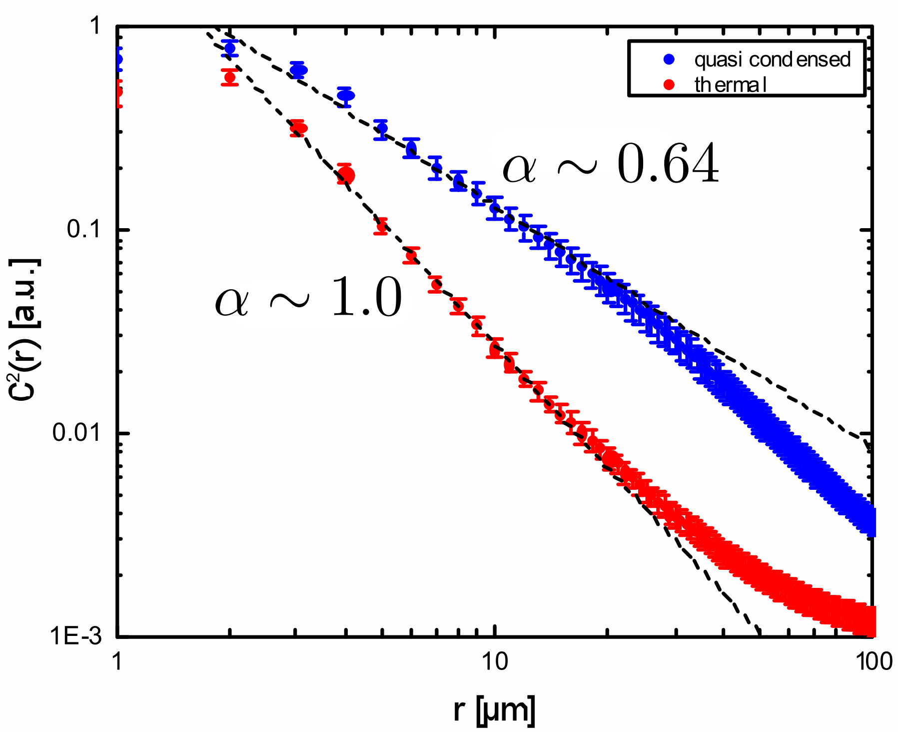
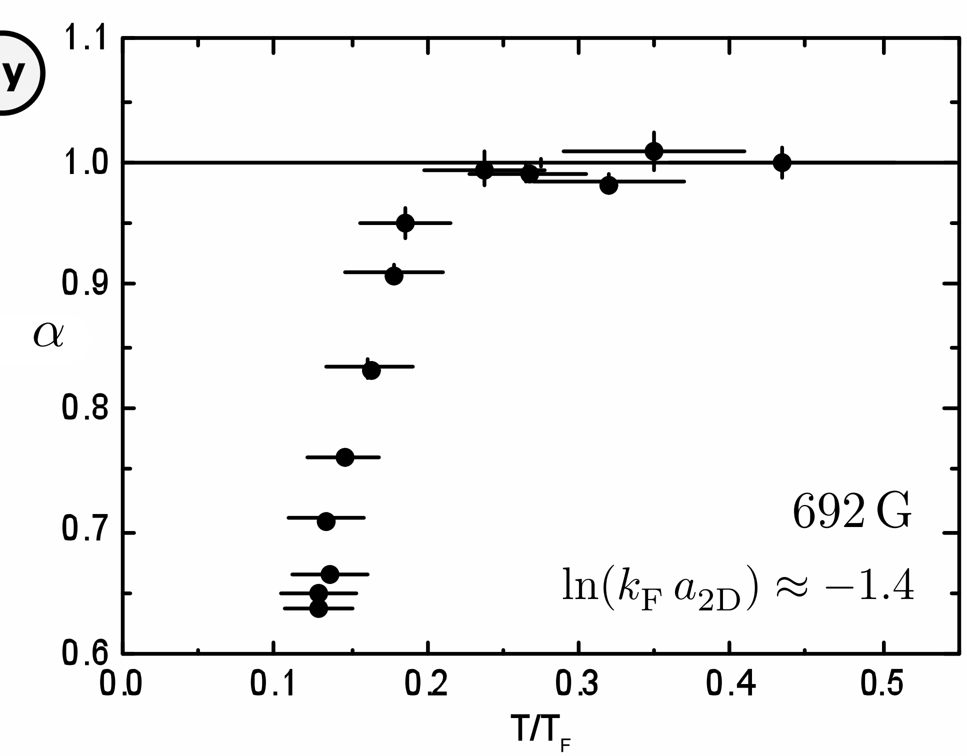


thermal: $\alpha = 1$ **exp. decay**

hom. BEC: $\alpha = 0$

hom. BKT: $\alpha = 0.25$

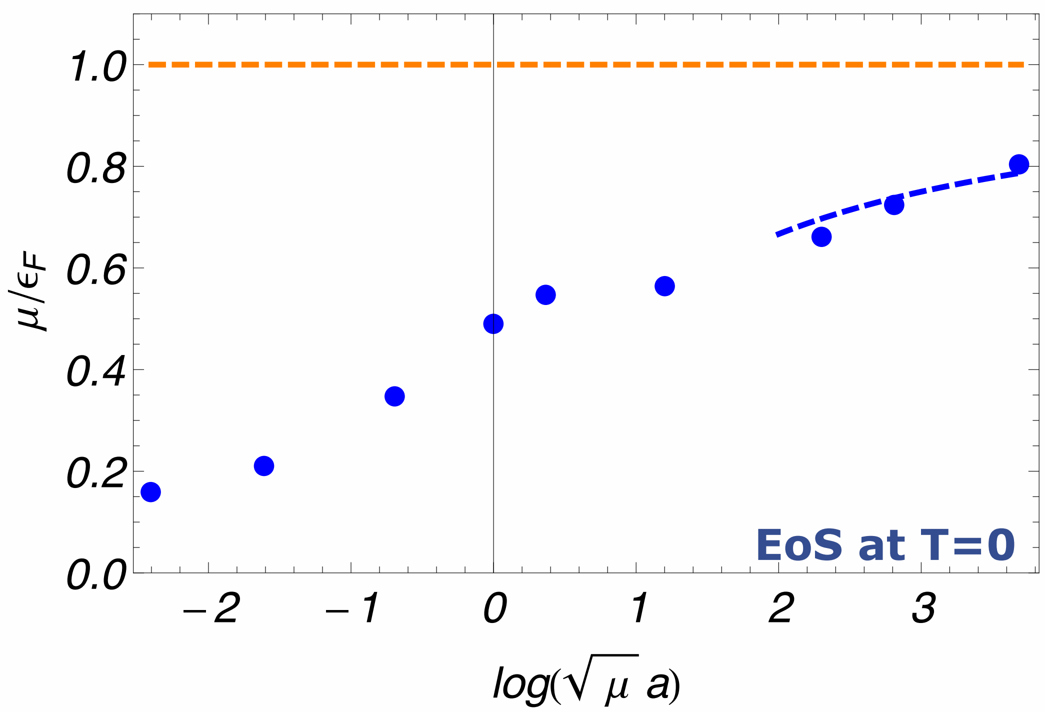
$\eta = \alpha$

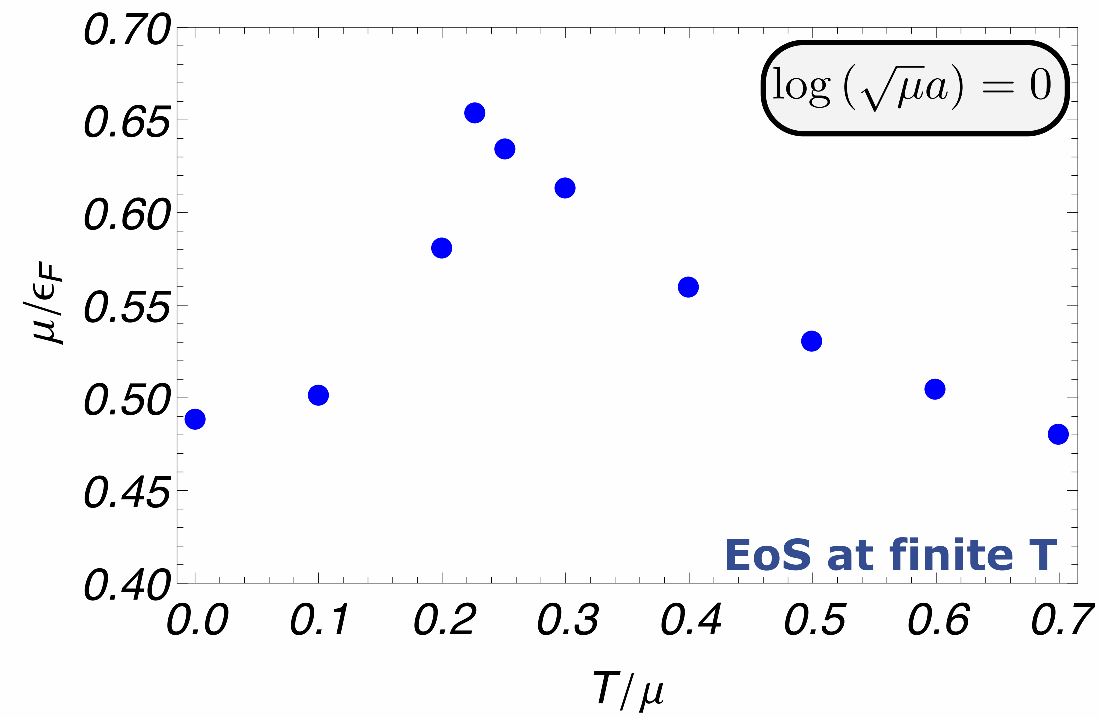


ultracold quantum gases in 2 dimensions

EoS & phase structure

Boettcher, JMP, Wetterich, in preparation





orange dashed line: mean field

blue dashed line: $$\frac{\mu}{\epsilon_F} = \frac{\log(k_F\, a)}{1 + \log(k_F\, a)}$$

$$\mu \to \mu_{\rm mb} = \mu - \frac{\epsilon_b}{2} = \mu + \frac{1}{a^2}$$

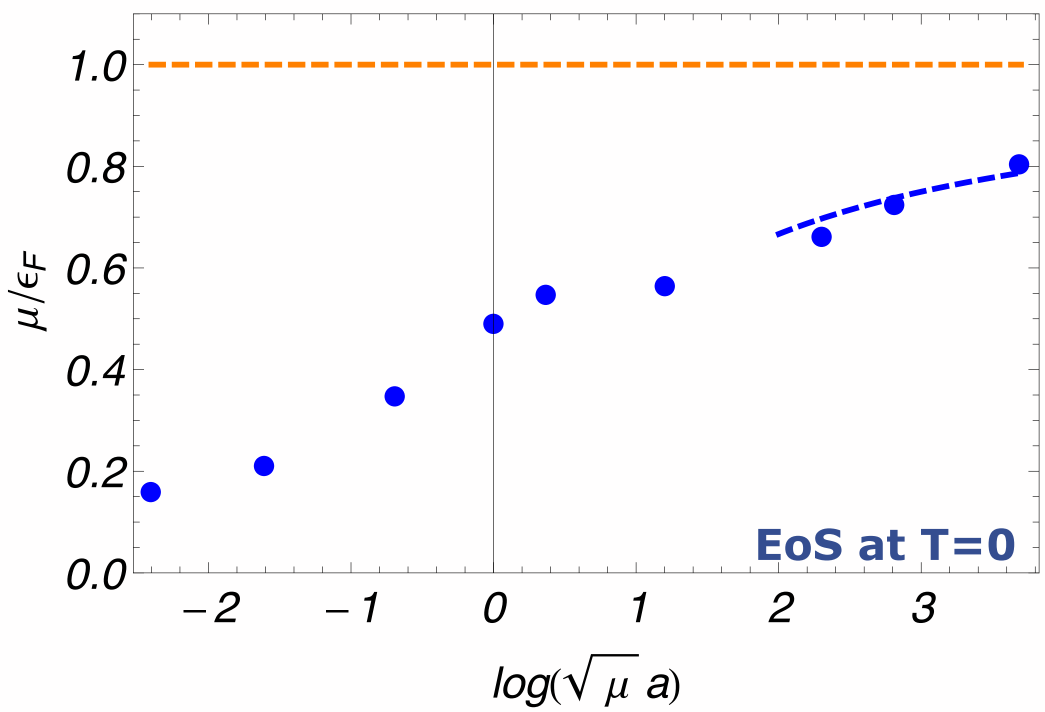
$$\epsilon_F = 2\pi\, n(\mu, T)$$

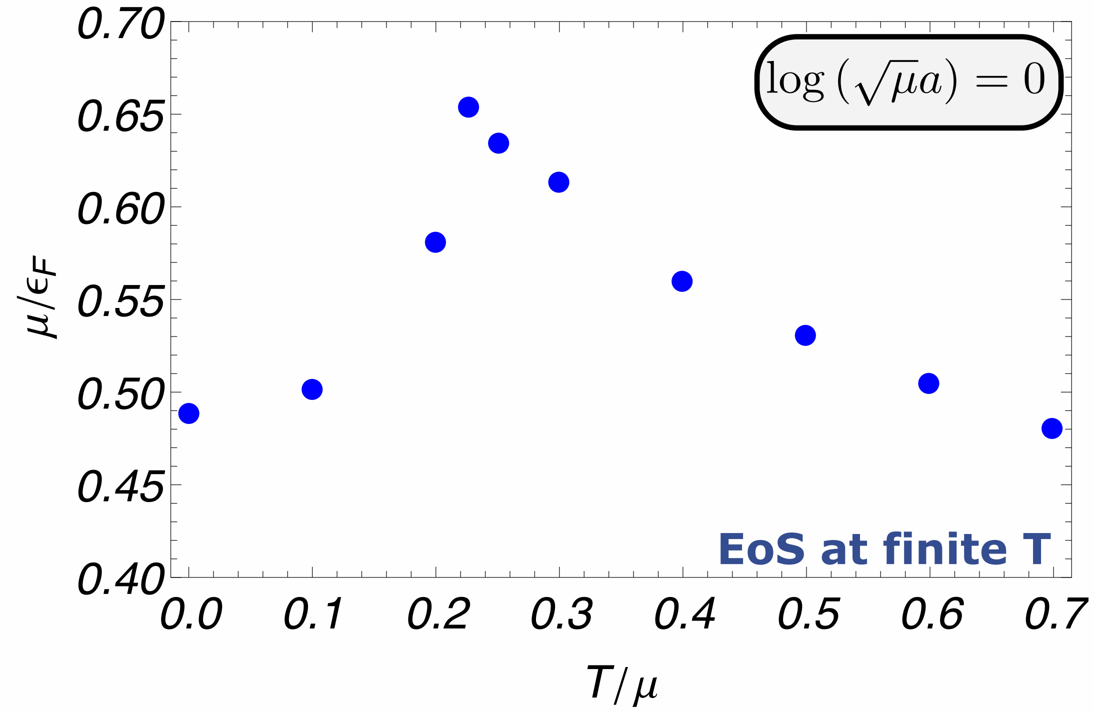
$$k_F = \sqrt{\epsilon_F}$$

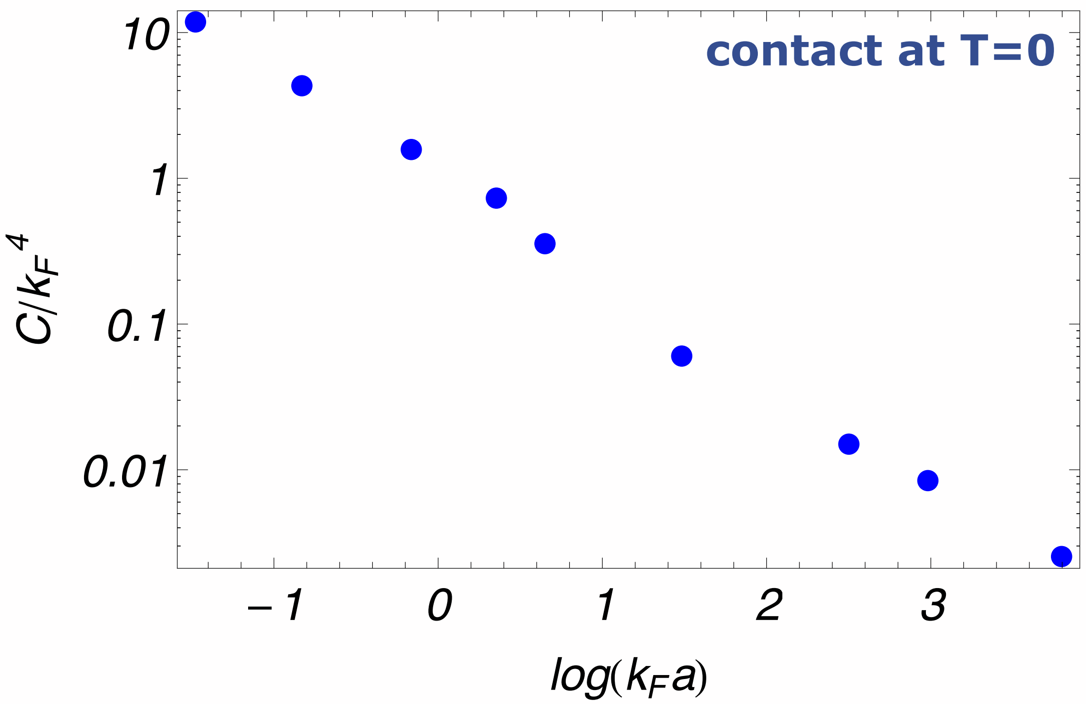
ultracold quantum gases in 2 dimensions

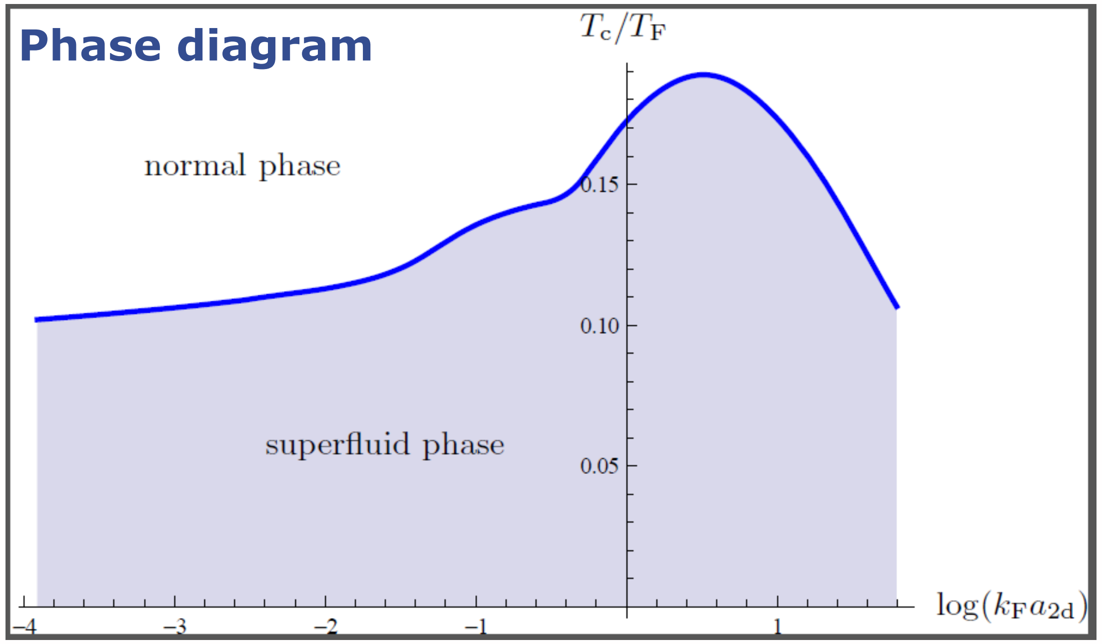
EoS & phase structure

Boettcher, JMP, Wetterich, in preparation









simpler approximation

ultracold quantum gases in 2 dimensions

Scaling

Boettcher, JMP, Wetterich, in preparation

$$\Gamma_k[\psi,\phi] = \cdots + \int_{\tau,\vec{x}} \phi^* \left(Z_{\phi,k}\partial_\tau - A_{\phi,k}\frac{\nabla^2}{2} \right) \phi + \cdots$$

$$\eta_k = -\frac{\partial_t A_{\phi,k}}{A_{\phi,k}}$$

$$g_1(r) \sim \langle \hat{\psi}^\dagger(r_0)\hat{\psi}(r_0+r)\rangle$$

$$C^2(r) = \frac{1}{\pi r^2}\int_0^r d^2r' \, |g_1(r')|^2 \propto r^{-2\eta}$$

thermal: $\alpha = 1$ — exp. decay

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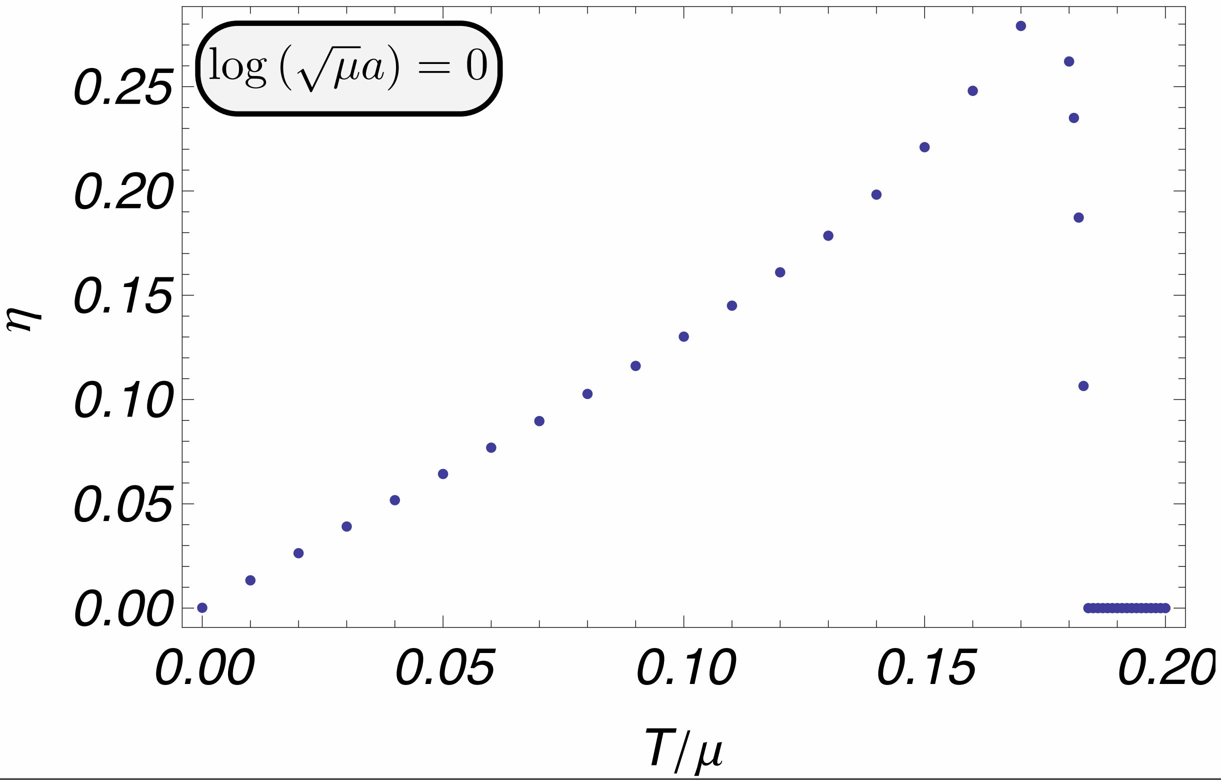
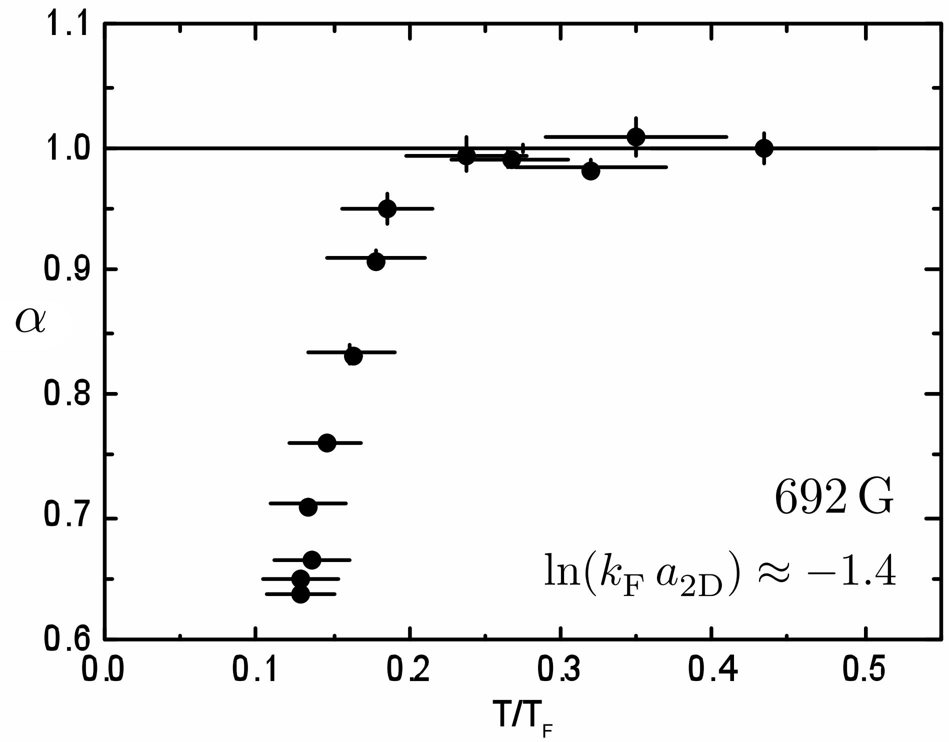
$\eta = \alpha$

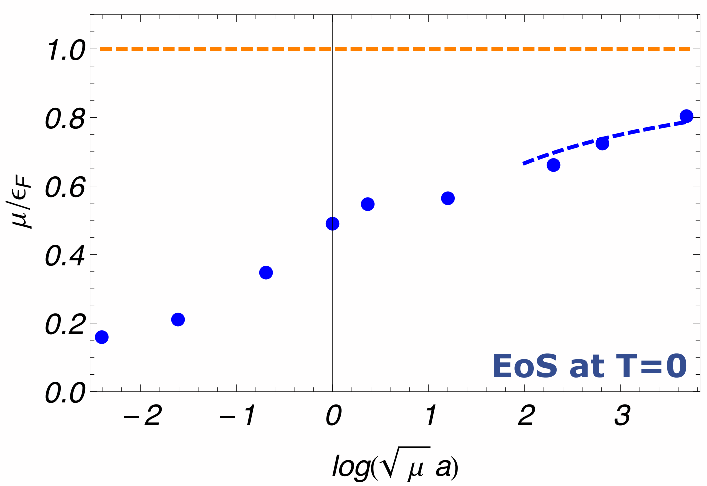


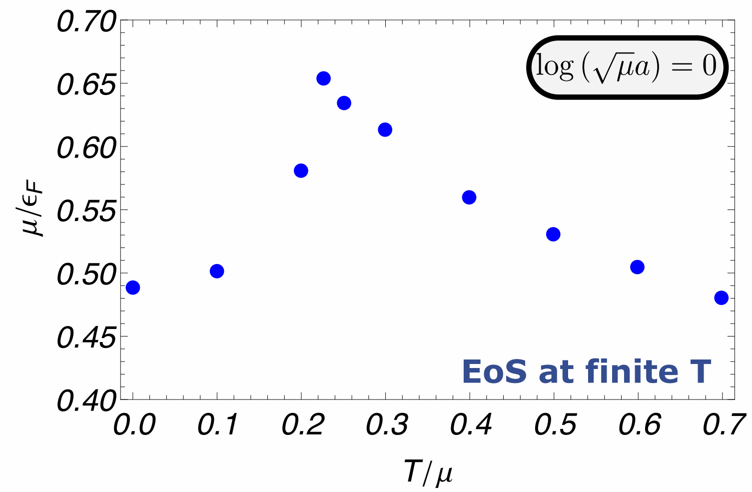


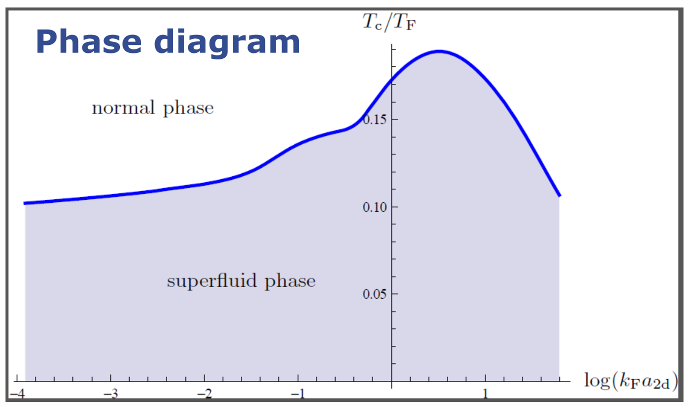
Summary & Outlook

Summary & outlook

- **Eos & phase structure in two dimensions**







- **Tan contact & BKT scaling**