Exploring the phase structure and dynamics of QCD

Jan M. Pawlowski

Universität Heidelberg & ExtreMe Matter Institute

Beijing, May 22th & June 1st 2015

Outline

- **Vacuum QCD & the hadron spectrum**
- **Phase structure of QCD**

Part I

- **Spectral Functions & Transport Coefficients**

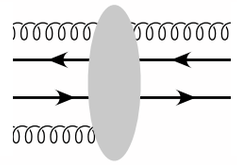
Part II

- **Outlook**

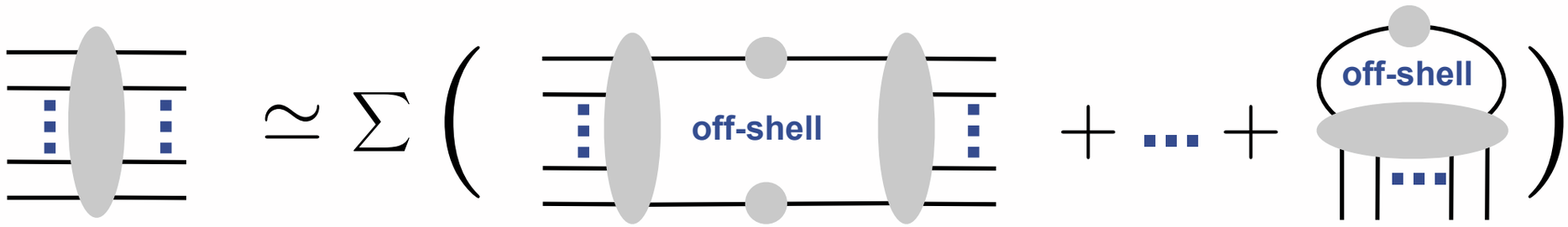
Continuum Methods for QCD

quark-gluon correlations

$$\langle q(x_1)\cdots\bar{q}(x_{2n})\,A_\mu(y_1)\cdots A_\mu(y_m)\rangle$$



functional relations

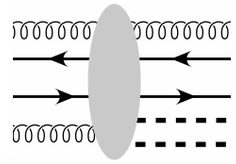


scattering amplitude/ vertex functions

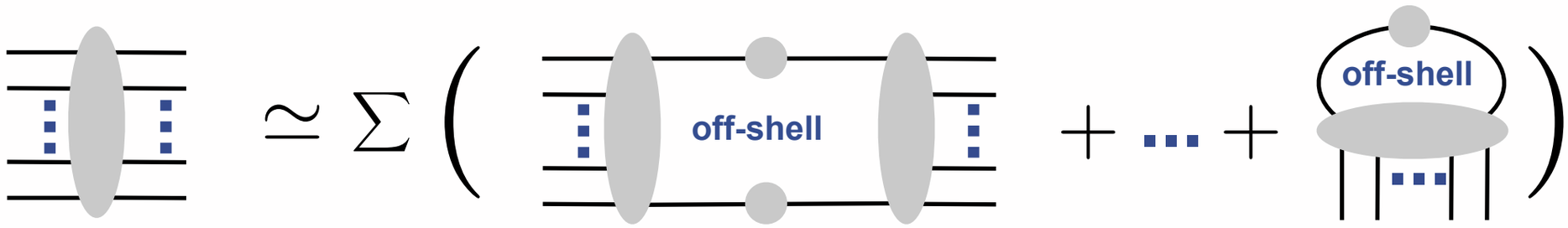
Continuum Methods for QCD

quark-gluon-hadron correlations

$$\langle q(x_1)\cdots\bar{q}(x_{2n})\,A_\mu(y_1)\cdots A_\mu(y_m)\,h(z_1)\cdots h(z_l)\rangle$$



functional relations



scattering amplitude/ vertex functions

Functional renormalisation group equations

Dyson-Schwinger equations

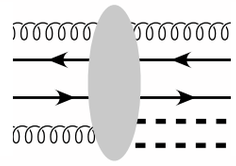
2PI/nPI hierarchies

Bethe-Salpeter equations

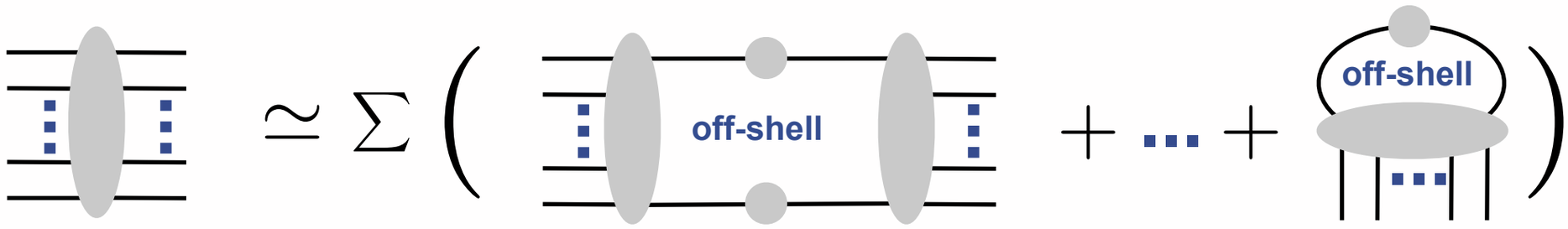
Continuum Methods for QCD

quark-gluon-hadron correlations

$$\langle q(x_1)\cdots\bar{q}(x_{2n})\,A_\mu(y_1)\cdots A_\mu(y_m)\,h(z_1)\cdots h(z_l)\rangle$$



functional relations



scattering amplitude/
vertex functions

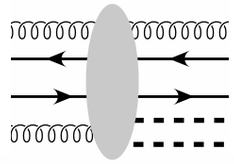
properties

- access to physics mechanisms 
- numerically tractable
  no sign problem
  systematic error control via closed form
- low energy models naturally encorporated

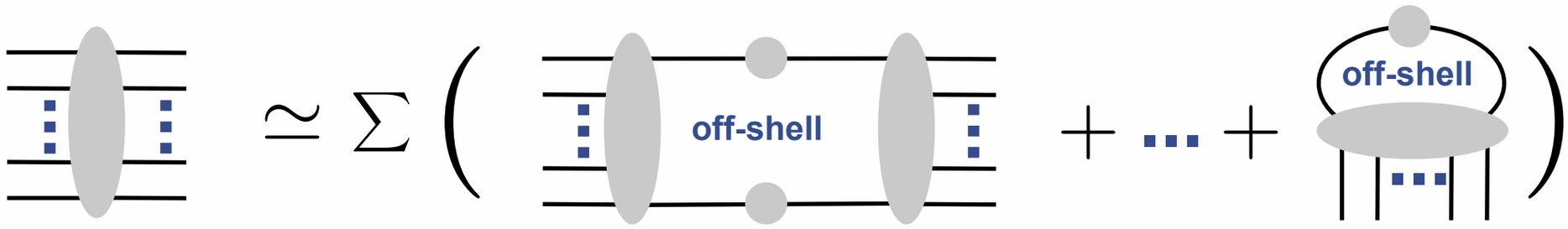
Continuum Methods for QCD

quark-gluon-hadron correlations

$$\langle q(x_1)\cdots\bar{q}(x_{2n})\,A_\mu(y_1)\cdots A_\mu(y_m)\,h(z_1)\cdots h(z_l)\rangle$$



functional relations



scattering amplitude/
vertex functions

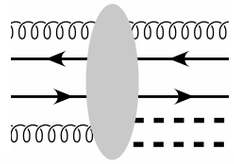
properties

- access to physics mechanisms 
- numerically tractable
  no sign problem
  systematic error control via closed form
- low energy models naturally encorporated

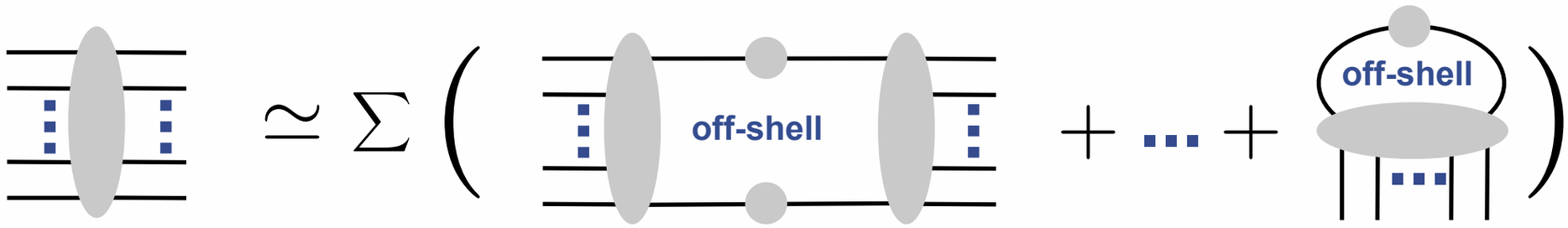
Continuum Methods for QCD

quark-gluon-hadron correlations

$$\langle q(x_1)\cdots\bar{q}(x_{2n})\,A_\mu(y_1)\cdots A_\mu(y_m)\,h(z_1)\cdots h(z_l)\rangle$$



functional relations



scattering amplitude/
vertex functions

e.g. lattice input on rhs

e.g. volume flucs., finite density,
dynamics, ...

properties

- access to physics mechanisms 
- numerically tractable
  no sign problem
  systematic error control via closed form
- low energy models naturally encorporated

FunMethods complementary to lattice

Functional Methods for QCD

Scales

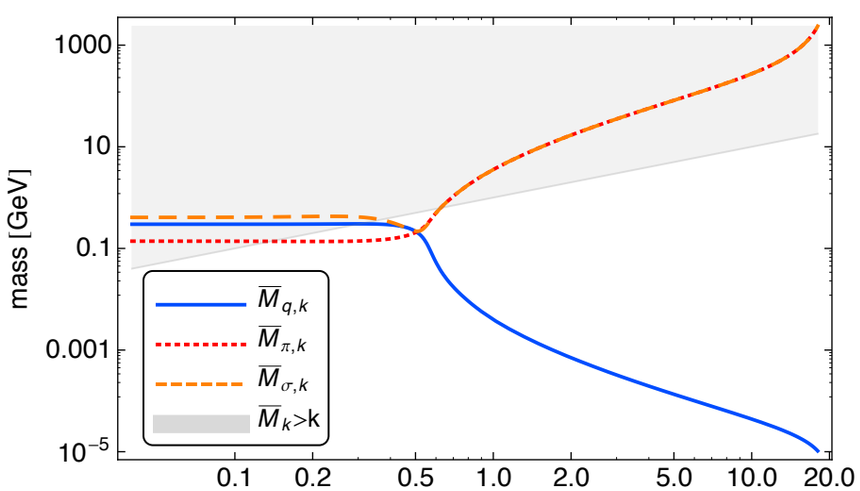
- **intrinsic scale of QCD:** $\Lambda_{\rm QCD} \approx 200\,{\rm MeV}$
  - **glue mass gap** **(Landau gauge: mass gap of glue propagator)** $\Delta m_{\rm glue} \approx \Lambda_{\rm QCD}$
  - **chiral symmetry breaking scale:** $\Delta m_\chi \approx m_{q,\rm constit.} - m_{q,\rm current}$ $\Delta m_\chi \approx 300\,{\rm MeV}$
  - **chiral/confinement critical temperatures:** $T_\chi \approx T_{\rm conf} \approx 150\,{\rm MeV}$

scales = $c(N_f, N_c)\Lambda_{\rm QCD}$

- **explicit mass scales of QCD:**
  - **current quark masses:** $\left.\dfrac{m_{q,\rm current}}{\Lambda_{\rm QCD}}\right|_{\rm light\ quarks} \approx 10^{-2}$ $m_\pi \approx 140\,{\rm MeV}$

meson masses

- **higher resonances:** $\dfrac{m_{\rm res}}{\Lambda_{\rm QCD}} \lesssim 10^{-1}$



Braun, Fister, Haas, JMP, Rennecke '14

Functional Methods for QCD

Scales

- **intrinsic scale of QCD:** $\Lambda_{\rm QCD} \approx 200\,{\rm MeV}$
  - **glue mass gap** **(Landau gauge: mass gap of glue propagator)** $\Delta m_{\rm glue} \approx \Lambda_{\rm QCD}$
  - **chiral symmetry breaking scale:** $\Delta m_\chi \approx m_{q,\rm constit.} - m_{q,\rm current}$ $\Delta m_\chi \approx 300\,{\rm MeV}$
  - **chiral/confinement critical temperatures:** $T_\chi \approx T_{\rm conf} \approx 150\,{\rm MeV}$

scales = $c(N_f, N_c)\Lambda_{\rm QCD}$

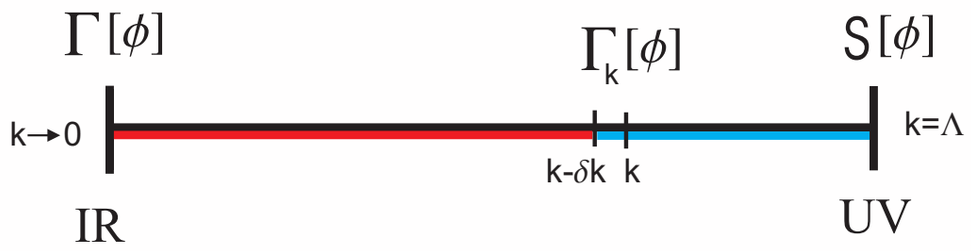
- **explicit mass scales of QCD:**
  - **current quark masses:** $\left.\dfrac{m_{q,\rm current}}{\Lambda_{\rm QCD}}\right|_{\rm light\ quarks} \approx 10^{-2}$ $m_\pi \approx 140\,{\rm MeV}$
- **higher resonances:** $\dfrac{m_{\rm res}}{\Lambda_{\rm QCD}} \lesssim 10^{-1}$
- **nuclear binding energy** $\approx 16\,{\rm MeV}$

best done with a combination of imaginary and real time flows

Functional RG for QCD

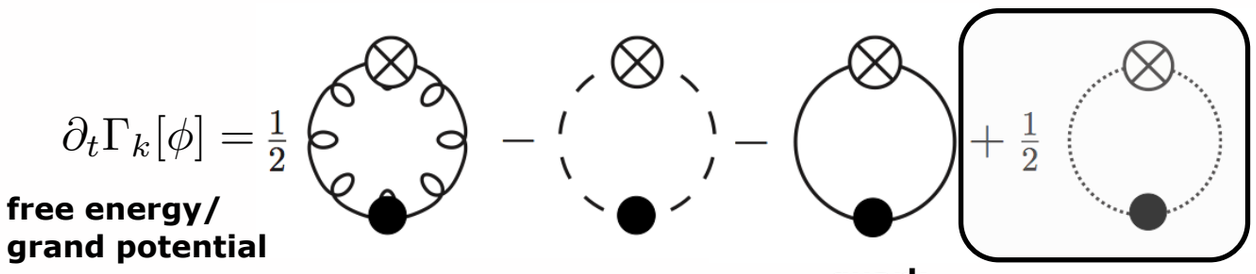
JMP, AIP Conf.Proc. 1343 (2011)
Nucl.Phys. A931 (2014) 113

free energy at momentum scale k



glue quantum fluctuations

hadronic quantum fluctuations



free energy/ grand potential

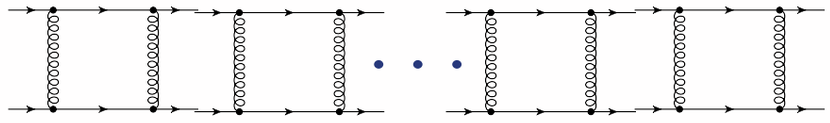
quark quantum fluctuations

Phase diagram survey

JMP, Schladming '13

ab initio

RG-scale k: $t = \ln k$



Dynamical hadronisation — **dynamical**

Gies, Wetterich '01
JMP '05
Flörchinger, Wetterich '09

Functional RG for QCD

fQCD collaboration: J. Braun, A. Cyrol, L. Fister, W.-j. Fu, T.K. Herbst, M. Mitter, N. Mueller, JMP, S. Rechenberger, F. Rennecke, N. Strodthoff

TARDIS, ERGE, DoFun2.0

DoFun

Huber, Braun, Comput.Phys.Commun. 183 (2012) 1290-1320

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

Braun, Fister, Haas, JMP, Rennecke, arXiv:1412.1045

Outline

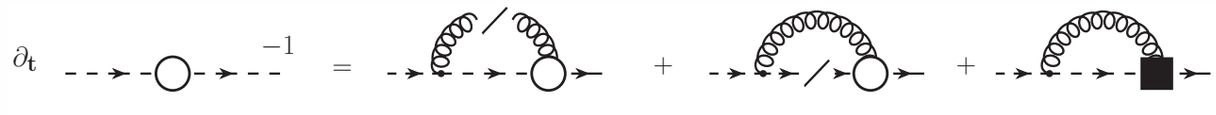
- **Vacuum QCD & the hadron spectrum**

- **Phase structure of QCD**

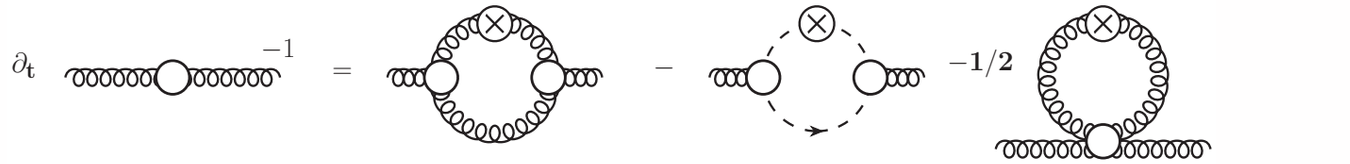
- **Spectral Functions & Transport Coefficients**

- **Outlook**

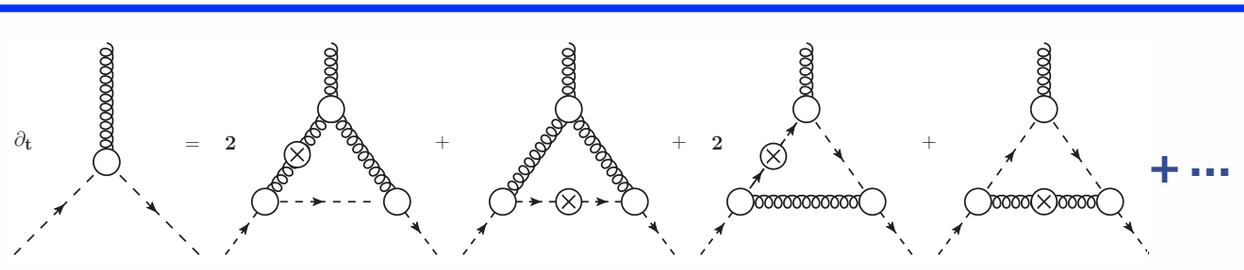
Glue sector

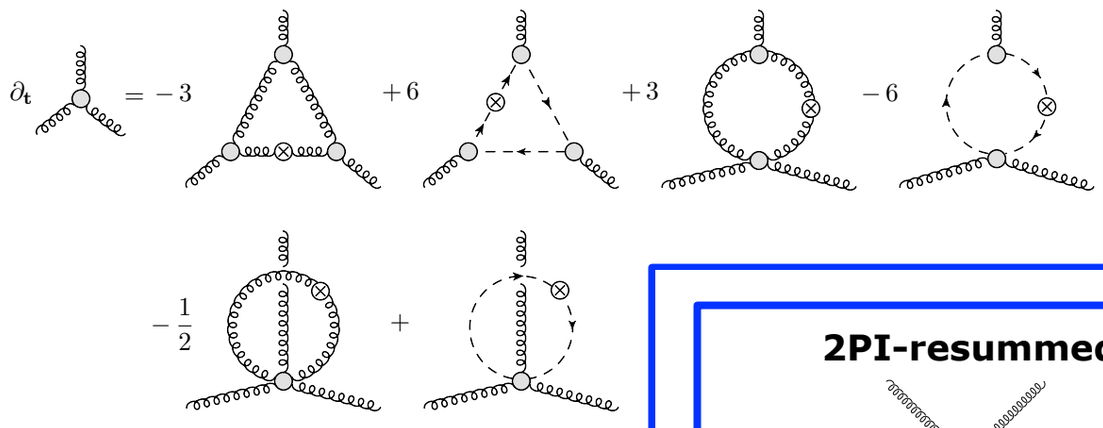


DSE-flow

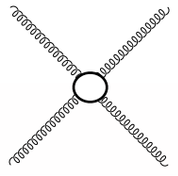


2PI-resummation



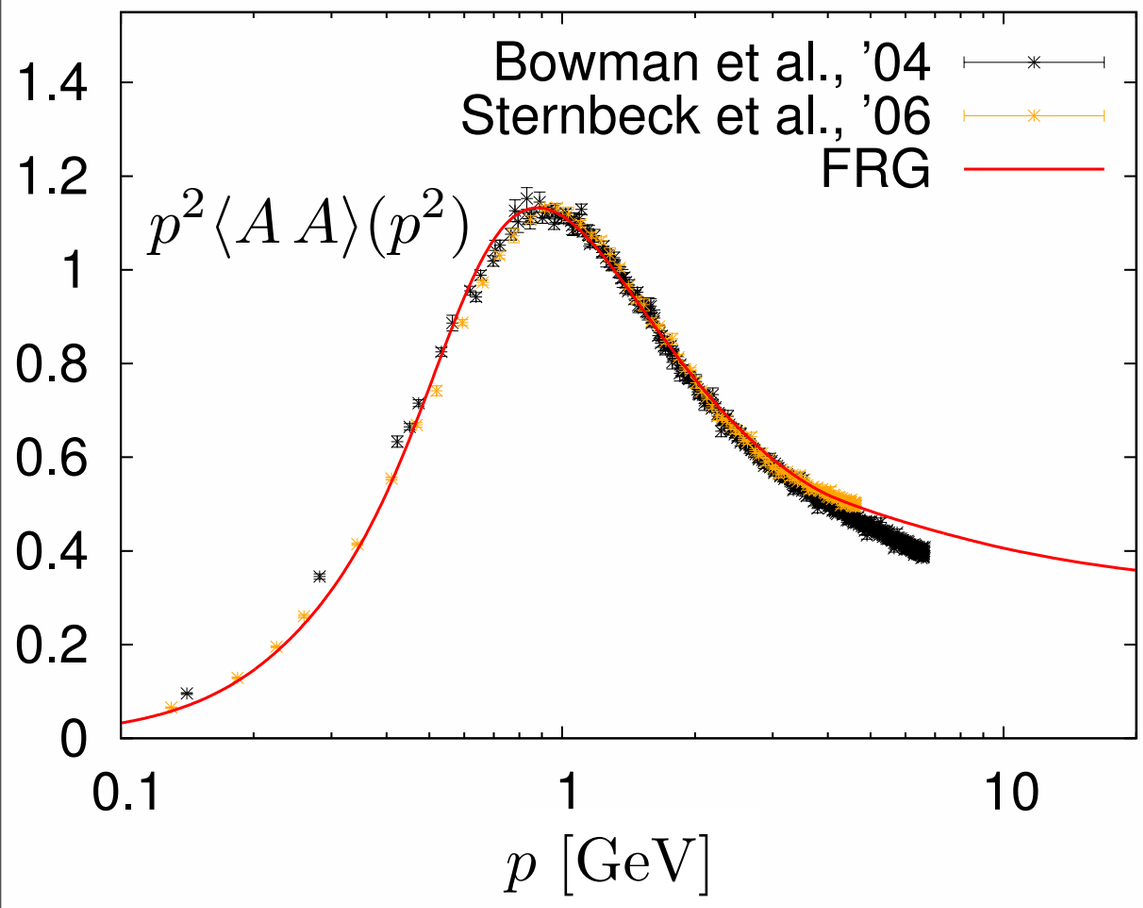


2PI-resummed



Glue sector

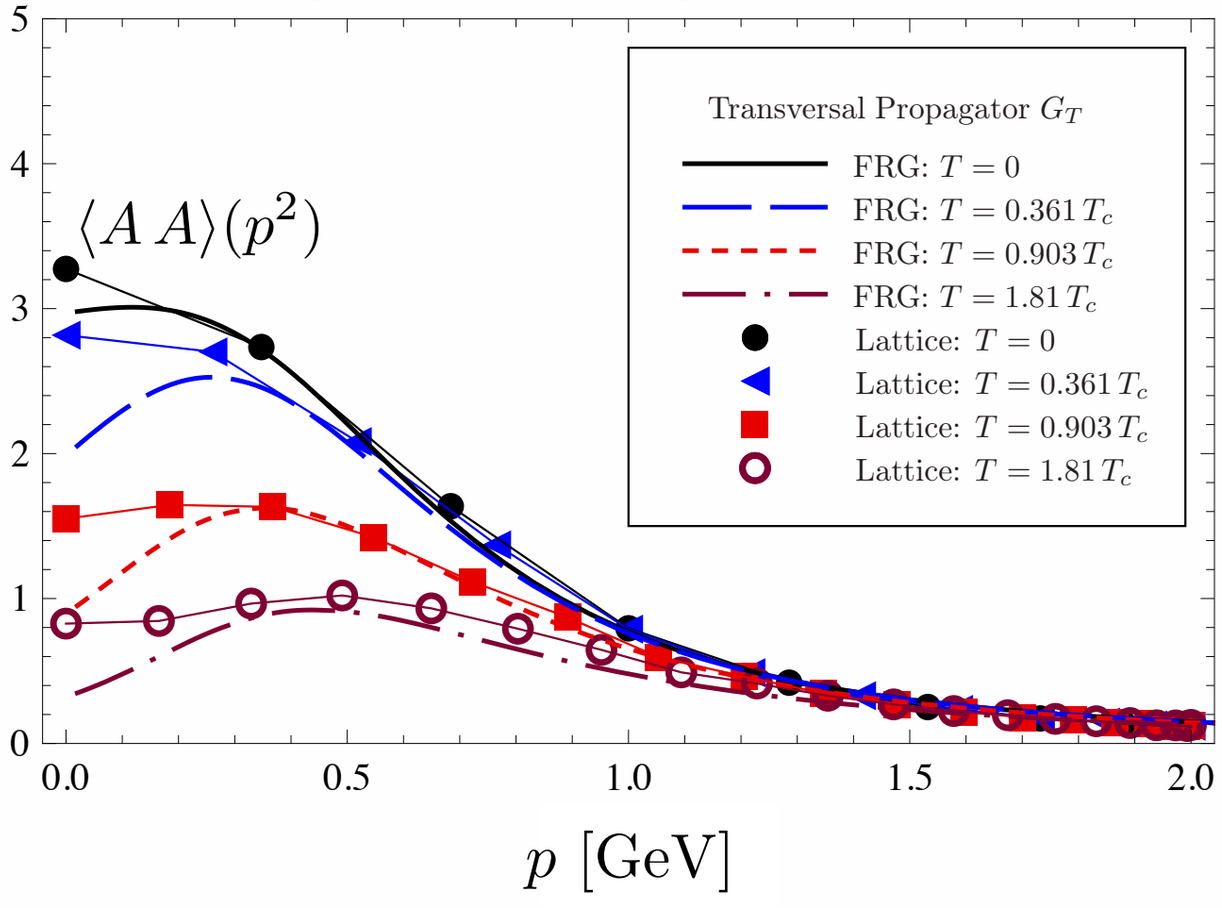
Yang-Mills propagators, T=0



Fischer, Maas, JMP, Annals Phys. 324 (2009) 2408

Fister, JMP '14

Yang-Mills propagators, finite T

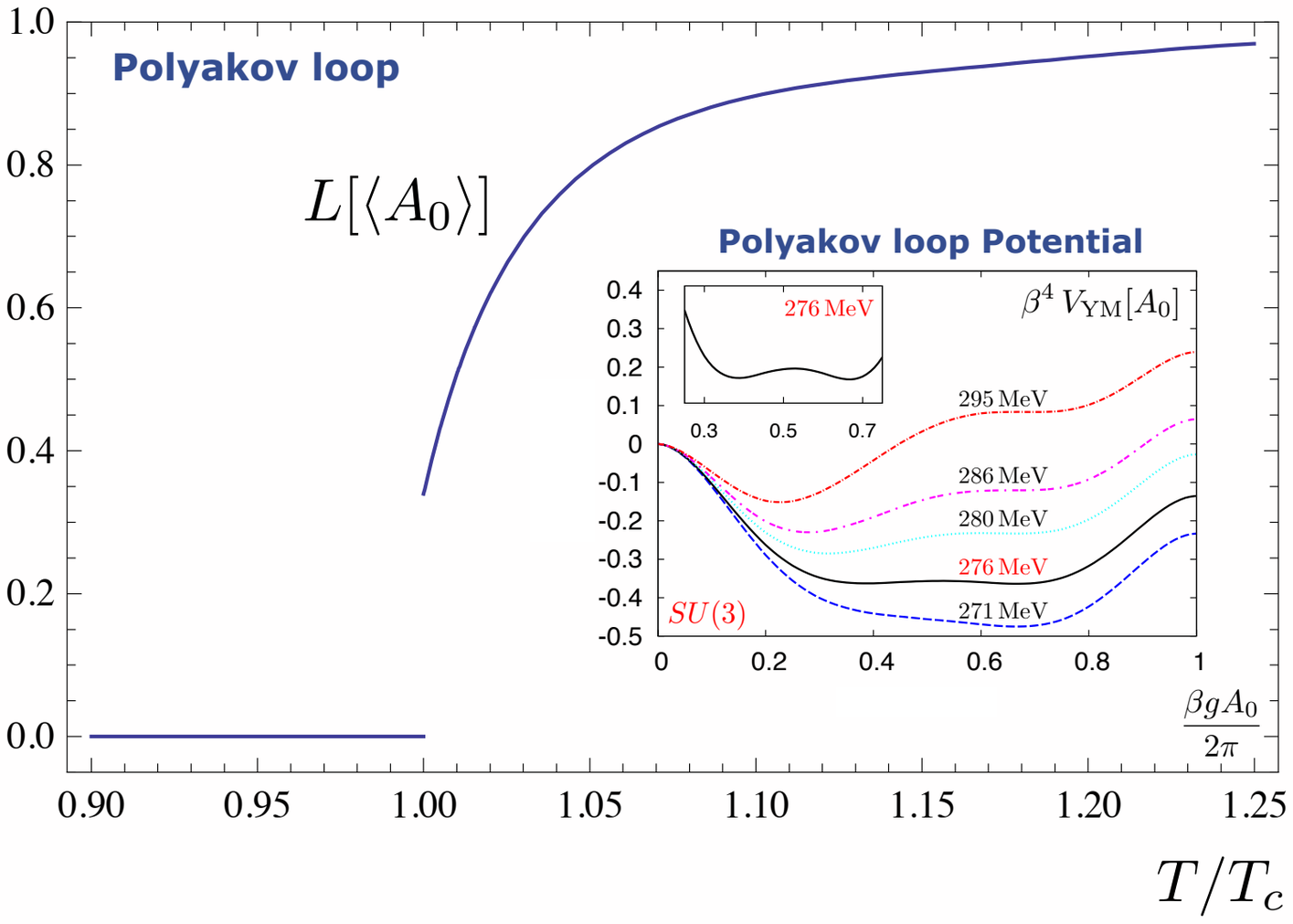


Fister, JMP, arXiv:1112.5440

Confinement

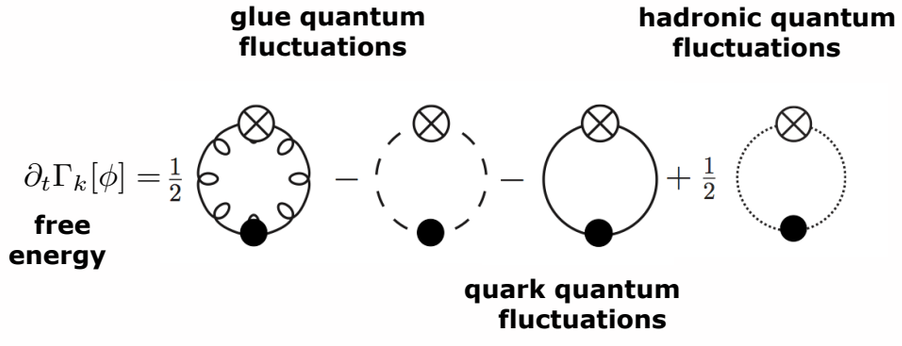
FRG: Braun, Gies, JMP, PLB 684 (2010) 262

FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010



$$T_c/\sqrt{\sigma} = 0.658 \pm 0.023$$

lattice : $T_c/\sqrt{\sigma} = 0.646$



confinement

gluon propagator

gapped relative to

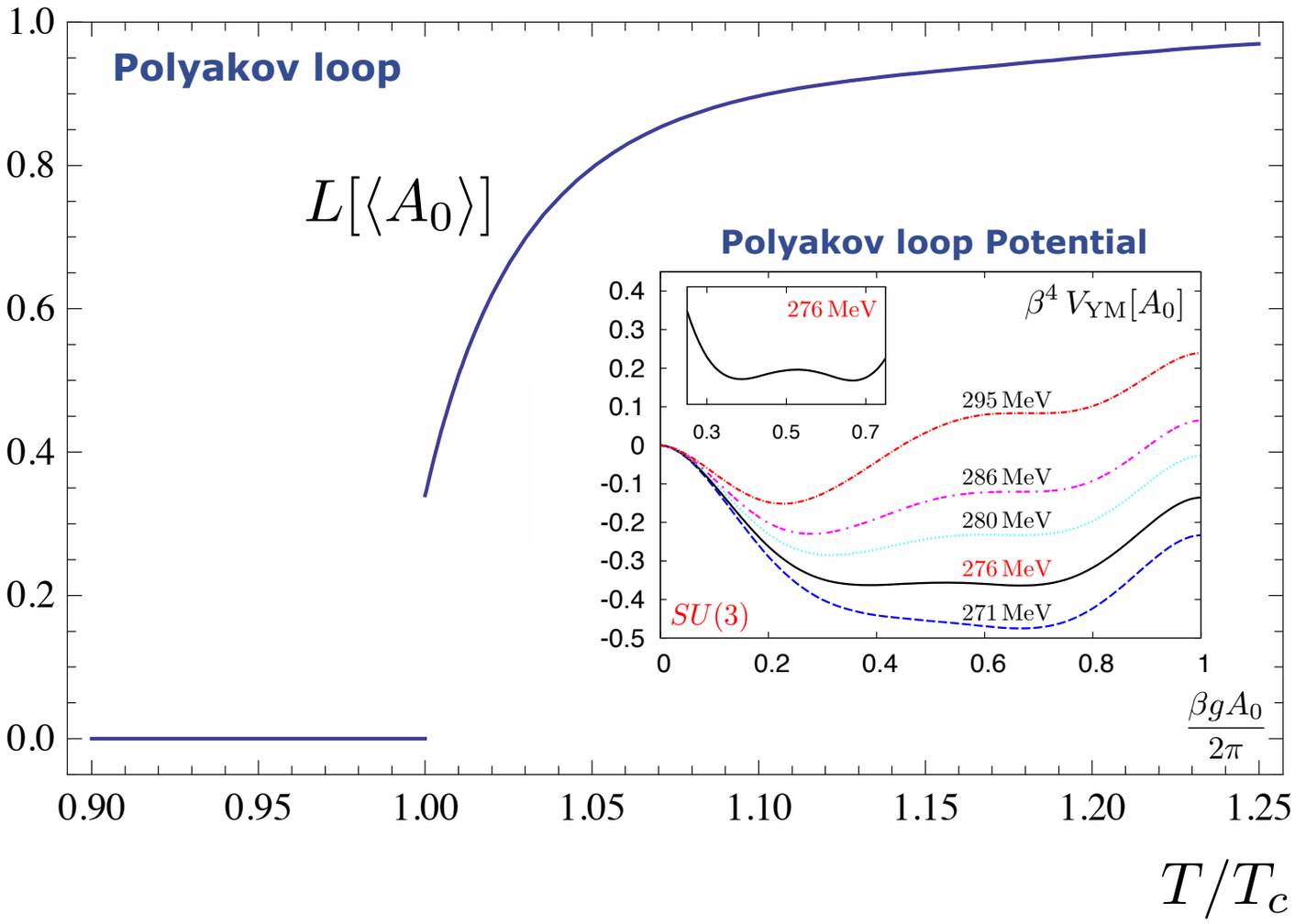
ghost propagator

Braun, Gies, JMP ’07
Marhauser, JMP ’08
Fister, JMP ’13

Confinement

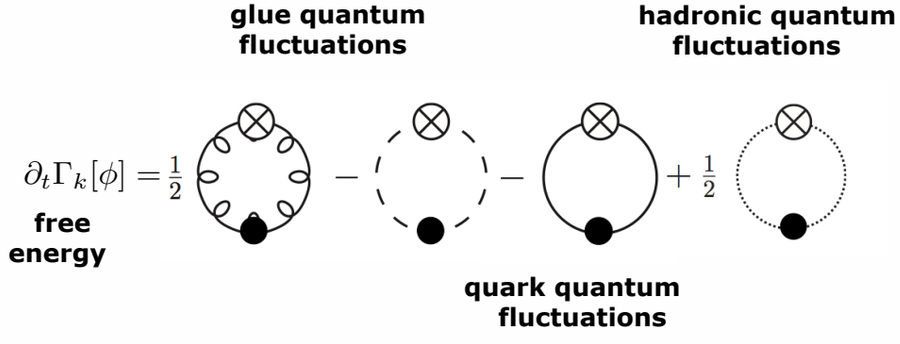
FRG: Braun, Gies, JMP, PLB 684 (2010) 262

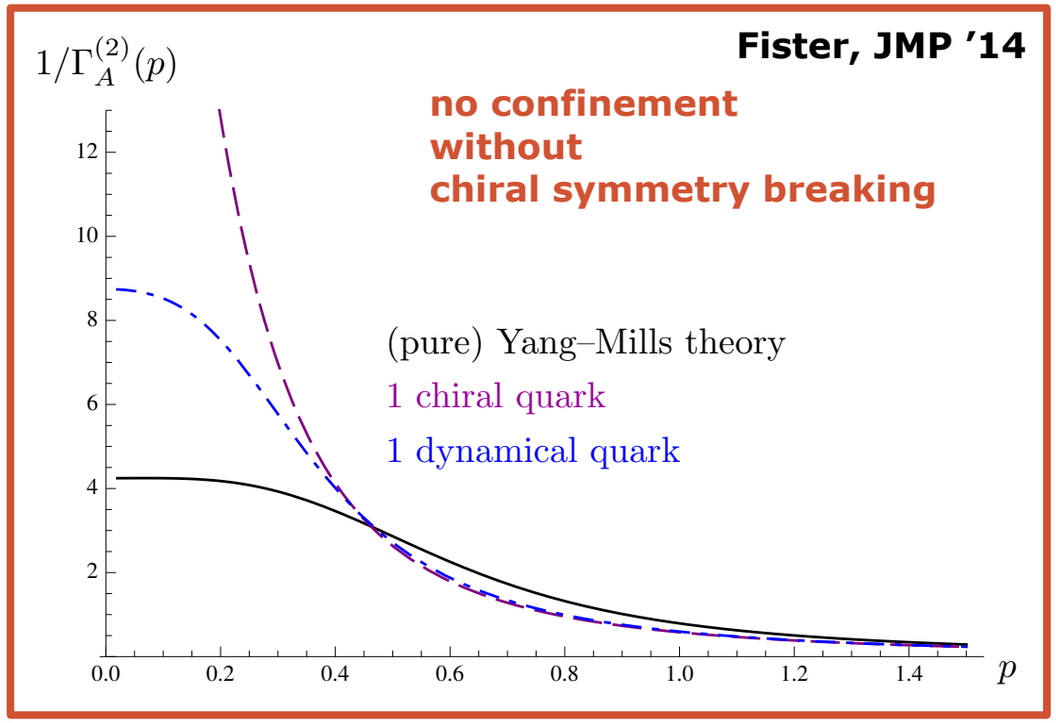
FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010



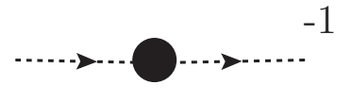
$$T_c/\sqrt{\sigma} = 0.658 \pm 0.023$$

lattice : $T_c/\sqrt{\sigma} = 0.646$

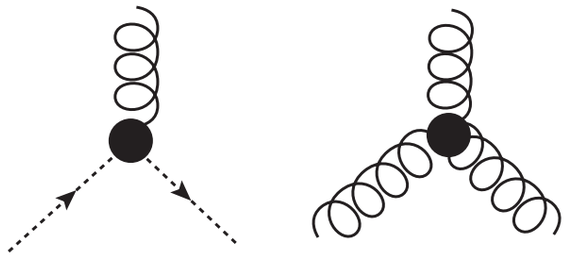




Chiral symmetry breaking

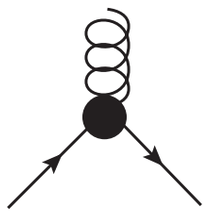


FRG Yang-Mills results

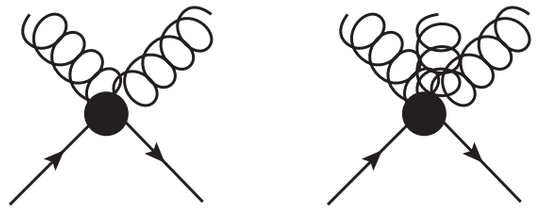


mom. dep.
classical tensor
structure

full mom. dep.

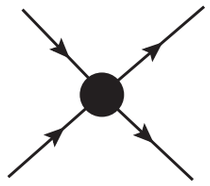


full mom. dep.
all tensor structures



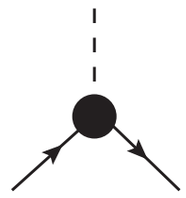
STI-consistent extension

mom. dep.



Fierz-complete basis
at $p = 0$ and mom. dep.

mom. dep.



mom. dep.

full effective
potential

fQCD

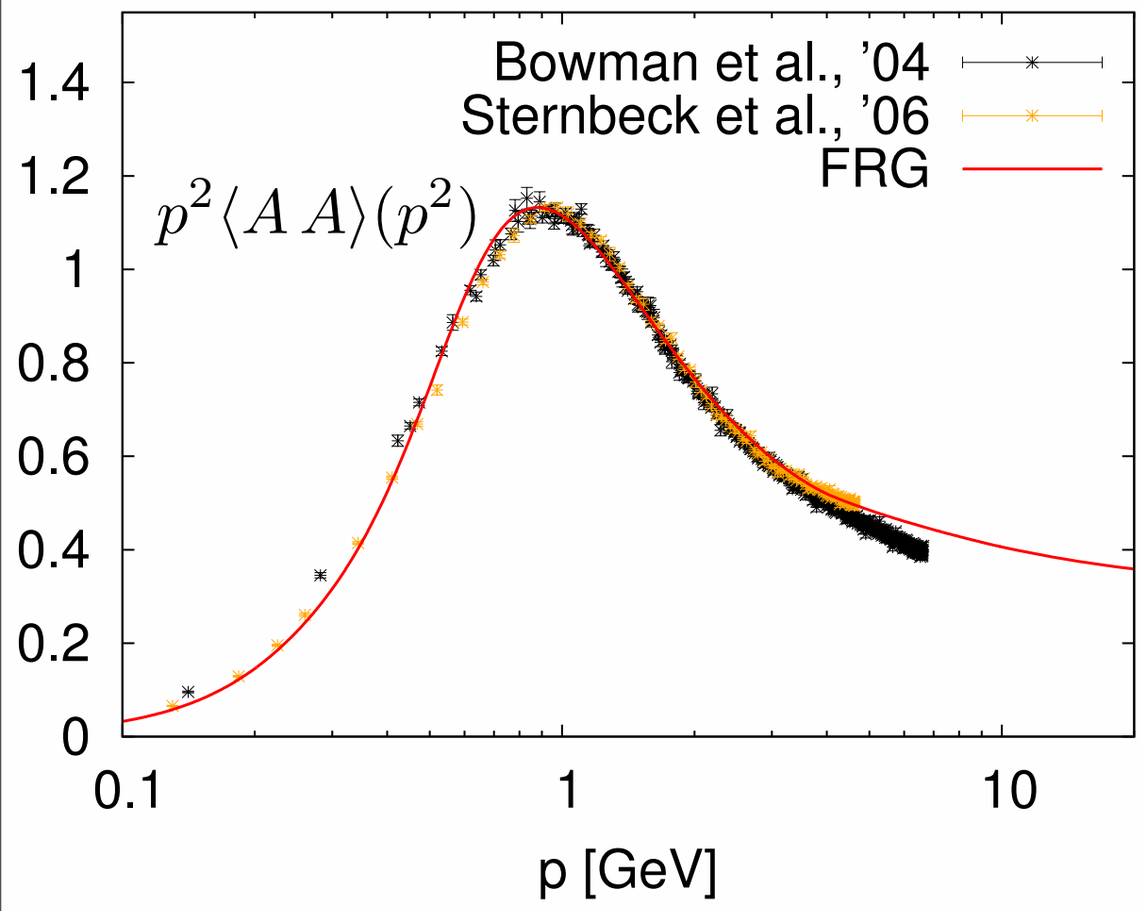
Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

see also Williams, Eur.Phys.J. A51 (2015) 5, 57

Chiral symmetry breaking

FRG-quenched QCD vs lattice-quenced QCD

quenched gluon dressing

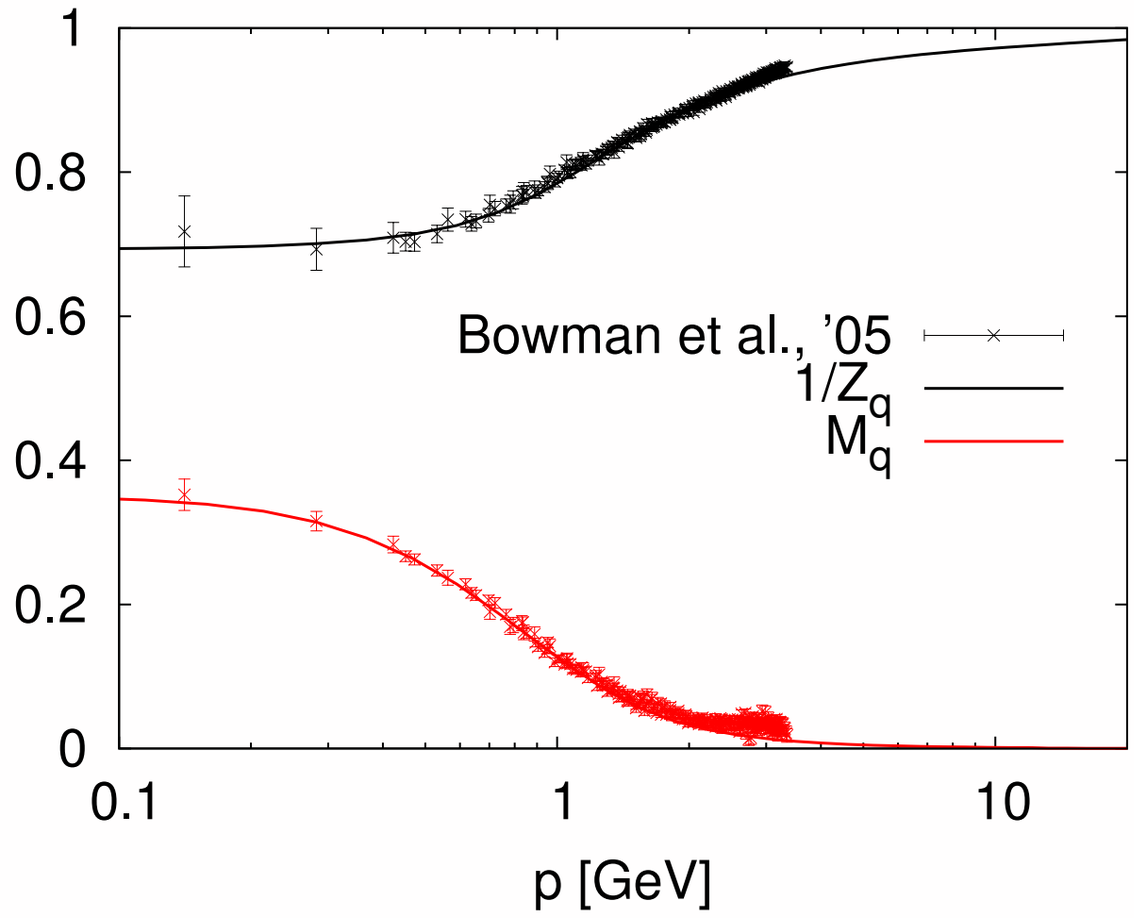


JMP, Rennecke, PRD 90, 076002

Helmboldt, JMP, Strodthoff, PRD 91 (2015) 5, 054010

Braun, Fister, Haas, JMP, Rennecke, arXiv:1412.1045

quark propagator



$\mathbf{N_f = 2}$

systematic error estimate: ~10%

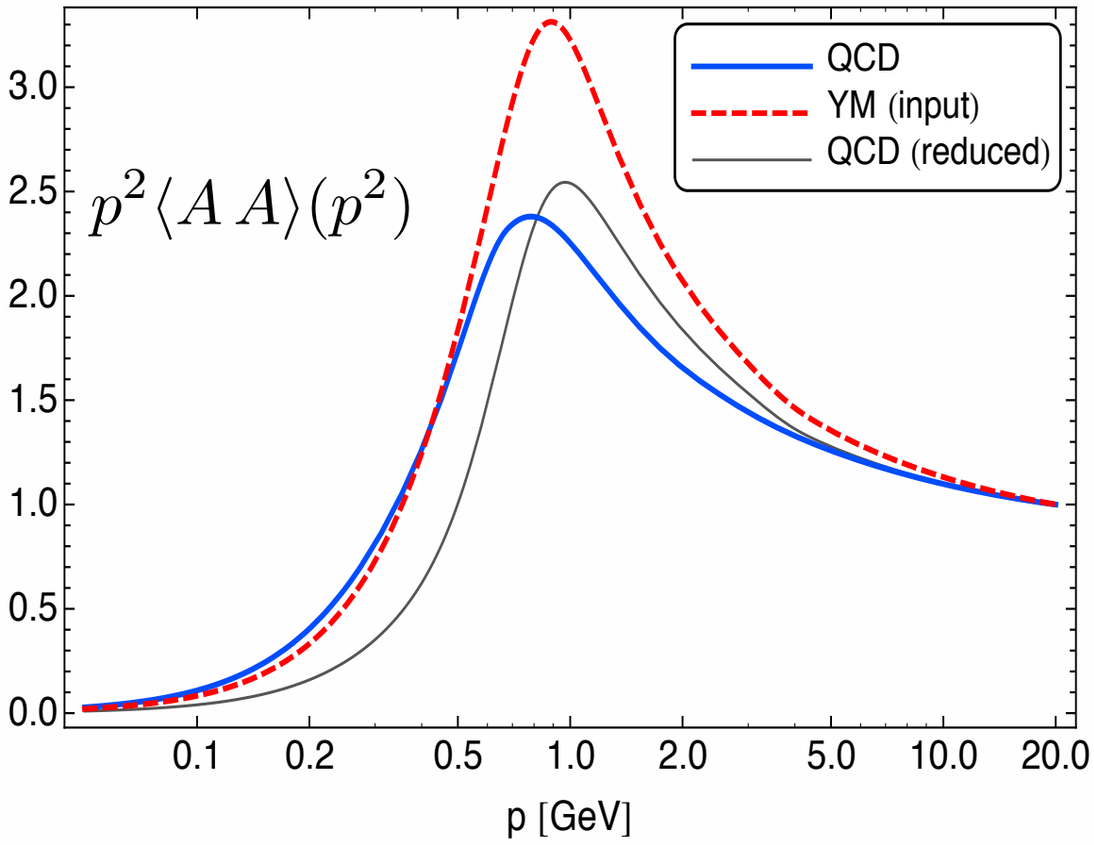
JMP

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

Chiral symmetry breaking

FRG-quenched QCD vs lattice-quenced QCD

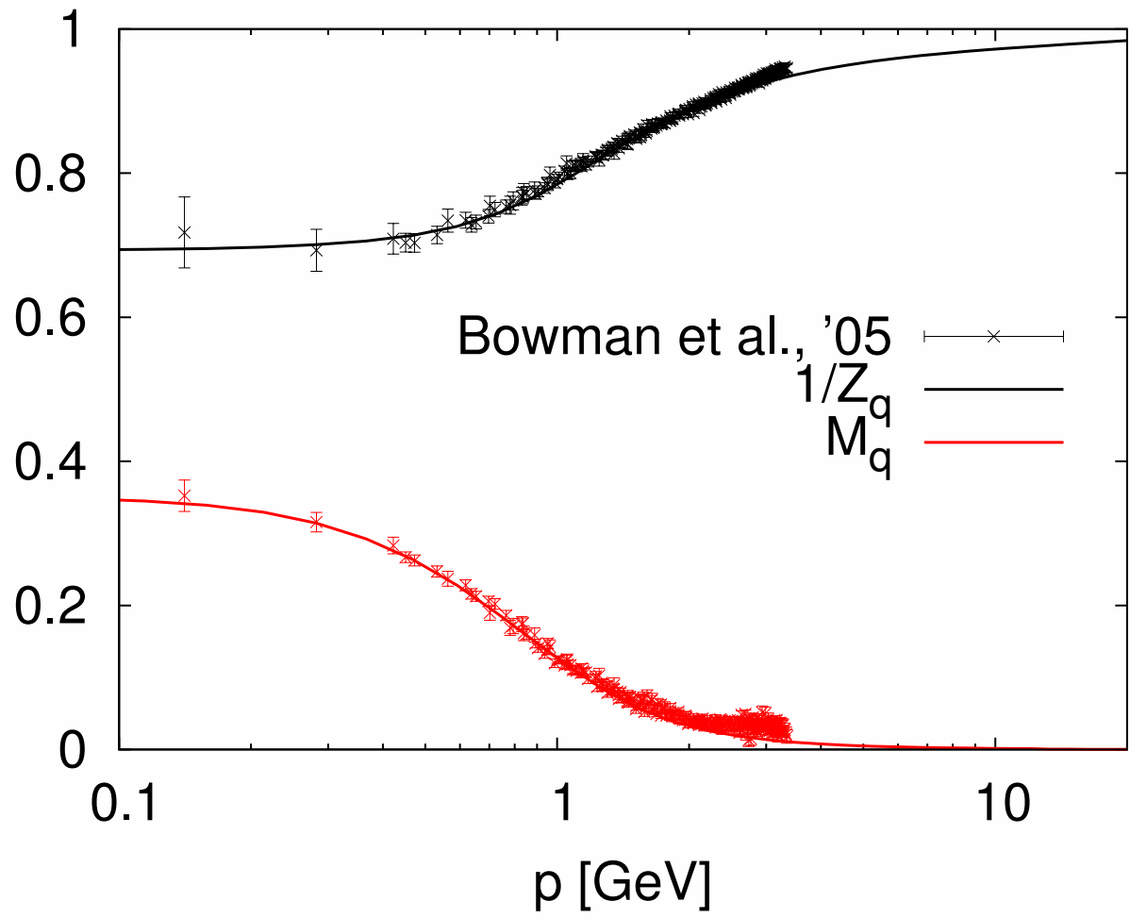
unquenched gluon dressing



Braun, Fister, Haas, JMP, Rennecke, arXiv:1412.1045

Rennecke, arXive:1504.03585

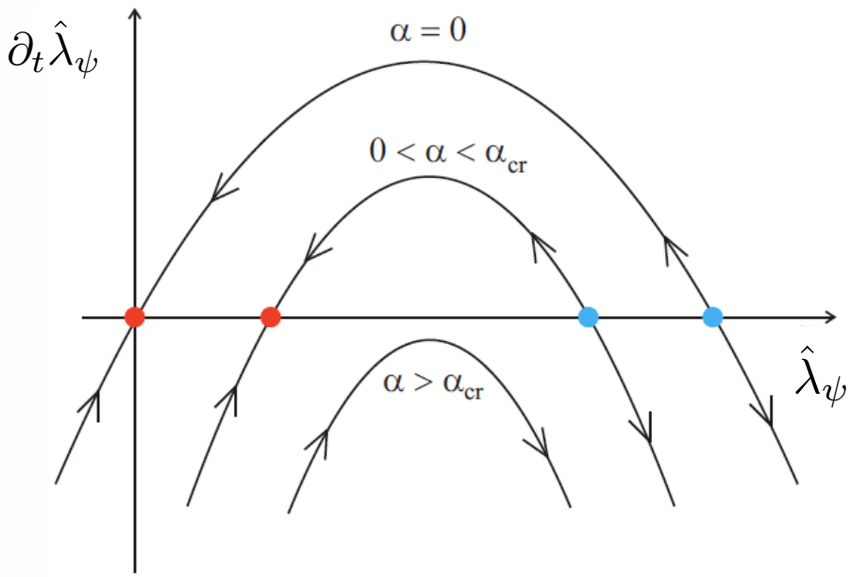
quark propagator

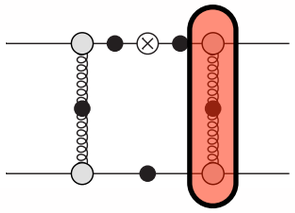


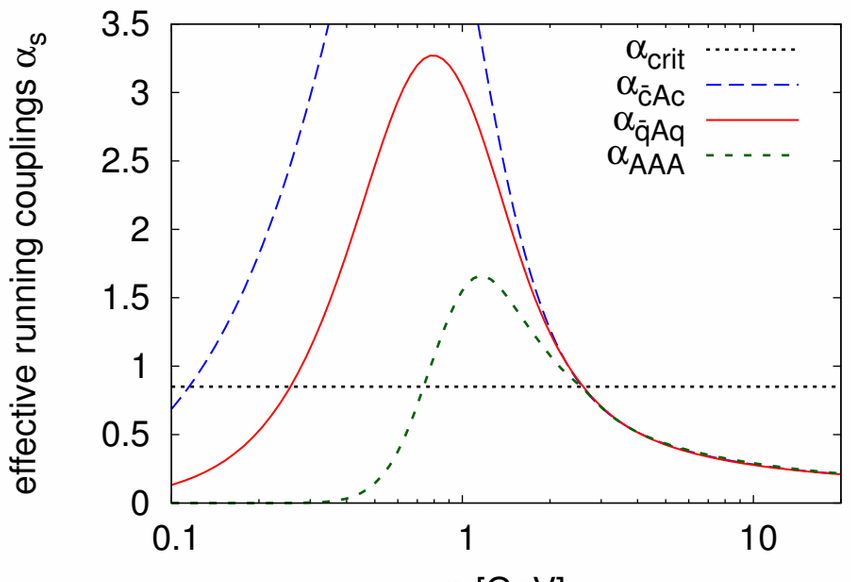
$\mathbf{N_f = 2}$ **systematic error estimate: ~10%**

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

Confinement & symmetry breaking



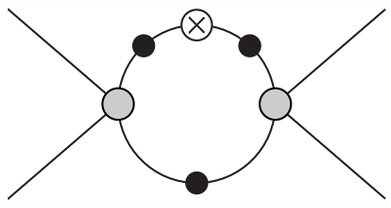




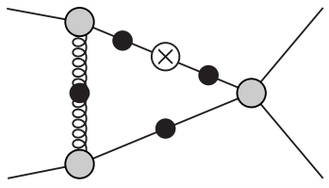
Mitter, JMP, Strodthoff ’14

Braun, Fister, Haas, JMP, Rennecke ’14

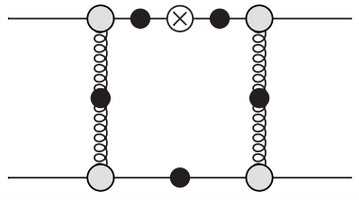
$$k\partial_k \hat{\lambda}_\psi = 2\hat{\lambda}_\psi +$$



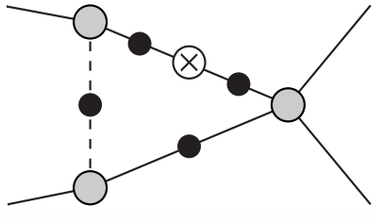
$+$



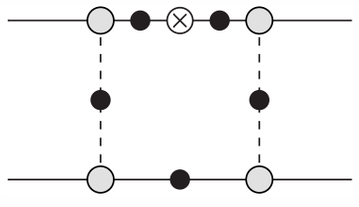
$+$



$+$

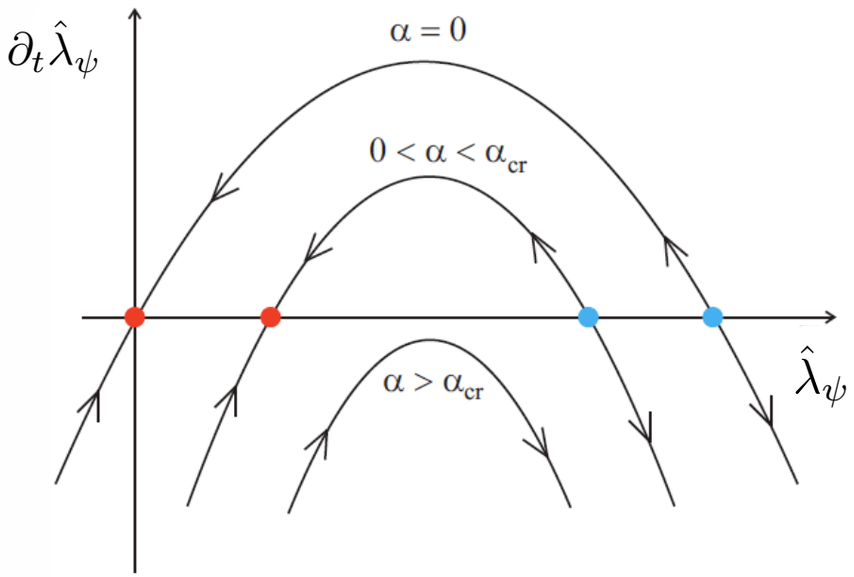


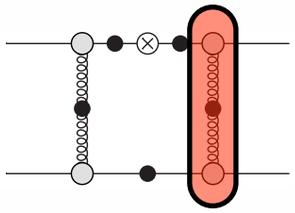
$+$

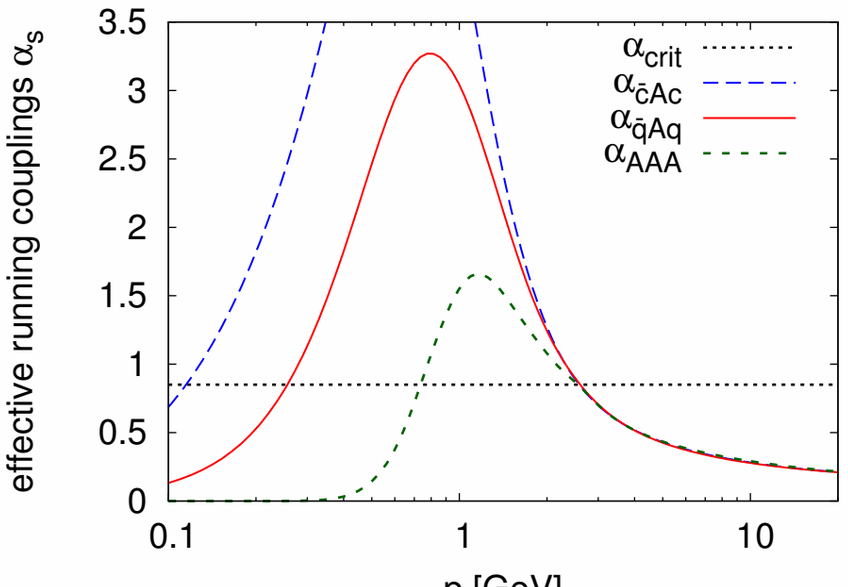


$+ \ \ldots$

Confinement & symmetry breaking







Mitter, JMP, Strodthoff ’14

Braun, Fister, Haas, JMP, Rennecke ’14

dynamical correlation of confinement and chiral symmetry breaking

confinement	chiral symmetry breaking
gluon propagator gapped relative to ghost propagator	gluon propagator not gapped too much

Fister, Mitter, JMP, Strodthoff ’14

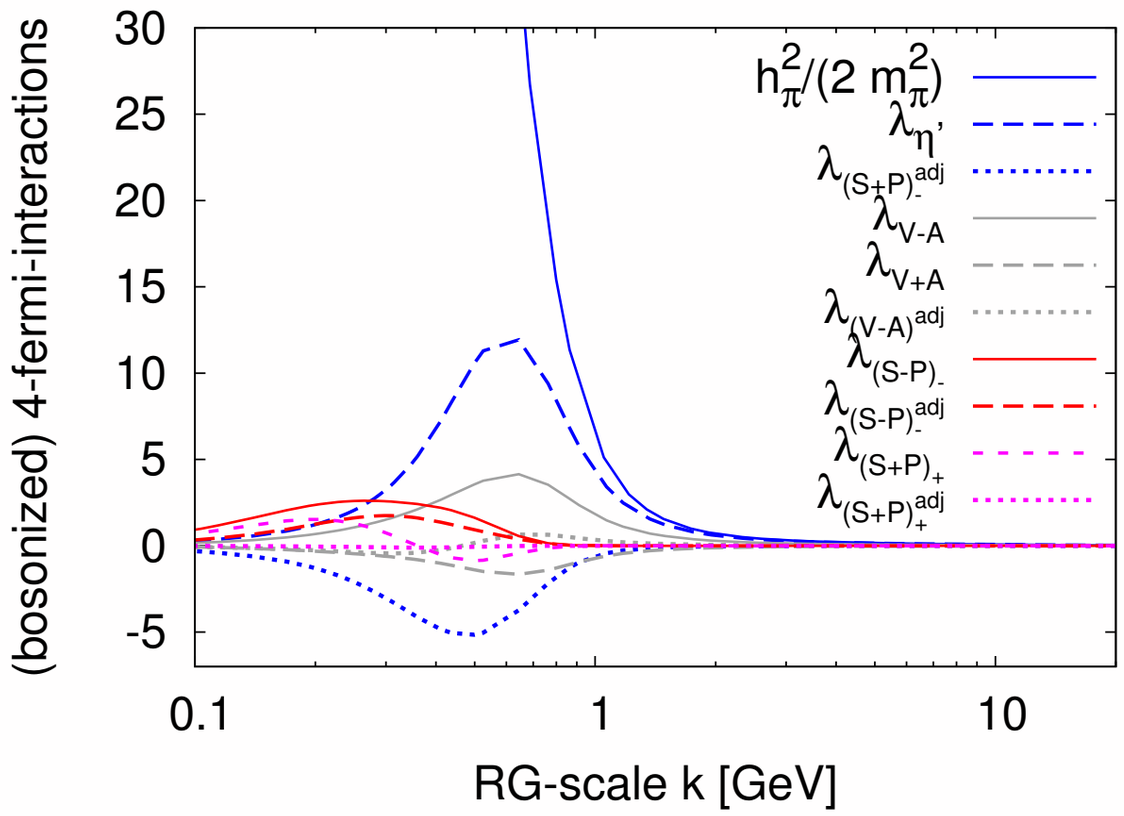
A glimpse at the hadron spectrum

preliminary

four-fermi scattering amplitude at pion pole

$$\langle \bar{q}\vec{\sigma}\gamma_5 q(p) \quad \bar{q}\vec{\sigma}\gamma_5 q(-p)\rangle \quad \rightarrow \quad \frac{\chi_{\bar{q}\pi q}\bar{\chi}_{\bar{q}\pi q}}{p^2 - m_\pi^2} \quad + \quad \text{finite terms}$$

$$\Gamma^{(4)}(p_1, p_2, p_3, p_4)$$



Mitter, JMP, Strodthoff, in preparation

A glimpse at the hadron spectrum

preliminary

four-fermi scattering amplitude at pion pole

$$\langle \bar{q}\vec{\sigma}\gamma_5 q(p) \quad \bar{q}\vec{\sigma}\gamma_5 q(-p)\rangle \;\rightarrow\; \frac{\chi_{\bar{q}\pi q}\bar{\chi}_{\bar{q}\pi q}}{p^2 - m_\pi^2} \;+\; \text{finite terms}$$

$$\Gamma^{(4)}_{(\bar{q}\gamma_5\vec{\sigma}q)^2}(p,p,-p,-p) \qquad \frac{\Gamma^{(3)}_{\bar{q}\pi q}\Gamma^{(3)}_{\bar{q}\pi q}}{p^2 - m_\pi^2}$$

pion decay constant f_π **via normalisation of** $\Gamma^{(3)}_{\bar{q}\pi q}$

aka BSE wave function

recent mini-review on DSE-BSE
Sanchis-Alepuz, Williams, arXiv:1503.05896

Mitter, JMP, Strodthoff, in preparation

A glimpse at the hadron spectrum

preliminary

four-fermi scattering amplitude at pion pole

$$\langle \bar{q}\vec{\sigma}\gamma_5 q(p) \quad \bar{q}\vec{\sigma}\gamma_5 q(-p)\rangle \;\rightarrow\; \frac{\chi_{\bar{q}\pi q}\bar{\chi}_{\bar{q}\pi q}}{p^2 - m_\pi^2} \;+\; \text{finite terms}$$

$$\Gamma^{(4)}_{(\bar{q}\gamma_5\vec{\sigma}q)^2}(p,p,-p,-p) \qquad \frac{\Gamma^{(3)}_{\bar{q}\pi q}\Gamma^{(3)}_{\bar{q}\pi q}}{p^2 - m_\pi^2}$$

pion decay constant f_π **via normalisation of** $\Gamma^{(3)}_{\bar{q}\pi q}$

real-time:

- **Spectral Fcts & Transport Coeffs**

Mitter, JMP, Strodthoff, in preparation

A glimpse at the hadron spectrum

preliminary

four-fermi scattering amplitude at pion pole

$$\langle \bar{q}\vec{\sigma}\gamma_5 q(p) \quad \bar{q}\vec{\sigma}\gamma_5 q(-p)\rangle \;\to\; \frac{\chi_{\bar{q}\pi q}\bar{\chi}_{\bar{q}\pi q}}{p^2-m_\pi^2} \;+\; \text{finite terms}$$

$$\Gamma^{(4)}_{(\bar{q}\gamma_5\vec{\sigma}q)^2}(p,p,-p,-p) \qquad \frac{\Gamma^{(3)}_{\bar{q}\pi q}\Gamma^{(3)}_{\bar{q}\pi q}}{p^2-m_\pi^2}$$

pion decay constant $\mathbf{f_\pi}$ **via normalisation of** $\boldsymbol{\Gamma^{(3)}_{\bar{q}\pi q}}$

$$\mathbf{f_\pi \simeq 99\,MeV}$$
quenched QCD

$$\mathbf{f_\pi \simeq 89\,MeV}$$
unquenched QCD

lattice **Davies et al., PRL 92 (2004) 022001**

$$\frac{\mathbf{f}_\pi^{\text{quenched}}}{\mathbf{f}_\pi^{\text{unquenched}}} \simeq 1.1$$

Mitter, JMP, Strodthoff, in preparation

A glimpse at the hadron spectrum

preliminary

four-fermi scattering amplitude at pion pole

$$\langle \bar{q}\vec{\sigma}\gamma_5 q(p) \quad \bar{q}\vec{\sigma}\gamma_5 q(-p)\rangle \;\rightarrow\; \frac{\chi_{\bar{q}\pi q}\bar{\chi}_{\bar{q}\pi q}}{p^2 - m_\pi^2} \;+\; \text{finite terms}$$

$$\Gamma^{(4)}_{(\bar{q}\gamma_5\vec{\sigma}q)^2}(p,p,-p,-p) \qquad \frac{\Gamma^{(3)}_{\bar{q}\pi q}\Gamma^{(3)}_{\bar{q}\pi q}}{p^2 - m_\pi^2}$$

pion decay constant $\mathbf{f_\pi}$ **via normalisation of** $\boldsymbol{\Gamma^{(3)}_{\bar{q}\pi q}}$

$\mathbf{f_\pi \simeq 99\,MeV}$ **quenched QCD**

$\mathbf{f_\pi \simeq 89\,MeV}$ **unquenched QCD**

lattice **Davies et al., PRL 92 (2004) 022001**

unquenched **e.g. Horsley et al., PLB 732, 41 (2014)** $\mathbf{f_\pi^{lattice} \simeq 89\,MeV}$

Mitter, JMP, Strodthoff, in preparation

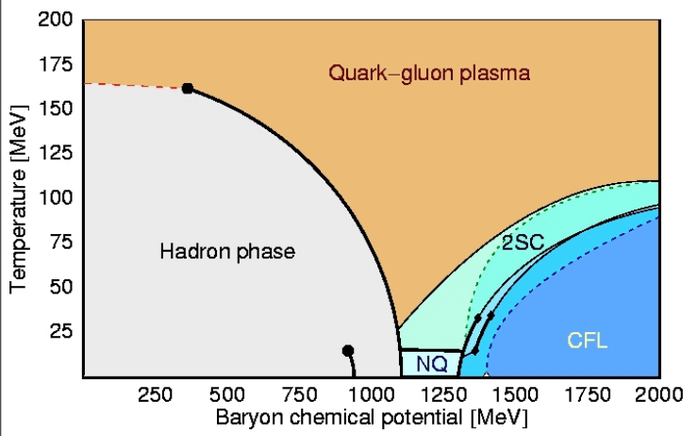
Outline

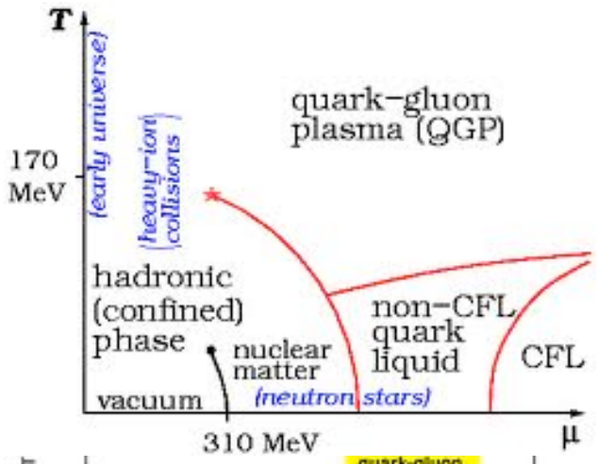
- Vacuum QCD & the hadron spectrum

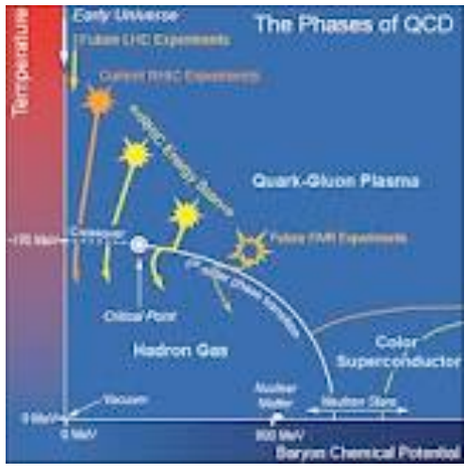
- Phase structure of QCD

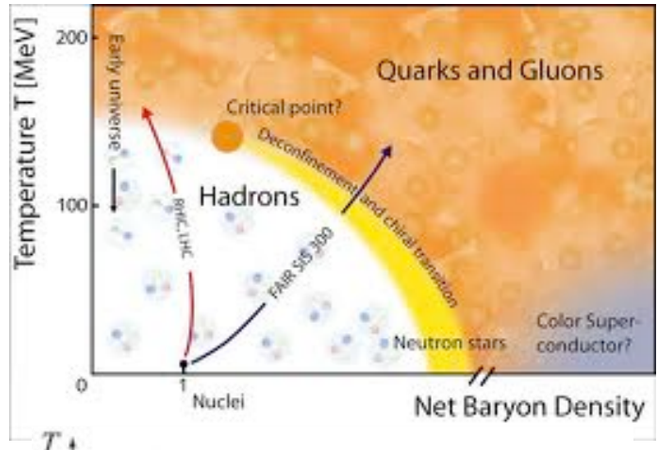
- Spectral Functions & Transport Coefficients

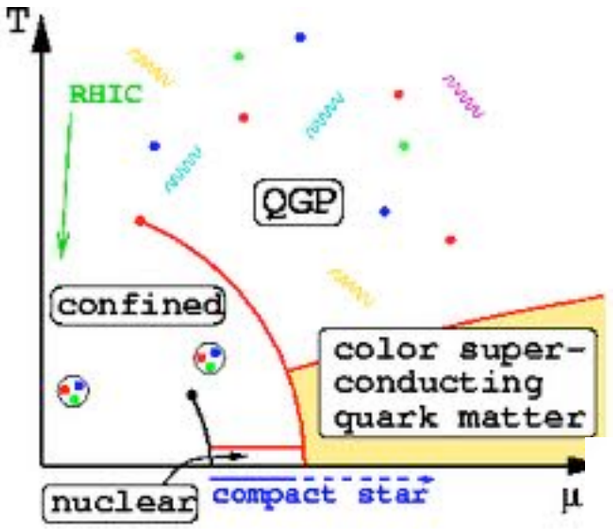
- Outlook

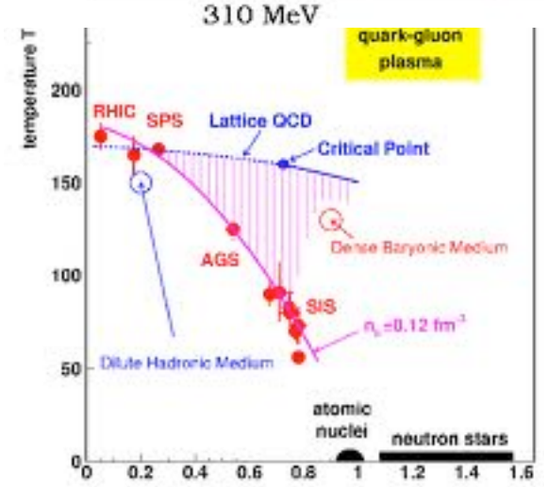

Quark–gluon plasma
Hadron phase
2SC
CFL
NQ
Temperature [MeV]
Baryon chemical potential [MeV]

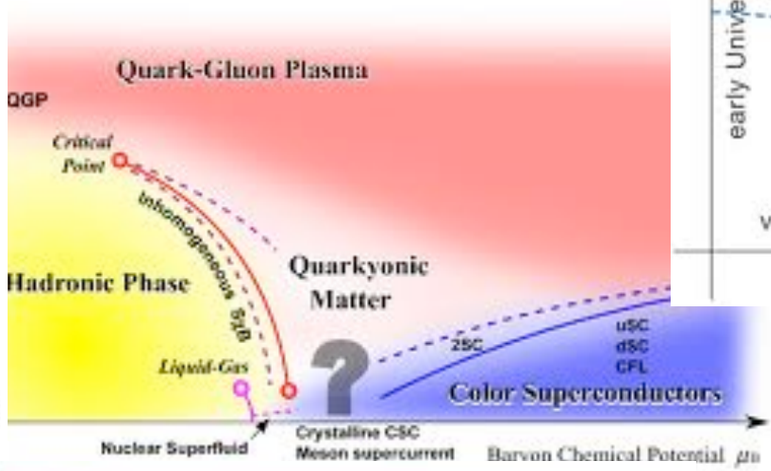

T
quark–gluon plasma (QGP)
170 MeV
(early universe)
(heavy–ion collisions)
hadronic (confined) phase
nuclear matter
(neutron stars)
non–CFL quark liquid
CFL
vacuum
310 MeV
μ

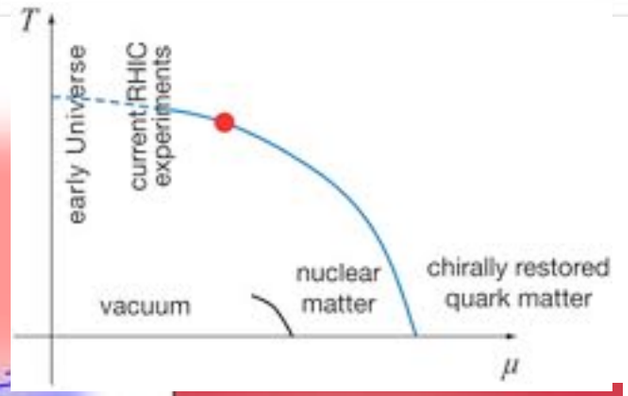

The Phases of QCD
Temperature
Early Universe
Quark-Gluon Plasma
Critical Point
Hadron Gas
Color Superconductor
Vacuum
Nuclear Matter
Neutron Stars
Baryon Chemical Potential


Temperature T [MeV]
Quarks and Gluons
Critical point?
Deconfinement and chiral transition
Hadrons
RHIC, LHC
FAIR SIS 300
Neutron stars
Color Super-conductor?
Nuclei
Net Baryon Density

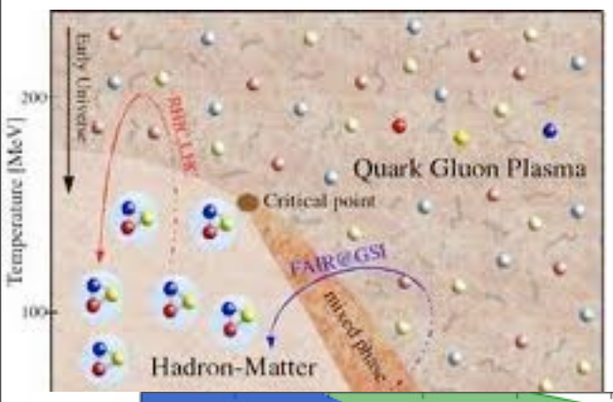

T
RHIC
QGP
confined
color super-conducting quark matter
nuclear
compact star
μ

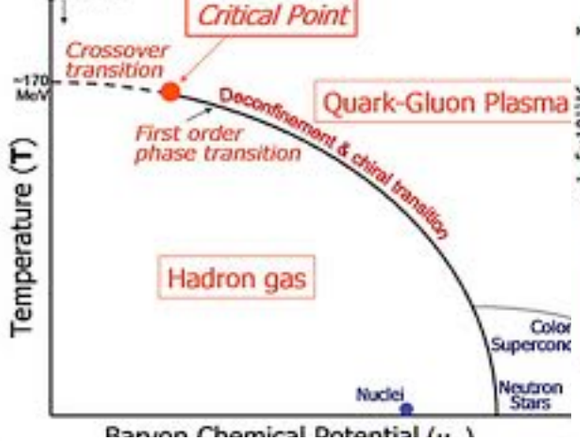

quark-gluon plasma
temperature T
RHIC
SPS
Lattice QCD
Critical Point
Dense Baryonic Medium
AGS
SIS
Dilute Hadronic Medium
atomic nuclei
neutron stars

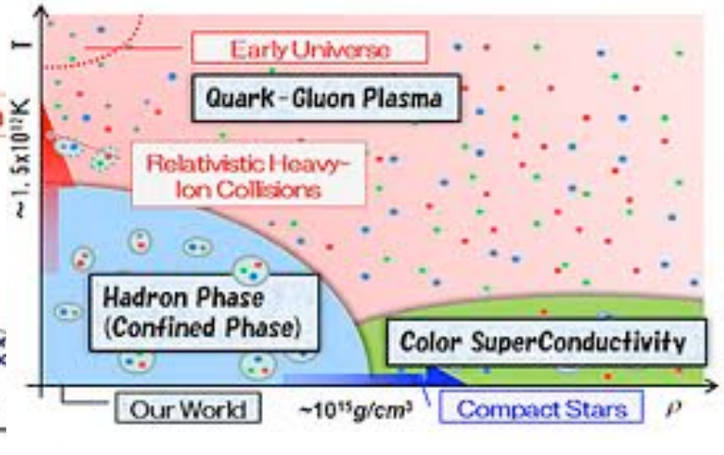

Quark-Gluon Plasma
QGP
Critical Point
Inhomogeneous χSB
Liquid-Gas
Hadronic Phase
Quarkyonic Matter
2SC
uSC
dSC
CFL
Color Superconductors
Nuclear Superfluid
Crystalline CSC
Meson supercurrent
Baryon Chemical Potential μB

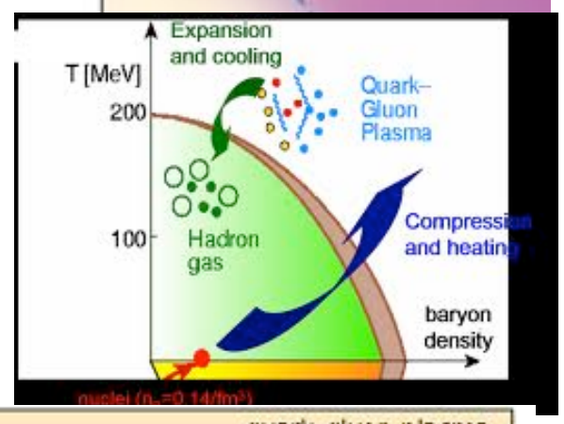

T
early Universe
current RHIC experiments
vacuum
nuclear matter
chirally restored quark matter
μ

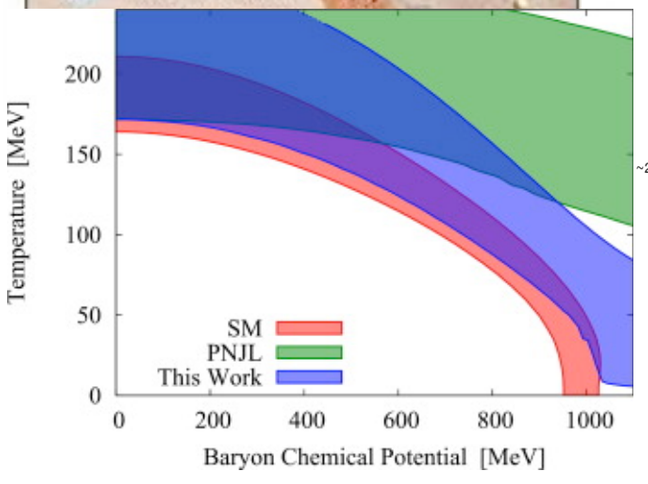

Quark-Gluon Plasma
T
Triple Point
Deconfinement
Tc
Chiral?

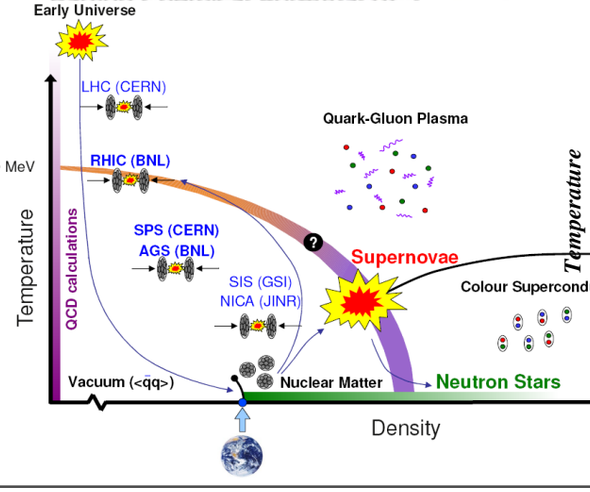

Early Universe
RHIC, LHC
Quark Gluon Plasma
Critical point
FAIR@GSI
mixed phase
Hadron-Matter
Temperature [MeV]

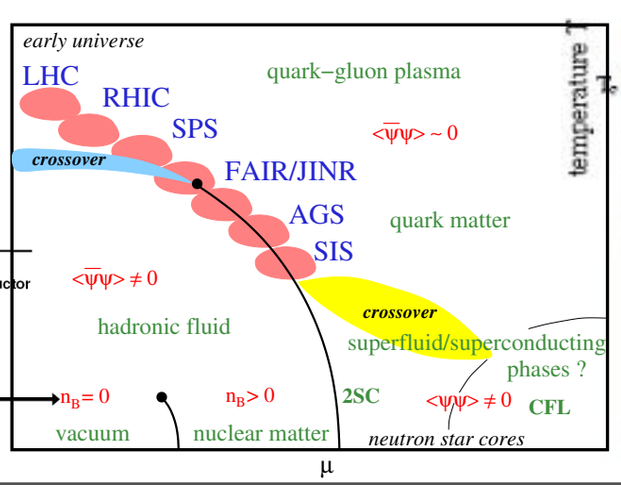

Critical Point
Crossover transition
First order phase transition
Deconfinement & chiral transition
Quark-Gluon Plasma
Hadron gas
Temperature (T)
Nuclei
Neutron Stars
Baryon Chemical Potential (μB)

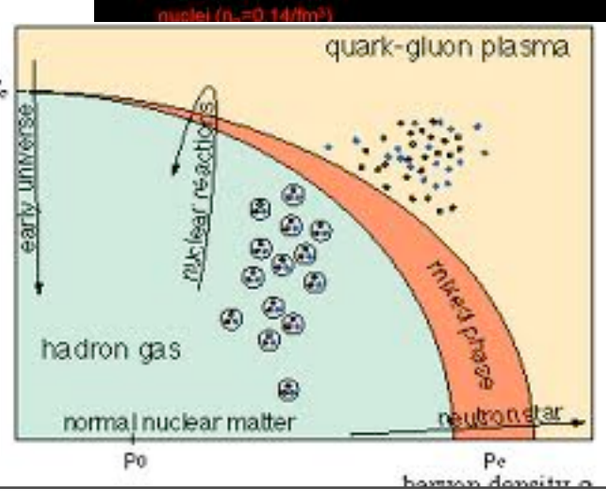

Early Universe
Quark - Gluon Plasma
Relativistic Heavy-Ion Collisions
Hadron Phase (Confined Phase)
Color SuperConductivity
Our World
~10^15 g/cm^3
Compact Stars
ρ


Expansion and cooling
T [MeV]
Quark–Gluon Plasma
Hadron gas
Compression and heating
baryon density
nuclei (n0=0.14/fm3)

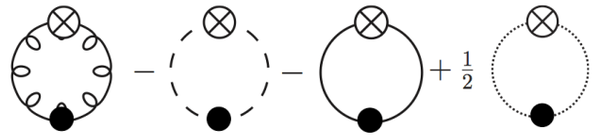

Temperature [MeV]
SM
PNJL
This Work
Baryon Chemical Potential [MeV]


Early Universe
LHC (CERN)
Quark-Gluon Plasma
RHIC (BNL)
~200 MeV
SPS (CERN)
AGS (BNL)
Supernovae
SIS (GSI)
NICA (JINR)
Colour Superconductor
QCD calculations
Temperature
Vacuum (<qq>)
Nuclear Matter
Neutron Stars
Density

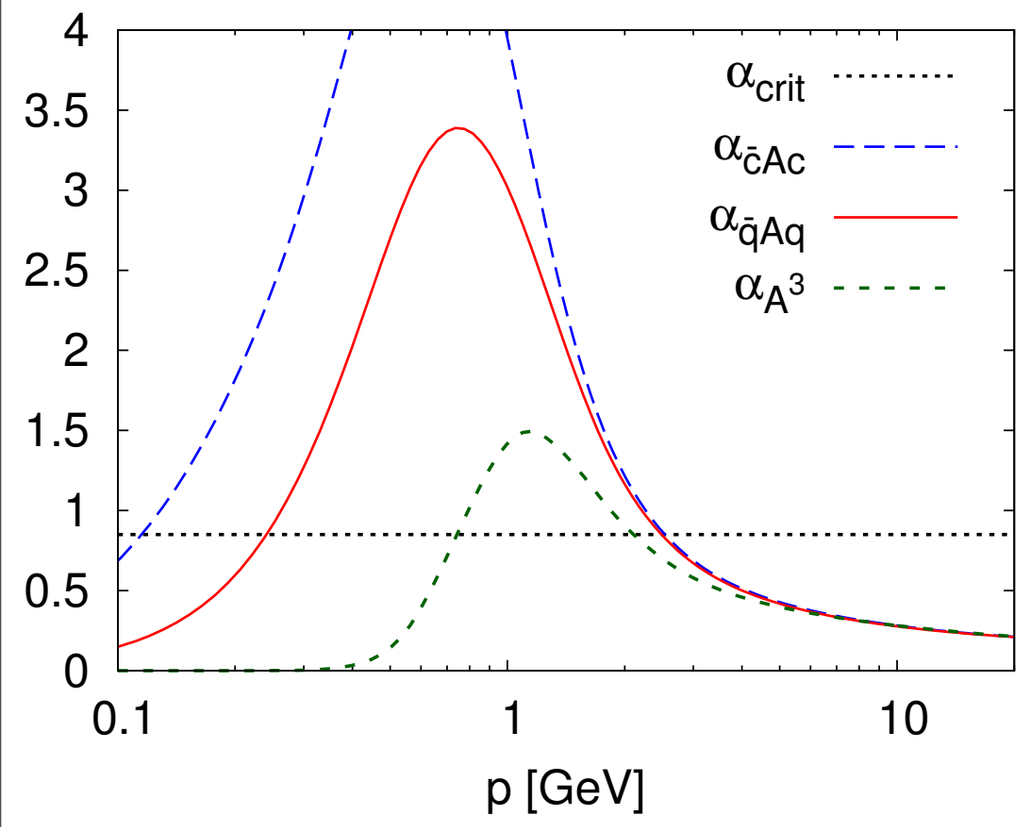

early universe
LHC
RHIC
SPS
FAIR/JINR
AGS
SIS
quark–gluon plasma
quark matter
crossover
<ψ̄ψ> ~ 0
<ψ̄ψ> ≠ 0
hadronic fluid
superfluid/superconducting phases ?
nB= 0
nB> 0
2SC
CFL
vacuum
nuclear matter
neutron star cores
temperature
μ


quark-gluon plasma
early universe
nuclear reactions
mixed phase
hadron gas
normal nuclear matter
neutron star
P0
Pc

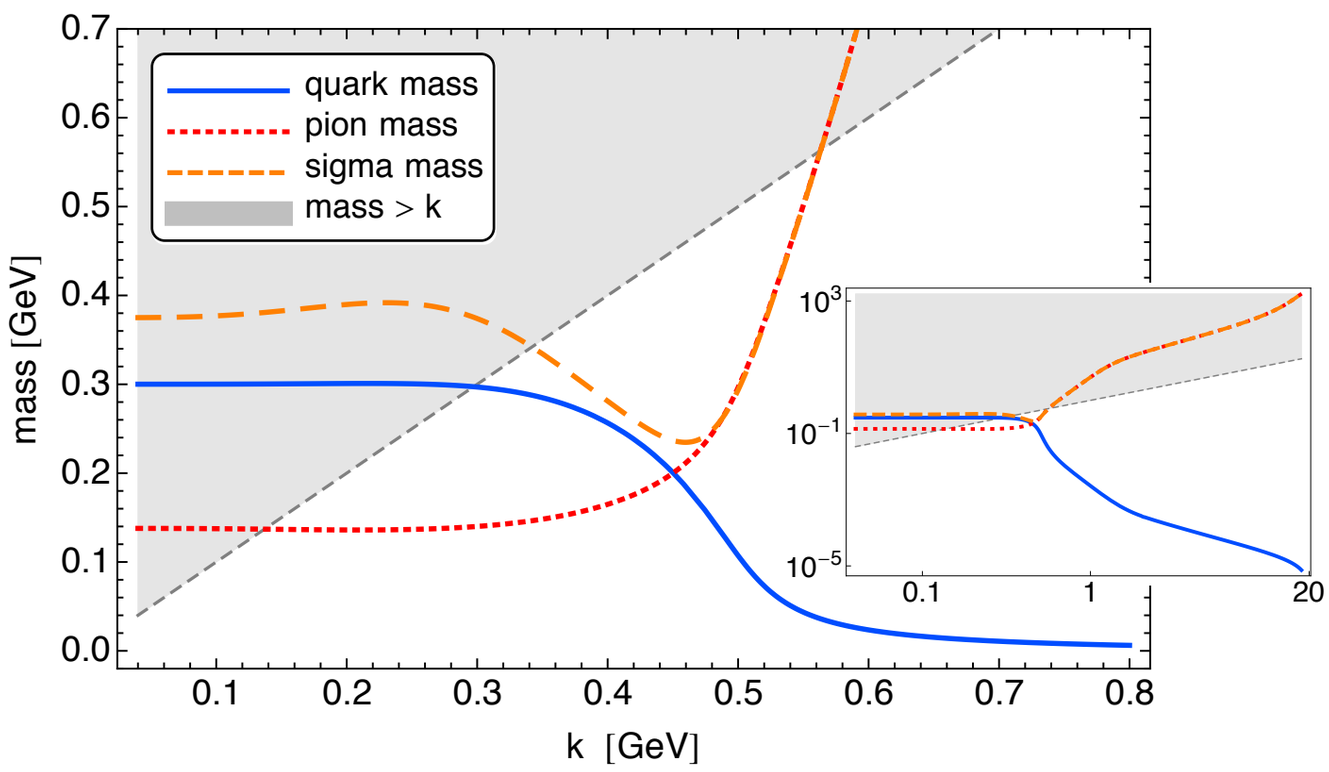
QCD

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}$$  $$+ \frac{1}{2}$$

Sequential decoupling of gluon, quark, sigma, pion fluctuations



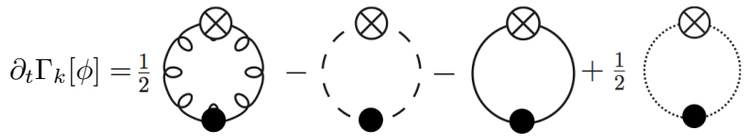
Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

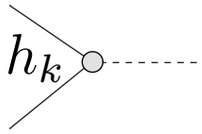


Braun, Fister, Haas, JMP, Rennecke, arXiv:1412.1045

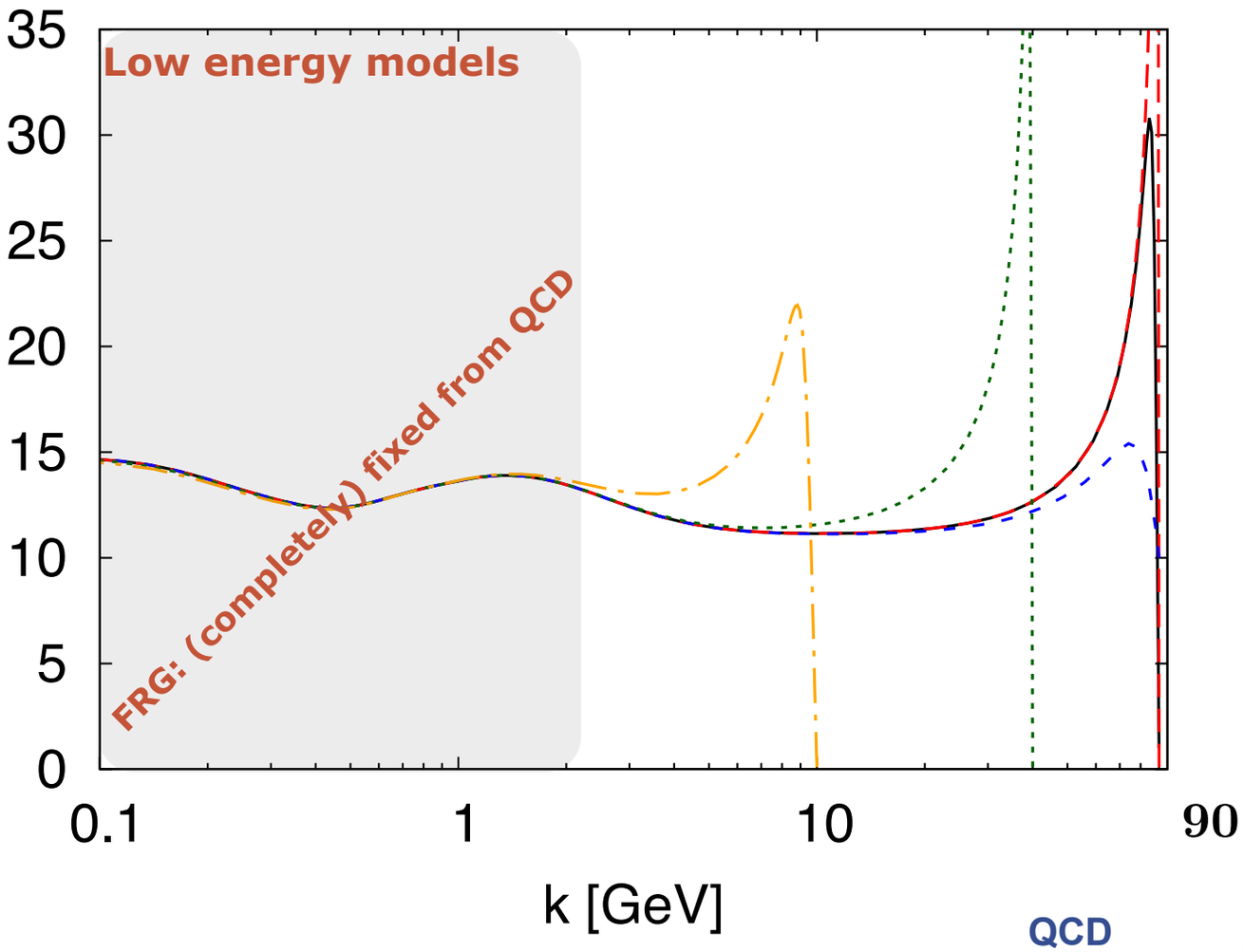
Rennecke, arXiv:1504.03585

QCD





quark-meson coupling

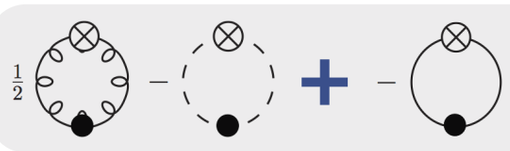


fQCD

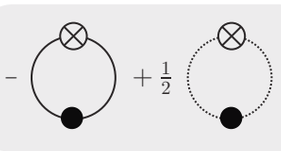
QCD — low energy models

PQM-model

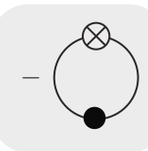
PNJL-model



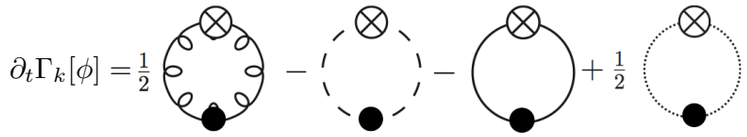
QM-model



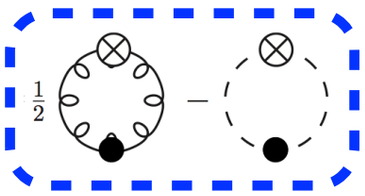
NJL-model

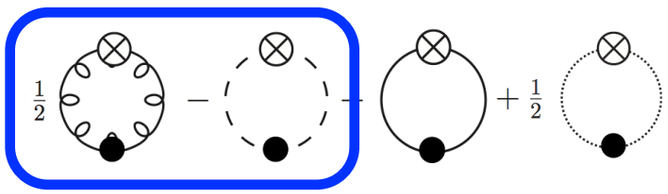


QCD



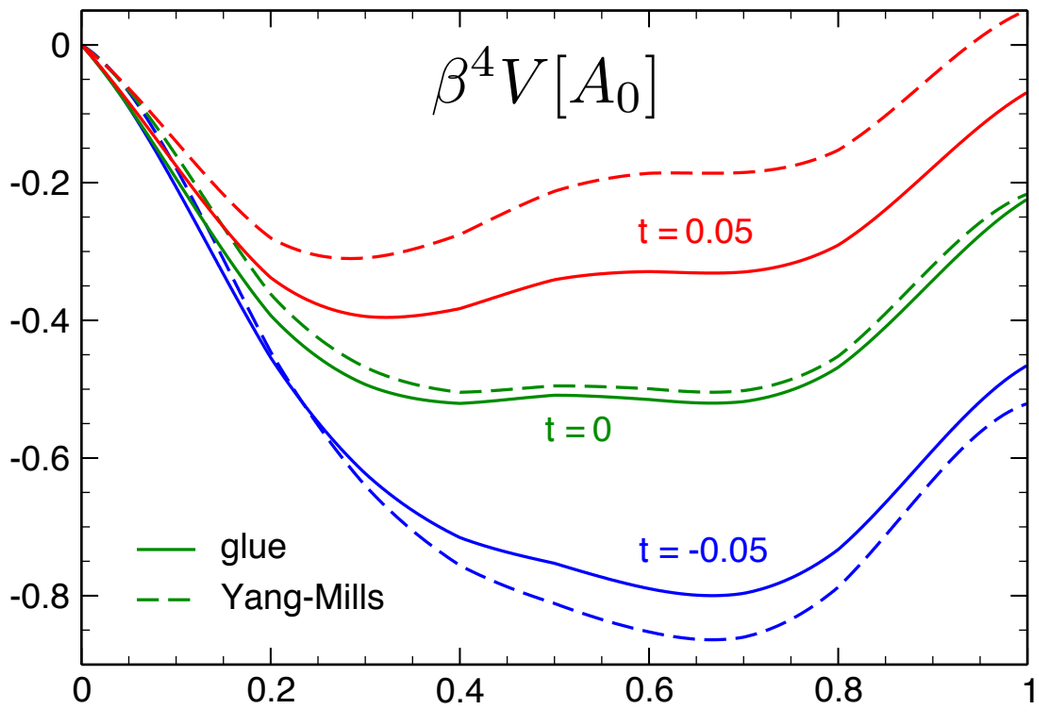
Yang-Mills





glue

Polyakov loop potential in full QCD

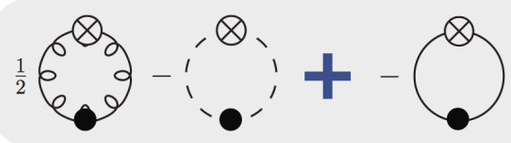


JMP, AIP Conf.Proc. 1343 (2011)

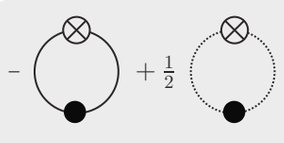
Haas, Stiele et al, PRD 87 (2013) 076004

PQM-model

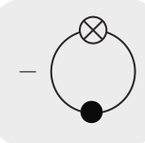
PNJL-model



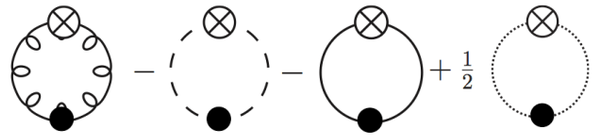
QM-model



NJL-model

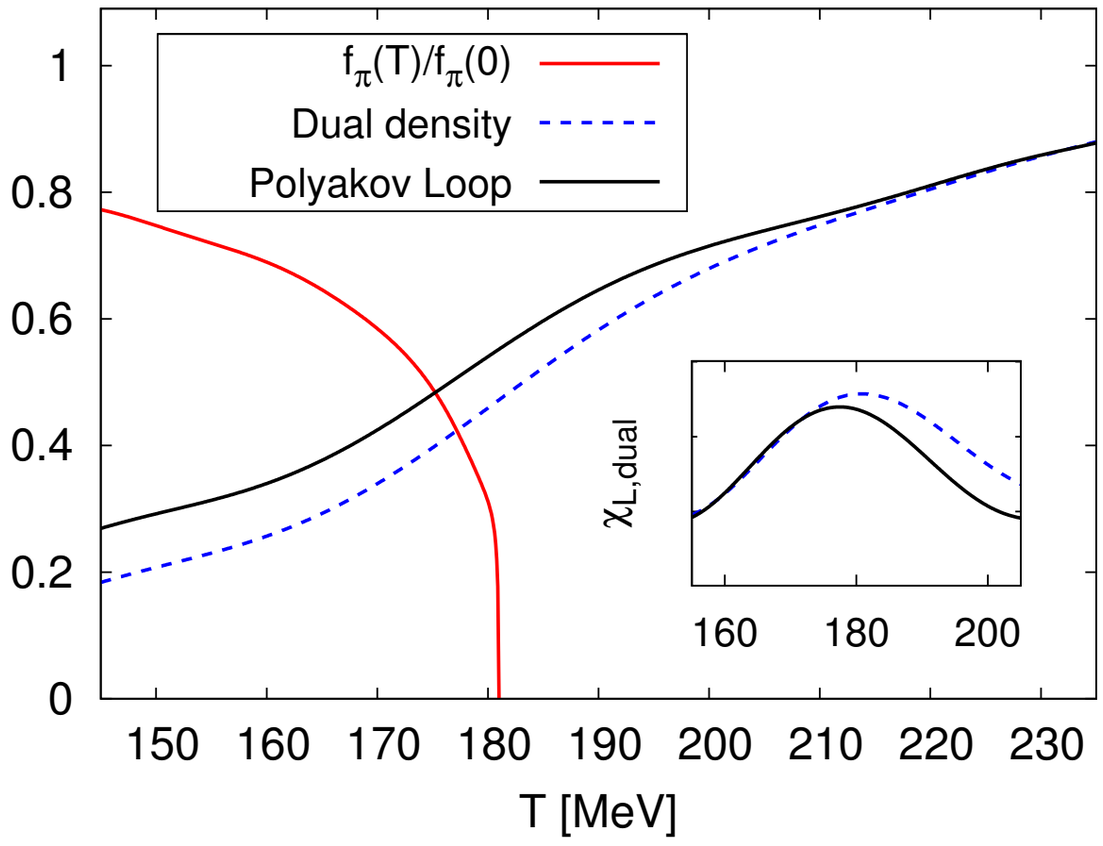


QCD

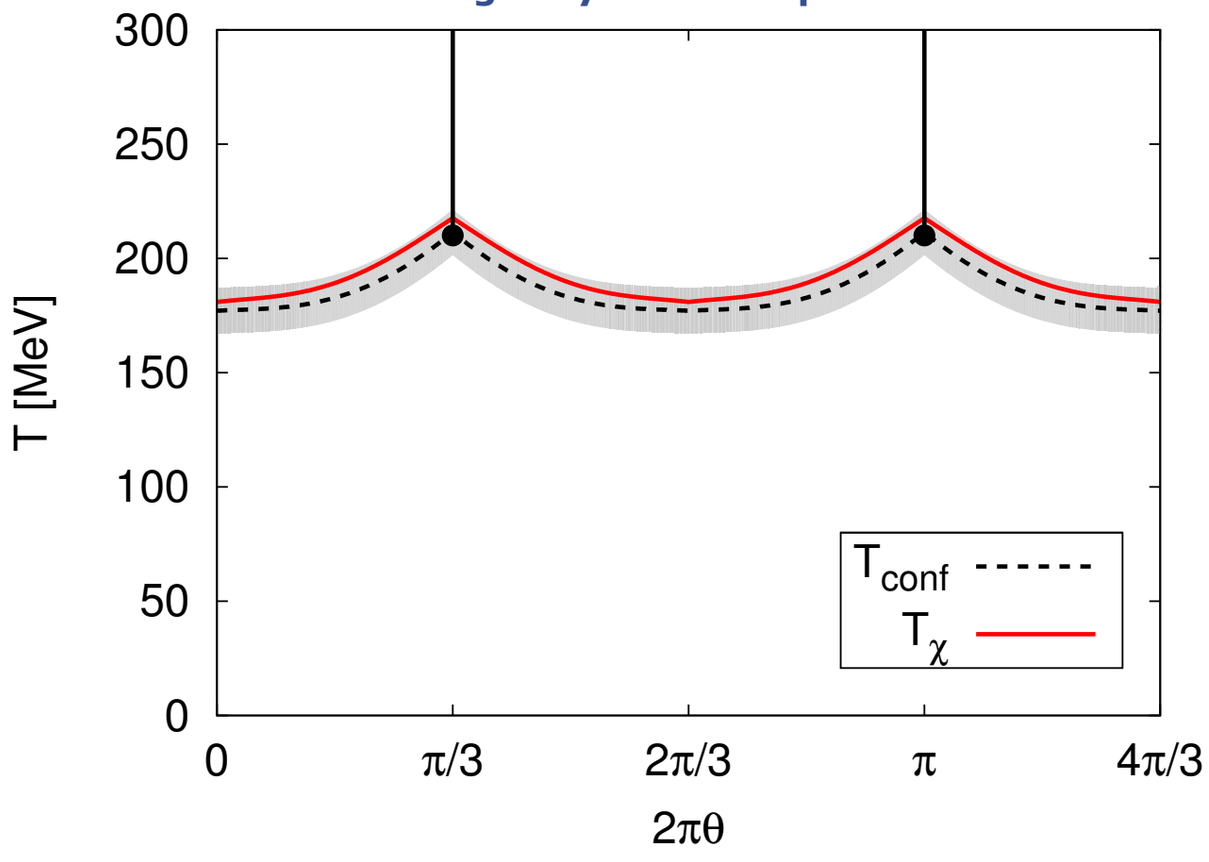
$$\partial_t \Gamma_k[\phi] = \tfrac{1}{2}$$  $$+ \tfrac{1}{2}$$

2 flavors & chiral limit

vanishing density



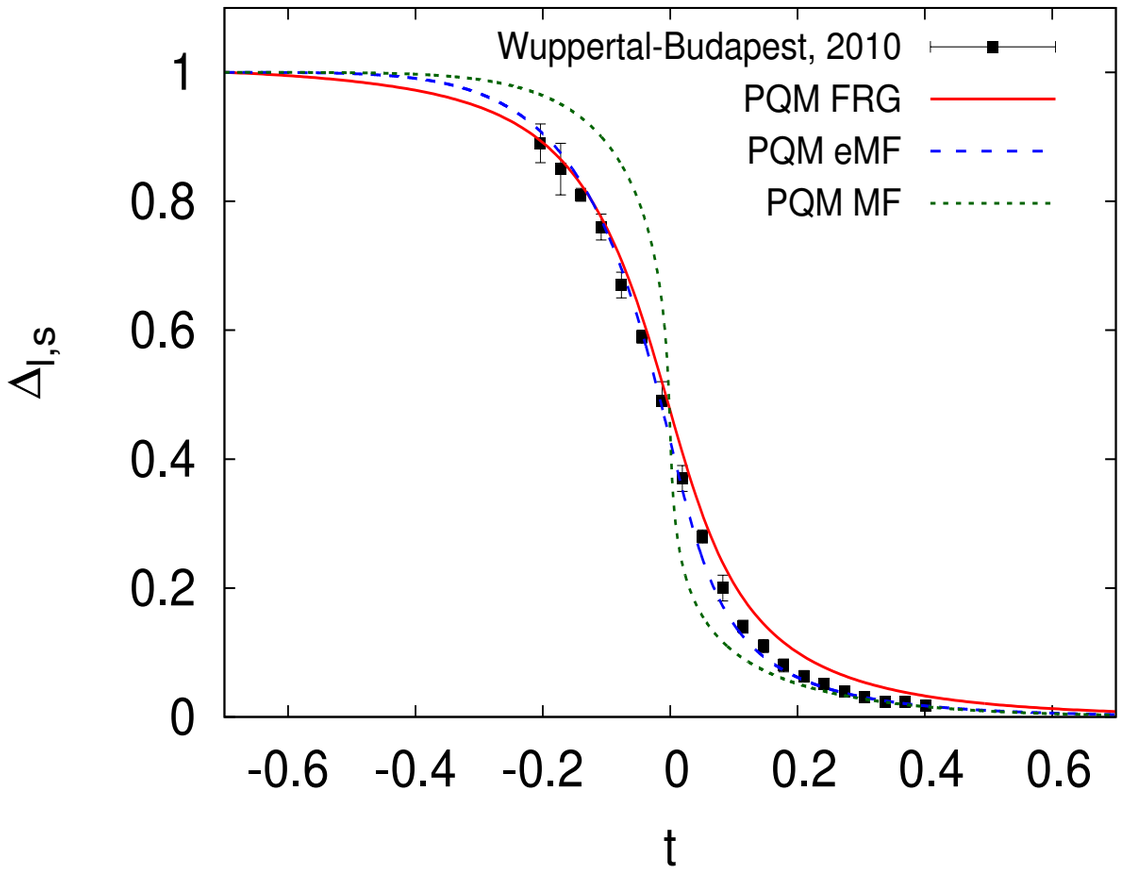
imaginary chemical potential



Braun, Haas, Marhauser, JMP, PRL 106 (2011) 022002

Thermodynamics

reduced chiral condensate

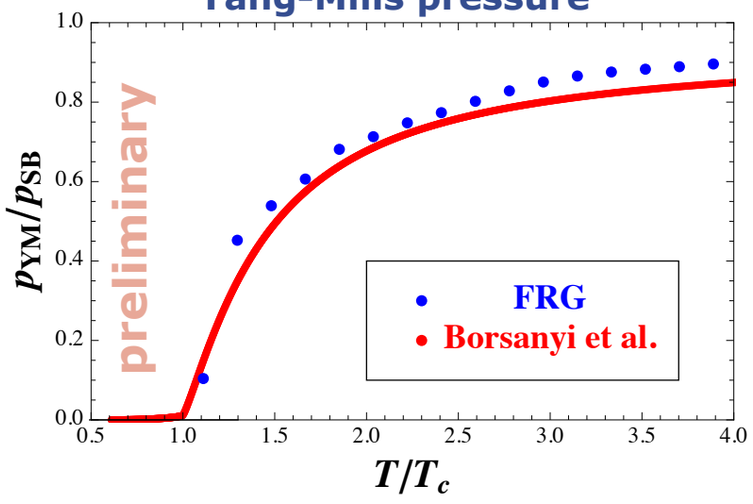


see talk of B.-J. Schaefer

2+1 flavor QCD - enhanced PQM-model

Kurtosis: Fu, JMP, in preparation

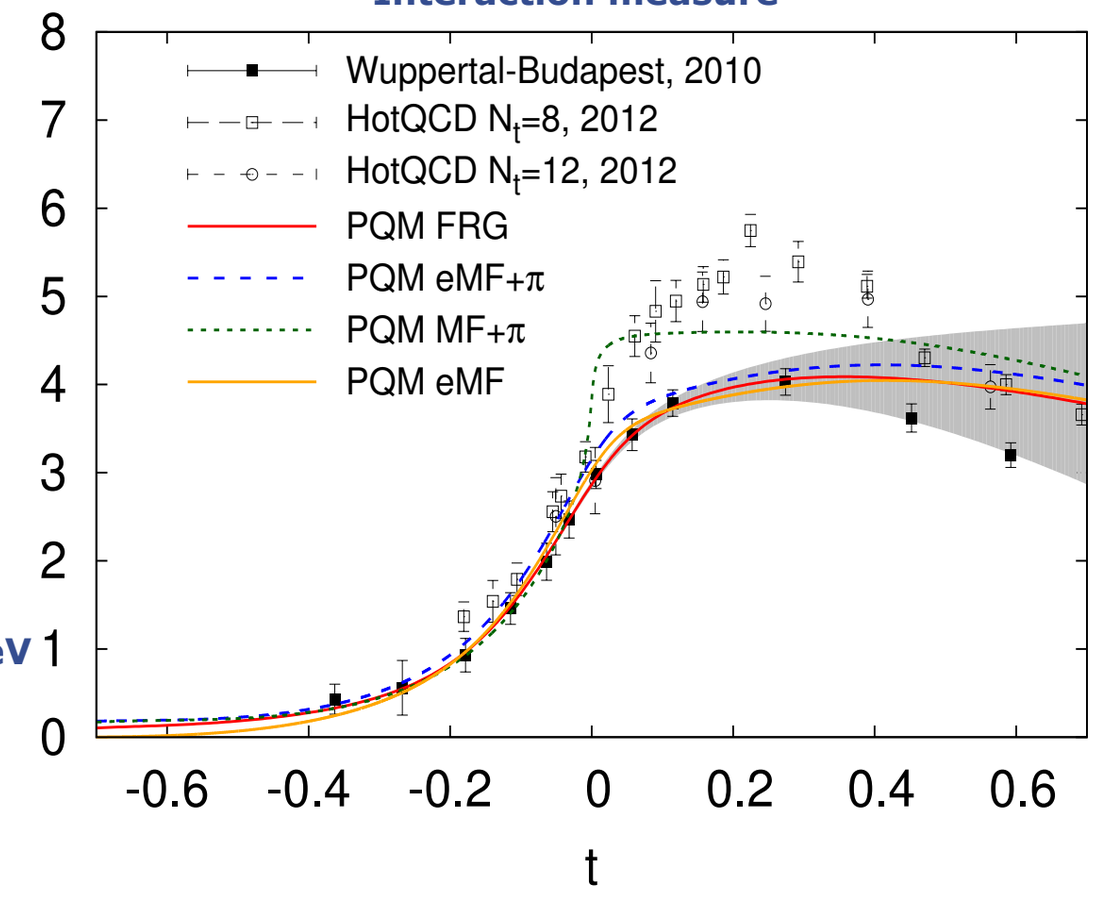
Yang-Mills pressure



Fister, JMP '11

Shaded area:
systematic error estimate
due to low initial UV scale 1 GeV

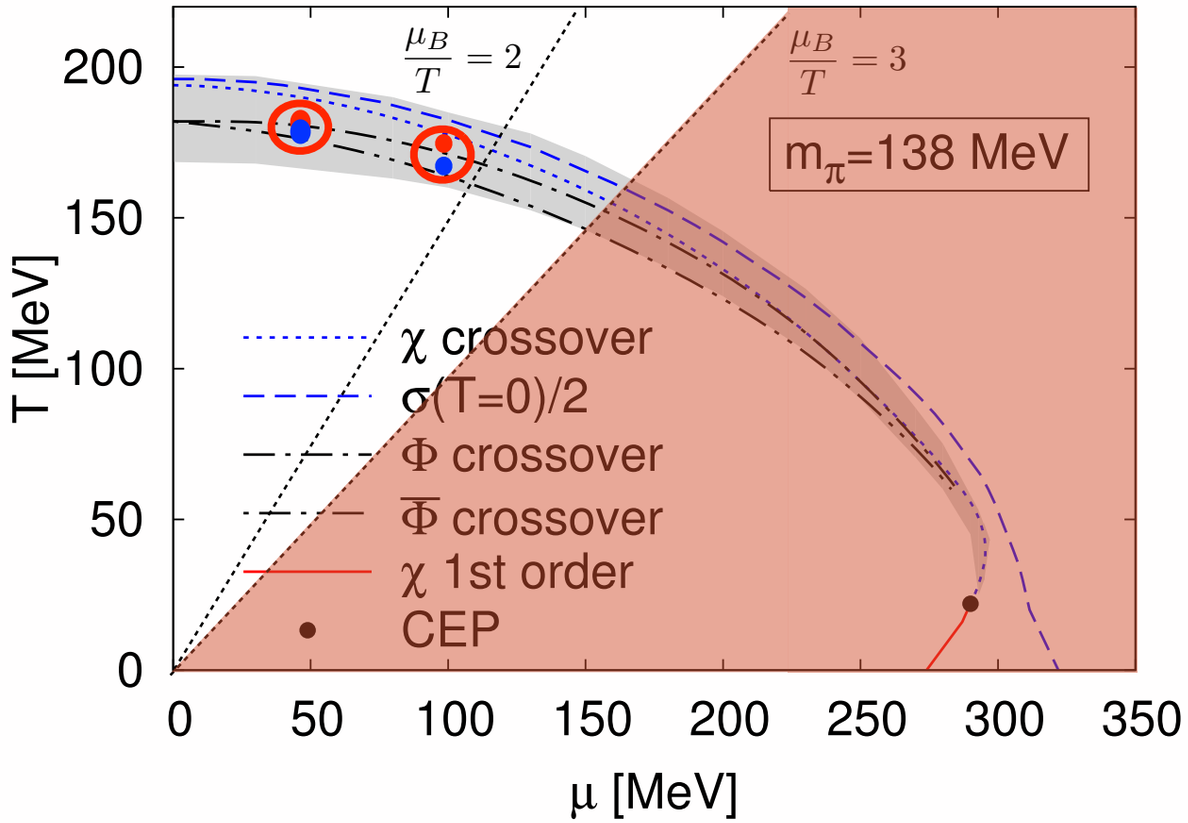
Interaction measure



Herbst, Mitter et al, PLB 731 (2014) 248-256

Phase structure at finite density

Phase diagram of quantised 2-flavor PQM-model

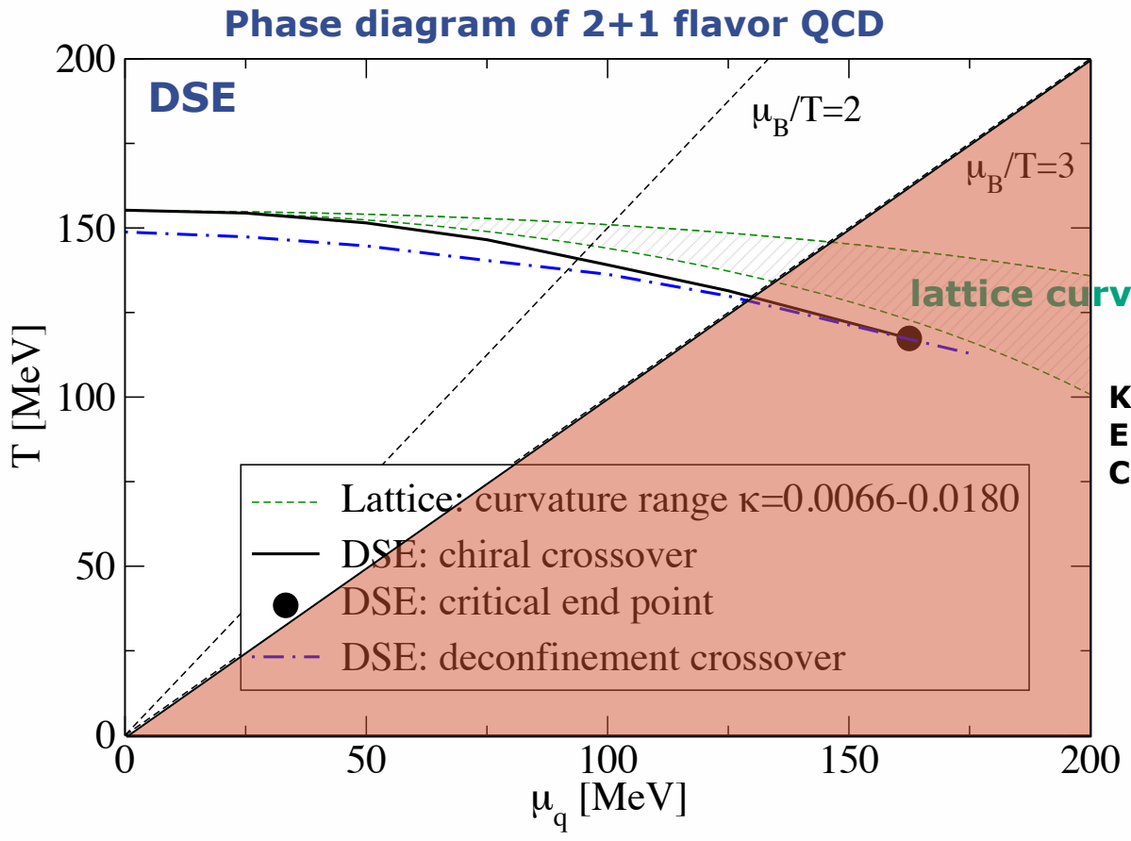


Herbst, JMP, Schaefer, PLB 696 (2011) 58-67
PRD 88 (2013) 1, 014007

FRG QCD results at finite density

Haas, Braun, JMP '09, unpublished

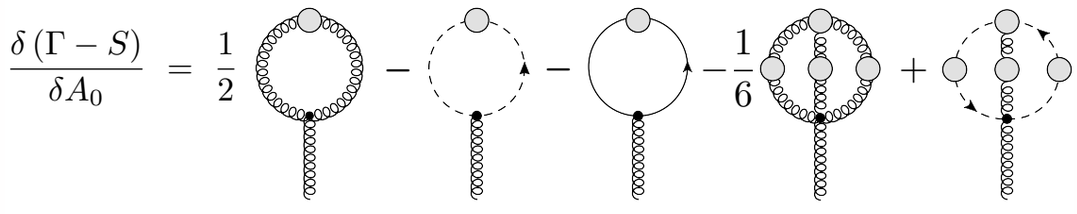
Phase structure at finite density



Kaczmarek at al. ’11
Endrodi, Fodor, Katz, Szabo ’11
Cea, Cosmai, Papa ’14

Fischer, Fister, Luecker, JMP, PLB732 (2014) 248

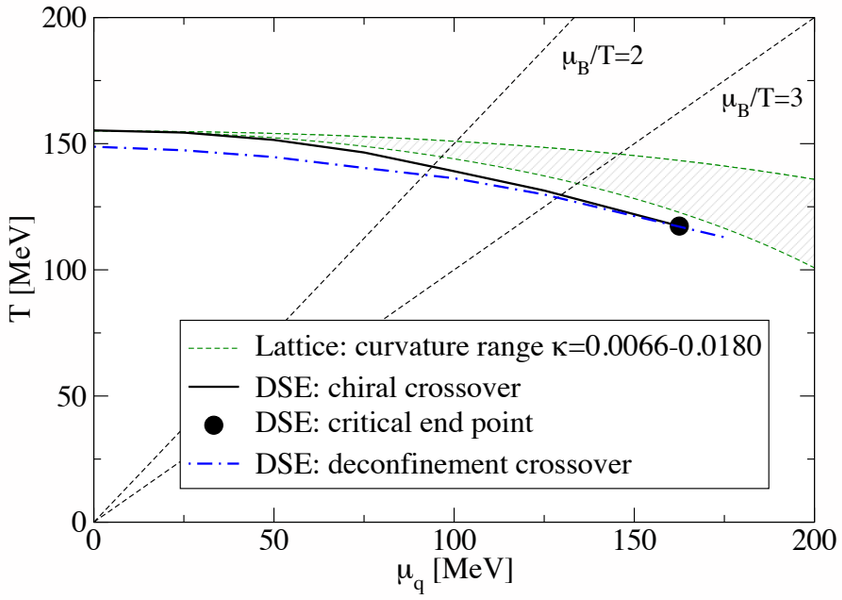
Fischer, Luecker, Welzbacher, PRD 90 (2014) 034022



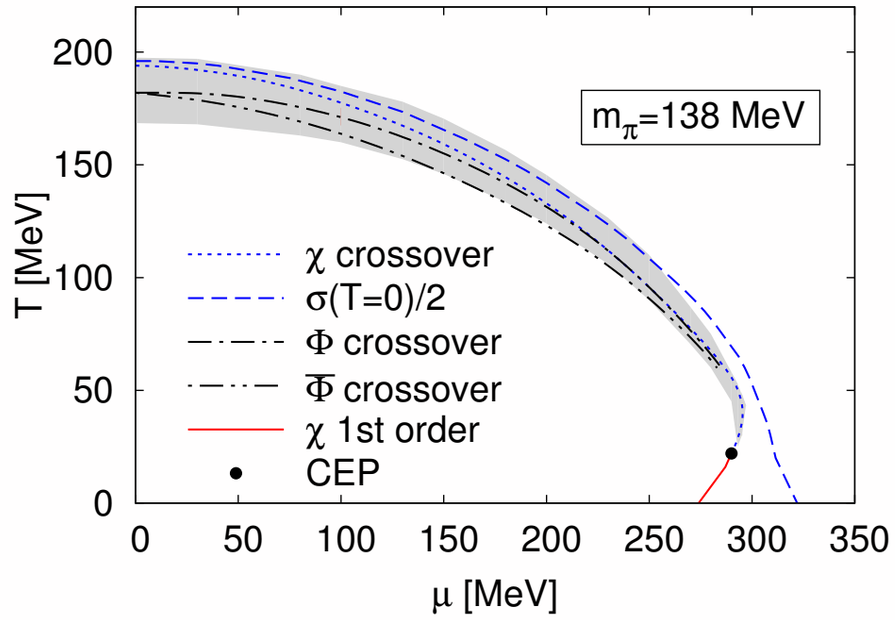
Fister, JMP, PRD 88 (2013) 045010

Phase structure at finite density

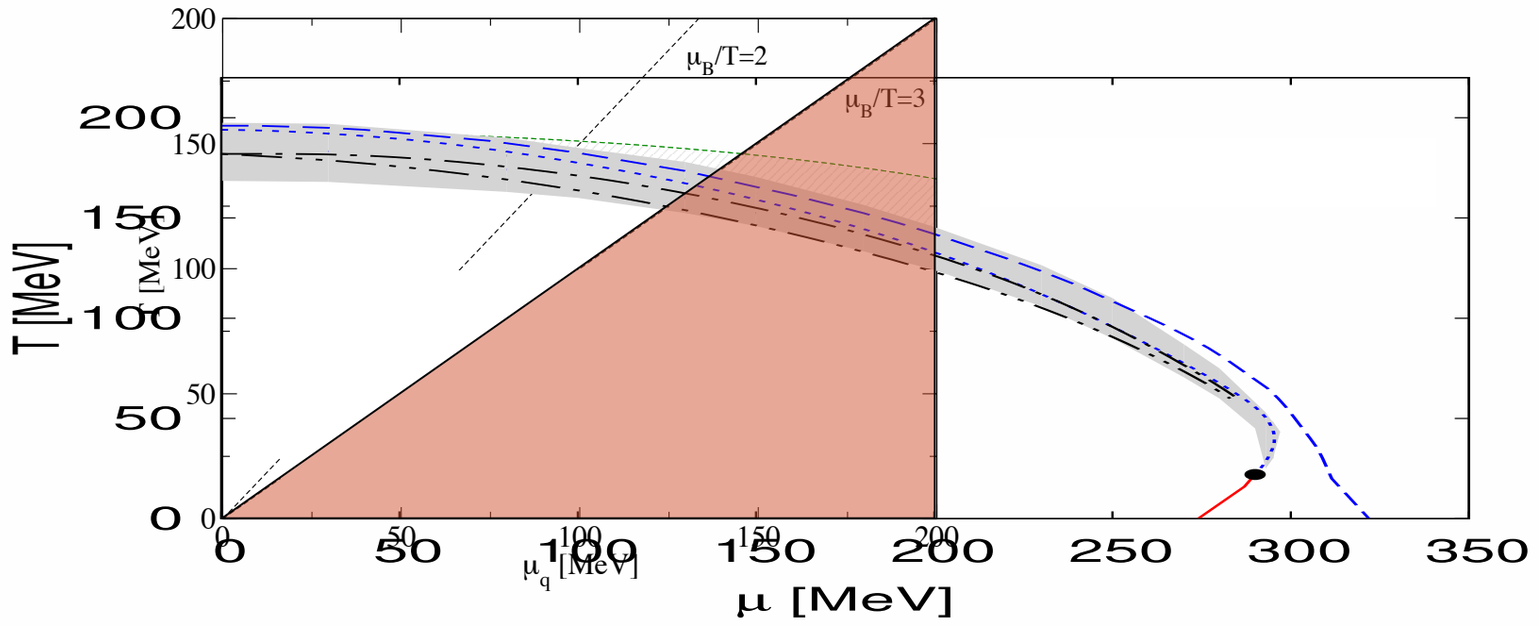
Phase diagram of 2+1 flavor QCD



Phase diagram of quantised 2-flavor PQM-model

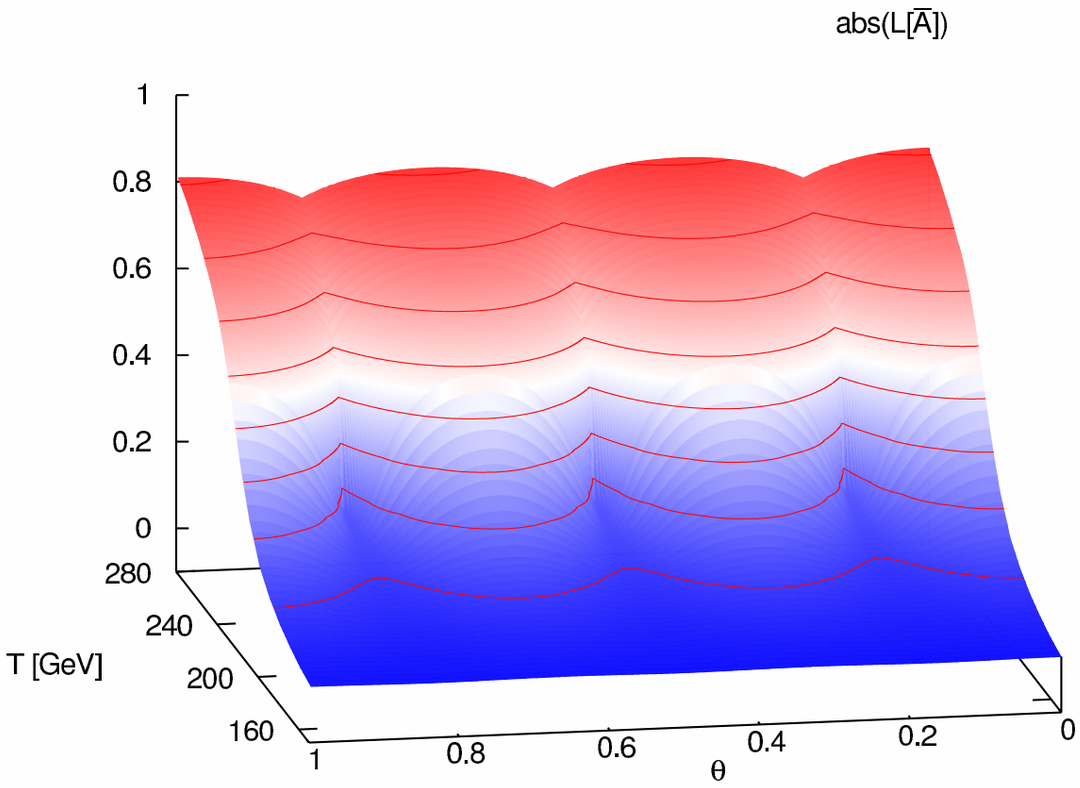


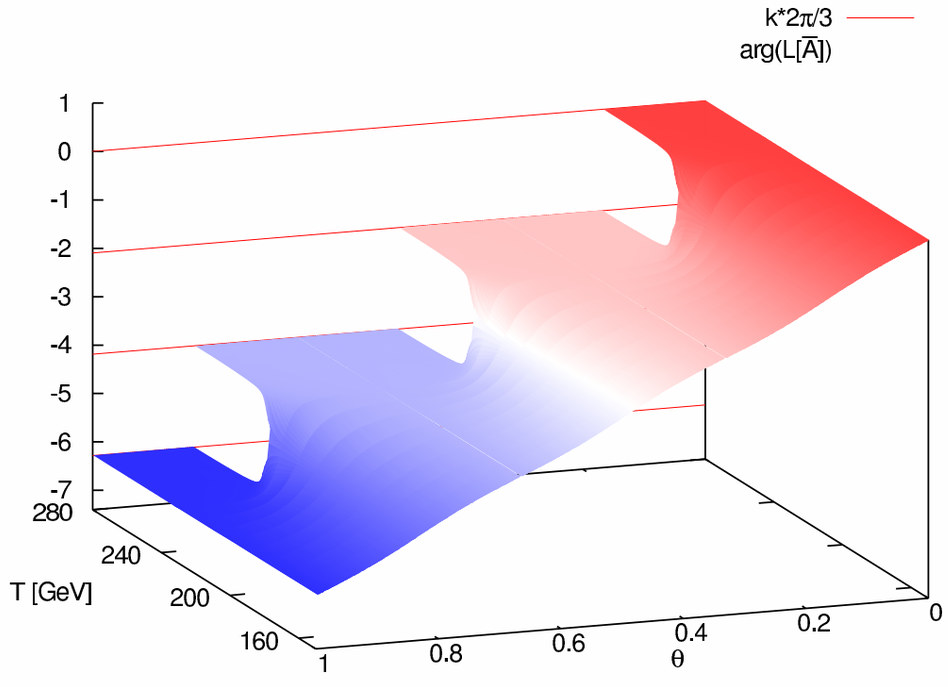
Comparison with 2 flavor vs 2+1 flavor scale matching of T_c



Phase structure at finite density for heavy quarks

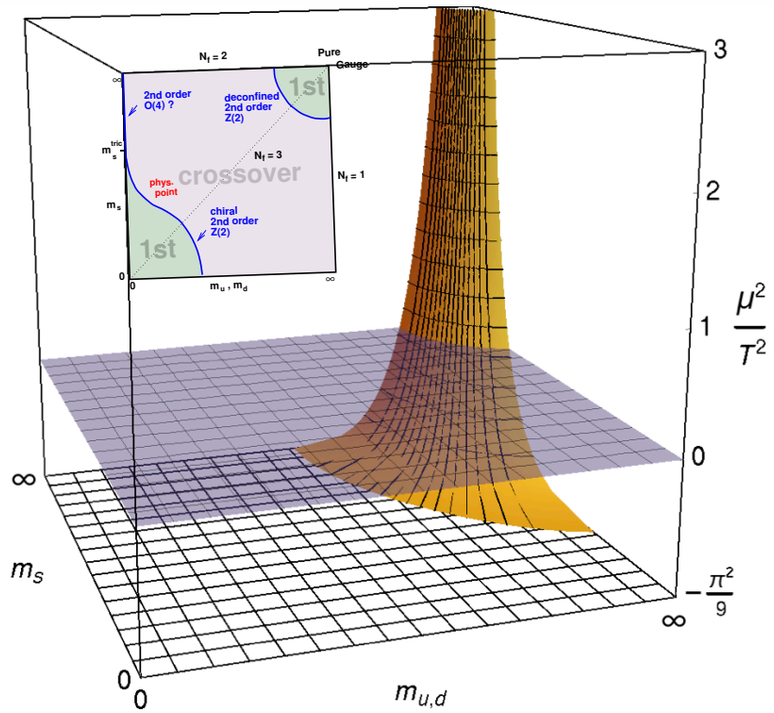
Polyakov loop at finite density



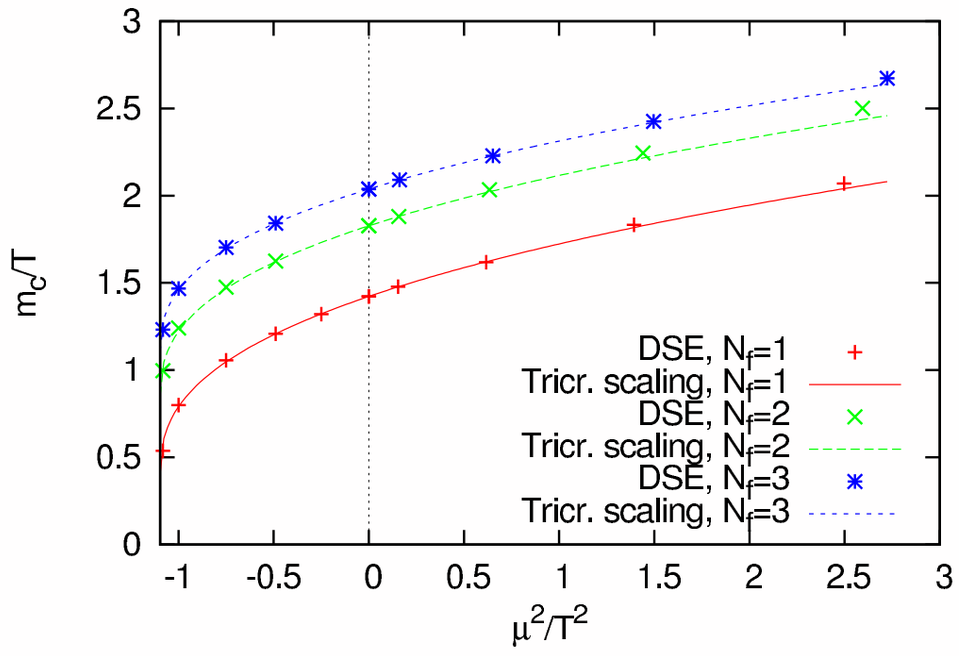


lattice see eg: Bonati, de Forcrand, D'Elia, Philipsen, Sanfilippo,PRD 90 (2014) 7, 074030

Critical surface at large masses



Tricritical scaling



Fischer, Luecker, JMP, PRD 91 (2015) 1, 014024

Outline

- **Vacuum QCD & the hadron spectrum**

- **Phase structure of QCD**

- **Spectral Functions & Transport Coefficients**

  Part II

- **Outlook**

... and now for something completely different ...

Gauge dynamics far from equilibrium

Gasenzer, McLerran, JMP, Sexty, Nucl.Phys. A930 (2014) 163

see also talks of J. Berges, A. Dumitru, S. Schlichting

Abelian Higgs model in 2+1 dimensions

Classical action:

$$S[A_\mu, \phi] = -\int_x \left[\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^* D^\mu \phi + V(\phi) \right]$$

ϕ **Higgs**

phase $\dfrac{\phi}{|\phi|} = e^{i\,\varphi}$

Abelian Higgs model in 2+1 dimensions

Classical action:

$$S[A_\mu, \phi] = -\int_x \left[\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^* D^\mu \phi + V(\phi)\right]$$

ϕ **Higgs**

phase $\dfrac{\phi}{|\phi|} = e^{i\,\varphi}$

Classical action of Yang-Mills theory in diagonalisation gauges:

$$S_{\rm YM} \simeq \frac{1}{2}\int_x \mathrm{tr} F^2_{\bar\mu\bar\nu} + \frac{1}{2}\int_x \mathrm{tr}\,(D_{\bar\mu} A_2)^2$$

$A_2 = A_2^c(x_0, x_1)$

Wilson loop

$$\mathcal{W}_2 = \mathcal{P}\exp\left\{ i \int_0^{L_2} dx_2\, A_2(x)\right\} = \exp\{i\,\phi\}$$

Vortex winding

$$n(\mathcal{S}) = \frac{1}{16\pi\, i}\oint_{\mathcal{S}} d^2x\, \epsilon_{ij} \mathrm{tr}\,\hat\phi\,\partial_i\hat\phi\,\partial_j\hat\phi$$

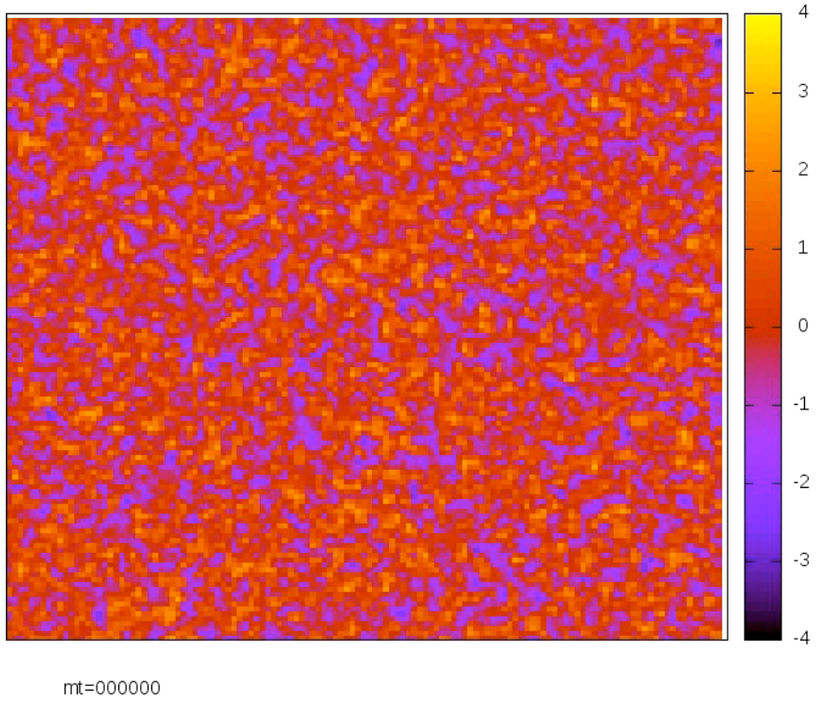
phase

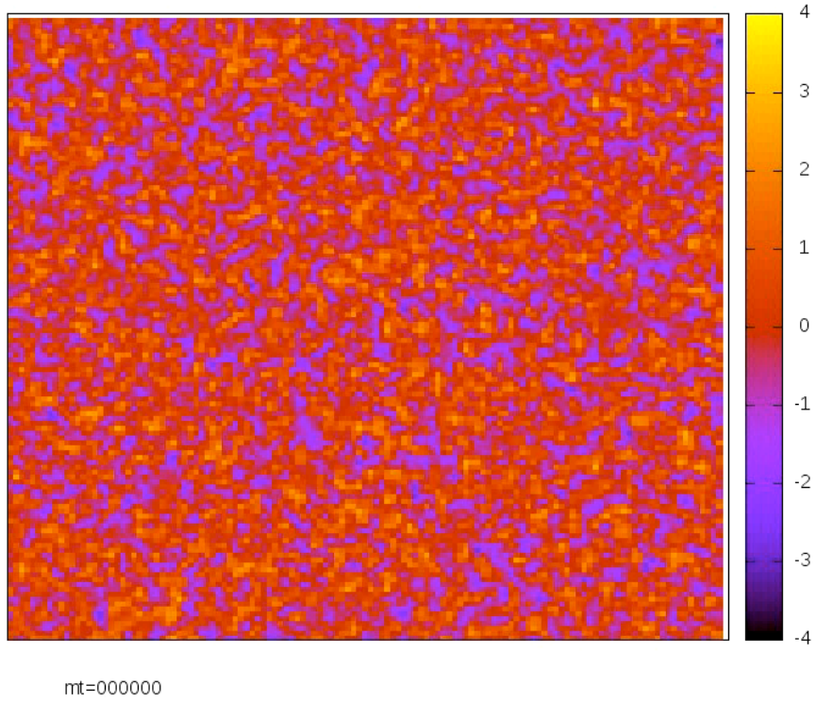
$$\hat\phi = \frac{\phi}{\|\phi\|}$$

Quiz

Complex scalar vs Abelian Higgs

phase φ of scalar field





'tachyonic' initial conditions

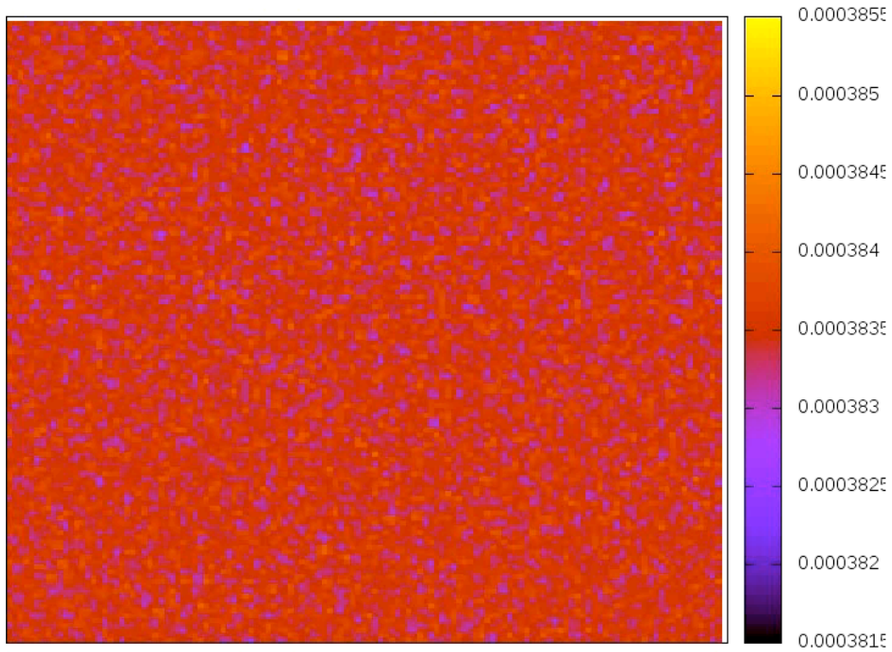
classical statistical lattice simulations

Which is which?

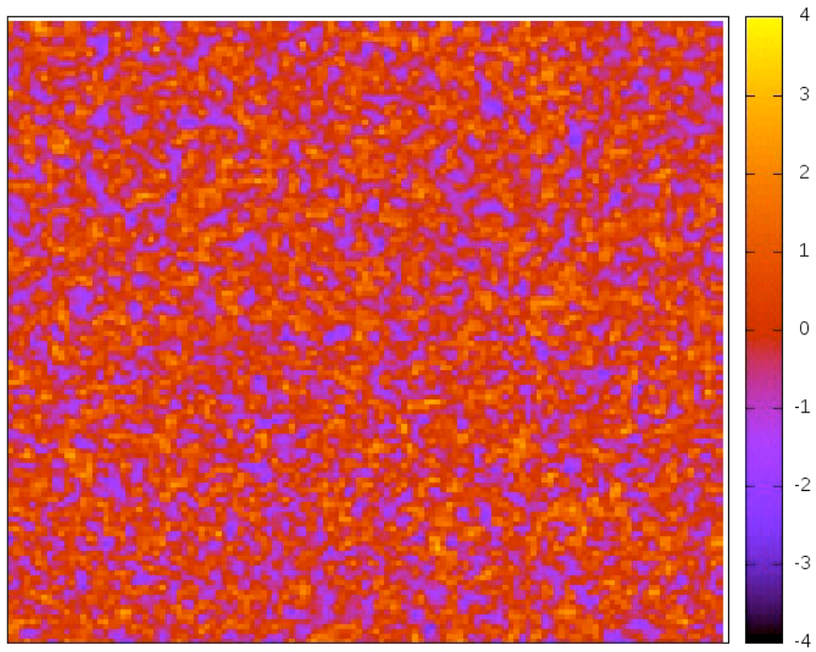
Gasenzer, McLerran, JMP, Sexty, Nucl.Phys. A930 (2014) 163

Vortex dynamics

magnetic field



phase of Higgs



'tachyonic' initial conditions

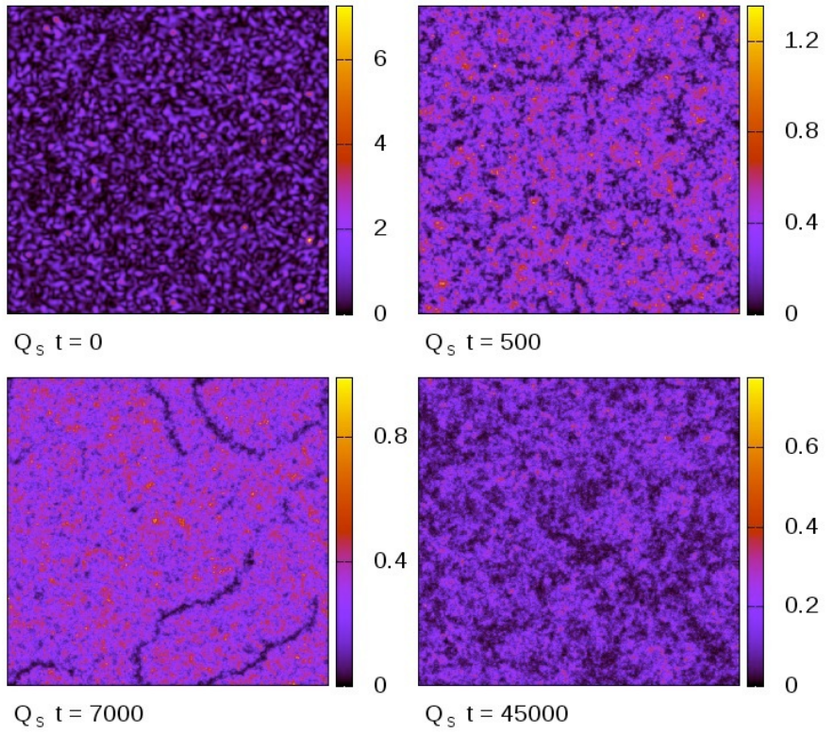
classical statistical lattice simulations

Gasenzer, McLerran, JMP, Sexty, Nucl.Phys. A930 (2014) 163

Charge

'overpopulation' initial conditions

modulus of Higgs

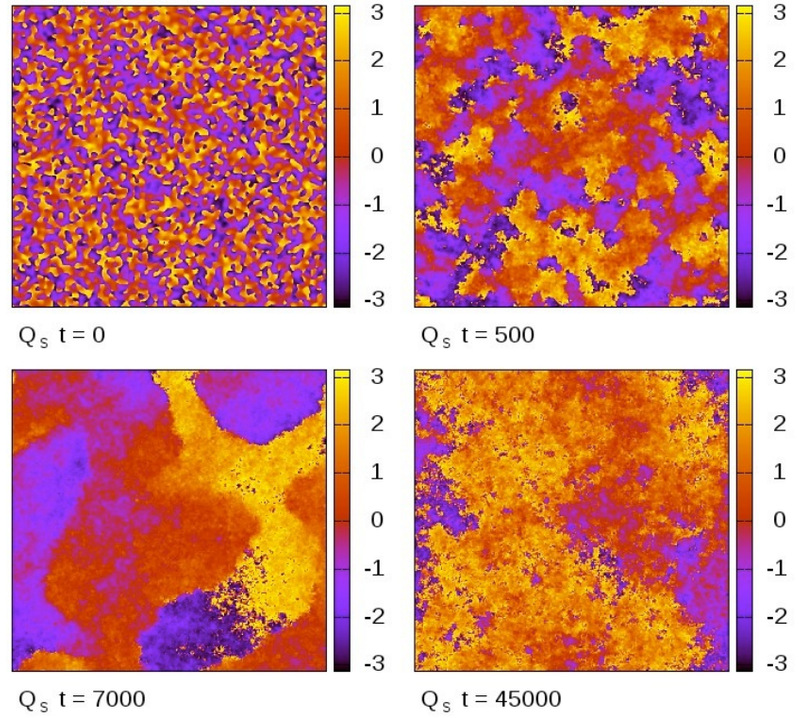


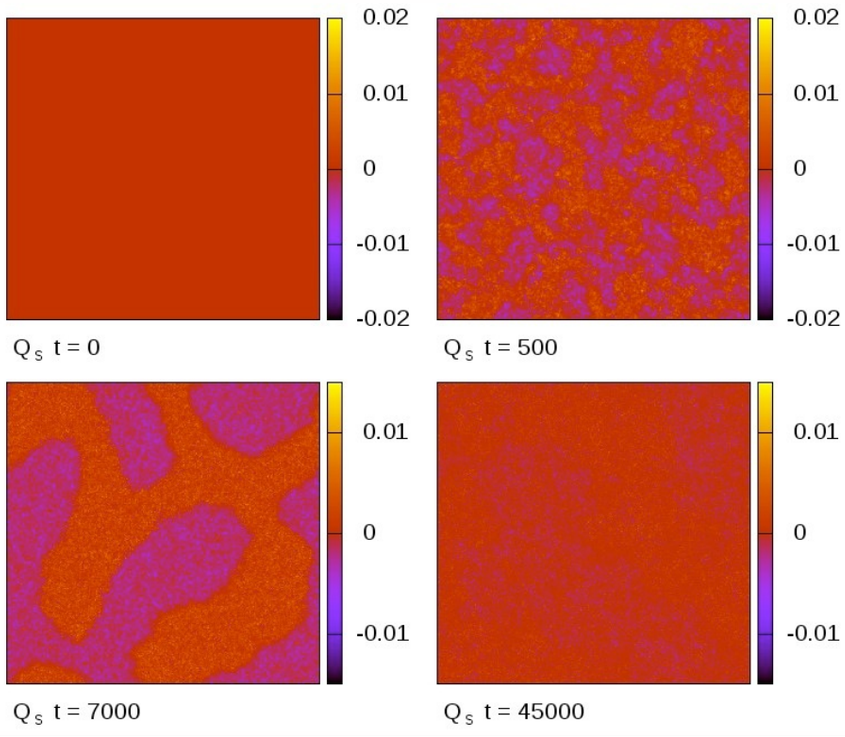
$$\xi = 0.025$$

coupling

$$\xi = \frac{6e^2}{\lambda}$$

relative phase





charge

$$\varphi^U(\vec{x},t) = \arg(G^U(\vec{0},\vec{x},t))$$

relative phase

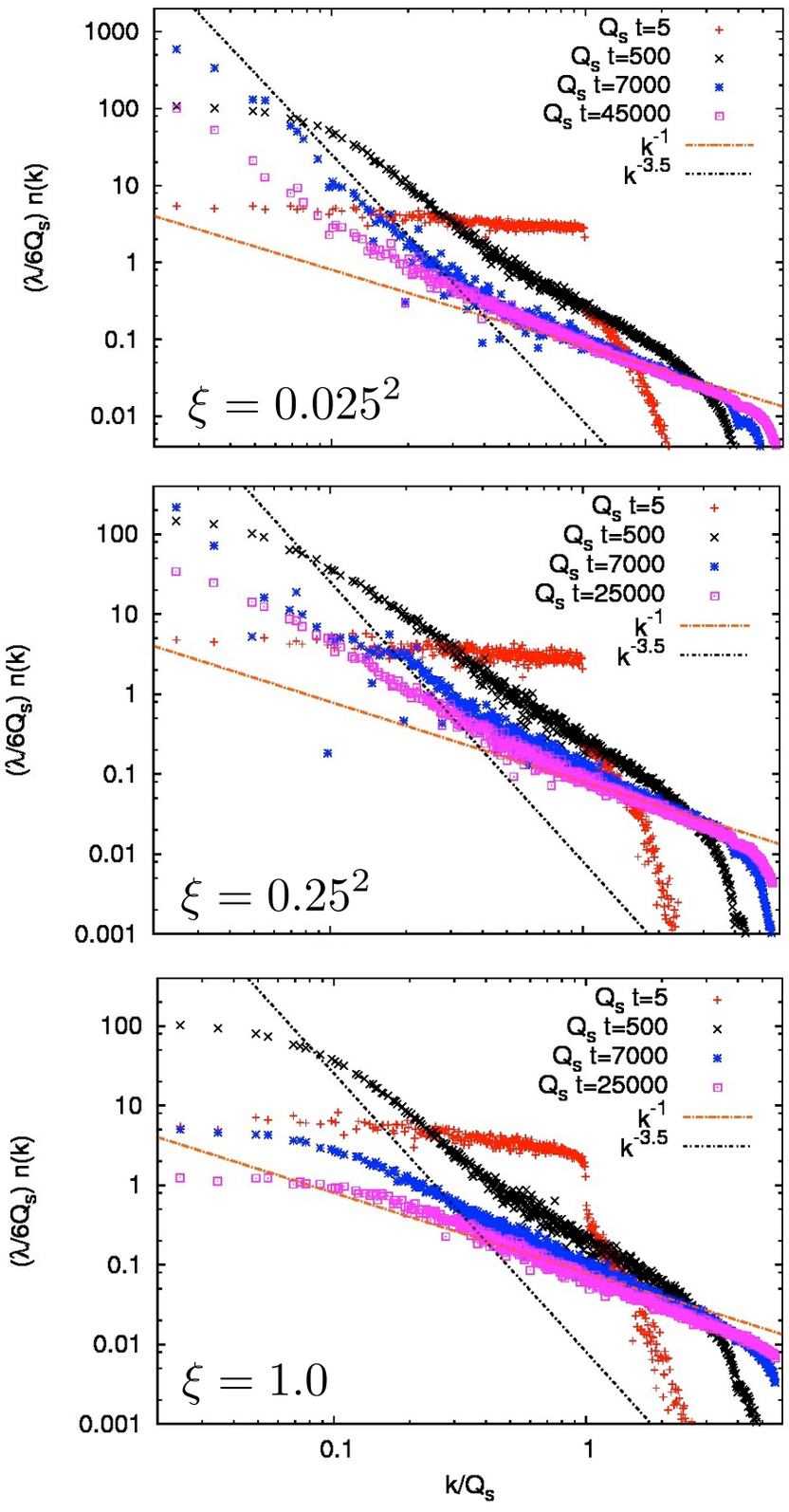
$$G^U(\vec{x},\vec{y},t) = \langle \phi(\vec{x},t) U(\vec{x},\vec{y},t) \phi(\vec{y},t)^* \rangle_{\mathrm{cl}}$$

parallel transport U

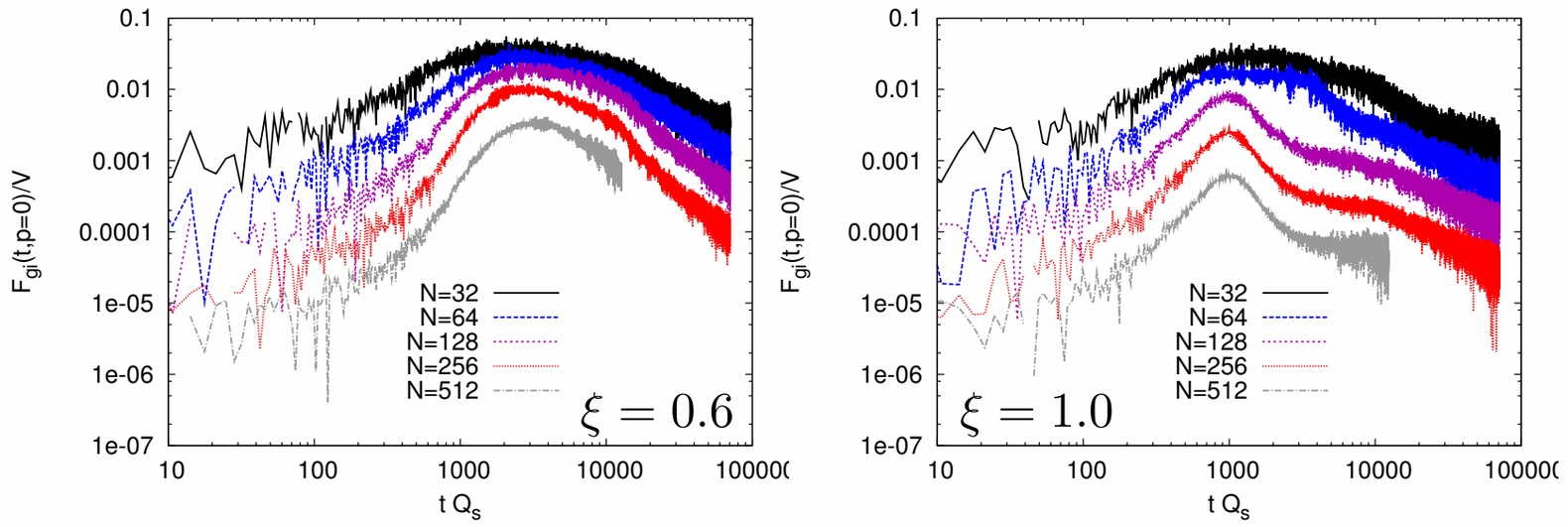
Gasenzer, McLerran, JMP, Sexty, Nucl.Phys. A930 (2014) 163

Occupation number & 'condensate'

'overpopulation' initial conditions

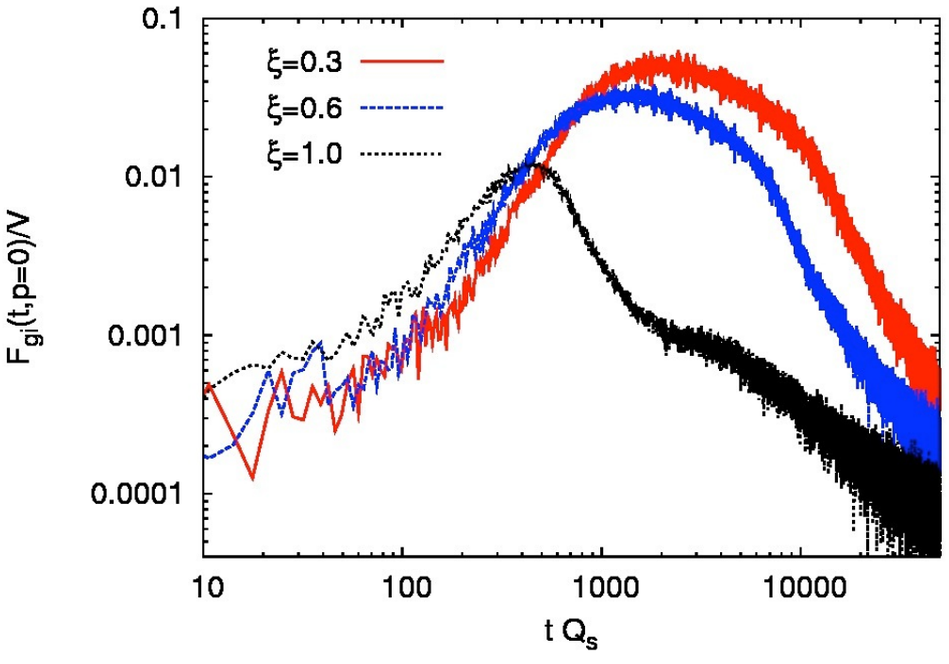


$$\frac{F_{\rm gi}(p=0)}{V} = \frac{1}{V^2}\int dxdy\phi^*(x)U(x,y)\phi(y)$$



coupling

$$\xi = \frac{6e^2}{\lambda}$$

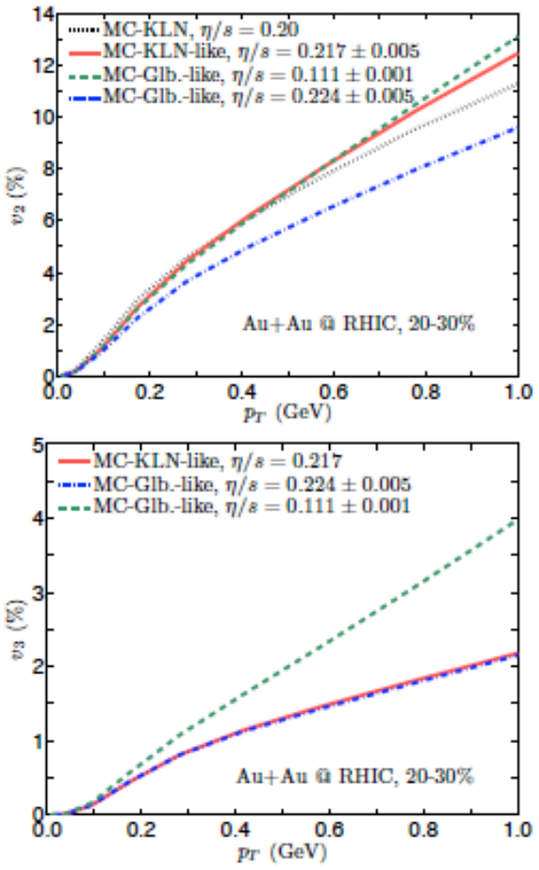


Gasenzer, McLerran, JMP, Sexty, Nucl.Phys. A930 (2014) 163

Dynamics

Shooting the elephant

Zhi Qiu and U. Heinz, to be published



Proof of principle calculation:

- Take ensemble of sum of deformed Gaussian profiles, $s(\boldsymbol{r}_\perp) = s_2(\boldsymbol{r}_\perp; \tilde{\varepsilon}_2, \psi_2) + s_3(\boldsymbol{r}_\perp; \tilde{\varepsilon}_3, \psi_3)$, with
  1. equal Gaussian radii $R_2^2 = R_3^2 = 8\,\mathrm{fm}^2$ to reproduce $\langle r_\perp^2 \rangle$ of MC-KLN source for 20-30% AuAu
  2. $\tilde{\varepsilon}_2$ and $\tilde{\varepsilon}_3$ adjusted such that
     - $\bar{\varepsilon}_{2,3} = \langle \varepsilon_{2,3} \rangle_{\mathrm{KLN}}^{20-30\%}$ ("MC-KLN-like")
     - $\bar{\varepsilon}_{2,3} = \langle \varepsilon_{2,3} \rangle_{\mathrm{Gl}}^{20-30\%}$ ("MC-Glauber-like")
  3. $\psi_2 = 0$, ψ_3 (direction of triangularity) distributed randomly
- Use $v_2^\pi(p_T)$ from VISH2+1 for $\eta/s = 0.20$ with MC-KLN initial conditions for 20-30% AuAu as "mock data"
- Fit mock $v_2^\pi(p_T)$ data with VISH2+1 for "MC-Glauber-like" or "MC-KLN-like" Gaussian initial conditions with both elliptic and triangular deformations by adjusting η/s
  $\Longrightarrow (\eta/s)_{\mathrm{KLN}} = 0.217 \pm 0.005$ for "MC-KLN-like",
  $(\eta/s)_{\mathrm{Gl}} = 0.111 \pm 0.001$ for "MC-Glauber-like"
- Compute $v_3^\pi(p_T)$ for "MC-KLN-like" fit with $(\eta/s)_{\mathrm{Gl}}$=0.217 and reproduce it with "MC-Glauber-like" initial condition by readjusting η/s
  $\Longrightarrow (\eta/s)_{\mathrm{Gl}}^{v_3} = 0.224 \pm 0.005$ for "MC-Glauber-like"
- Compute $v_2^\pi(p_T)$ for "MC-Glauber-like" initial profiles with readjusted $(\eta/s)_{\mathrm{Gl}}^{v_3} = 0.224$ and compare with "MC-Glauber-like" fit to original mock data $\Longrightarrow$ clearly visible (and measurable) difference!

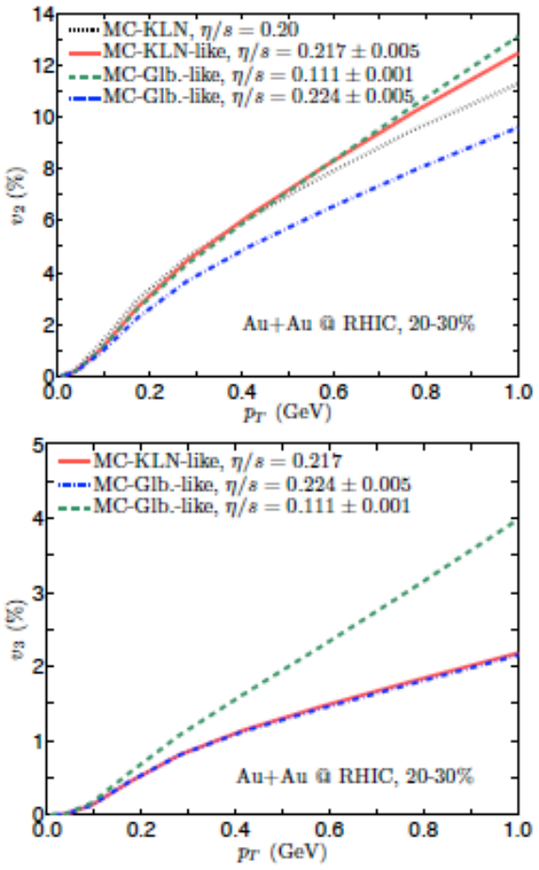
This exercise proves: (i) Fitting $v_3(p_T)$ data with MC-Glauber and MC-KLN initial conditions yields the same η/s (within narrow error band); (ii) The corresponding $v_2(p_T)$ fits are quite different, and only one (more precisely: at most one!) of the models will fit the corresponding $v_2(p_T)$ data.

U. Heinz, talk at RETUNE '12

Dynamics

Computing the elephant

Zhi Qiu and U. Heinz, to be published



Proof of principle calculation:

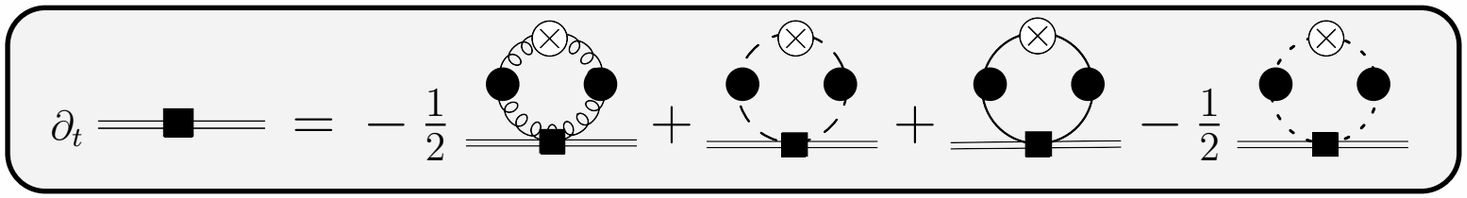
- Take ensemble of sum of deformed Gaussian profiles, $s(\boldsymbol{r}_\perp) = s_2(\boldsymbol{r}_\perp; \tilde{\varepsilon}_2, \psi_2) + s_3(\boldsymbol{r}_\perp; \tilde{\varepsilon}_3, \psi_3)$, with
  1. equal Gaussian radii $R_2^2 = R_3^2 = 8\,\mathrm{fm}^2$ to reproduce $\langle r_\perp^2 \rangle$ of MC-KLN source for 20-30% AuAu
  2. $\tilde{\varepsilon}_2$ and $\tilde{\varepsilon}_3$ adjusted such that
     - $\bar{\varepsilon}_{2,3} = \langle \varepsilon_{2,3} \rangle_{\mathrm{KLN}}^{20-30\%}$ ("MC-KLN-like")
     - $\bar{\varepsilon}_{2,3} = \langle \varepsilon_{2,3} \rangle_{\mathrm{Gl}}^{20-30\%}$ ("MC-Glauber-like")
  3. $\psi_2 = 0$, ψ_3 (direction of triangularity) distributed randomly
- Use $v_2^\pi(p_T)$ from VISH2+1 for $\eta/s = 0.20$ with MC-KLN initial conditions for 20-30% AuAu as "mock data"
- Fit mock $v_2^\pi(p_T)$ data with VISH2+1 for "MC-Glauber-like" or "MC-KLN-like" Gaussian initial conditions with both elliptic and triangular deformations by adjusting η/s
  $\Longrightarrow (\eta/s)_{\mathrm{KLN}} = 0.217 \pm 0.005$ for "MC-KLN-like",
  $(\eta/s)_{\mathrm{Gl}} = 0.111 \pm 0.001$ for "MC-Glauber-like"
- Compute $v_3^\pi(p_T)$ for "MC-KLN-like" fit with $(\eta/s)_{\mathrm{Gl}}$=0.217 and reproduce it with "MC-Glauber-like" initial condition by readjusting η/s
  $\Longrightarrow (\eta/s)_{\mathrm{Gl}}^{v_3} = 0.224 \pm 0.005$ for "MC-Glauber-like"
- Compute $v_2^\pi(p_T)$ for "MC-Glauber-like" initial profiles with readjusted $(\eta/s)_{\mathrm{Gl}}^{v_3} = 0.224$ and compare with "MC-Glauber-like" fit to original mock data $\Longrightarrow$ clearly visible (and measurable) difference!

This exercise proves: (i) Fitting $v_3(p_T)$ data with MC-Glauber and MC-KLN initial conditions yields the same η/s (within narrow error band); (ii) The corresponding $v_2(p_T)$ fits are quite different, and only one (more precisely: at most one!) of the models will fit the corresponding $v_2(p_T)$ data.

computing transport coefficients

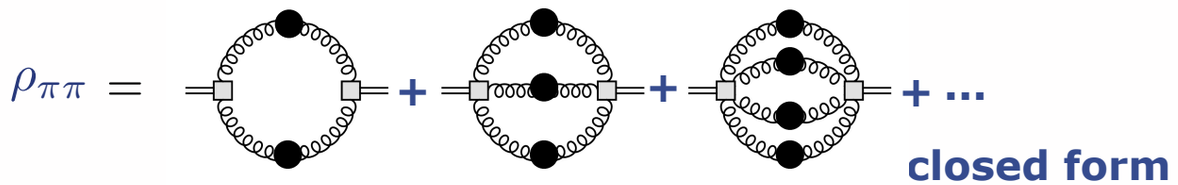
Correlations of the energy-momentum tensor

Flow

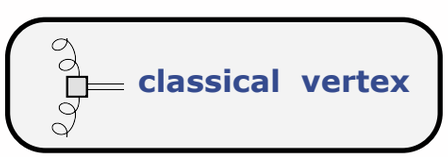


$\rho_{\pi\pi}$

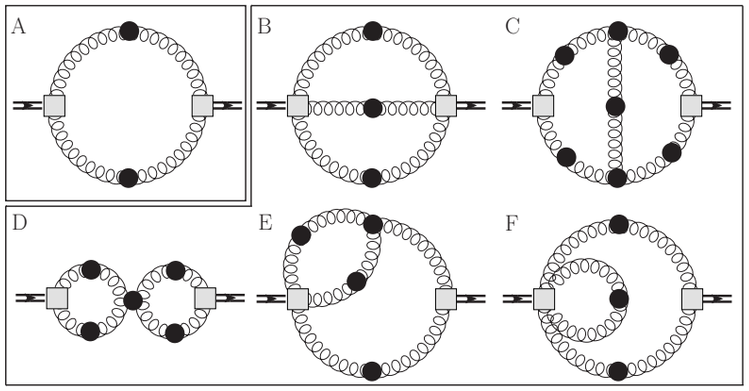
Diagrammatic representation



closed form



Complete 2-loop corrections

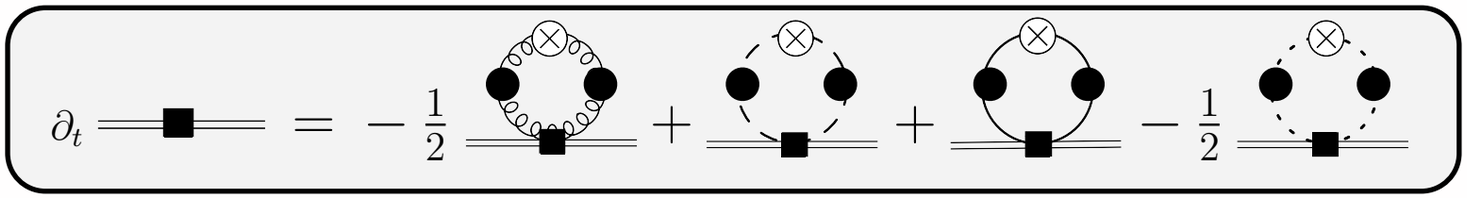


Haas, Fister, JMP, PRD 90 (2014) 9, 091501

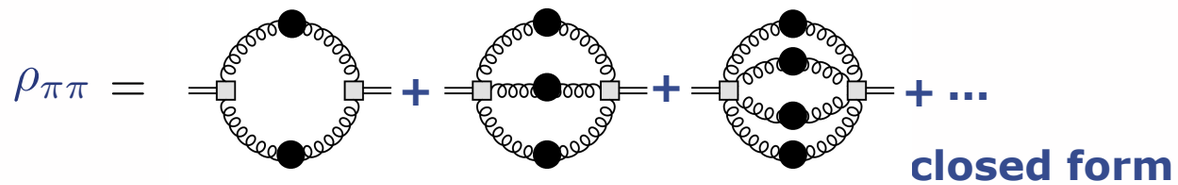
Christiansen, Haas, JMP, Strodthoff, arXiv:1411.7986

Correlations of the energy-momentum tensor

Flow

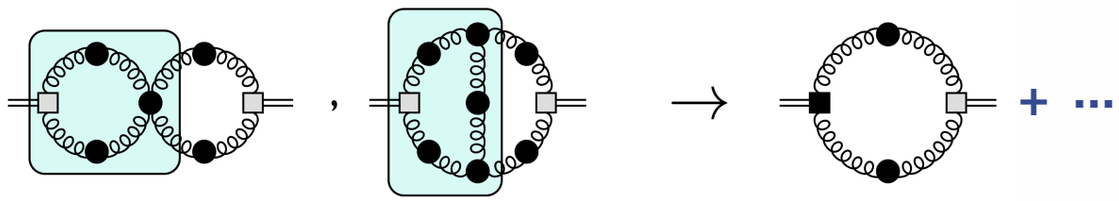


Diagrammatic representation



closed form

Vertex corrections

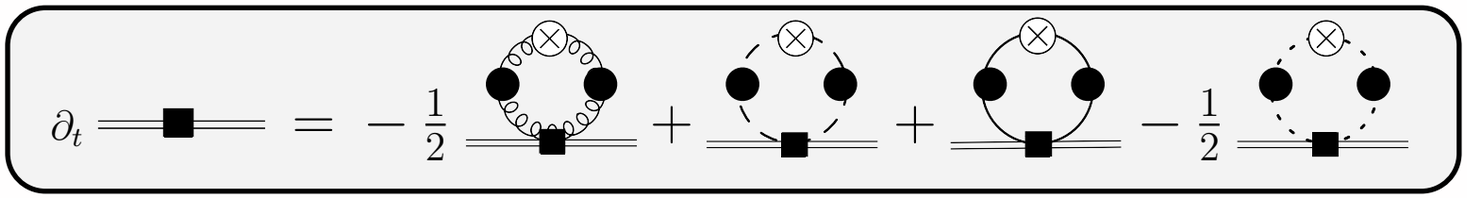


Haas, Fister, JMP, PRD 90 (2014) 9, 091501

Christiansen, Haas, JMP, Strodthoff, arXiv:1411.7986

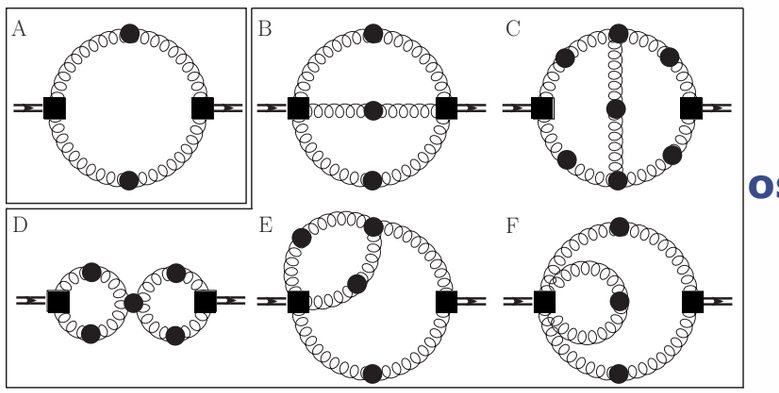
Correlations of the energy-momentum tensor

Flow



Current approximation

$\rho_{\pi\pi} =$



full vertex

with optimised RG-scheme from

osed form Fister, JMP 'Phys.Rev. D88 (2013) 045010

$\rho_{T/L}$ with MEM

Haas, Fister, JMP, PRD 90 (2014) 9, 091501

Christiansen, Haas, JMP, Strodthoff, arXiv:1411.7986

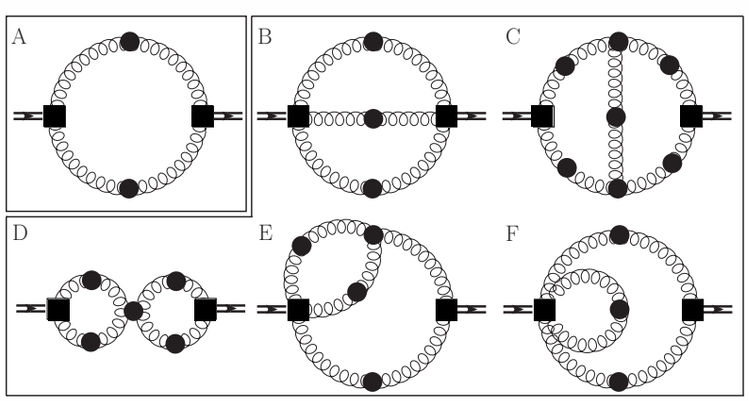
Transport coefficients

Shear viscosity

$$\eta = \frac{1}{20}\,\frac{d}{d\omega}\bigg|_{\omega=0}\rho_{\pi\pi}(\omega,0)$$ **Kubo relation**

Current approximation

$\rho_{\pi\pi}\ =$



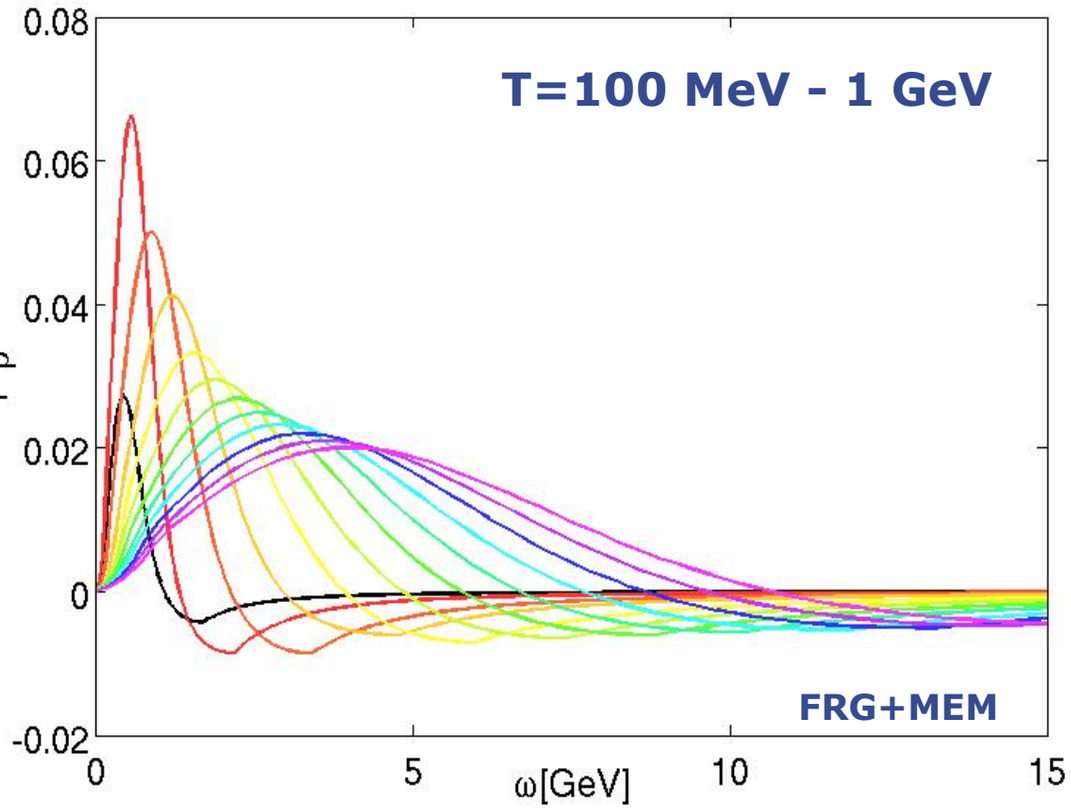
$\rho_{T/L}$ **with MEM**

$$\rho_{\pi\pi}(p) = \frac{2d_A}{3}\int\frac{d^4k}{(2\pi)^4}\left[n(k^0)-n(k^0+p_0)\right]\left(V_{TT}\rho_T(k)\rho_T(k+p)+V_{TL}\rho_T(k)\rho_L(k+p)+V_{LL}\rho_L(k)\rho_L(k+p)\right)$$ **+ ...**

Haas, Fister, JMP, PRD 90 (2014) 9, 091501

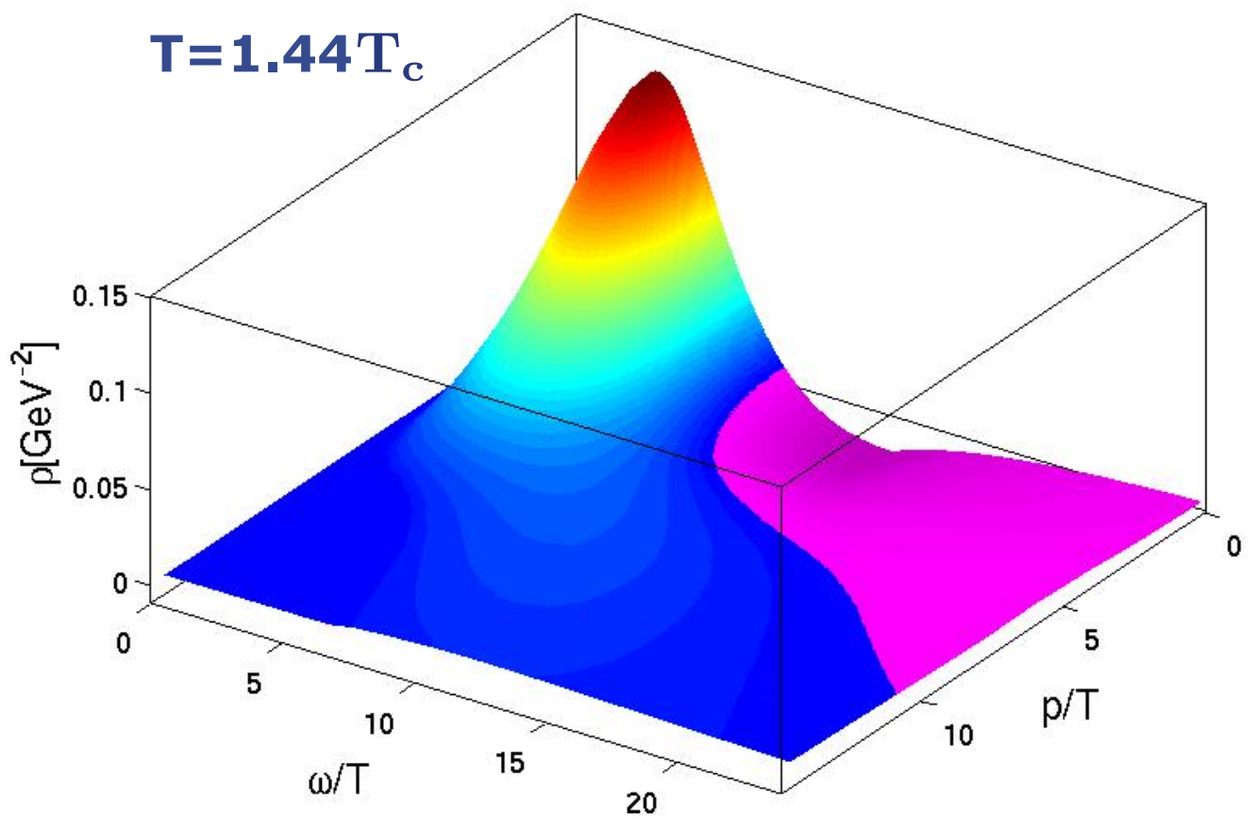
Christiansen, Haas, JMP, Strodthoff, arXiv:1411.7986

Gluon spectral function at finite T



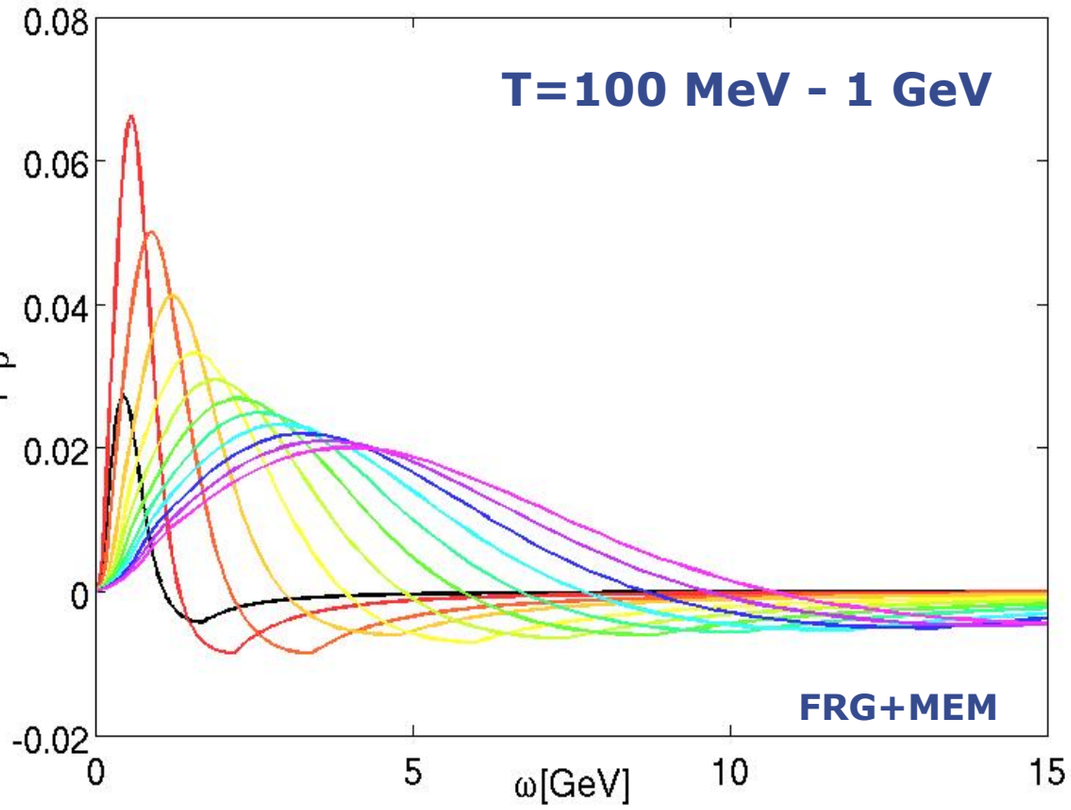
MEM

transversal

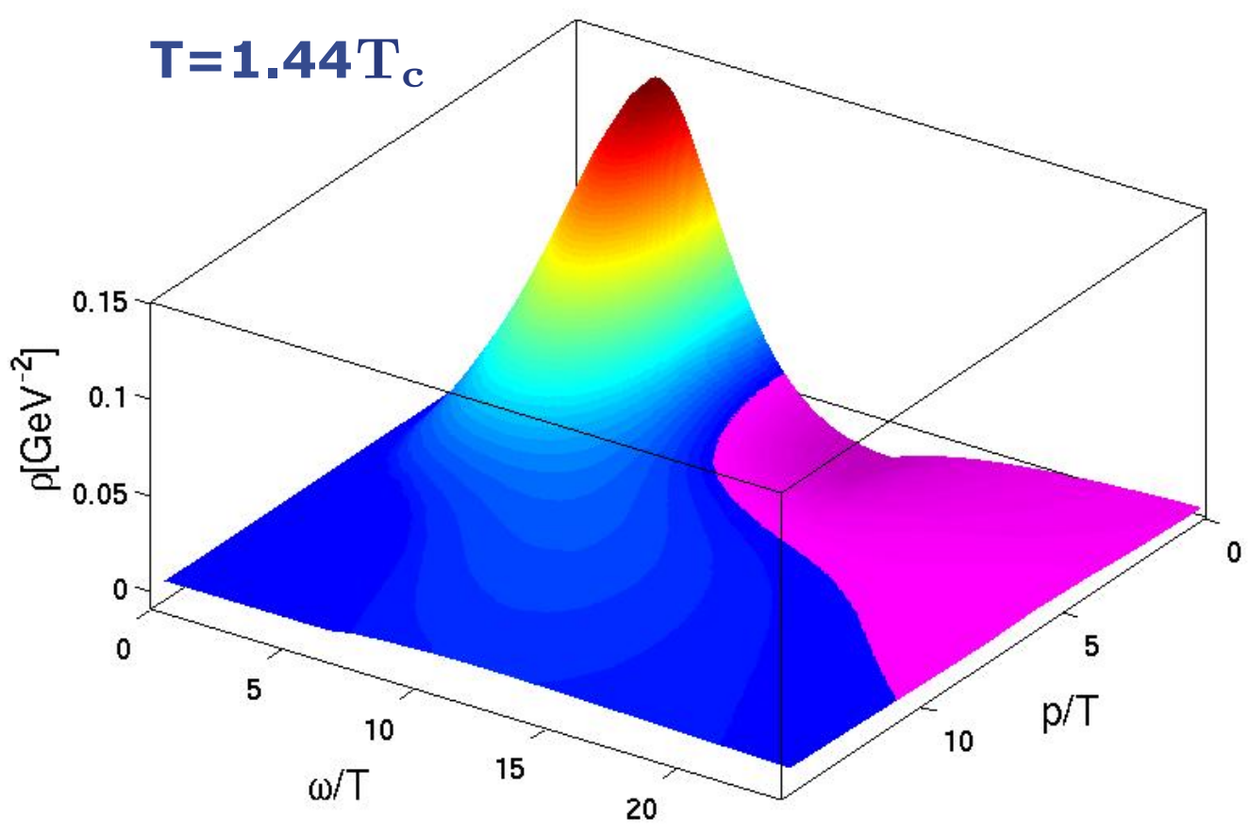


Haas, Fister, JMP, PRD 90 (2014) 9, 091501

Gluon spectral function at finite T



transversal



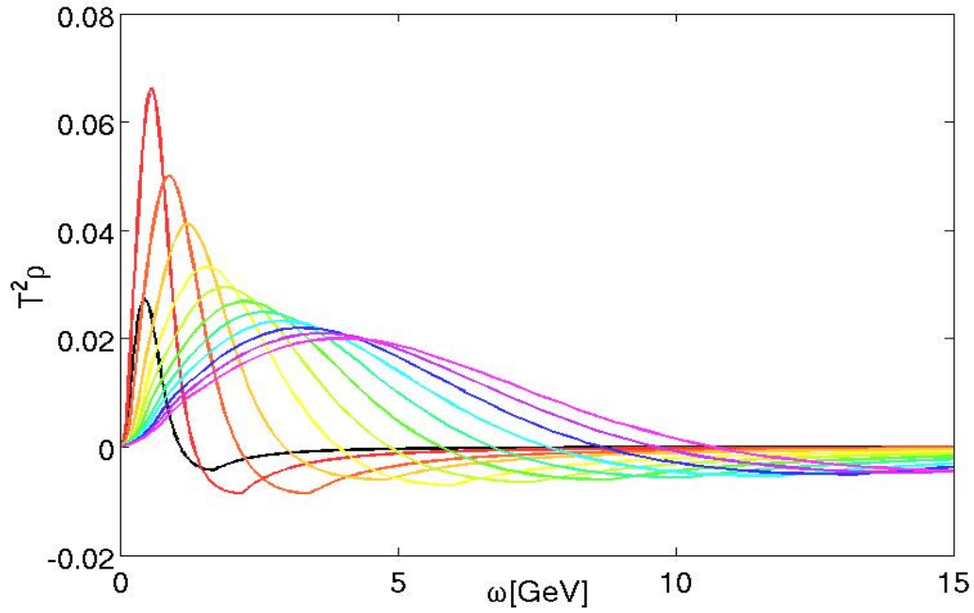
'Those are my methods (principles), and if you don't like them…well, I have others'

direct computation **Groucho Marx**

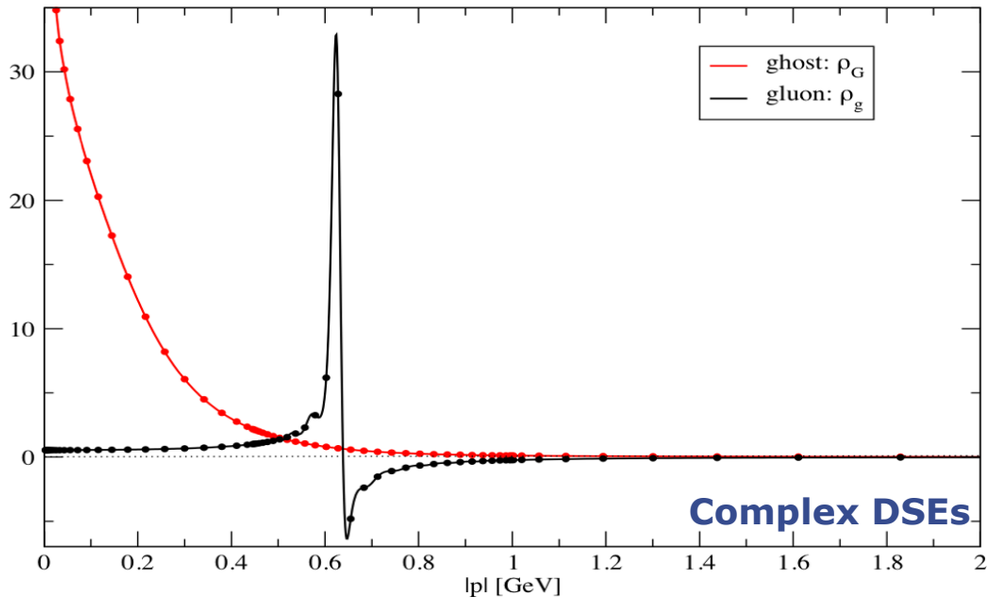
Haas, Fister, JMP, PRD 90 (2014) 9, 091501

Dynamics

gluon spectral functions

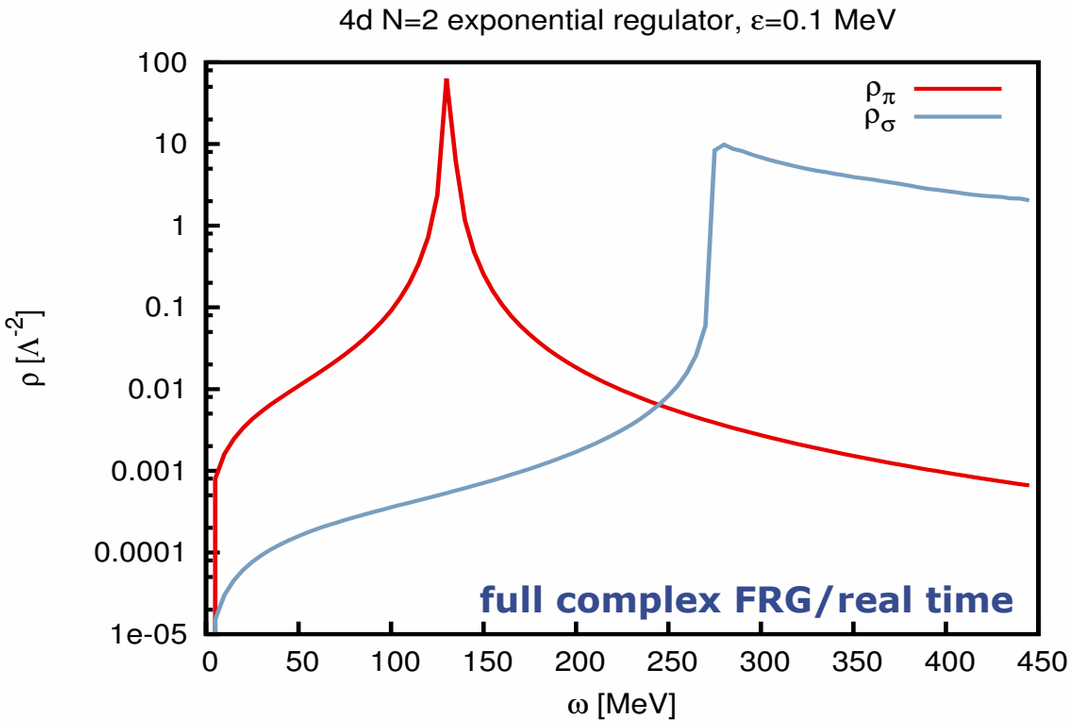


Haas, Fister, JMP, PRD 90 (2014) 9, 091501

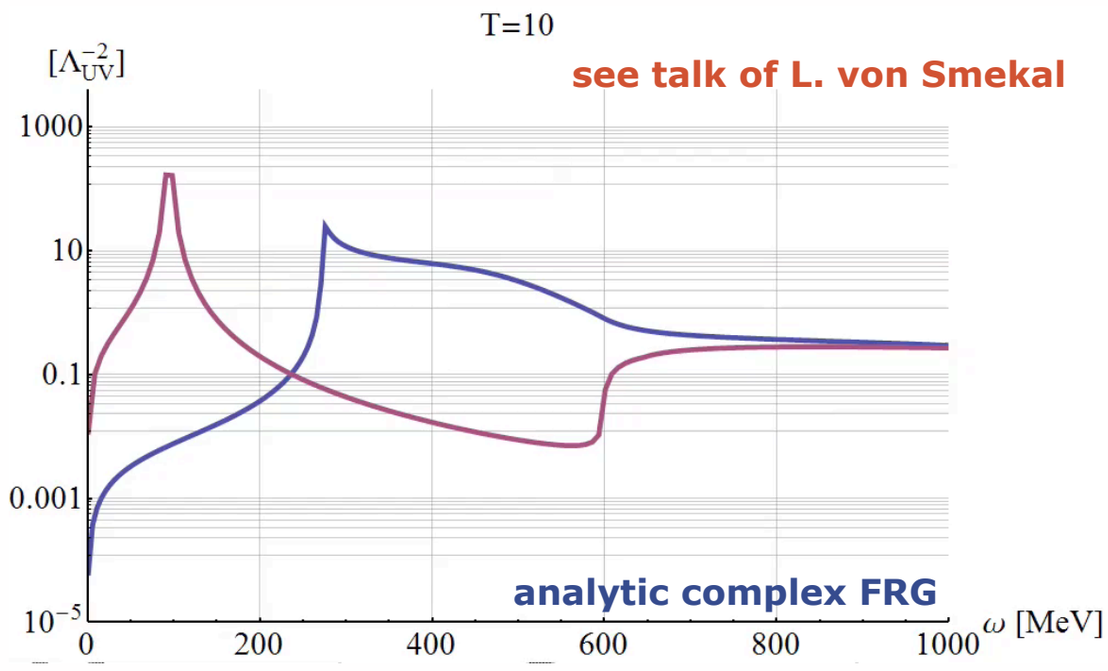


Strauss, Fischer, Kellermann, PRL 109 (2012) 252001

pion and sigma spectral functions



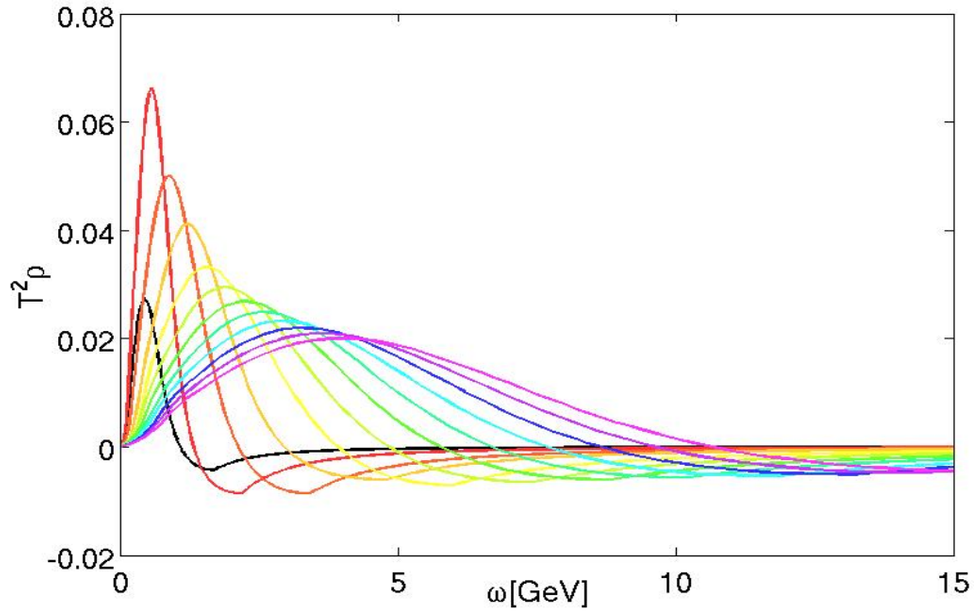
JMP, Strodthoff, in preparation



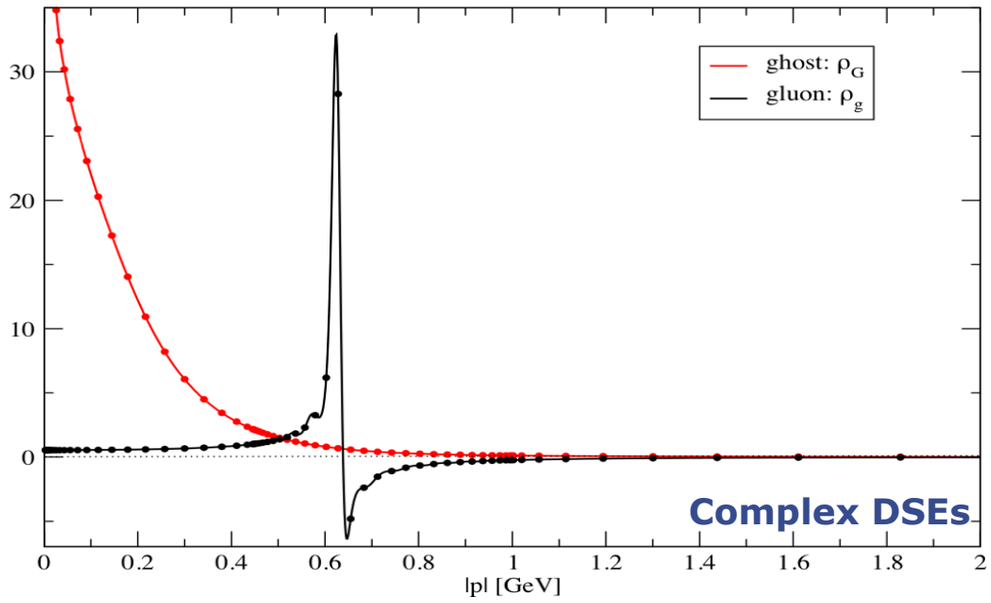
Tripolt, Strodthoff, von Smekal, Wamach, PRD 89 (2014) 034010
Kamikado, Strodthoff, von Smekal, Wambach, EPJ C74 (2014) 2806

Dynamics

gluon spectral functions



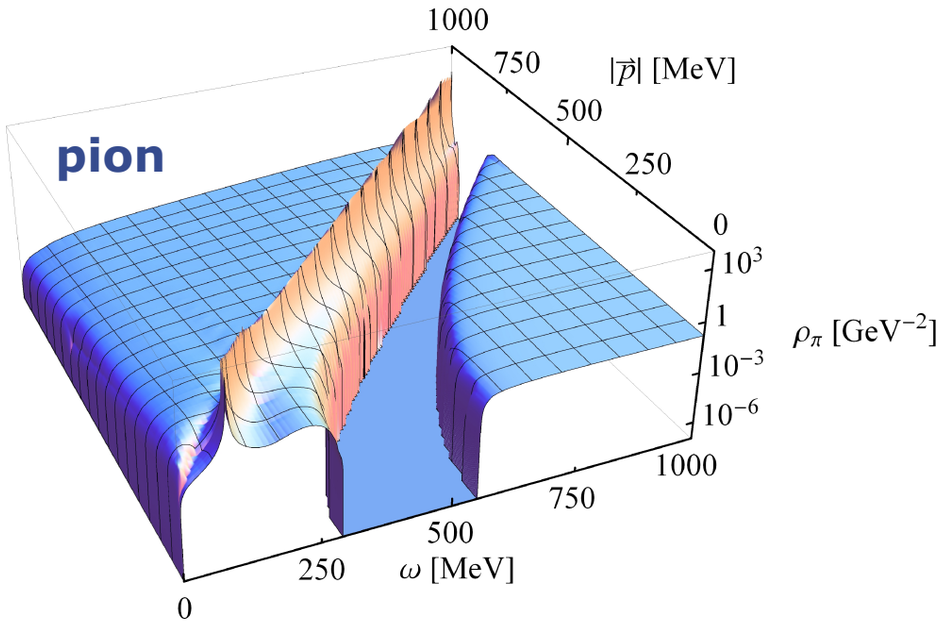
Haas, Fister, JMP, PRD 90 (2014) 9, 091501

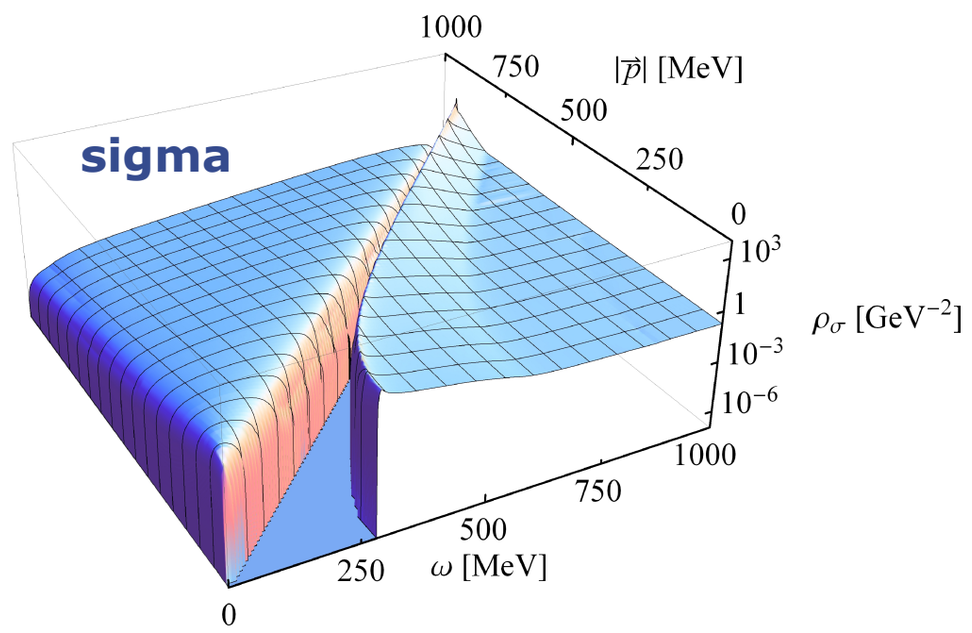


Strauss, Fischer, Kellermann, PRL 109 (2012) 252001

pion and sigma spectral functions

see talk of L. von Smekal



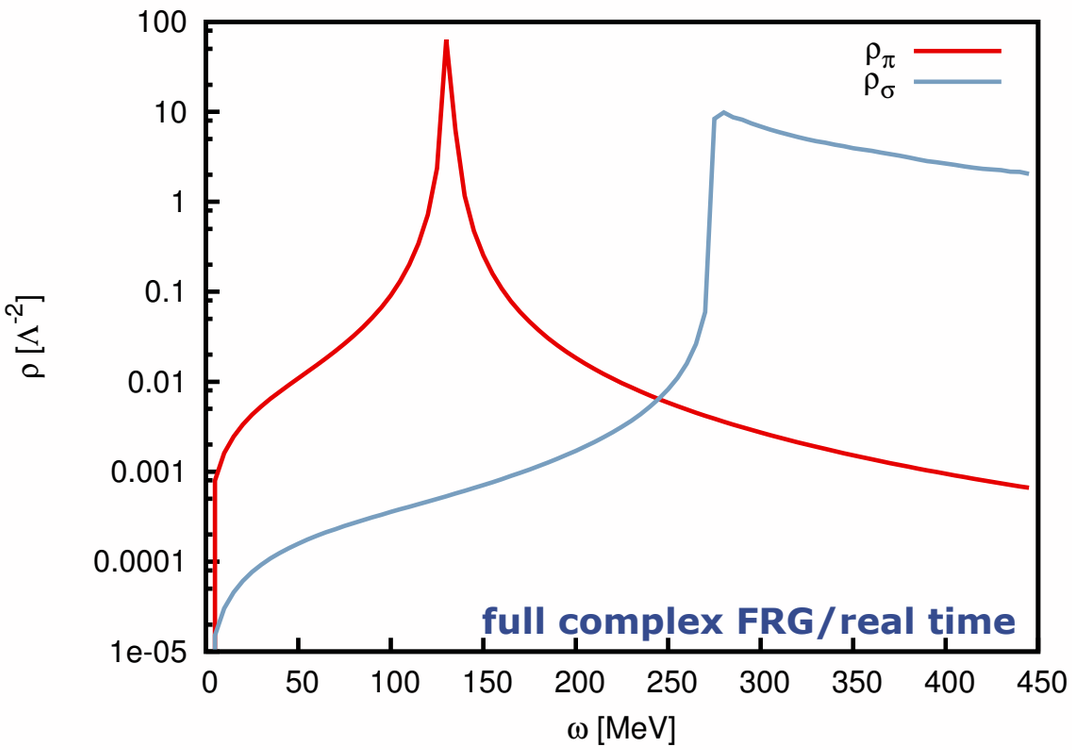


Tripolt, von Smekal, Wambach, Phys.Rev. D90 (2014) 7, 074031
Tripolt, PhD-thesis

Viscosity in pure glue

spectral functions

pion and sigma spectral functions



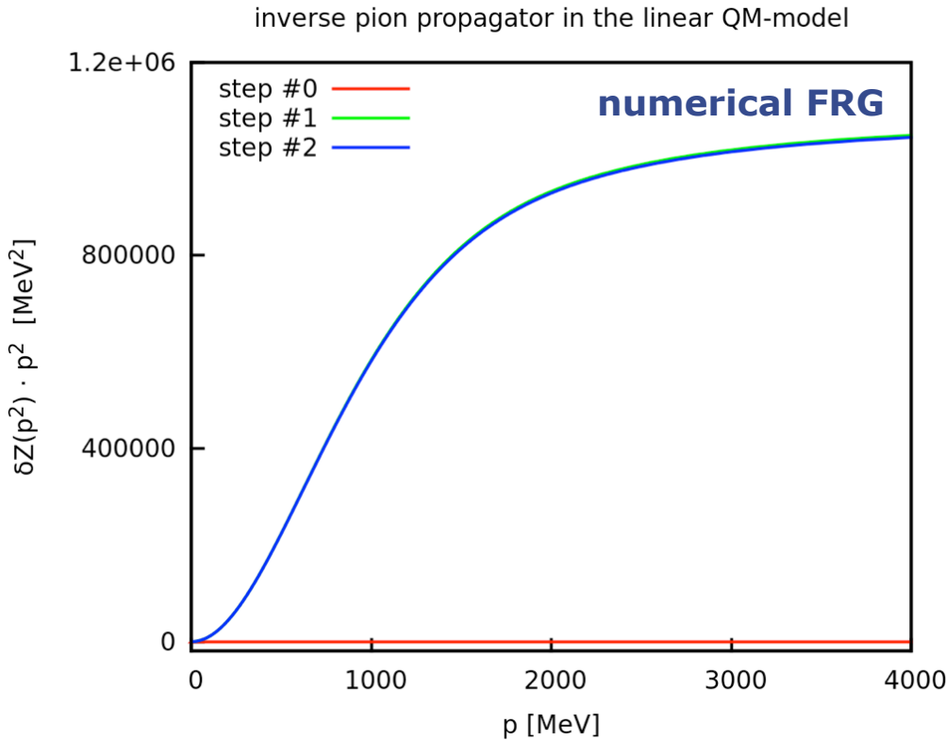
JMP, Strodthoff, in preparation

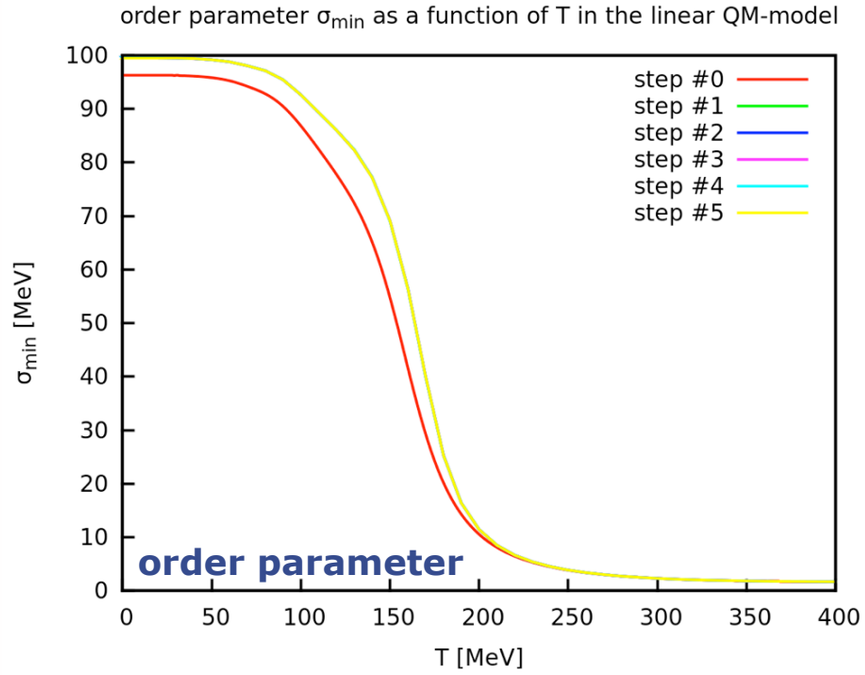
O(N)-model

iteration step	σ_0 [MeV]	δ_ρ [%]	$m_{\rm pole}$ [MeV]	$m_{\rm cur}$ [MeV]	δ_m [%]
0	93.55	0.0043	130.3113	136.7593	4.9
1	100.05	0.0028	126.6390	126.4590	0.14
5	99.38	0.0043	127.0347	127.0110	0.019

iteration step	σ_0 [MeV]	δ_ρ [%]	$m_{\rm pole}$ [MeV]	$m_{\rm cur}$ [MeV]	δ_m [%]
0	96.25	0.0052	91.4911	134.8281	47
1	99.56	0.0044	90.8841	91.1611	0.30
5	99.56	0.0073	90.9244	91.1551	0.25

QM-model



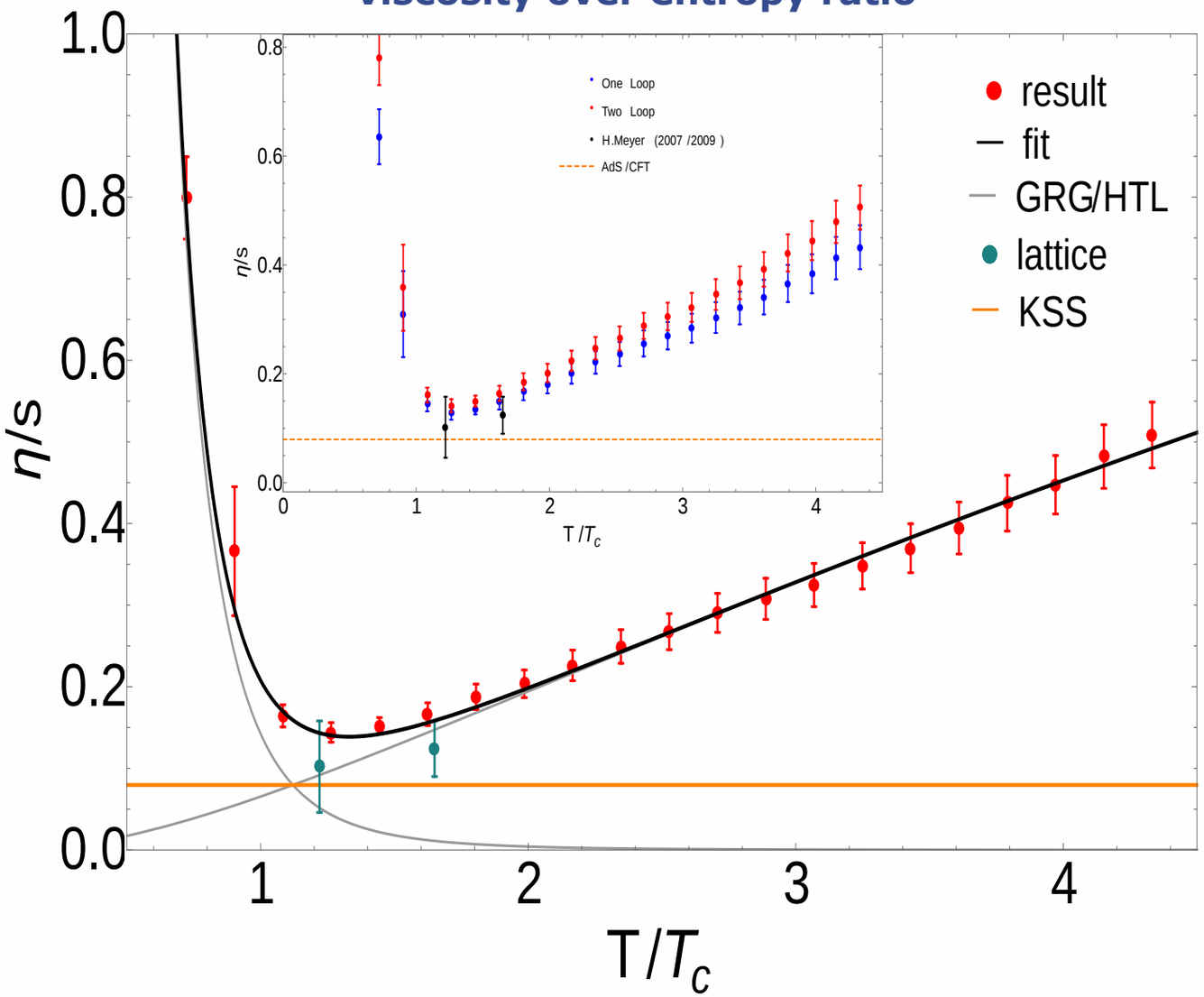


Helmboldt, JMP, Strodthoff, Phys.Rev. D91 (2015) 5, 054010

Dynamics

transport coefficients

viscosity over entropy ratio

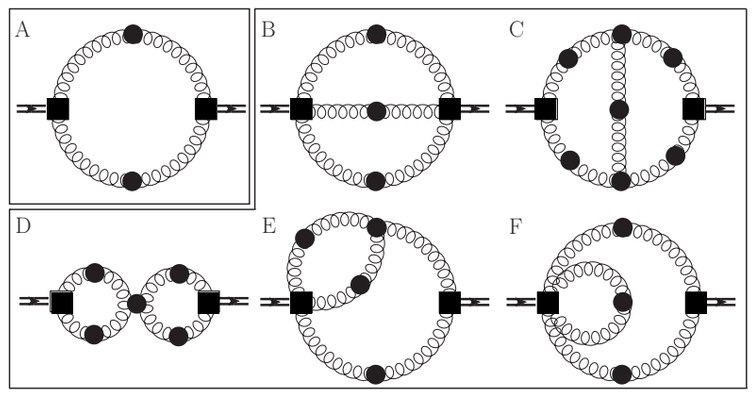


Kubo relation

$$\eta = \frac{1}{20} \frac{d}{d\omega}\bigg|_{\omega=0} \rho_{\pi\pi}(\omega, 0)$$

'3-loop' exact functional relation for $\rho_{\pi\pi}$

1 & 2-loop terms



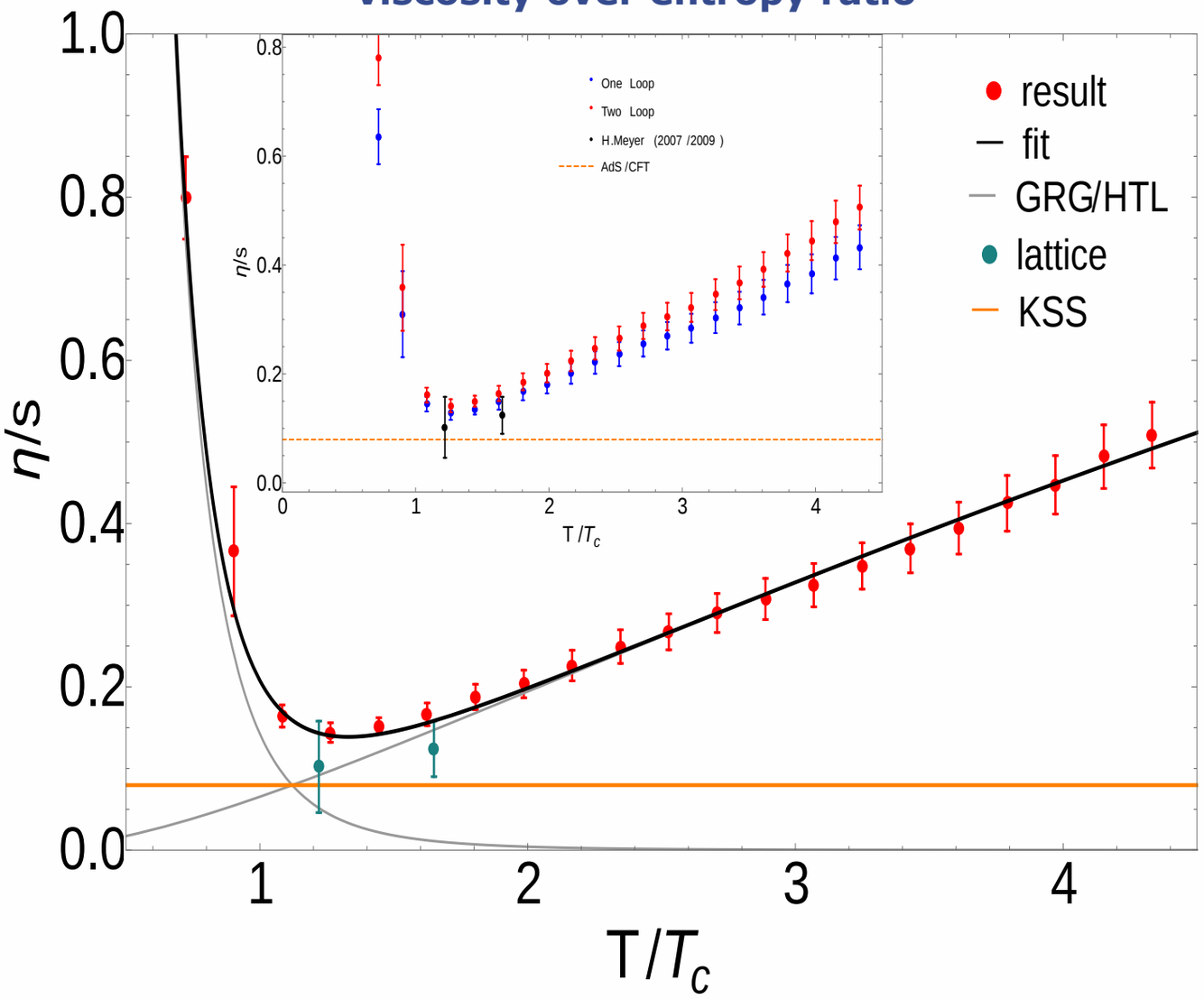
Haas, Fister, JMP, PRD 90 (2014) 9, 091501

Christiansen, Haas, JMP, Strodthoff, arXiv:1411.7986

Dynamics

QCD - estimate for viscosity over entropy ratio

viscosity over entropy ratio



$$\frac{\eta}{s}(T) = \frac{a_{\rm qgp}}{\alpha_s^{\gamma_{\rm qgp}}(cT/T_c)} + \frac{a_{\rm grg}}{(T/T_c)^{\gamma_{\rm grg}}}$$

$$\gamma_{\rm grg} \approx 5$$

$$\gamma_{\rm qgp} \approx 1.6$$

pure glue

$$a_{\rm qgp} \approx 0.15$$

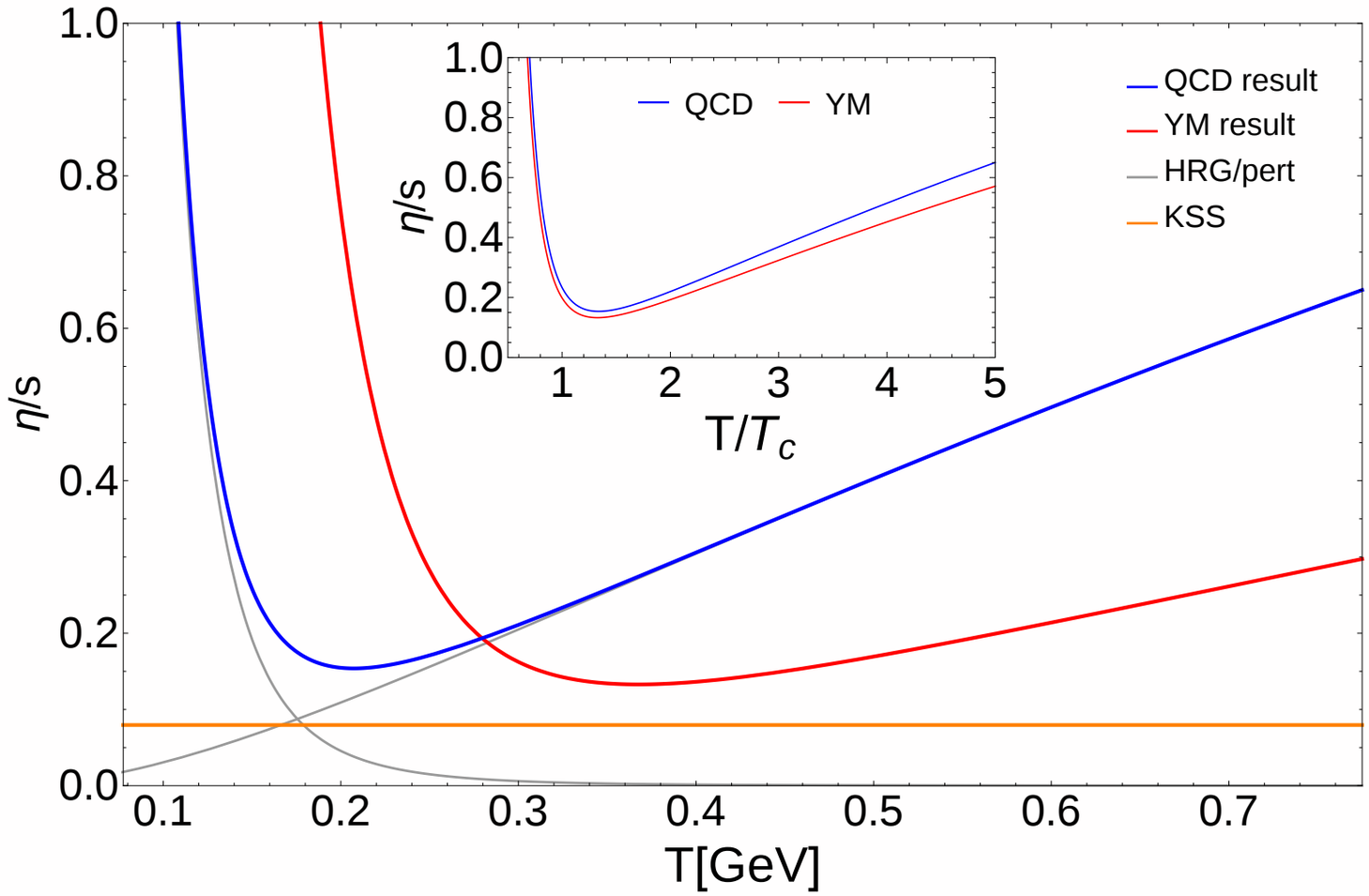
$$a_{\rm hrg} \approx 0.14$$

$$c \approx 0.66$$

Christiansen, Haas, JMP, Strodthoff, arXiv:1411.7986

Dynamics

QCD - estimate for viscosity over entropy ratio



$$a_{\rm qgp} \approx 0.2$$

$$a_{\rm hrg} \approx 0.16$$

$$c \approx 0.79$$

QCD

$$\gamma_{\rm grg} \approx 5$$

$$\gamma_{\rm qgp} \approx 1.6$$

pure glue

$$a_{\rm qgp} \approx 0.15$$

$$a_{\rm hrg} \approx 0.14$$

$$c \approx 0.66$$

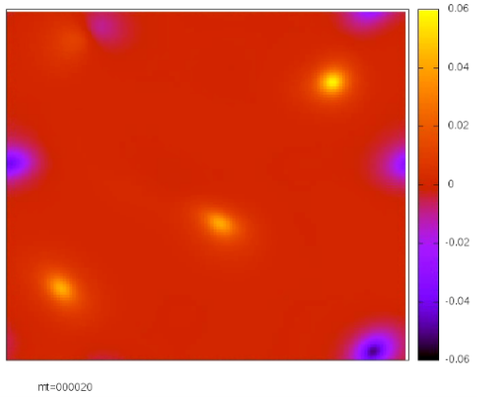
$$\frac{\eta}{s}(T) = \frac{a_{\rm qgp}}{\alpha_s^{\gamma_{\rm qgp}}(cT/T_c)} + \frac{a_{\rm grg}}{(T/T_c)^{\gamma_{\rm grg}}}$$

Christiansen, Haas, JMP, Strodthoff, arXiv:1411.7986

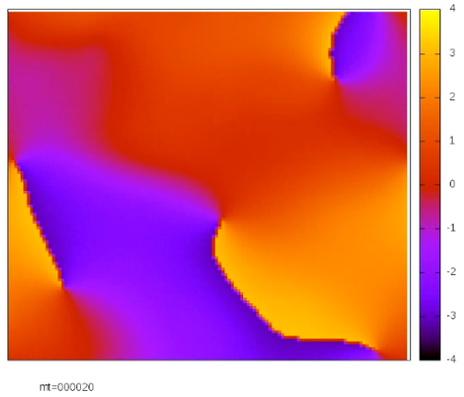
Summary

- **Gauge dynamics far from equilibrium**

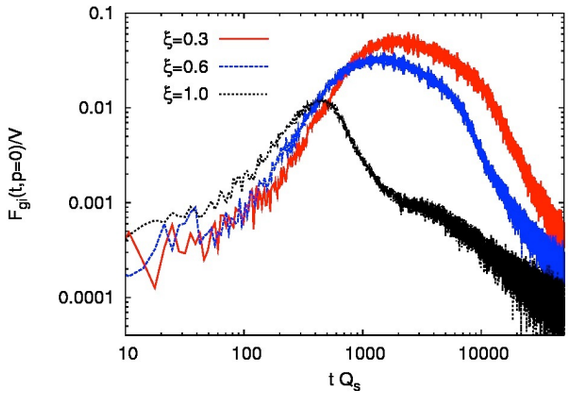
Abelian Higgs



magnetic field

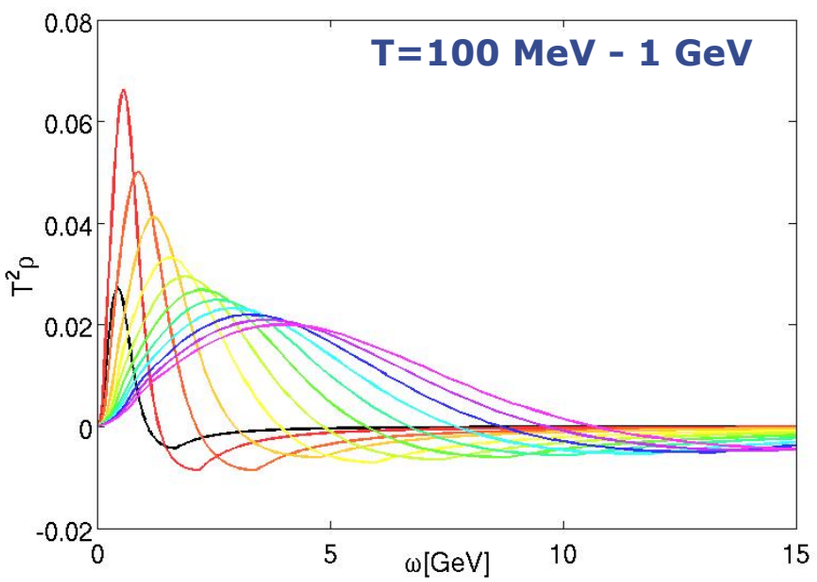


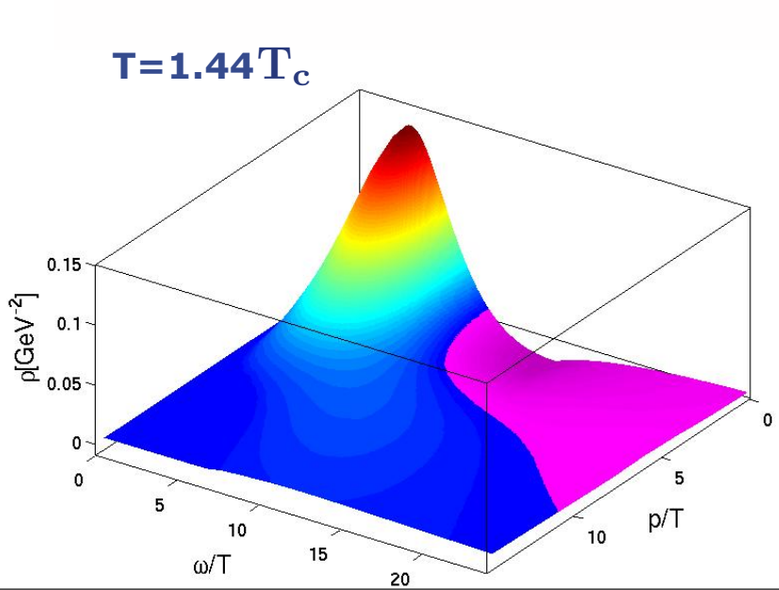
phase of Higgs



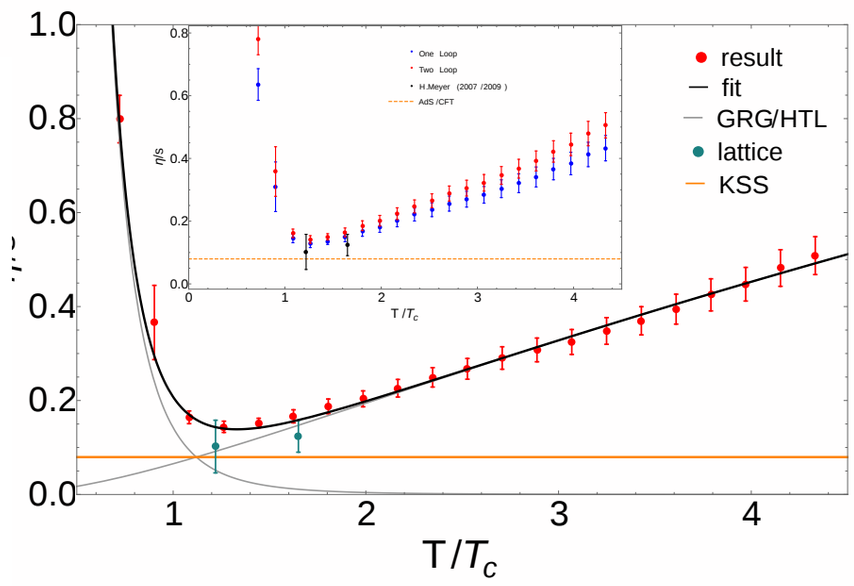
- **Spectral functions and transport coefficients**

spectral functions





viscosity over entropy ratio



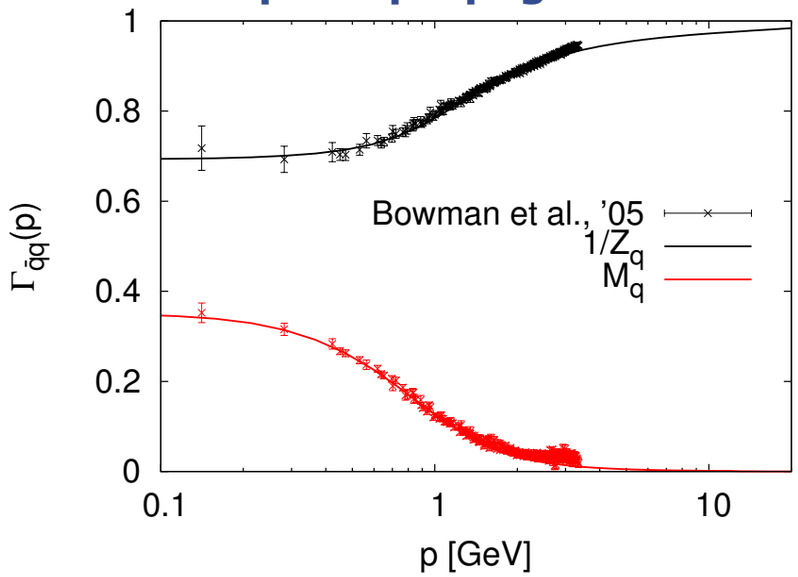
Summary & Outlook

Summary & Outlook

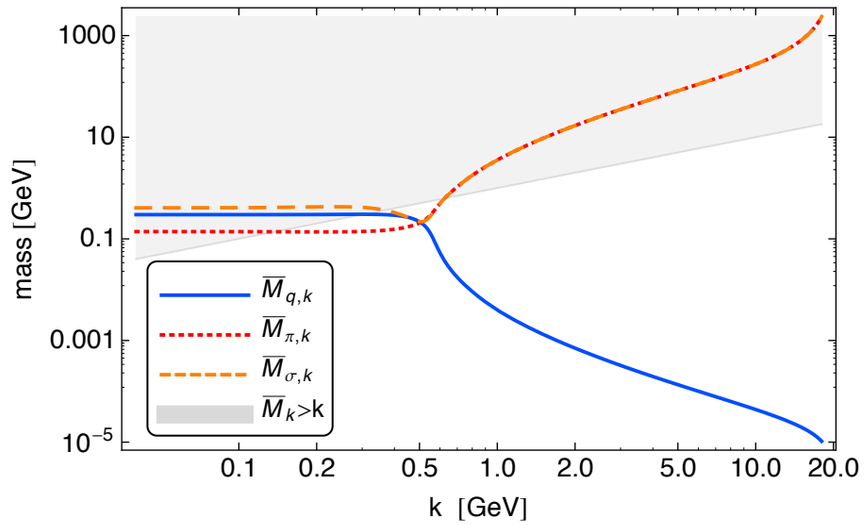
- **Chiral Symmetry Breaking and Confinement**

$$\frac{f_{\pi,\mathrm{FRG}}}{f_{\pi,\mathrm{lattice}}} = 0.99$$

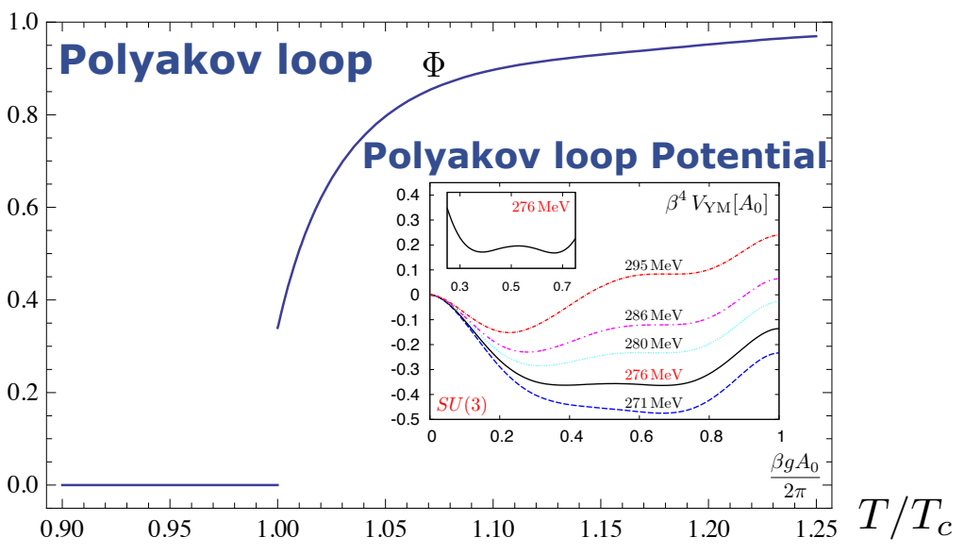
quark propagator



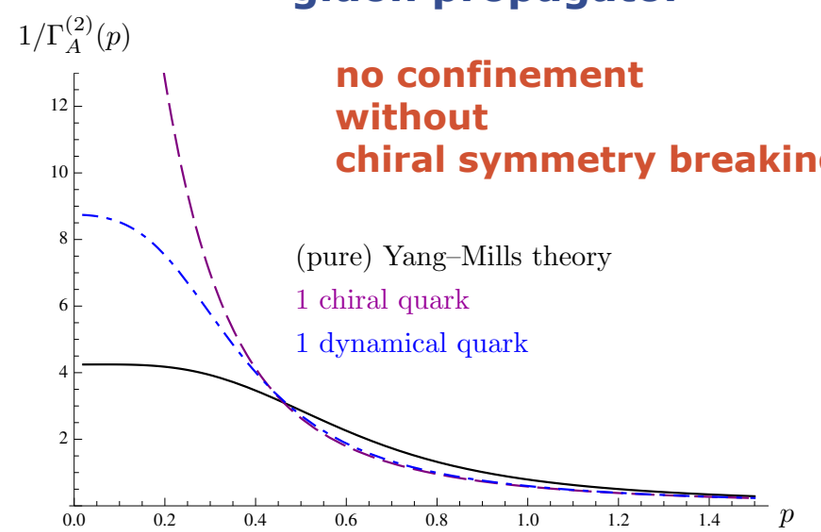
meson masses & decoupling



fQCD



gluon propagator



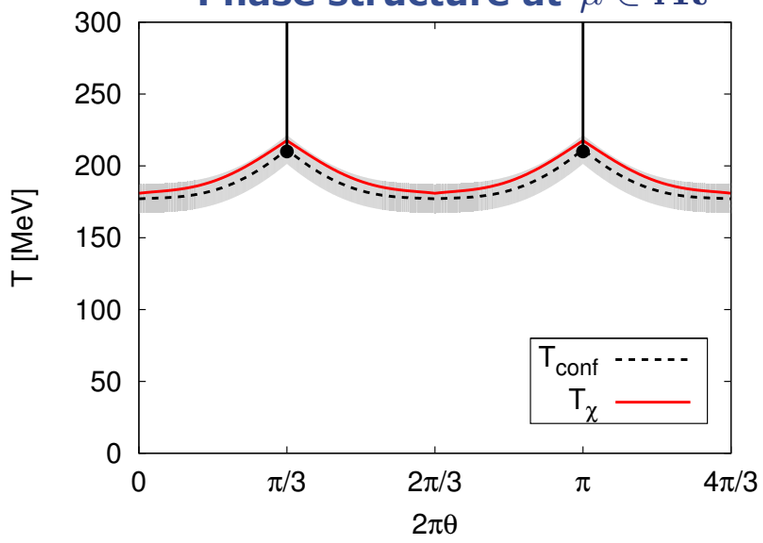
no confinement without chiral symmetry breaking

(pure) Yang–Mills theory
1 chiral quark
1 dynamical quark

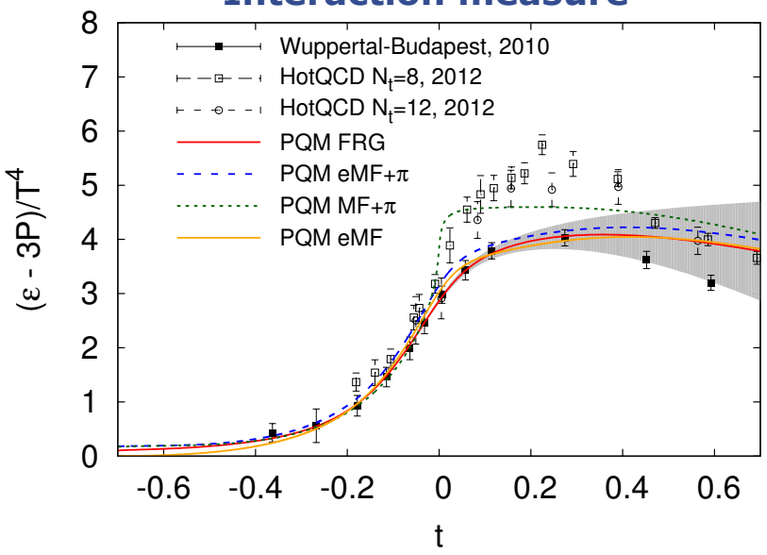
Summary & Outlook

Phase structure and Transport

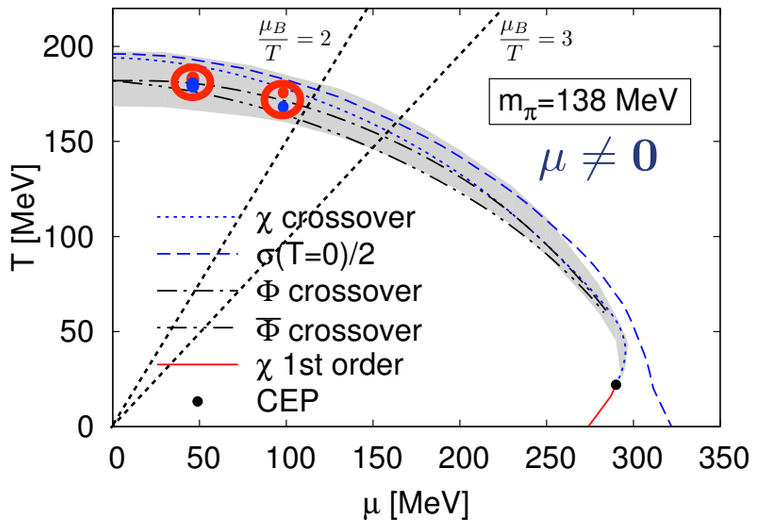
Phase structure at $\mu \in \mathrm{i}\,\mathbb{R}$



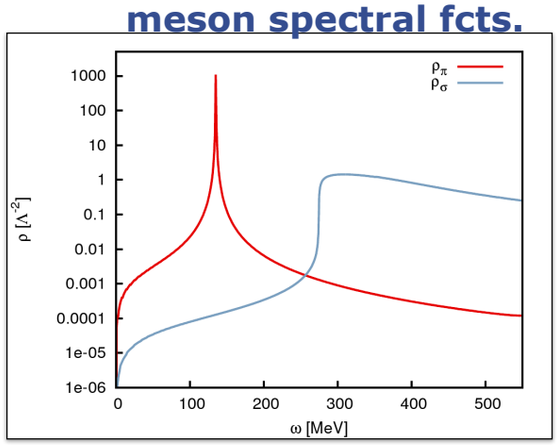
Interaction measure



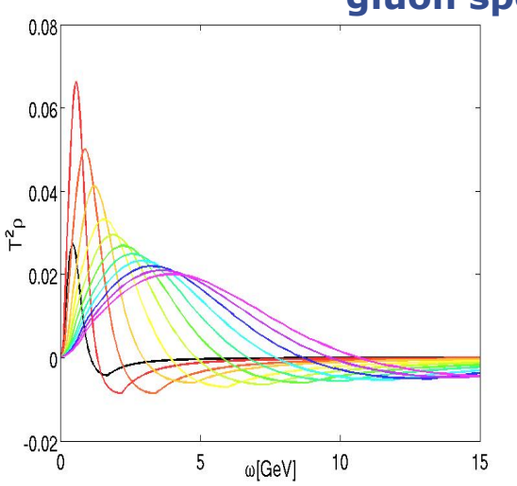
Phase structure at finite density

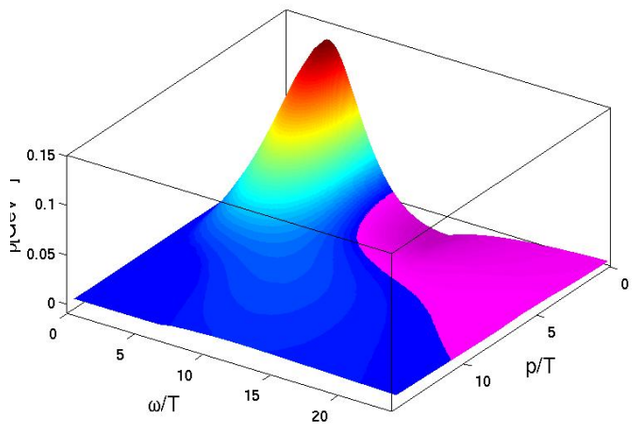


meson spectral fcts.

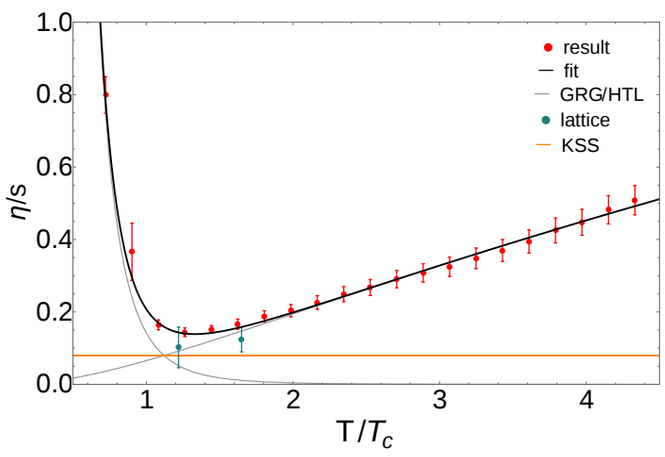


gluon spectral fcts.





glue viscosity/entropy



Summary & Outlook

- Chiral Symmetry Breaking and Confinement
- Phase Structure and Transport
- Towards quantitative precision
- Baryons, high density regime & CEP, dynamics
- Hadronic properties
  - hadron spectrum & in medium modifications
  - low energy constants