

Equation of state and phase diagram of strongly interacting matter

Jan M. Pawłowski

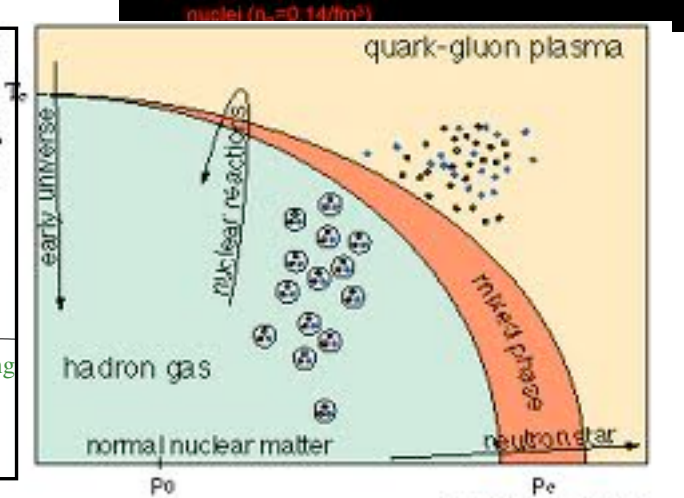
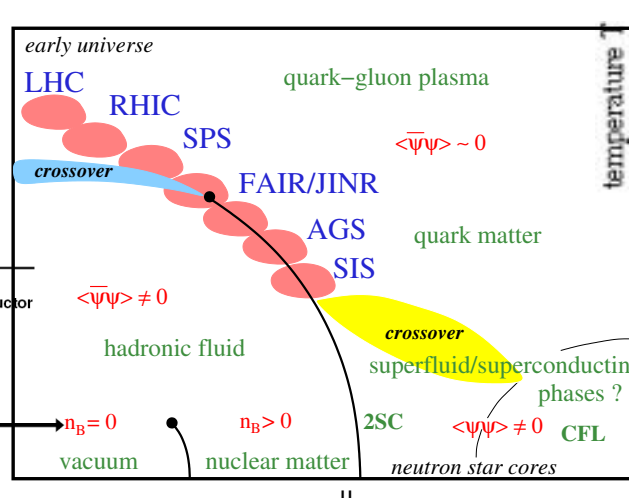
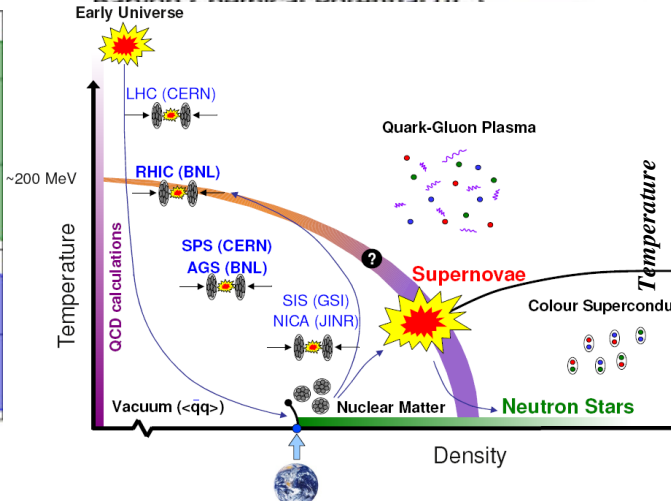
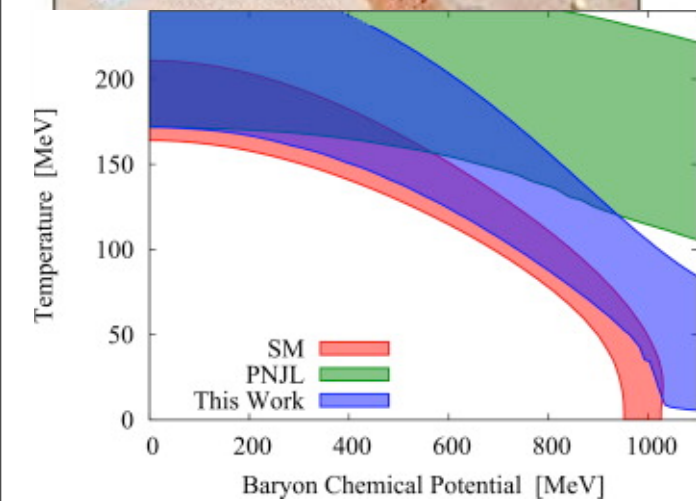
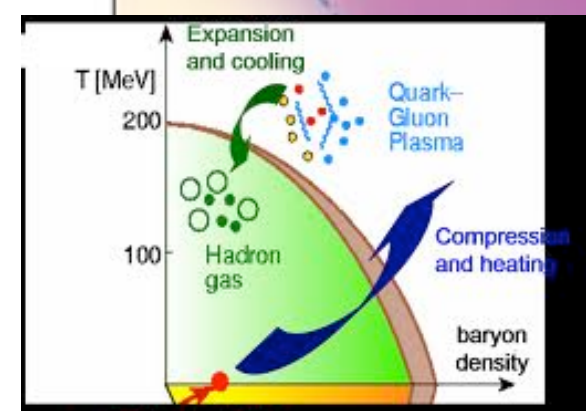
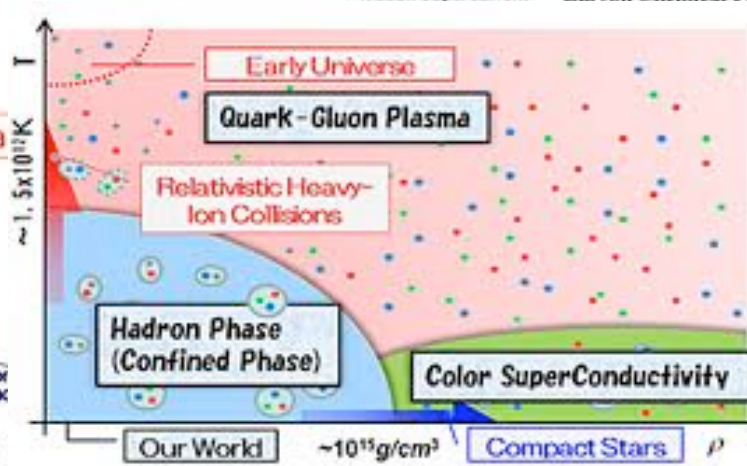
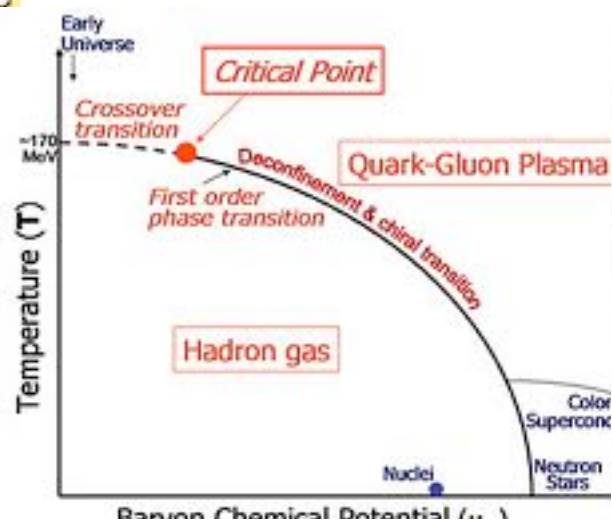
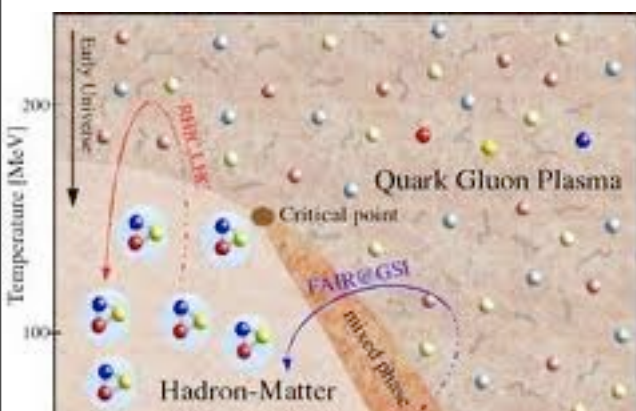
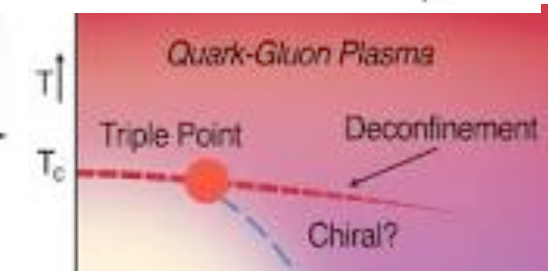
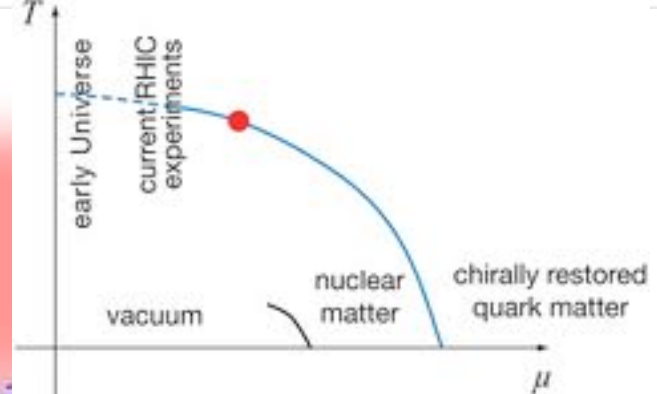
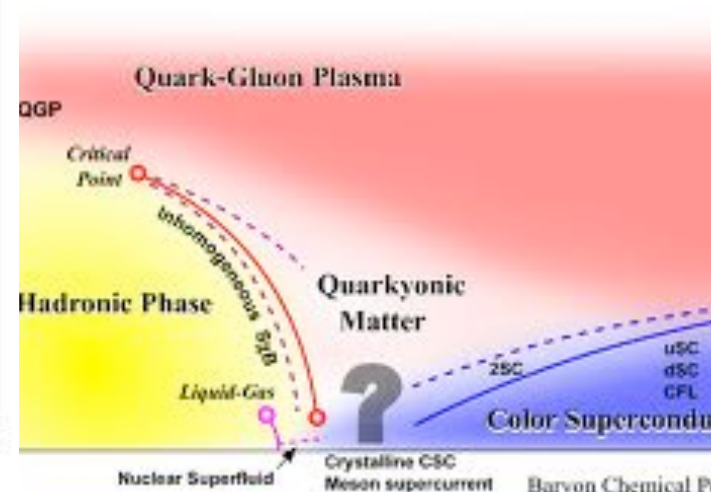
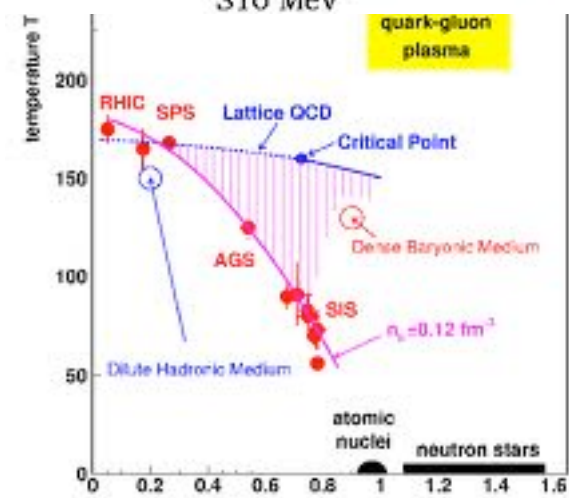
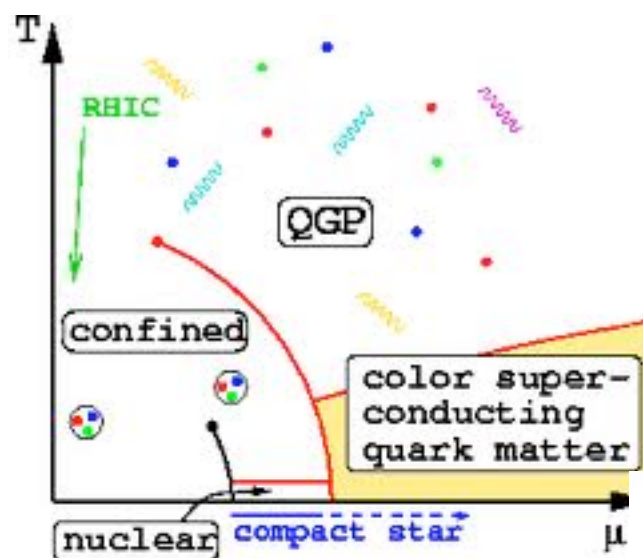
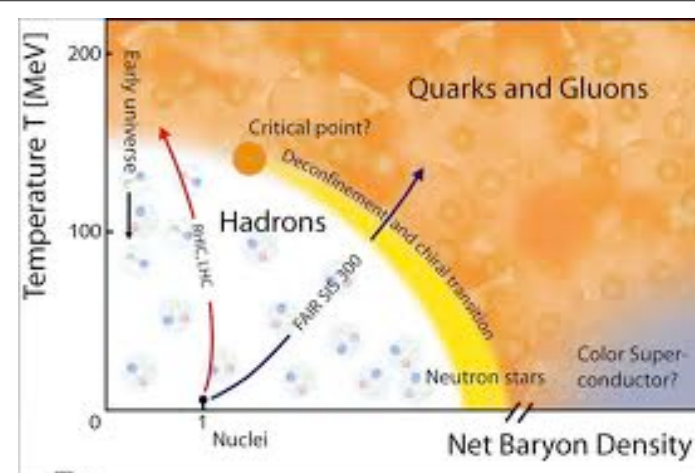
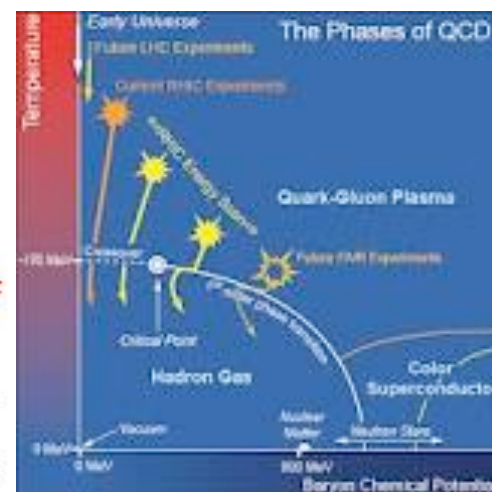
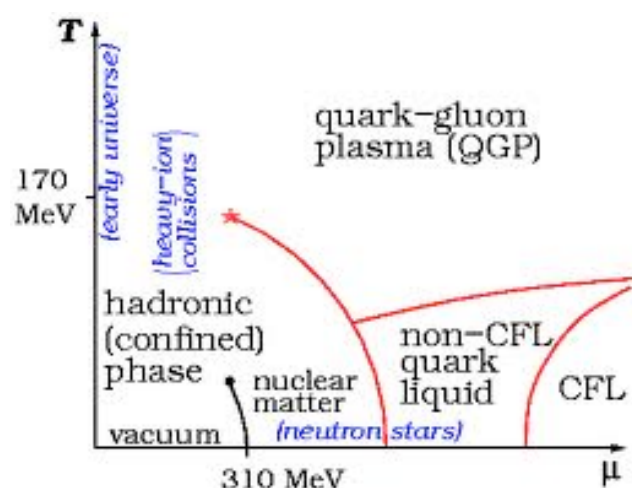
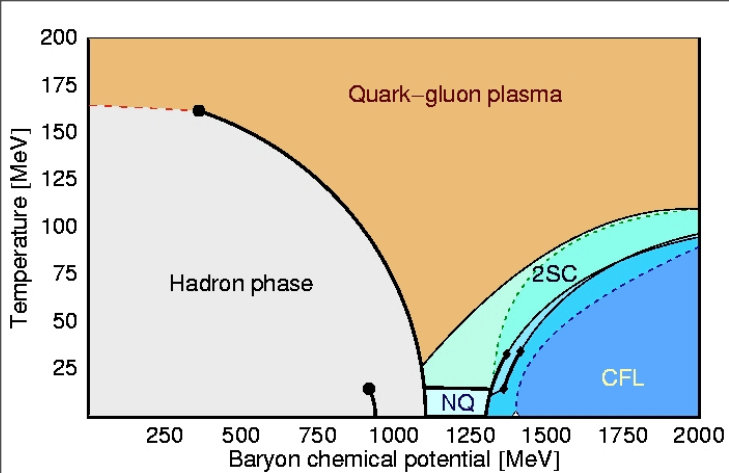
Universität Heidelberg & ExtreMe Matter Institute

Darmstadt, May 22th 2014



Related Talks & Posters

- C. Fischer **'Locating the CEP'**
- L. Fister **'On the phase structure and dynamics of QCD'**
- M. Hopfer **'The role of the quark-gluon vertex function in the QCD phase transition'**
- M. Huber **'Nonperturbative gluonic three-point correlations'**
- M. Mitter **'Phase Structure of Strongly Interacting Matter: Thermodynamics and Chiral Anomaly'**
- K. Morita **'The Chiral Criticality in the Probability Distribution of Conserved Charges'**
- R. Stiele **'Thermodynamics and phase structure of strongly-interacting matter'**
- M. Strickland **'Three loop HTL perturbation theory at finite temperature and chemical potential'**
- N. Strodthoff **'QCD-like theories at finite density'**
- A. Tripolt **'Spectral functions from the functional renormalization group'**



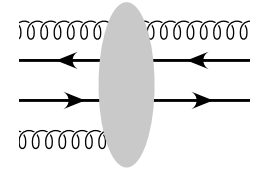
Outline

- **Phase Structure of QCD and Equation of State**
- **Spectral Functions & Transport Coefficients**
- **Outlook**

Functional Methods for QCD

quark-gluon correlations

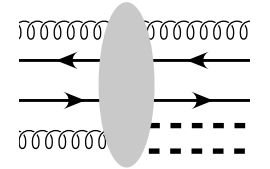
$$\langle q(x_1) \cdots \bar{q}(x_{2n}) A_\mu(y_1) \cdots A_\mu(y_m) \rangle$$



Functional Methods for QCD

quark-gluon-hadron correlations

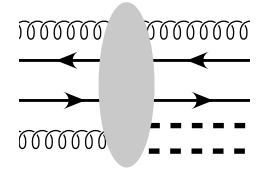
$$\langle q(x_1) \cdots \bar{q}(x_{2n}) A_\mu(y_1) \cdots A_\mu(y_m) h(z_1) \cdots h(z_l) \rangle$$



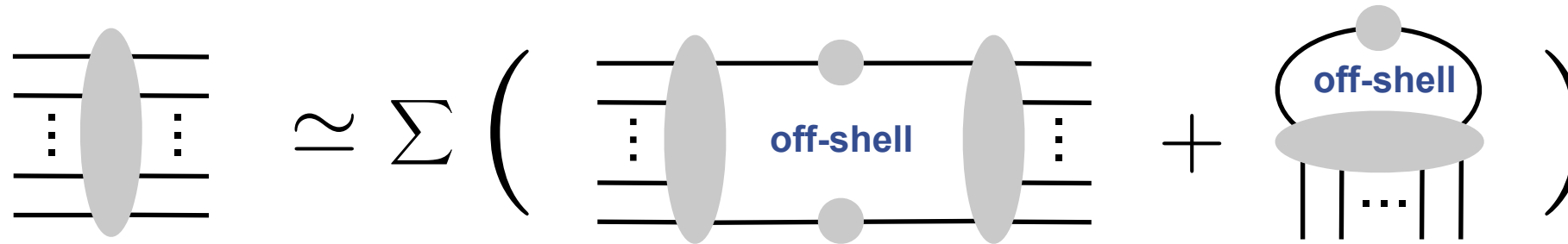
Functional Methods for QCD

quark-gluon-hadron correlations

$$\langle q(x_1) \cdots \bar{q}(x_{2n}) A_\mu(y_1) \cdots A_\mu(y_m) h(z_1) \cdots h(z_l) \rangle$$



functional relations



scattering amplitude/
vertex functions

Functional renormalisation group equations

Dyson-Schwinger equations

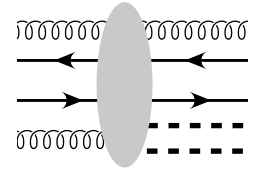
2PI/nPI hierarchies

⋮

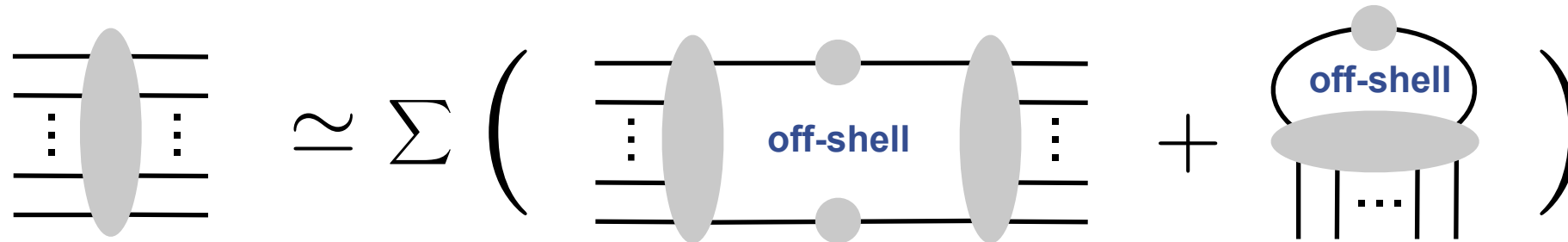
Functional Methods for QCD

quark-gluon-hadron correlations

$$\langle q(x_1) \cdots \bar{q}(x_{2n}) A_\mu(y_1) \cdots A_\mu(y_m) h(z_1) \cdots h(z_l) \rangle$$




functional relations



scattering amplitude/
vertex functions

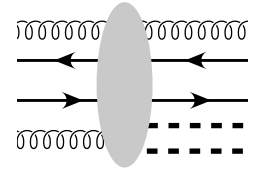
properties

- access to physics mechanisms 
- numerically tractable
no sign problem lattice: see talk of D. Sexty
systematic error control via closed form
- low energy models naturally incorporated

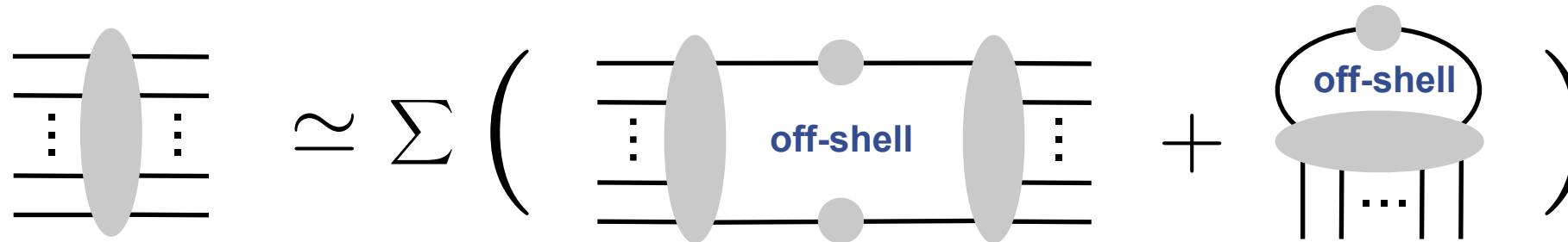
Functional Methods for QCD

quark-gluon-hadron correlations

$$\langle q(x_1) \cdots \bar{q}(x_{2n}) A_\mu(y_1) \cdots A_\mu(y_m) h(z_1) \cdots h(z_l) \rangle$$




functional relations



scattering amplitude/
vertex functions

properties

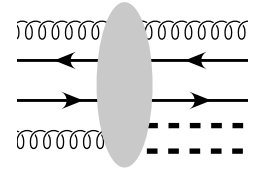
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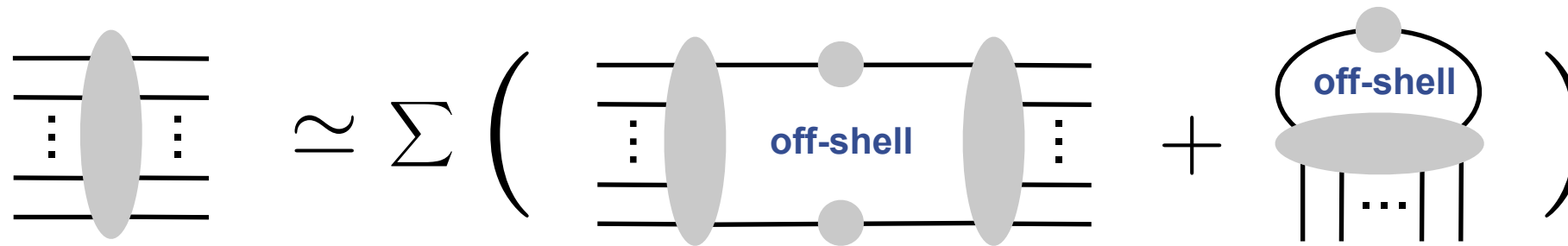
Functional Methods for QCD

quark-gluon-hadron correlations

$$\langle q(x_1) \cdots \bar{q}(x_{2n}) A_\mu(y_1) \cdots A_\mu(y_m) h(z_1) \cdots h(z_l) \rangle$$



functional relations




scattering amplitude/
vertex functions

e.g. lattice input on rhs

e.g. volume flucs., finite density,
dynamics, ...

properties

- access to physics mechanisms 
- numerically tractable
no sign problem lattice: see talk of D. Sexty
systematic error control via closed form
- low energy models naturally incorporated

FunMethods complementary to lattice



Functional Methods for QCD

Functional RG

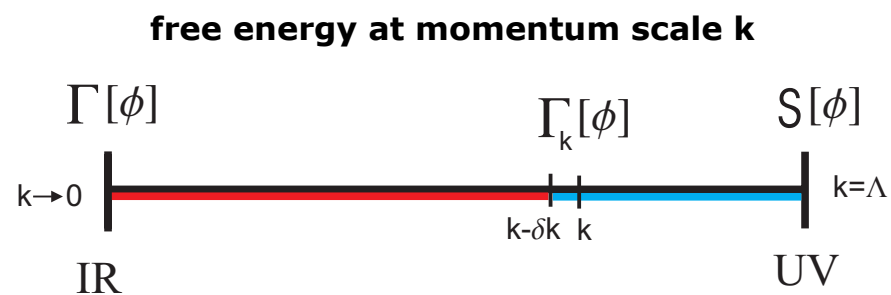
JMP, AIP Conf.Proc. 1343 (2011)

FRG QCD survey

JMP, Aussois '12

Phase diagram survey

JMP, Schladming '13



RG-scale k : $t = \ln k$

glue quantum fluctuations

hadronic quantum fluctuations

quark quantum fluctuations

free energy/ grand potential

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{glue loop} - \text{hadronic loop} - \text{quark loop} + \frac{1}{2} \text{ghost loop} \right)$$

Functional Methods for QCD

Functional RG

JMP, AIP Conf.Proc. 1343 (2011)

FRG QCD survey

JMP, Aussois '12

Phase diagram survey

JMP, Schladming '13

free energy at momentum scale k

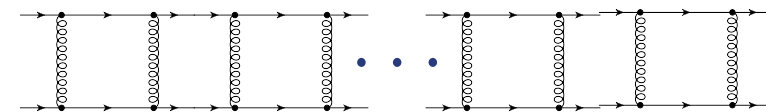
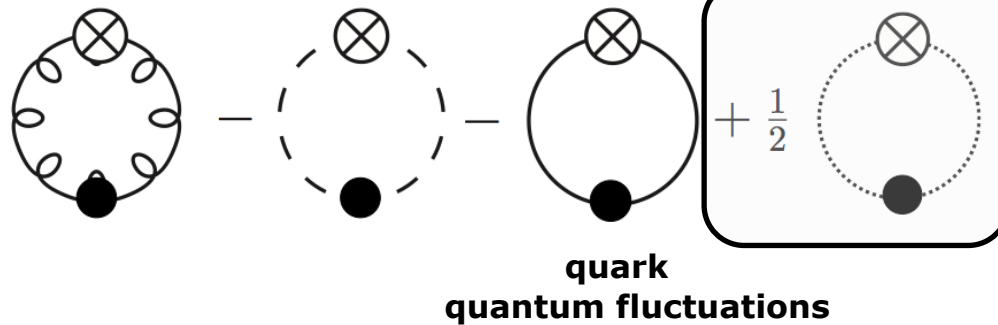


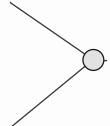
RG-scale k : $t = \ln k$

glue
quantum fluctuations

hadronic
quantum fluctuations

$\partial_t \Gamma_k[\phi] = \frac{1}{2}$
free energy/
grand potential



Dynamical hadronisation  **dynamical**

Gies, Wetterich '01
JMP '05
Flörchinger, Wetterich '09

Functional Methods for QCD

Yang-Mills

$$\partial_t \text{---} \circ \text{---}^{-1} = \text{---} \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---}$$

DSE-flow

$$\partial_t \text{---} \text{---} \text{---}^{-1} = \text{---} \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} \text{---}^{-1/2} \text{---} \text{---} \text{---} \text{---}$$

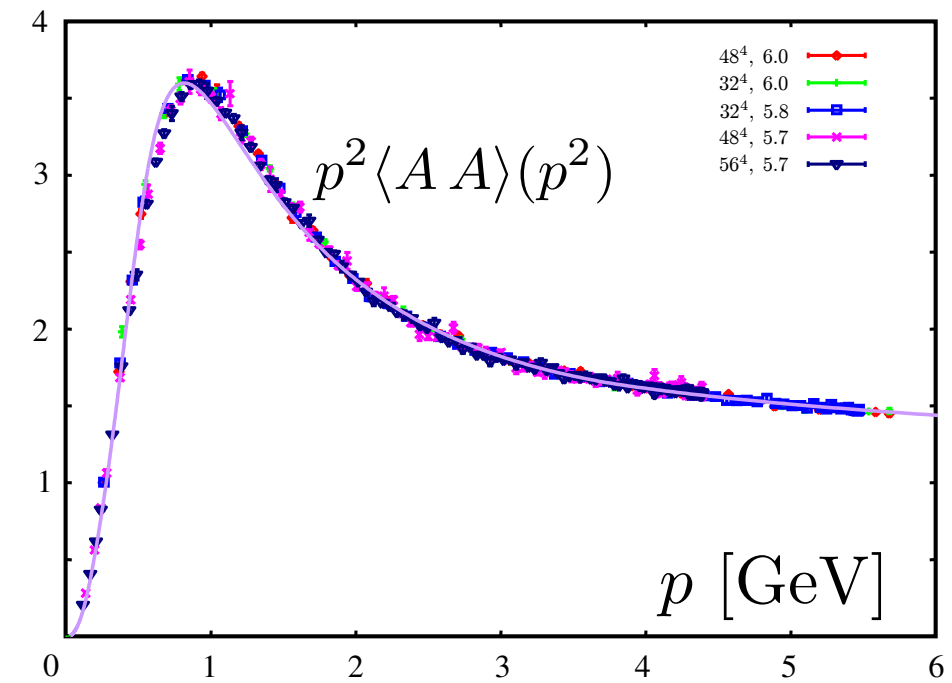
2PI-resummation

$$\partial_t \text{---} \text{---} \text{---} = 2 \text{---} \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---} + 2 \text{---} \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---} + \dots$$

$$\partial_t \text{---} \text{---} \text{---} = -3 \text{---} \text{---} \text{---} \text{---} + 6 \text{---} \text{---} \text{---} \text{---} + 3 \text{---} \text{---} \text{---} \text{---} - 6 \text{---} \text{---} \text{---} \text{---}$$

$$-\frac{1}{2} \text{---} \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---}$$

Yang-Mills propagators

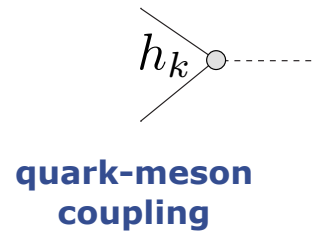


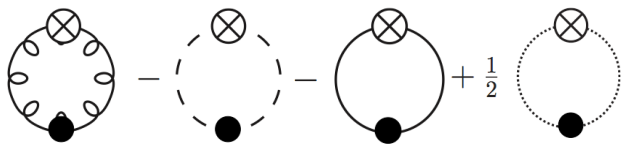
FRG: Fischer, Maas, JMP, Annals Phys. 324 (2009) 2408

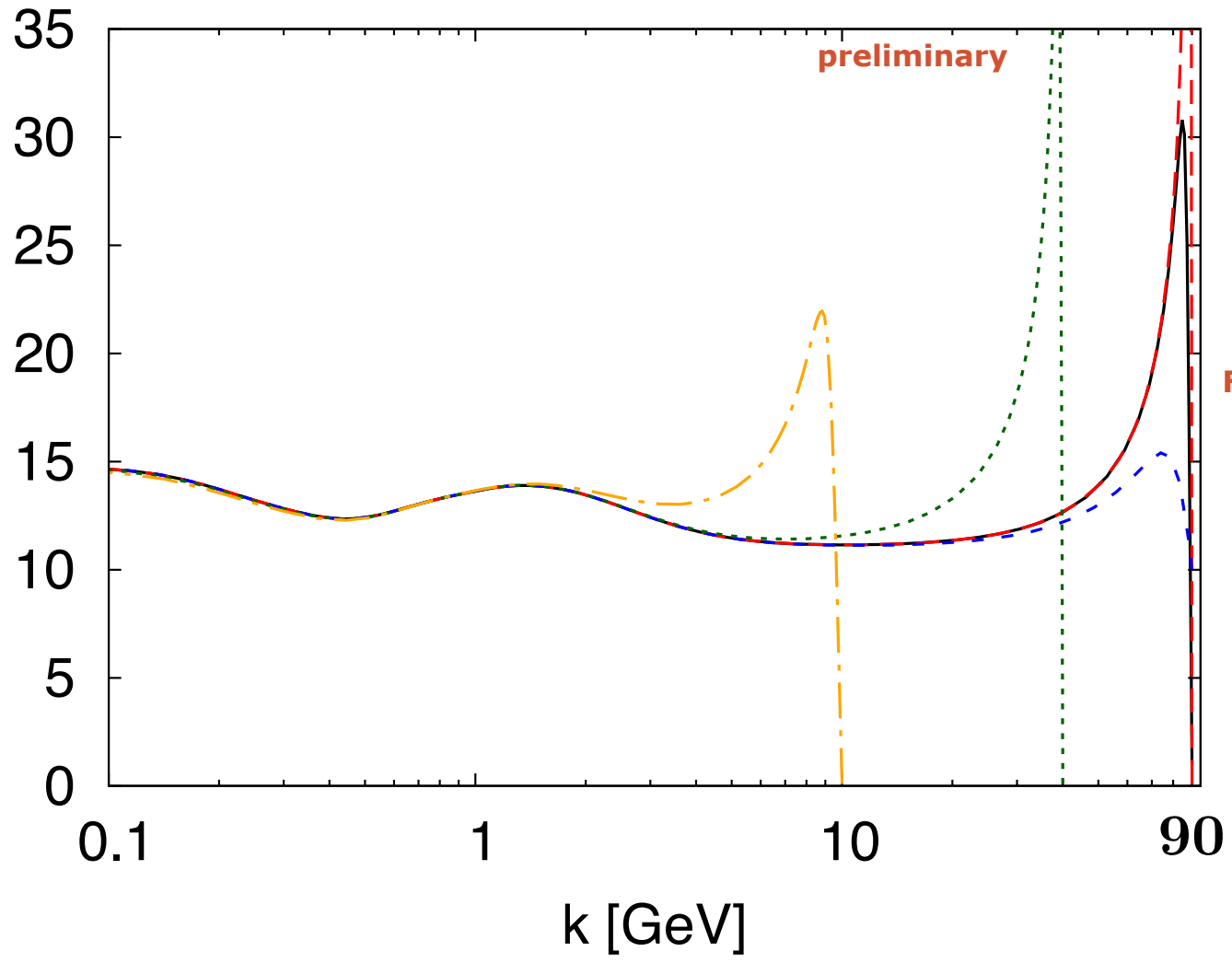
lattice: Sternbeck et al, PoS LAT2006 (2006) 076

see poster of M. Huber

QCD

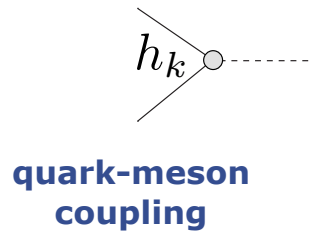


$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{diagram 1} - \text{diagram 2} - \text{diagram 3} + \frac{1}{2} \text{diagram 4} \right)$$


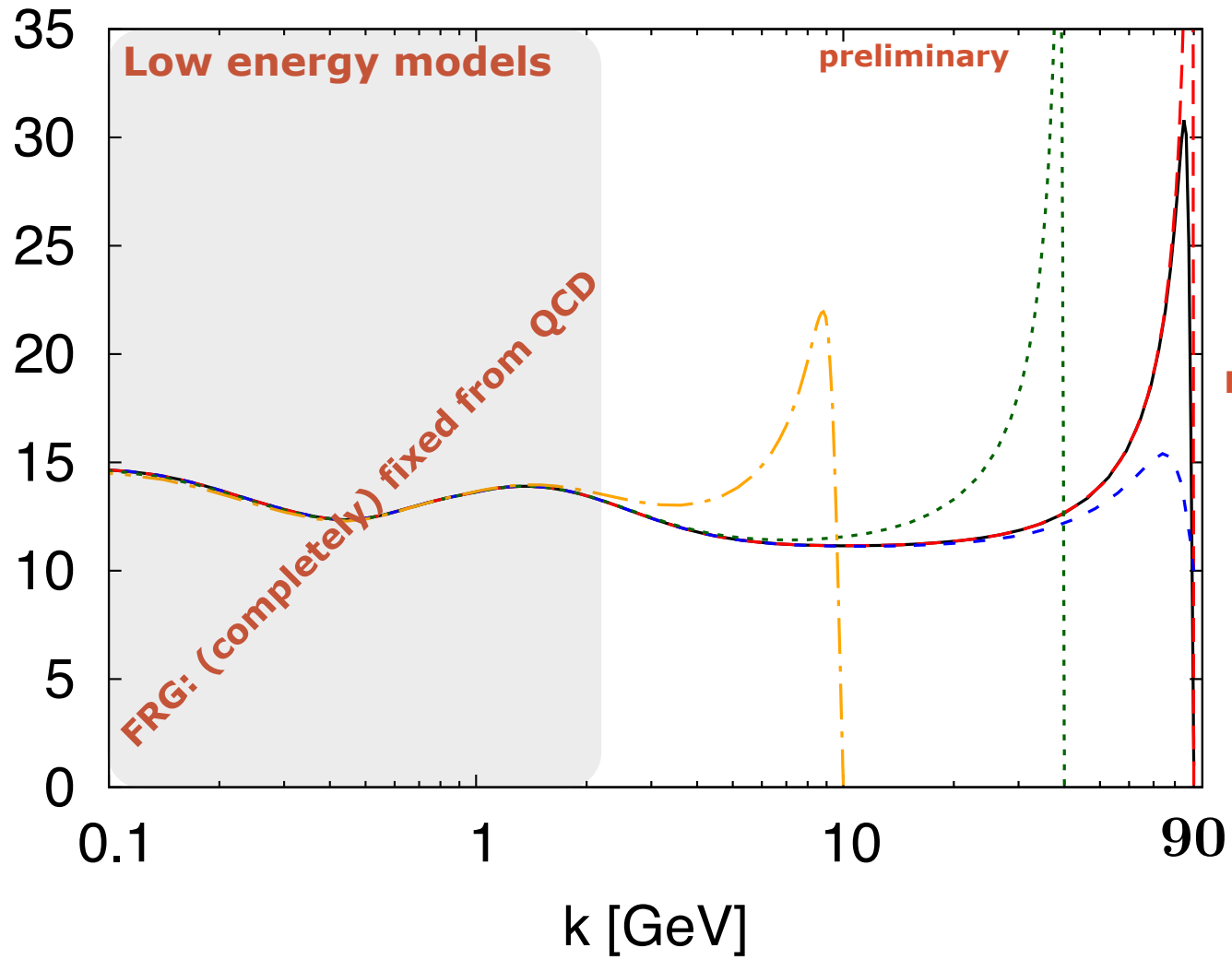


FRG-QCD: Fister, Herbst, Mitter, Rennecke, Strodtzoff, JMP

QCD



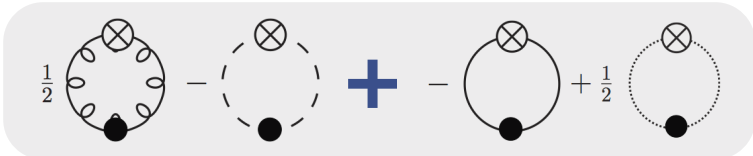
$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{[diagram 1]} - \text{[diagram 2]} - \text{[diagram 3]} + \frac{1}{2} \text{[diagram 4]}$$



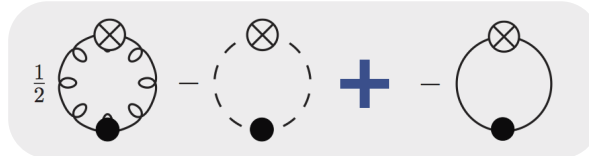
FRG-QCD: Fister, Herbst, Mitter, Rennecke, Strothoff, JMP

Model results on the phase structure of QCD

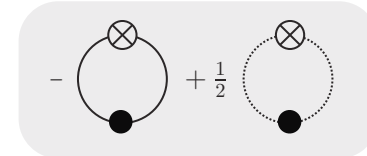
PQM-model



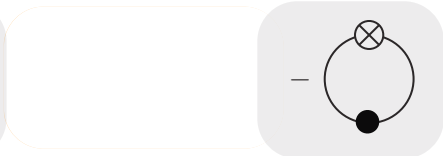
PNJL-model



QM-model



NJL-model

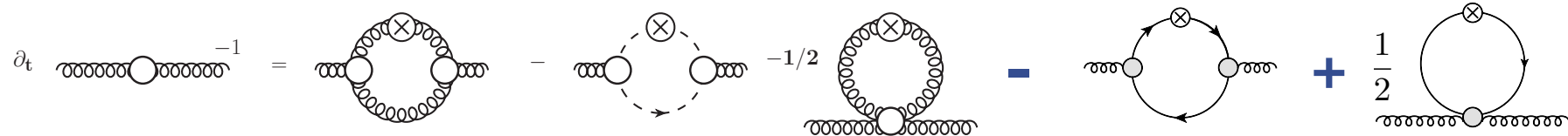


Functional Methods for QCD

present best approximation



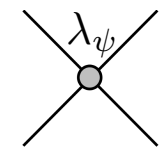
momentum dependence



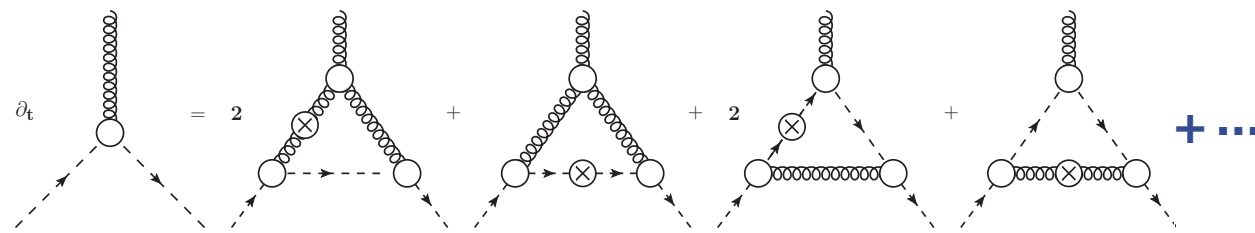
momentum dependence



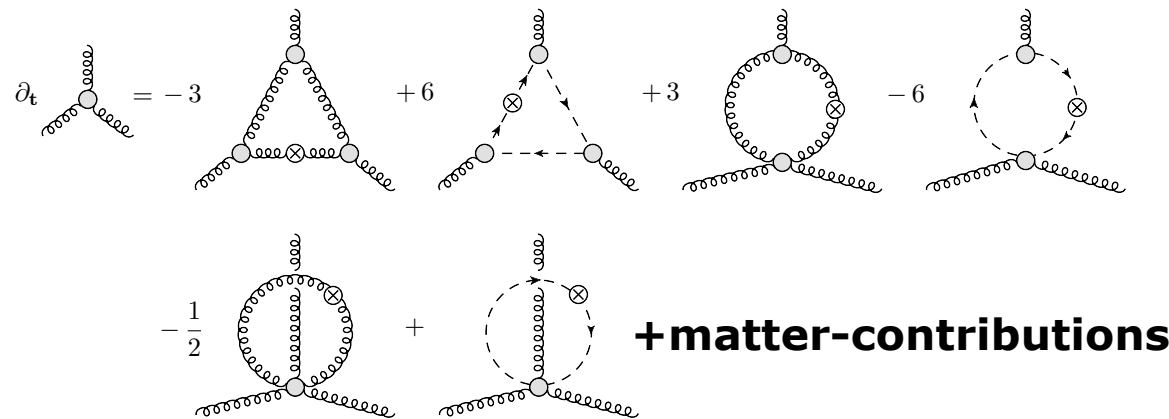
momentum dependence



all tensor structures



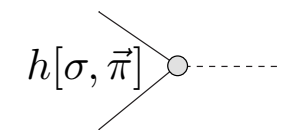
momentum dependence



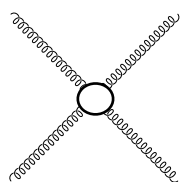
momentum dependence



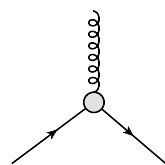
full mesonic field-dependence



2PI-resummed



momentum dependence

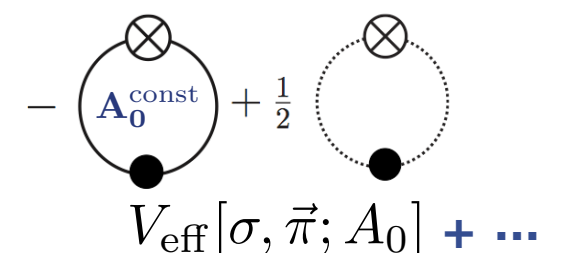


all tensor structures

FRG-QCD: Fister, Herbst, Mitter, Rennecke, Strodtzoff, JMP

DSE: see poster of M. Hopfer

full field-dependence

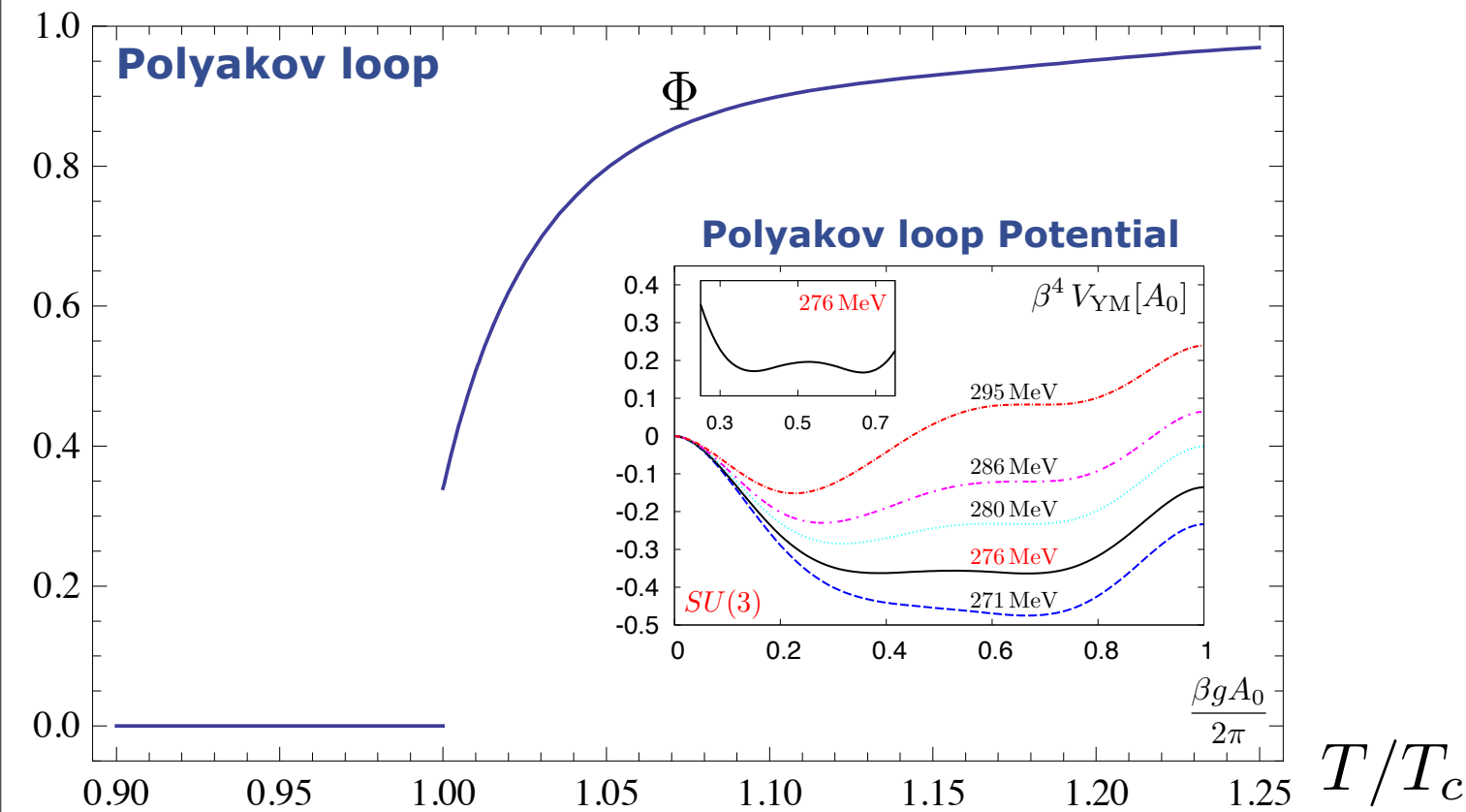


Phase Structure and Equation of State

Confinement

FRG: Braun, Gies, JMP, PLB 684 (2010) 262

FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010

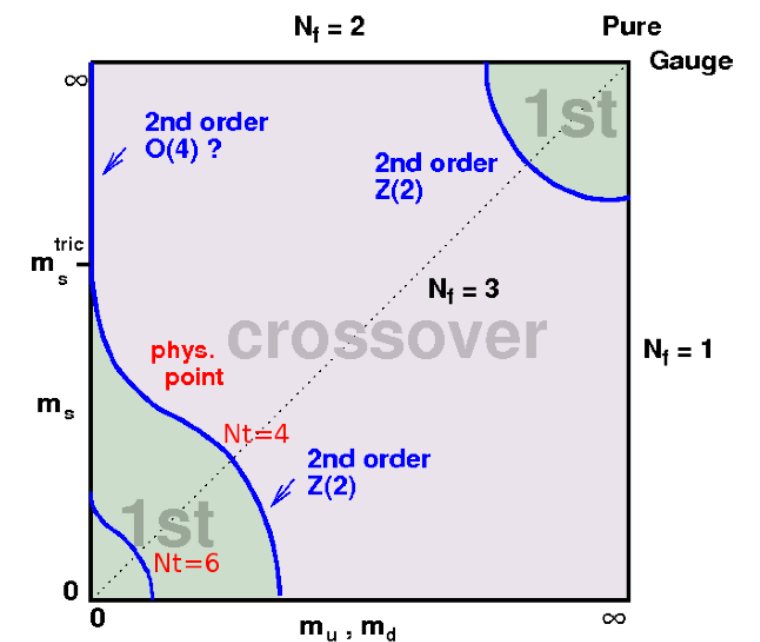


$$T_c/\sqrt{\sigma} = 0.658 \pm 0.023$$

$$\text{lattice : } T_c/\sqrt{\sigma} = 0.646$$

see also talk of C. Sasaki

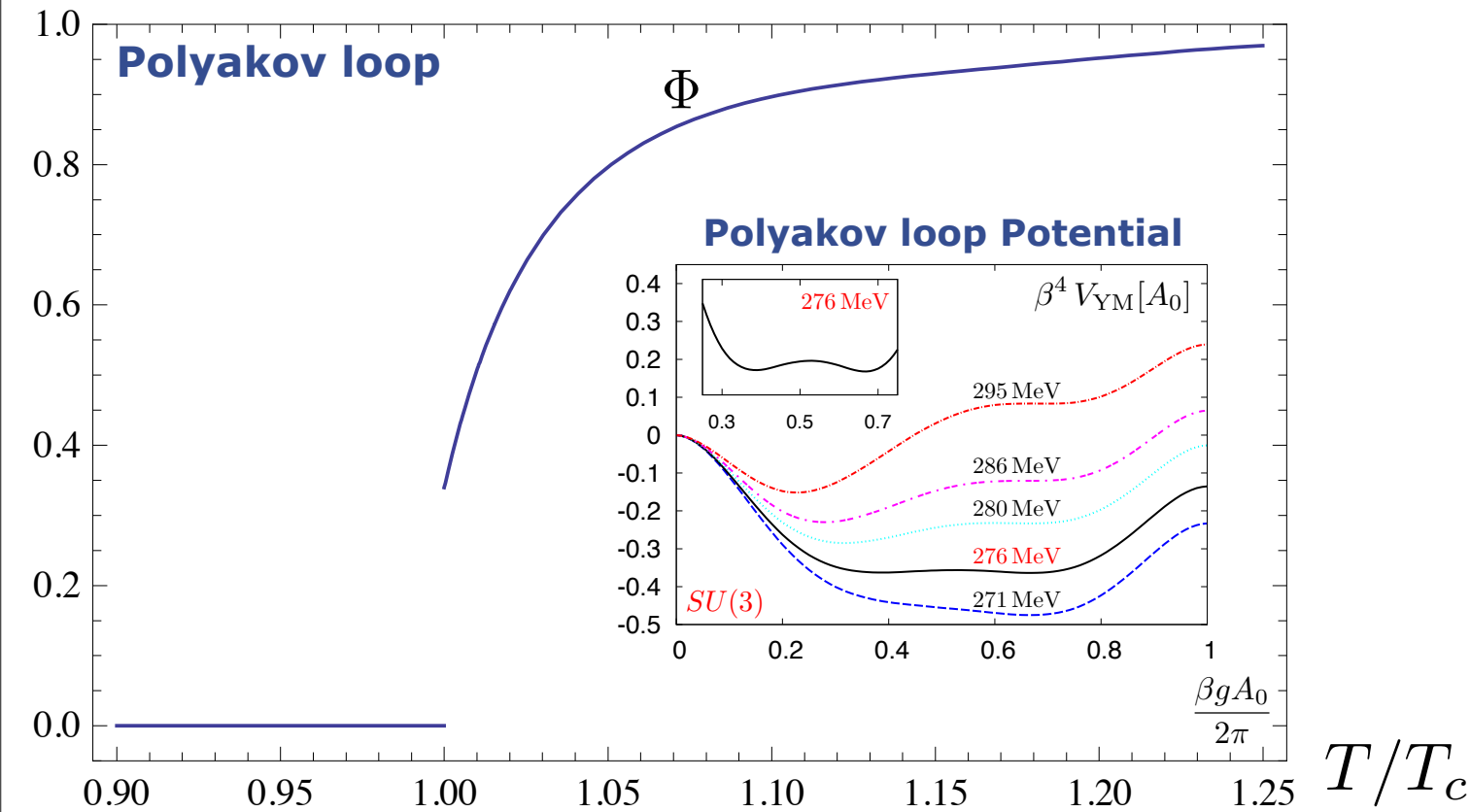
$$\Phi[A_0] = \frac{1}{3} \left(1 + 2 \cos \frac{1}{2} \beta g A_0 \right)$$



Confinement

FRG: Braun, Gies, JMP, PLB 684 (2010) 262

FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010



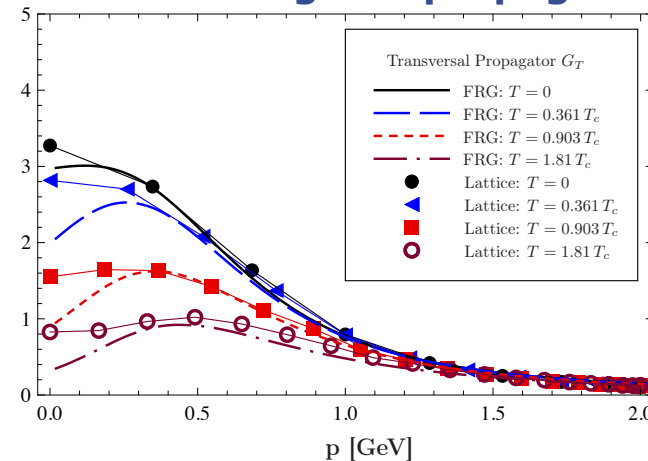
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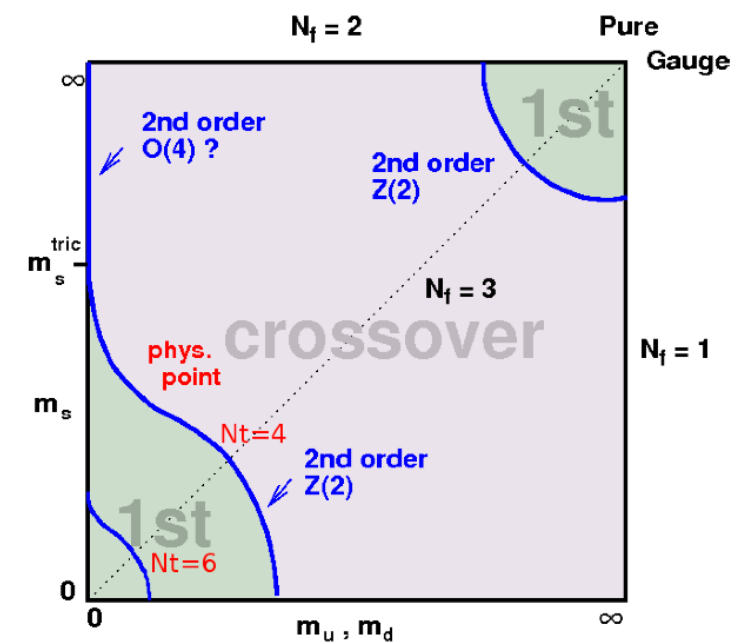
see also talk of C. Sasaki

$$\Phi[A_0] = \frac{1}{3} \left(1 + 2 \cos \frac{1}{2} \beta g A_0 \right)$$

transversal gluon propagator



Fister, JMP, arXiv:1112.5440

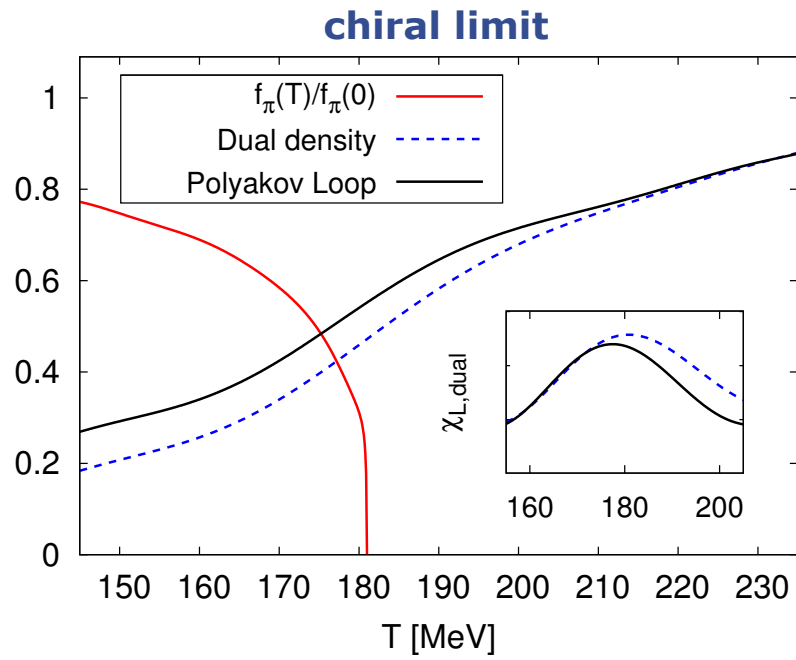


- from the full propagators
- gauge independence
- confinement criteria

Full dynamical QCD

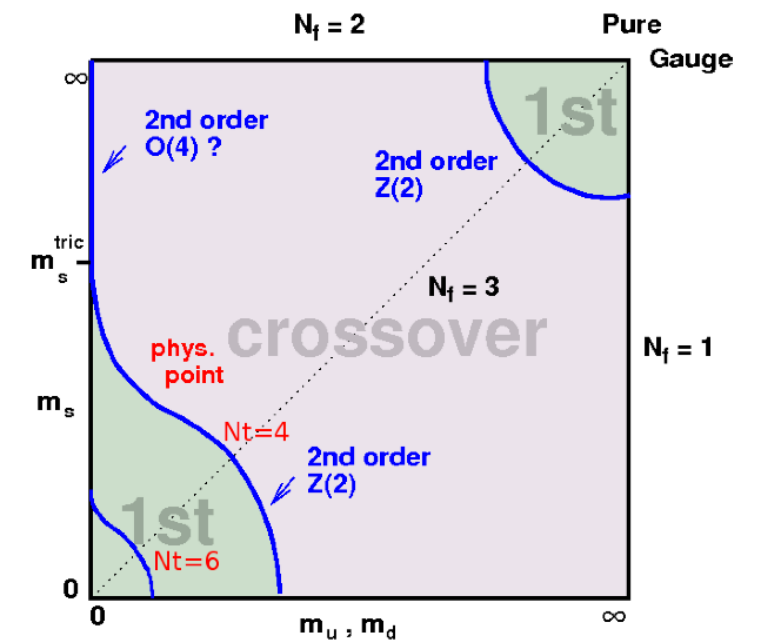
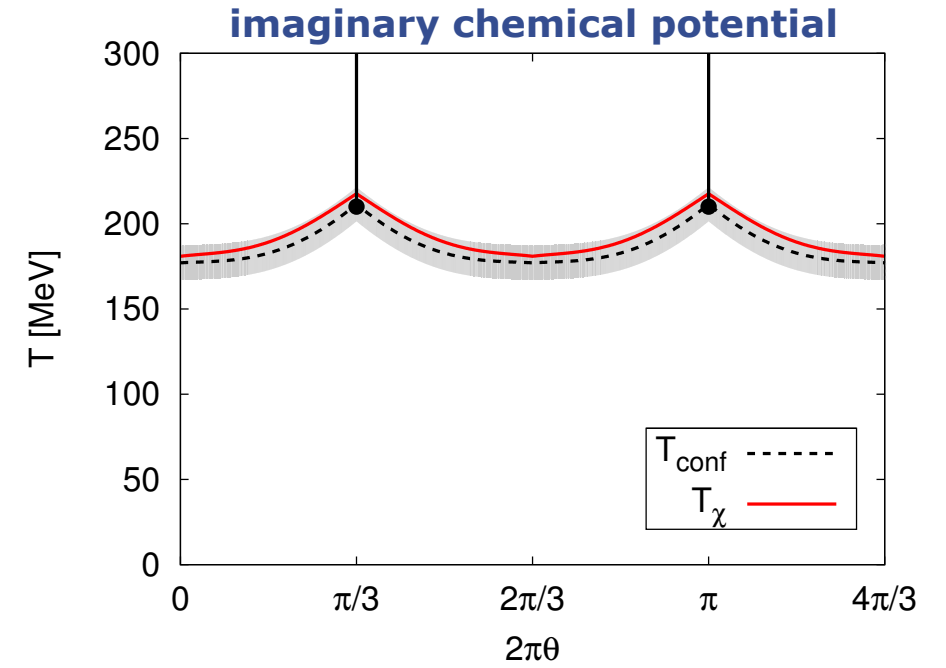
Phase structure

Braun, Haas, Marhauser, JMP, PRL 106 (2011) 022002



$$N_f = 2$$

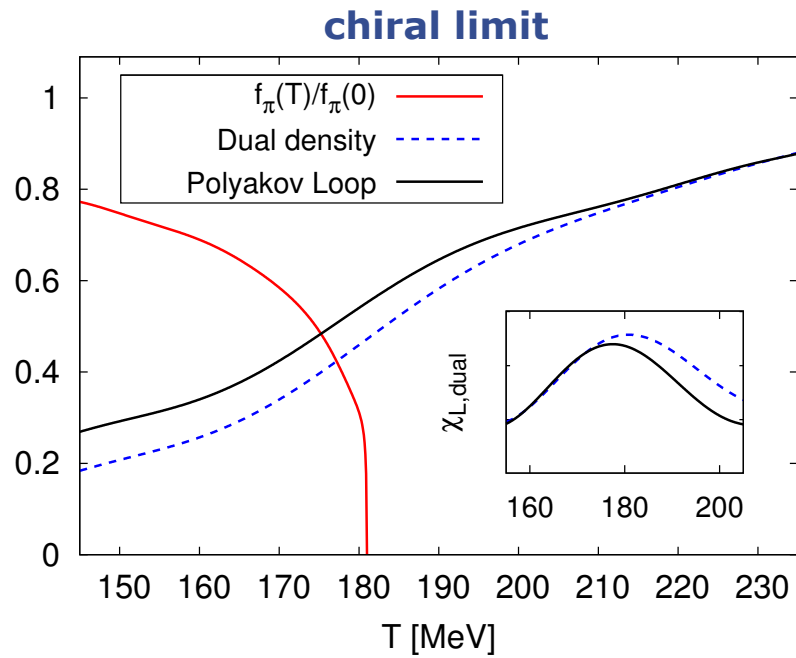
$$T_\chi \simeq T_{conf}$$



Full dynamical QCD

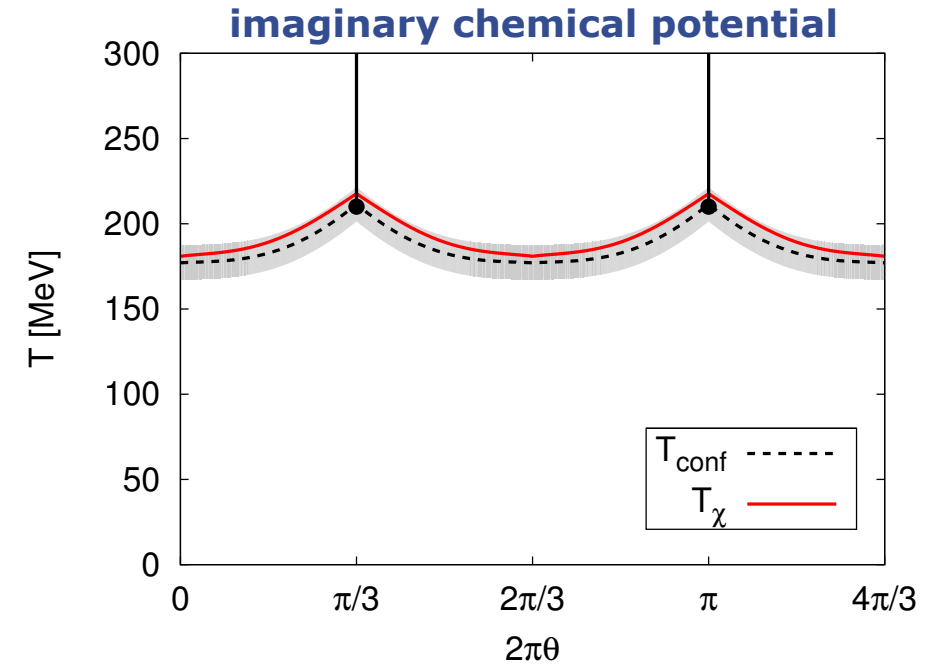
Phase structure

Braun, Haas, Marhauser, JMP, PRL 106 (2011) 022002

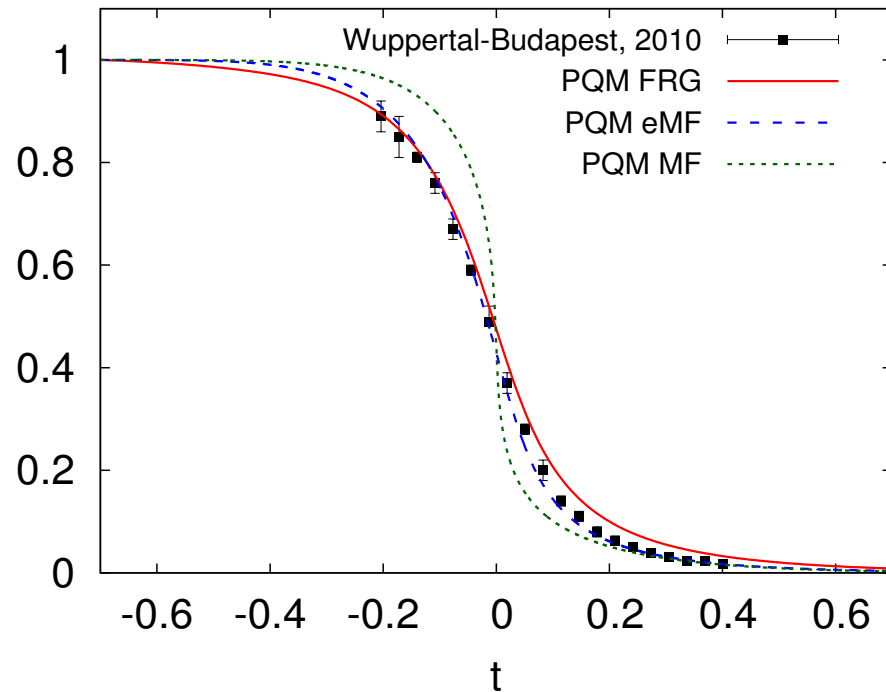


$$N_f = 2$$

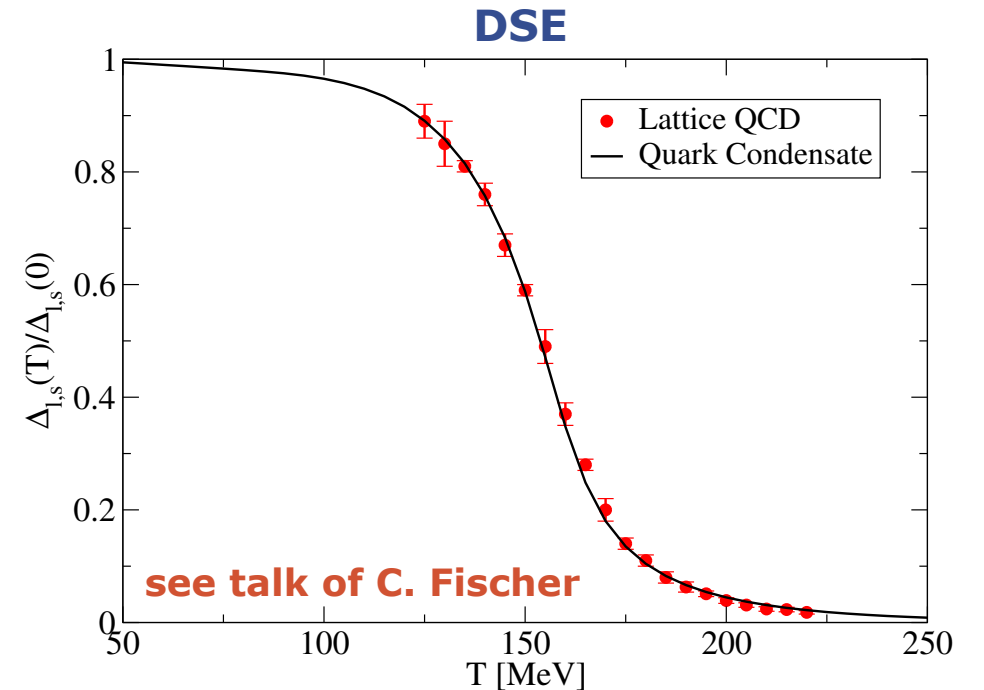
$$T_\chi \simeq T_{conf}$$



QCD-improved PQM model



$$N_f = 2+1$$



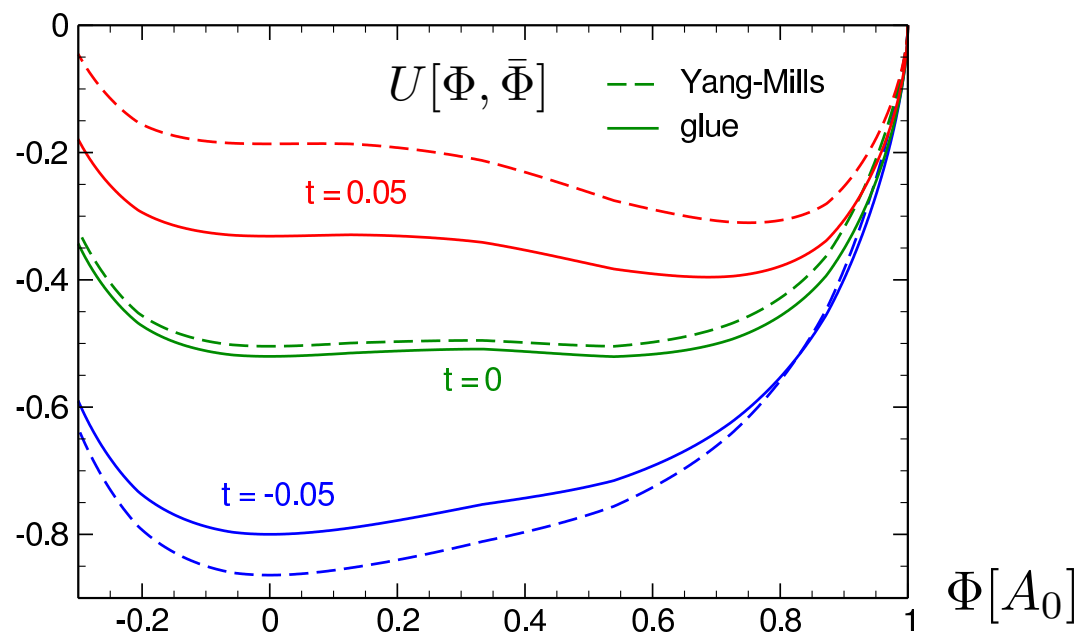
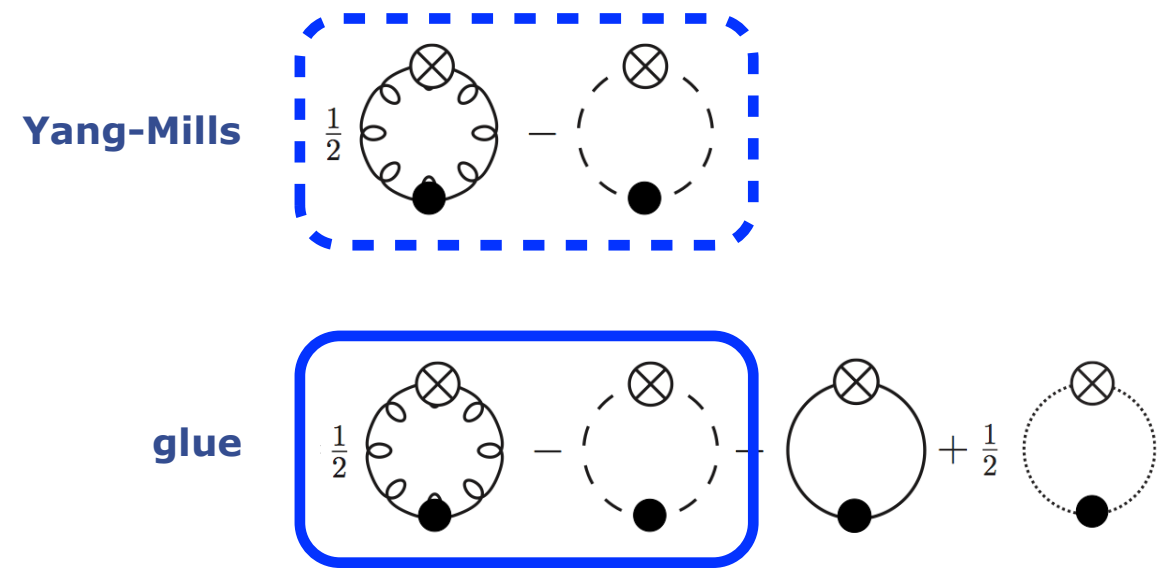
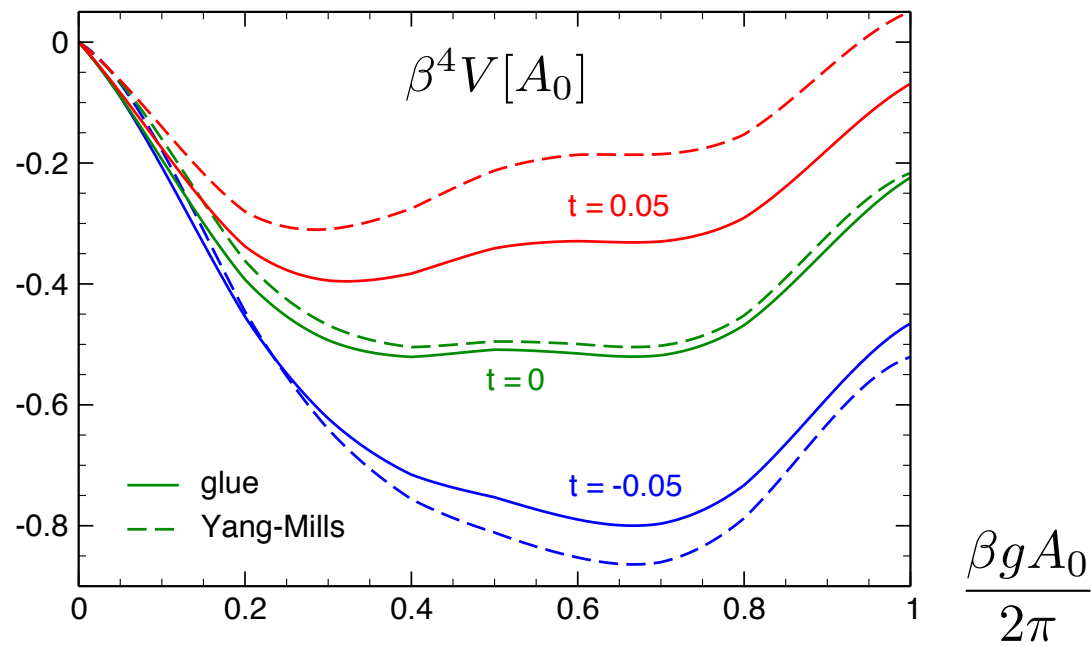
Luecker, Fischer, Fister, JMP, PoS CPOD2013 (2013) 057
 Fischer, Luecker, Welzbacher, arXiv:1405.4762

Herbst, Mitter et al, PLB 731 (2014) 248-256

Full dynamical QCD

Improving models towards QCD

Polyakov loop potential in full QCD



see poster of R. Stiele

JMP, AIP Conf.Proc. 1343 (2011)

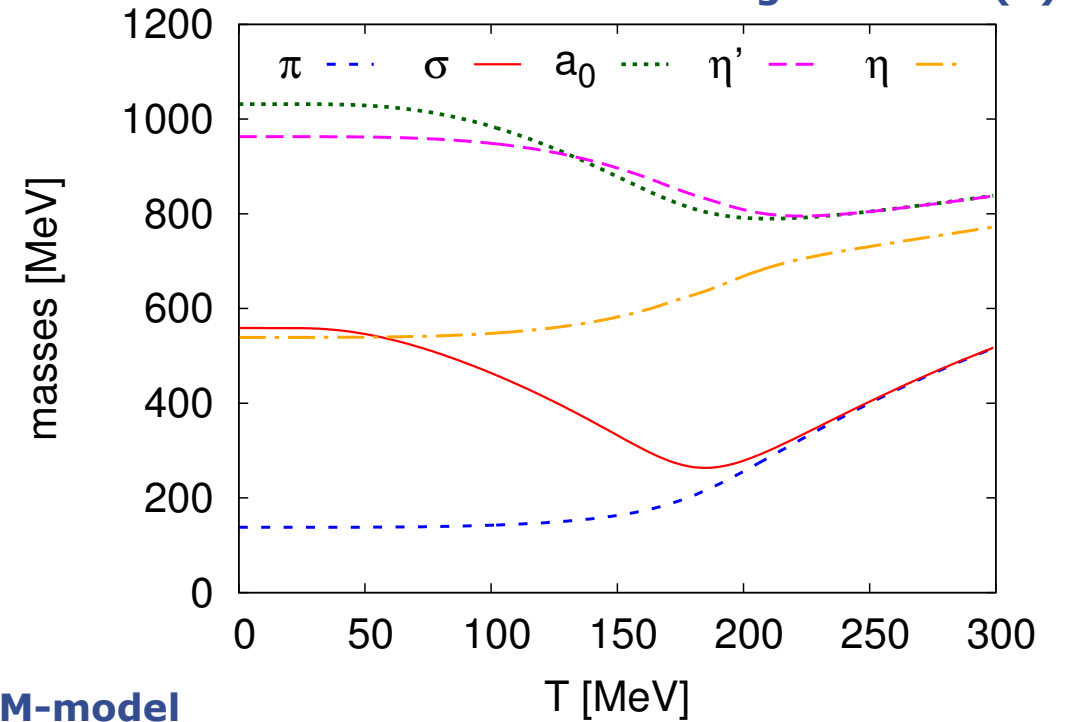
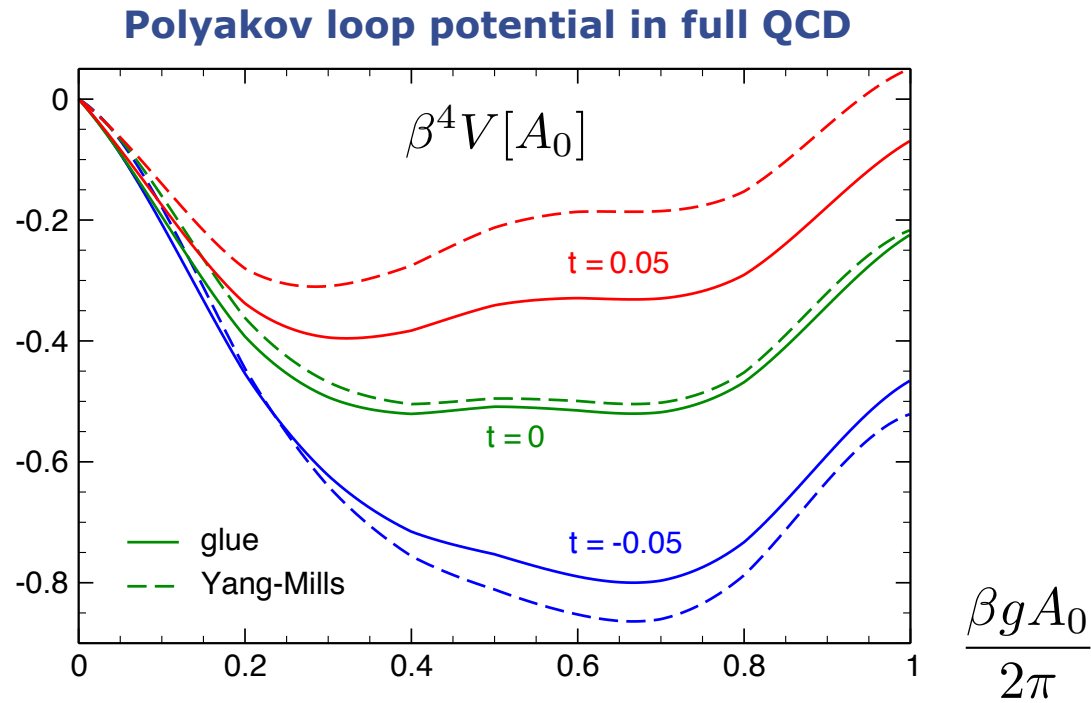
Haas, Stiele et al, PRD 87 (2013) 076004

Full dynamical QCD

Improving models towards QCD

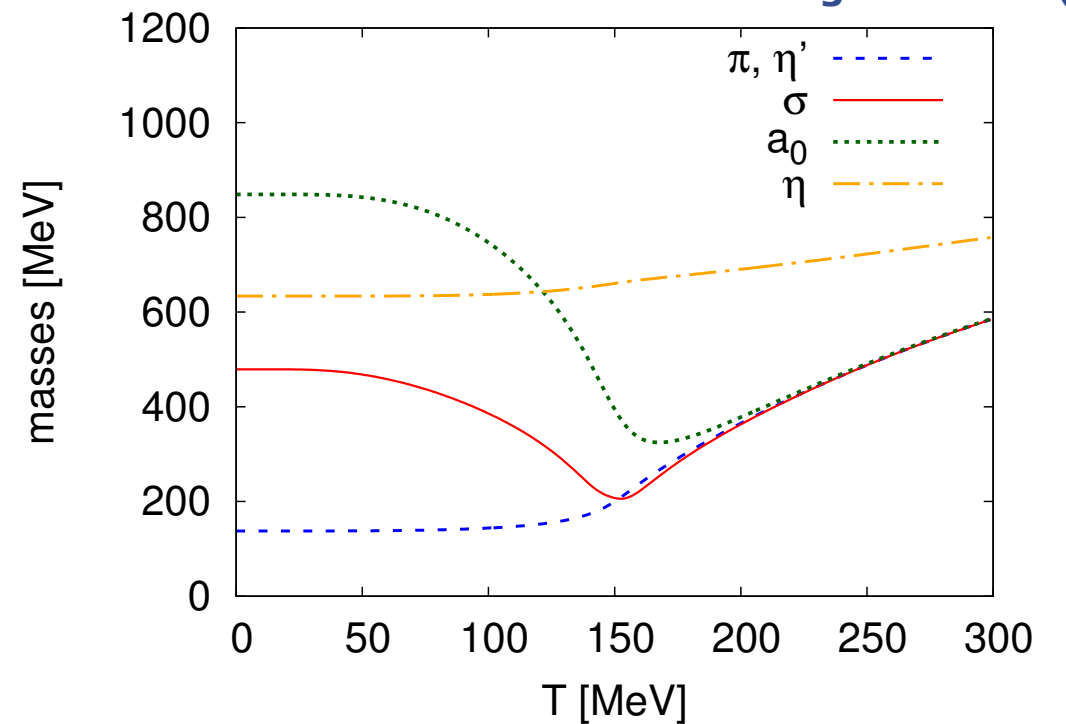
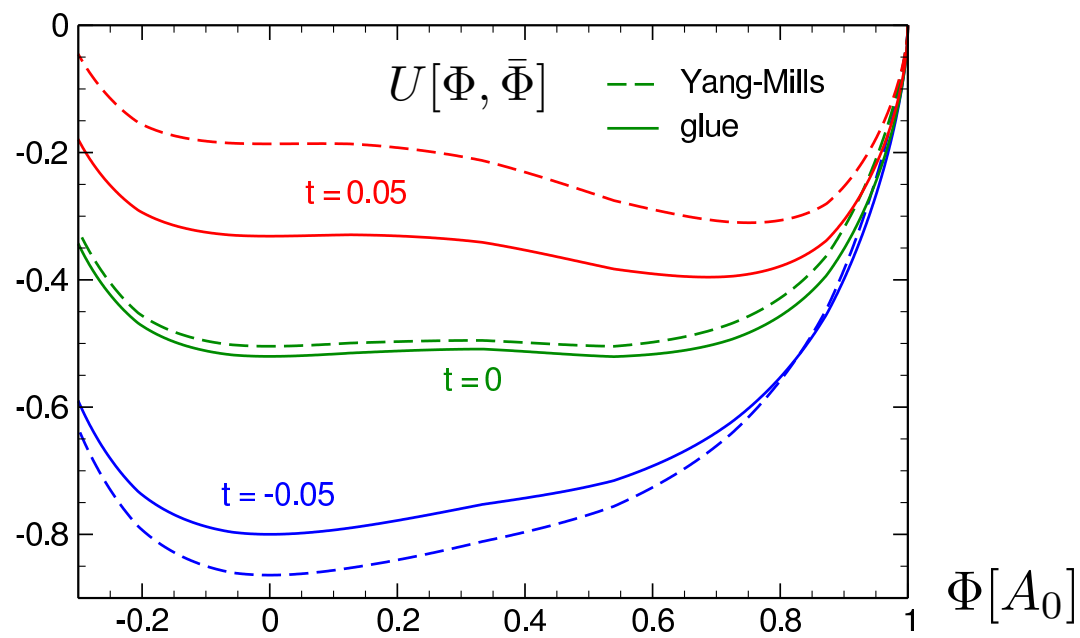
Mitter, Schaefer, Phys.Rev. D89 (2014) 054027

with anomalous breaking of axial U(1)



QM-model

without anomalous breaking of axial U(1)

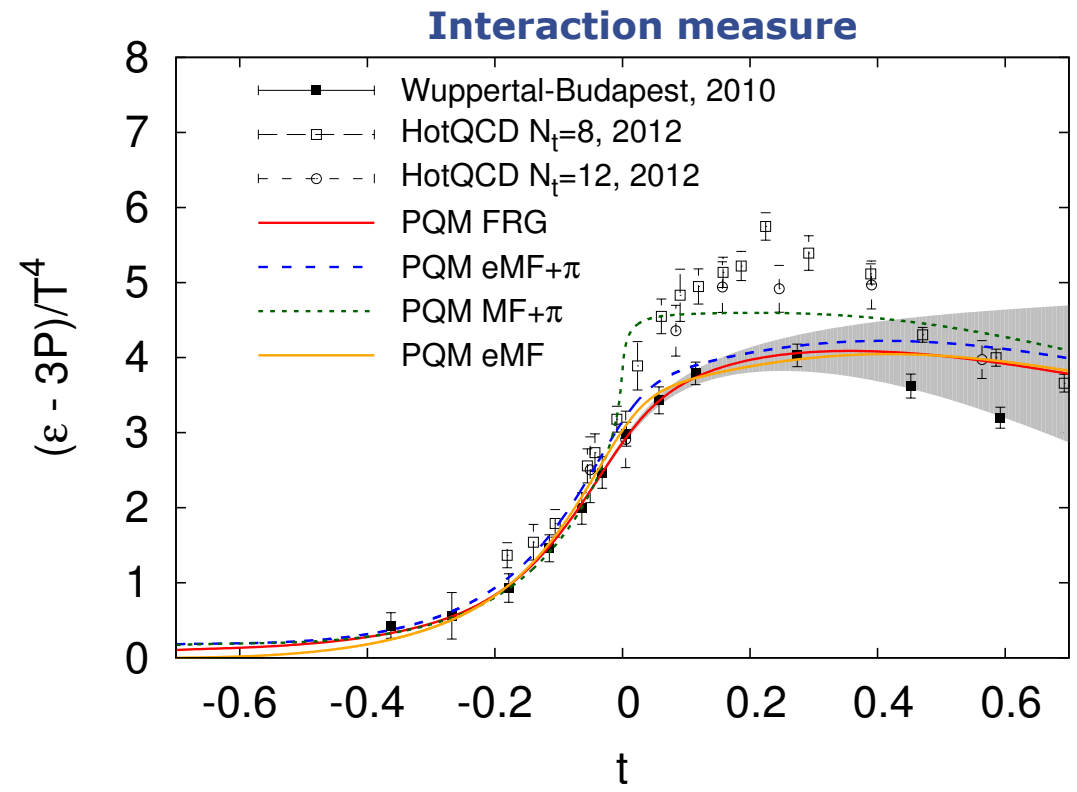
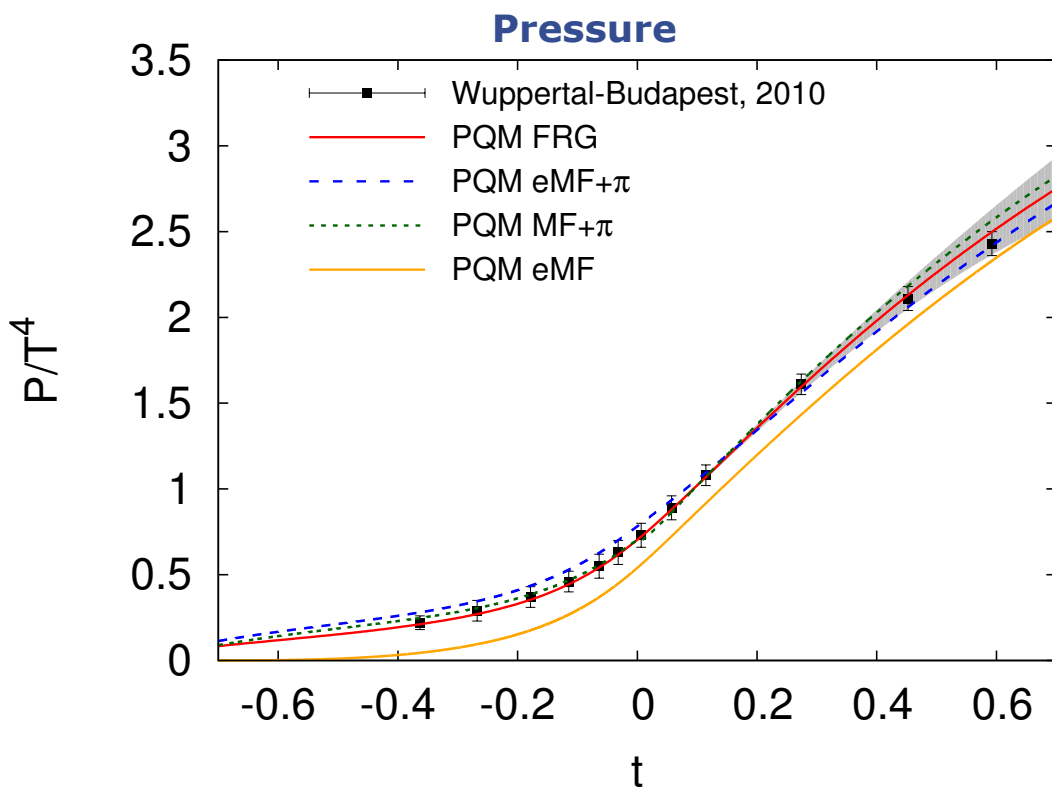


see poster of M. Mitter

Full dynamical QCD

Equation of state

Herbst, Mitter, JMP, Schaefer, Stiele, Phys.Lett. B731 (2014) 248-256



Shaded area: systematic error estimate due to low initial scale 1 GeV

lattice: see talk of A. Bazavov

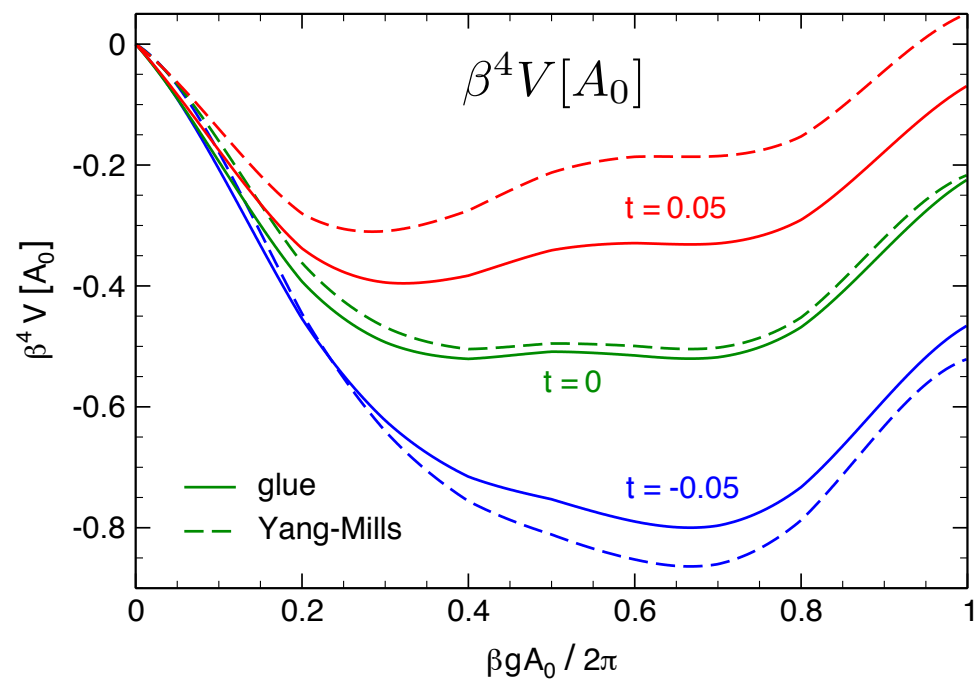
high T: see talk of M. Strickland

see poster of M. Mitter

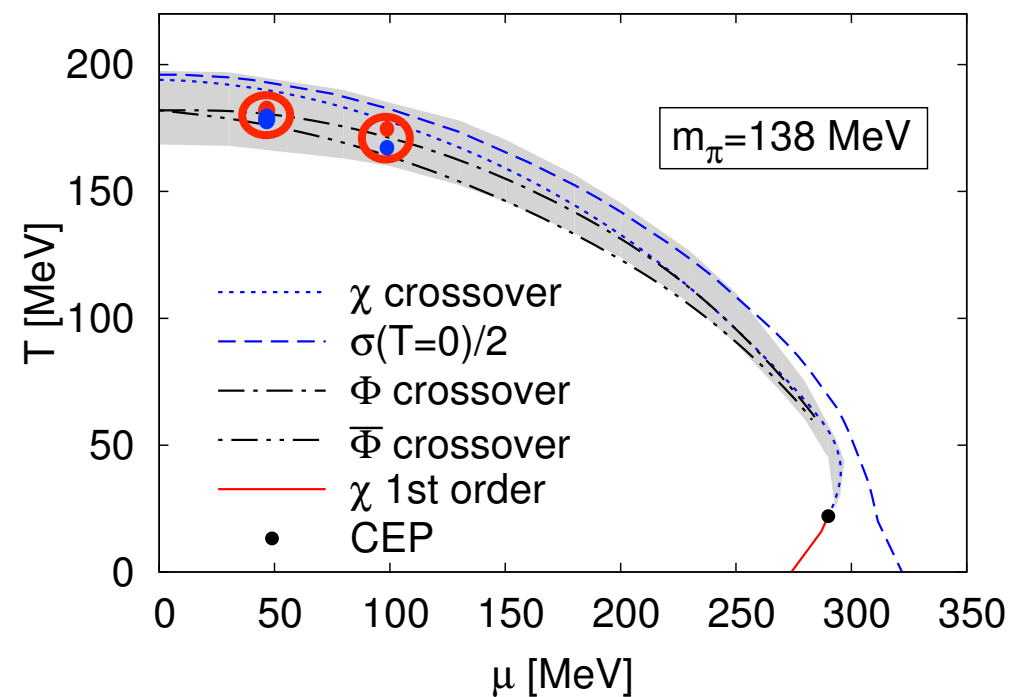
Full dynamical QCD

Phase structure at finite density

Polyakov loop potential in full QCD



Phase diagram of quantised PQM-model



Herbst, JMP, Schaefer, PLB 696 (2011) 58-67
PRD 88 (2013) 1, 014007



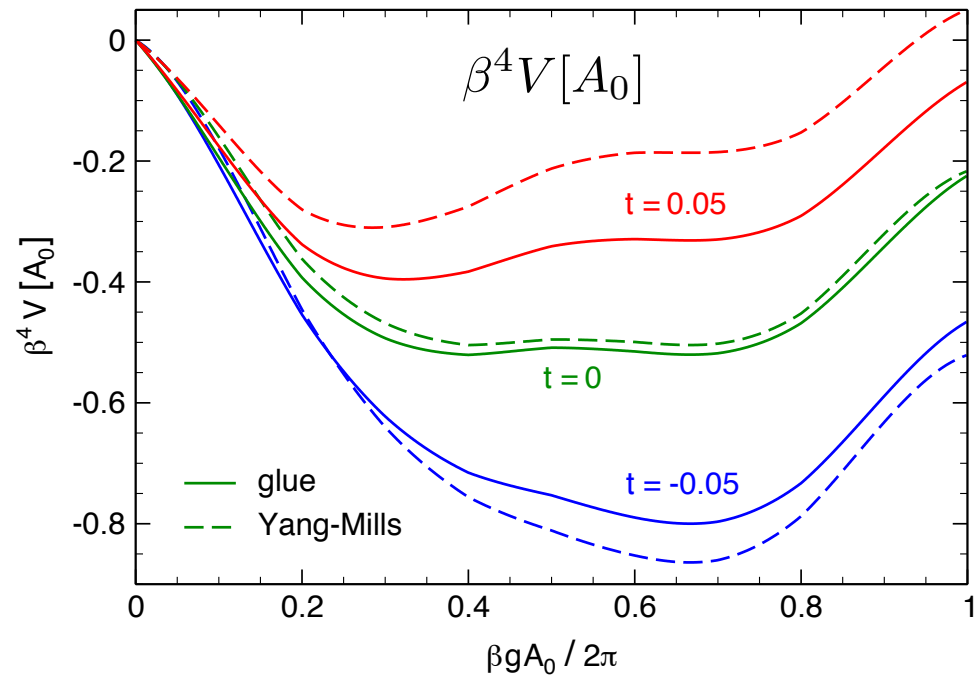
FRG QCD results at finite density

Haas, Braun, JMP '09, unpublished

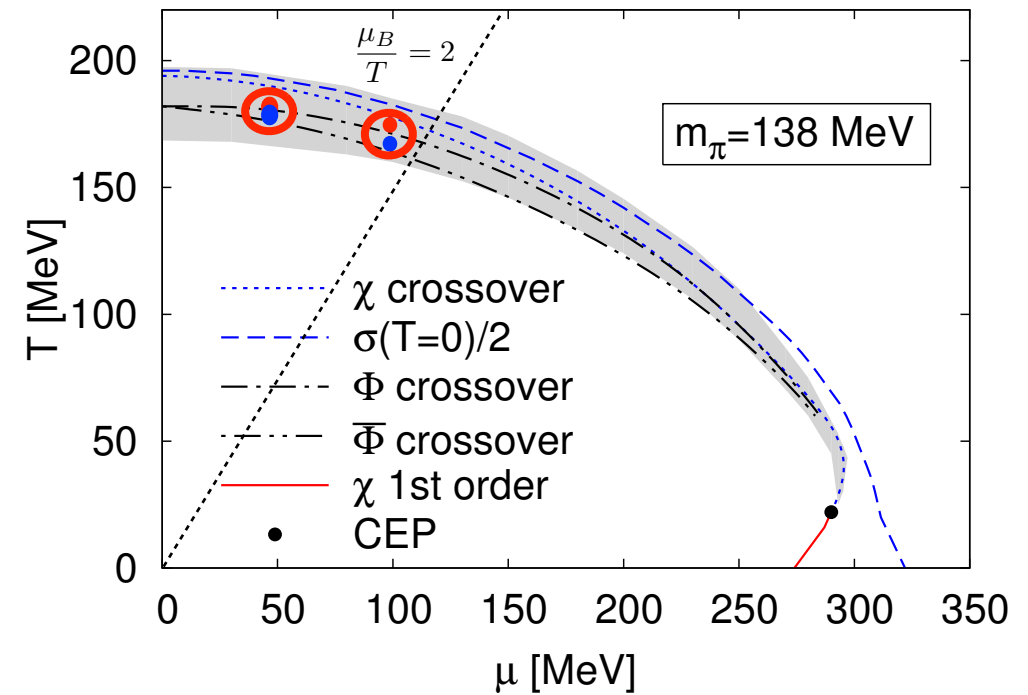
Full dynamical QCD

Phase structure at finite density

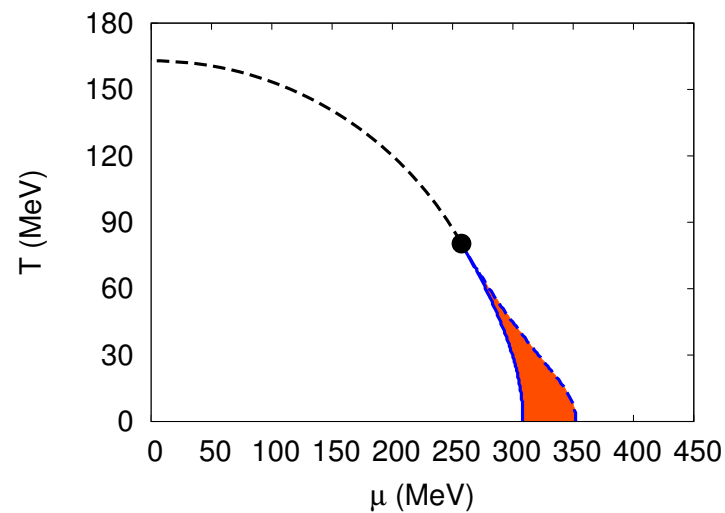
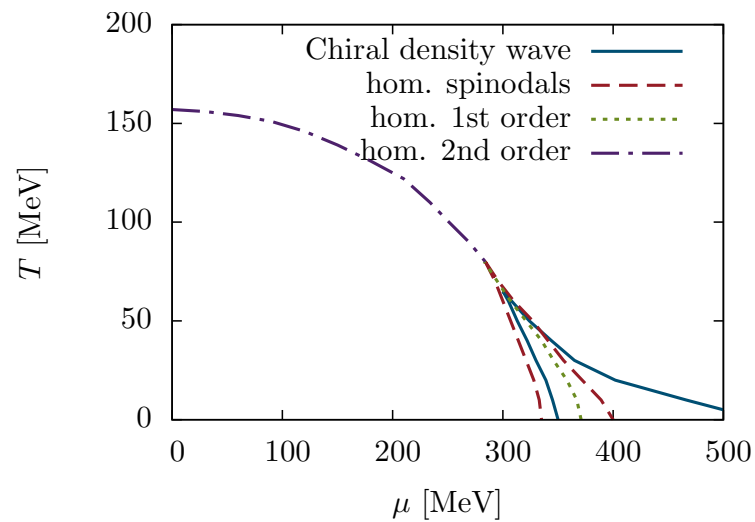
Polyakov loop potential in full QCD



Phase diagram of quantised PQM-model



inhomogeneous phases



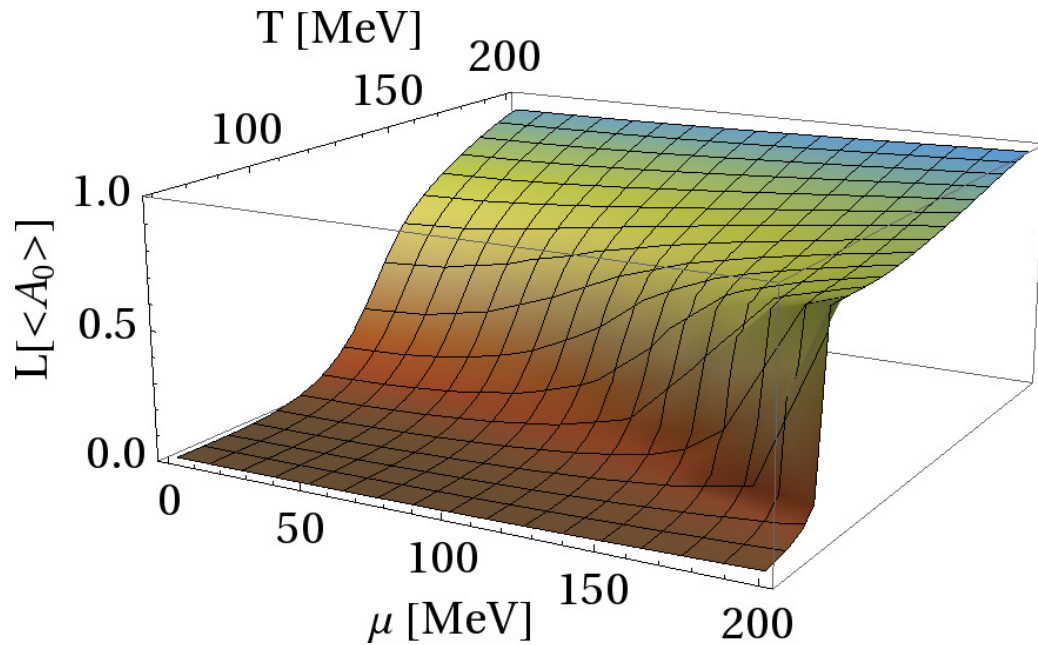
diquarks, baryons,
see poster of N. Strodthoff

higher moments
see poster of K. Morita

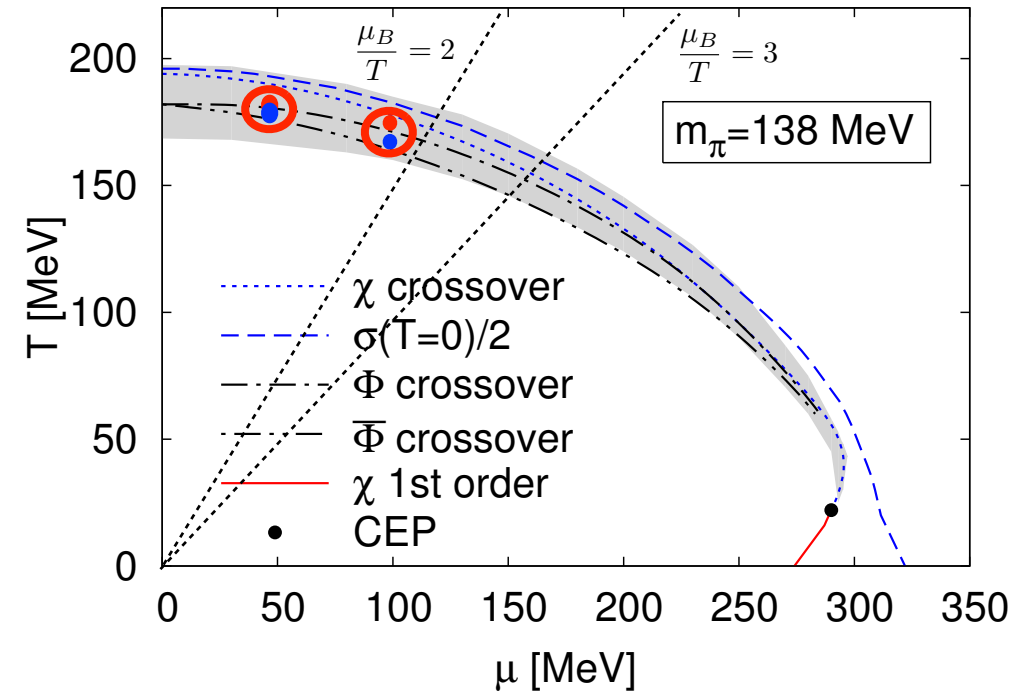
Full dynamical QCD

Phase structure at finite density

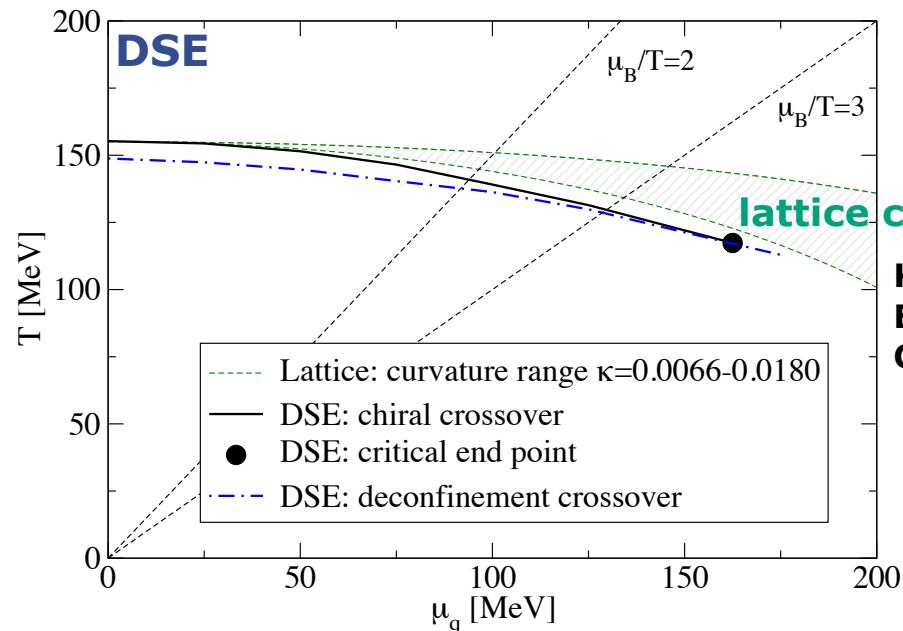
Polyakov loop at finite density



Phase diagram of quantised PQM-model



Phase diagram of 2+1 flavor QCD



Kaczmarek et al. '11
Endrodi, Fodor, Katz, Szabo '11
Cea, Cosmai, Papa '14

see talk of C. Fischer

Critical point unlikely for

$$\frac{\mu_B}{T} < 2$$

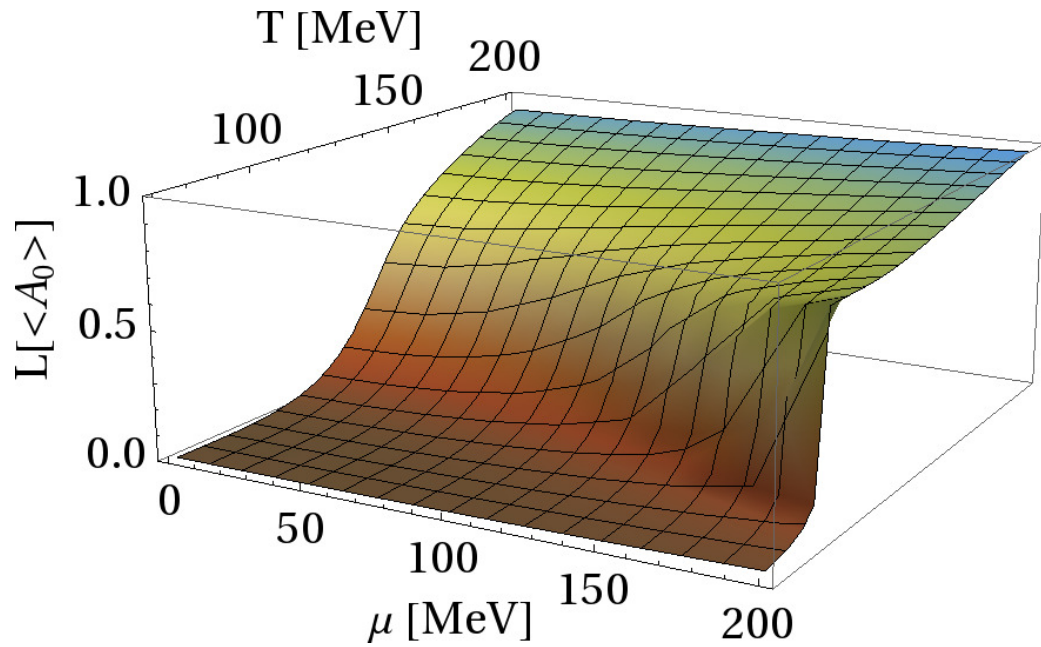
Fischer, Luecker, PLB 718 (2013) 1036

Fischer, Fister, Luecker, JMP, PLB732 (2014) 248

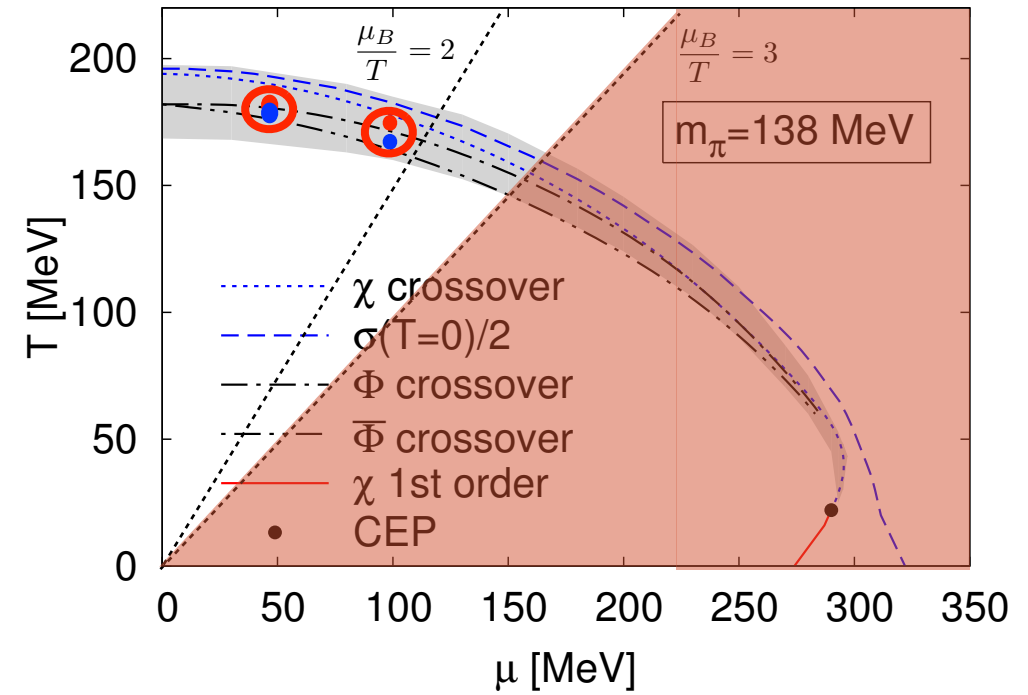
Full dynamical QCD

Phase structure at finite density

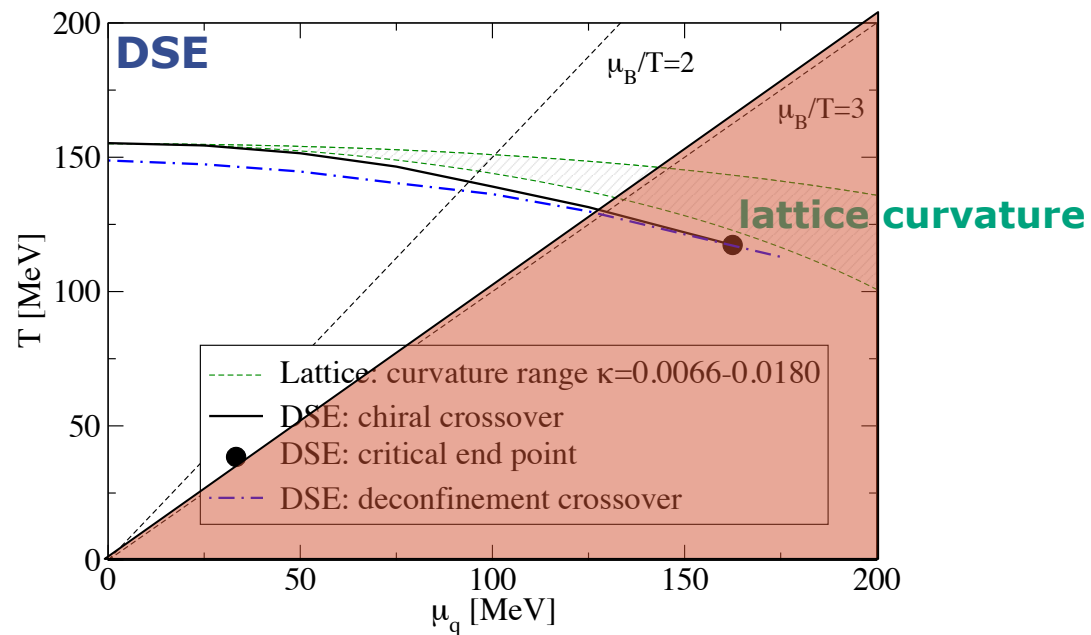
Polyakov loop at finite density



Phase diagram of quantised PQM-model



Phase diagram of 2+1 flavor QCD



Critical point unlikely for

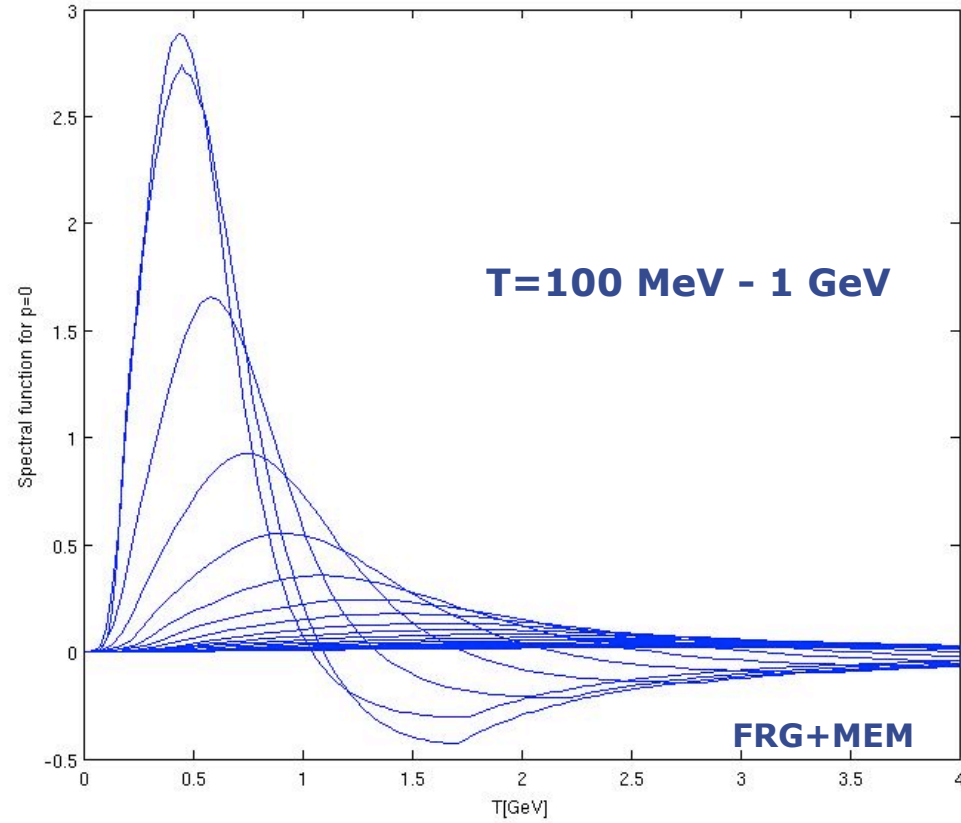
$$\frac{\mu_B}{T} < 2$$

Spectral functions & Transport Coefficients

Viscosity in pure glue

spectral functions

gluon spectral function

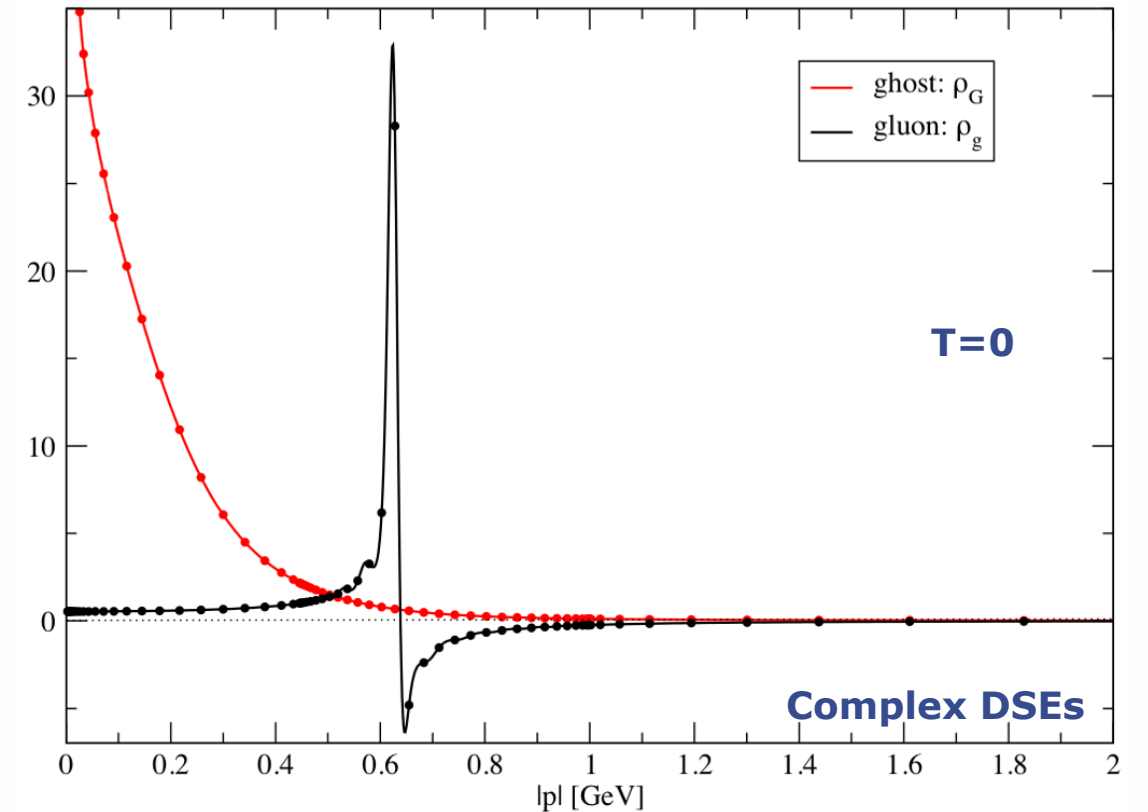


M. Haas, Fister, JMP, arXiv:1308.4960

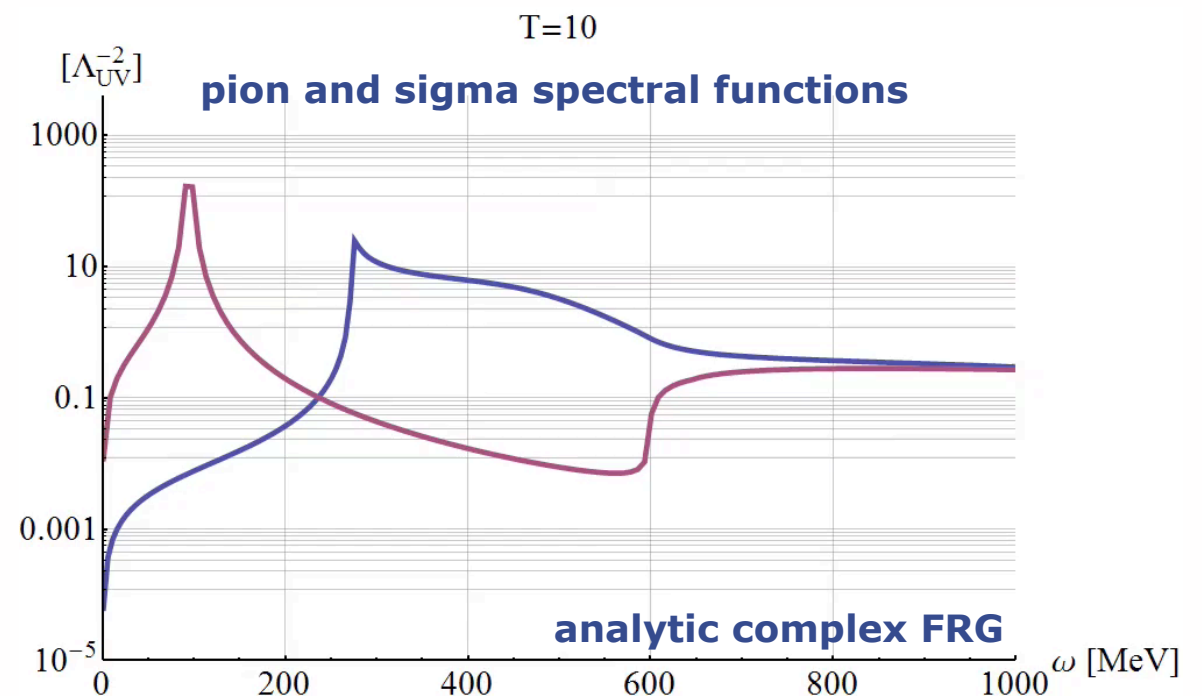
see poster of L. Fister

see talk of A. Tripolt

Tripolt, Strodthoff, von Smekal, Wamach, PRD 89 (2014) 034010
Kamikado, Strodthoff, von Smekal, Wambach, EPJ C74 (2014) 2806



Strauss, Fischer, Kellermann, PRL 109 (2012) 252001



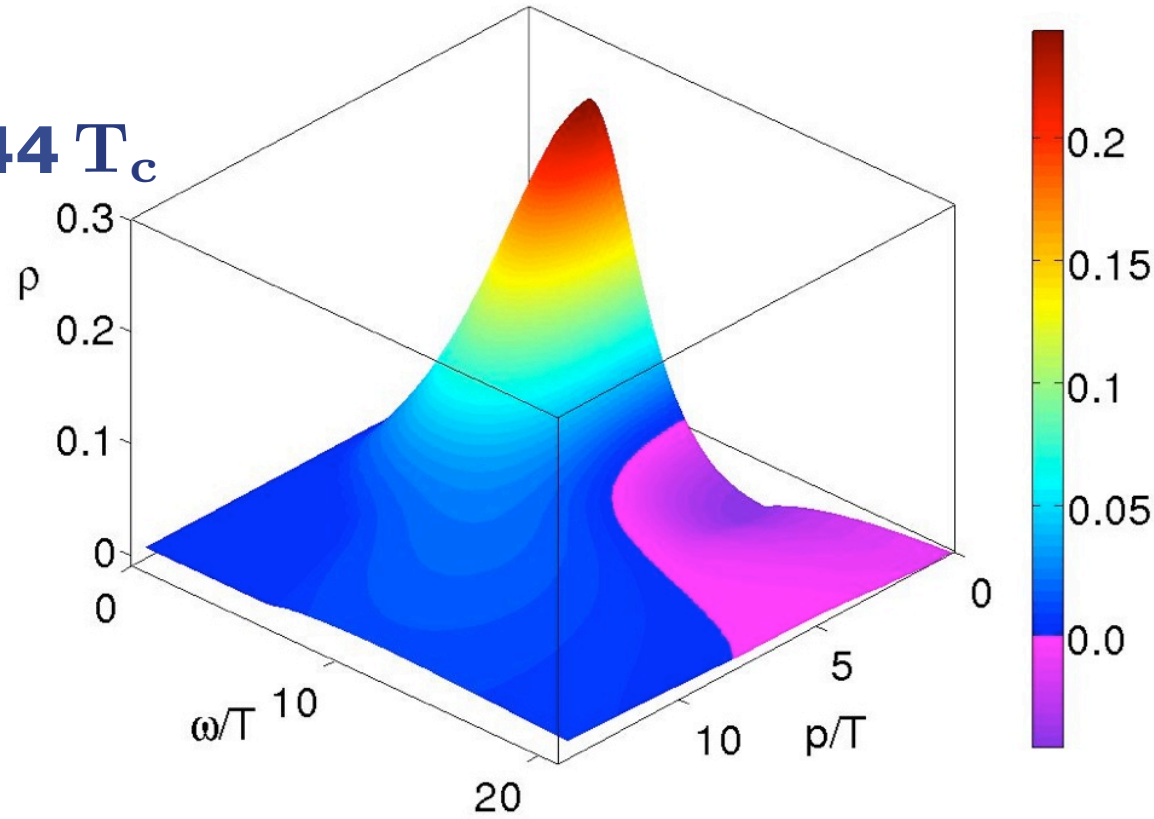
Viscosity in pure glue

spectral functions

M. Haas, Fister, JMP, arXiv:1308.4960

transversal

$T = 1.44 T_c$

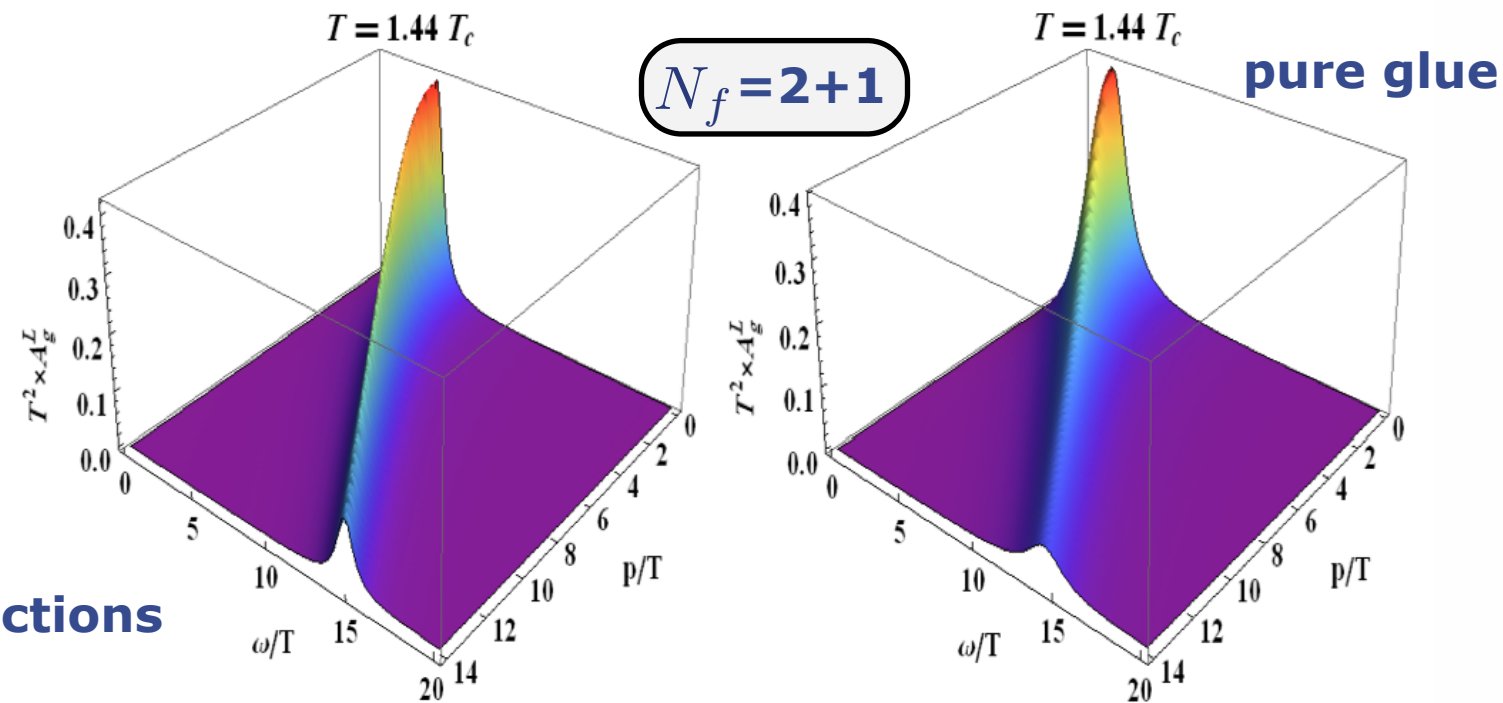


$T = 1.44 T_c$

see poster of L. Fister

see talk of E. Bratkovskaya

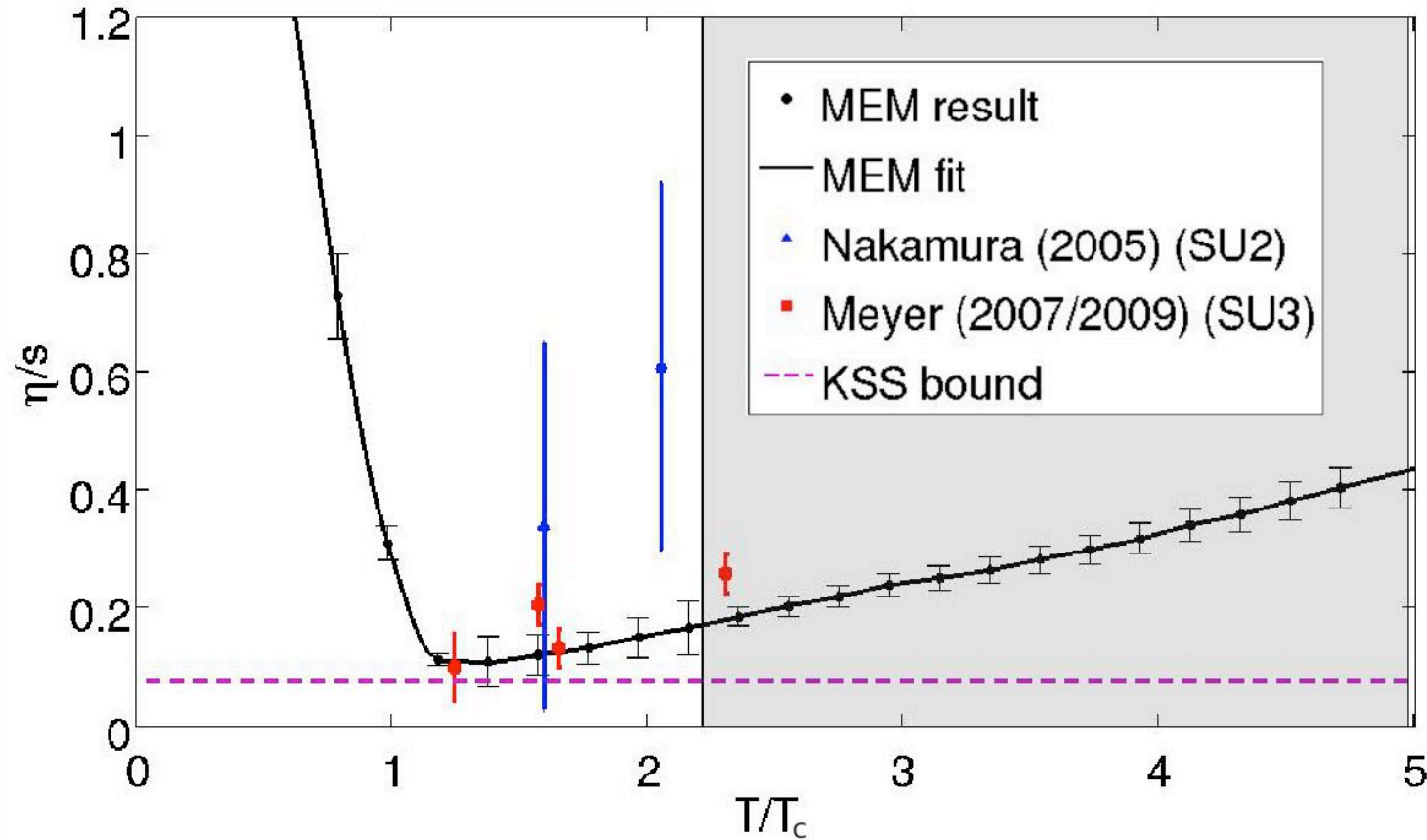
PHSD spectral functions



Viscosity in pure glue

shear viscosity

M. Haas, Fister, JMP, arXiv:1308.4960



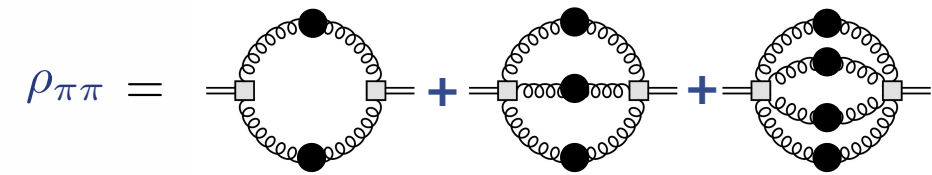
$T \lesssim 2T_c$: MEM+optimised RG-scheme systematic error estimates

Shaded area: MEM error estimates

Kubo relation

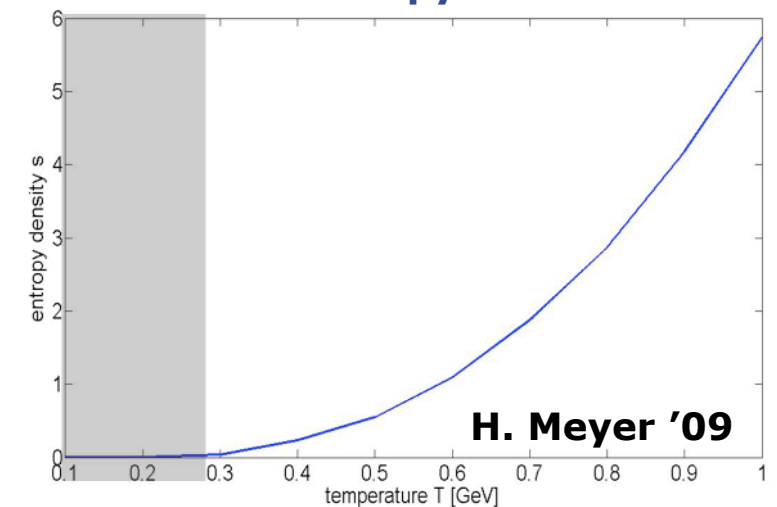
$$\eta = \frac{1}{20} \left. \frac{d}{d\omega} \right|_{\omega=0} \rho_{\pi\pi}(\omega, 0)$$

Diagrammatic representation



+ ... 3-loop closed form

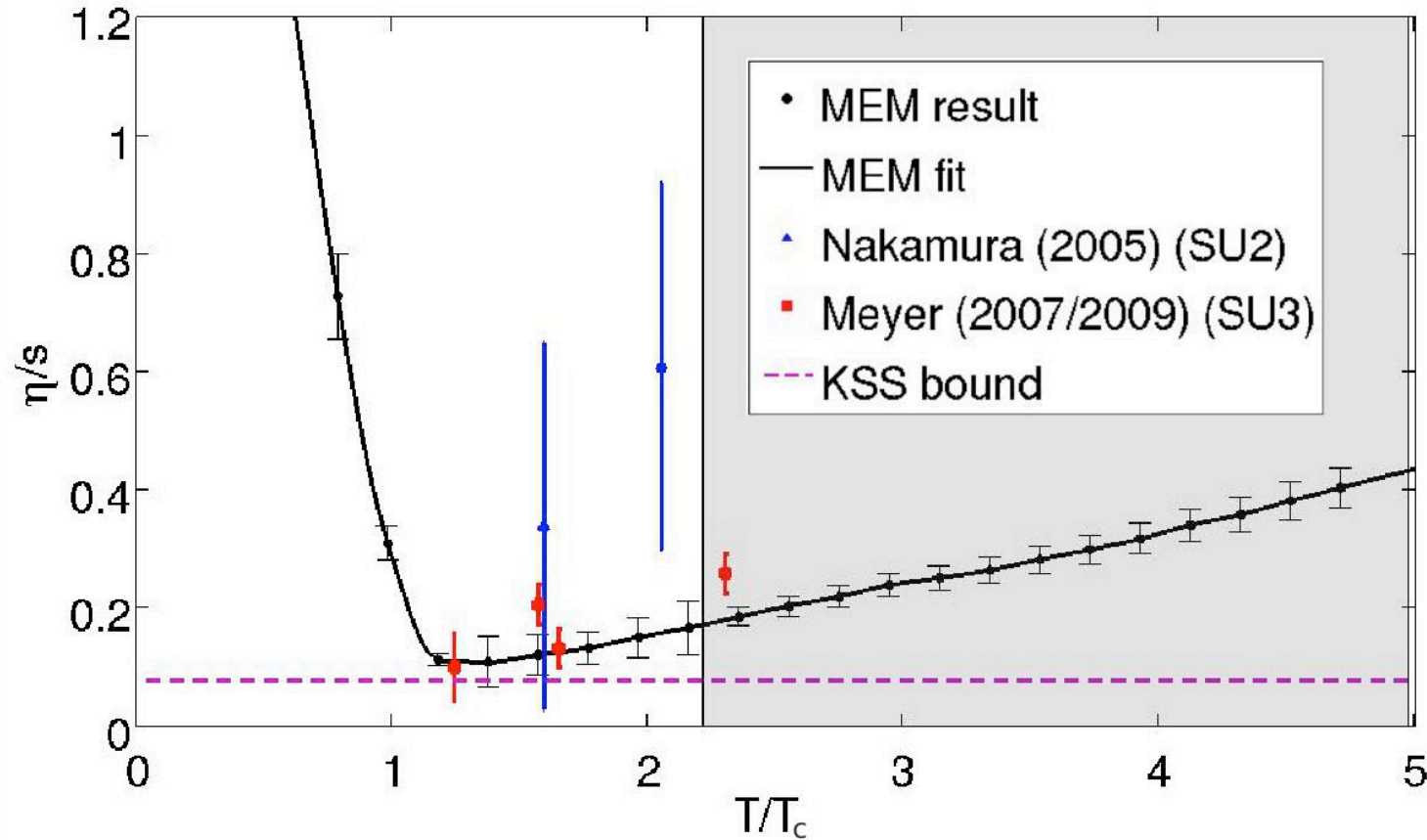
entropy lattice



Viscosity in pure glue

shear viscosity

M. Haas, Fister, JMP, arXiv:1308.4960



$T \lesssim 2T_c$: MEM+optimised RG-scheme systematic error estimates

Shaded area: MEM error estimates

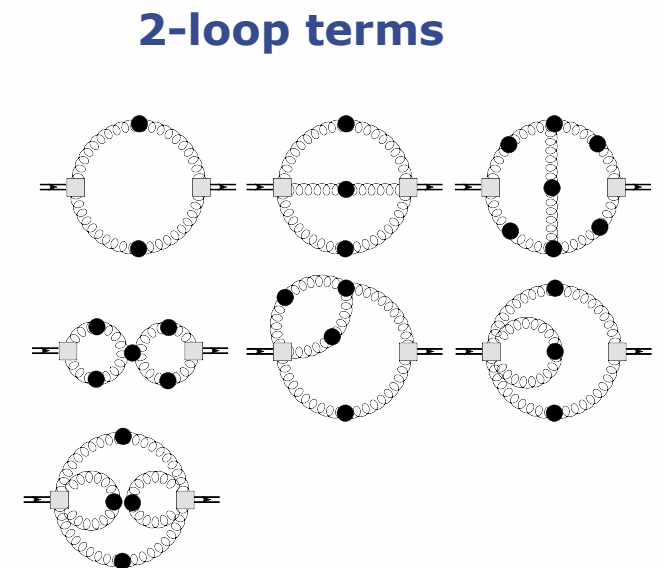
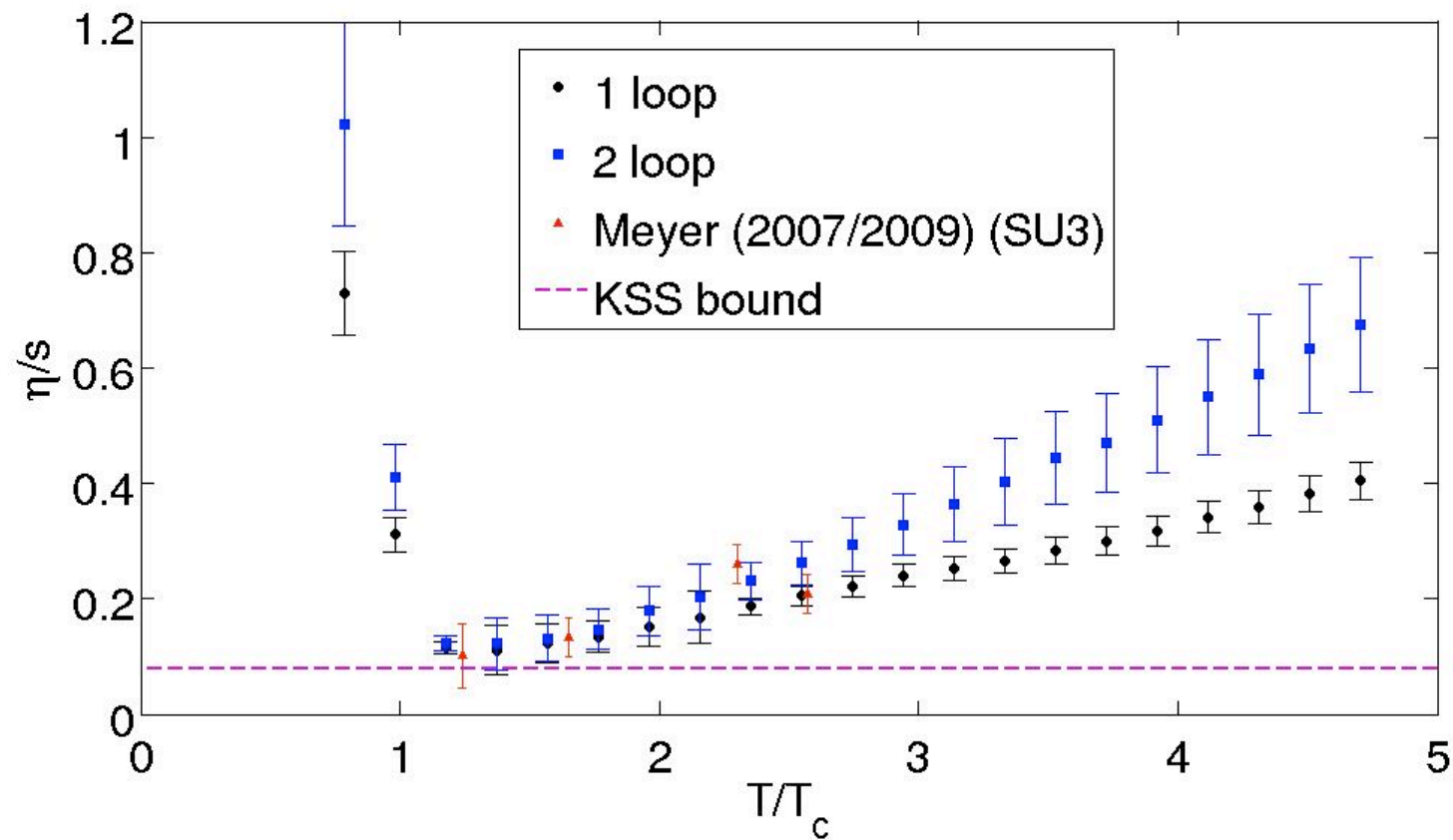
minimum at $T = 1.25T_c$: $\frac{\eta}{s} = 1.45 \frac{1}{4\pi}$

scale matching with QCD: $\frac{\eta}{s} = 2.27 \frac{1}{4\pi}$

Viscosity in pure glue

shear viscosity

Christiansen, M. Haas, JMP, Strodthoff, in prep.



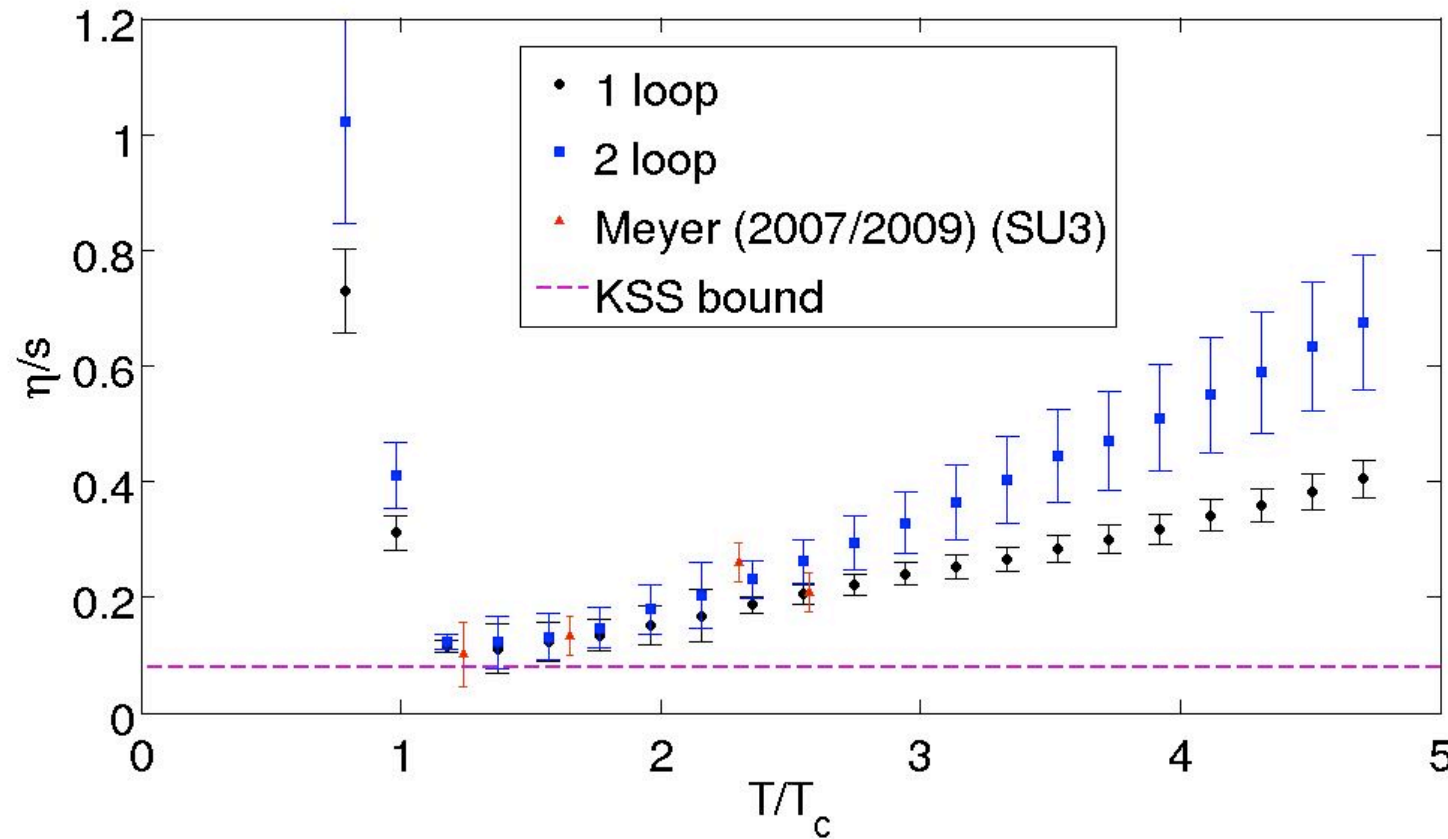
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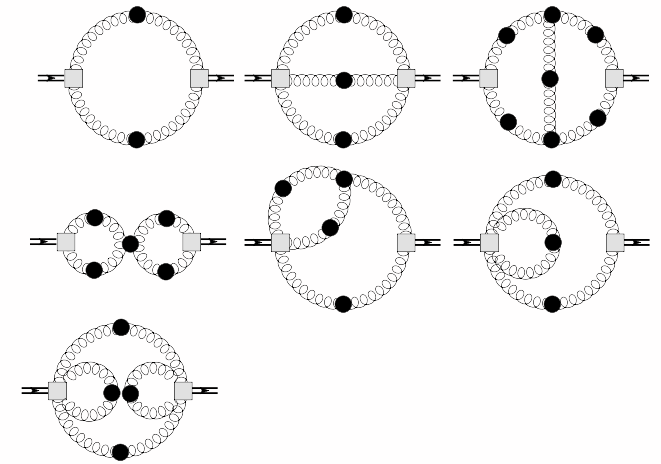
Viscosity in pure glue

shear viscosity

Christiansen, M. Haas, JMP, Strodthoff, in prep.

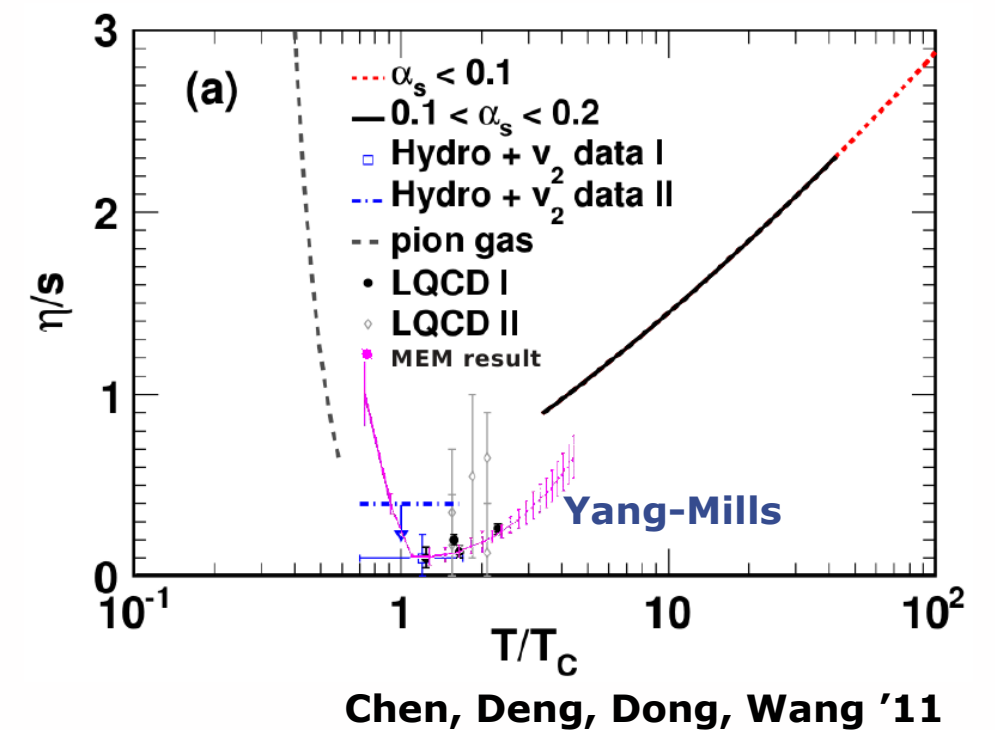


2-loop terms



minimum at $T = 1.25T_c$: $\frac{\eta}{s} = 1.45 \frac{1}{4\pi}$

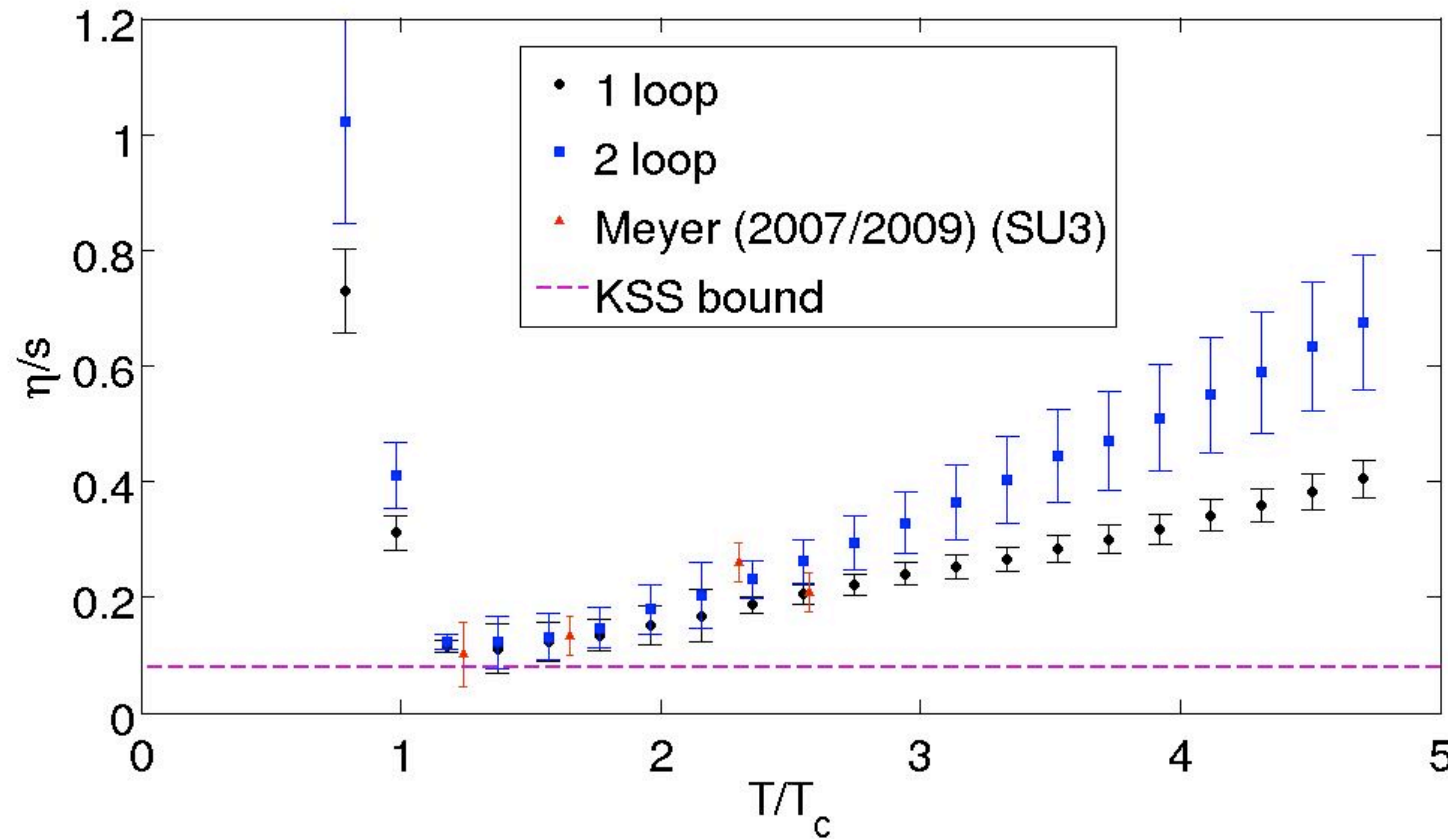
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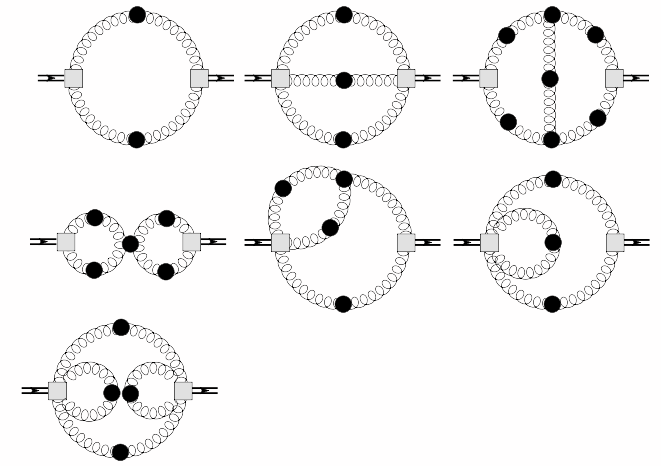
Viscosity in pure glue

shear viscosity

Christiansen, M. Haas, JMP, Strodthoff, in prep.

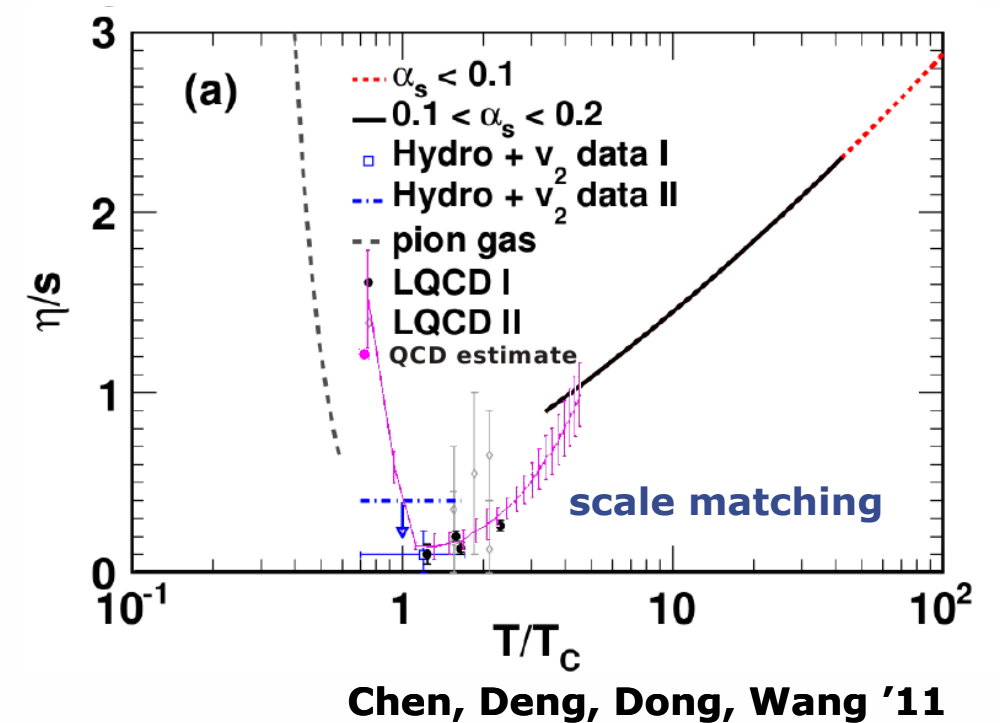


2-loop terms



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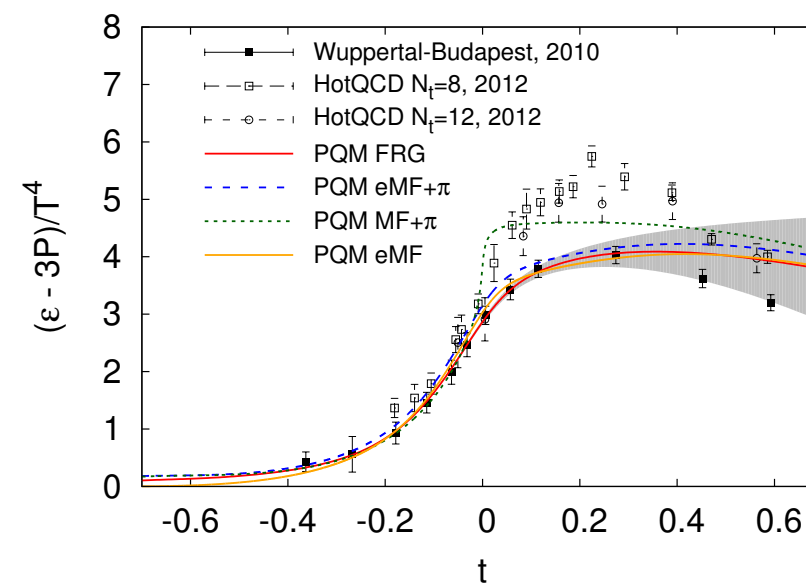
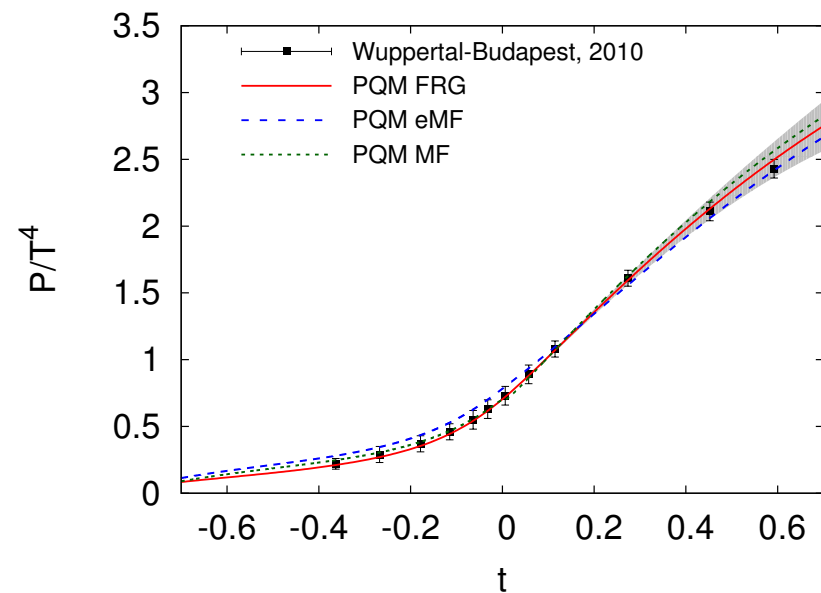
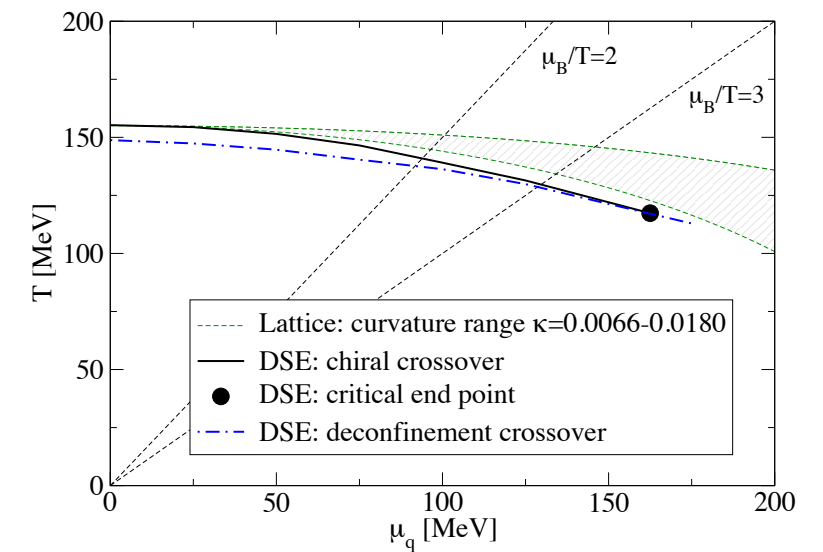
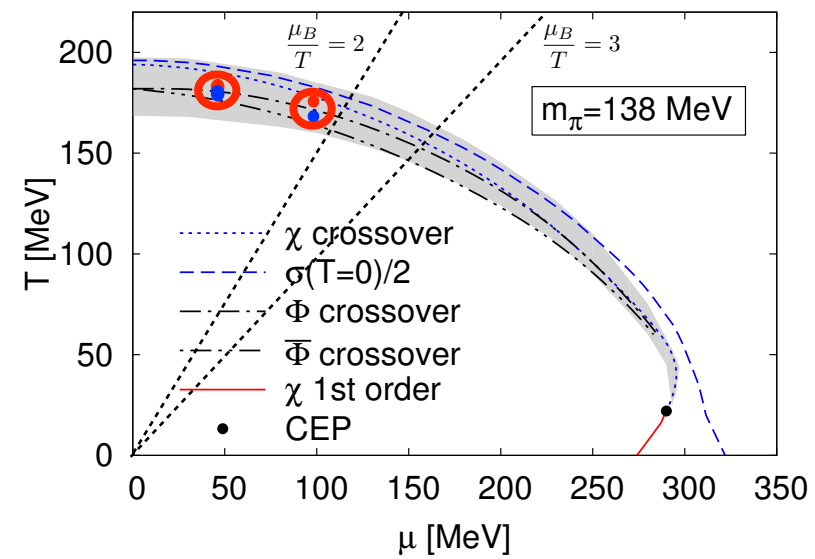
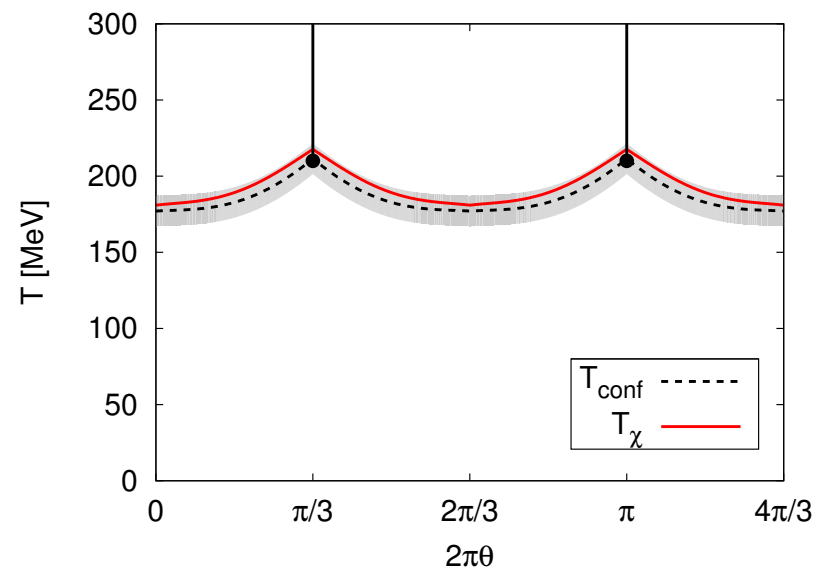
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Summary & Outlook

Summary & outlook

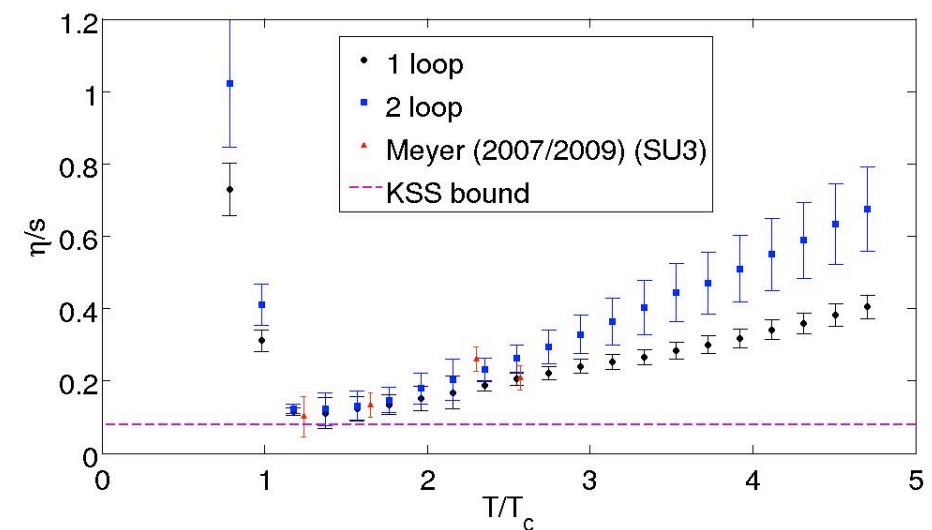
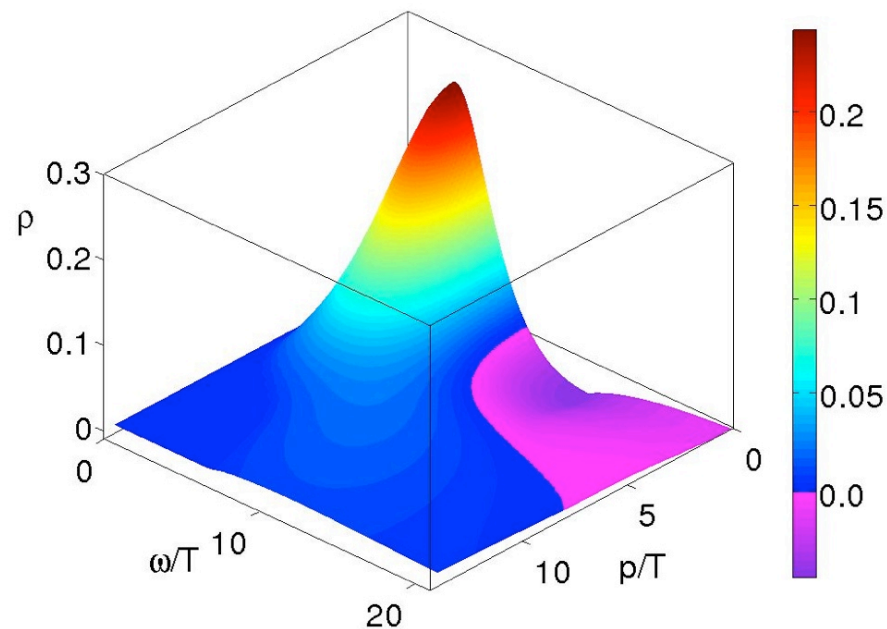
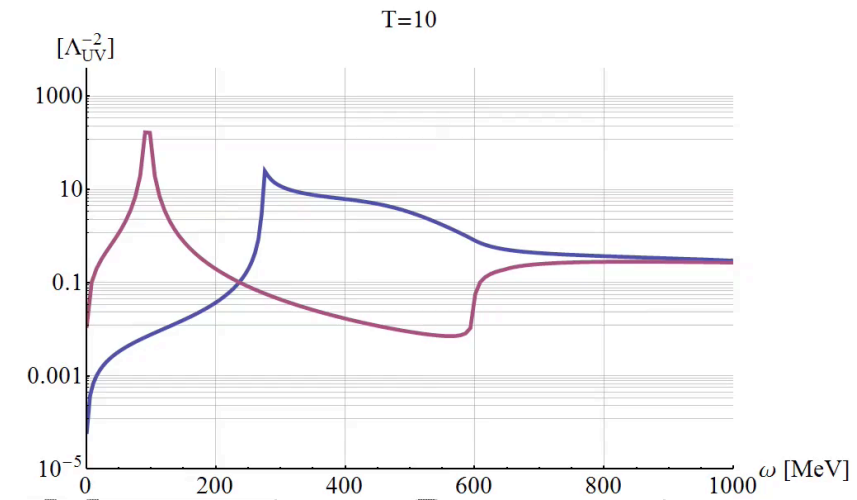
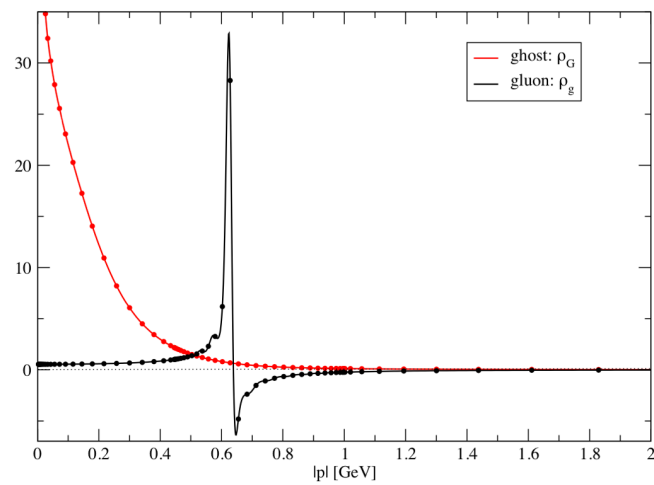
Phase structure and Equation of State



Summary & outlook

Phase structure and Equation of State

Spectral functions and Transport Coefficients



Summary & outlook

- **Phase structure and Equation of State**
- **Spectral functions and Transport Coefficients**
- **Towards quantitative precision**
- **Baryons, high density regime, dynamics**
- **Hadronic properties**
 - **hadron spectrum & in medium modifications**
 - **low energy constants**