RG methods for ultracold gases and non-equilibrium physics

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outline

1. Physics of ultracold atoms
   - BEC-BCS crossover
   - RG-flows for ultracold atoms
2. Results
   - many-body effects
   - phase diagram
3. Non-equilibriums physics
   - non-equilibrium time evolution
   - time evolution from RG-flows

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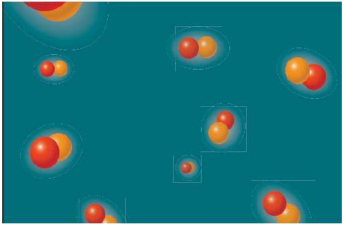
BEC-BCS crossover

(Eagles'69; Leggett'80)

- Bound molecules of two atoms on microscopic scale

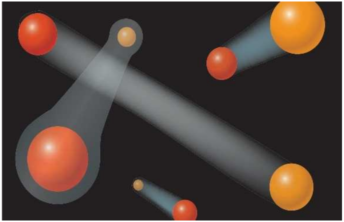
- Fermions with attractive interactions

BEC at low T



(Cho@Science'03)

BCS superfluidity at low T

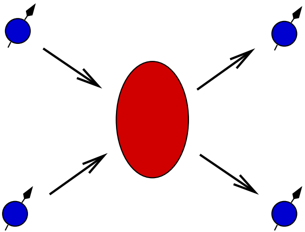


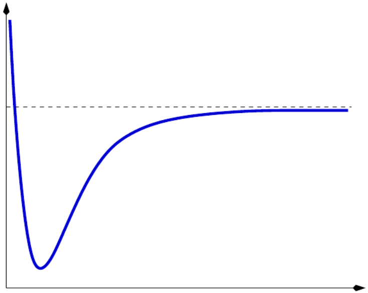
Crossover by means of a Feshbach resonance

(Regal&'04; Bartenstein&'04; Zwierlein&'04; Kinast&'04; Bourdel&'04)

Feshbach resonance

- 2-atom scattering





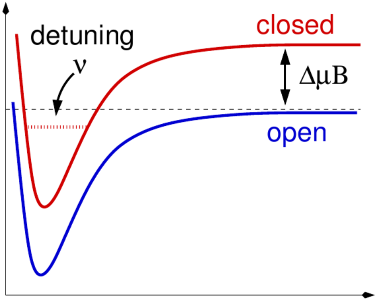
- s-wave scattering length

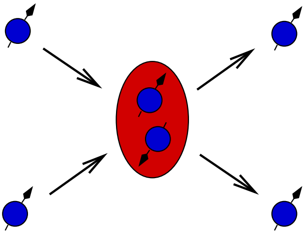
a

Feshbach resonance

- 2-atom scattering

resonant hyperfine interaction
between interaction channels





- s-wave scattering length

$$a(B)$$

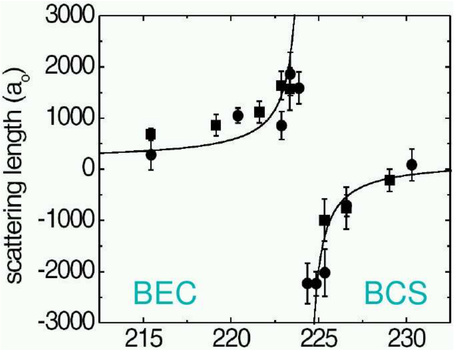
near a Feshbach resonance

microscopic interaction

- tunable interaction strength

$$a(B) = a_{\text{bg}}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

e.g., ^{40}K



(REGAL, JIN'03)

microscopic theory

- microscopic action

$$S_F = \int d\tau \int d^3x \left[\psi^\dagger \left(\partial_\tau - \frac{\nabla^2}{2M}\right)\psi + \frac{1}{2}\lambda_\psi(\psi^\dagger\psi)(\psi^\dagger\psi)\right]$$

- fermionic interaction parameter

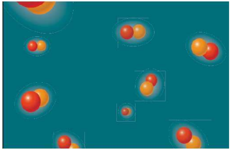
$$\lambda_\psi = \frac{4\pi}{M}\, a(B)$$

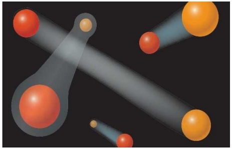
- further parameters of 2-atom physics encoded in momentum-dependence of λ_ψ

relevant degrees of freedom

ψ: stable fermionic atom field

ϕ: bosonic molecule field / Cooper pair





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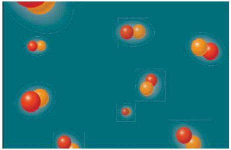
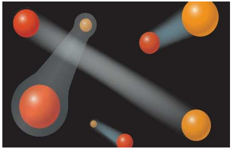




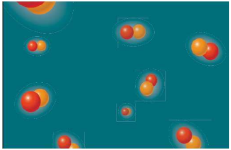
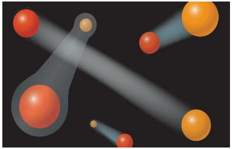
- include all relevant dof as propagating fields

$$\Gamma[\psi] \longrightarrow \Gamma[\psi, \phi]$$

relevant degrees of freedom

ψ: stable fermionic atom field

ϕ: bosonic molecule field / Cooper pair

$$\Gamma[\psi,\phi] = \int d\tau \int d^3x \,\{Z_\psi \psi^\dagger(\partial_\tau - A_\psi \nabla^2 - \sigma)\psi + \lambda_\psi (\psi^\dagger\psi)^2$$

$$+ Z_\phi \phi^*(\partial_\tau - A_\phi \nabla^2)\phi + U(\phi) - \frac{h_\phi}{2}(\phi^* \psi^{\,T} \epsilon \psi - \phi \psi^\dagger \epsilon \phi^*) + \dots \}$$

relevant degrees of freedom

ψ: stable fermionic atom field

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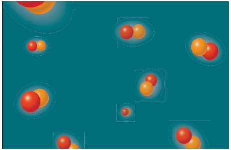

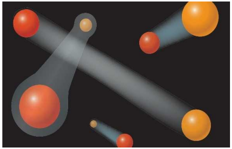

- computation with flow equation

$$k\partial_k \Gamma_k[\psi,\phi] = \frac{1}{2}\,\mathrm{STr}\,\frac{1}{\Gamma_k^{(2)}[\psi,\phi]+R_k}\,k\partial_k R_k$$

a glimpse at the functional RG

Callan-Symanzik equation

$$k\partial_k \Gamma_k[\phi] = \frac{1}{2}\,\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi] + k^2}\,2k^2$$

with

$$\frac{1}{\Gamma_k^{(2)}[\phi] + k^2} = \langle \phi(p)\phi(-p)\rangle$$

a glimpse at the functional RG

Wetterich, Phys. Lett. B **301** (1993) 90

$$k\partial_k \Gamma_k[\phi] = \frac{1}{2}\,\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi] + R_k(p^2)}\,k\partial_k R_k(p^2)$$

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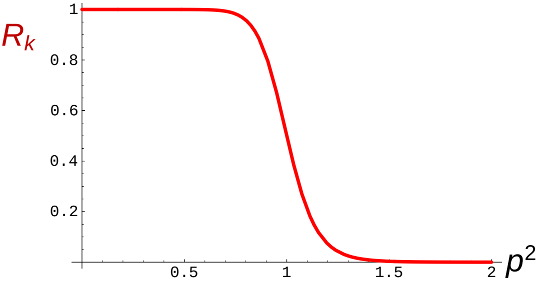
- flow $k\partial_k \Gamma_k[\phi]$ is infrared and ultraviolet finite

IR-structure



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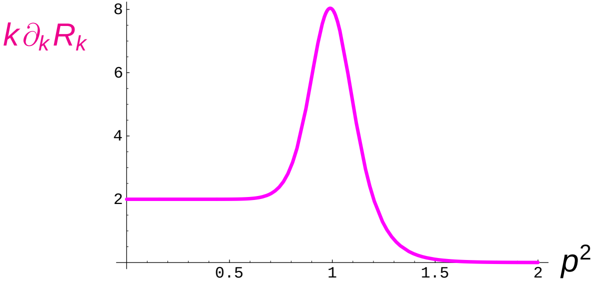
- flow $k\partial_k\Gamma_k[\phi]$ is infrared and ultraviolet finite

UV-structure



a glimpse at the functional RG

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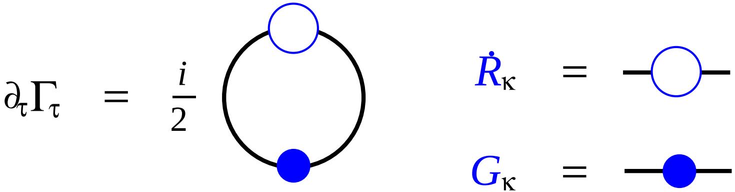
$$k\partial_k \Gamma_k[\phi] = \frac{1}{2}\,\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi] + R_k(p^2)}\,k\partial_k R_k(p^2)$$

- diagrammatic representation with $\tau = \ln k$



$$G_k = \frac{1}{\Gamma_k^{(2)}[\phi] + R_k(p^2)}$$

a glimpse at the functional RG

Wetterich, Phys. Lett. B **301** (1993) 90

$$k\partial_k \Gamma_k[\phi] = \frac{1}{2}\,\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi] + R_k(p^2)}\,k\partial_k R_k(p^2)$$

- self-similarity, reparameterisation & projections
- fermions straightforward though 'physically' complicated
  - no sign problem numerics as in scalar theories!
  - chiral fermions reminder: Ginsparg-Wilson fermions from RG argument!
  - bound states via (re-)bosonisation effective field theory techniques applicable!

effective action

- interaction terms

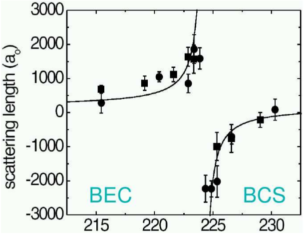
$$\int d\tau\, d^3x \left(m_\phi^2 \phi^* \phi + \frac{\lambda_\phi}{2}(\phi^*\phi)^2 - \frac{h_\phi}{2}(\phi^* \psi^{\mathsf{T}} \epsilon \psi - \phi \psi^\dagger \epsilon \phi^*) + \dots \right)$$

- relation to microphysics (via Hubbard-Stratonovich)

$$\lambda_\psi = \frac{4\pi a_{bg}}{M}$$

$$m_\phi^2 = \bar{\mu}(B - B_0) - 2\sigma$$

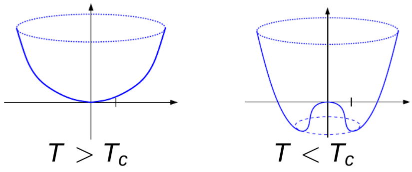
$$h_\phi^2 \sim \Delta B$$



IR physics

Effective potential $U(\phi)$

- determines symmetry status



- order parameter $\rho_0 = \phi^*_{\text{min}} \phi_{\text{min}}$

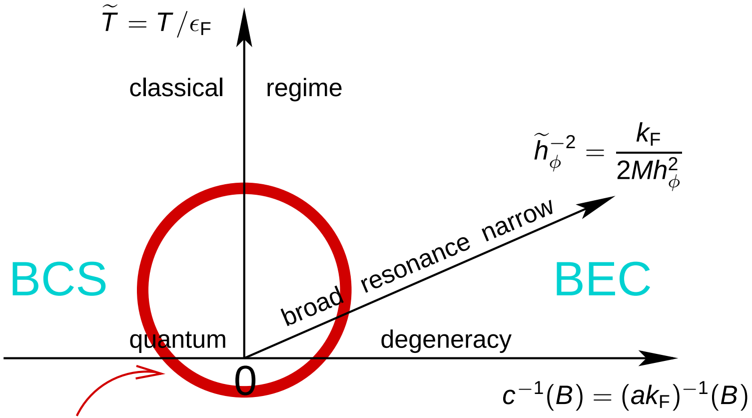
condensate fraction Ω_c, fermionic gap Δ

- U'' determines correlation length ξ

- $U(\phi, \sigma)$ determines (flow of) density $\partial_k n = -\dfrac{\partial\, \partial_k U(\phi_{\text{min}}, \sigma)}{\partial \sigma}$

crossover diagram

in units of Fermi momentum $k_\mathrm{F} = (3\pi^2 n)^{1/3}$ and energy $\epsilon_\mathrm{F} = k_\mathrm{F}^2/(2M)$



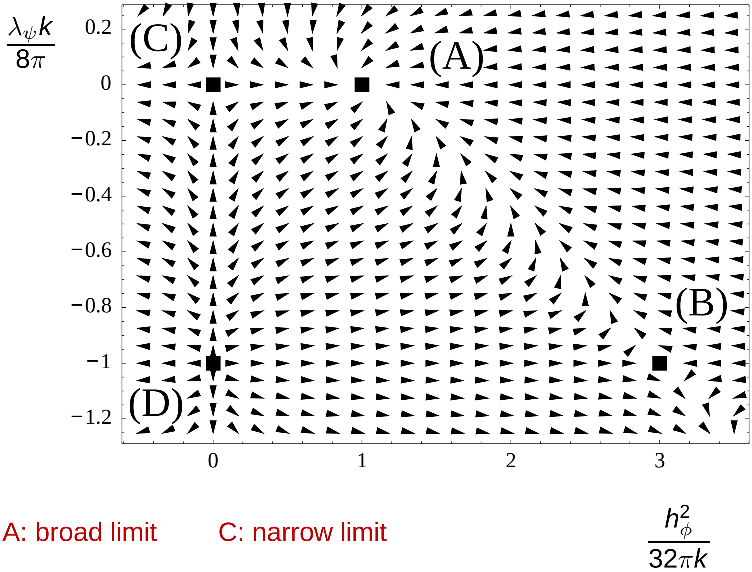
"concentration" parameter strong coupling, "broad-resonance universality" $h_\phi \sim \Delta B \to \infty$

Universal long-distance physics for ^{6}Li and ^{40}K ?

(Diehl,Wetterich'05; Nicolić,Sachdev'06)

universality

Diehl,Gies,Pawlowski,Wetterich'07



A: broad limit   C: narrow limit

re-bosonisation

- derivative expansion for Γ_k

$$\Gamma_k[\psi,\phi] = \int d\tau \int d^3x Z_\psi \left\{ \psi^\dagger(\partial_\tau - A_\psi \nabla^2 - \sigma)\psi + \lambda_\psi (\psi^\dagger \psi)^2 \right.$$

$$\left. + Z_\phi \phi^* (\partial_\tau - A_\phi \nabla^2)\phi + U(\phi) - \frac{h_\phi}{2}(\phi^* \psi^T \epsilon \psi - \phi \psi^\dagger \epsilon \phi^*) + \dots \right\}$$

re-bosonisation

- optimised re-bosonised derivative expansion for Γ_k

$$\Gamma_k[\psi,\phi] = \int d\tau \int d^3x \, \{\psi^\dagger(\partial_\tau - A_\psi \nabla^2 - \sigma)\psi + \lambda_\psi(\psi^\dagger\psi)^2$$

$$+\phi^*(\partial_\tau - A_\phi \nabla^2)\phi + U(\phi) - \frac{h_\phi}{2}(\phi^*\psi^T\epsilon\psi - \phi\psi^\dagger\epsilon\phi^*) + \ldots \}$$

- ψ, ϕ normalised fields via k-dependent field transformation

re-bosonisation

- optimised re-bosonised derivative expansion for Γ_k

$$\Gamma_k[\psi,\phi] = \int d\tau \int d^3x \, \{\psi^\dagger(\partial_\tau - A_\psi \nabla^2 - \sigma)\psi + \lambda_\psi(\psi^\dagger\psi)^2$$

$$+\phi^*(\partial_\tau - A_\phi\nabla^2)\phi + U(\phi) - \frac{h_\phi}{2}(\phi^*\psi^{\,T}\epsilon\psi - \phi\psi^\dagger\epsilon\phi^*) + \ldots\Big\}$$

- ψ, ϕ normalised fields

- 'ultra'-locality: implicit momentum dependence

re-bosonisation

- optimised re-bosonised derivative expansion for Γ_k

$$\Gamma_k[\psi,\phi] = \int d\tau \int d^3x \, \Big\{ \psi^\dagger(\partial_\tau - A_\psi \nabla^2 - \sigma)\psi + \lambda_\psi (\psi^\dagger\psi)^2$$

$$+\phi^*(\partial_\tau - A_\phi \nabla^2)\phi + U(\phi) - \frac{h_\phi}{2}(\phi^* \psi^{\,T} \epsilon \psi - \phi \psi^\dagger \epsilon \phi^*) + \ldots \Big\}$$

- ψ, ϕ normalised fields

- 'ultra'-locality: implicit momentum dependence

- $\rightarrow$ optimisation

  - physical cut-off scales $k_\psi \simeq k_\phi$
  - optimal cut-off: $k_\psi = k_\phi$ $+$ minimal flow

Litim '00, Pawlowski '05

re-bosonisation

- optimised re-bosonised derivative expansion for Γ_k

$$\Gamma_k[\psi,\phi] = \int d\tau \int d^3x \, \{\psi^\dagger(\partial_\tau - A_\psi \nabla^2 - \sigma)\psi + \lambda_\psi(\psi^\dagger\psi)^2$$

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- ψ, ϕ normalised fields

- 'ultra'-locality: implicit momentum dependence

- optimisation

- full re-bosonisation

re-bosonisation

- optimised re-bosonised derivative expansion for Γ_k

$$\Gamma_k[\psi,\phi] = \int d\tau \int d^3x \, \{ \psi^\dagger(\partial_\tau - A_\psi \nabla^2 - \sigma)\psi + \lambda_\psi (\psi^\dagger\psi)^2$$

$$+\phi^*(\partial_\tau - A_\phi \nabla^2)\phi + U(\phi) - \frac{h_\phi}{2}(\phi^*\psi^{\,T}\epsilon\psi - \phi\psi^\dagger\epsilon\phi^*) + \dots \}$$

- ψ, ϕ normalised fields

- 'ultra'-locality: implicit momentum dependence

- optimisation

- full re-bosonisation, passive:

H. Gies, C. Wetterich '01

$$\Gamma_k^{\rm pass}[\psi_k,\phi_k] = \Gamma_k[\psi(\phi_k,\psi_k),\phi(\phi_k,\psi_k)]$$

re-bosonisation

- optimised re-bosonised derivative expansion for Γ_k

$$\Gamma_k[\psi,\phi] \;=\; \int d\tau \int d^3x \,\{\psi^\dagger(\partial_\tau - A_\psi \nabla^2 - \sigma)\psi + \lambda_\psi(\psi^\dagger\psi)^2$$

$$+\phi^*(\partial_\tau - A_\phi \nabla^2)\phi + U(\phi) - \frac{h_\phi}{2}(\phi^*\psi^{\,T}\epsilon\psi - \phi\psi^\dagger\epsilon\phi^*) + \ldots\Big\}$$

- ψ, ϕ normalised fields

- 'ultra'-locality: implicit momentum dependence

- optimisation

- full re-bosonisation, active: only ϕ_k

Pawlowski '05

$$-\int \phi^* R_\phi \phi + \int J_\phi \phi + J_\psi \psi \to -\int \phi_k^* R_\phi \phi_k + \int J_\phi \phi_k + \int J_\psi \psi$$

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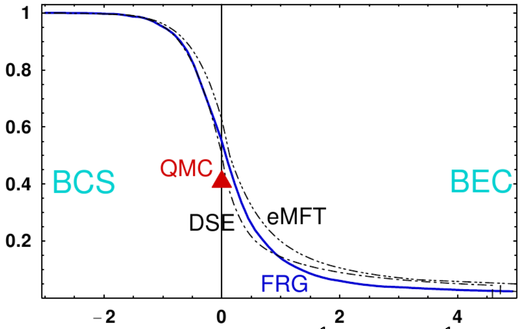
many-body effects

Diehl,Gies,Pawlowski,Wetterich'07

chemical potential $\tilde{\sigma}$ minus molecular binding energy $\tilde{\epsilon}_{\mathrm{M}}$ at $T = 0$

$\tilde{\sigma} - \tilde{\epsilon}_{\mathrm{M}}$



$c^{-1} = (ak_{\mathrm{F}})^{-1}$

many-body effects

Diehl,Gies,Pawlowski,Wetterich'07

chemical potential $\widetilde{\sigma}$ minus molecular binding energy $\widetilde{\epsilon}_{\mathrm{M}}$ at $T = 0$

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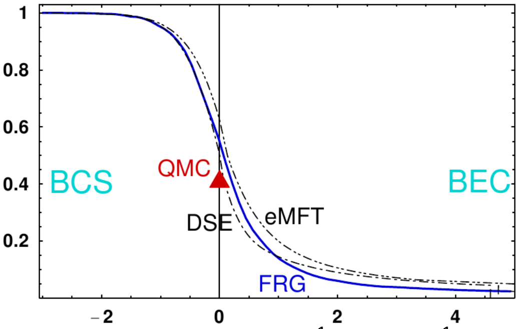


$c^{-1} = (ak_{\mathrm{F}})^{-1}$

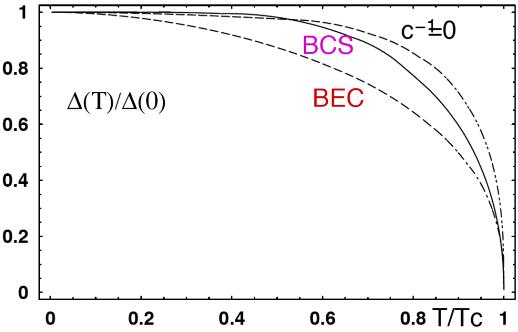
QMC (Giorgini&'04)	FRG (Diehl et al)	ϵNLO (Son&06)	DSE (Diehl&'05)	MFT	2PI (Zwerger&'06)	Padé (Baker'99)	NSR (Hu&'06)	Exp.
0.42(2)	0.55	0.475	0.50	0.63	0.36	0.33	0.40	0.32 -0.51

second-order phase transition

Diehl,Gies,Pawlowski,Wetterich'07

Fermionic gap

$\Delta = h_\phi \sqrt{\rho_{\rm m}}$



$T\,/\,T_{\rm c}$

BEC
$c^{-1} = 4$

BCS
$c^{-1} = -2$

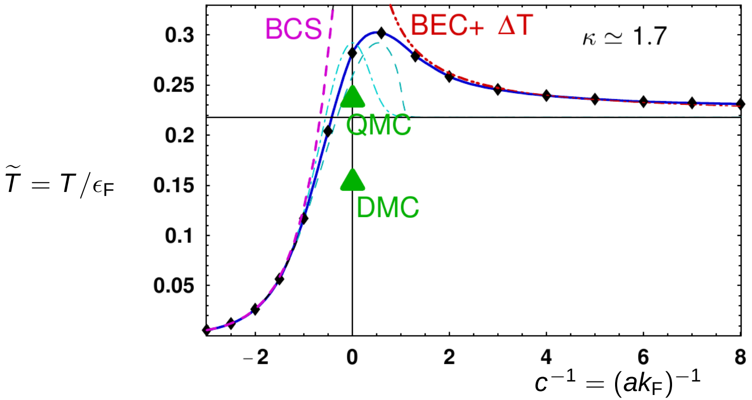
resonance
$c^{-1} = 0$

2nd order phase transition

phase diagram

Diehl,Gies,Pawlowski,Wetterich'07

broad resonance limit



shift ΔT of T_c in BEC regime:

(Baym,Blaizot,Holzmann,Laloë,Vautherin'99)
(Baym,Blaizot,Zinn-Justin'00)
(Blaizot,Mendez-Galain,Wschebor'05,06)

$$\frac{T_c - T_c^{BEC}}{T_c^{BEC}} = \kappa\, a_B n^{1/3}, \quad \kappa \simeq 1.3$$

summary and outlook

- phase diagram of ultracold fermionic gases
- fermion-dimer scattering: include momentum-dep. $\psi^\dagger\psi\phi^*\phi$
  Diehl, Krahl, Scherer '07
- particle-hole fluctuations (T_c): a lesson in re-bosonisation
- re-bosonisation at low and high scales
- quantum phase transition
- non-equilibrium time evolution

Gasenzer, Pawlowski '07

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motivation

Non-perturbative effects in non-equilibrium systems

- strong coupling g

- late time behaviour: effective coupling $\Delta t\, g$

- far from equilibrium: all correlation functions important

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Non-perturbative effects in non-equilibrium systems

- strong coupling g
- late time behaviour: effective coupling $\Delta t\, g$
- far from equilibrium: all correlation functions important

Non-equilibrium shopping list

- method with built-in energy conservation: $\partial_t \int_{\vec{x}} \langle T^{00} \rangle = 0$
- large coupling & far from equilibrium accessible
- control

Time evolution of non-equilibrium systems

- Equation of motion approach BBGKY

$$i\partial_t \langle \mathcal{O} \rangle_t = \langle [\mathcal{O} \, , H] \rangle_t$$

  - Gross-Pitaevskii equation Mean field approximation to $\partial_t \langle \phi \rangle_t$

$$\partial_t \phi_x = \left[-\frac{\Delta}{2m} + V(x) + g|\phi_x|^2 \right] \phi_x$$

  - Green functions $\mathcal{O} = W^{(n)} = \prod_{i=1}^{n} \phi_{x_i}$

$$\partial_t W^{(n)} = \sum c_i(H) \prod_{j_i} W^{(j_i)}$$

    - no memory kernels
    - 'polynomial' structure
    - potential numerical instability

Time evolution of non-equilibrium systems

- Equation of motion approach BBGKY

$$i\partial_t \langle \mathcal{O} \rangle_t = \langle [\mathcal{O}\,, H] \rangle_t$$

- *n*PI evolutions
  - gap equation dynamic equation for propagator $G = \langle \phi\phi \rangle_{\mathrm{connected}}$

$$G_0^{-1} G = \mathbb{1} - \frac{i}{2} (S^{(4)} G) G + \Delta G_{n\mathrm{PI}}$$

  - higher correlation functions $\Gamma^{(n)} = \langle \phi \cdots \phi \rangle_{\mathrm{amputated}}$

$$\Gamma^{(n)} = \sum c_i(S_{\mathrm{cl}}) \prod_{j_i} \Gamma^{(j_i)}_{n\mathrm{PI}} G^{g_i}$$

  - transport equations derivative expansion

Time evolution of non-equilibrium systems

- Equation of motion approach BBGKY

$$i\partial_t \langle \mathcal{O} \rangle_t = \langle [\mathcal{O}\,,\,H] \rangle_t$$

- nPI evolutions
  - gap equation

$$G_0^{-1} G = \mathbb{1} - \frac{i}{2} (S^{(4)} G) G + \Delta G_{n\mathrm{PI}}$$

  - higher correlation functions

$$\Gamma^{(n)} = \sum c_i (S_{\mathrm{cl}}) \prod_{j_i} \Gamma^{(j_i)}_{n\mathrm{PI}} G^{g_i}$$

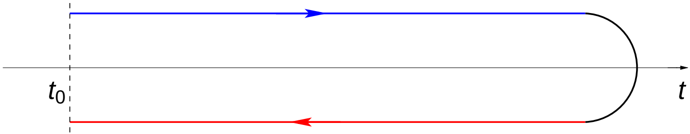
    - memory kernels
    - 'non-algebraic' structure
    - potential numerical (in-)stability? functional form stable?

closed time path

Generating functional for real time correlation functions

$$Z[J] \qquad \text{with} \qquad \frac{\delta^n Z[J]}{\delta J(t_1)\cdots\delta J(t_n)} = \langle \phi(t_1)\cdots\phi(t_n)\rangle$$

- the fields ϕ live on the closed time path Schwinger-Keldysch



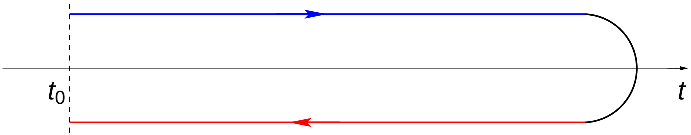
from $\langle t|\mathcal{O}|t\rangle = \langle t_0|U^\dagger(t)\,\mathcal{O}\,U(t)|t_0\rangle$

closed time path

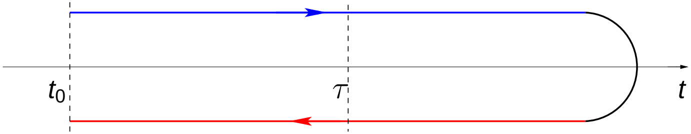
Generating functional for real time correlation functions

$$Z[J] \qquad \text{with} \qquad \frac{\delta^n Z[J]}{\delta J(t_1)\cdots\delta J(t_n)} = \langle \phi(t_1)\cdots\phi(t_n)\rangle$$

- the fields ϕ live on the closed time path



- causality: $\langle \phi(t_1)\cdots\phi(t_n)\rangle$ with $t < \tau$



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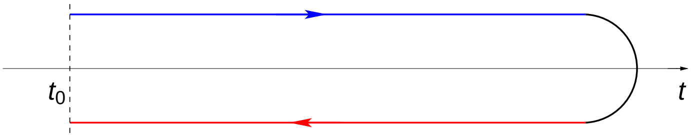

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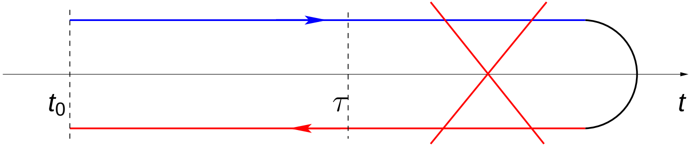

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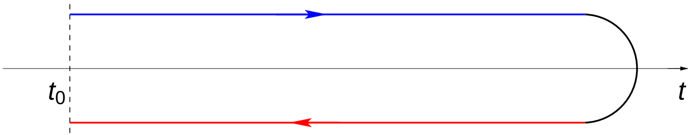

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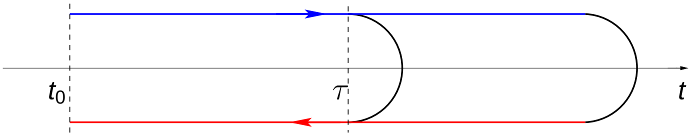

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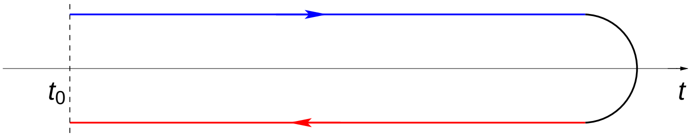

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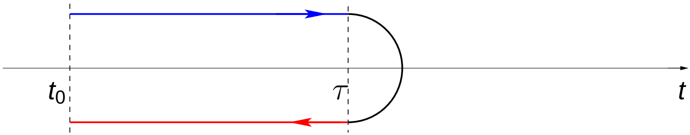

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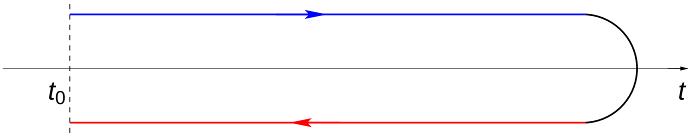

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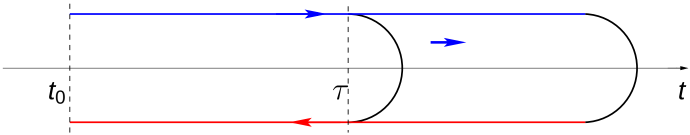

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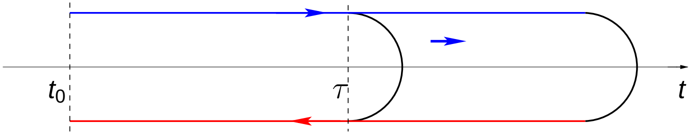
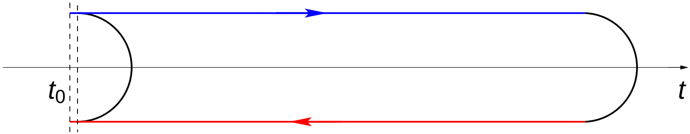


- initial conditions at t_0



temporal flows

Generating functional for real time correlation functions with $t_i < \tau$

$$Z_{\tau}[J]$$

temporal flows

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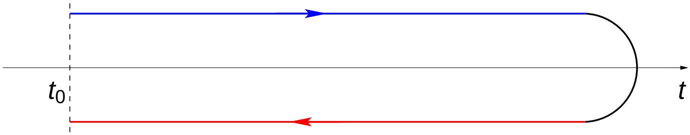

but vanish for $t > \tau$: $\phi(t > \tau) = 0$

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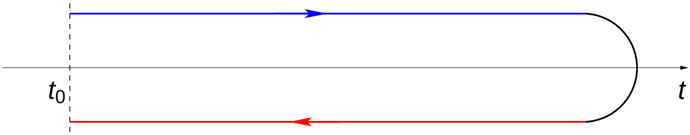

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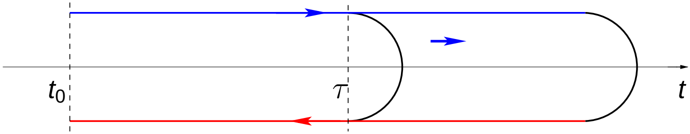


temporal flows

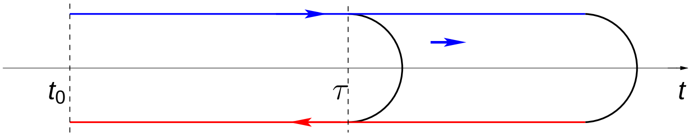
Generating functional for real time correlation functions with $t_i < \tau$

$$Z_\tau[J] = \exp\left\{-\frac{i}{2}\int_{t_a t_b,\mathcal{C}} \frac{\delta}{\delta J(t_a)} R_\tau(t_a,t_b) \frac{\delta}{\delta J(t_b)}\right\} \; Z[J]$$

- regulator function R_τ

$$-i\,R_\tau(t_a,t_b) = \begin{cases} 0 & t_a \text{ and } t_b < \tau \\ \infty & \text{else} \end{cases}$$

- The fields ϕ effectively live on the closed time path



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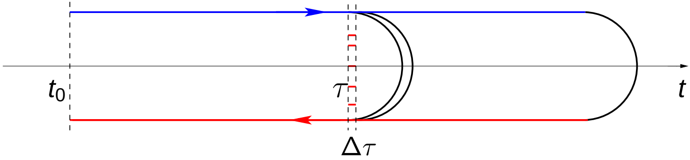
- infinitesimal change of τ



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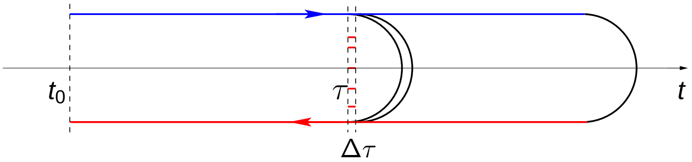

- τ-evolution of effective action $\Gamma_\tau[\phi]$

$\Gamma_\tau[\phi] \simeq \log Z_\tau - \int J\phi$

$$\partial_\tau \Gamma_\tau[\phi] = \frac{i}{2}\int_{\mathcal{C}} \left[\frac{1}{\Gamma_\tau^{(2)}[\phi] + R_\tau}\right]_{ab} \partial_\tau R_{\tau,ab}$$

temporal flows

Generating functional for real time correlation functions with $t_i < \tau$

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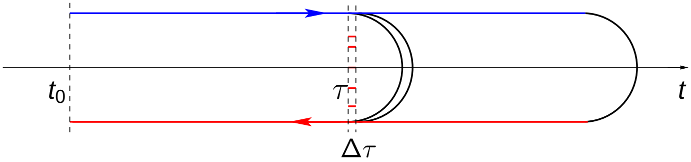
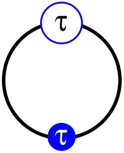
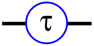
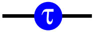


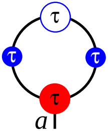
- τ-evolution of effective action $\Gamma_\tau[\phi]$

$\partial_\tau \Gamma_\tau \;=\; \frac{i}{2}$ 

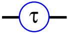
$\dot{R}_{\tau,ab} \;=$ 

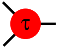
$G_{\tau,ab} \;=$ 

τ-evolution of correlation functions

$\partial_\tau \Gamma^{(1)}_{\tau,a} \;=\; \frac{i}{2}$ 

,

$\dot{R}_{\tau,ab} =$ 

$\Gamma^{(3)}_{\tau,abc} =$ 

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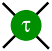
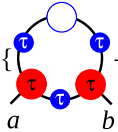
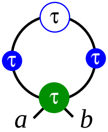
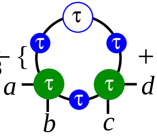
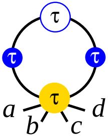
$\Gamma^{(4)}_{\tau,abc} =$ 

$\partial_\tau \Gamma^{(2)}_{\tau,ab} = -\frac{1}{2}\,\{$ 

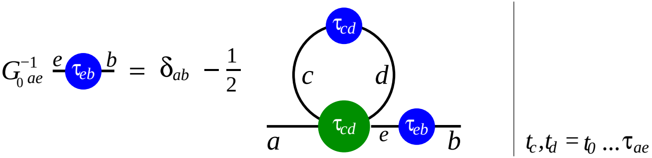
$+\, P\,(a,b\,)\ \} + \frac{i}{2}$ 

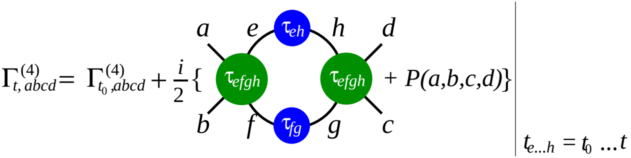
,

$\partial_\tau \Gamma^{(4)}_{\tau,abcd}\,[\phi = 0] = -\frac{1}{8}\,\{$ 

$+\, P\,(a,b,c,d\,)\ \} + \frac{i}{2}$ 

integrated τ-evolution of correlation functions





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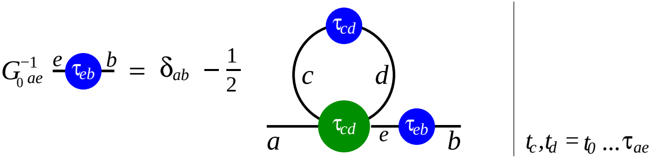
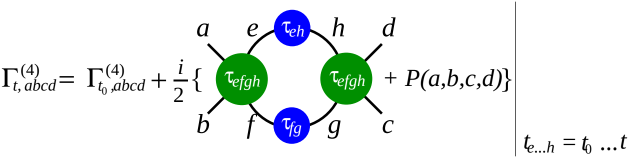
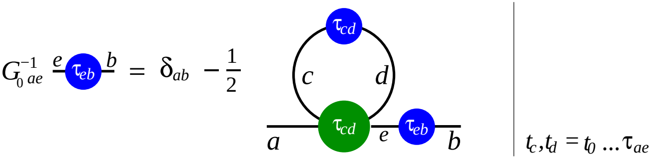
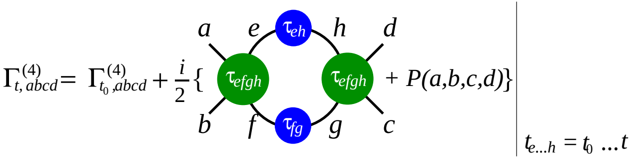




- *s*-channel approximation with energy conservation

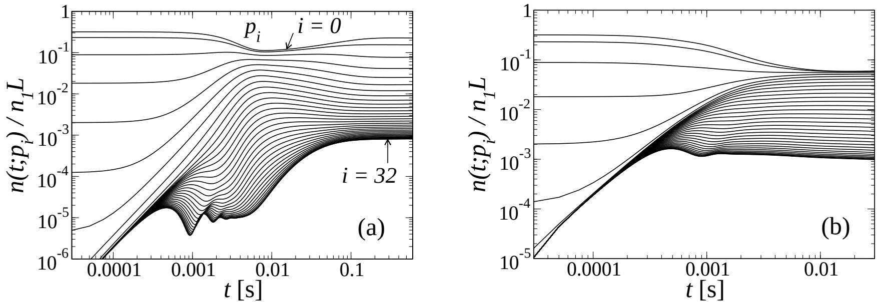
integrated τ-evolution of correlation functions





- *s*-channel approximation with energy conservation
- full self-consistency check possible

numerical results



$\gamma = 1.5 \times 10^{-3}$ $\gamma = 15$

$g = \hbar^2 \gamma n_1 / m$ $n_1 = 10^7 \mathrm{atoms/m}$

Conclusion

- time evolution equation for non-equilibrium effective action
- applicable for strong couplings & far from equilibrium
- energy conservation
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Shopping list

- renormalisation in higher dimensions
- gauge theories
- memory kernels