

# The FRG approach to gauge theories & applications to QCD

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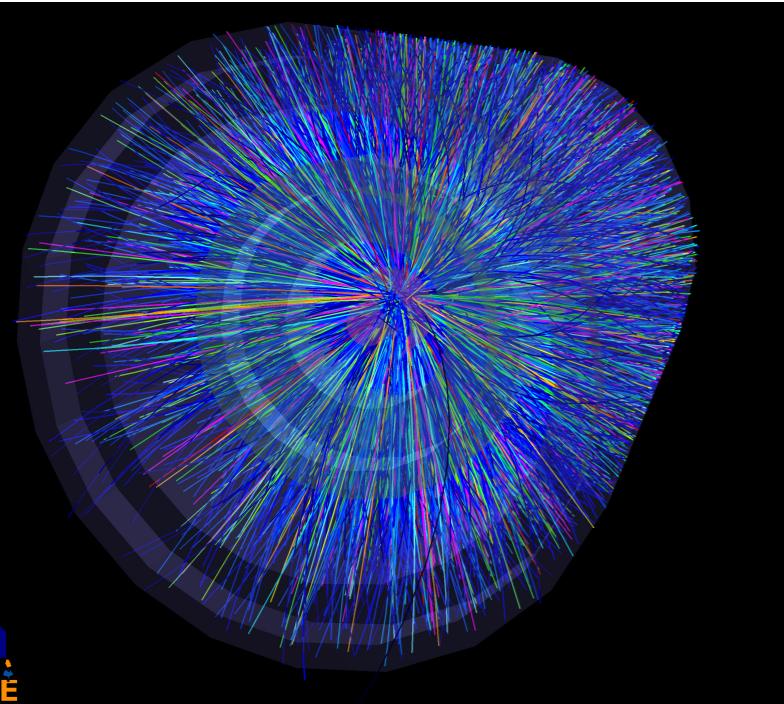
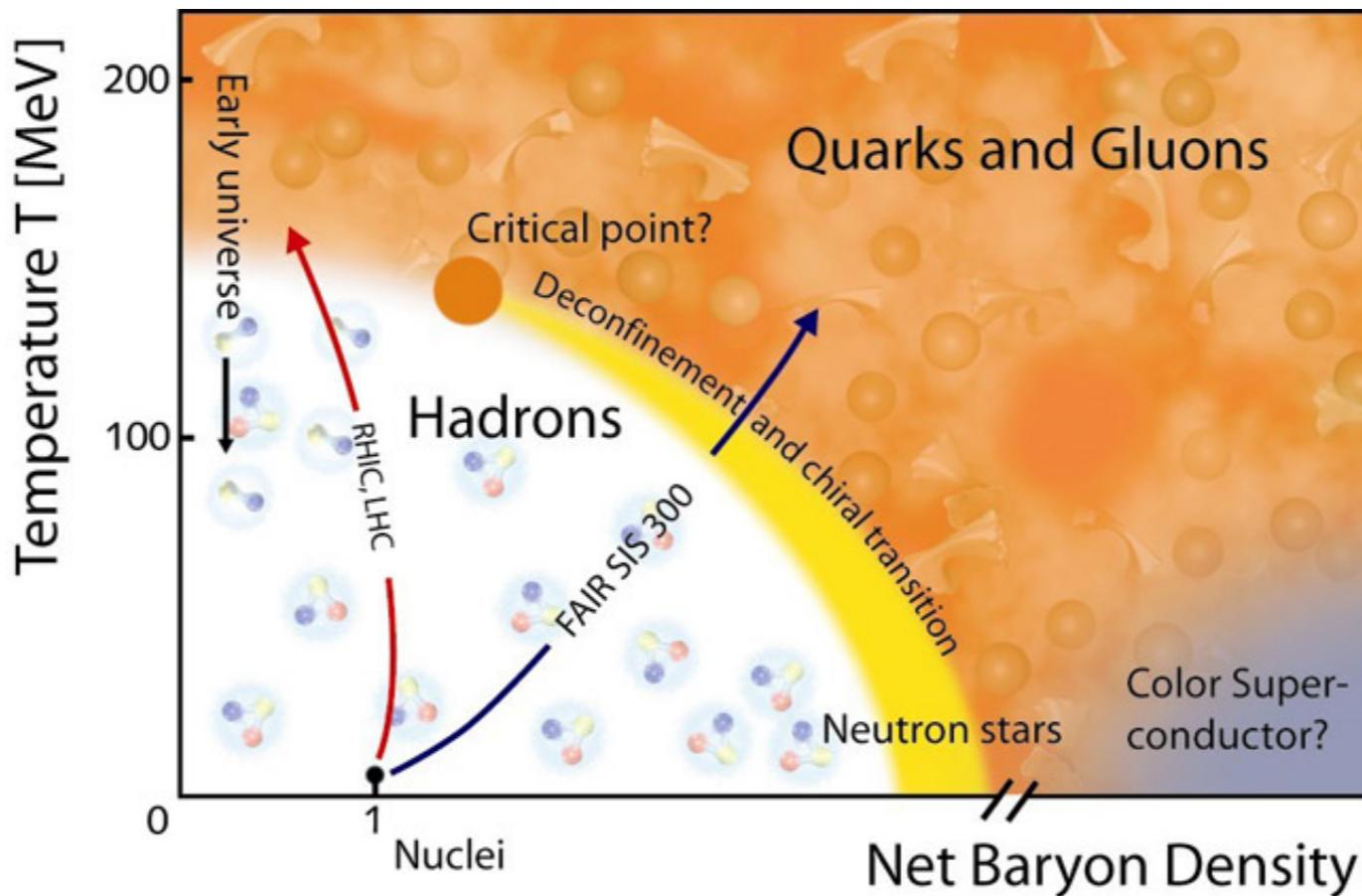


# Outline

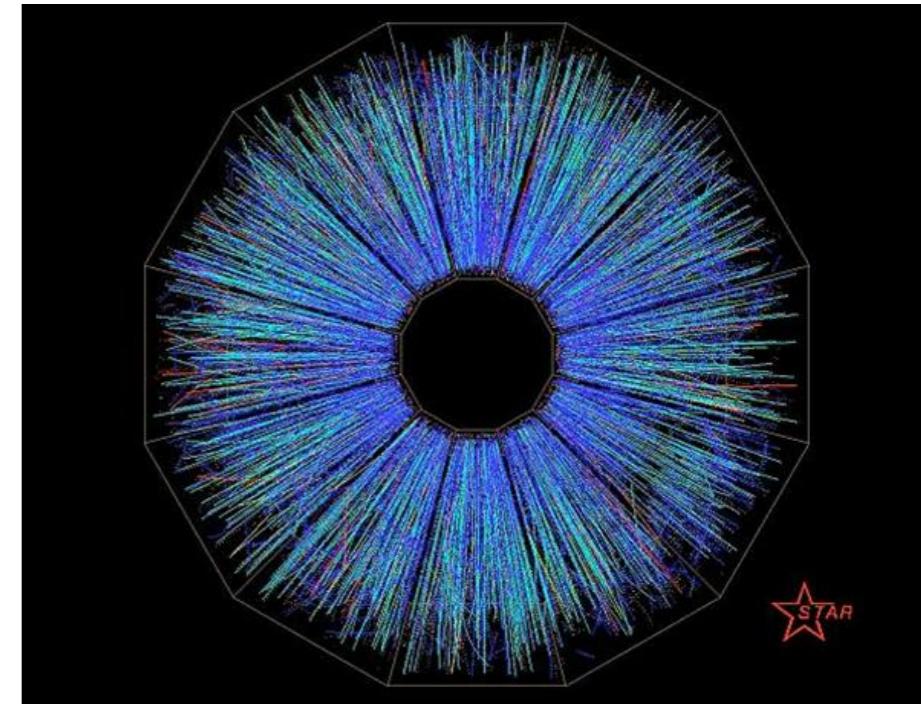
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- **Motivation**
- **QCD**
  - Asymptotic freedom and all that
  - confinement
  - chiral symmetry breaking
- **Functional methods for QCD**
  - FRG for QCD
  - Dynamical hadronisation
  - Gauge symmetry, gauge fixing and regularisation
  - Approximation scheme
- **Results**
  - Yang-Mills theory at zero and finite temperature
  - Many-flavour QCD
  - Phase diagram of QCD
- **Summary & outlook**

# Heavy ion collisions



**ALICE, LHC**

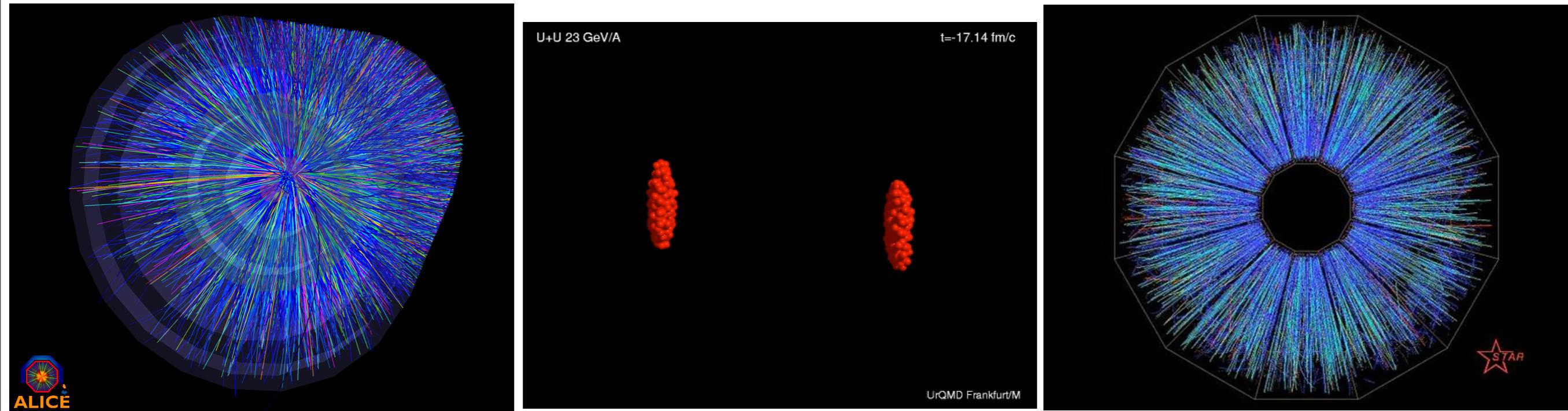
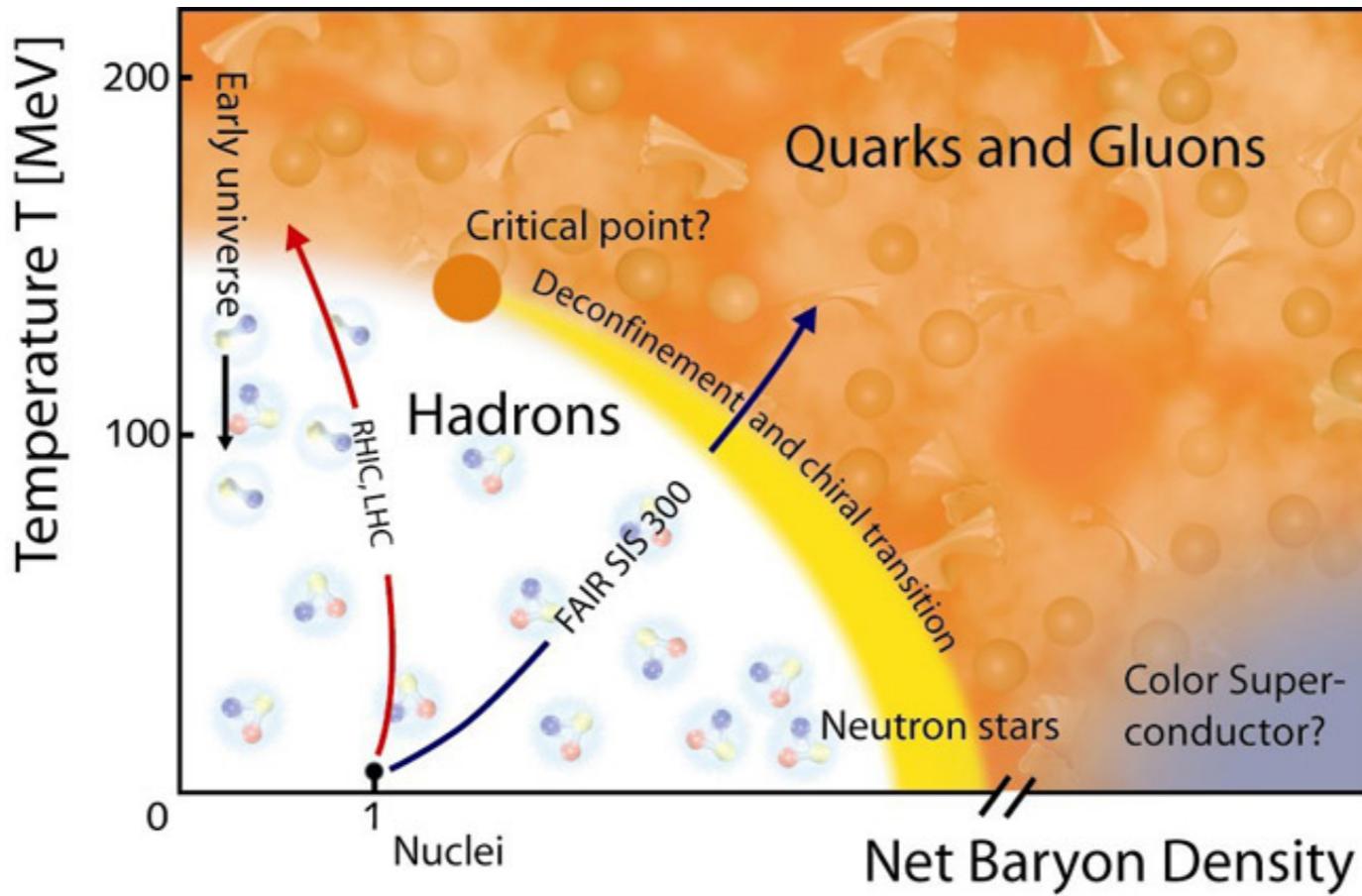


**Simulation of a heavy ion collision**

UrQMD Frankfurt/M

**STAR, RHIC**

# Heavy ion collisions



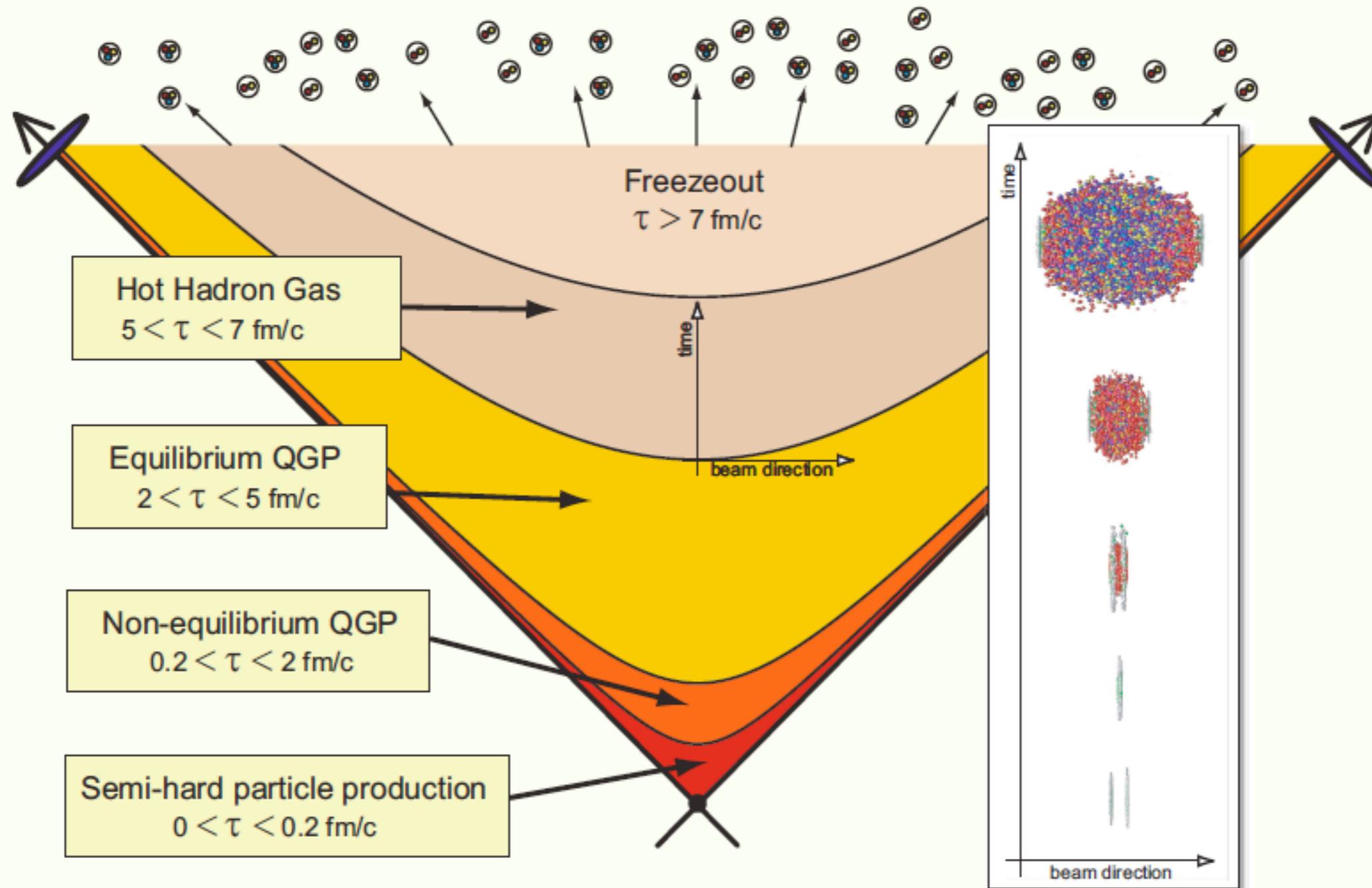
**ALICE, LHC**

**Simulation of a heavy ion collision**

**STAR, RHIC**

# Heavy ion collisions

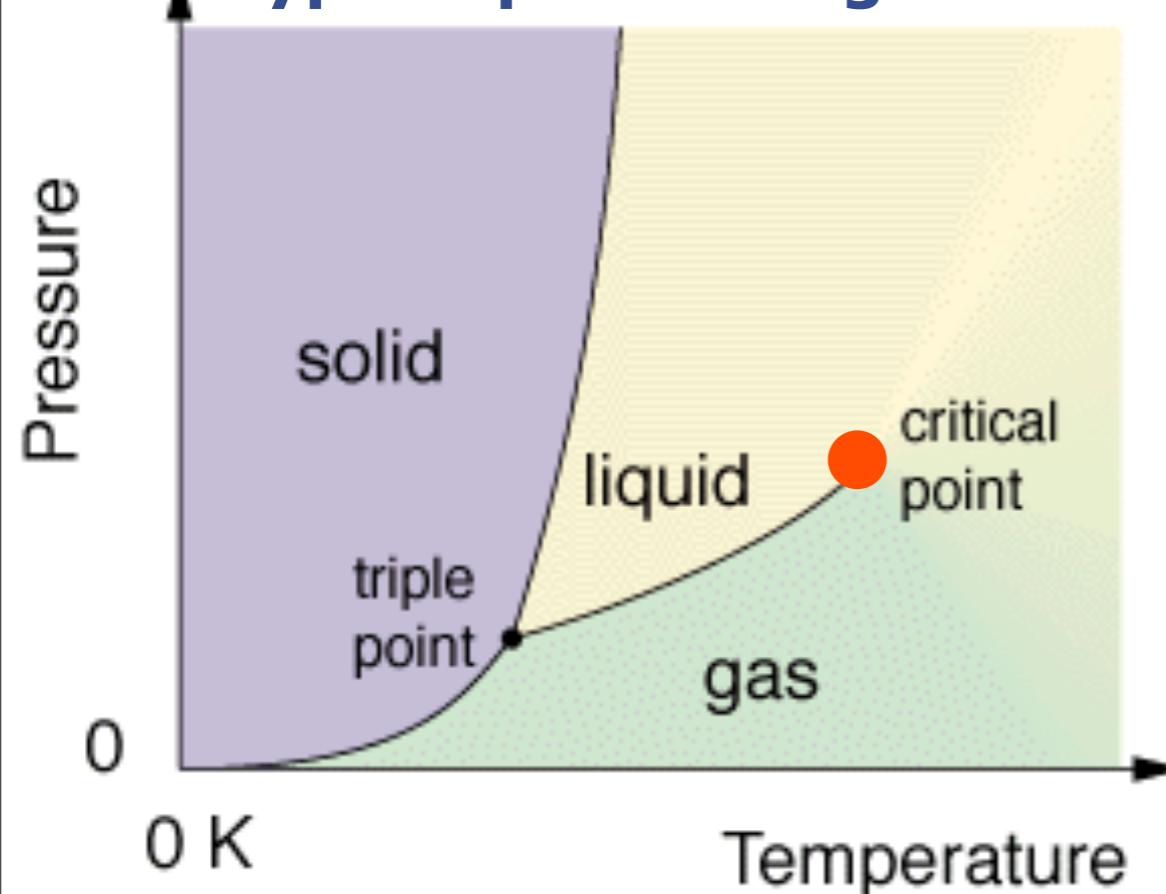
## Heavy-ion collision timescales and “epochs” @ RHIC



\* $1 \text{ fm/c} \simeq 3 \times 10^{-24} \text{ seconds}$

# Phase diagrams & order parameters

typical phase diagram



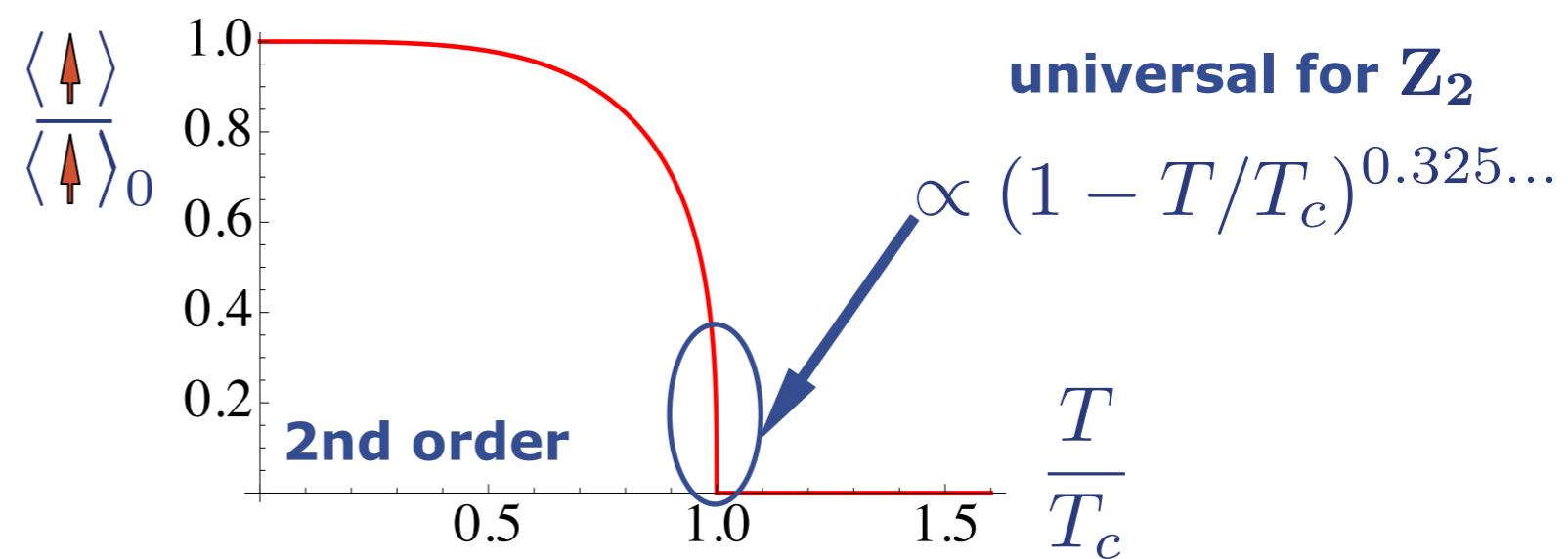
<http://ltl.tkk.fi/research/theory/TypicalPD.gif>

Order parameter: density  $n$

- density jumps  $\rightarrow$  1st order phase transition
- derivative of density jumps  $\rightarrow$  2nd order phase transition
- density smooth  $\rightarrow$  cross-over

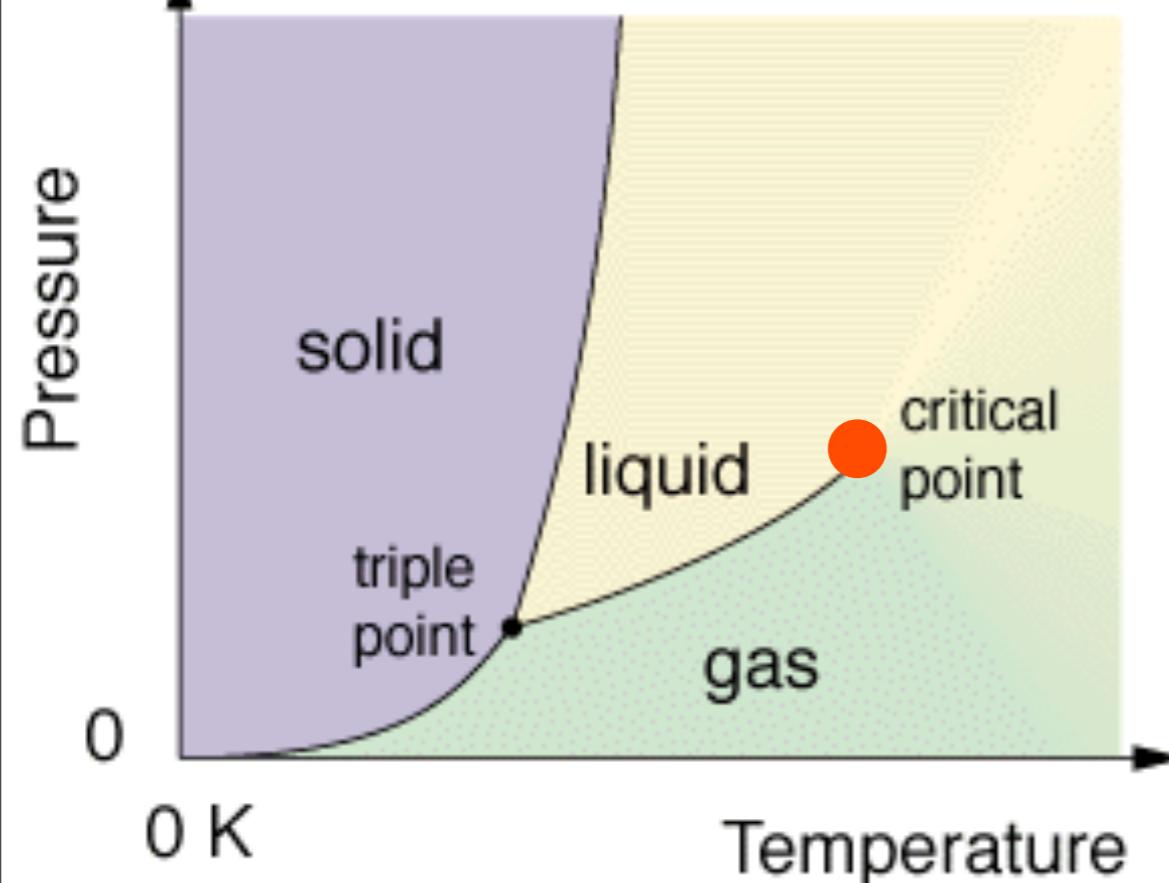
Ising model in 3d:  $(\downarrow \uparrow)$ -spin system

Order parameter:  $\langle \uparrow \rangle$



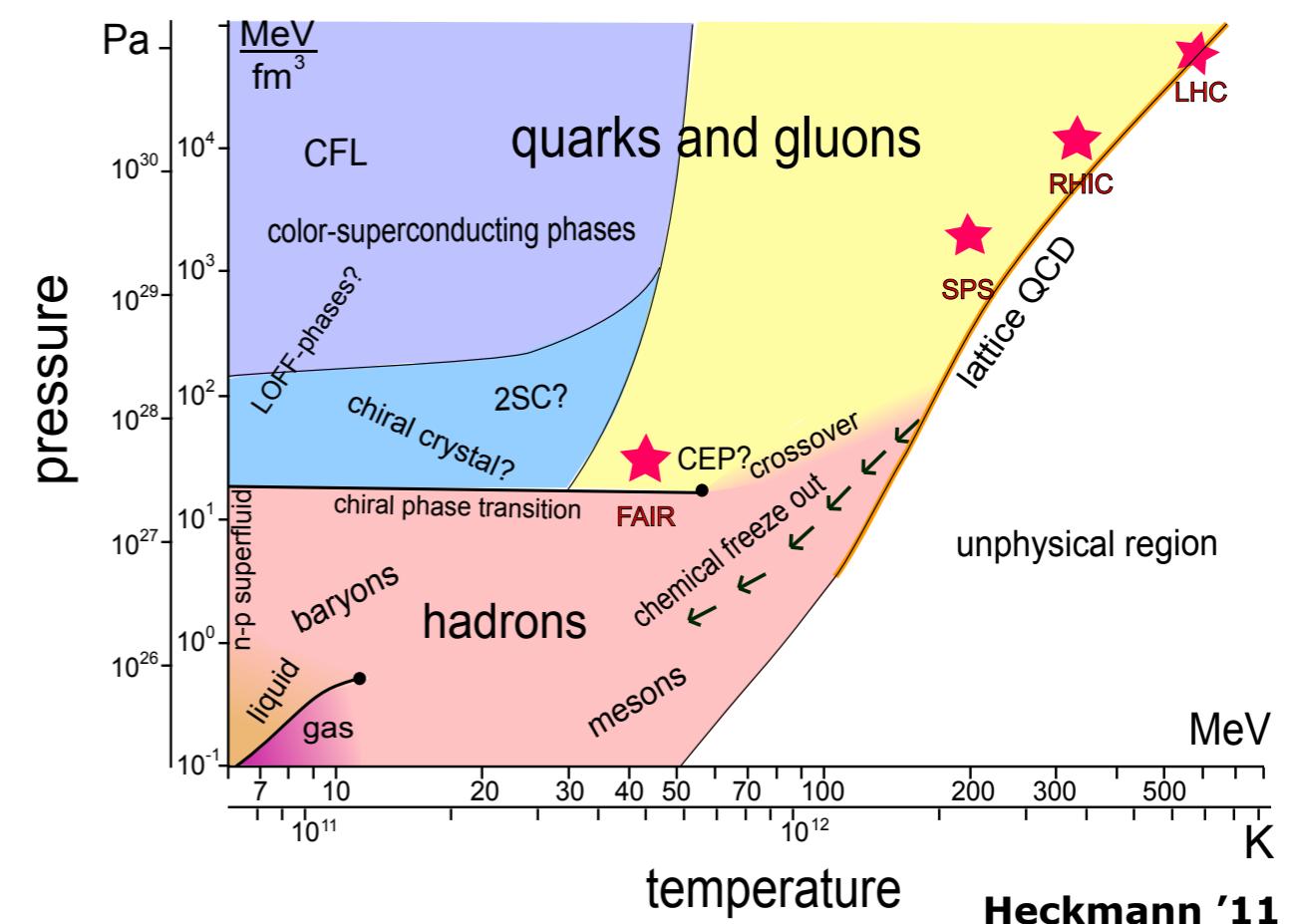
# Phase diagrams & order parameters

typical phase diagram



<http://ltl.tkk.fi/research/theory/TypicalPD.gif>

phase diagram of QCD



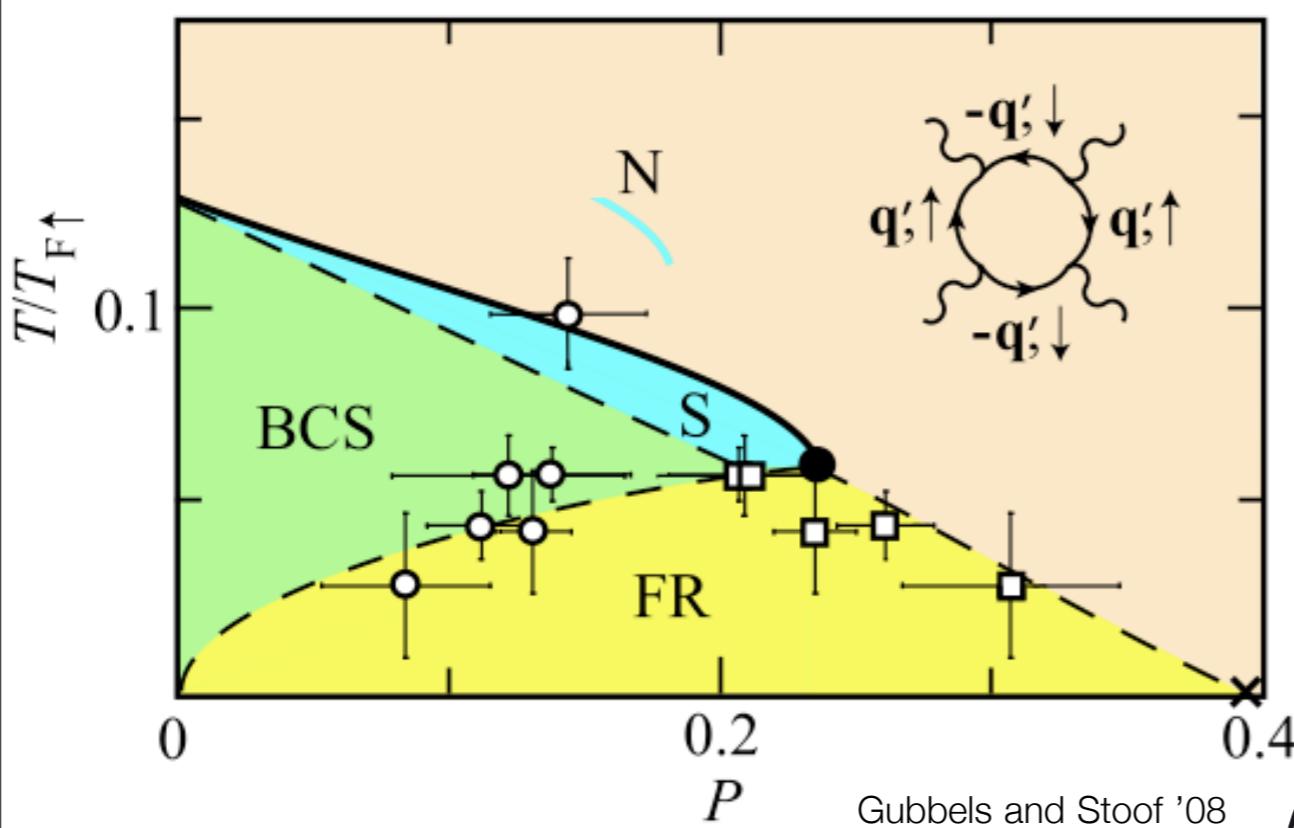
Phases in QCD

quarks massless - massive

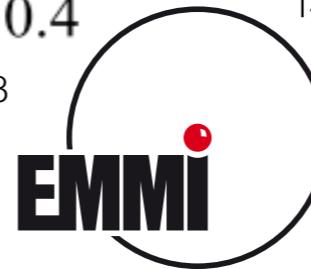
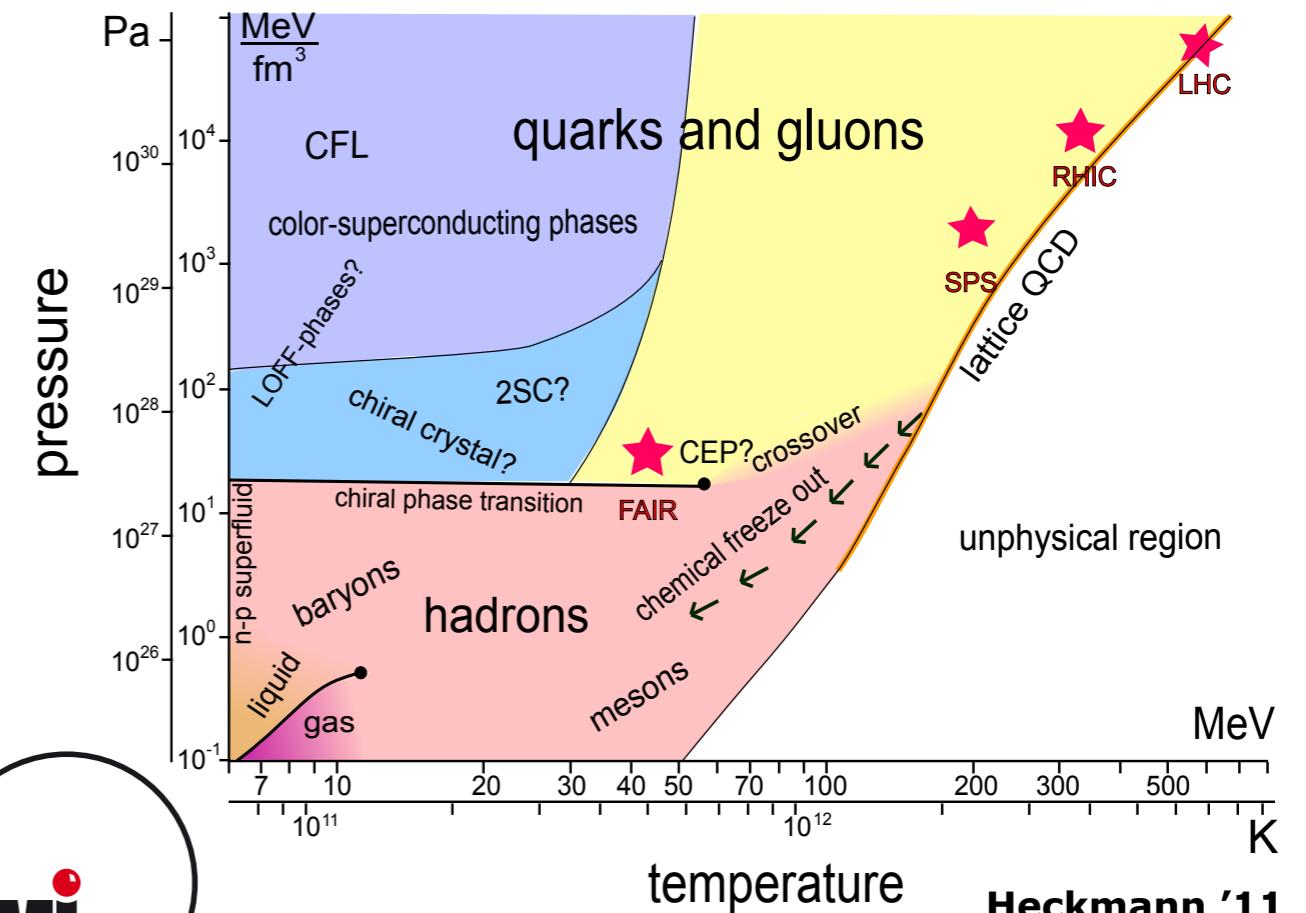
quarks confined - deconfined

# Phase diagrams & order parameters

## Phase diagram of cold atoms



## phase diagram of QCD

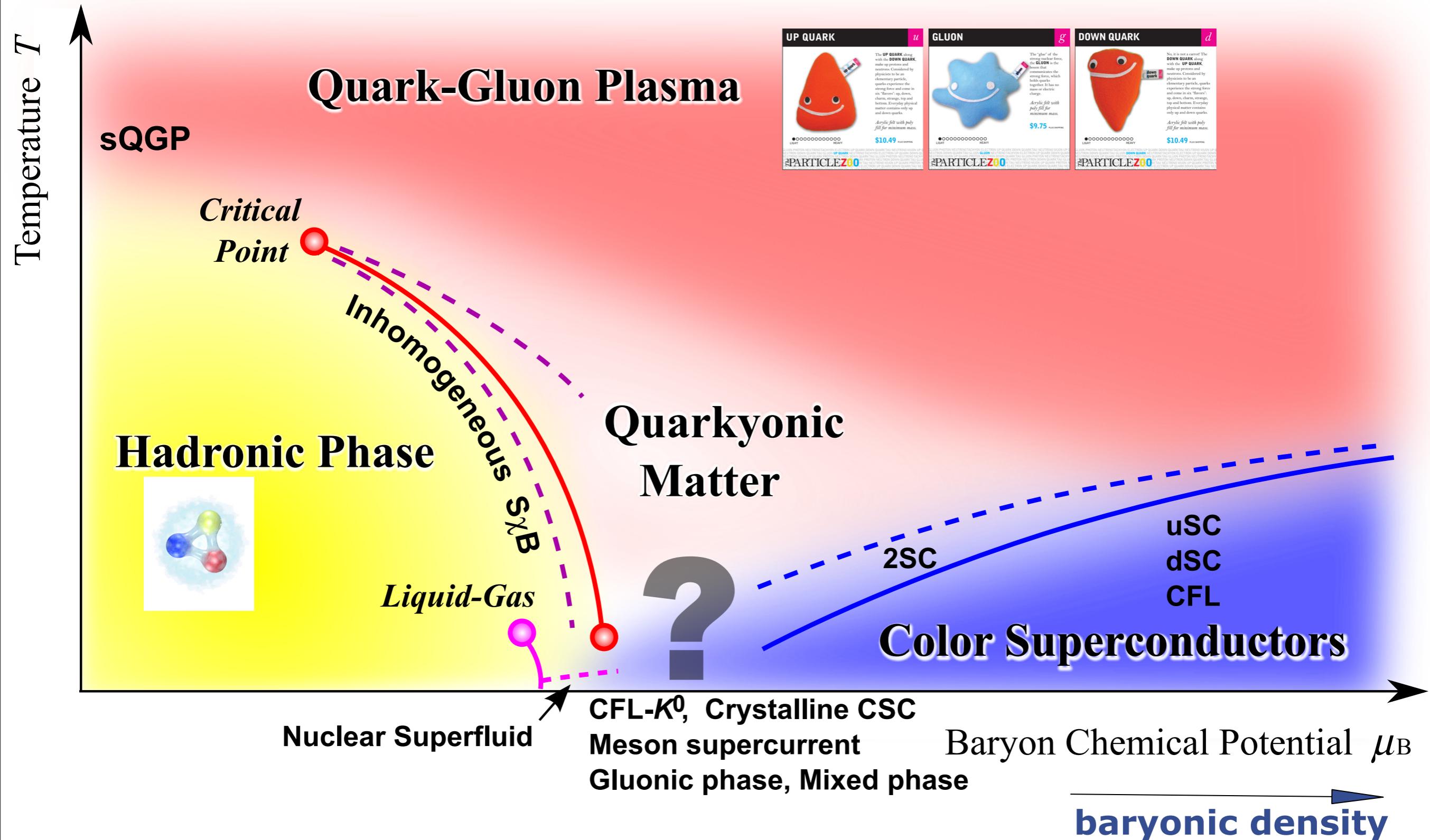


## Phases in QCD

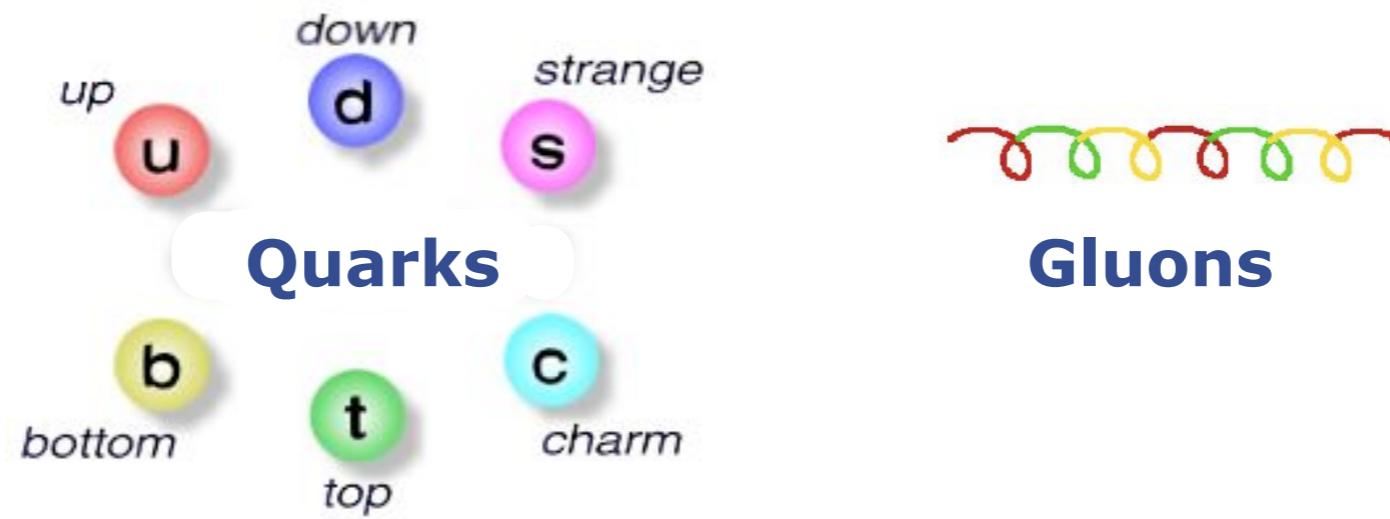
**quarks massless - massive**

**quarks confined - deconfined**

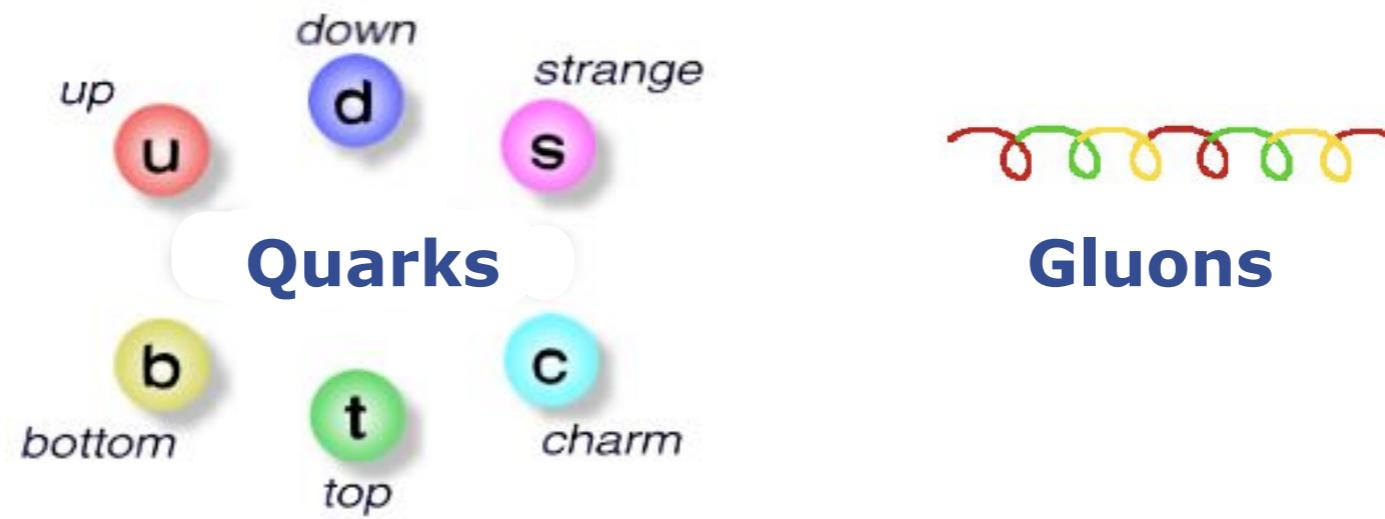
# Phase diagram of QCD



# QCD



# QCD, asymptotic freedom and all that



# QCD, asymptotic freedom and all that

## Action and interactions

**QCD action**  $S_{\text{QCD}}$

**Yang-Mills**

**gauge fixing**

$$\frac{1}{4} \int_x F_{\mu\nu}^a F_{\mu\nu}^a + \frac{1}{2\xi} \int_x (\partial_\mu A_\mu^a)^2 + \int_x \bar{c}^a \partial_\mu D_\mu^{ab} c^b + \int_x \bar{q} \cdot (i \not{D} + i m_\psi + i \mu \gamma_0) \cdot q$$

**gluon**                                   **ghost**                                   **quarks**

**Pure gauge theory**

**matter sector**

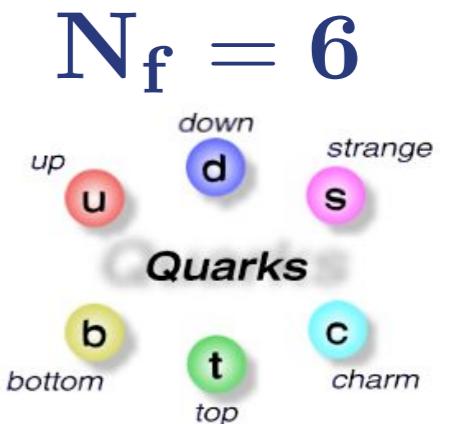
$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + ig f^{abc} A_\mu^b A_\nu^c$$

$$\not{D} = \gamma_\mu D_\mu$$

$$a, b, c = 1, \dots, N_c^2 - 1$$



$$D_\mu^{ab}(A) = \partial_\mu \delta^{ab} - ig f^{abc} A_\mu^c$$



# QCD, asymptotic freedom and all that

## Action and interactions

**QCD action**  $S_{\text{QCD}}$

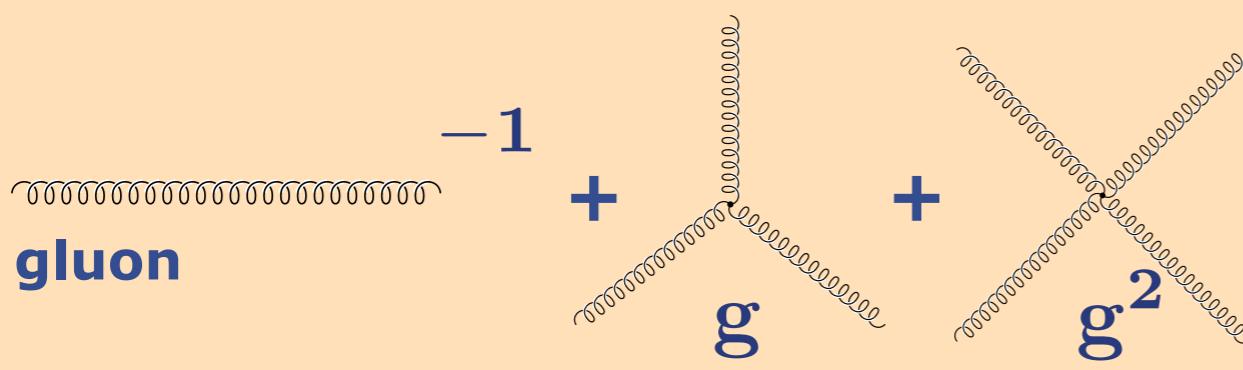
**Yang-Mills**

**gauge fixing**

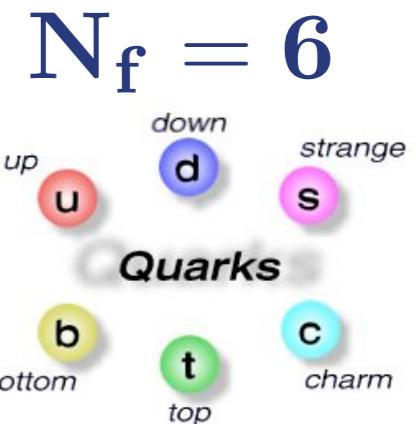
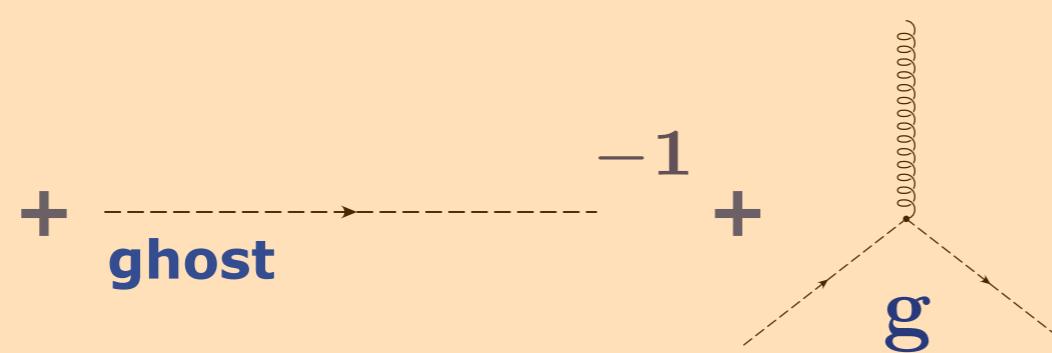
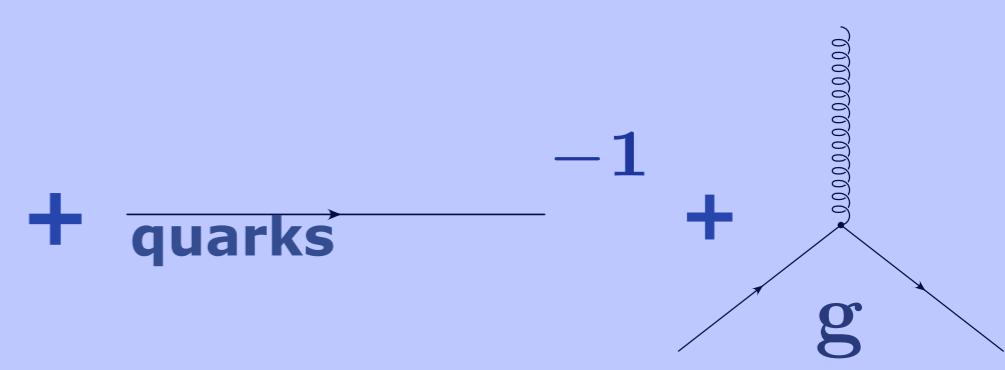
$$\frac{1}{4} \int_x F_{\mu\nu}^a F_{\mu\nu}^a + \frac{1}{2\xi} \int_x (\partial_\mu A_\mu^a)^2 + \int_x \bar{c}^a \partial_\mu D_\mu^{ab} c^b + \int_x \bar{q} \cdot (i \not{D} + i m_\psi + i \mu \gamma_0) \cdot q$$

**gluon**   **ghost**   **quarks**

**Pure gauge theory**



**matter sector**



# **QCD, asymptotic freedom and all that**

## Action and interactions

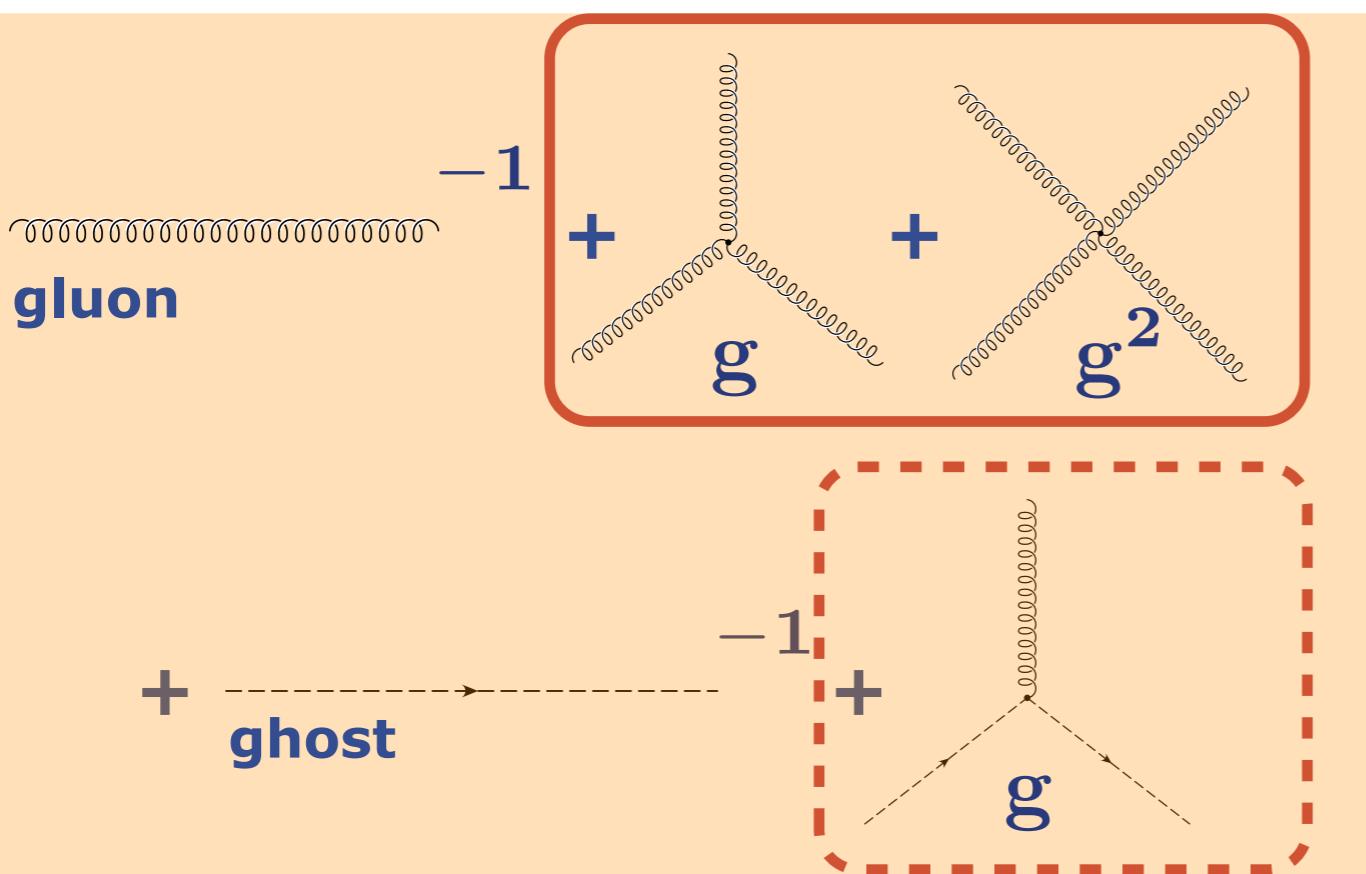
# **QCD action** $S_{\text{QCD}}$

# Yang-Mills

# gauge fixing

# Pure gauge theory

## **matter sector**



# purely non-Abelian

# **QCD, asymptotic freedom and all that**

## Action and interactions

# **QCD action** $S_{\text{QCD}}$

# Yang-Mills

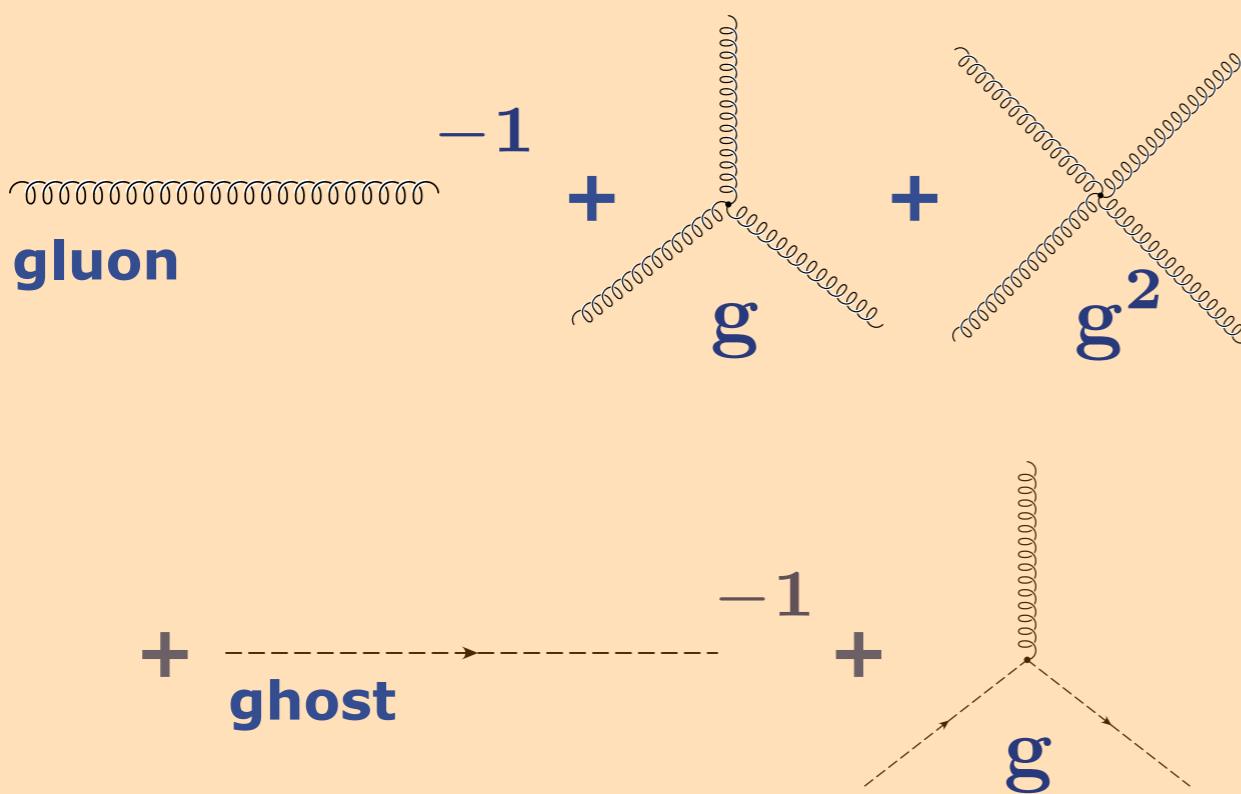
# gauge fixing

$$\frac{1}{4} \int_x F_{\mu\nu}^a F_{\mu\nu}^a + \frac{1}{2\xi} \int_x (\partial_\mu A_\mu^a)^2 + \int_x \bar{c}^a \partial_\mu D_\mu^{ab} c^b + \int_x \bar{q} \cdot (i \not{D} + i m_\psi + i \mu \gamma_0) \cdot q$$

**gluon**                                    **ghost**                                    **quarks**

# Pure gauge theory

# matter sector

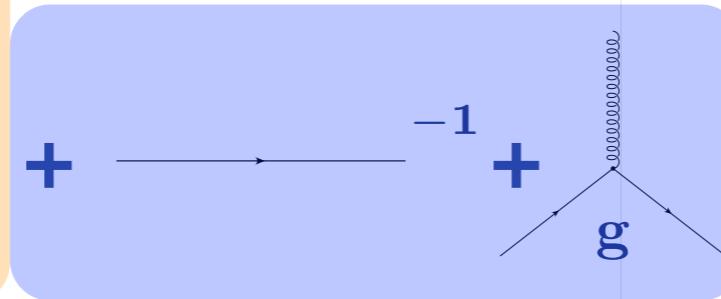
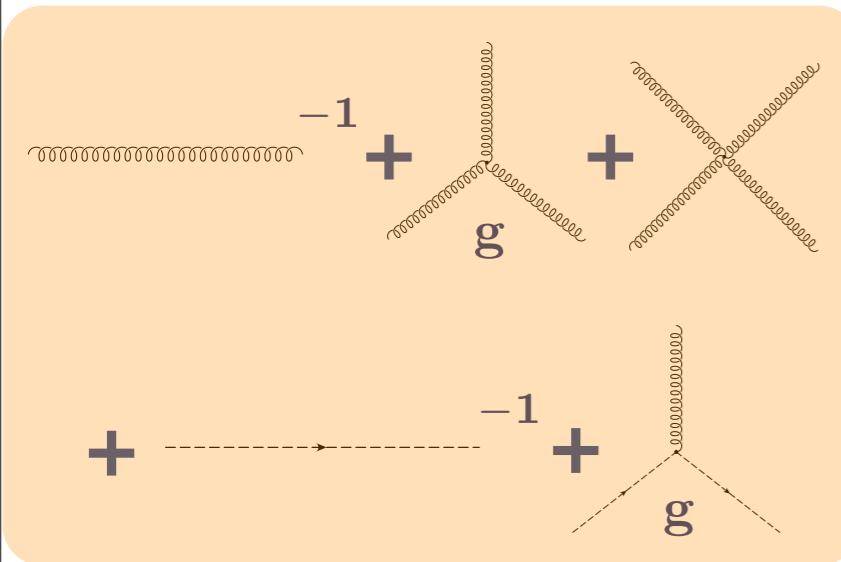


# parameters

- 1 coupling  $g$
  - mass matrix  $m_\psi$   $N_f \times N_f$

# QCD, asymptotic freedom and all that

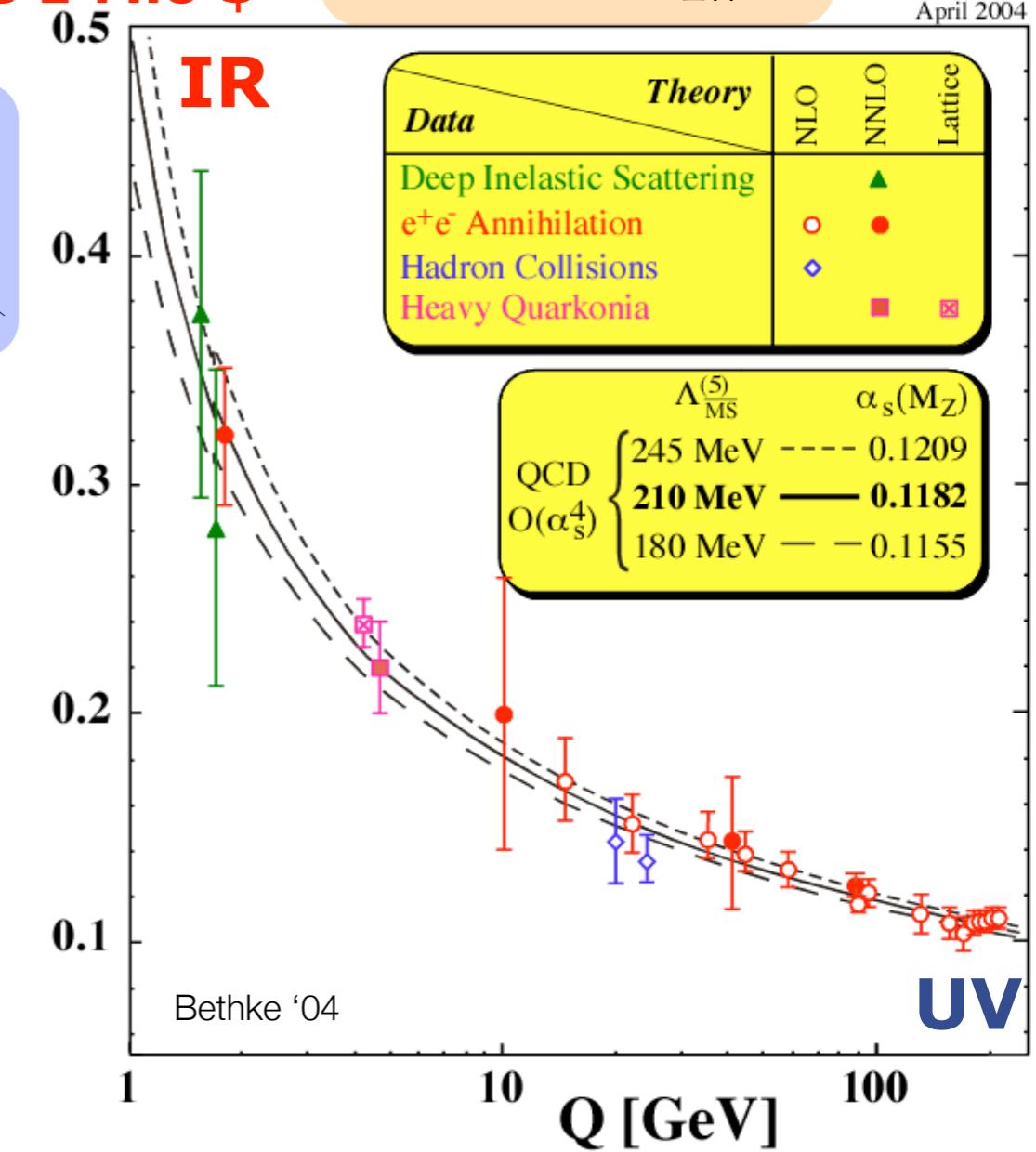
## Running coupling at low and high energies



**Millenium Prize 1 Mio \$**

$$\alpha_s(Q) = \frac{g^2(Q)}{4\pi}$$

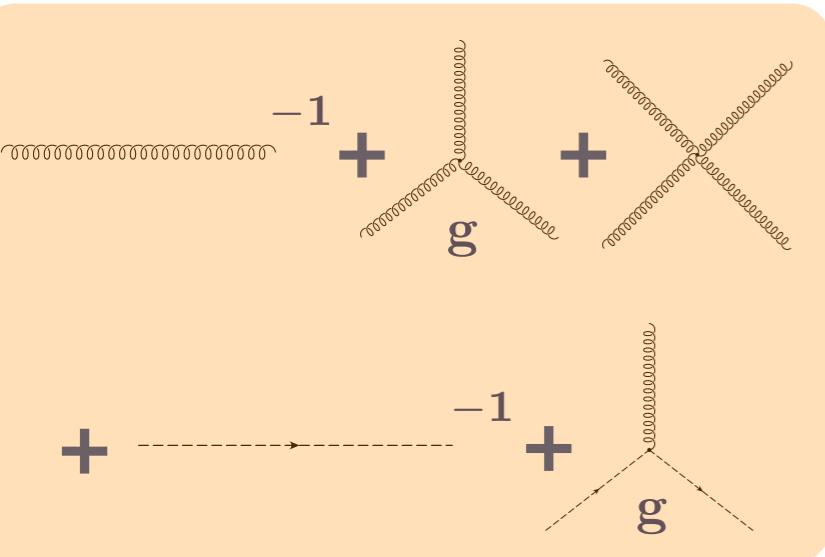
April 2004



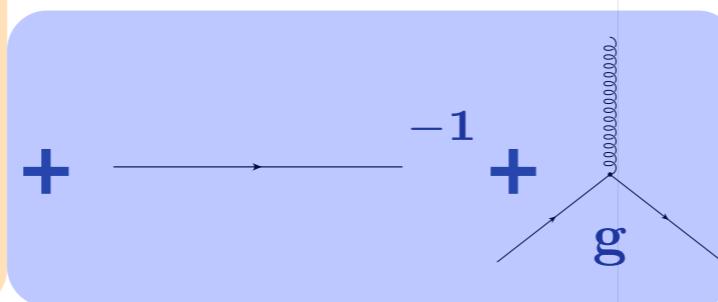
**Nobel Prize '04**  
Gross, Politzer, Wilczek

# QCD, asymptotic freedom and all that

## Running coupling at low and high energies



Pure gauge theory

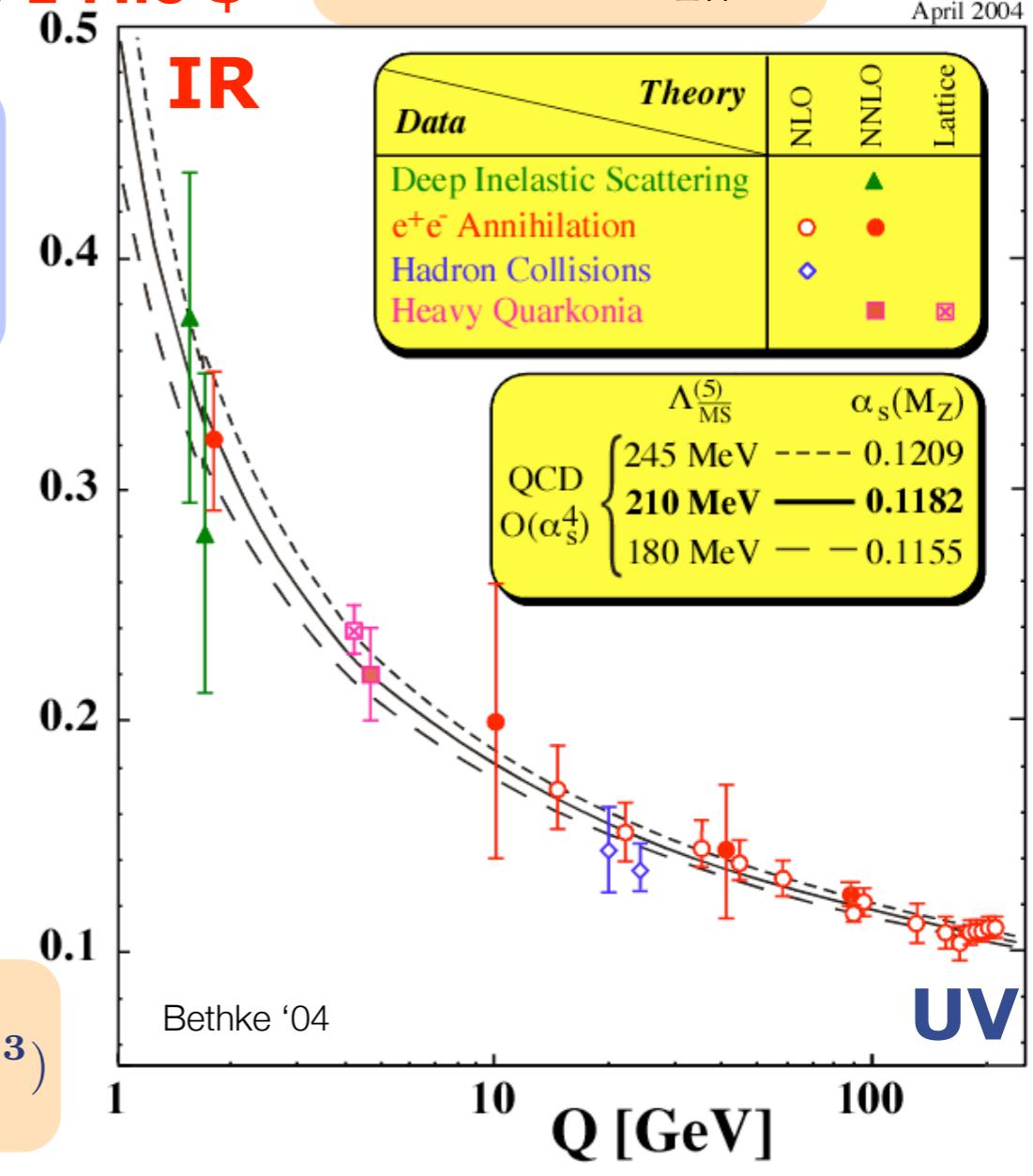


matter sector

**Millenium Prize 1 Mio \$**

$$\alpha_s(Q) = \frac{g^2(Q)}{4\pi}$$

April 2004



$$\alpha_s(Q) = \frac{\alpha_s(\mu)}{1 - \alpha_s(\mu)\beta_0 \log(Q^2/\mu^2)}$$

- **running coupling (1-loop)**

$$\beta = Q^2 \frac{\partial \alpha_s(Q)}{\partial Q^2} = \beta_0 \alpha_s(\mu)^2 + \mathcal{O}(\alpha_s(\mu)^3)$$

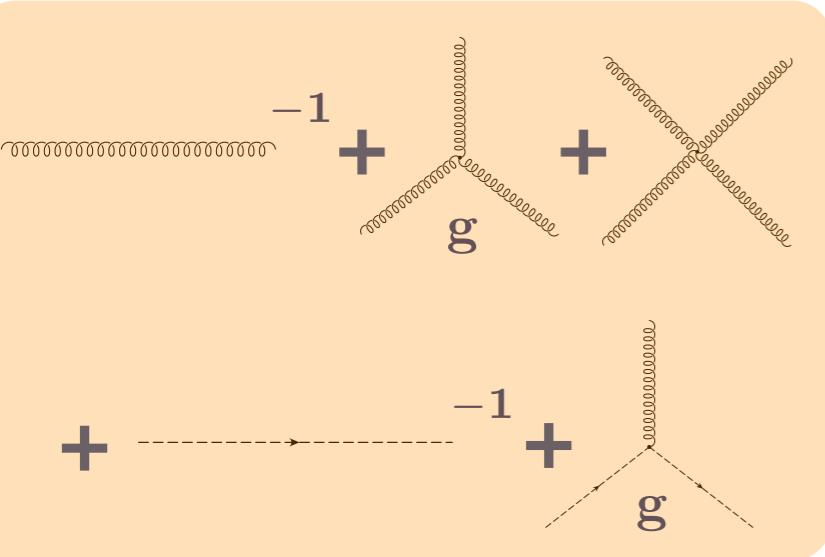
$$\beta = -\frac{1}{12\pi} (33 - 2N_f) \alpha_s^2 + \mathcal{O}(\alpha_s^3)$$

**Nobel Prize '04**

Gross, Politzer, Wilczek

# QCD, asymptotic freedom and all that

## Running coupling at low and high energies



**Millenium Prize 1 Mio \$**

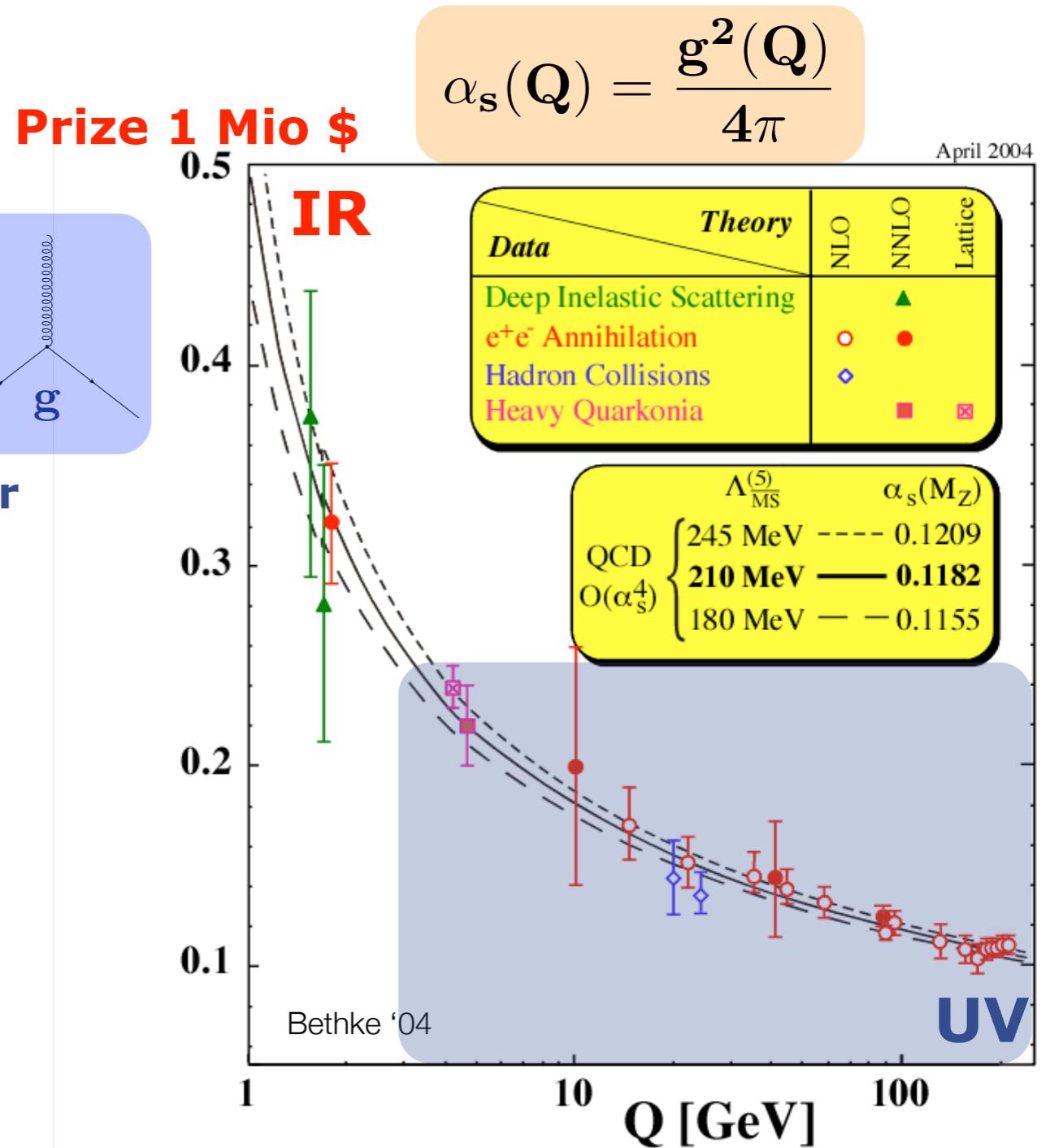
**matter sector**

- **running coupling (1-loop)**

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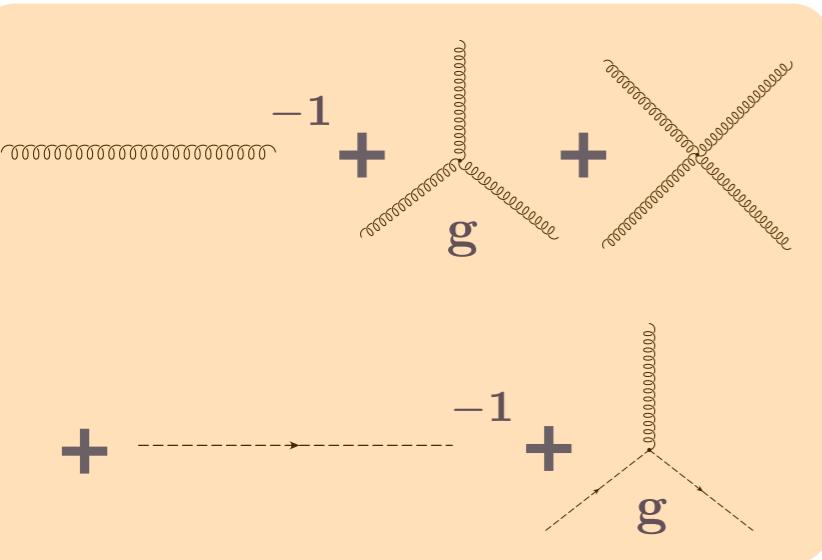
- **UV: asymptotic freedom**

$$\alpha_s(Q \rightarrow \infty) = 0$$

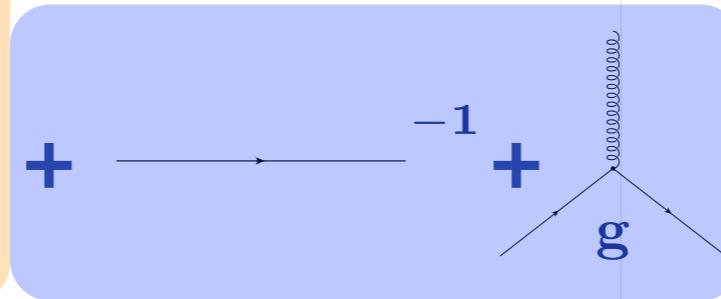


# QCD, asymptotic freedom and all that

## Running coupling at low and high energies



Pure gauge theory

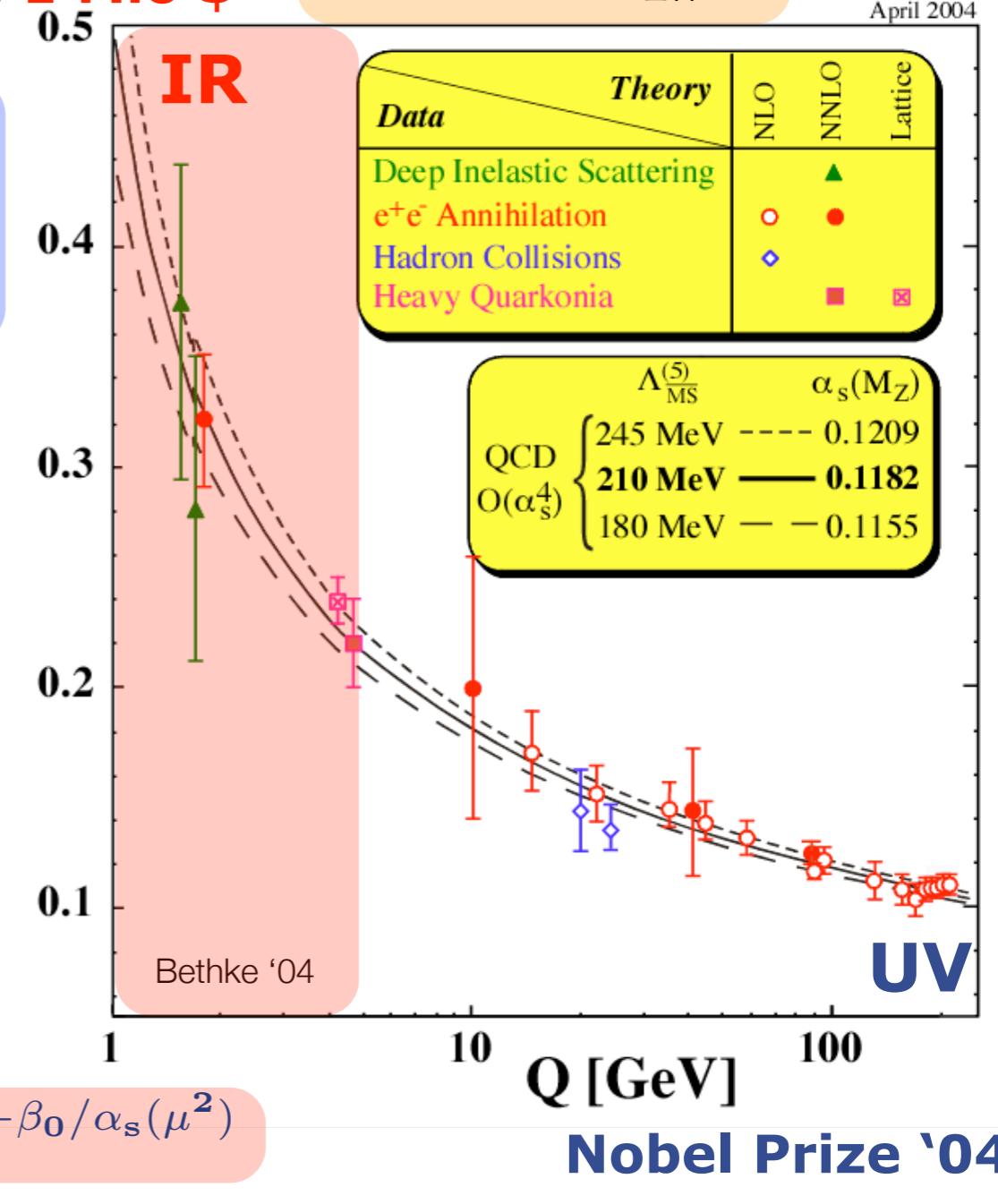


matter sector

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Nobel Prize '04

Gross, Politzer, Wilczek

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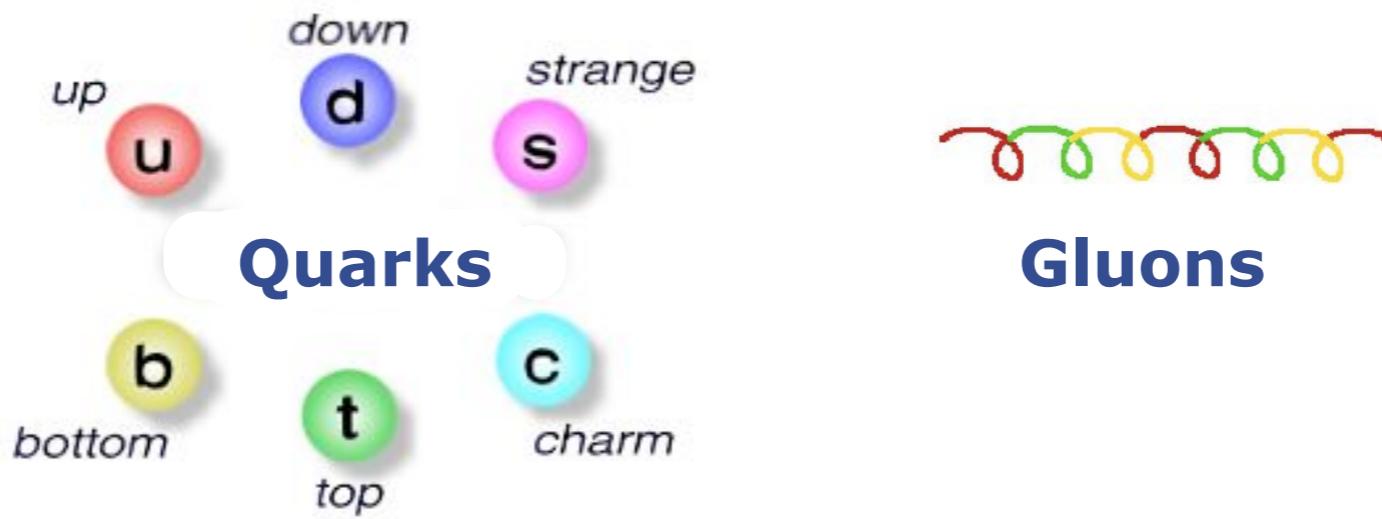
- running coupling (1-loop)

$$\alpha_s(\Lambda_{QCD}^2) = \infty$$

at  $\Lambda_{QCD}^2 = \mu^2 e^{-\beta_0/\alpha_s(\mu^2)}$

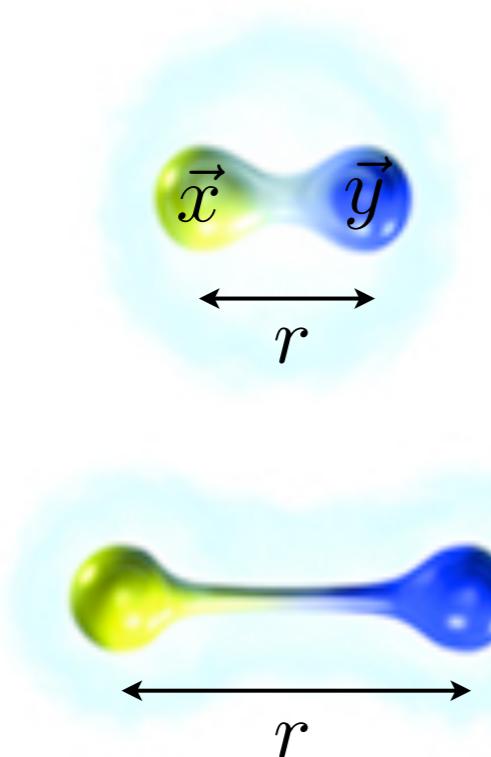
$$\Lambda_{QCD} = 217^{+25}_{-23} \text{ MeV}$$

# Confinement



# Confinement

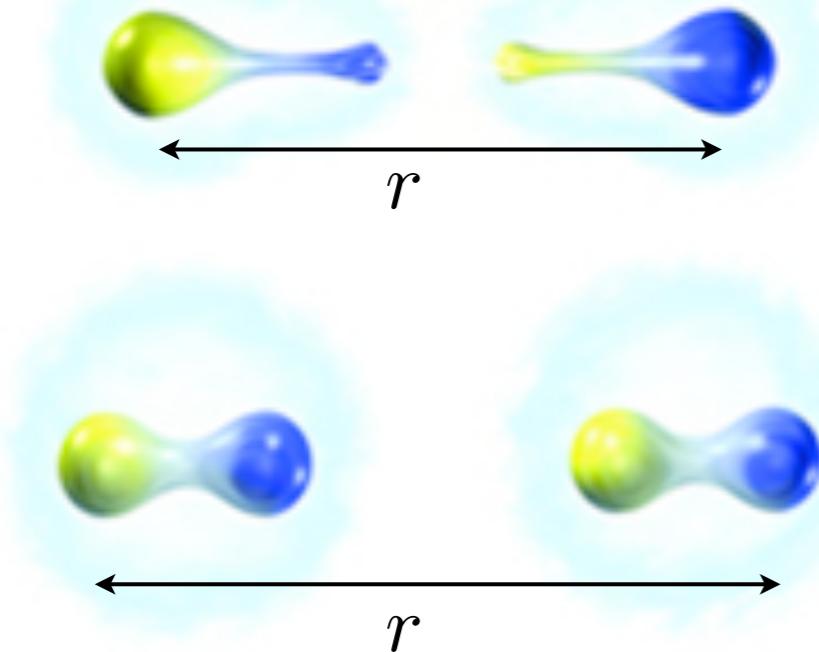
Free energy  $F_{q\bar{q}}$  of a quark - antiquark pair



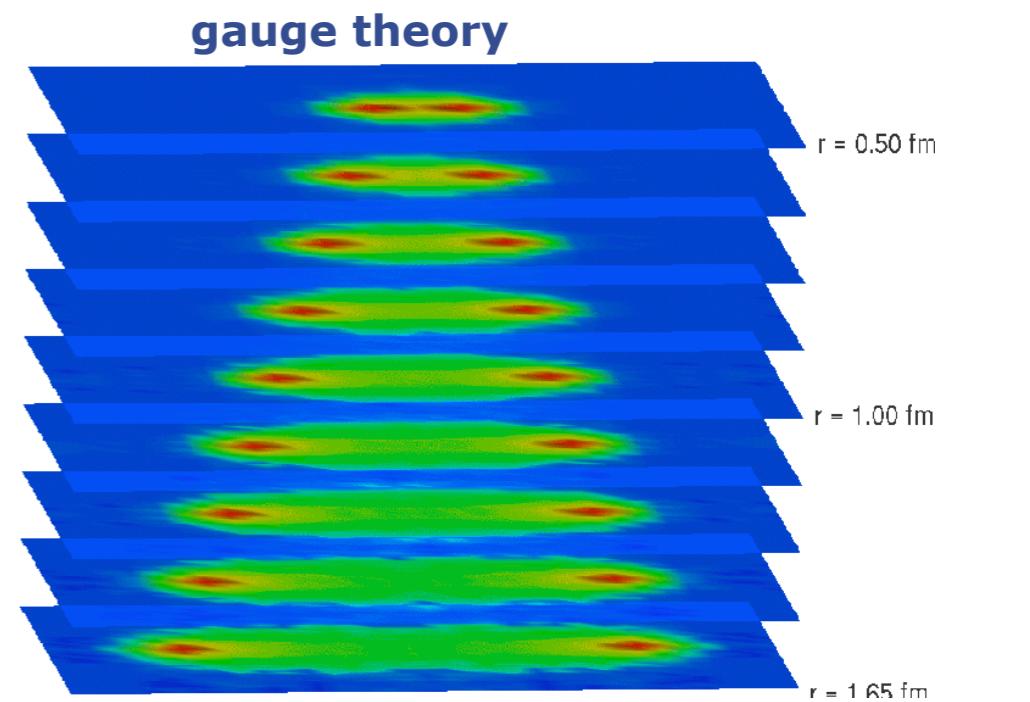
$$F_{q\bar{q}} \simeq -\frac{1}{r}$$

$$F_{q\bar{q}} \simeq \sigma r$$

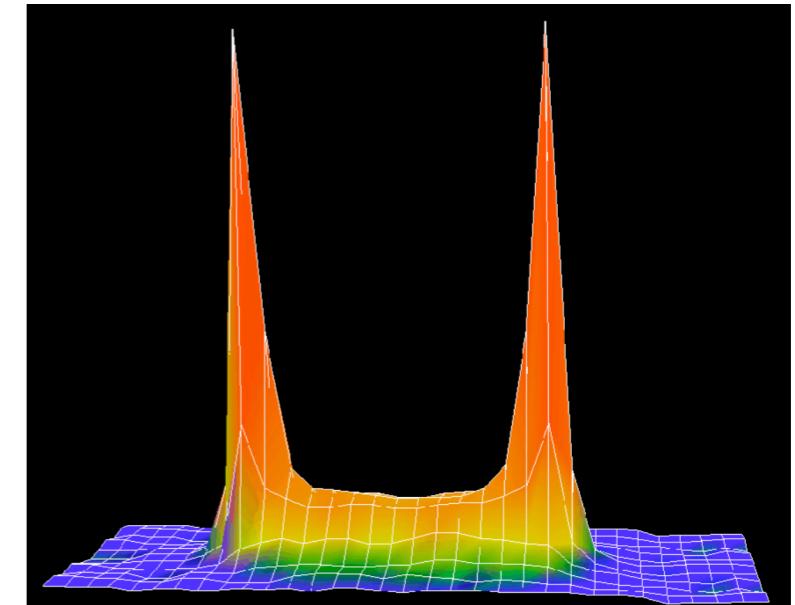
string breaking at  $r \approx 1\text{fm}$



$$F_{q\bar{q}} \simeq \text{const.}$$

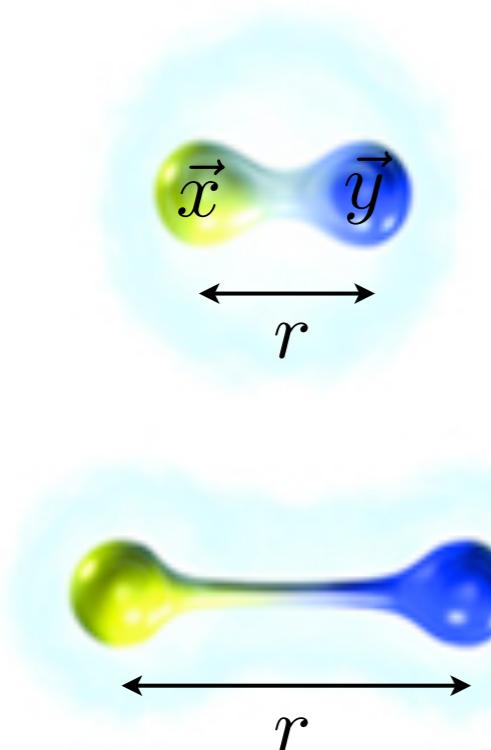


Energy density      Bali et al. '94



# Confinement

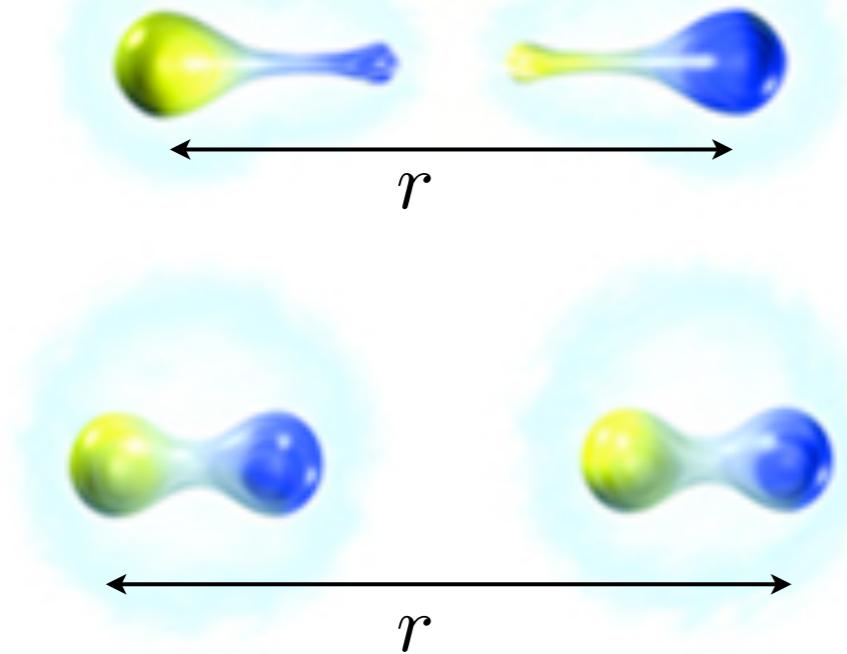
Free energy  $F_{q\bar{q}}$  of a quark - antiquark pair



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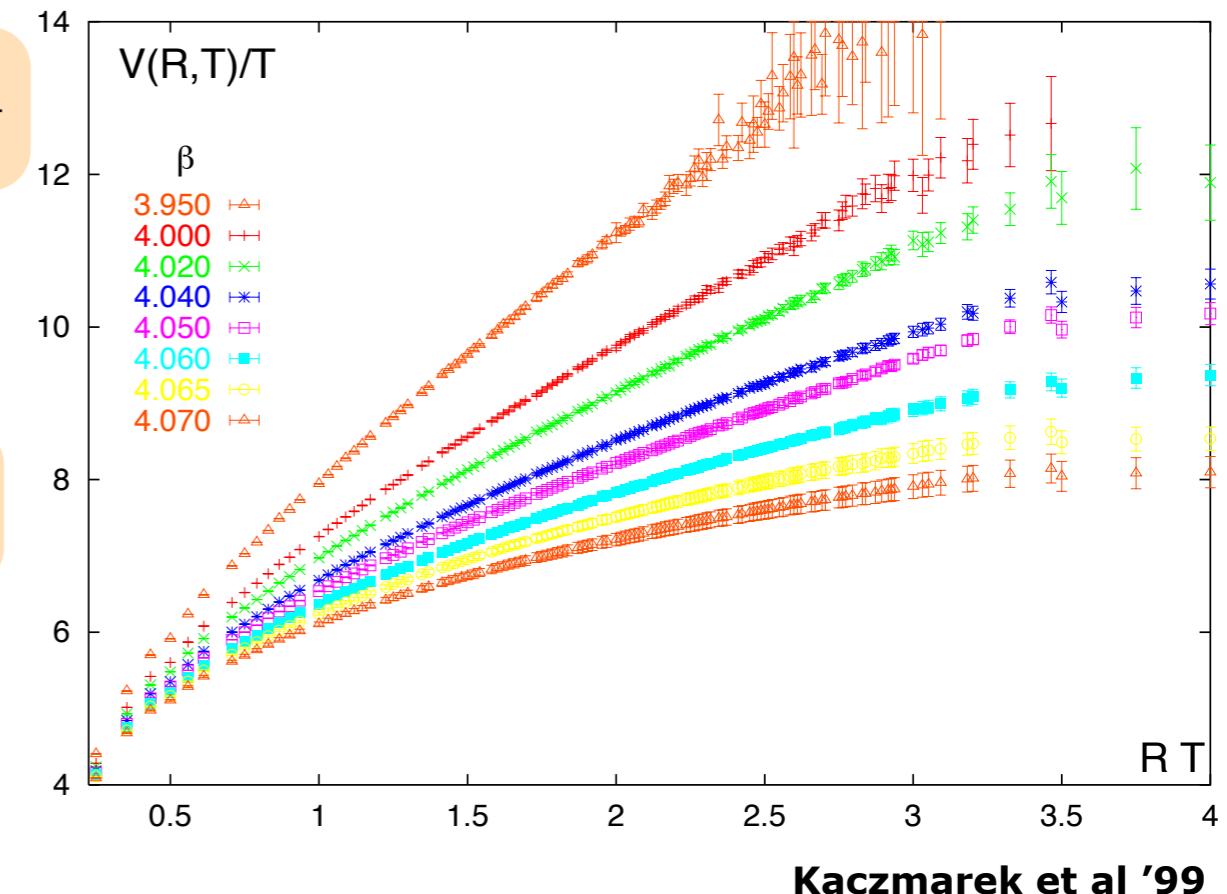
$$F_{q\bar{q}} \simeq \sigma r$$

string breaking at  $r \approx 1\text{fm}$



$$F_{q\bar{q}} \simeq \text{const.}$$

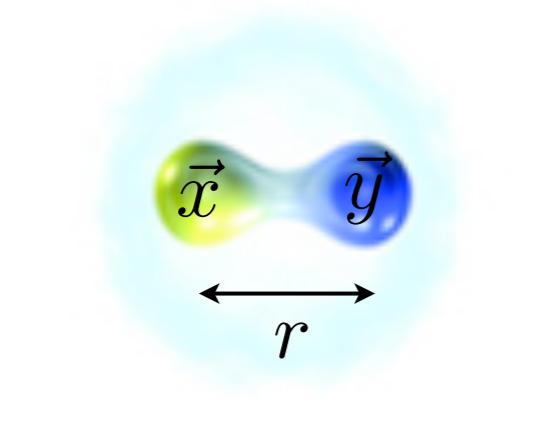
pure gauge theory



Kaczmarek et al '99

# Confinement

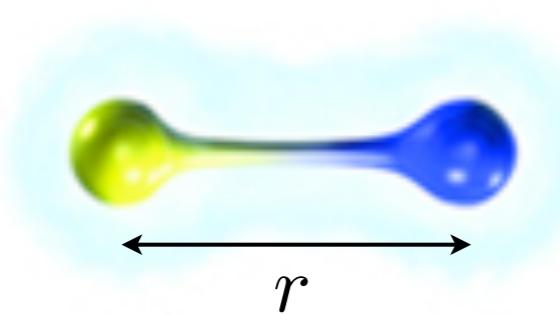
Free energy  $F_{q\bar{q}}$  of a quark - antiquark pair



$$F_{q\bar{q}} \simeq -\frac{1}{r}$$

**Order parameter**  $\sim \langle q \rangle'$

$$\Phi = e^{-\frac{1}{2T} F_{q\bar{q}}(\infty)}$$



$$F_{q\bar{q}} \simeq \sigma r$$

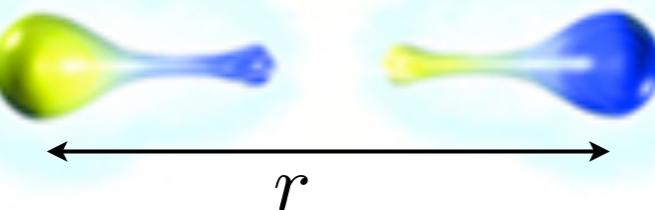
- **Confinement**

$$\Phi = 0$$

- **Deconfinement**

$$\Phi \neq 0$$

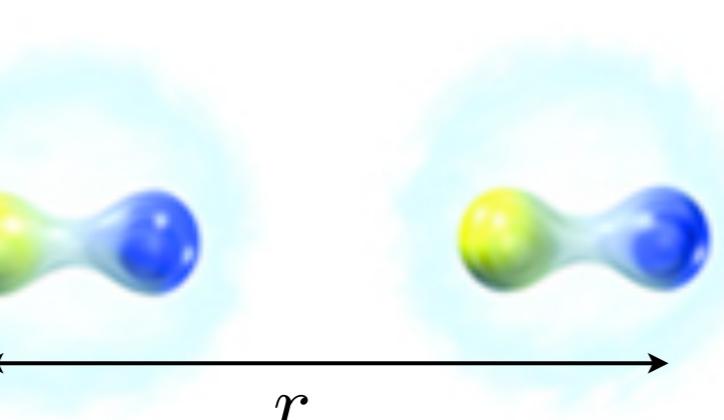
string breaking at  $r \approx 1\text{fm}$



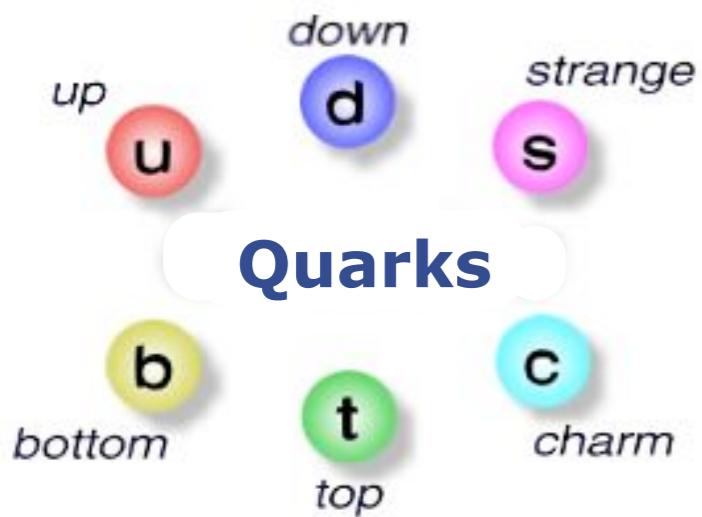
$$F_{q\bar{q}} \simeq \text{const.}$$

**Polyakov loop**

$$\Phi = \frac{1}{3} \langle \text{Tr } \mathcal{P} \exp\{ig \int_0^{1/T} dx_0 A_0\} \rangle$$



# Chiral symmetry breaking $\Delta m_{\chi SB} \approx 400 \text{ MeV}$



$$N_f = 2 + 1$$

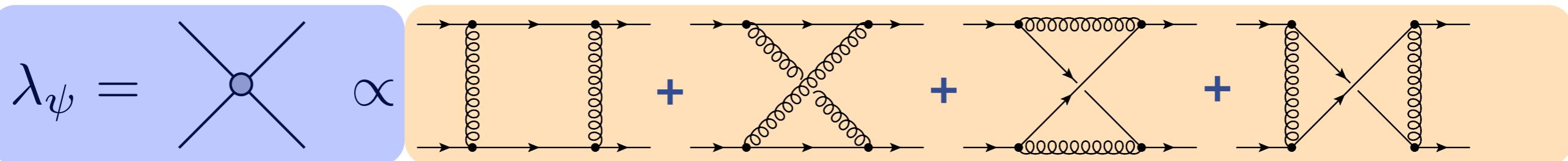
Generation	first	second	third	Charge
Mass [MeV]	1.5-4	1150-1350	$170 \times 10^3$	
Quark	u	c	t	$\frac{2}{3}$
Quark	d	s	b	$-\frac{1}{3}$
Mass [MeV]	4-8	80-130	$(4.1-4.4) \times 10^3$	

$$\Lambda_{\text{QCD}} = 217^{+25}_{-23} \text{ MeV}$$

# Chiral symmetry breaking

- Perturbative four-fermi coupling

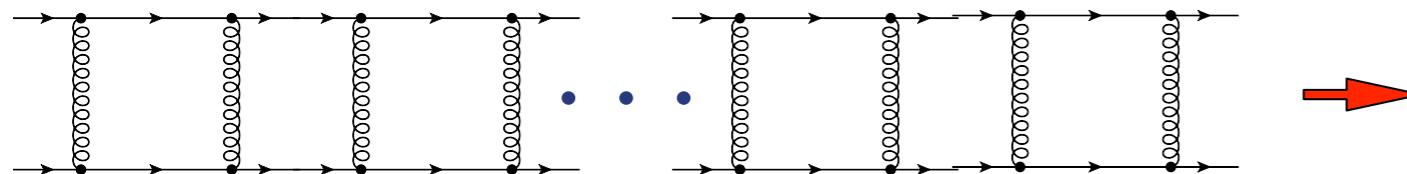
$$\frac{\lambda_\psi}{2} \int [(\bar{q}q)^2 + (i\bar{q}\gamma_5\vec{\tau}q)^2]$$



$$\lambda_\psi \propto \alpha_s^2$$

$$N_f = 2 : \vec{\tau} = (\sigma_1, \sigma_2, \sigma_3)$$

- Fermionic mass term for  $\langle \bar{q}q \rangle \neq 0$



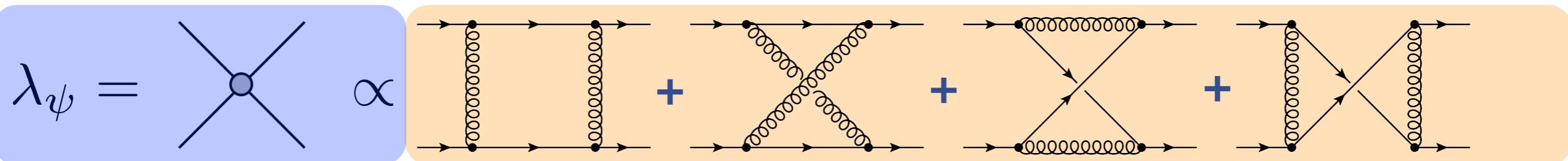
$$\frac{\lambda_\psi}{2} \int (\bar{q}q)^2 \longrightarrow \frac{\lambda_\psi}{2} \int \langle \bar{q}q \rangle \bar{q}q$$

mean field

# Chiral symmetry breaking

- Perturbative four-fermi coupling

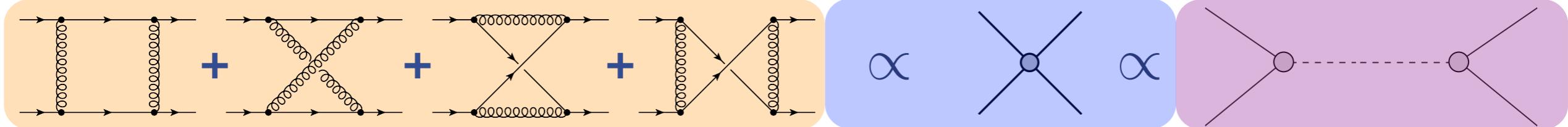
$$\frac{\lambda_\psi}{2} \int [(\bar{q}q)^2 + (i\bar{q}\gamma_5\vec{\tau}q)^2]$$



$$\lambda_\psi \propto \alpha_s^2$$

$$N_f = 2 : \vec{\tau} = (\sigma_1, \sigma_2, \sigma_3)$$

- Bosonisation (Hubbard-Stratonovich)  $\langle \sigma \rangle \neq 0$



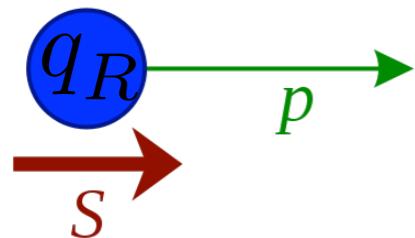
$$\frac{\lambda_\psi}{2} \int [(\bar{\psi}\psi)^2 + (i\bar{\psi}\gamma_5\vec{\tau}\psi)^2] = \frac{m_\sigma^2}{2} \int_x (\sigma^2 + \vec{\pi}^2) + i h \int_x \bar{\psi}(\sigma + i\gamma_5\vec{\tau}\vec{\pi})\psi$$

EOM( $\sigma$ )

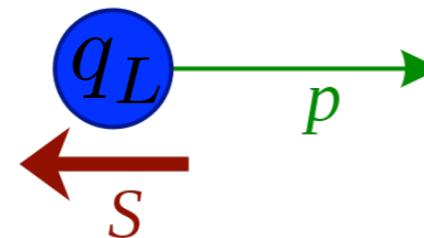
# Chiral symmetry breaking

- Chirality for massless particles

Right-handed:



Left-handed:



- Order parameter



$$\sigma = \langle \bar{q}q \rangle_{\text{chiral condensate}}$$

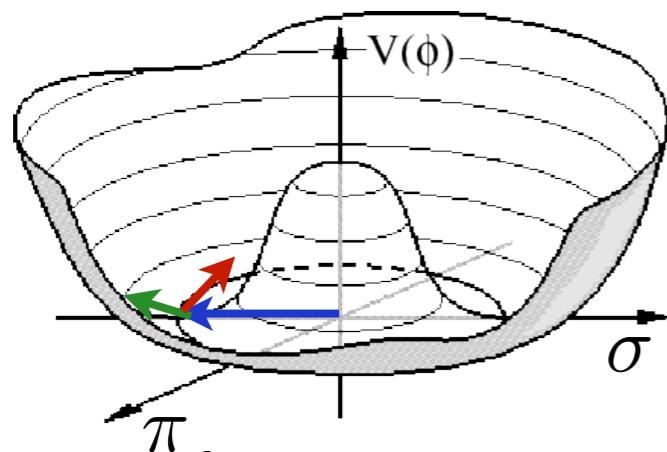
- Chiral symmetry

$$\sigma = 0$$

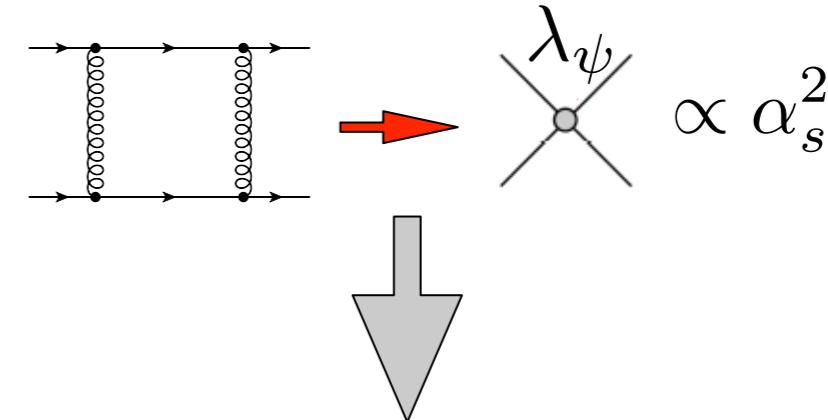
- Symmetry broken

$$\sigma \neq 0$$

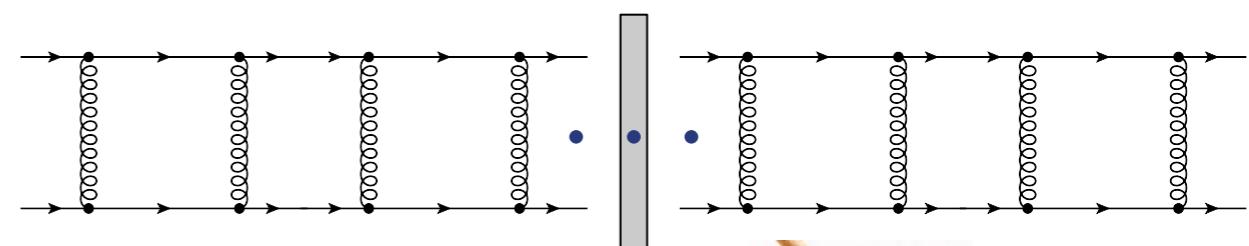
- Meson potential



## chiral symmetry



$$\int d^4x \lambda_\psi [(\bar{q}q)^2 - (\bar{q}\gamma_5 q)^2]$$



$$\langle \bar{q}q \rangle \neq 0$$



mass term:  $\langle \bar{q}q \rangle \bar{q}q$

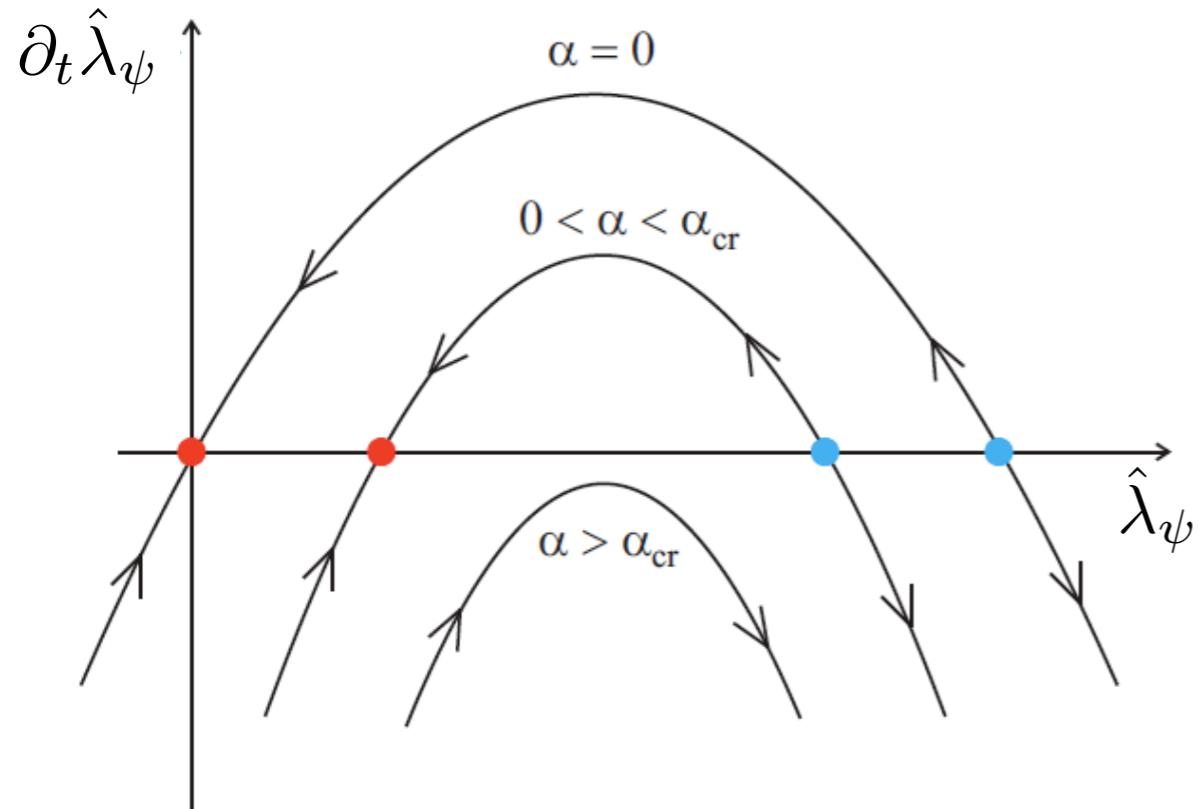
chiral symmetry broken

# Chiral symmetry breaking

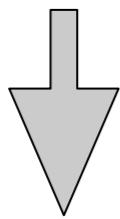
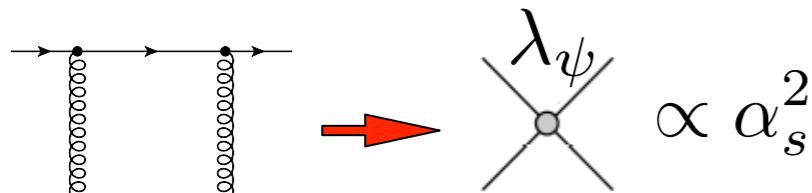
## A glimpse at chiral symmetry breaking in QCD within the FRG

Flow for four-fermion coupling  $\hat{\lambda}_\psi = \lambda_\psi k^2$  with infrared scale  $k$

$$k \partial_k \hat{\lambda}_\psi = 2 \hat{\lambda}_\psi + A \left( \frac{T}{k} \right) \hat{\lambda}_\psi^2 + B \left( \frac{T}{k} \right) \hat{\lambda}_\psi \alpha_s + C \left( \frac{T}{k} \right) \alpha_s^2 + \dots$$

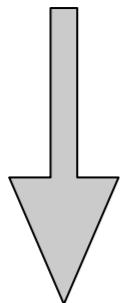


# Chiral symmetry breaking



$$\int d^4x \lambda_\psi [(\bar{q}q)^2 - (\bar{q}\gamma_5 q)^2]$$

$$\langle \bar{q}q \rangle \neq 0$$



mass term:  $\langle \bar{q}q \rangle \bar{q}q$

$$\alpha_s > \alpha_{s,\text{crit}}$$

Order parameter 

$$\sigma = \langle \bar{q}q \rangle$$

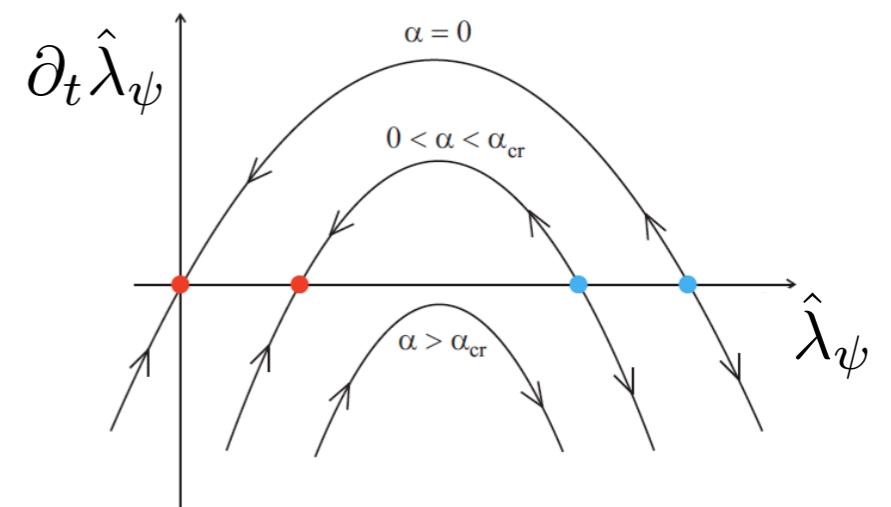
chiral condensate

- Chiral symmetry  $\sigma = 0$

- Symmetry broken  $\sigma \neq 0$

$$\sigma = 0$$

$$\sigma \neq 0$$



Chiral symmetry breaking directly sensitive to size of  $\alpha_s$

# Chiral symmetry breaking

## anomalous chiral symmetry breaking

- Axial U(1)

$$q \rightarrow e^{i\gamma_5 \alpha} q$$

with current

$$J_{5,\mu} \propto \bar{q} \gamma_5 \gamma_\mu q$$

classically

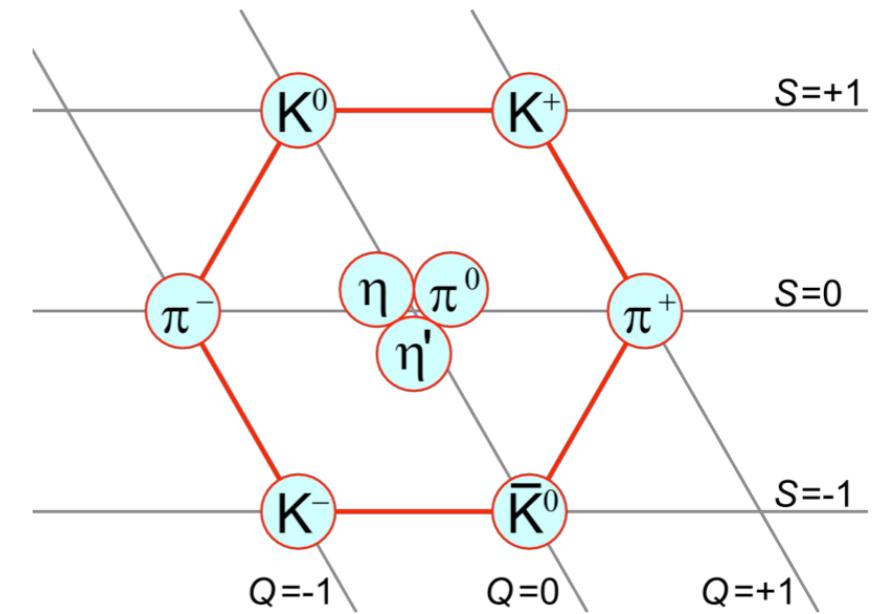
$$\partial_\mu J_{5,\mu} = 0$$

- Anomalous breaking of the axial U(1)

quantum

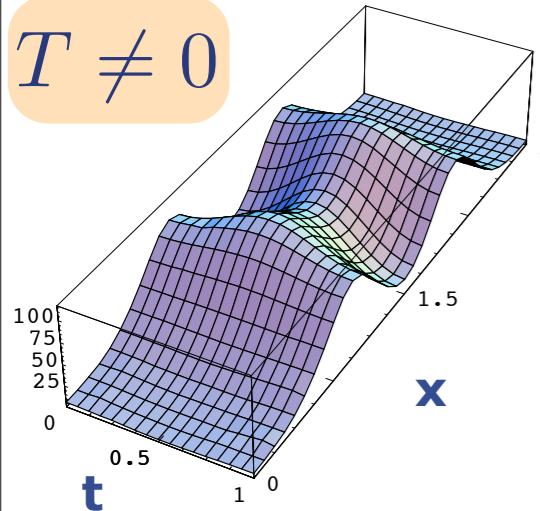
$$\partial_\mu \langle J_{5,\mu} \rangle = \frac{N_f}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \langle F_{\mu\nu}^a F_{\rho\sigma}^a \rangle$$

axial anomaly



Nonet of pseudoscalar mesons

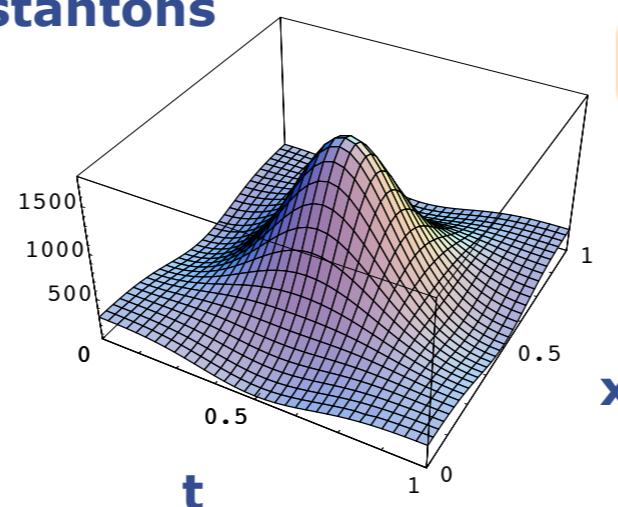
$$m_{\eta'} \simeq 960 \text{ MeV}$$



induced by instantons

$$-\frac{1}{2} \text{tr} F^2$$

SU(2)



$$T = 0$$

Plots from Ford, JMP '05

# Chiral symmetry breaking

## anomalous chiral symmetry breaking

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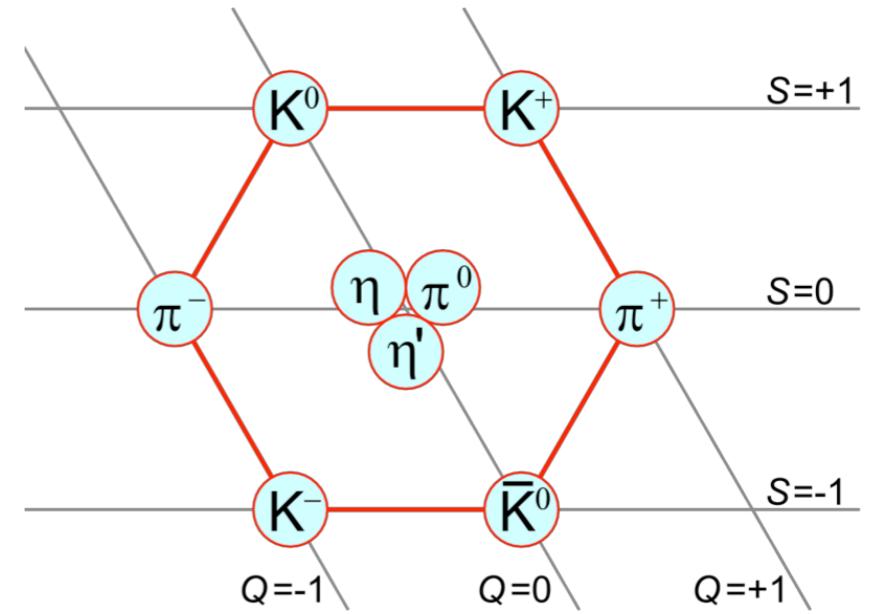
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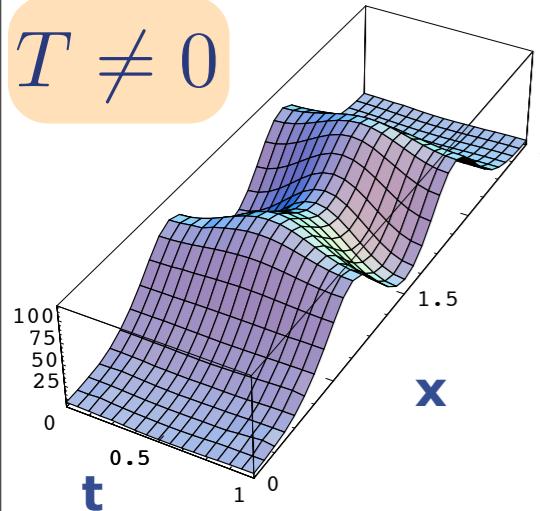


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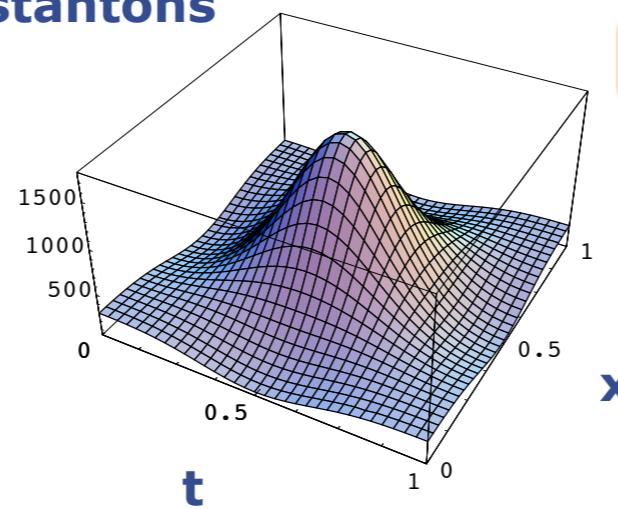
fermionic zero modes



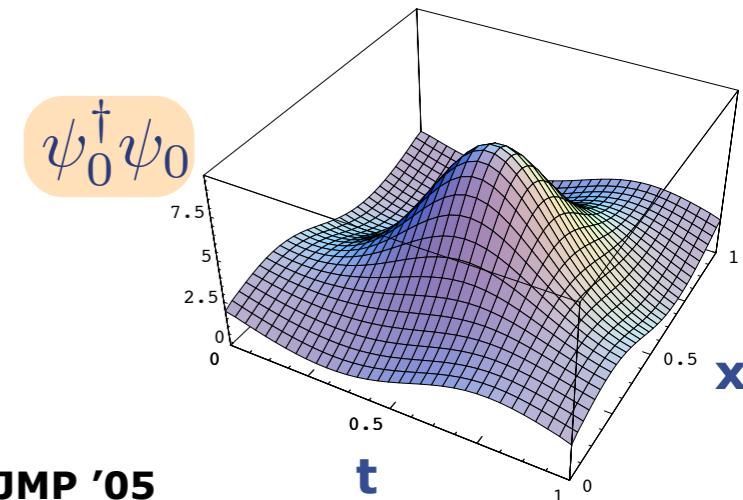
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# Chiral symmetry breaking

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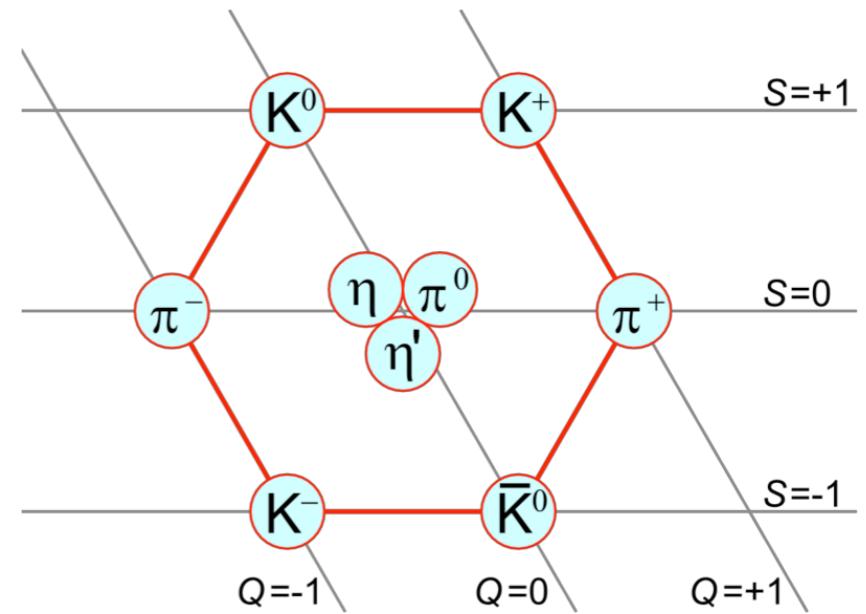
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axial anomaly



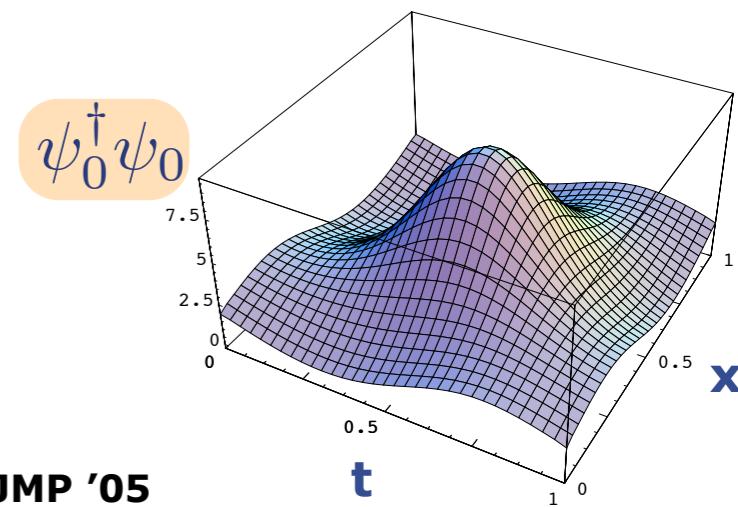
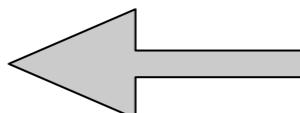
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't Hooft determinant

$$\Delta(k, \theta) \left( \det_{flav.} \bar{q}_L q_R + \det_{flav.} \bar{q}_R q_L \right)$$



Plots from Ford, JMP '05

# Chiral symmetry breaking

## anomalous chiral symmetry breaking

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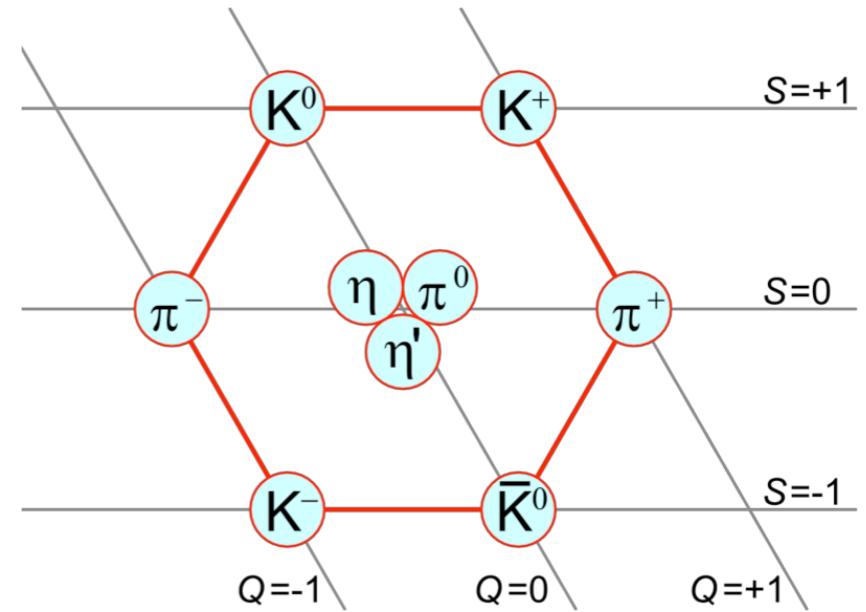
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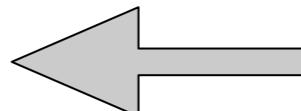
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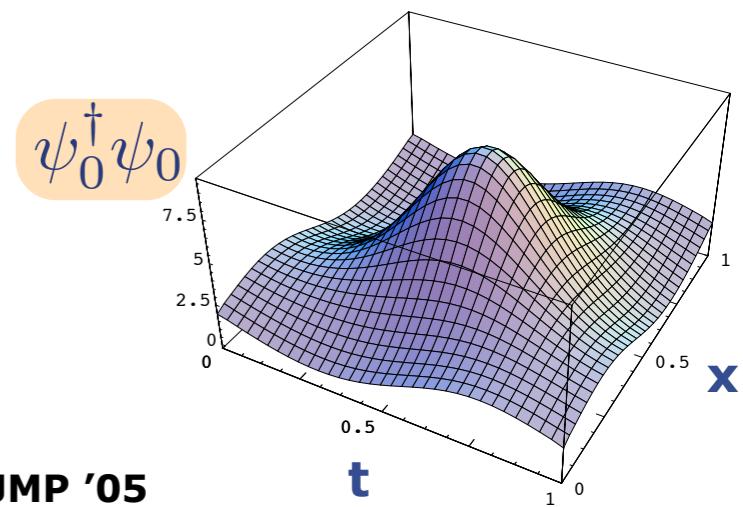
fermionic zero modes

't Hooft determinant

$$\Delta(k, \theta) \left( \det_{flav.} \bar{q}_L q_R + \det_{flav.} \bar{q}_R q_L \right)$$



$$\Delta(k, \theta) \propto (k^2 + c_k \Delta m_{\chi sb}^2)^{-\frac{3}{2} N_f + 2} e^{-2\pi/\alpha_{s,k}}$$



# Chiral symmetry breaking

## physical masses

chiral symmetry breaking:  $\Delta m_{\chi SB} \approx 400 \text{ MeV}$



up



charm



top



down



strange



bottom



2 light flavours, one heavy flavour 2+1

# **Functional Methods for QCD**

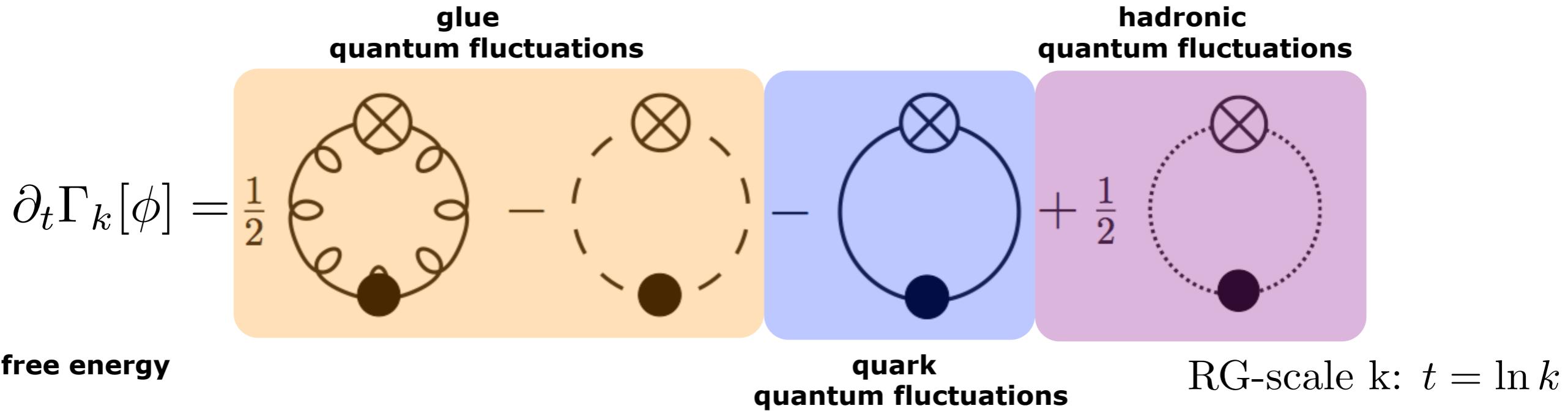
**FunMethods: FRG-DSE-2PI-...**

# **FRG for QCD**

**FunMethods: FRG-DSE-2PI-...**

# Functional Methods for QCD

JMP, AIP Conf. Proc. 1343 (2011)



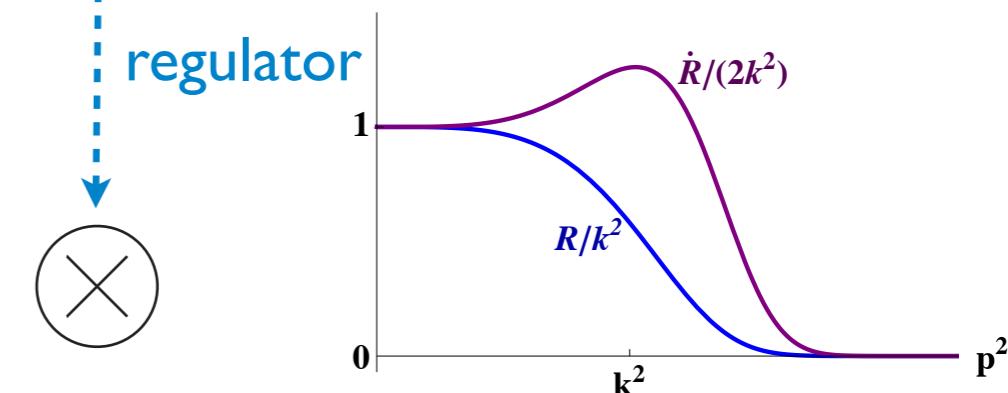
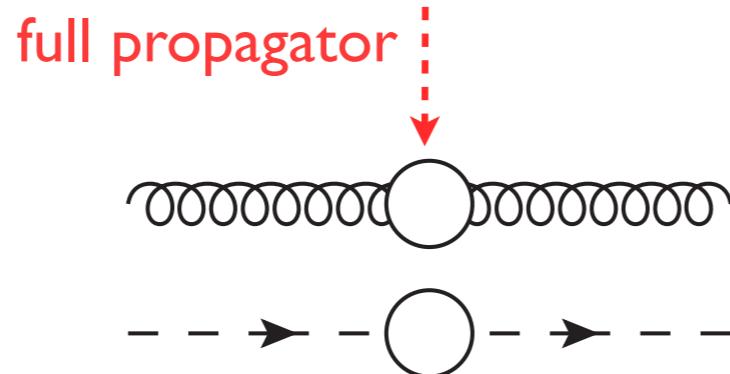
## Yang-Mills:

$$\partial_t \Gamma_k[A, \bar{c}, c] = \frac{1}{2} \text{Tr} \left\{ \frac{1}{\Gamma^{(2)}[A, \bar{c}, c] + R_k} \partial_t R_k \right\} - \partial_t C_k$$

$\downarrow$

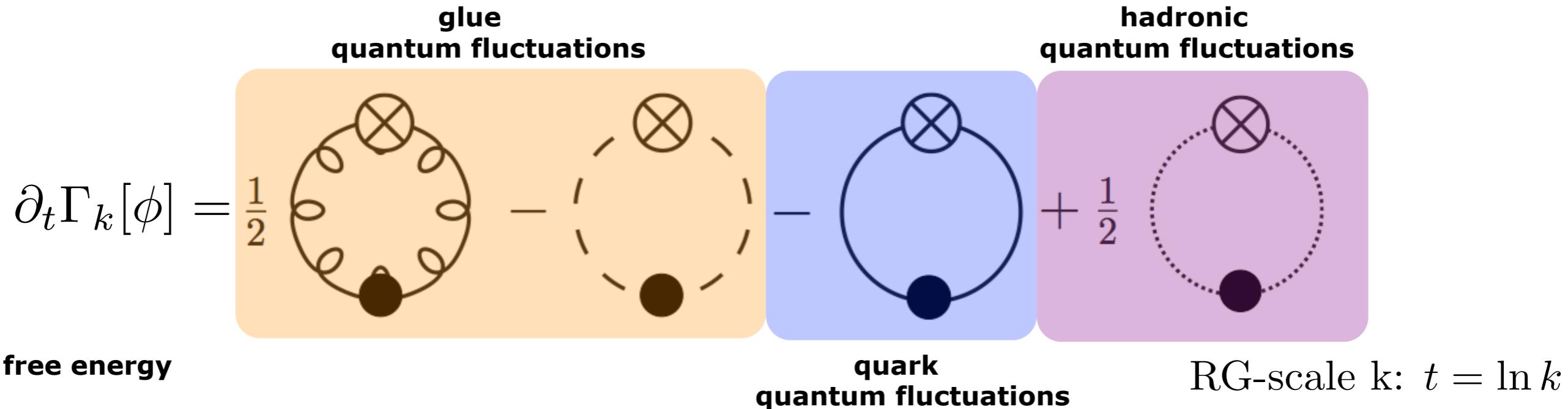
$\partial_t = k \partial_k$

by L. Fister



# Functional Methods for QCD

JMP, AIP Conf.Proc. 1343 (2011)



▪ **Gluons have cost us decades**

▪ **Fermions are straightforward** though 'physically' complicated

- no sign problem
- chiral fermions

▪ **bound states via dynamical hadronisation**

**Complementary to lattice!**

# Functional Methods for QCD

JMP, AIP Conf.Proc. 1343 (2011)

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left( \text{solid circle with } \otimes \text{ and wavy lines} - \text{dashed circle with } \otimes \text{ and wavy lines} \right)$$

glue  
quantum fluctuations

free energy

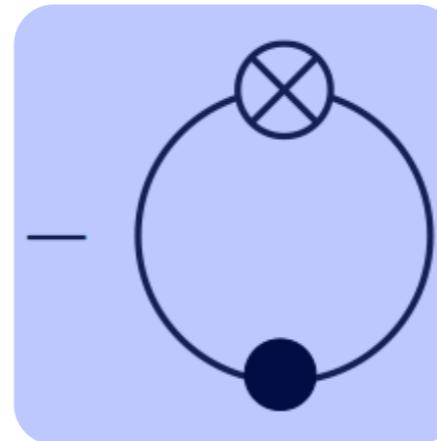
Yang-Mills theory

# Functional Methods for QCD

JMP, AIP Conf.Proc. 1343 (2011)

$$\partial_t \Gamma_k[\phi] =$$

**free energy**



**quark  
quantum fluctuations**

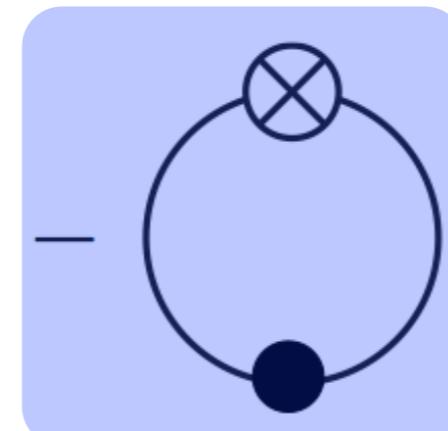
**NJL-type models**

# Functional Methods for QCD

JMP, AIP Conf.Proc. 1343 (2011)

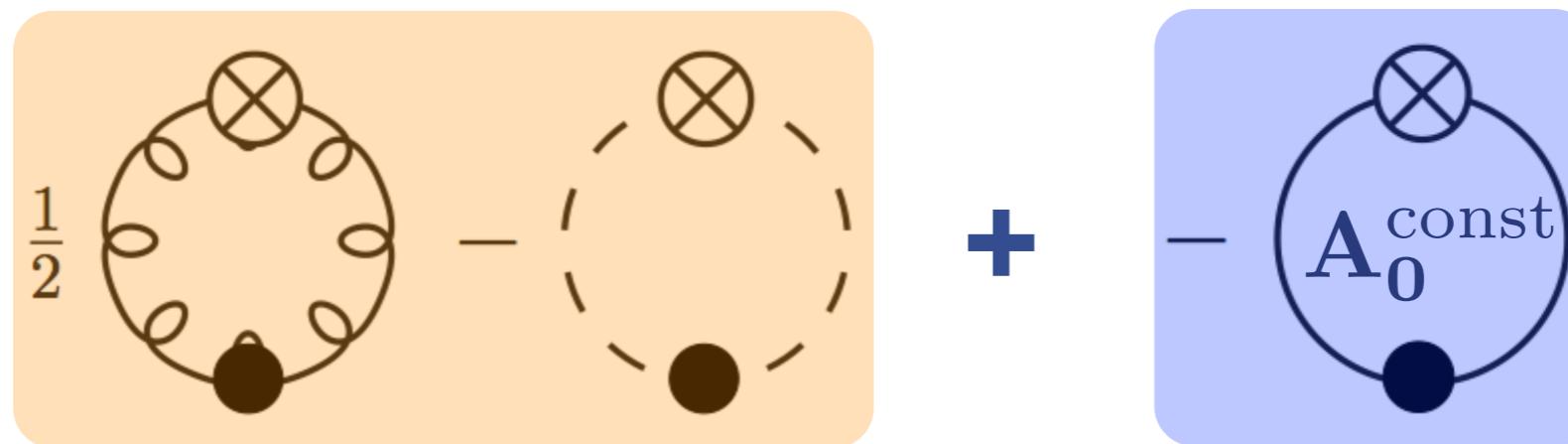
$$\partial_t \Gamma_k[\phi] =$$

**free energy**



**quark  
quantum fluctuations**

**NJL-type models**



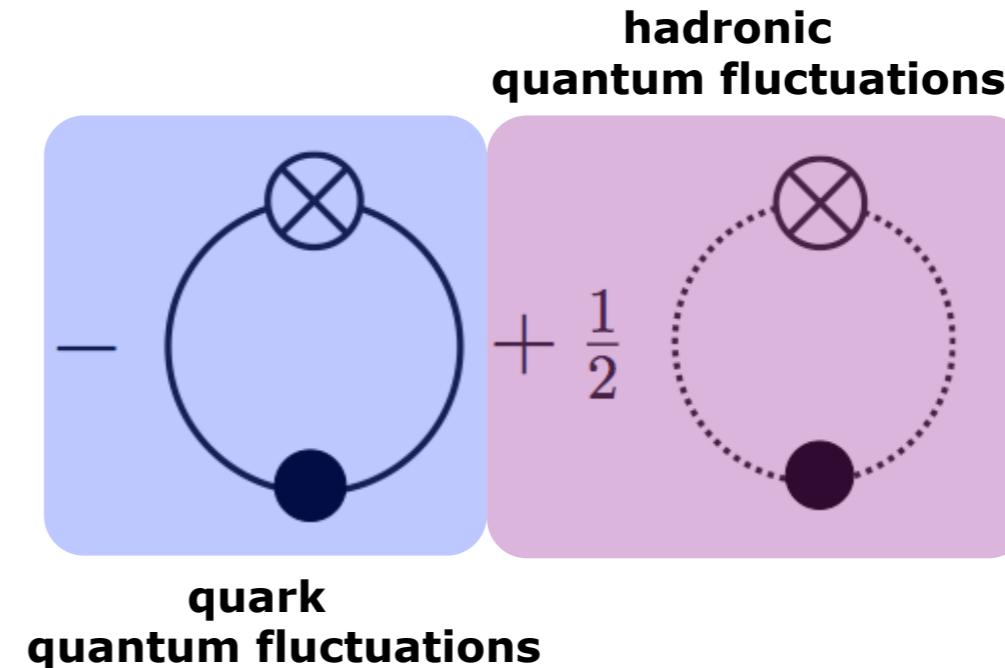
**PNJL models**

# Functional Methods for QCD

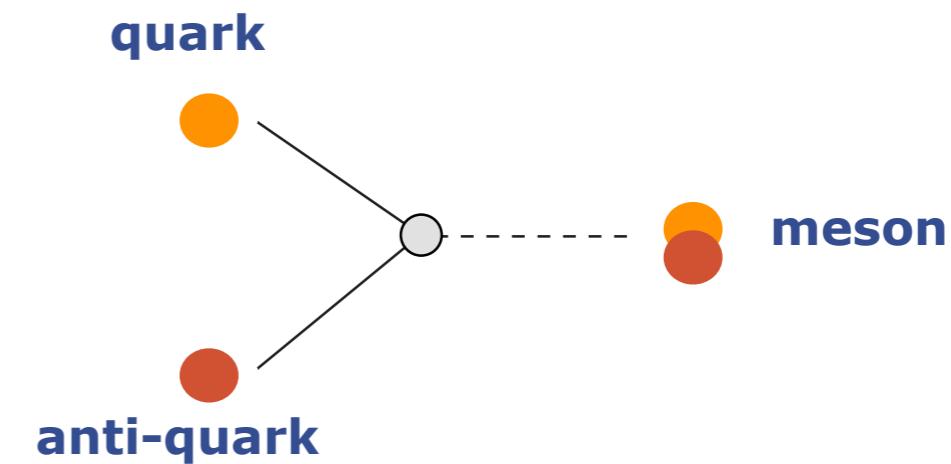
JMP, AIP Conf.Proc. 1343 (2011)

$$\partial_t \Gamma_k[\phi] =$$

free energy



## Quark-hadron models



- **bound states via dynamical hadronisation**

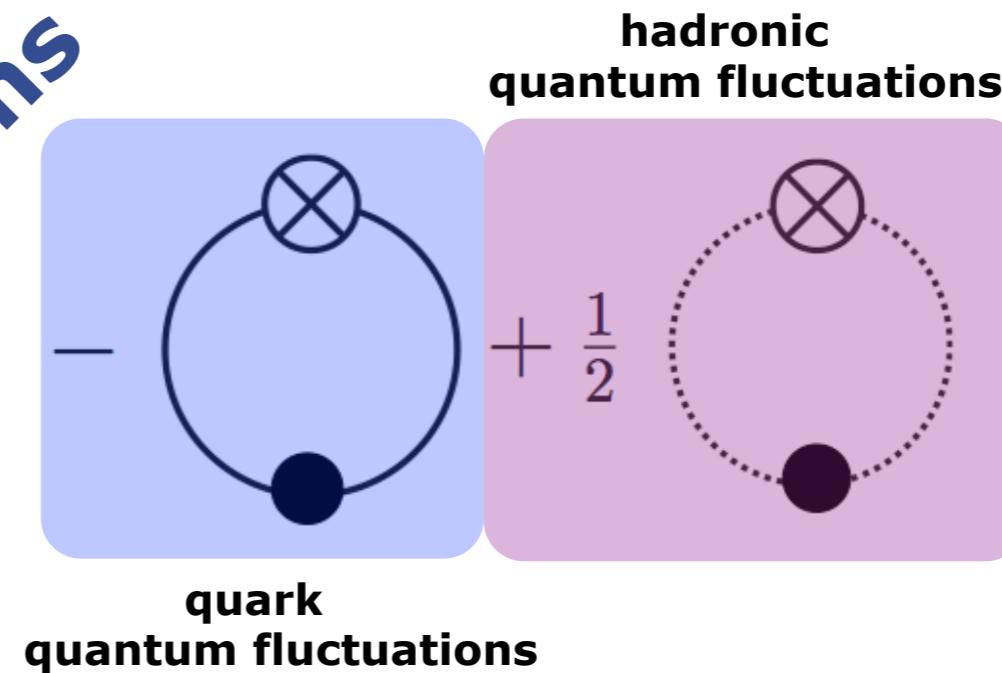
# Functional Methods for QCD

JMP, AIP Conf.Proc. 1343 (2011)

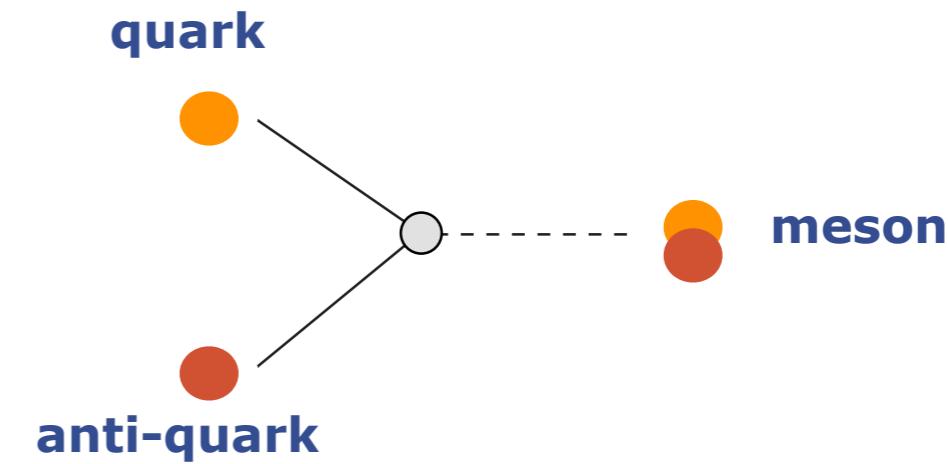
$$\partial_t \Gamma_k[\phi] =$$

free energy

benchmark in ultracold atoms



## Quark-hadron models



- bound states via dynamical hadronisation

# Functional Methods for QCD

JMP, AIP Conf.Proc. 1343 (2011)

$$\partial_t \Gamma_k[\phi] =$$

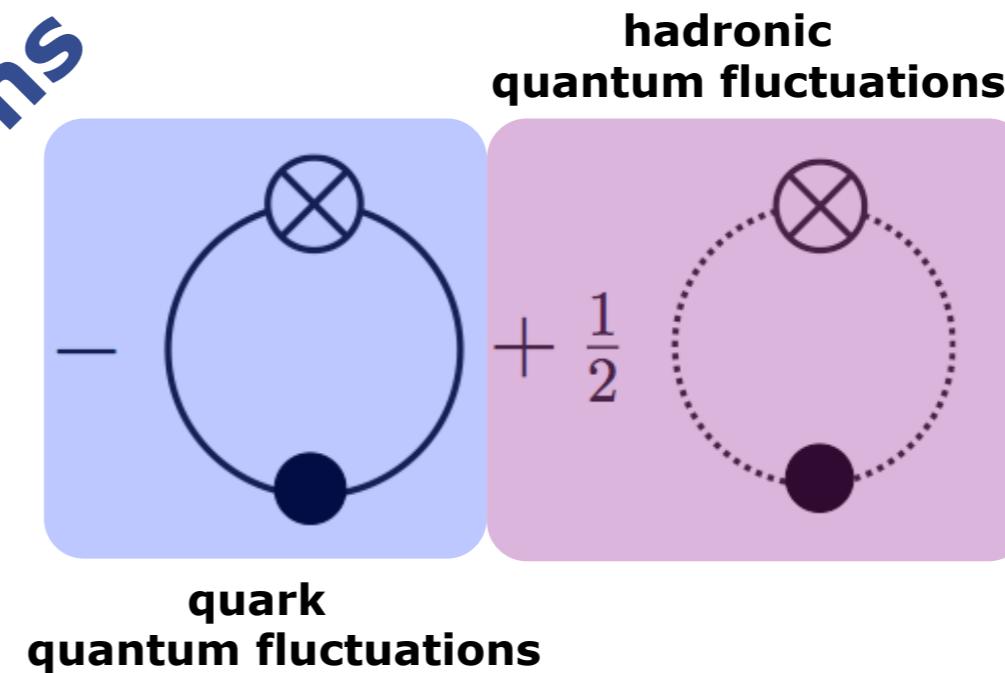
free energy



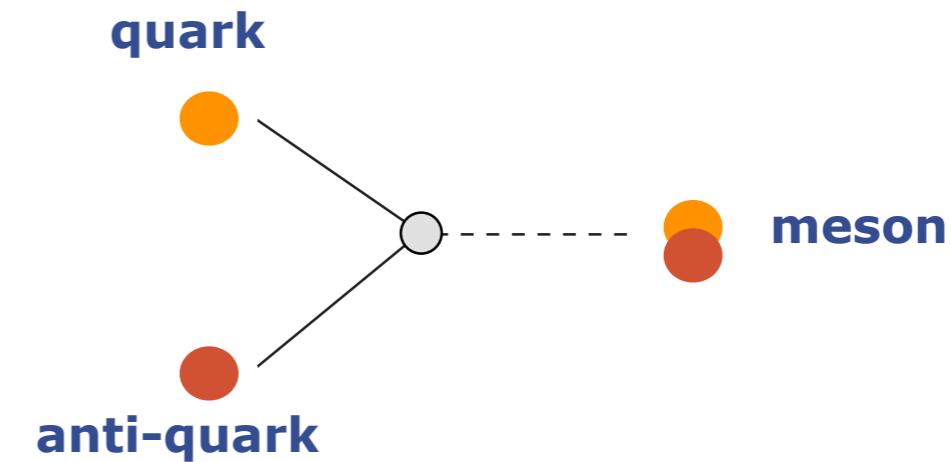
benchmark in ultracold atoms

'You name it, we do it'

John Thomas  
QGP meets cold atoms-Episode III



## Quark-hadron models



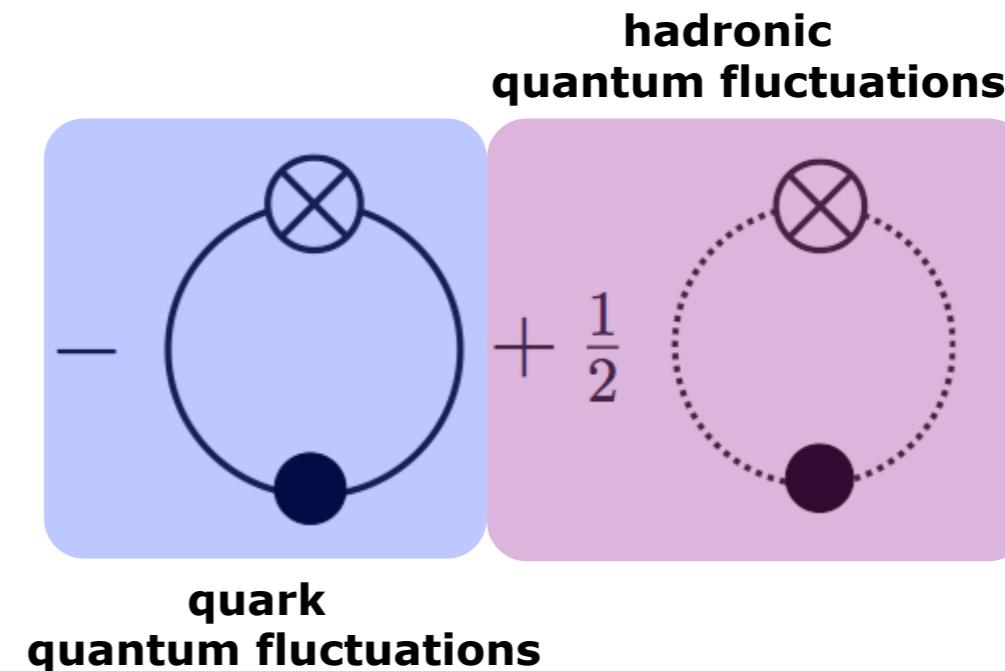
- bound states via dynamical hadronisation

# Functional Methods for QCD

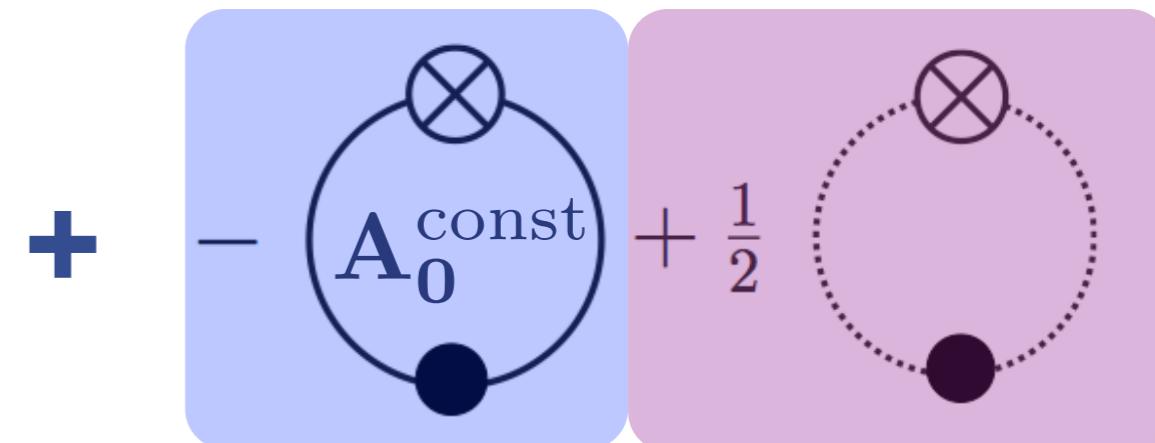
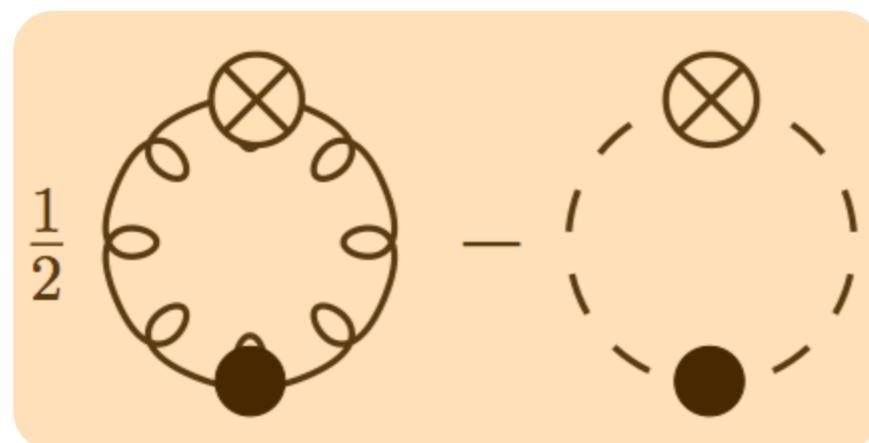
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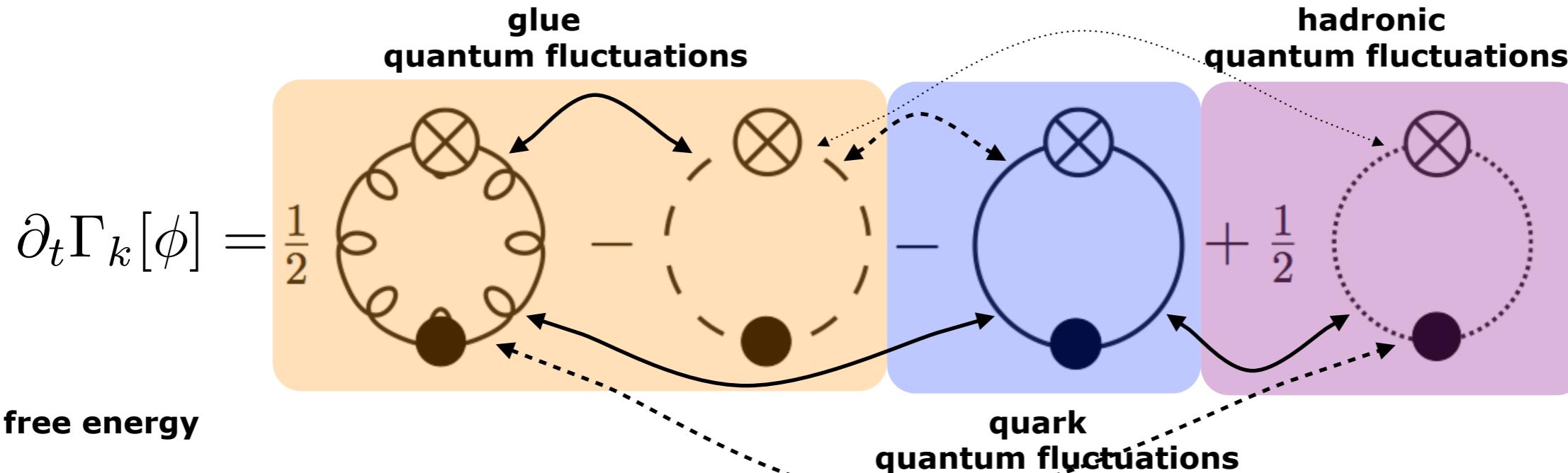
## Quark-hadron models



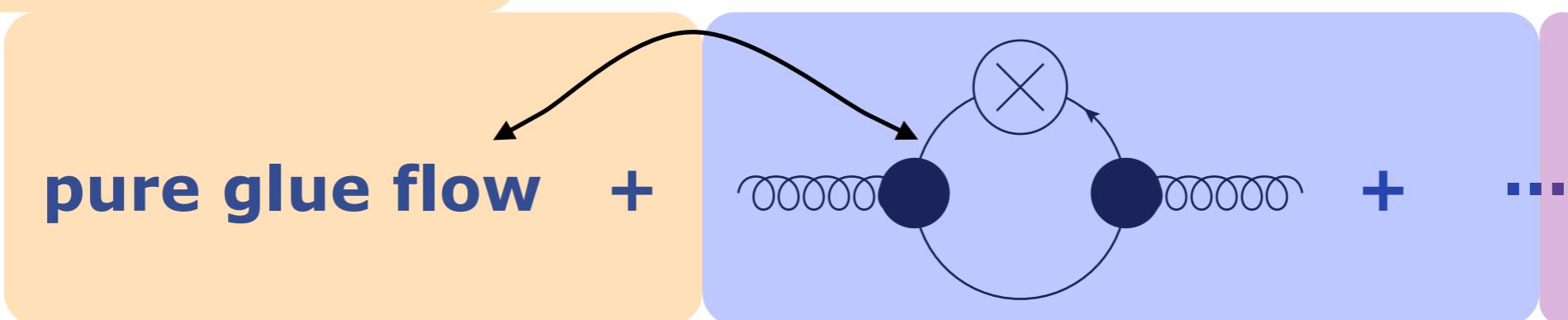
## PQM models

# Functional Methods for QCD

JMP, AIP Conf.Proc. 1343 (2011)



flow of gluon propagator

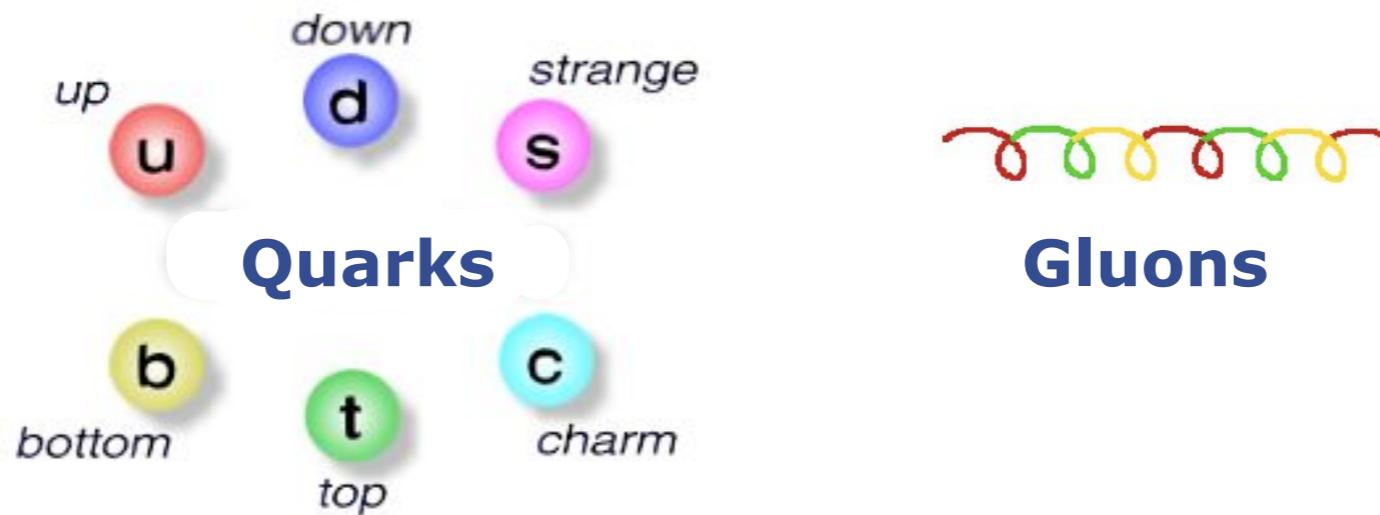


Naturally incorporates PQM/PNJL models as specific low order truncations

# Dynamical hadronisation

Gies, Wetterich '01  
JMP '05

Flörchinger, Wetterich '09

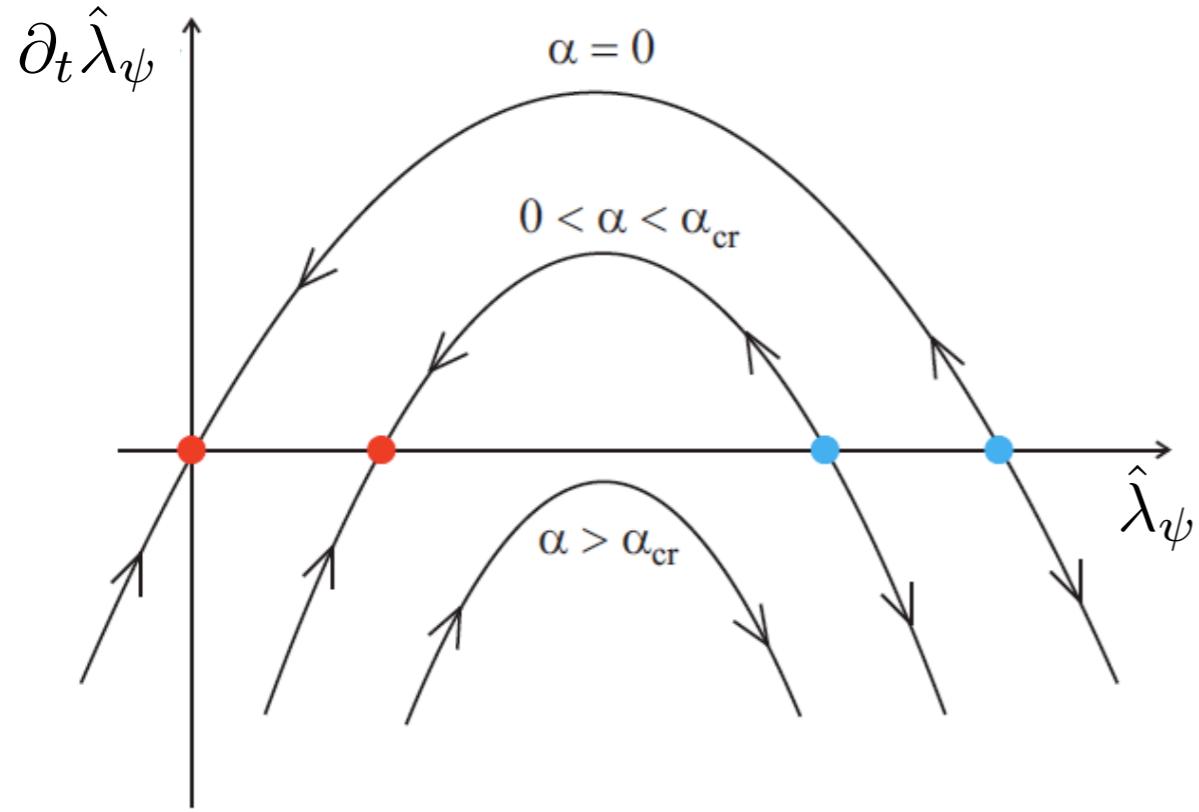


# Chiral symmetry breaking

## A glimpse at chiral symmetry breaking in QCD within the FRG

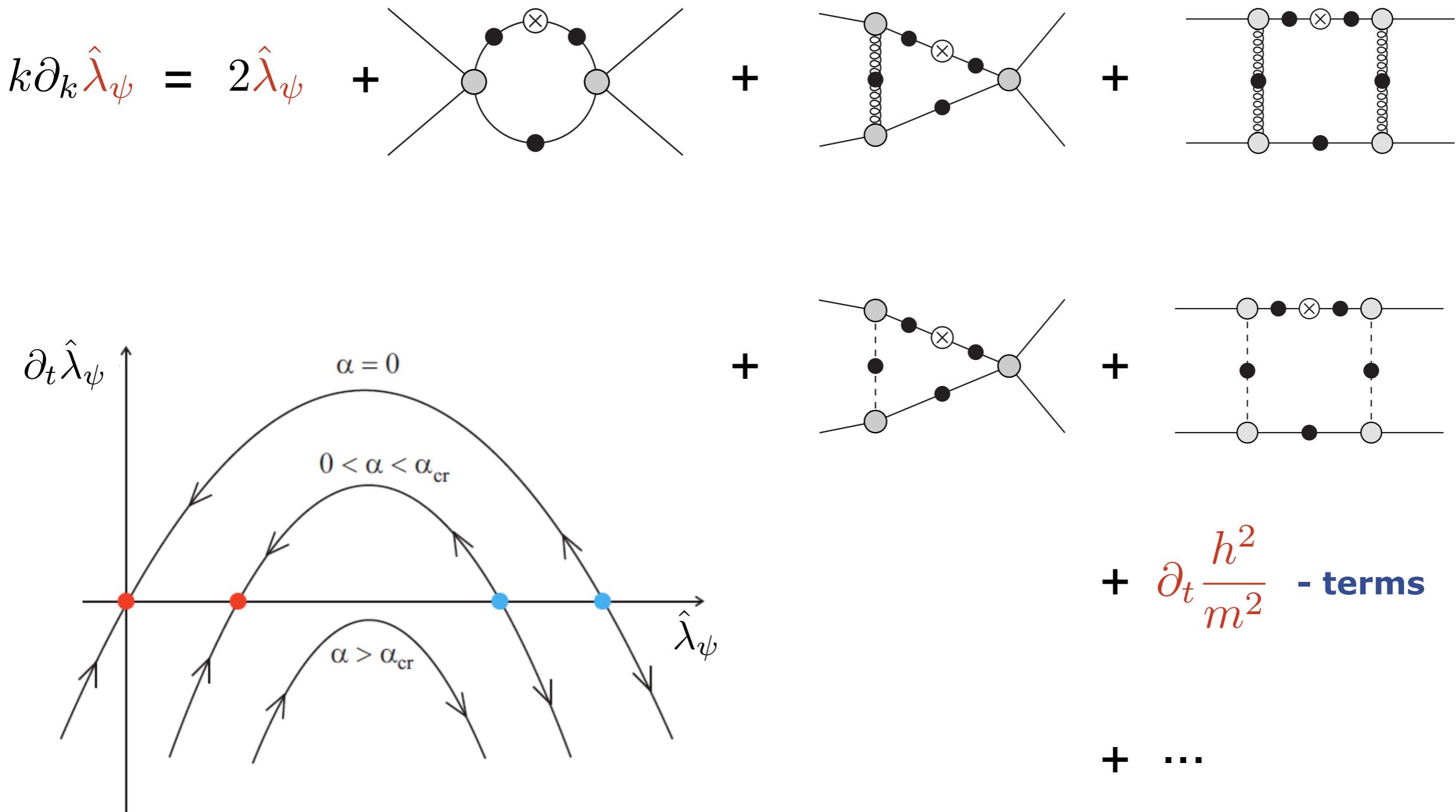
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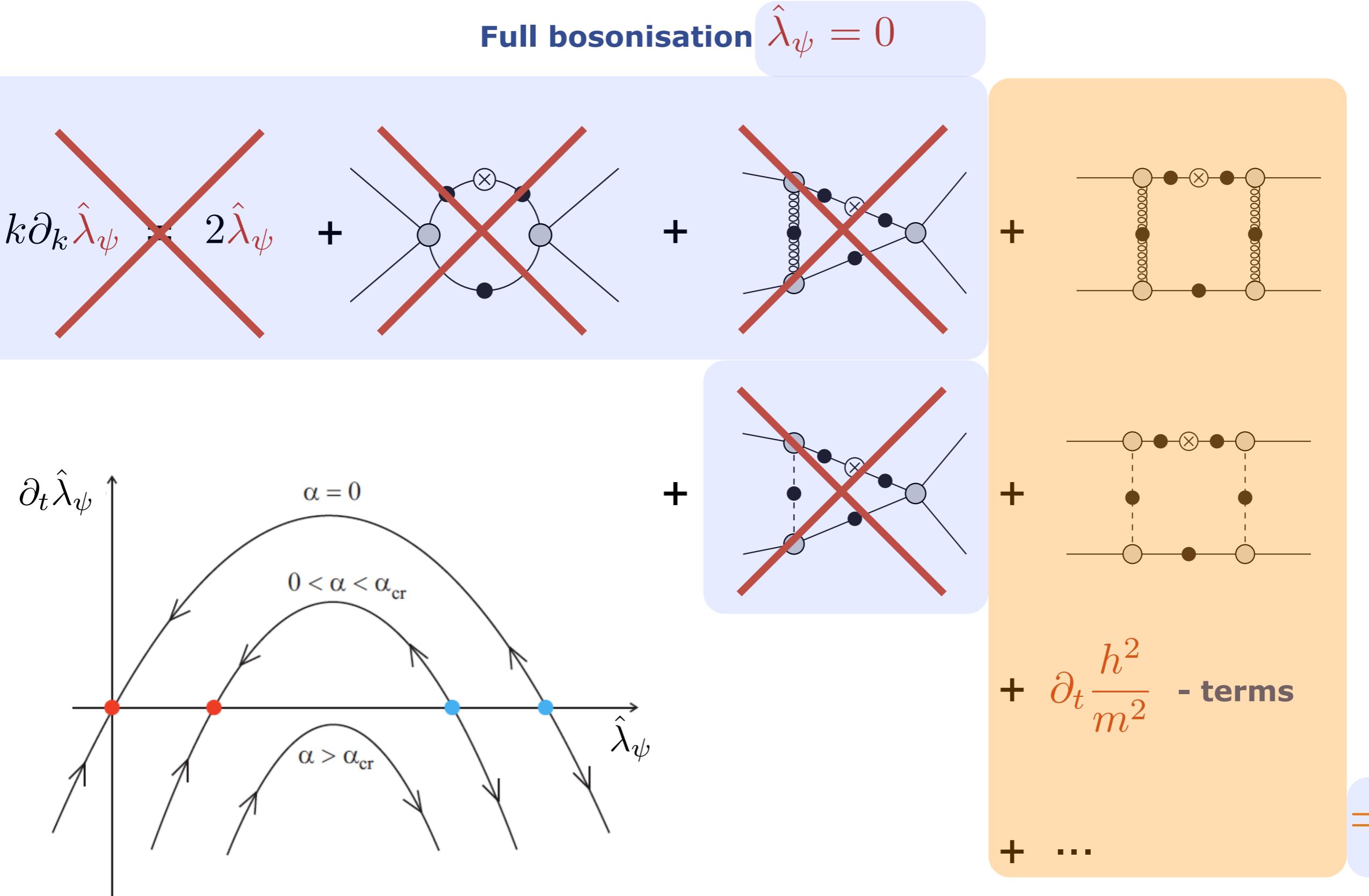


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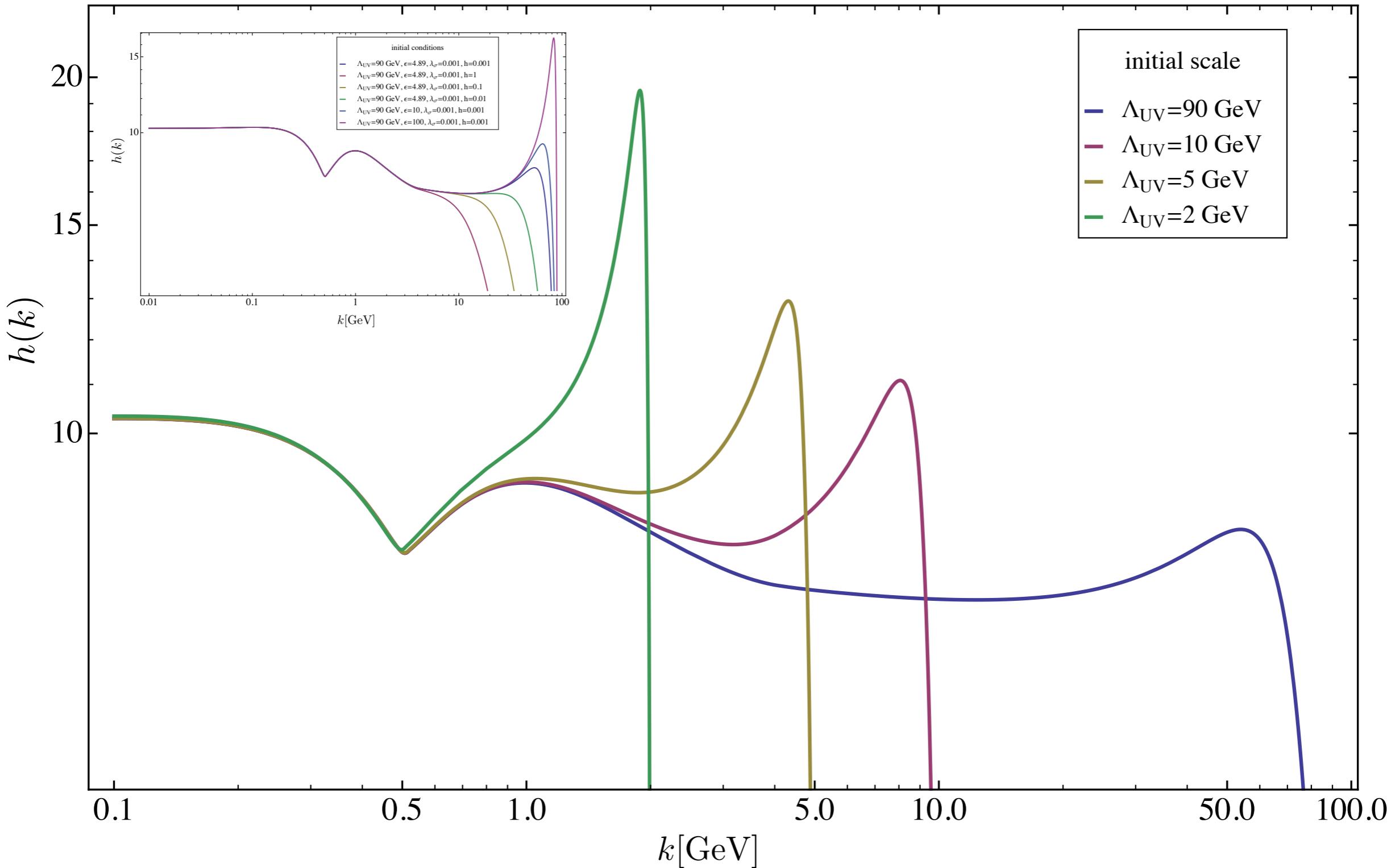
# Dynamical hadronisation



# Dynamical hadronisation

Full bosonisation  $\hat{\lambda}_\psi = 0$

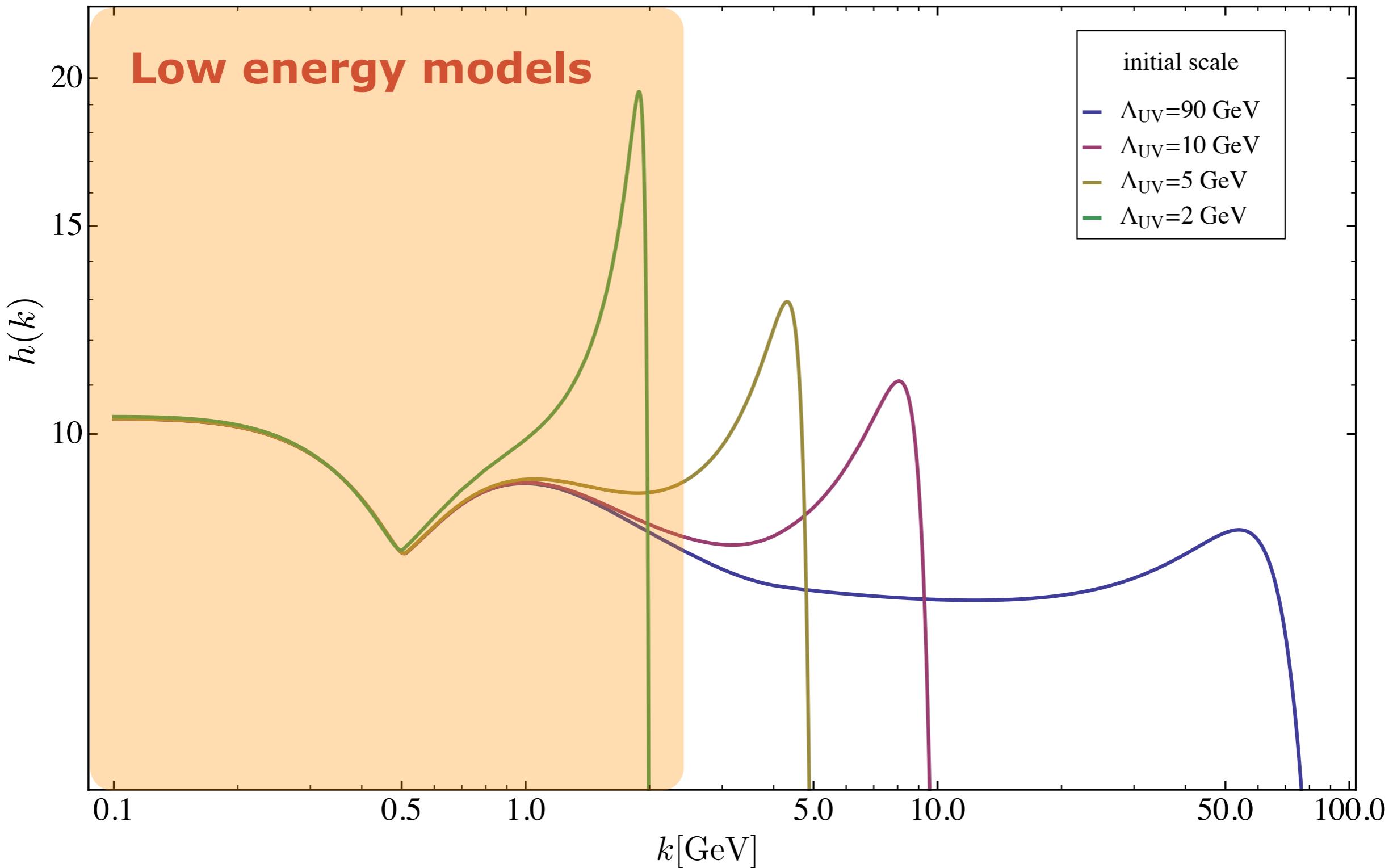
Braun, Fister, Haas, JMP,  
in prep



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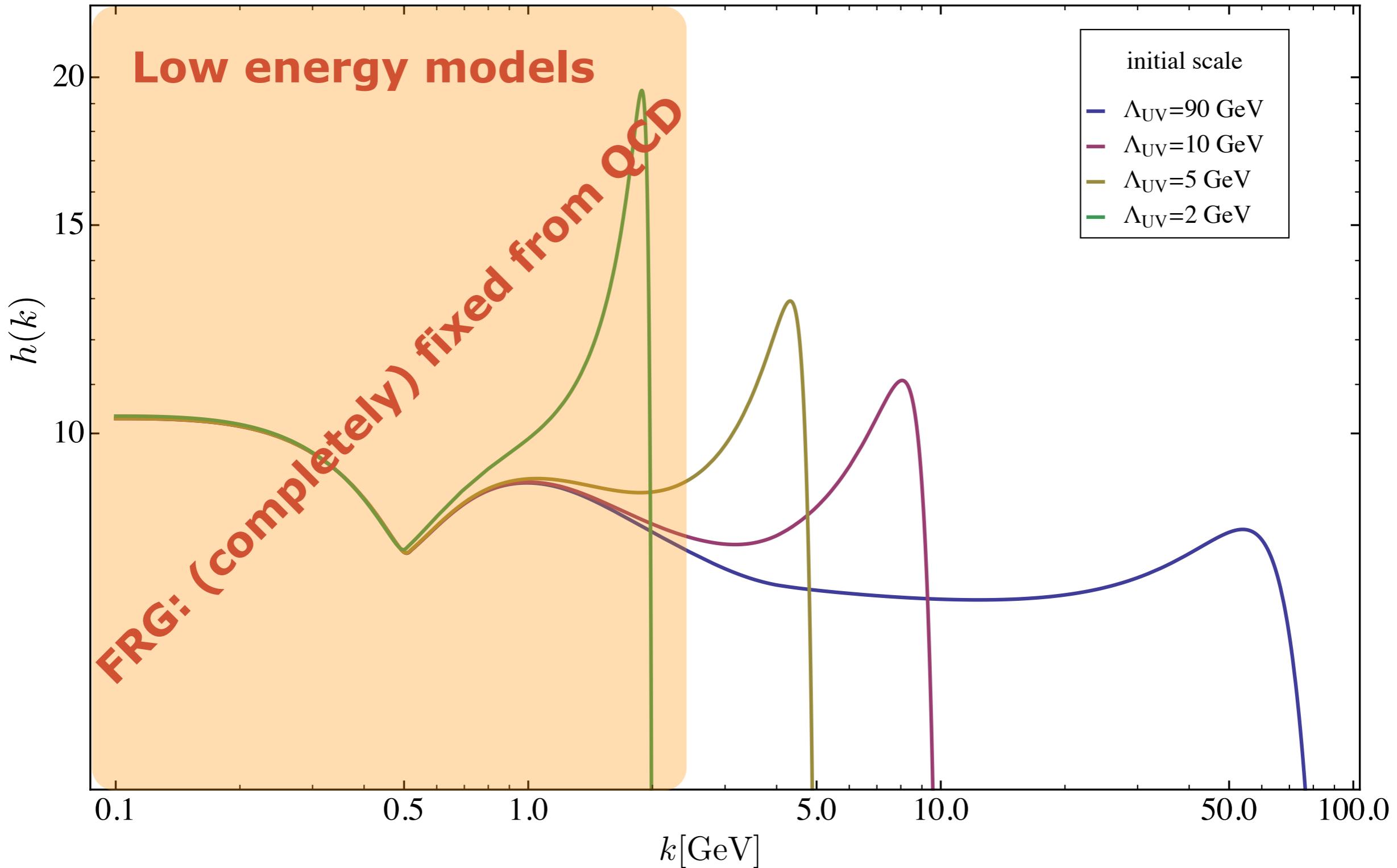
Braun, Fister, Haas, JMP,  
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# Dynamical hadronisation

Full bosonisation  $\hat{\lambda}_\psi = 0$

Braun, Fister, Haas, JMP,  
in prep



# Gauge symmetry, gauge fixing & regularisation

Corfu, ERG2010, JMP



**Gluons**

# Gauge symmetry

---

- **non-Abelian gauge symmetry:**  $U = e^{i\omega} \in SU(N)$

$$\delta_\omega : gA_\mu \rightarrow U^{-1}gA_\mu U - iU^{-1}\partial_\mu U$$

- **classical action is invariant under gauge transformations**

$$S_{\text{YM}}[A^U] = S_{\text{YM}}[A] \quad \text{with} \quad S_{\text{YM}}[A] = \frac{1}{2} \int_x \text{tr } F_{\mu\nu}^2$$

**and field strength**

$$F_{\mu\nu}^a = \partial_\mu A_\nu - \partial_\nu A_\mu + igf^{abc}A^b A^c$$

- **gauge symmetry**  **redundancy in field degrees of freedom**
- **gauge fixing (necessarily breaking of gauge invariance)**
- **gauge invariant variables (necessarily non-local)**

# Gauge fixing

---

- **gauge fixing and ghost term (Jacobian), e.g. covariant gauge**  $\partial_\mu A_\mu = 0$

$$\frac{1}{2\xi} \int_x (\partial A)^2 + \int_x \bar{C} \cdot \partial D \cdot C$$

- **Slavnov-Taylor identities for effective action**  $\Gamma[\phi]$  **with**  $\phi = (A, C, \bar{C})$

$$\delta_\omega(\Gamma - S_{\text{cl}}) + \text{loops} = 0$$

- **BRST Master equation with anti-fields**

$$\int_x \frac{\delta \Gamma}{\delta \phi} \frac{\delta \Gamma}{\delta \phi^*} = 0$$

**EoM for anti-fields**  $\phi^*$  **are the symmetry transformations**

# Regularisation

- **Cut-off term**

$$\frac{1}{2} \int_x A \cdot R_k^A \cdot A + \int_x \bar{C} \cdot R_k^C \cdot C$$

- **Slavnov-Taylor identities for effective action  $\Gamma[\phi]$**

$$\delta_\omega(\Gamma - S_{\text{cl}}) + \text{loops} = \frac{1}{2} \text{Tr} \left( U R_k U^{-1} \right) G_k[\phi]$$

Bonini, Ellwanger, Litim,  
Marchesini, Morris, JMP, Reuter,  
Weber, Wetterich, ....

- **BRST Master equation with anti-fields**

$$\int_x \frac{\delta \Gamma}{\delta \phi} \frac{\delta \Gamma}{\delta \phi^*} = \Delta \Gamma[\phi, \phi^*]$$

**STI/ME** → **modified STI/modified ME**

Igarashi, Itoh, Itou,  
Kugo, Sonoda, ...

# Gauge invariant flows

- **covariant cut-off**

Morris '99

Morris, Rosten'06

Aronne, Morris, Rosten'06

Rosten'10

$$\text{tr} \int F_{\mu\nu} c^{-1}(D^2/k^2) F_{\mu\nu}$$

Polchinski flow

- **SU(N)  $\rightarrow$  spontaneously broken SU(N|N)**

- **Pauli-Villars fields**

Locality?

# Gauge invariant flows

- **covariant cut-off**

Morris '99

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$$\text{tr} \int F_{\mu\nu} c^{-1}(D^2/k^2) F_{\mu\nu}$$

Polchinski flow

- **SU(N)  $\rightarrow$  spontaneously broken SU(N|N)**

- **Pauli-Villars fields**

Complicated

# Gauge invariant flows

- **covariant cut-off**

Morris '99

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Rosten'10

$$\text{tr} \int F_{\mu\nu} c^{-1}(D^2/k^2) F_{\mu\nu}$$

Polchinski flow

- **SU(N)  $\rightarrow$  spontaneously broken SU(N|N)**

- **Pauli-Villars fields**

## Remarks

- **Polchinski flow: well-suited for formal developments**

- **Wetterich flow: well-suited for numerics**

# Gauge invariant flows

---

- covariant cut-off

Morris '99

Morris, Rosten'06

Aronne, Morris, Rosten'06

Rosten'10

$$\text{tr} \int F_{\mu\nu} c^{-1}(D^2/k^2) F_{\mu\nu}$$

- $\text{SU}(N)$   spontaneously broken  $\text{SU}(N|N)$

- Pauli-Villars fields

## Results

- two-loop YM beta function & one-loop QCD beta function

Manifestly gauge invariant

- one loop computation for thin Wilson loops

- speculation of infrared slavery scenario

within ?renormalised? strong coupling expansion

# Gauge invariant flows

---

- **geometrical approach**

Branchina, Meissner, Veneziano '03  
JMP '03

$$\phi_\mu(A) = \bar{A}_\mu + a_\mu + O(a^2)$$

- **effective action only depends on gauge invariant part of  $\phi$**
- **Non-locality & modified Nielsen identity** JMP '03

# Gauge invariant flows

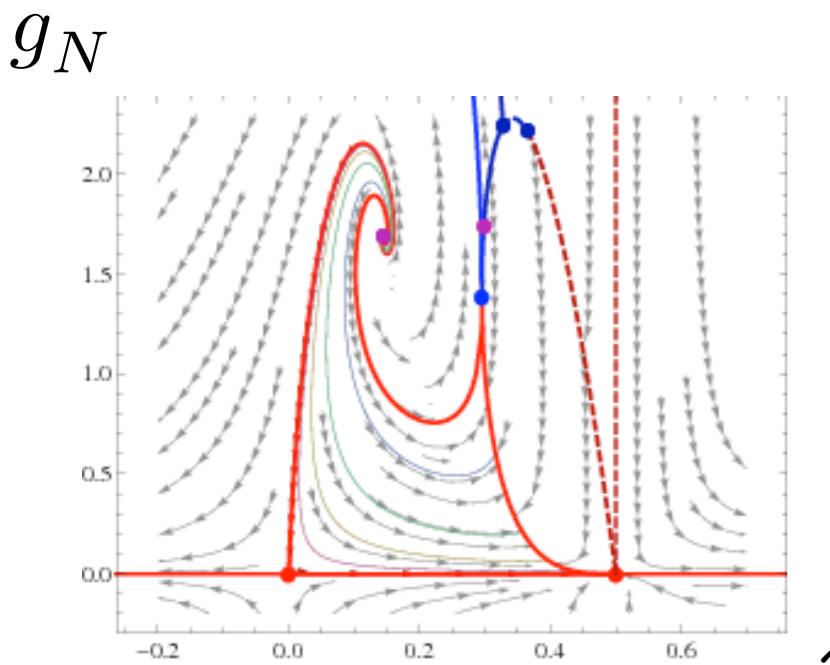
- **geometrical approach**

Branchina, Meissner, Veneziano '03  
JMP '03

$$\phi_\mu(A) = \bar{A}_\mu + a_\mu + O(a^2)$$

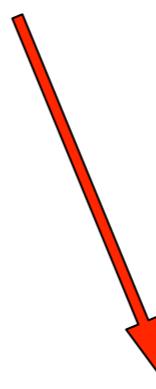
- **effective action only depends on gauge invariant part of  $\phi$**

- **Non-locality & modified Nielsen identity** JMP '03



## Results

UV-IR stable gravity  
Donkin, JMP, '12



bi-metric expansion

# Gauge invariant flows

---

- **background field approach**

Reuter, Wetterich '94

$$\phi_\mu(A) = \bar{A}_\mu + a_\mu$$

- **linear split in  $\phi$**

- **background gauge invariance & modified fluctuation STIs**

Reuter, Wetterich '97  
Freire, Litim, JMP '00

# Gauge invariant flows

---

- **background field approach**

Reuter, Wetterich '94

$$\phi_\mu(A) = \bar{A}_\mu + a_\mu$$

- **linear split in  $\phi$**

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Reuter, Wetterich '97  
Freire, Litim, JMP '00

→ **gauge-fixed setting**

**see gravity talks**

# Gauge invariant flows

---

- **background field approach**

Reuter, Wetterich '94

$$\phi_\mu(A) = \bar{A}_\mu + a_\mu$$

- **linear split in  $\phi$**

- **background gauge invariance & modified fluctuation STIs**

Reuter, Wetterich '97  
Freire, Litim, JMP '00

## Remarks

- **'single metric' truncation in YM**

**one loop beta function non-universal!**

Litim, JMP '02

- **'Einstein-Hilbert' truncation in YM**

**no confinement!**

see confinement section

# Gauge invariant flows

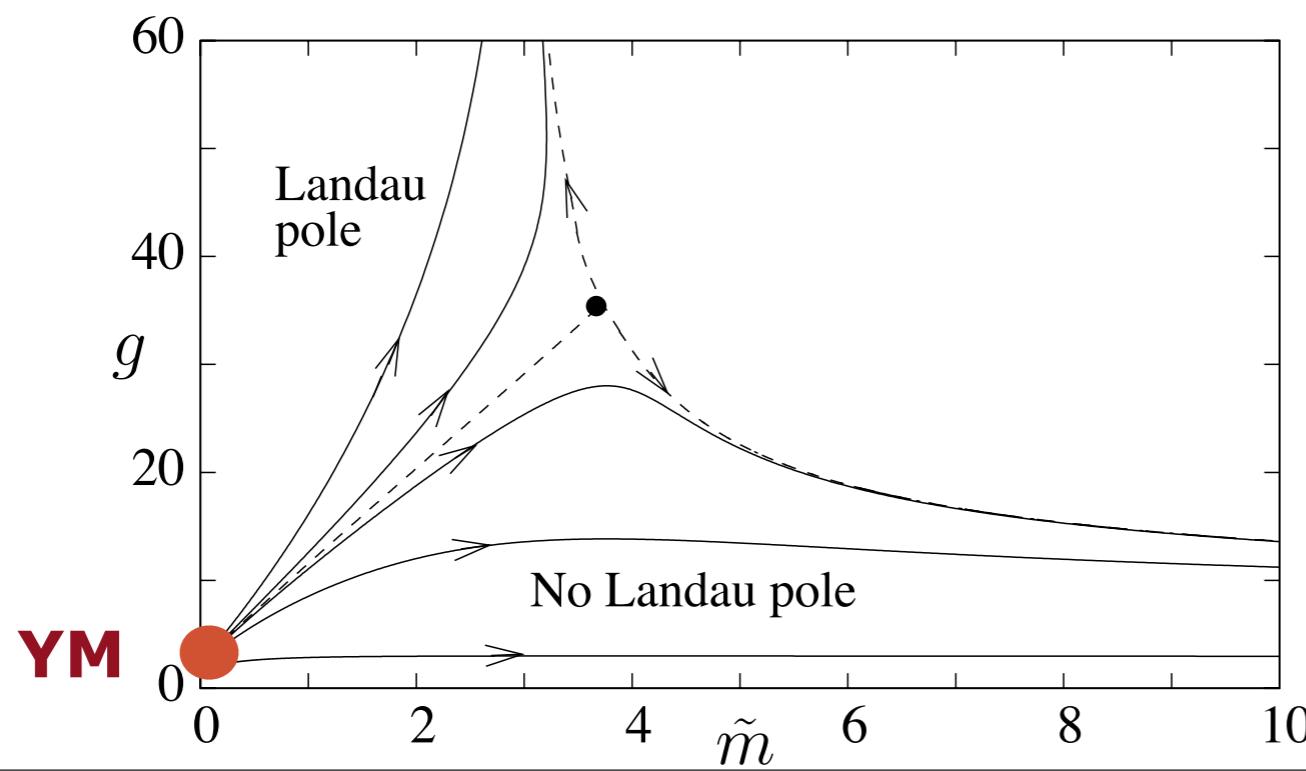
Curci-Ferrari-Delbourgo-Jarvis model

Tissier, Wschebor '08, '10

- Massive deformation of Yang-Mills theory

- Relation to Yang-Mills theory?

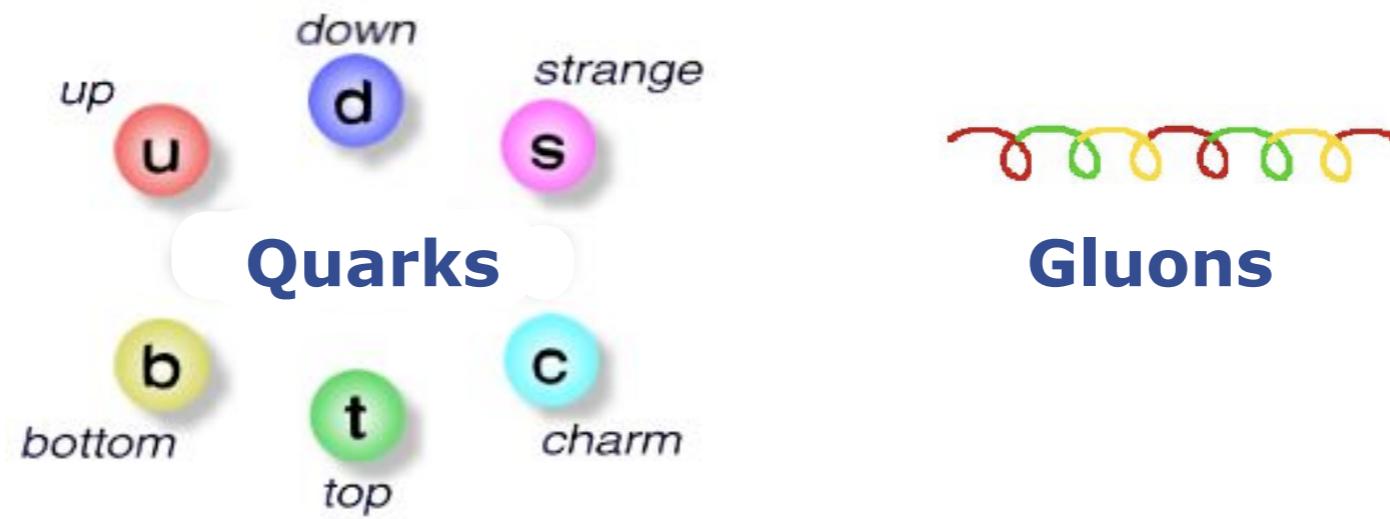
- 'Lifting the Gribov ambiguity'



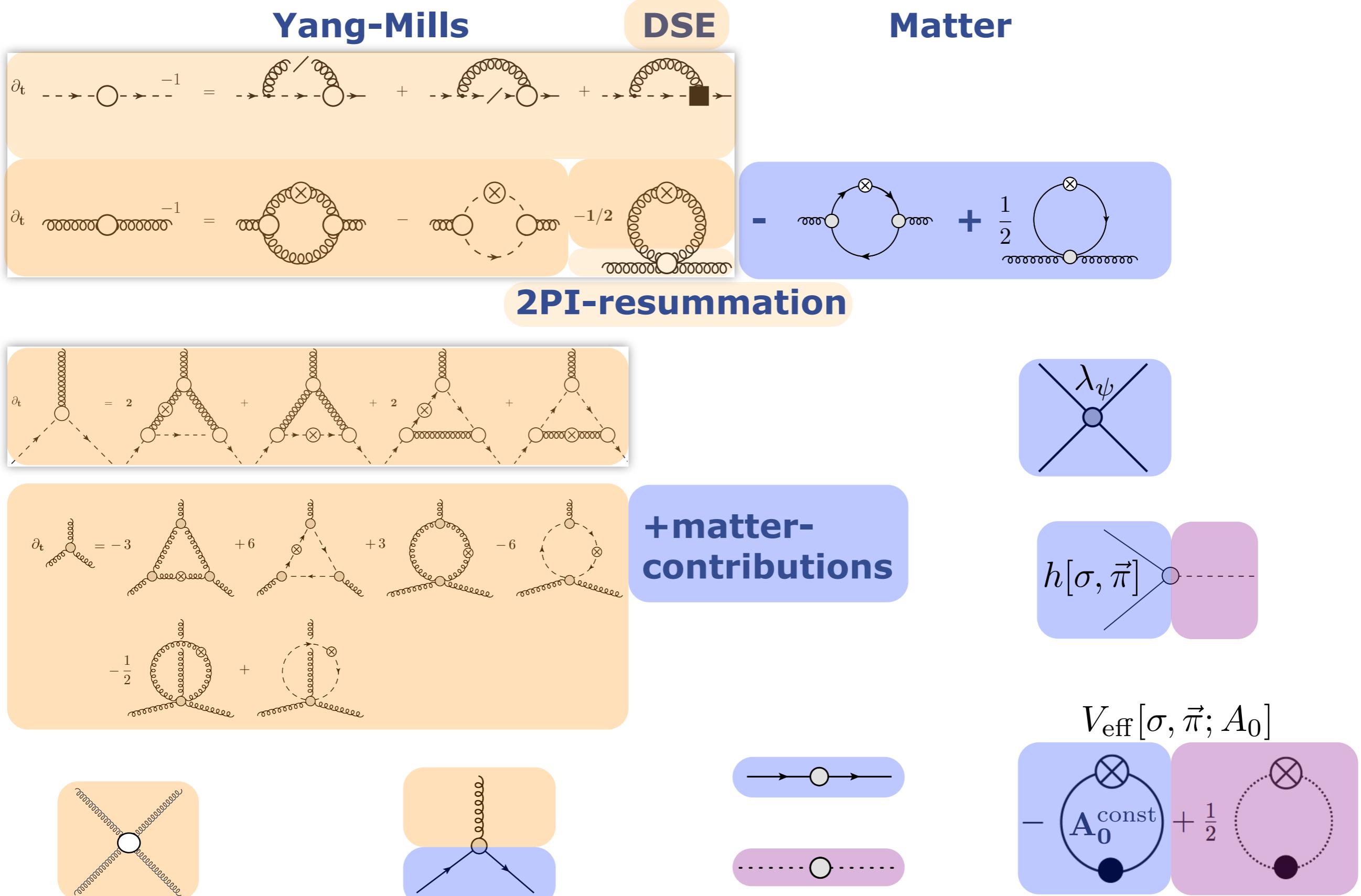
Serreau, Tissier '12

see talk of J. Serreau

# Approximation scheme



# Approximation scheme



# Outline

- **Motivation**
- **QCD**
  - Asymptotic freedom and all that
  - confinement
  - chiral symmetry breaking
- **Functional methods for QCD**
  - FRG for QCD
  - Dynamical hadronisation
  - Gauge symmetry, gauge fixing and regularisation
  - Approximation scheme
- **Results**
  - Yang-Mills theory at zero and finite temperature
  - Many-flavour QCD
  - Phase diagram of QCD
- **Summary & outlook**

**then you run ...**

**Corfu, ERG2010**

# **Results**

**DSE: see talk by R. Alkofer**

**then you run ...**

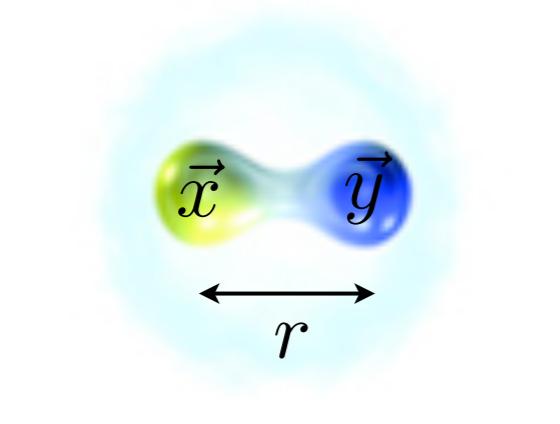
**Corfu, ERG2010**

# **Yang-Mills theory at zero and finite temperature**

# **Confinement**

# Confinement

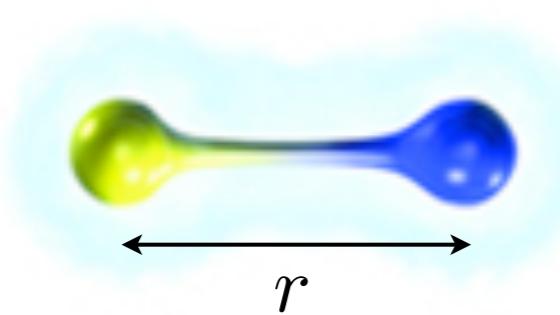
Free energy  $F_{q\bar{q}}$  of a quark - antiquark pair



$$F_{q\bar{q}} \simeq -\frac{1}{r}$$

**Order parameter**  $\sim \langle q \rangle'$

$$\Phi = e^{-\frac{1}{2T} F_{q\bar{q}}(\infty)}$$



$$F_{q\bar{q}} \simeq \sigma r$$

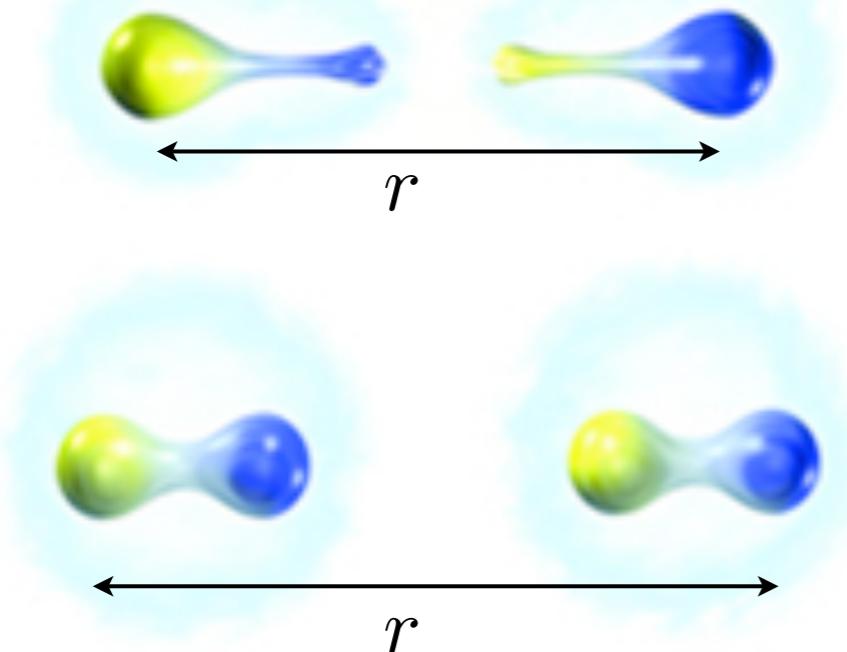
• **Confinement**

$$\Phi = 0$$

• **Deconfinement**

$$\Phi \neq 0$$

string breaking at  $r \approx 1\text{fm}$



$$F_{q\bar{q}} \simeq \text{const.}$$

**Polyakov loop**

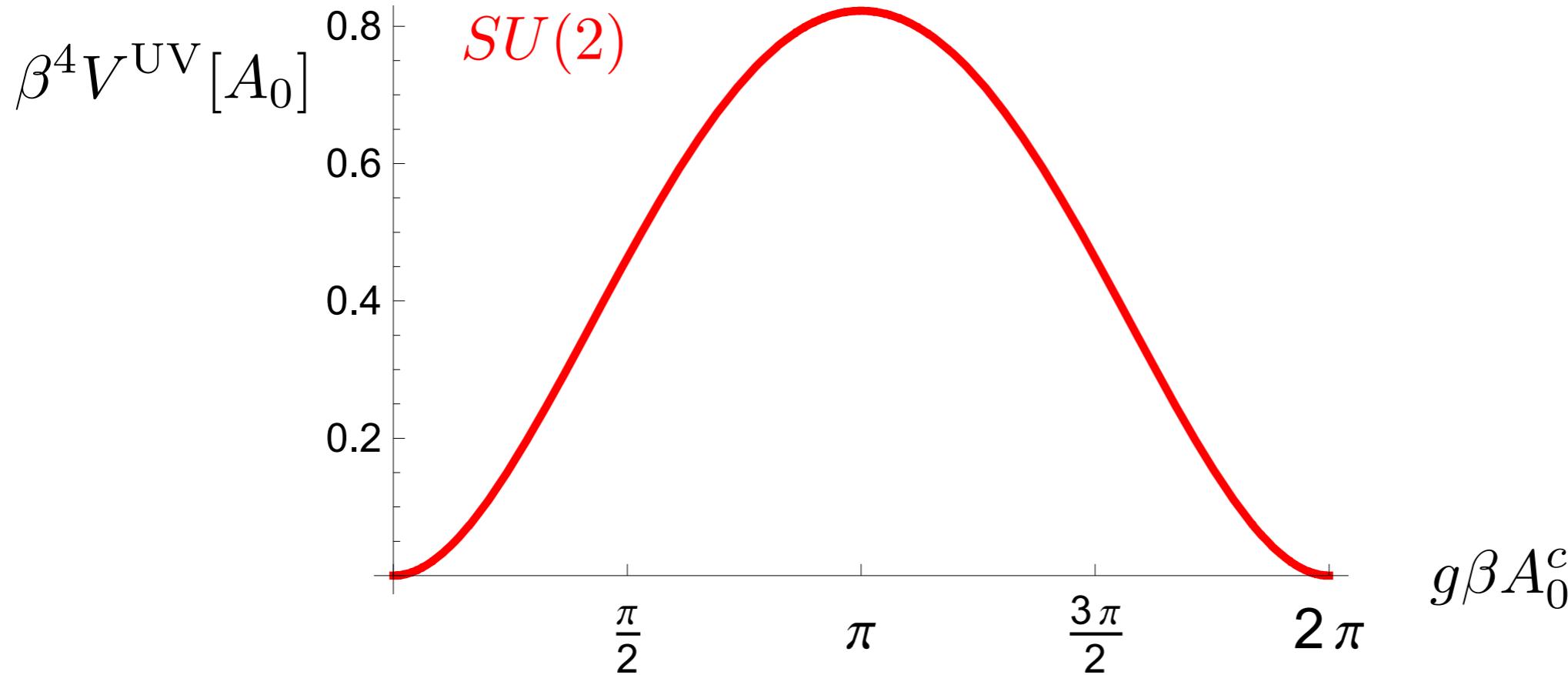
$$\Phi = \frac{1}{3} \langle \text{Tr } \mathcal{P} \exp\{ig \int_0^{1/T} dx_0 A_0\} \rangle$$

# Confinement

## Effective Polyakov loop potential

$$V^{\text{UV}}[A_0] = \frac{1}{2\Omega} \text{Tr} \log S_{AA}^{(2)}[A_0] - \frac{1}{\Omega} \text{Tr} \log S_{C\bar{C}}^{(2)}[A_0]$$

Gross, Pisarski, Yaffe '81  
Weiss '81



$$SU(2) : \Phi[A_0] = \cos \frac{1}{2}\beta g A_0^c \quad \text{with} \quad A_0 = A_0^c \frac{\sigma_3}{2}$$

## Effective potential

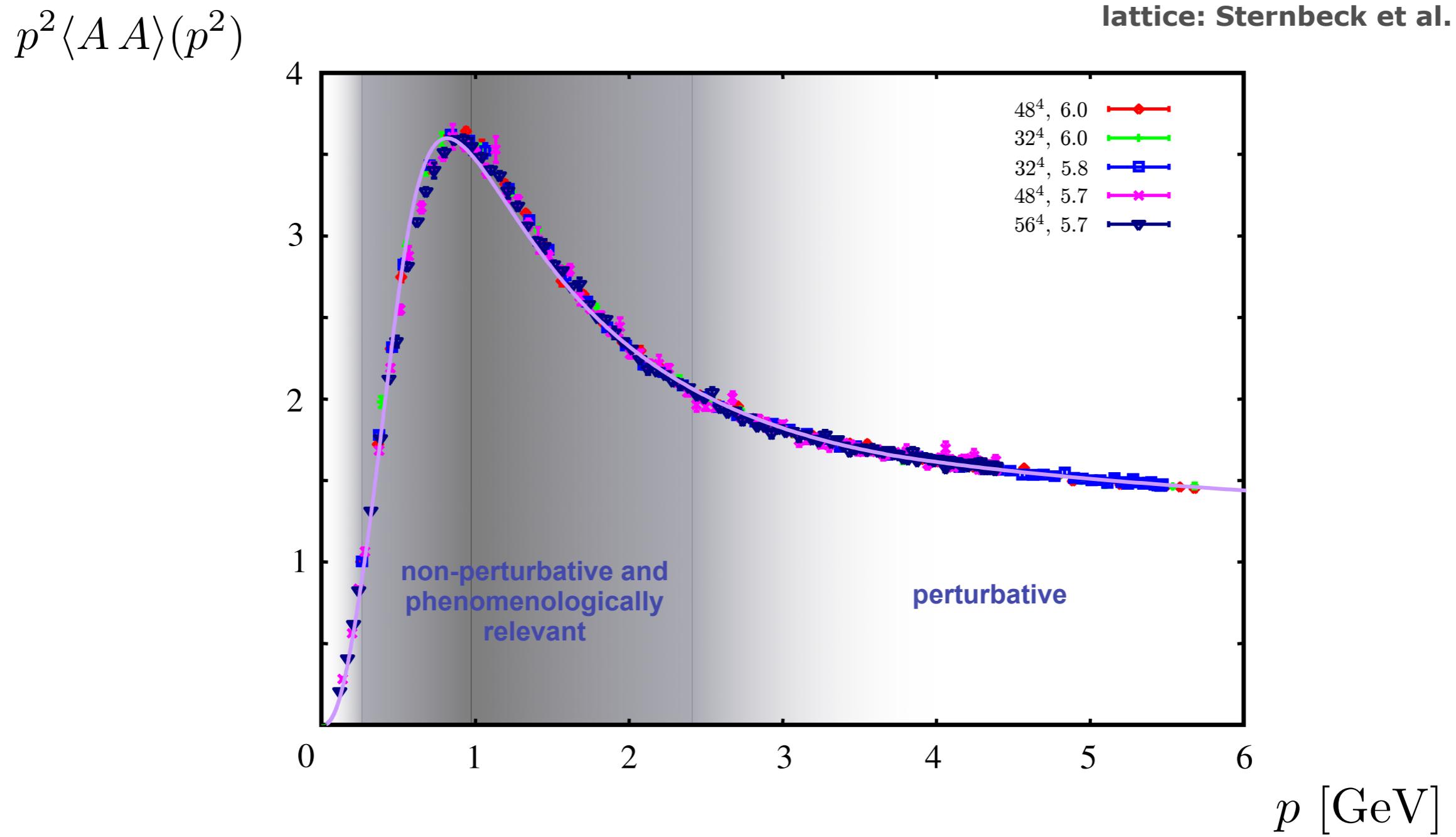
$$V[A_0] = -\frac{1}{2} \text{Tr} \log \langle AA \rangle [A_0] + O(\partial_t \langle AA \rangle) + \text{Tr} \log \langle C\bar{C} \rangle [A_0] + O(\partial_t \langle C\bar{C} \rangle)$$

free energy

# Propagators

FRG: Fischer, Maas, JMP '08

lattice: Sternbeck et al. '06

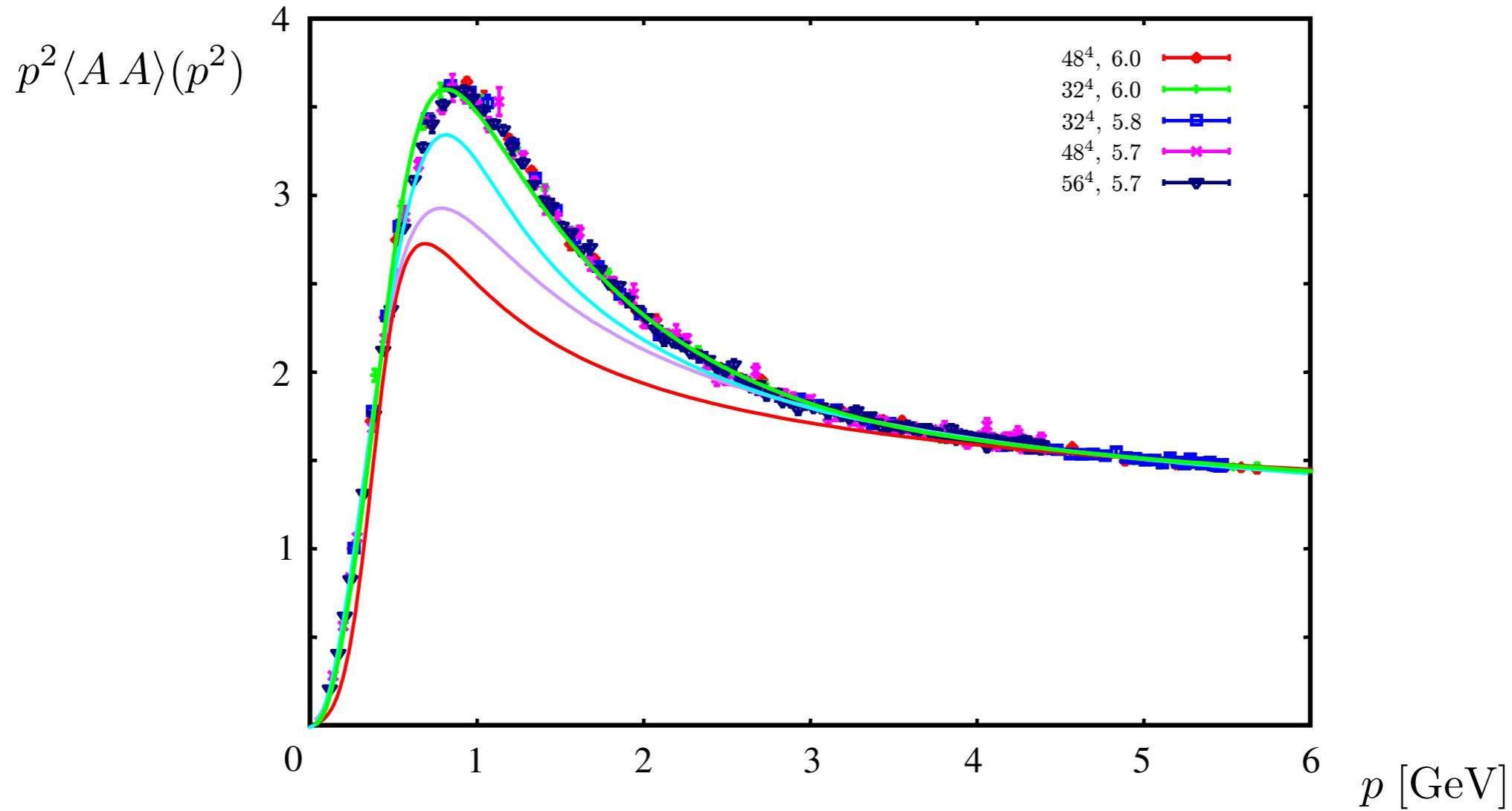


Propagators phenomenologically well described in 1/N expansion

# Propagators

Pure Yang-Mills,  $T = 0$

lattice: Sternbeck et al. '06



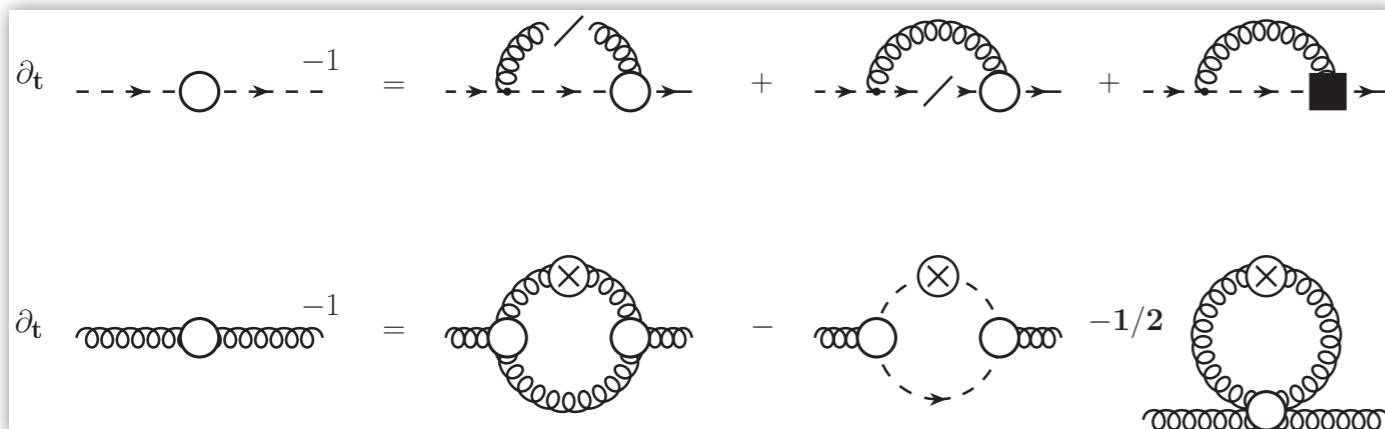
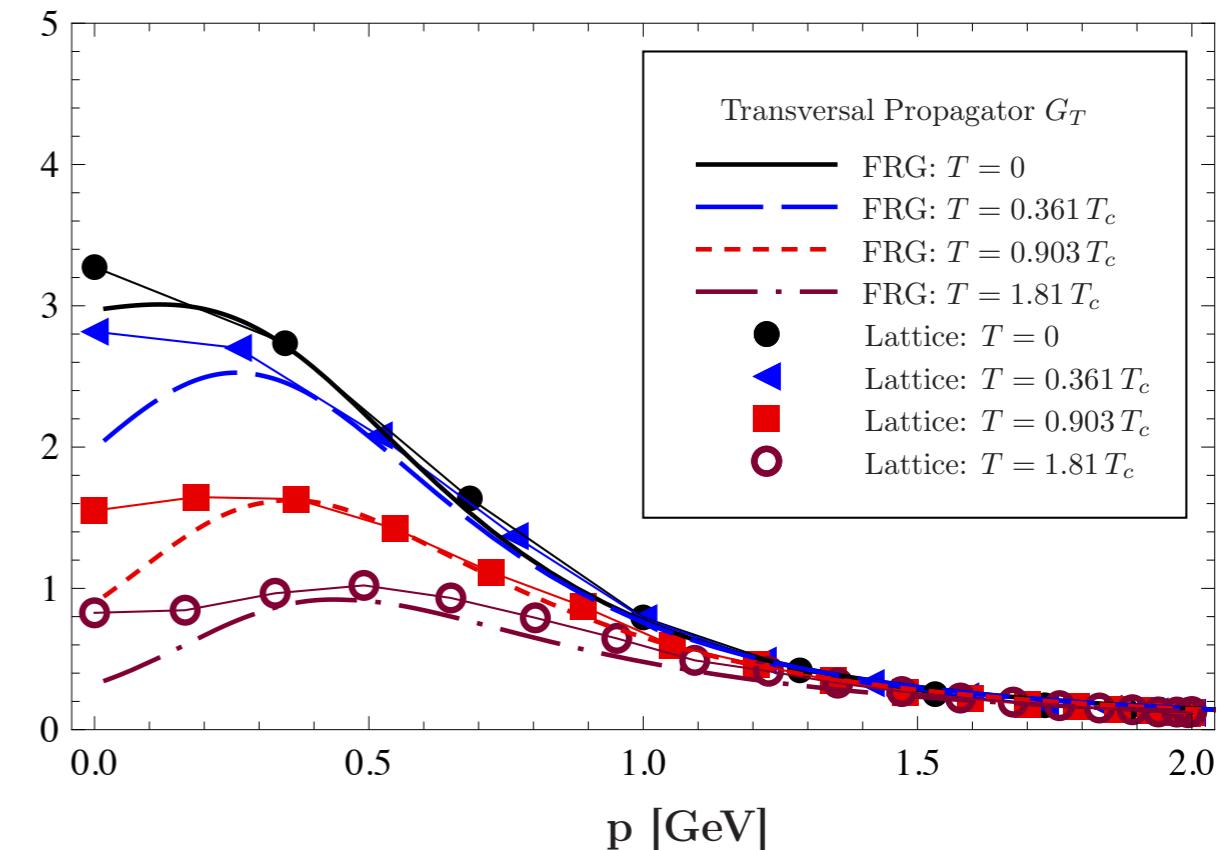
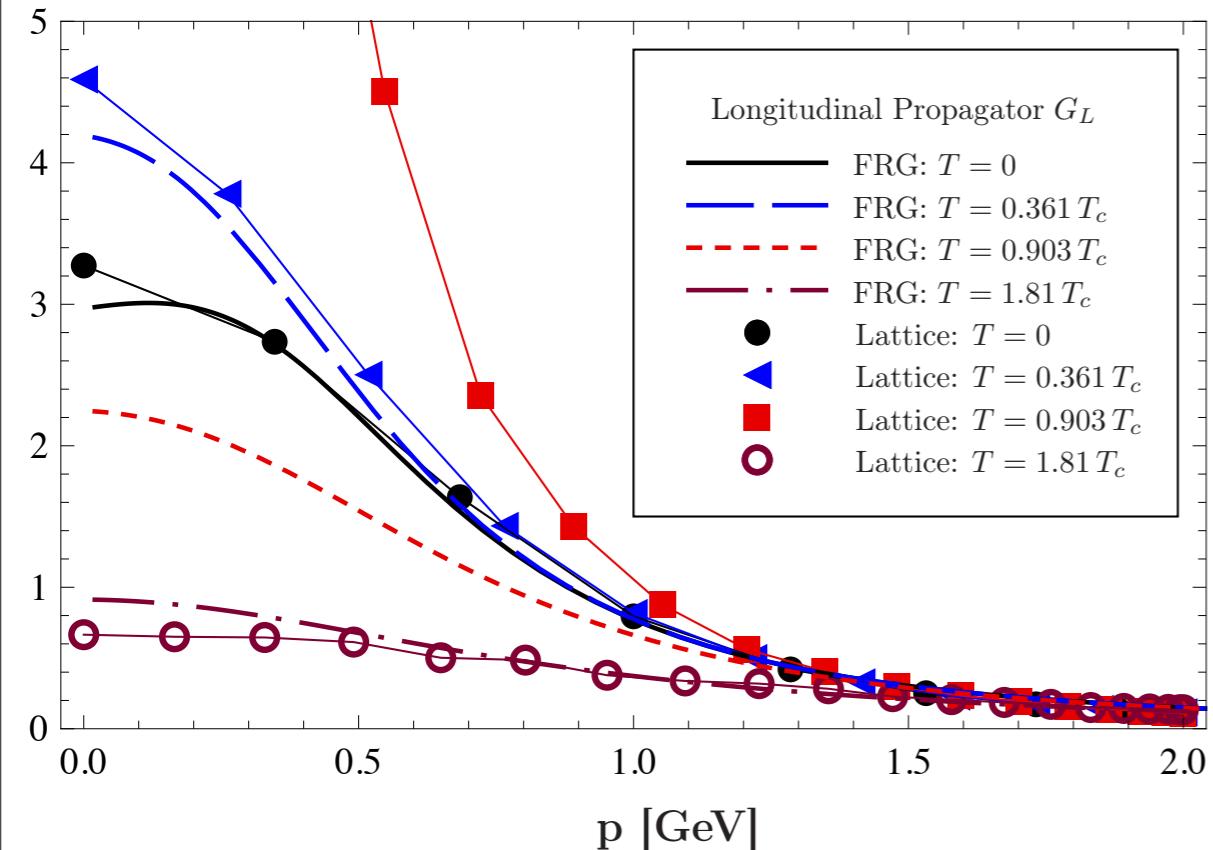
- von Smekal, Hauck, Alkofer '97
- Lerche, von Smekal, Phys. '02
- Fischer, Alkofer, Phys. Rev. '02
- JMP, Litim, Nedelko, von Smekal '03; JMP '06 (unpublished)
- Fischer, Maas, JMP'08

# Confinement

## Thermal gluon propagators

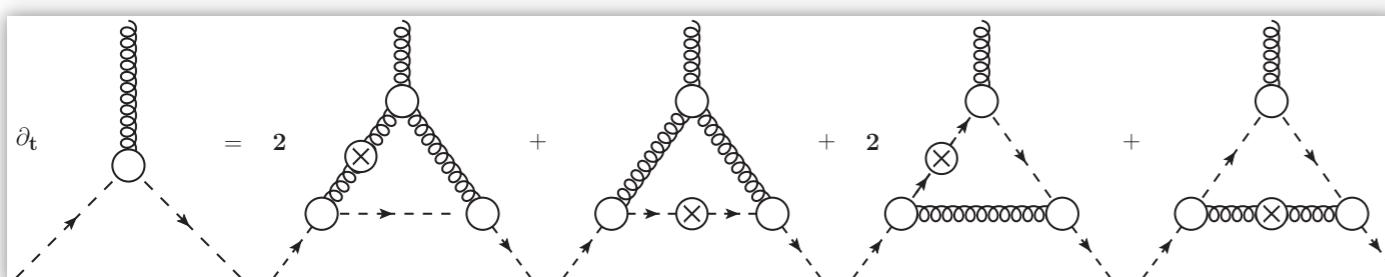


Fister, JMP '11



Lattice: Maas, JMP, Spielmann, von Smekal '11

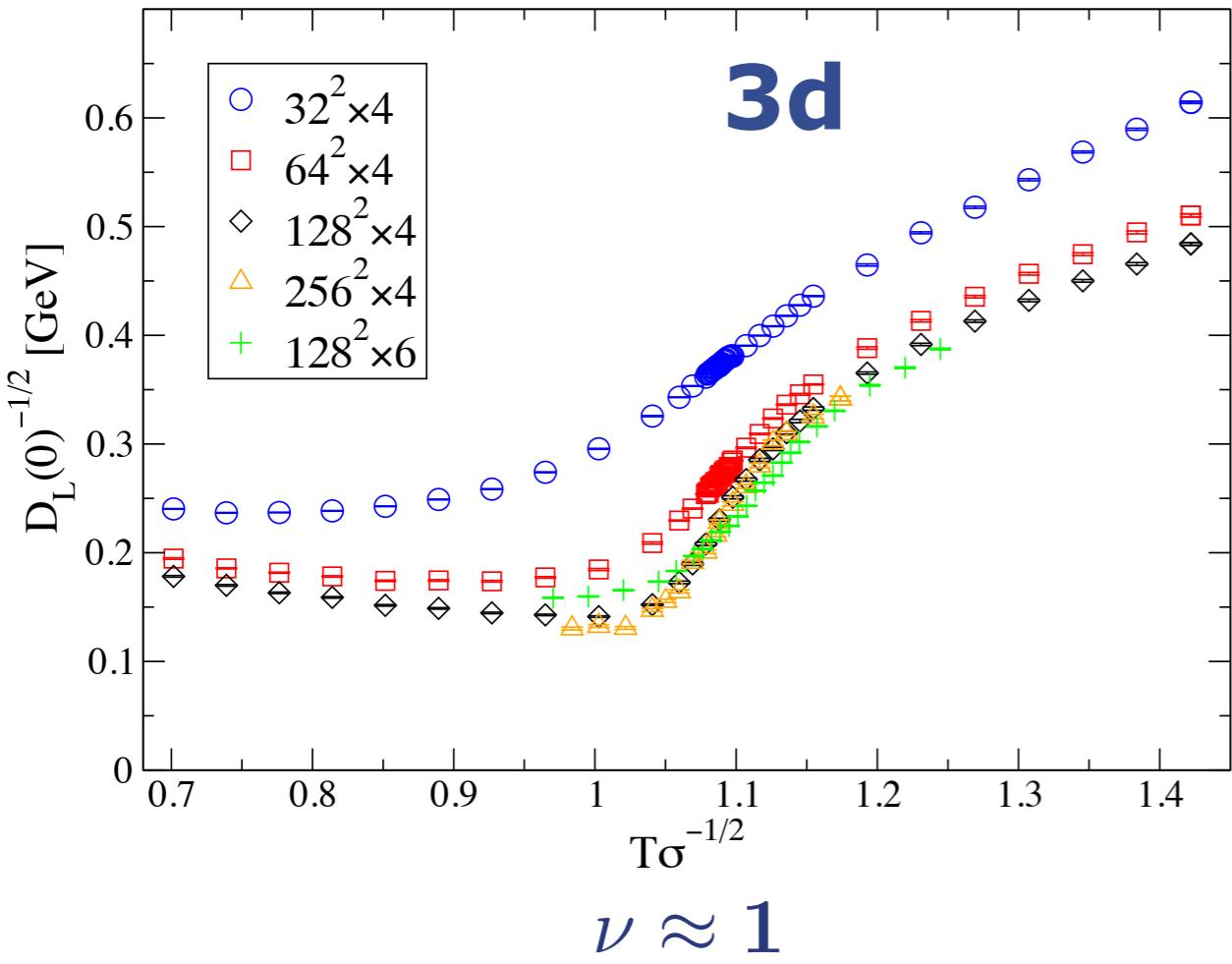
see talk of L.Fister



+ RG-dressed gluonic vertices  
confirmed with the full system, JMP, Fister, in prep

# Confinement

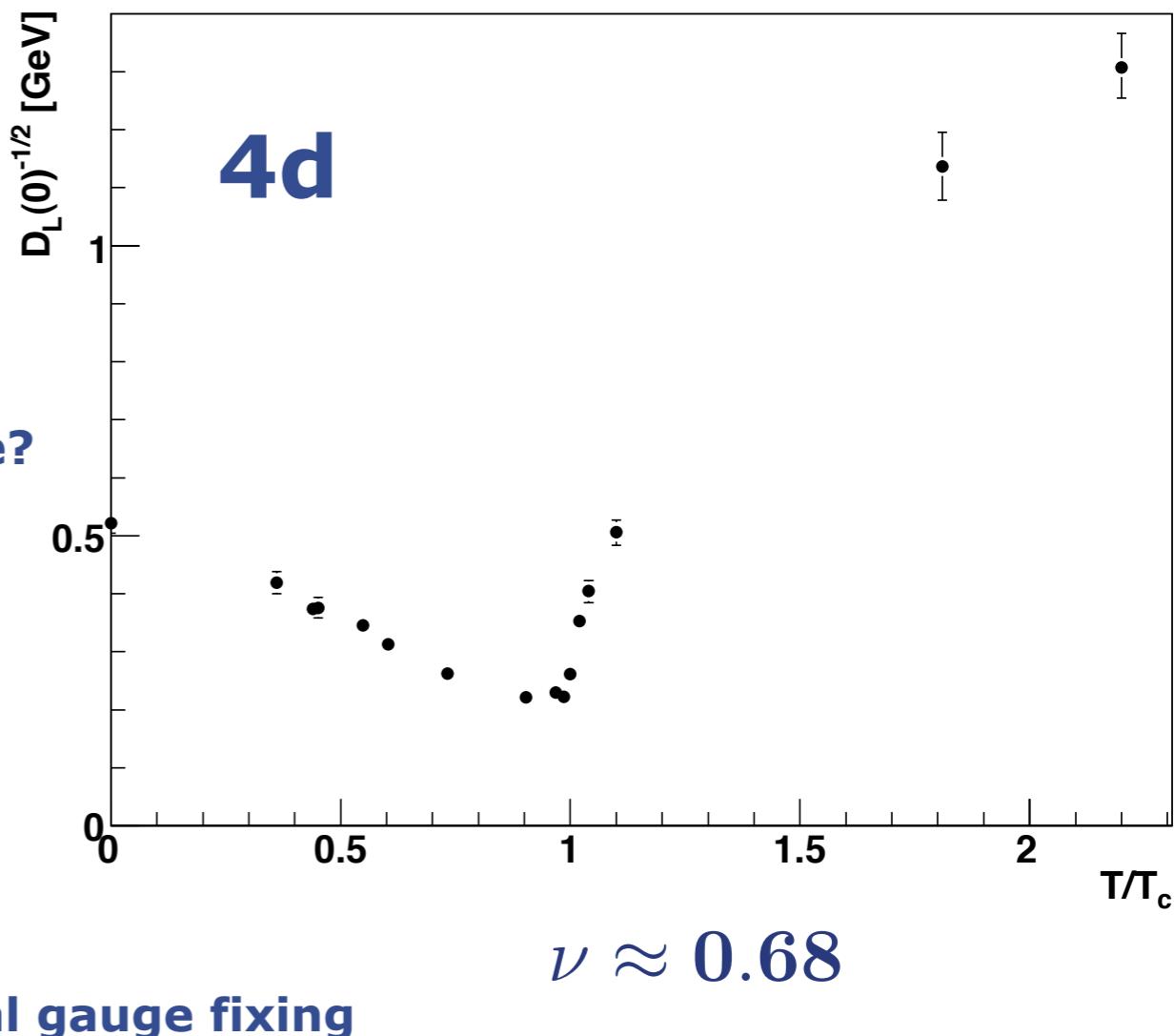
## Chromo-electric propagator



Maas, JMP, Spielmann, von Smekal '11

$$D_L(0) = \langle A A \rangle_T(0)$$

Electric screening mass for SU(2)



critical scaling in Landau gauge props on the lattice?

$$D_L(0)^{-1/2} \propto |T - T_c|^\nu + \dots$$

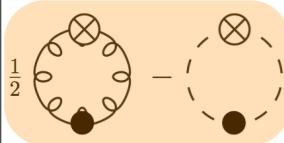
FRG

$$D_L(0)^{-1/2} \propto V''[A_0] + \dots$$

global gauge fixing

# Confinement

## Order parameter

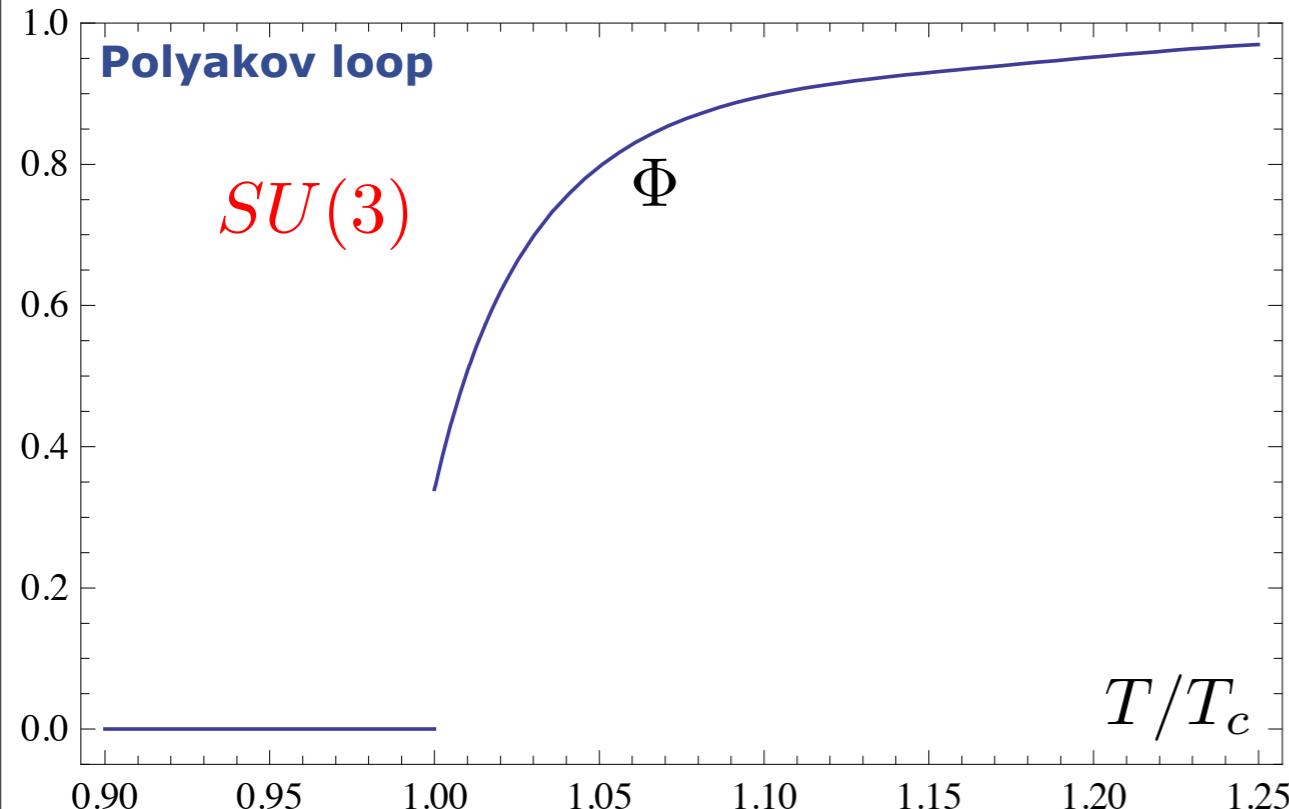


$$T_c = 276 \pm 10 \text{ MeV}$$

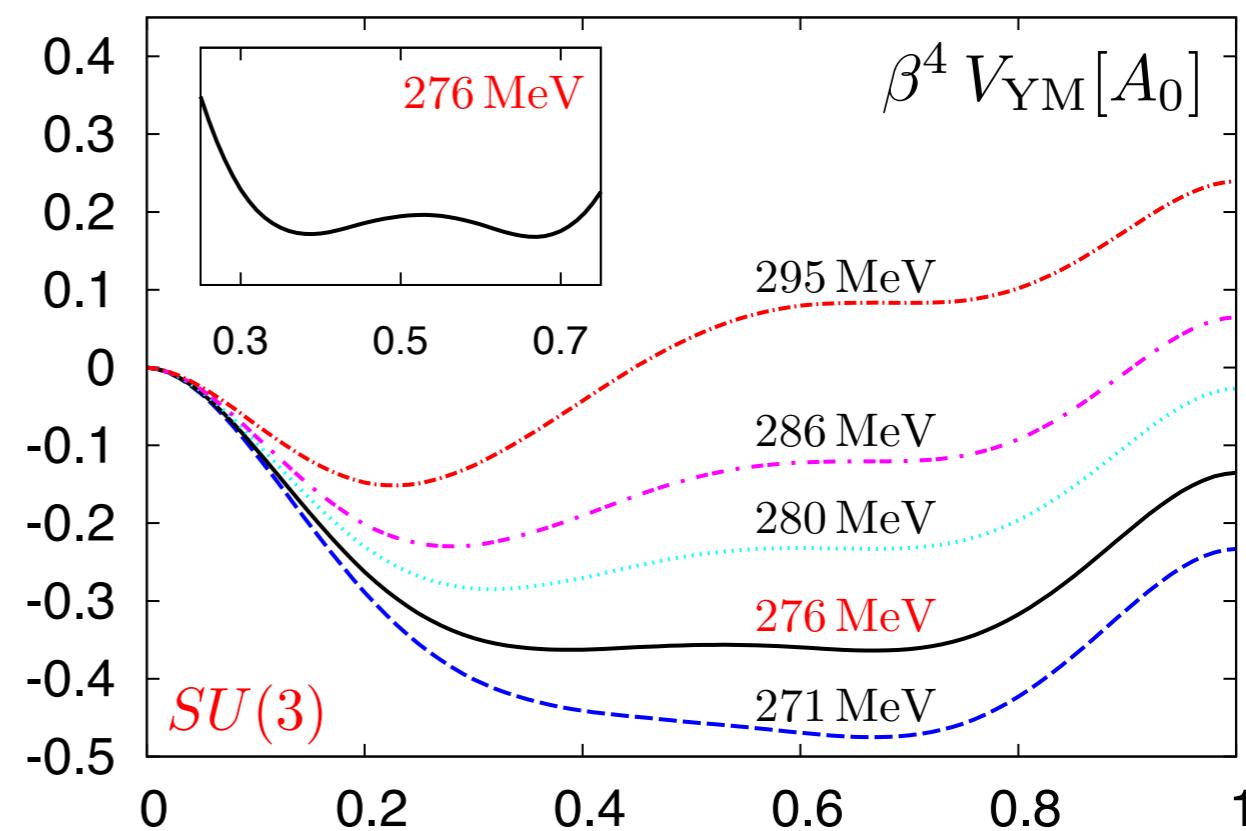
$$T_c/\sqrt{\sigma} = 0.658 \pm 0.023$$

**Braun, Gies, JMP '07**

$$\text{lattice : } T_c/\sqrt{\sigma} = 0.646$$



SU(2) & critical scaling: Marhauser, JMP '08  
SU(N), Sp(2), E(7): Braun, Eichhorn, Gies, JMP '10

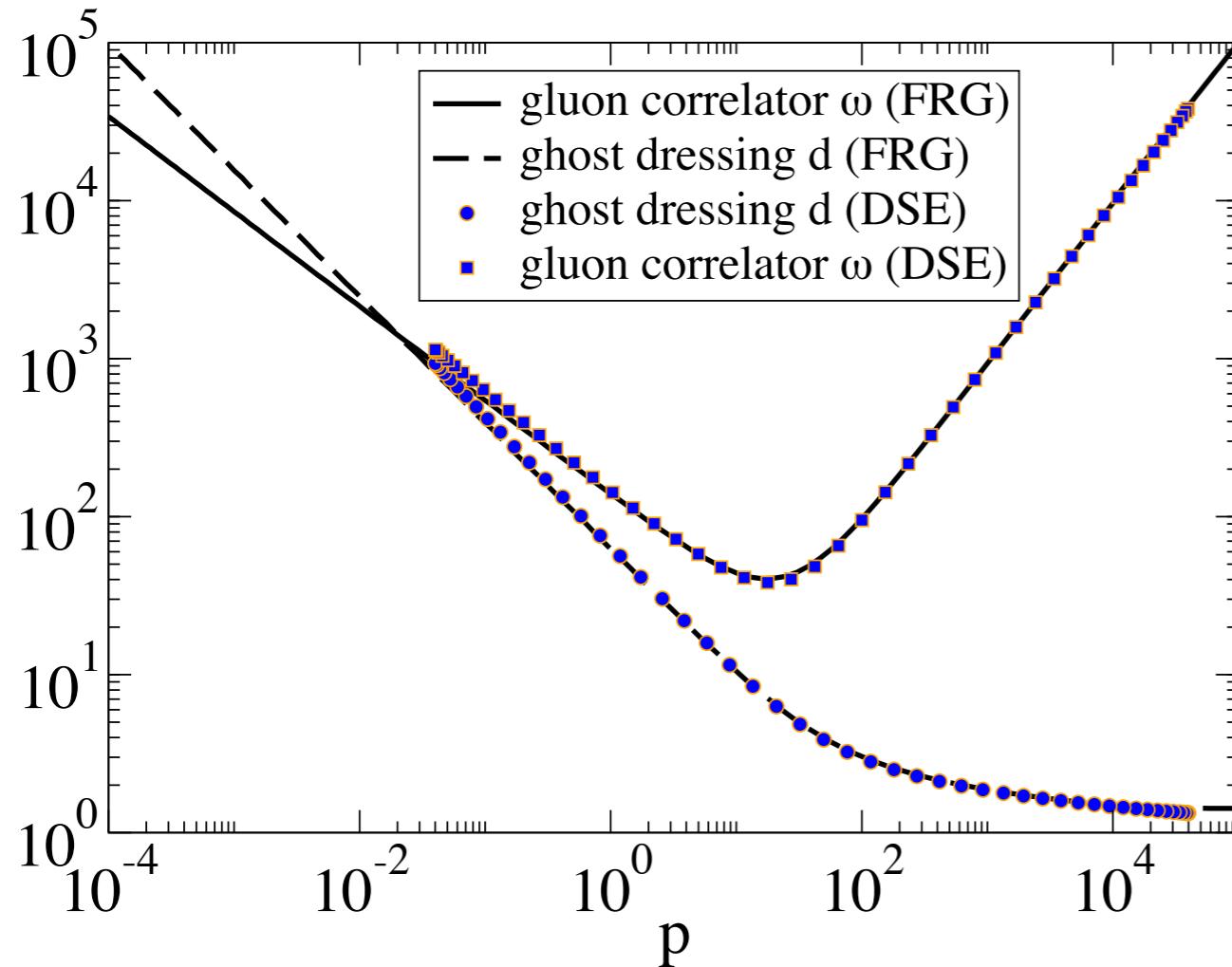


$$\Phi[A_0] = \frac{1}{3}(1 + 2 \cos \frac{1}{2}\beta g A_0)$$

$$\Phi[\frac{4}{3}\pi \frac{1}{\beta g}] = 0 \quad \frac{\beta g A_0}{2\pi}$$

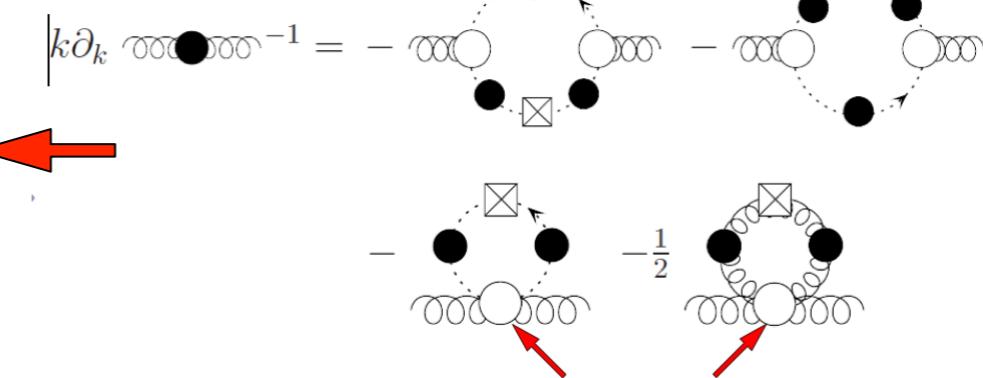
# Confinement

## Coulomb gauge



Braun, Gies, JMP '07

Confinement



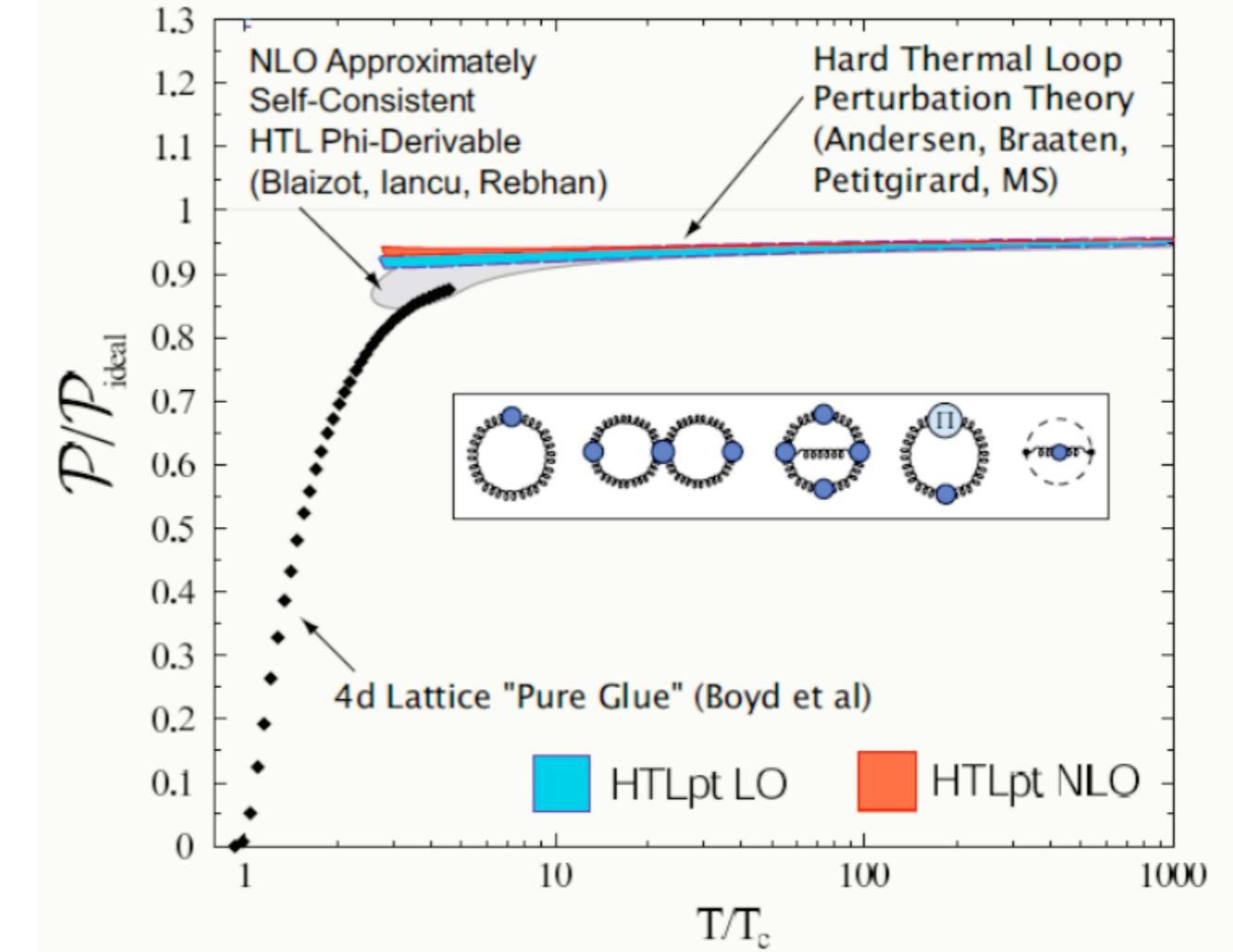
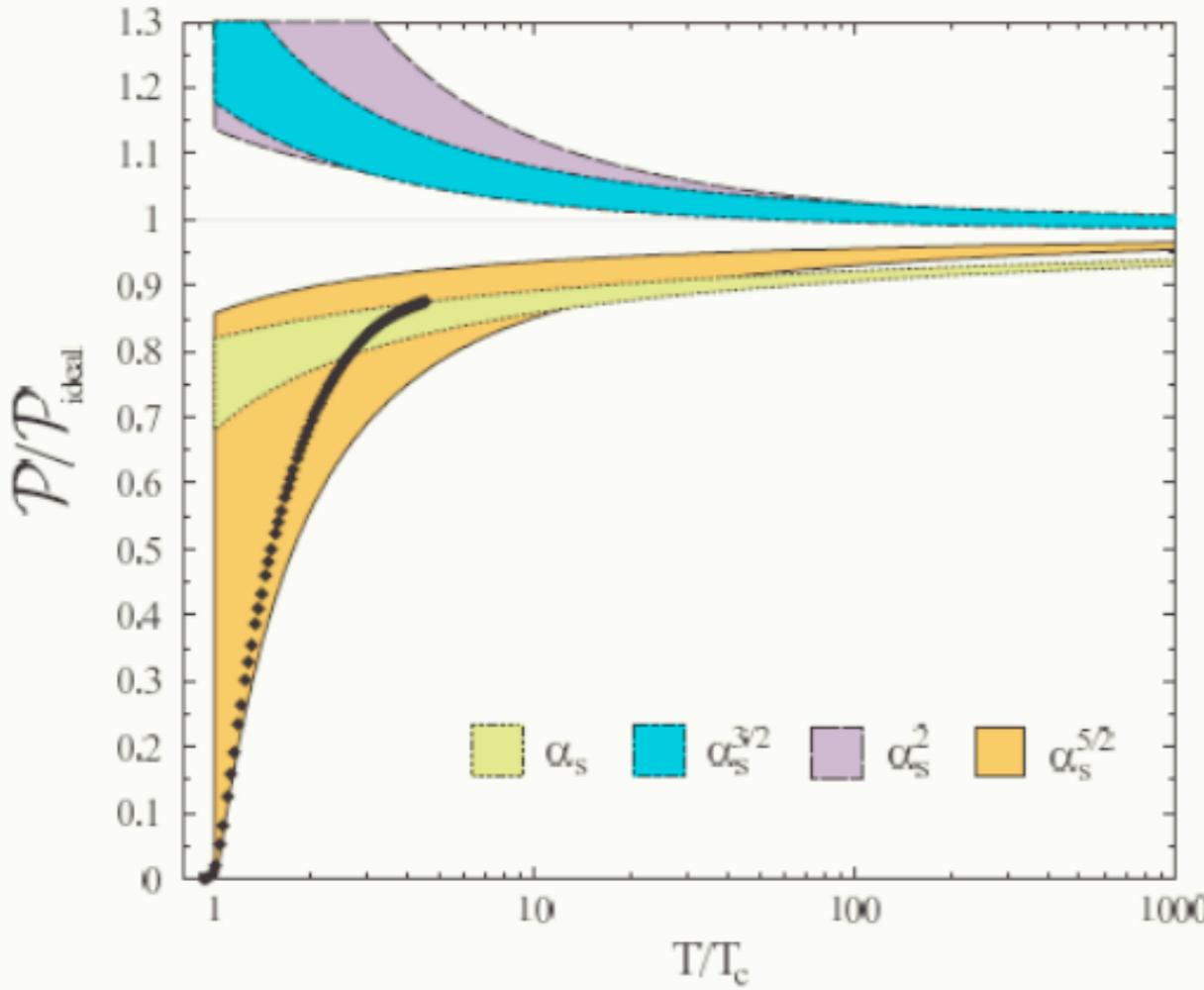
Insert full DSE's with ghost terms only

$T_c$ , work in progress

Leder, JMP, Reinhardt, Weber '10

# **Confinement & Thermodynamics**

# Confinement & Thermodynamics

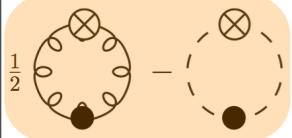


Strickland

$$-p(T; \bar{A}) = \int_{\Lambda}^0 \frac{dk}{k} \left\{ \begin{array}{c} \text{Diagram with } T \\ - \\ \text{Diagram with } T=0 \end{array} \right\} \Big|_{\bar{A}}$$

Fister, JMP

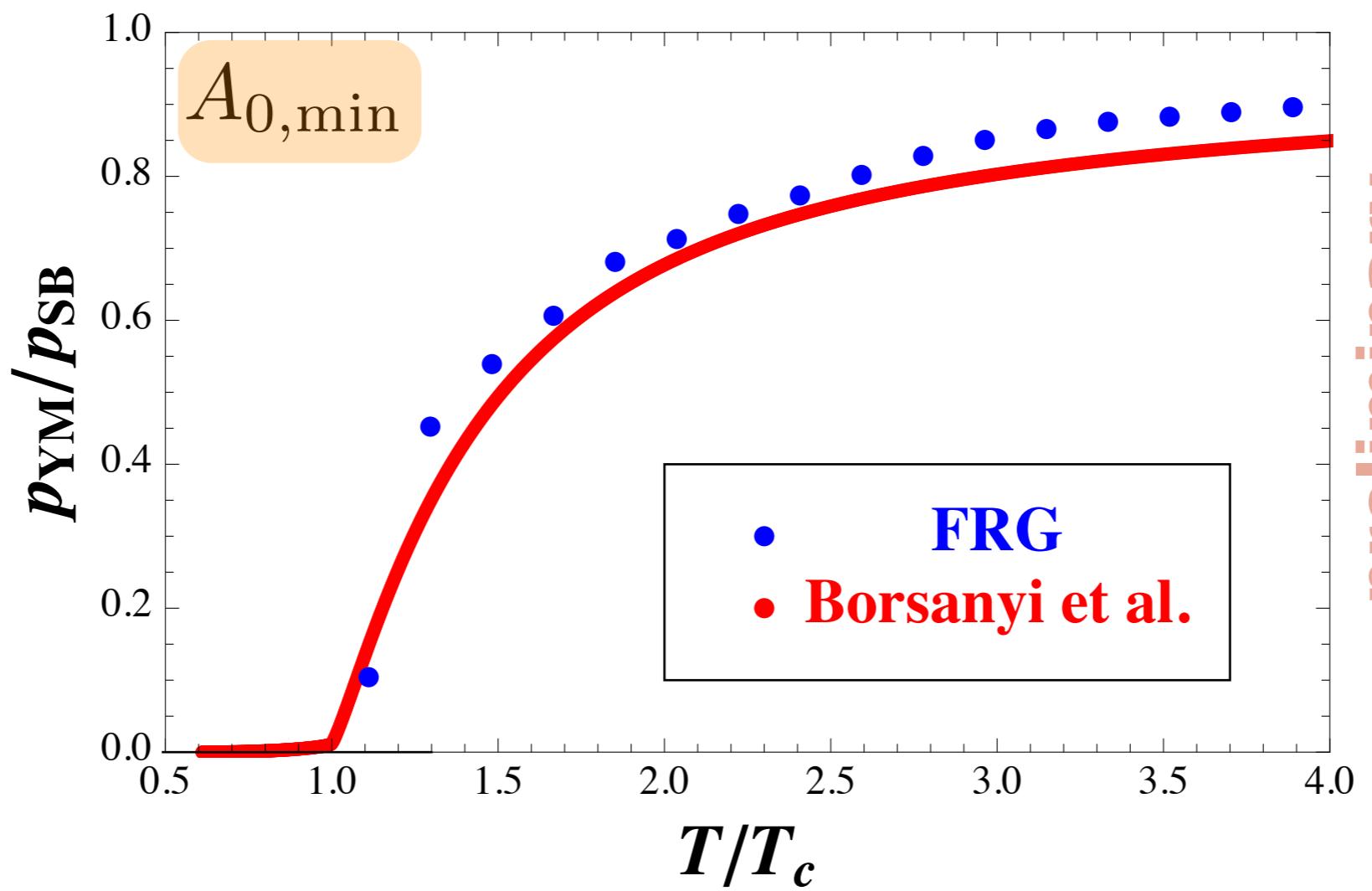
# Confinement & Thermodynamics



$$-p(T; \bar{A}) = \int_{\Lambda}^0 \frac{dk}{k} \left\{ \left. \text{Diagram with } T \right|_T - \left. \text{Diagram with } T=0 \right|_{T=0} \right\} \Big|_{\bar{A}}$$

Σ  
 $G_{T,k}$   $\partial_t R_k$       ∫  
 $G_{T=0,k}$   $\partial_t R_k$

see talk of L.Fister



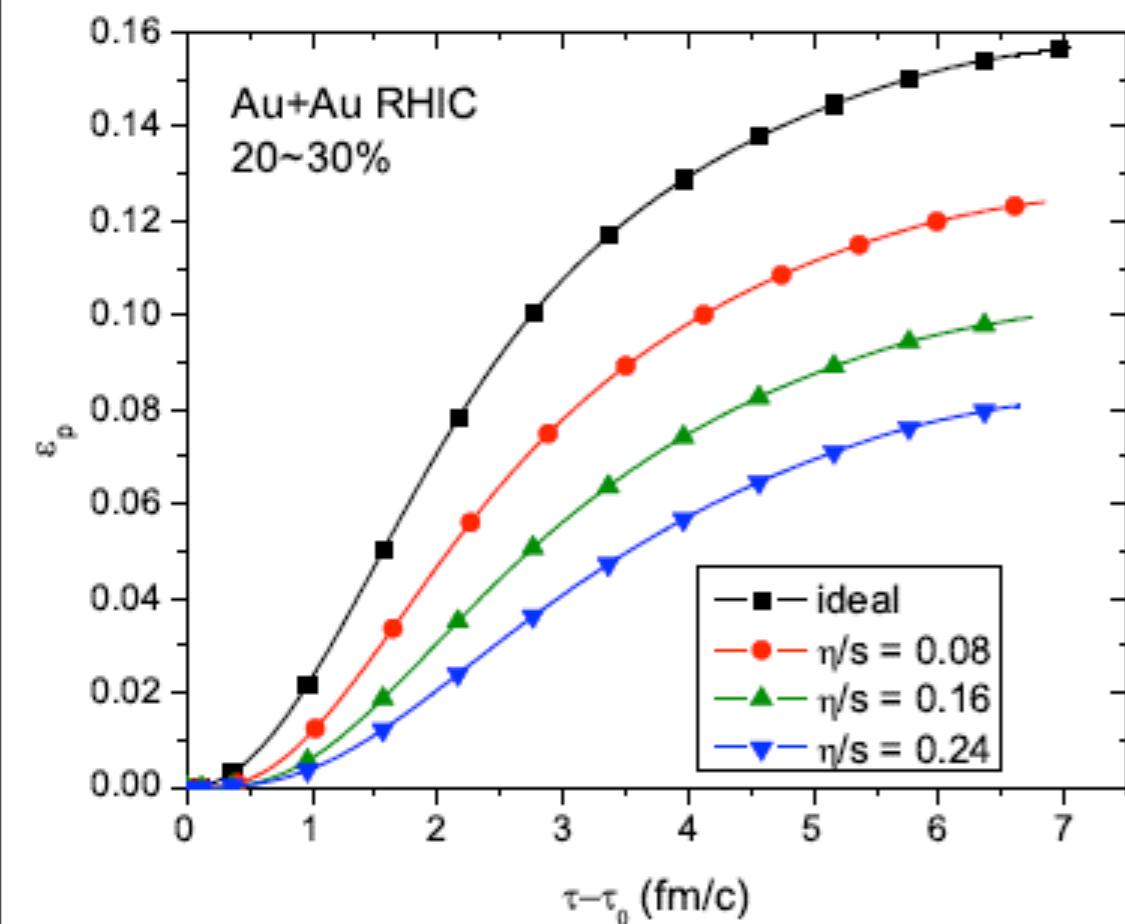
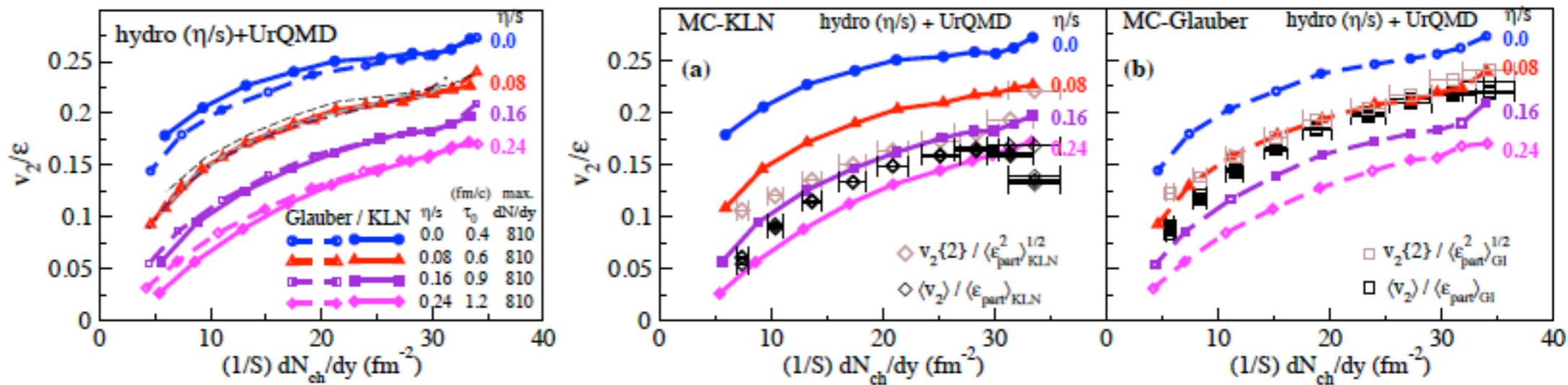
preliminary

# **Transport in QCD**

# Transport in QCD

## Extraction of $(\eta/s)_{QGP}$ from AuAu@RHIC

H. Song, S.A. Bass, U. Heinz, T. Hirano, C. Shen, PRL106 (2011) 192301



$$1 < 4\pi(\eta/s)_{QGP} < 2.5$$

U. Heinz, talk at RETUNE '12

# Transport in QCD

flow of  $\rho_{\pi\pi}$

$$\partial_t = \square = -\frac{1}{2} \left( \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} - \frac{1}{2} \text{Diagram 4} \right)$$

$$\rho_{\pi\pi} = \square$$

current approximation

$$\rho_{\pi\pi} = \square$$

'Those are my methods (principles), and if you don't like them...well, I have others'

Groucho Marx

$\rho_{T/L}$  with MEM

$$\rho_{\pi\pi}(p) = \frac{2}{3}(N_c^2 - 1) \int \frac{d^4 k}{(2\pi)^4} [n(k_0) - n(k_0 + p_0)] (V_{TT}(k)\rho_T(k)\rho_T(k+p) + V_{TL}\rho_T(k)\rho_L(k+p) + V_{LL}\rho_L(k)\rho_L(k+p))$$

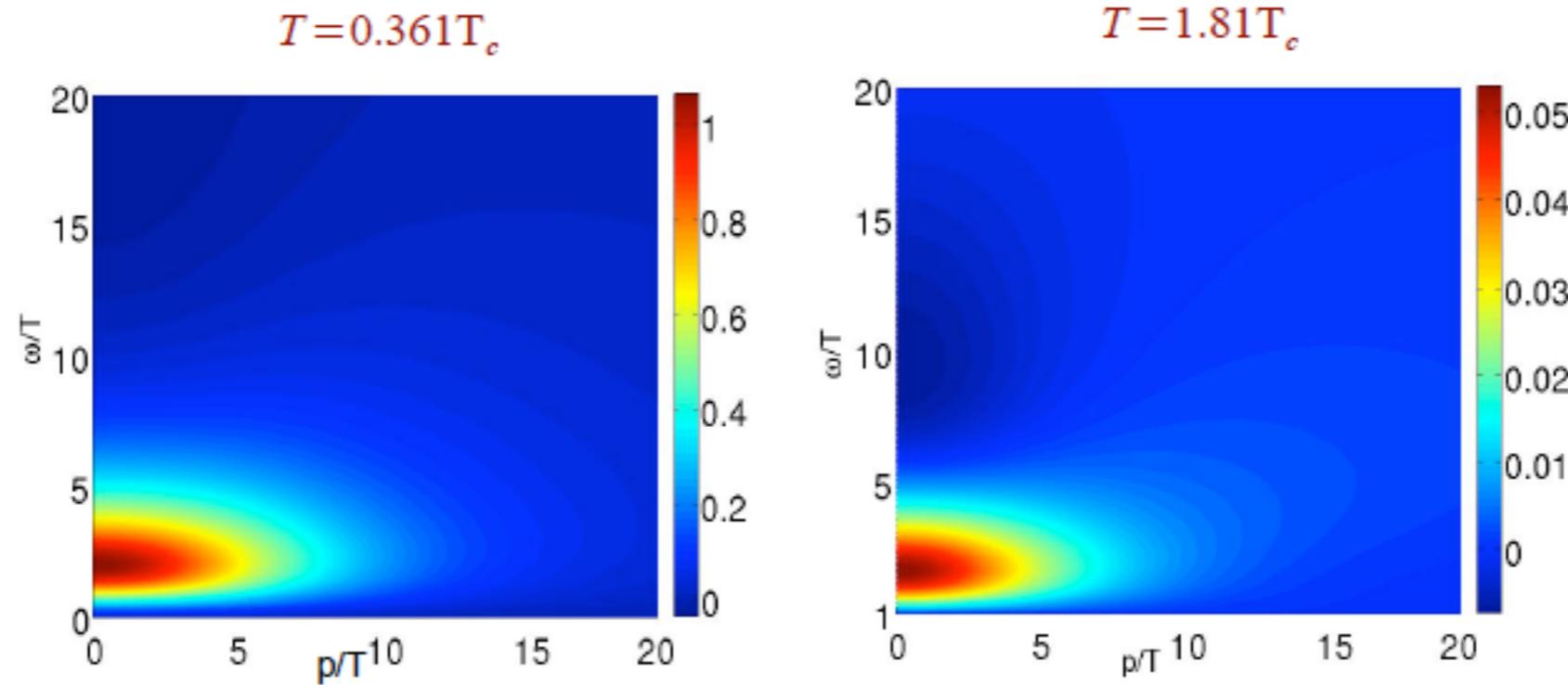
# Viscosity in YM

## spectral functions

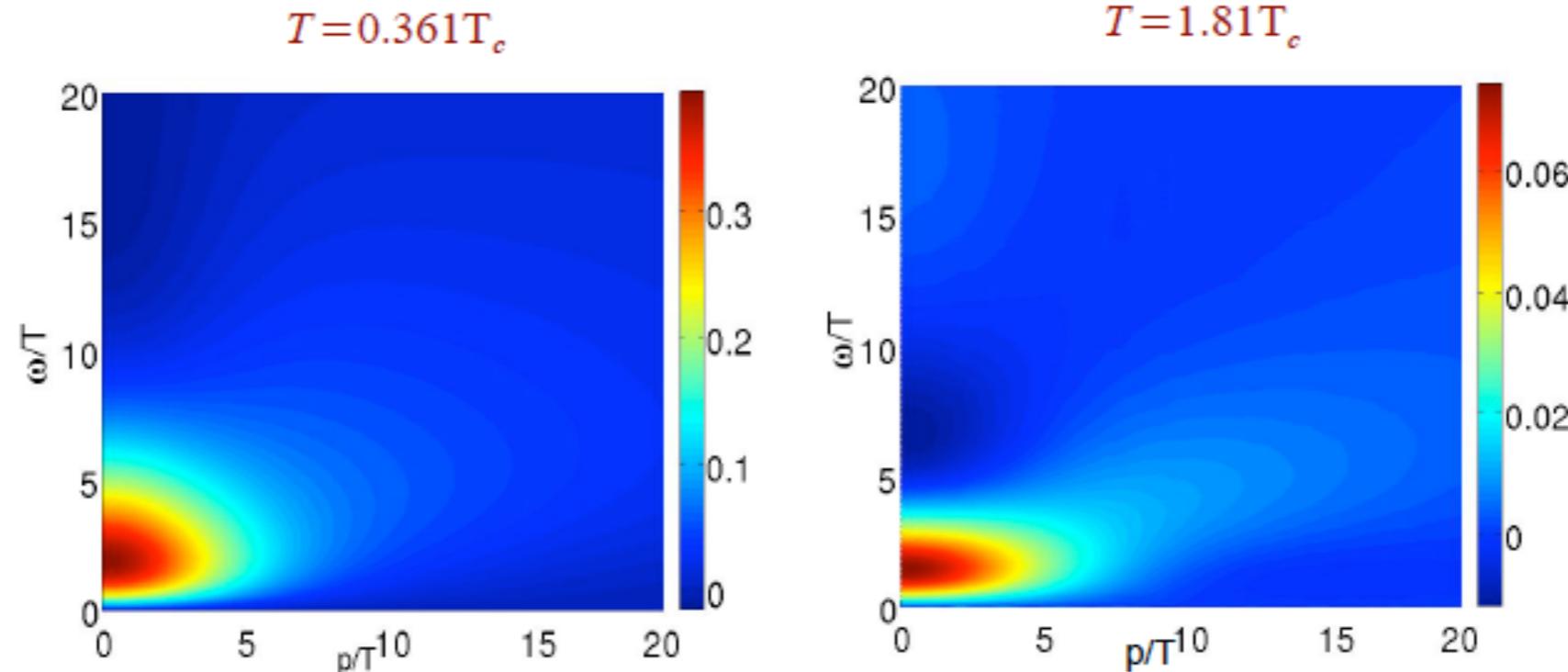


### transversal spectral functions

Fister, M. Haas, JMP, in prep

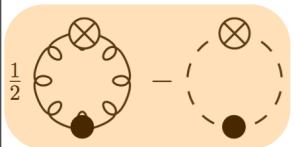


### longitudinal spectral functions



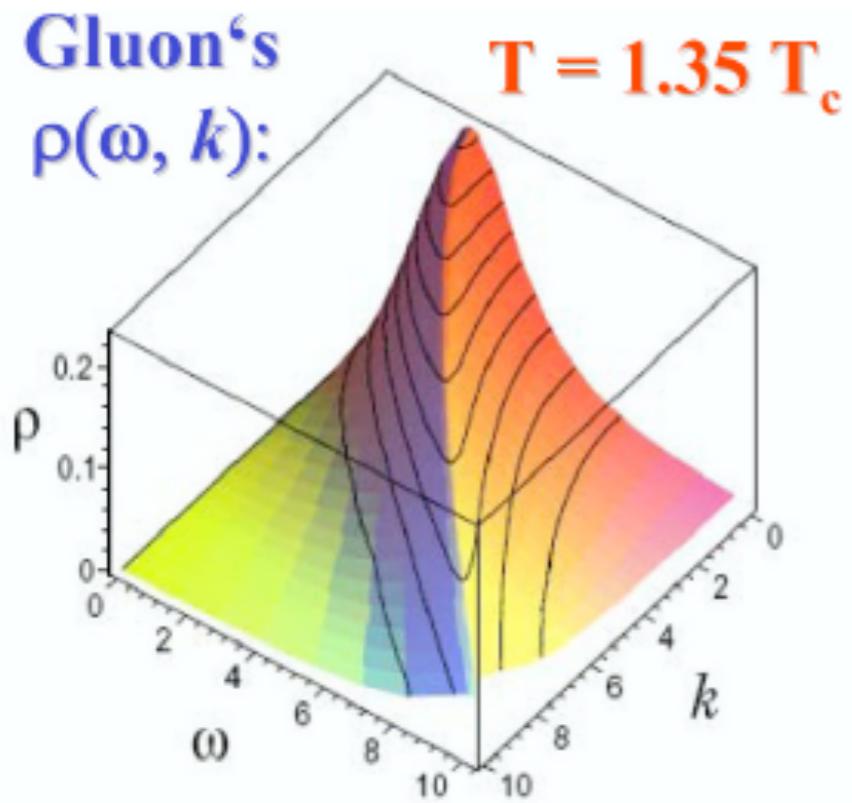
# Viscosity in YM

## spectral functions



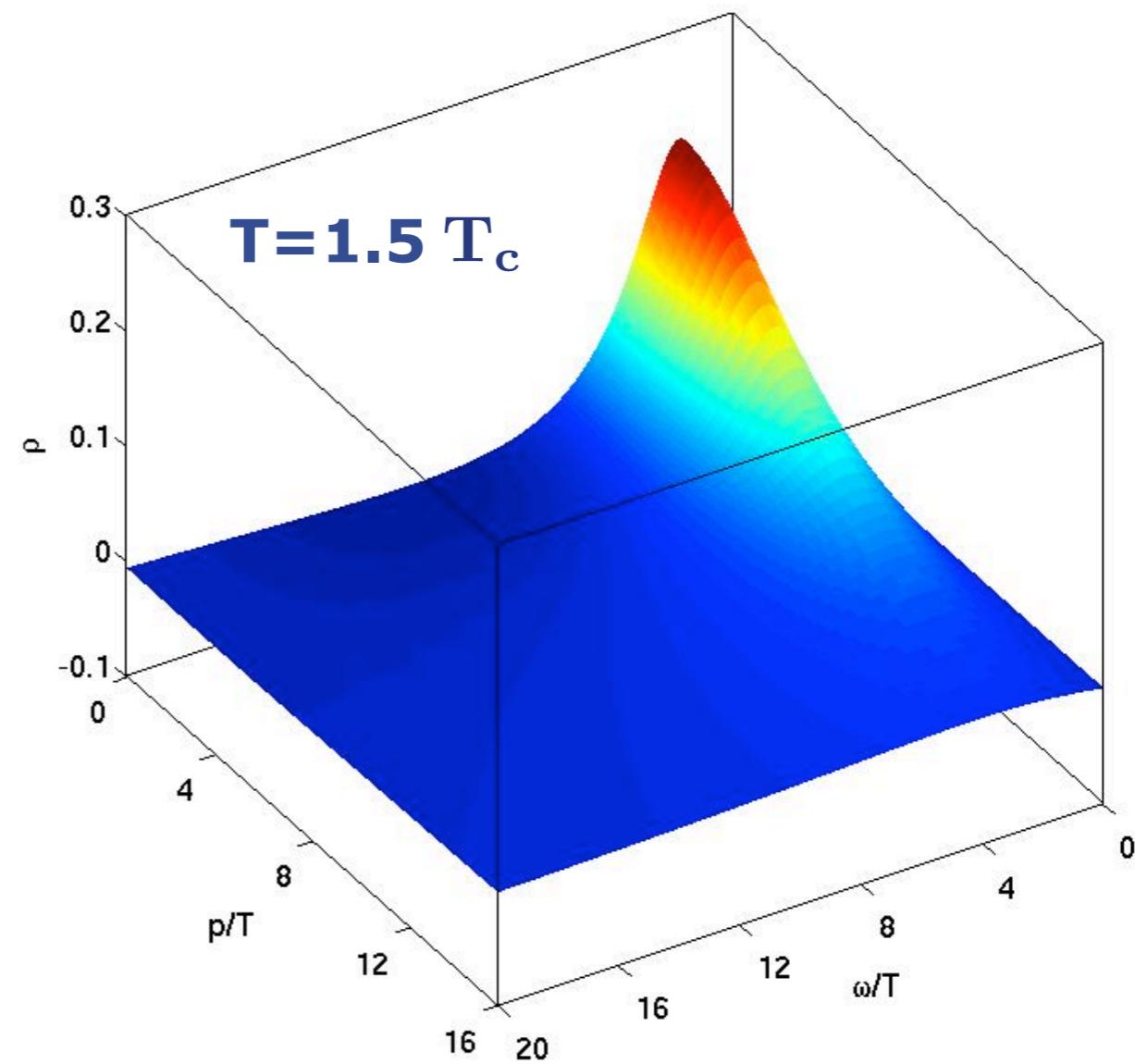
Fister, M. Haas, JMP, in prep

→ Broad spectral function :



E. Bratkovskaya, talk at RETUNE '12

transversal spectral function



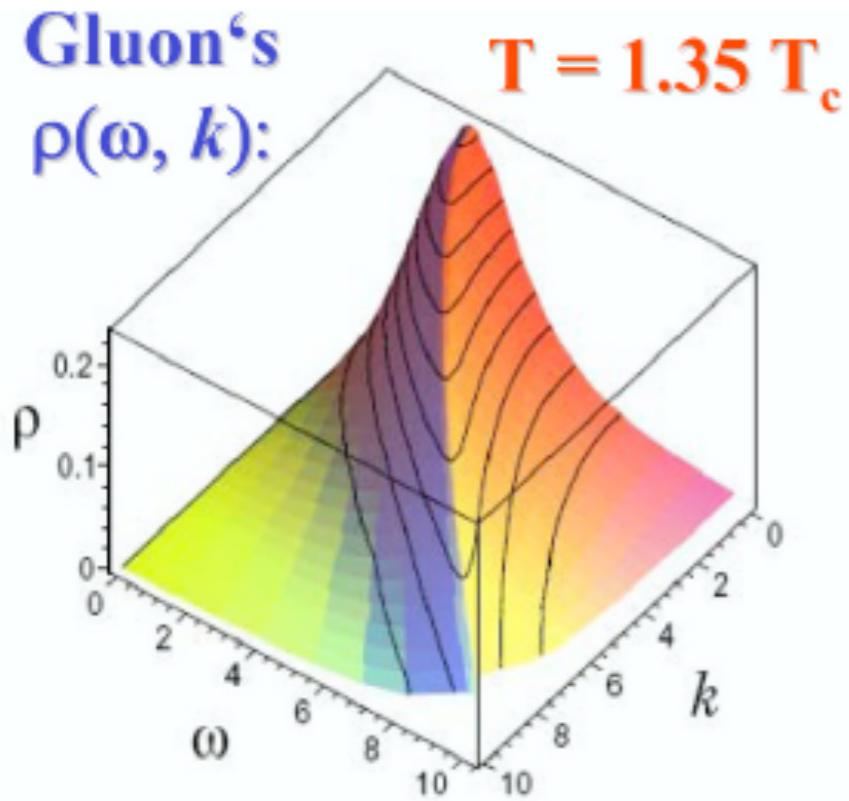
# Viscosity in YM

## spectral functions



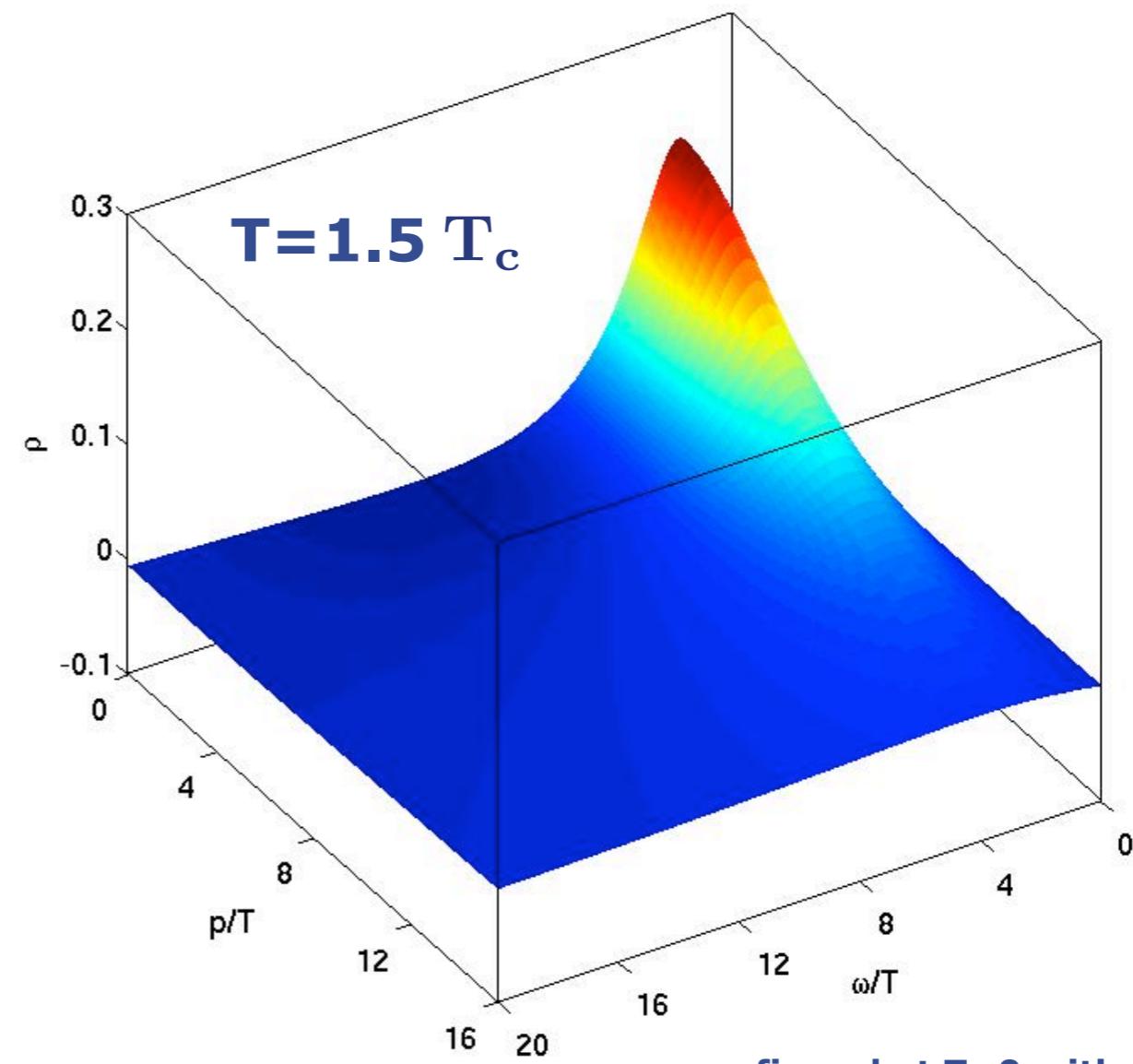
Fister, M. Haas, JMP, in prep

→ Broad spectral function :



E. Bratkovskaya, talk at RETUNE '12

transversal spectral function



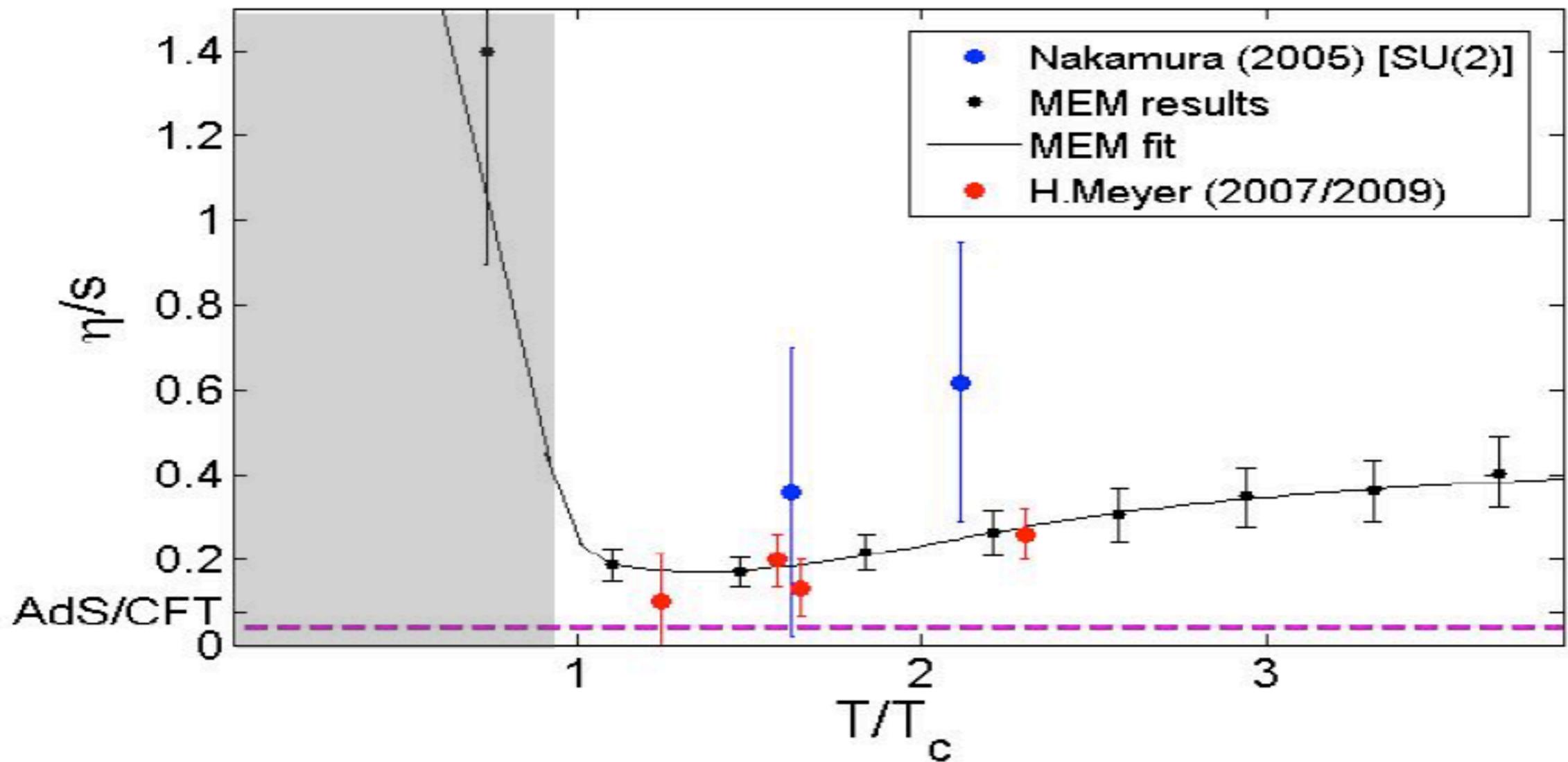
confirmed at  $T=0$  with complex DSEs  
Strauss, Fischer, Kellermann '12

# Viscosity in YM

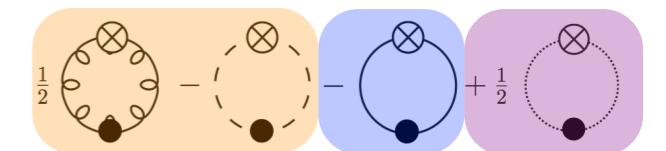


## shear viscosity

Fister, M. Haas, JMP, in prep

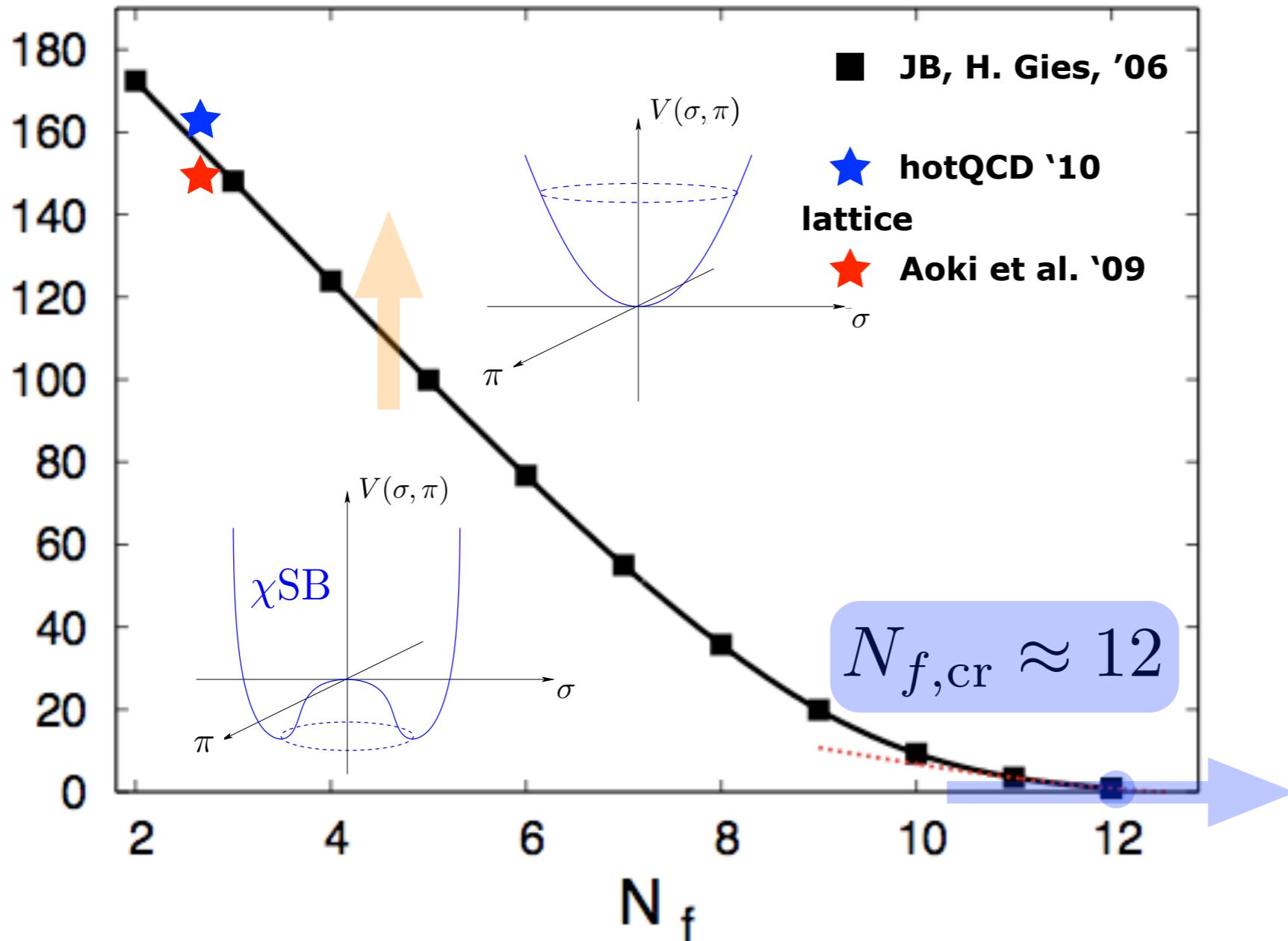
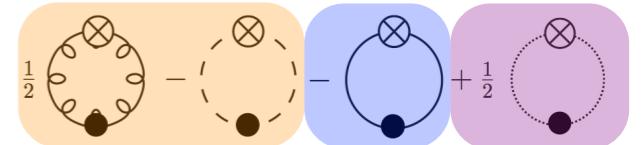


# Many-flavour QCD



**nearly conformal  $SU(N)$ , see talk of H. Terao**

# Many-flavour QCD



Braun, Gies '06 & '09  
Braun, Fischer, Gies '10

see also talk of H. Gies

$$\beta = -\frac{1}{12\pi} (33 - 2N_f) \alpha_s^2 + O(\alpha_s^3)$$

no chiral SB above

$$N_{f*} \approx 12$$

no asymptotic freedom

$$N_{f*} \approx 16$$

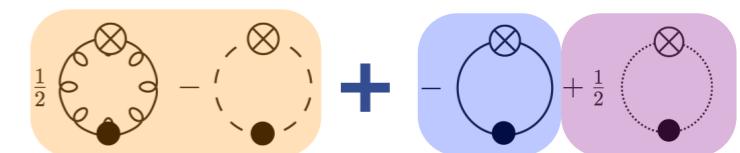
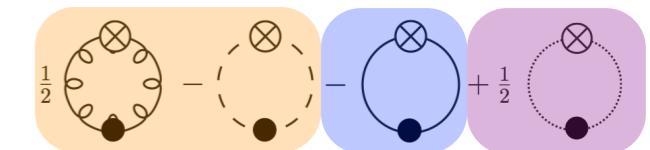
$$|\langle \bar{\psi} \psi \rangle|^{\frac{1}{3}} \sim |T - T_{\text{cr}}|^{\beta}$$

$$|\langle \bar{\psi} \psi \rangle|^{\frac{1}{3}} \sim |N_f - N_{f,\text{cr}}|^{\frac{1}{|\Theta|}} e^{-\frac{\text{const.}}{|N_f - N_{f,\text{cr}}|}}$$

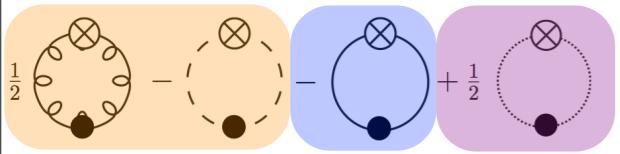
Quantum phase transition

keep running ...

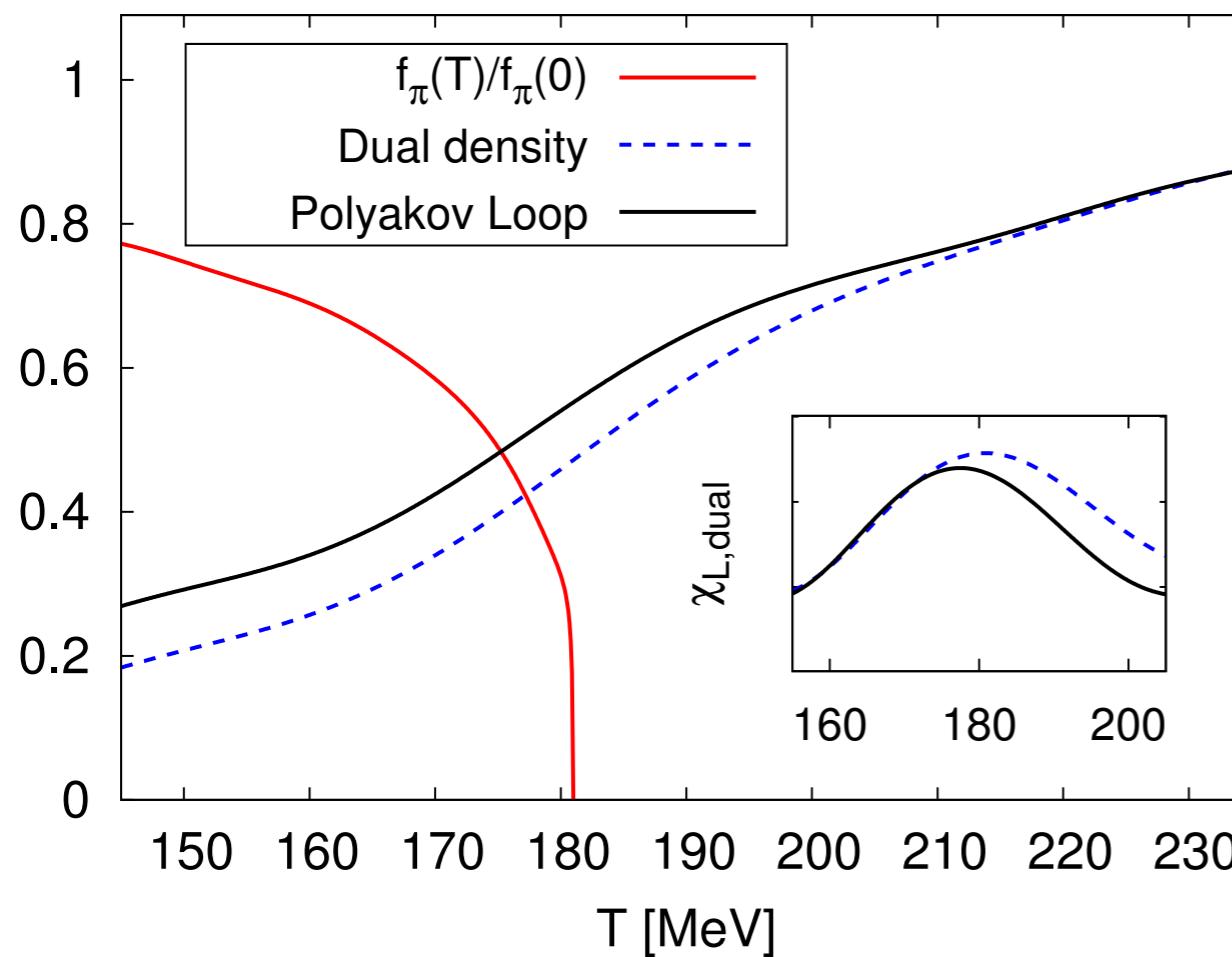
# Phase diagram



# Full dynamical QCD: $N_f = 2$ & chiral limit

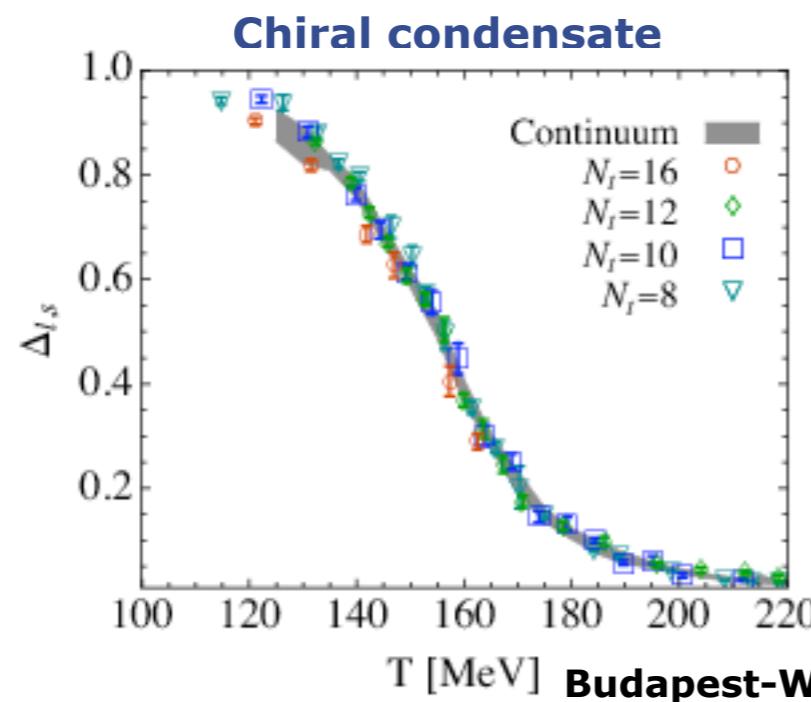


## Phase structure

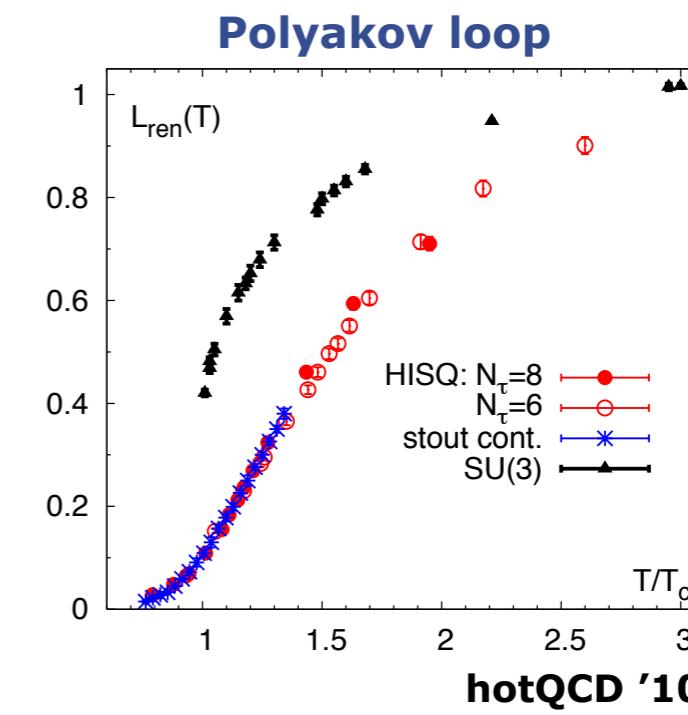


Braun, Haas, Marhauser, JMP '09

- $T_\chi \simeq T_{\text{conf}} \simeq 180 \text{ MeV}$
- **Width**  $\Delta T_{\text{conf}} \simeq \pm 20 \text{ MeV}$



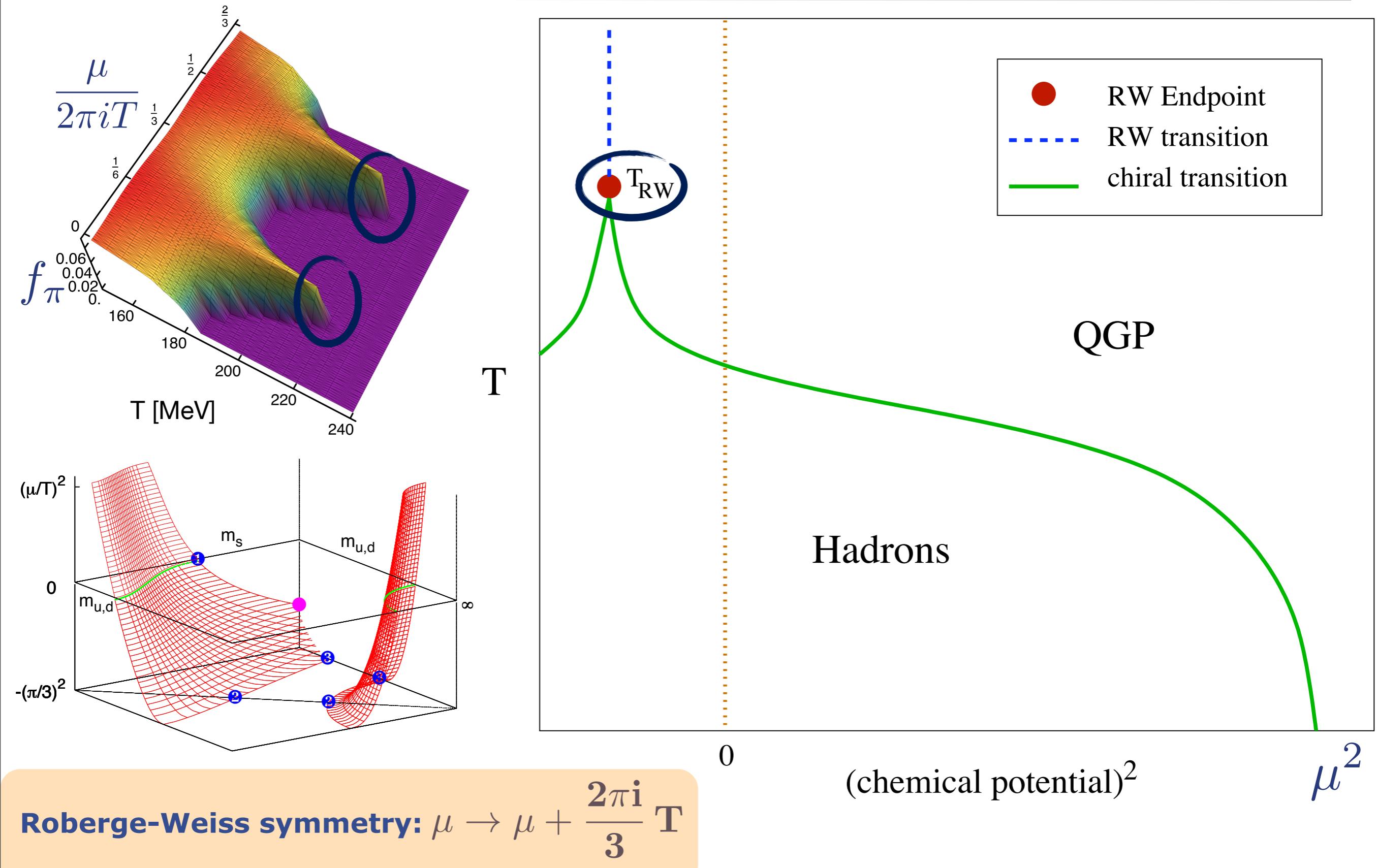
$N_f = 2+1$



Budapest-Wuppertal '10

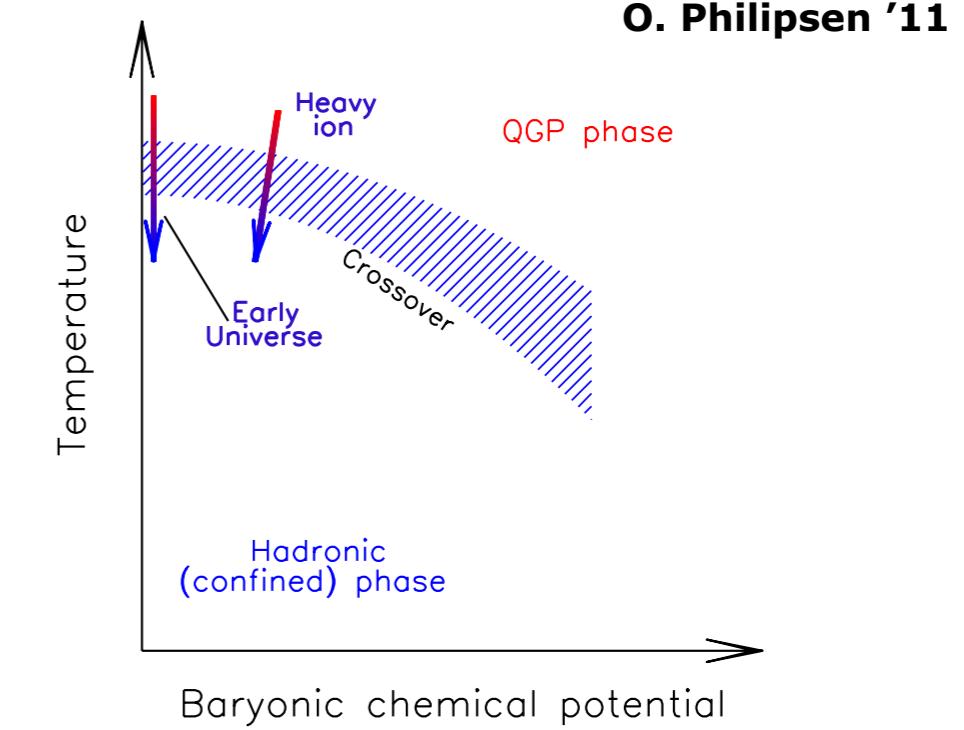
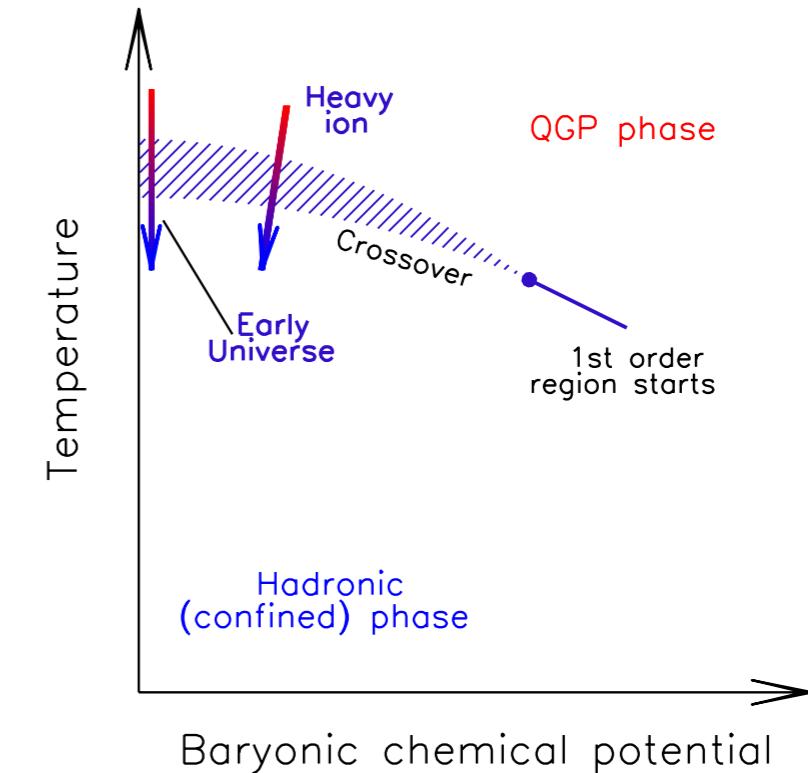
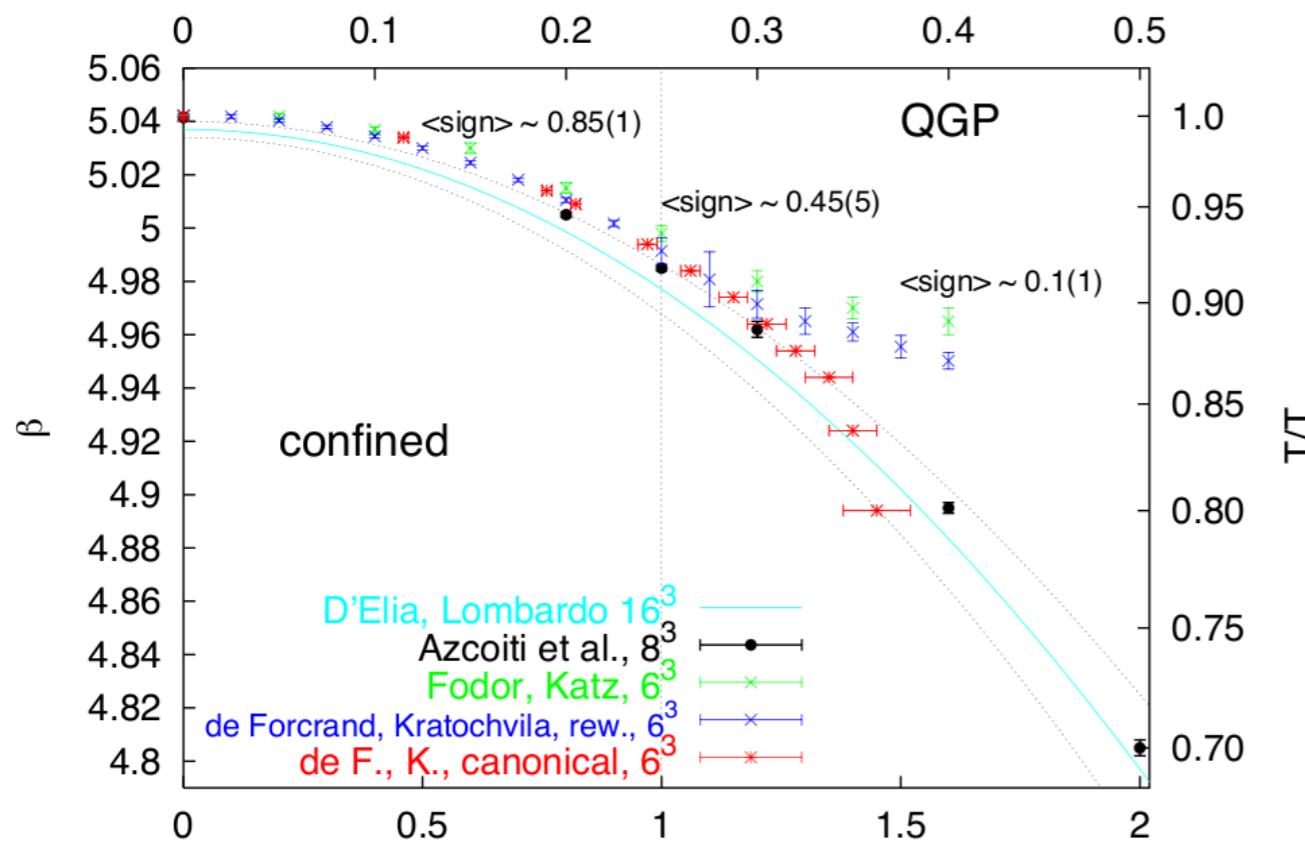
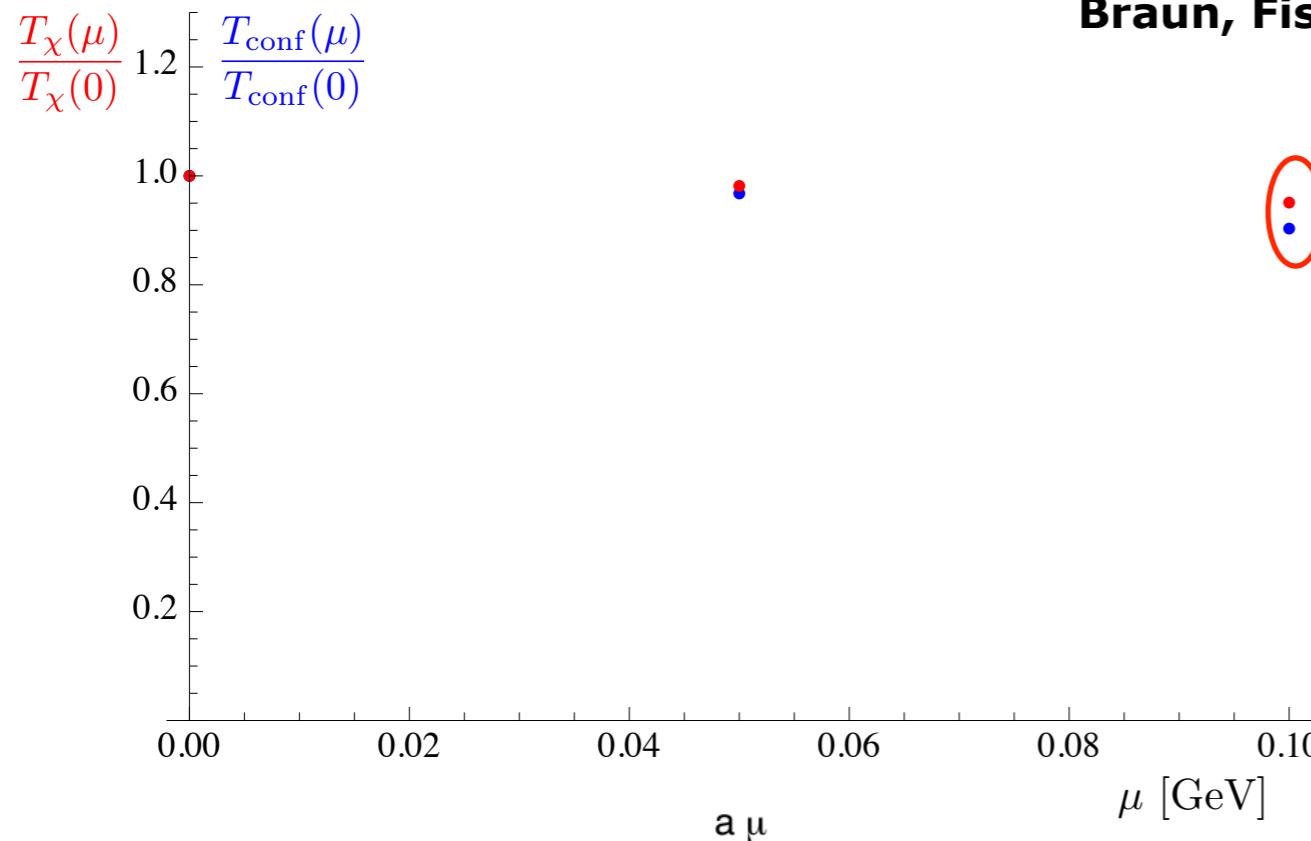
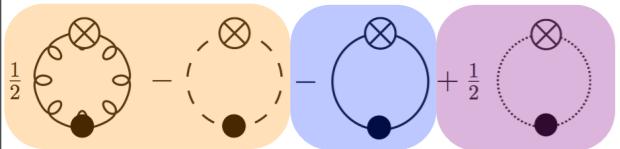
hotQCD '10

# Chemical potential



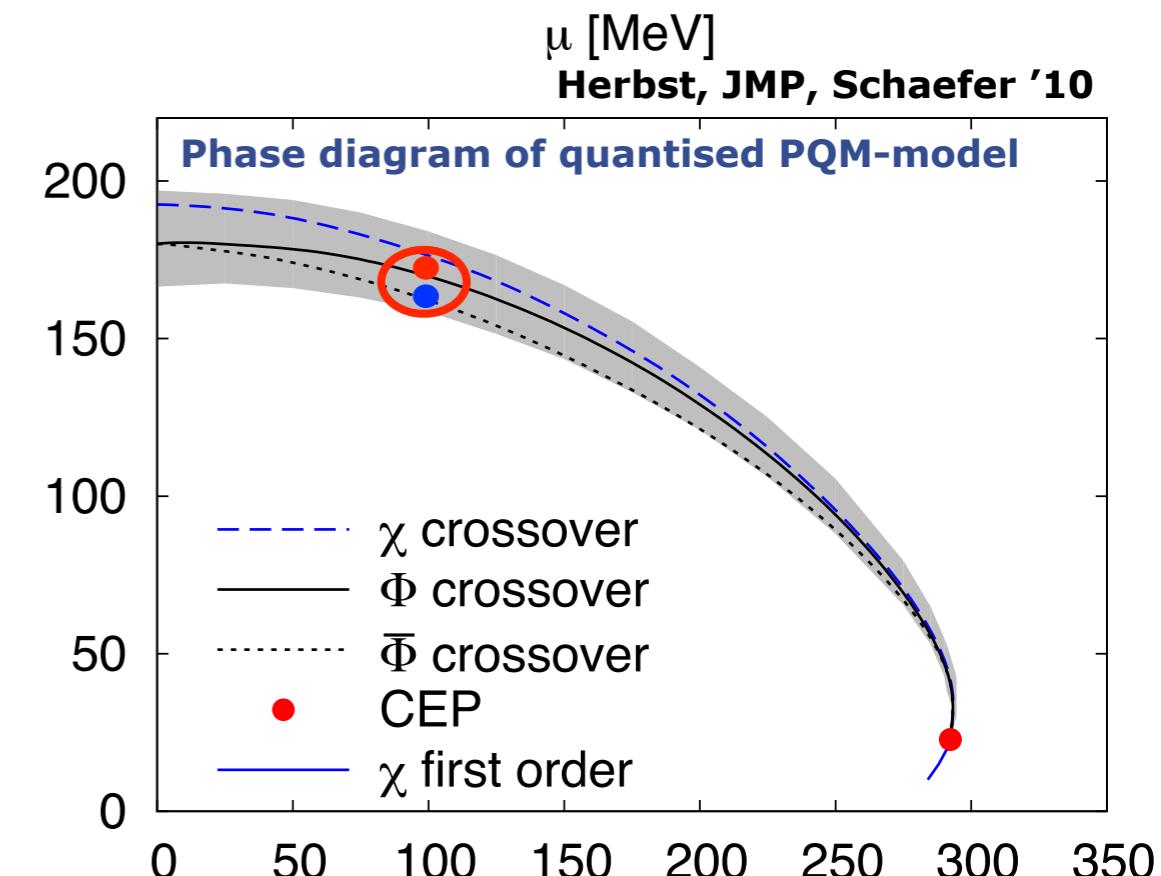
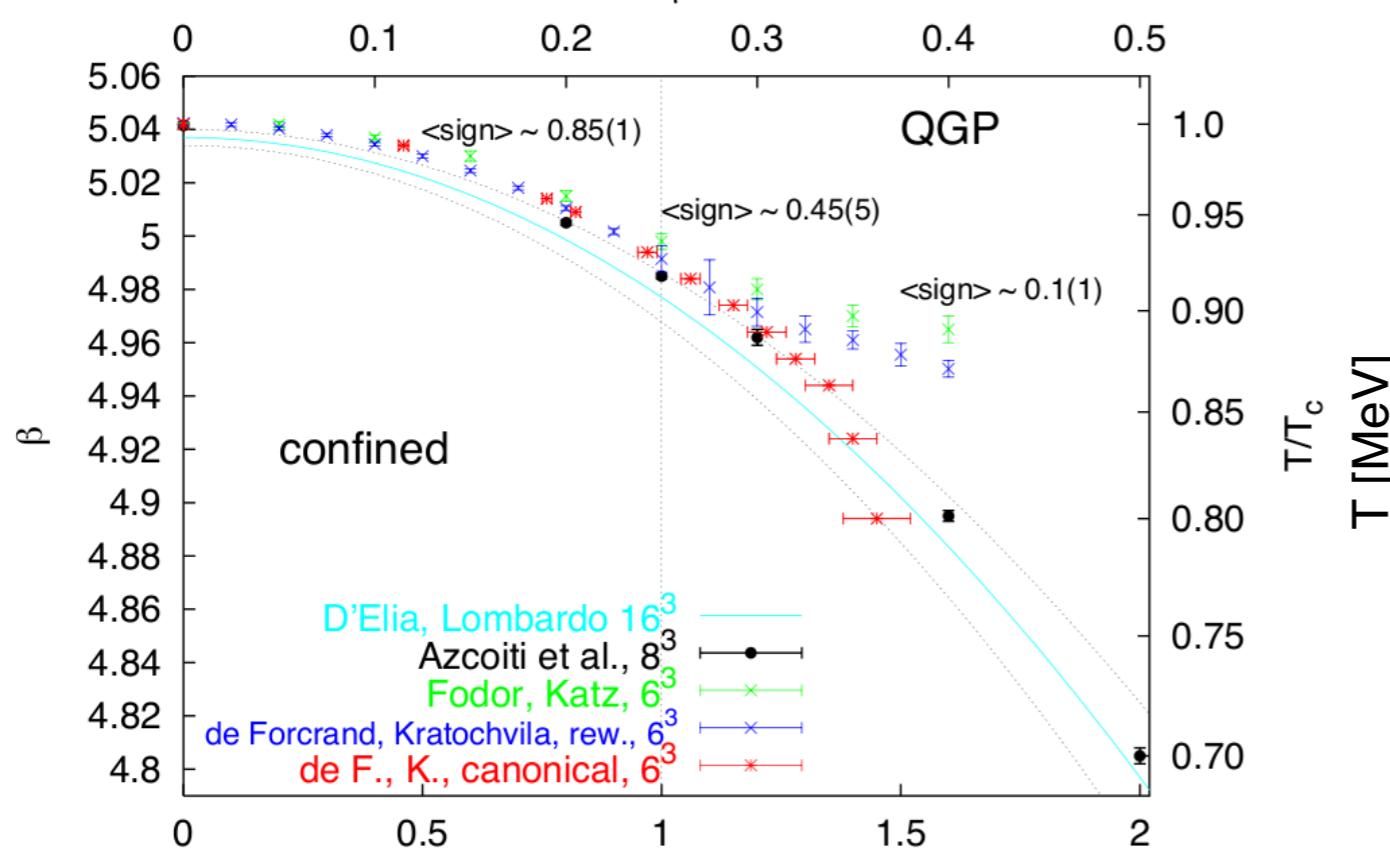
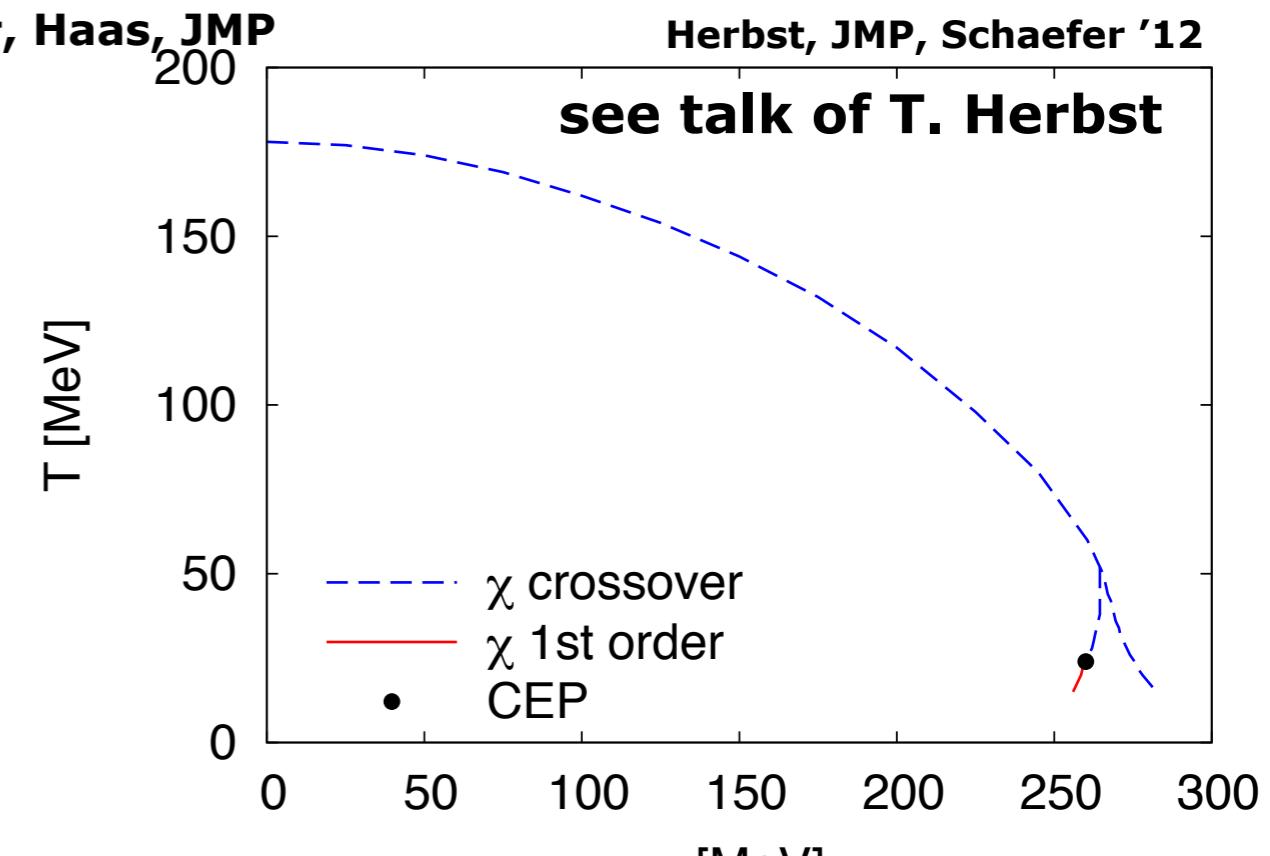
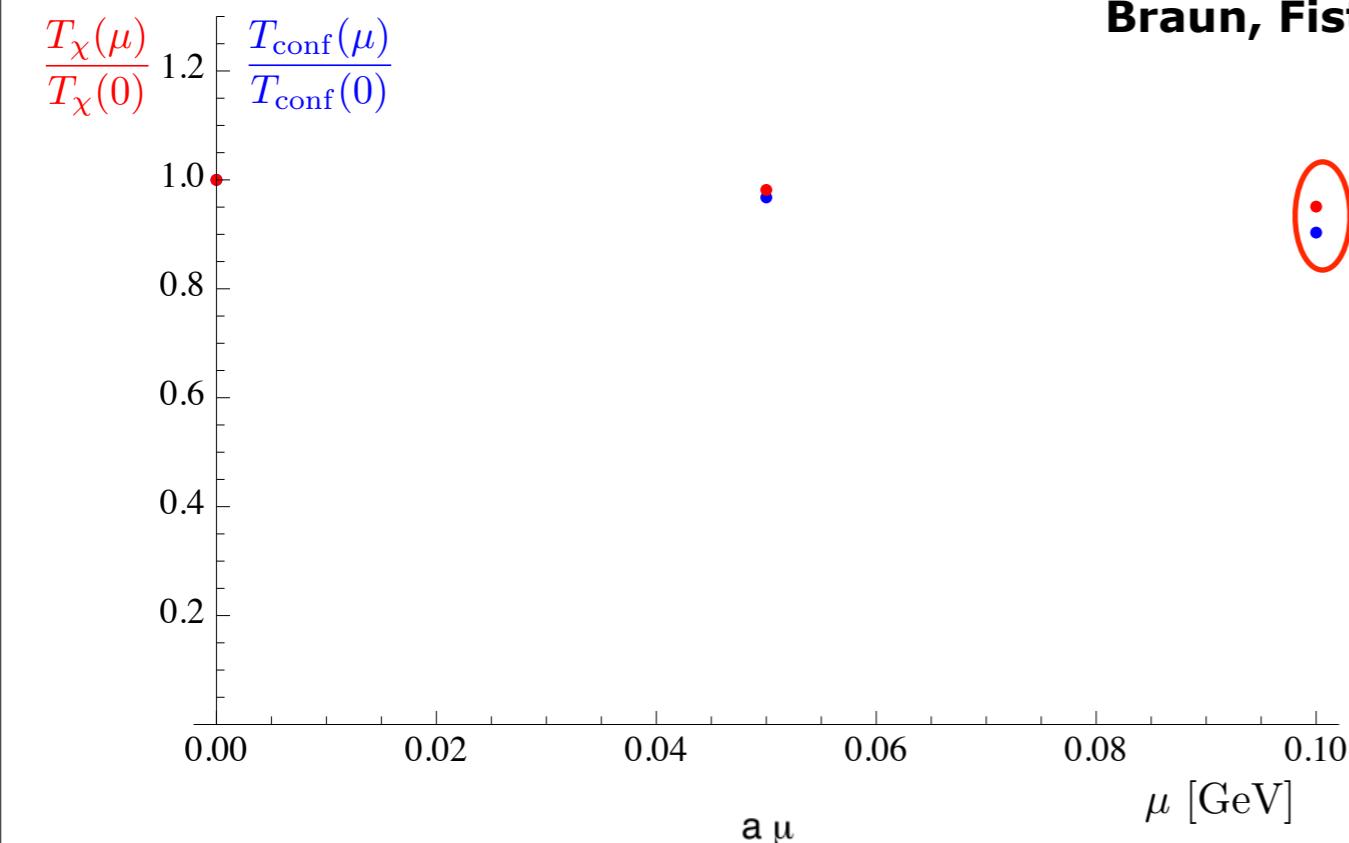
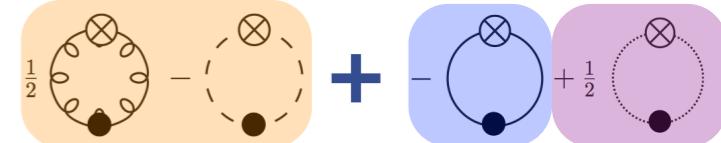
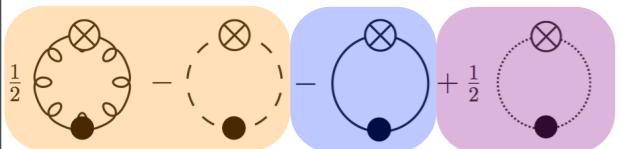
# Chemical potential

## Full dynamical QCD



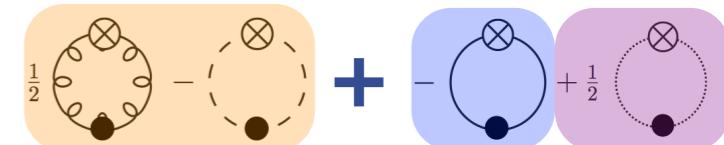
# Chemical potential

## Full dynamical QCD



# Chemical potential

## Polyakov-extended models



## Potential

Polyakov-loop Potential

$$U[\Phi, \bar{\Phi}]$$

Fit to YM-thermodynamics

Fermionic fluctuations

$$\Omega[\Phi, \bar{\Phi}, \sigma, \vec{\pi}]$$

One loop computation

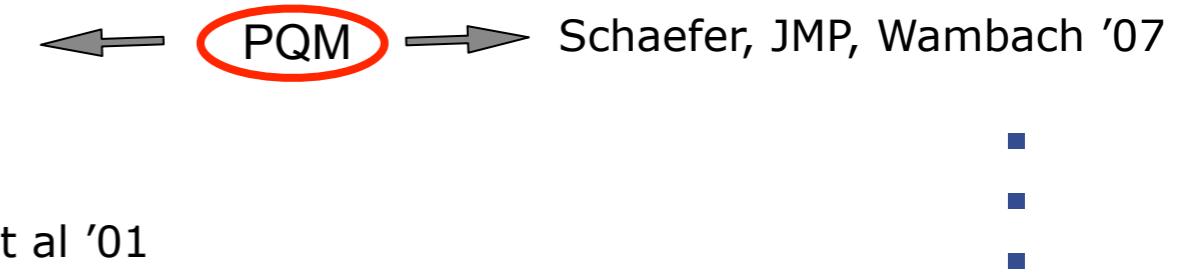
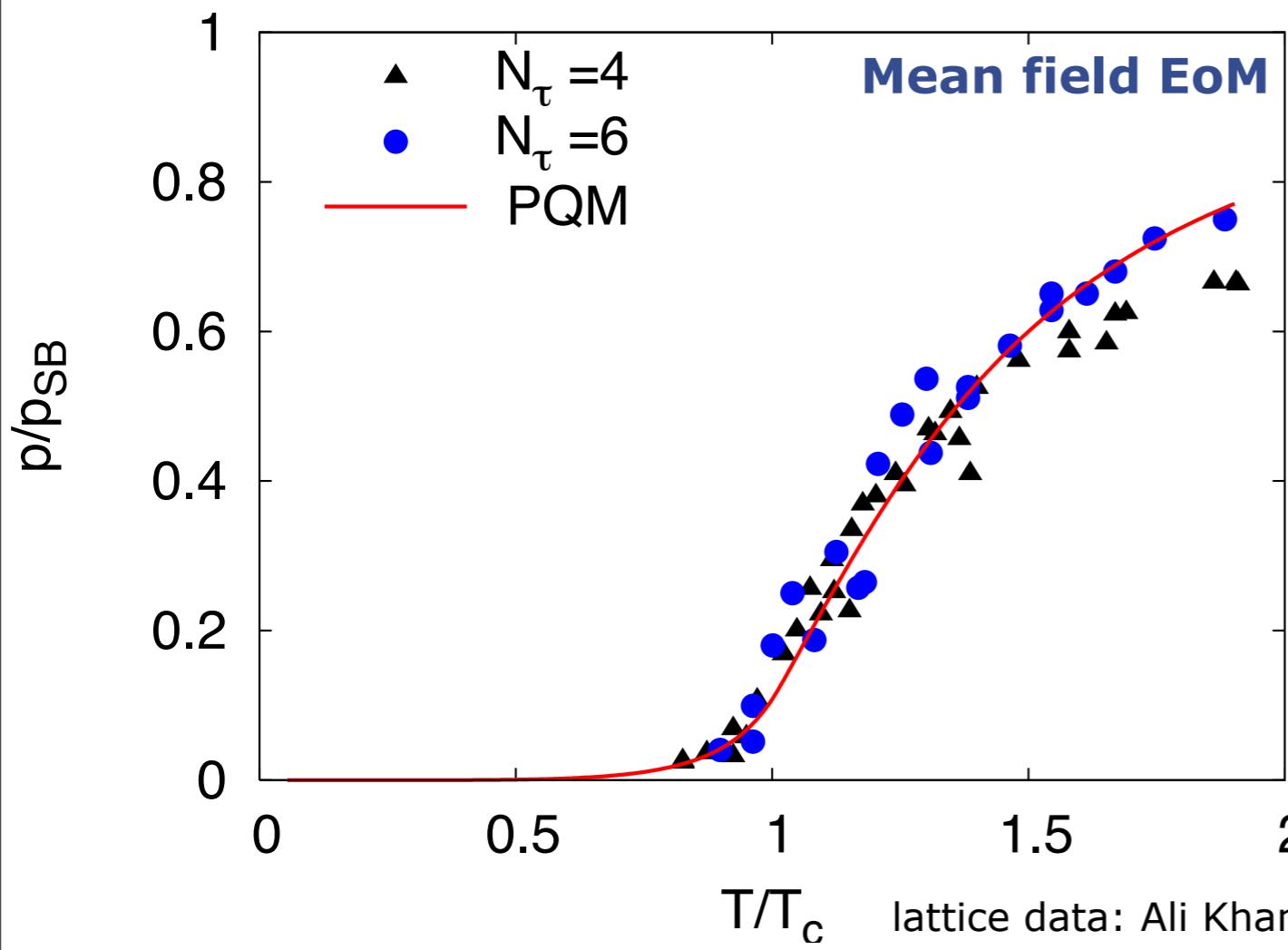
Mesonic potential

$$V[\sigma, \vec{\pi}]$$

Fit of meson phenomenology

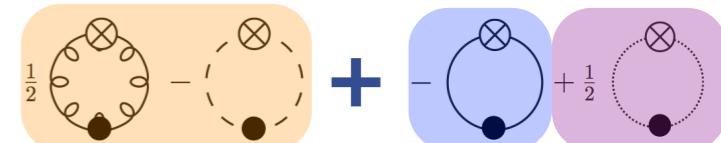
Meisinger, Ogilvie '96

Pisarski '00



# Chemical potential

Dynamical Polyakov-extended models



Herbst, JMP, Schaefer '10

## Potential

Polyakov-loop Potential

$$U[\Phi, \bar{\Phi}]$$

Fit to YM-thermodynamics

Fermionic fluctuations

$$\Omega[\Phi, \bar{\Phi}, \sigma, \vec{\pi}]$$

fermionic fluctuations

Mesonic potential

$$V[\sigma, \vec{\pi}]$$

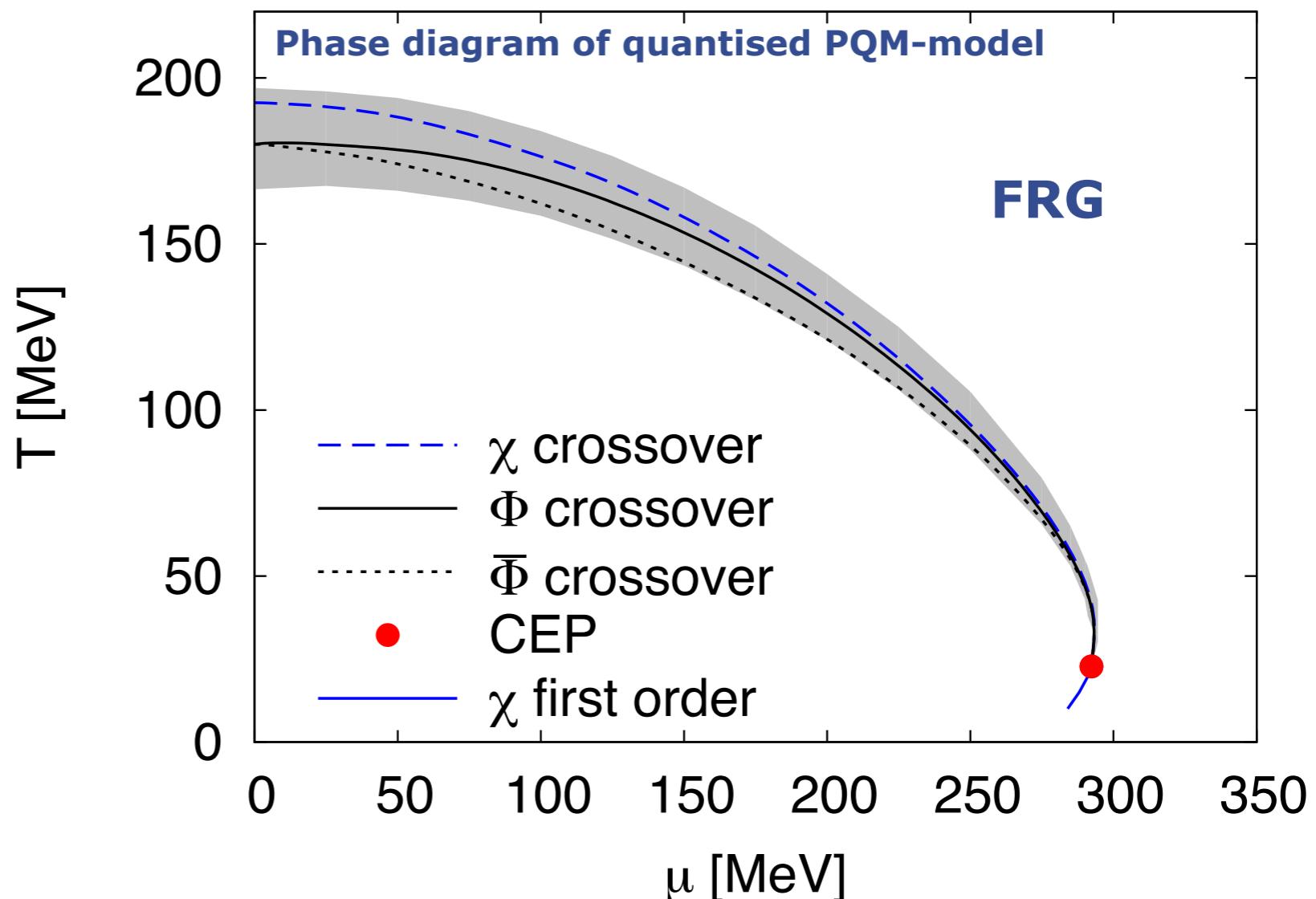
mesonic fluctuations

quark fluctuations change glue dynamics

$$T_{0\text{YM}} \rightarrow T_0(N_f, \mu; m_q)$$

estimated via HTL/HDL computation

Schaefer, JMP, Wambach '07



# Chemical potential

## Polyakov-extended models as reduced QCD

### Potential

Polyakov-loop Potential

$$U[\Phi, \bar{\Phi}]$$

Fermionic fluctuations

$$\Omega[\Phi, \bar{\Phi}, \sigma, \vec{\pi}]$$

Mesonic potential

$$V[\sigma, \vec{\pi}]$$

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{ (Diagram A)} - \text{ (Diagram B)} - \text{ (Diagram C)} + \frac{1}{2} \text{ (Diagram D)}$$

Flow equation for QCD

The flow equation for QCD is represented by a series of four diagrams. The first diagram is a solid circle with a cross at the top and a dot at the bottom. The second diagram is a dashed circle with a cross at the top and a dot at the bottom. The third diagram is a solid circle with a cross at the top and a dot at the bottom. The fourth diagram is a dotted circle with a cross at the top and a dot at the bottom. The equation is written as  $\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{ (Diagram A)} - \text{ (Diagram B)} - \text{ (Diagram C)} + \frac{1}{2} \text{ (Diagram D)}$ .

# Chemical potential

## Polyakov-extended models as reduced QCD

### Towards QCD

Braun, Haas, JMP, in prep  
JMP '10

Polyakov-loop Potential

$$U[\Phi, \bar{\Phi}]$$

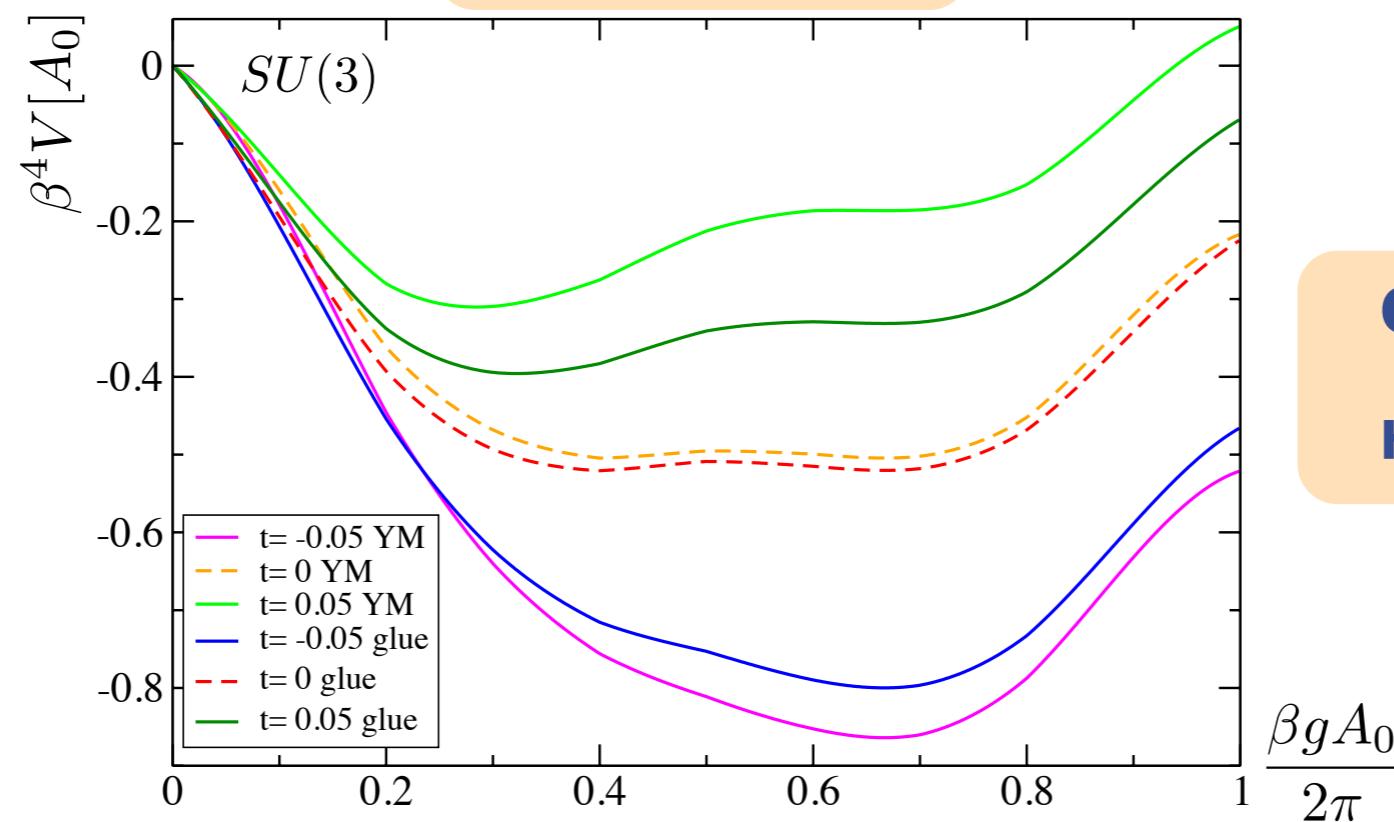
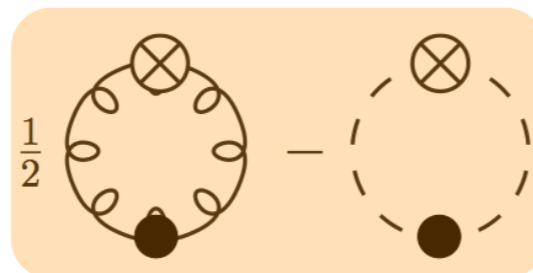
+

Fermionic fluctuations

$$\Omega[\Phi, \bar{\Phi}, \sigma, \vec{\pi}]$$

Mesonic potential

$$V[\sigma, \vec{\pi}]$$

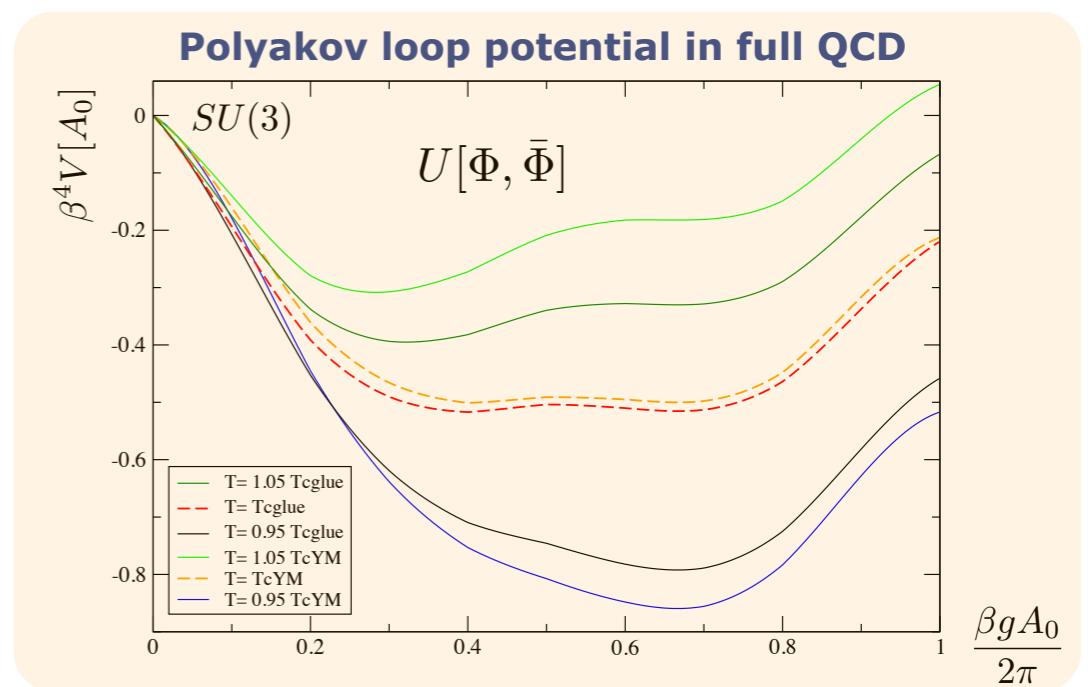
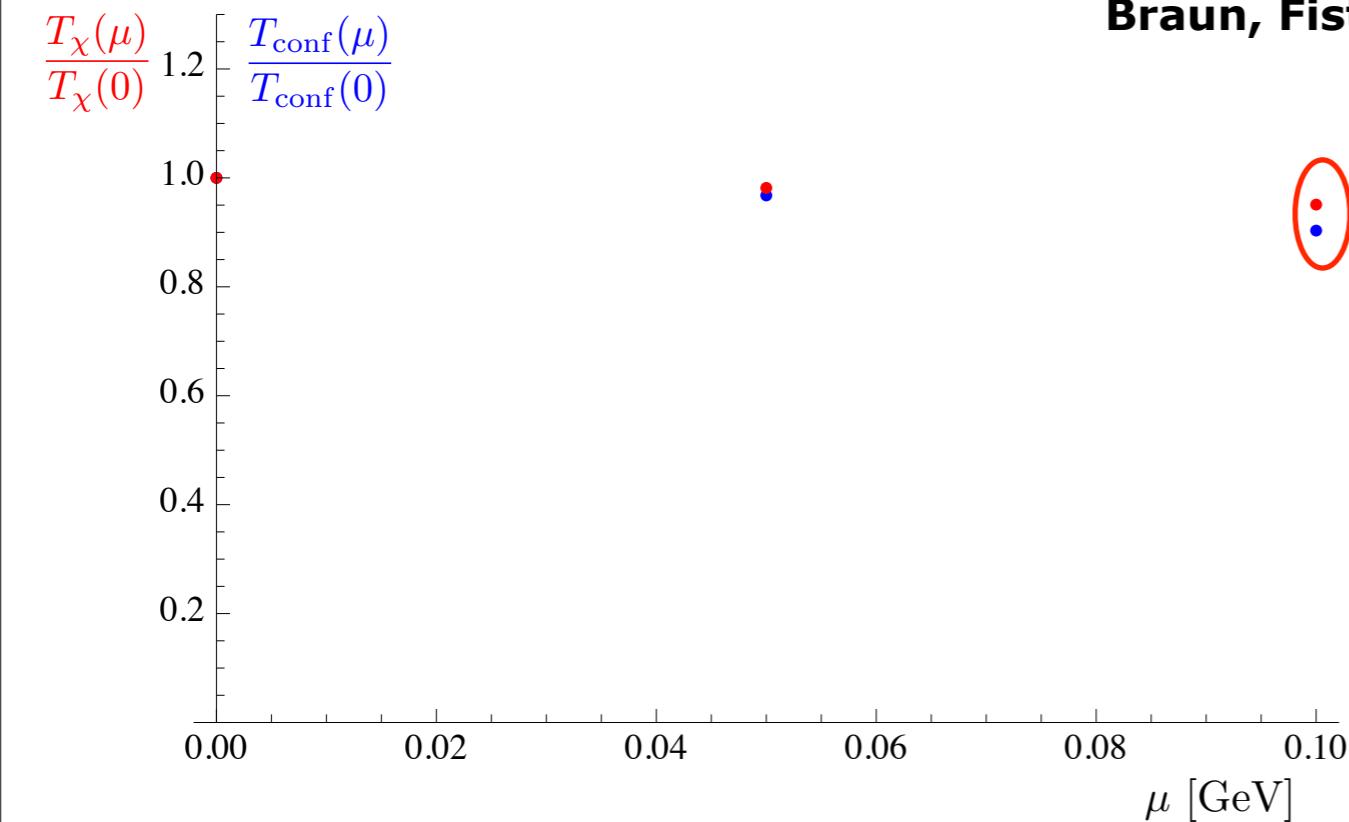
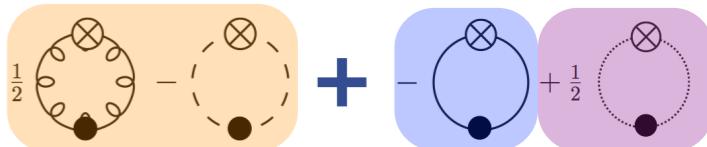
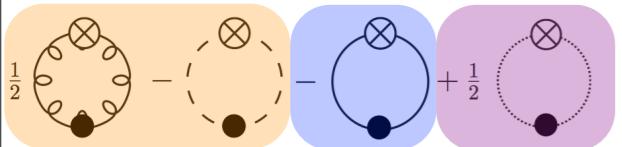


**QCD confirmation of  
HTL/HDL quark estimate**

$$(\beta^4 V)_{\text{glue}}[t, A_0] \simeq (\beta^4 V)_{\text{YM}}[t_{\text{YM}}(t), A_0]$$

# Chemical potential

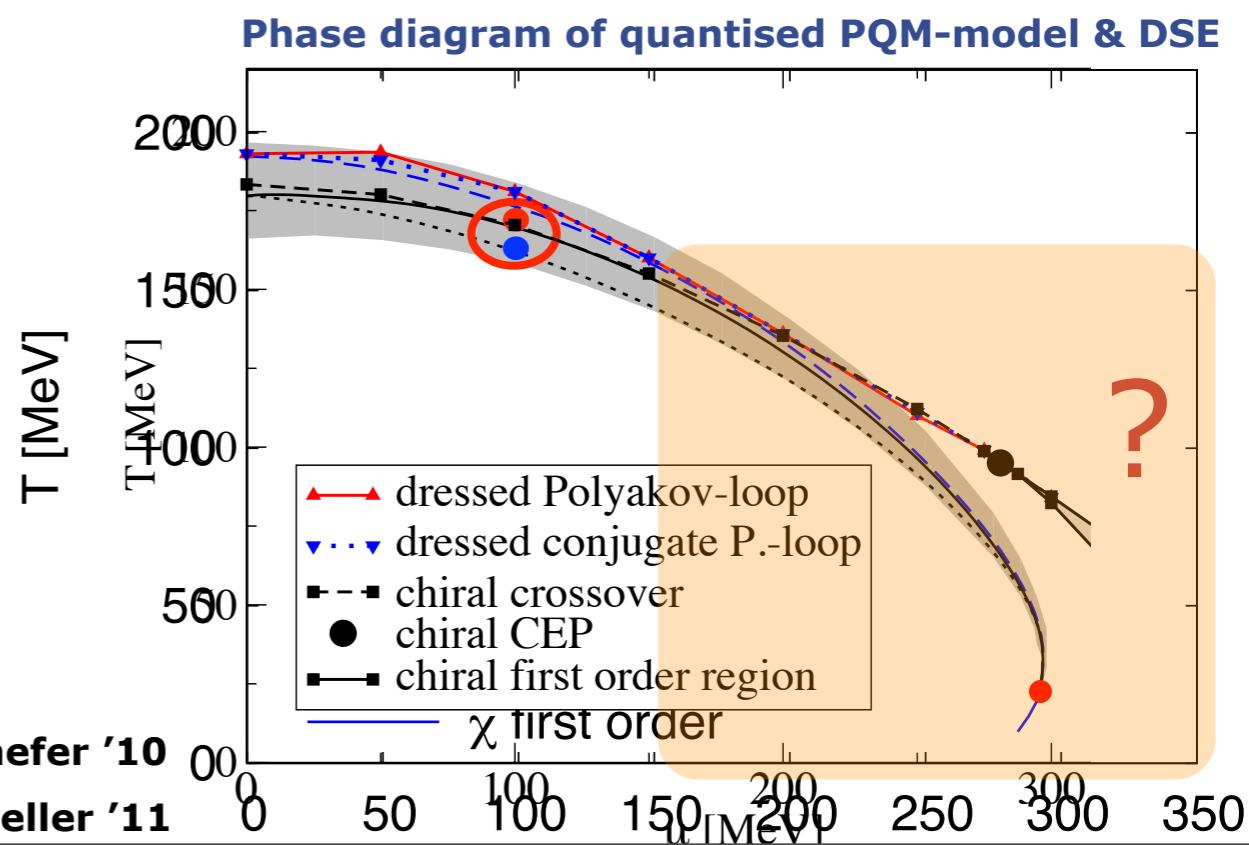
## Full dynamical QCD



**Critical point unlikely for**

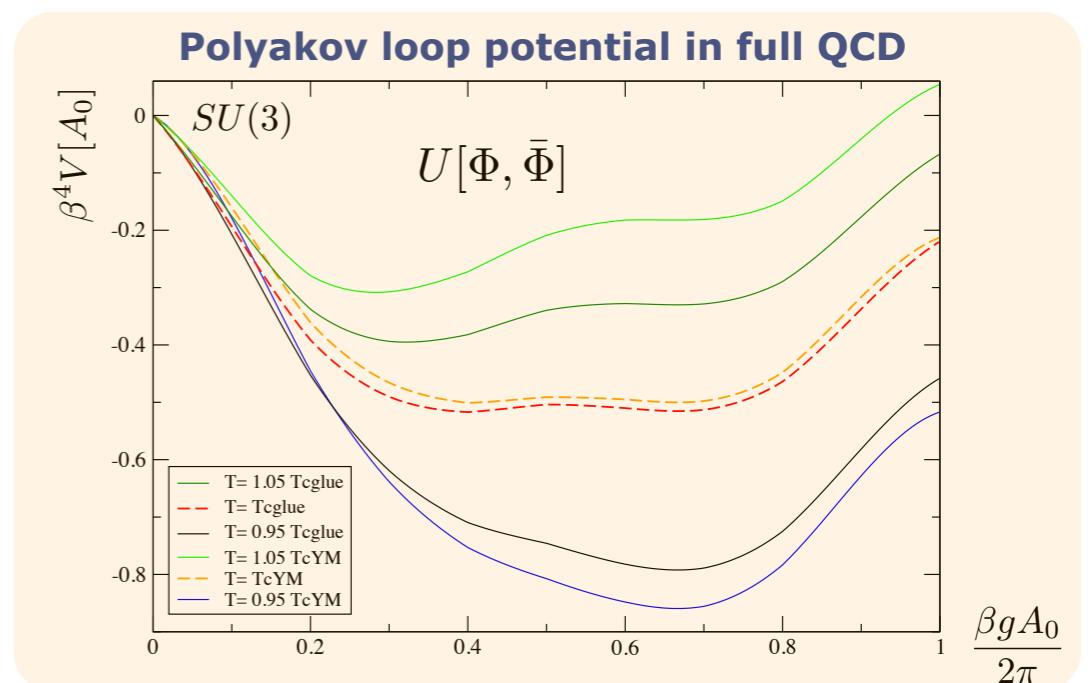
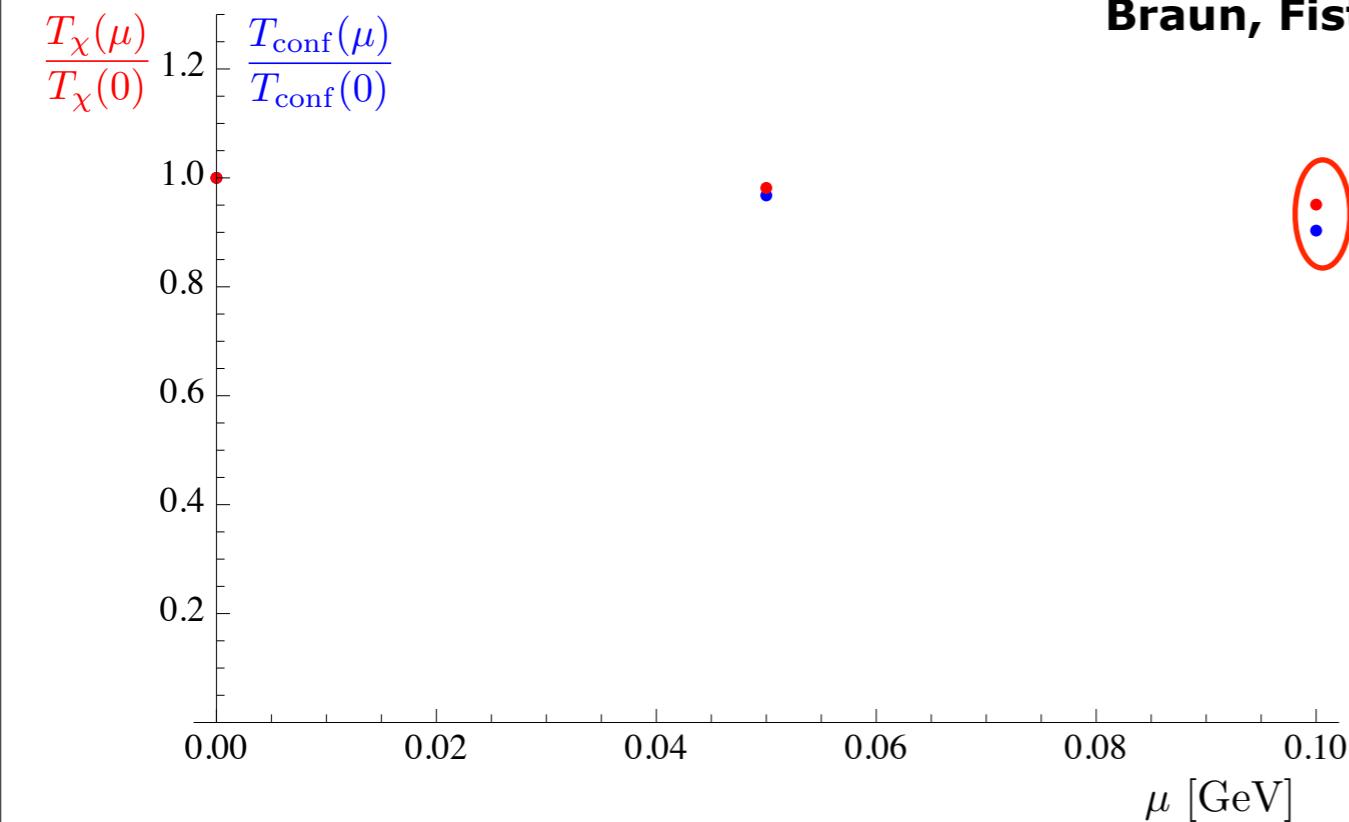
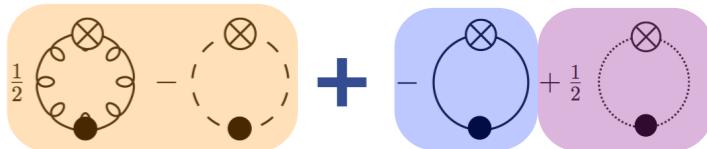
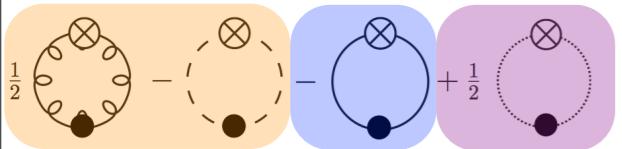
$$\frac{\mu_B}{T} < 2$$

**PQM:** Herbst, JMP, Schaefer '10  
**DSE:** Fischer, Lücker, Mueller '11



# Chemical potential

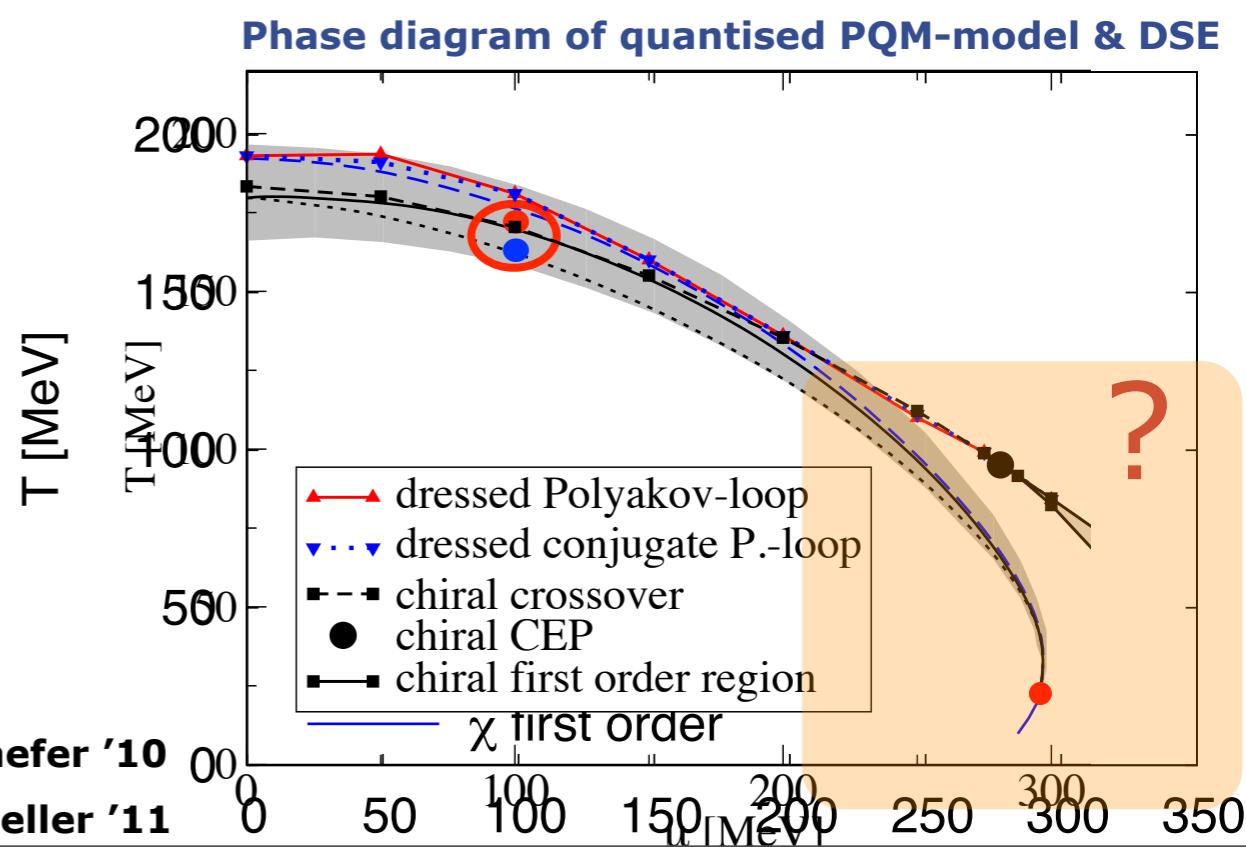
## Full dynamical QCD

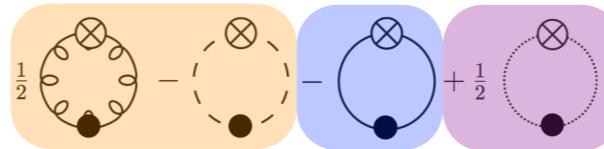


**Critical point unlikely for**

$$\frac{\mu_B}{T} < 4.5$$

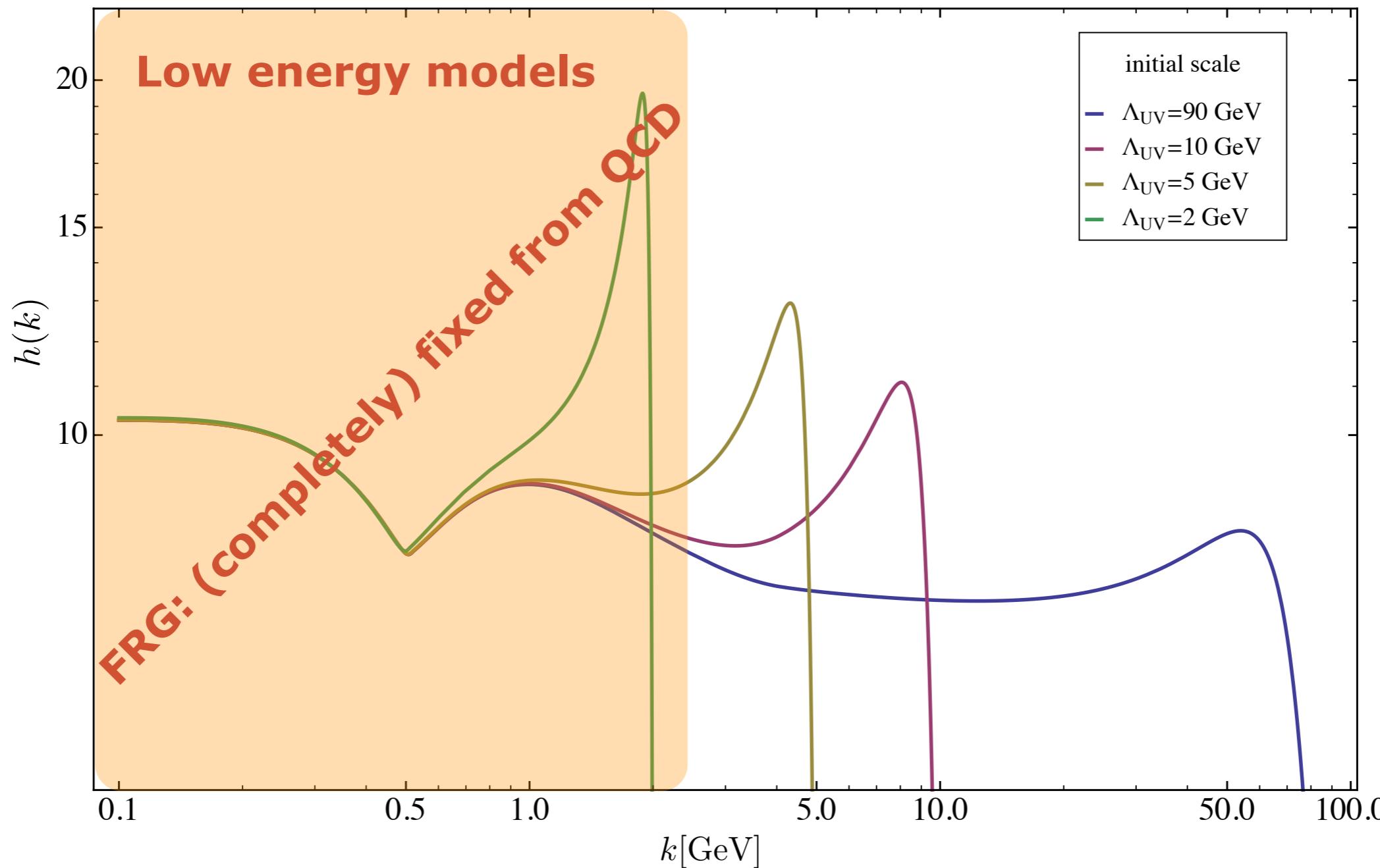
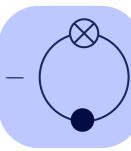
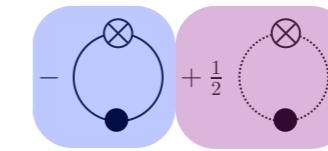
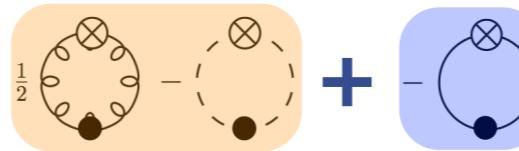
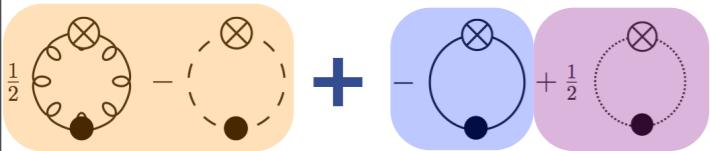
PQM: Herbst, JMP, Schaefer '10  
DSE: Fischer, Lücker, Mueller '11



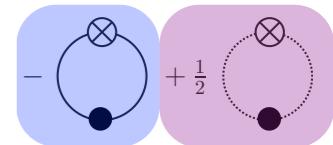


keep running ...

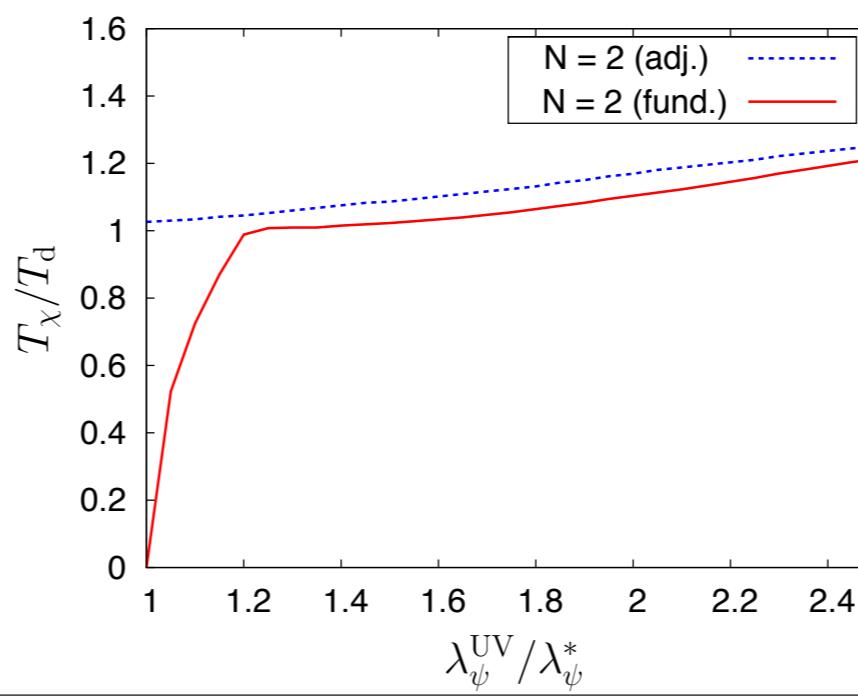
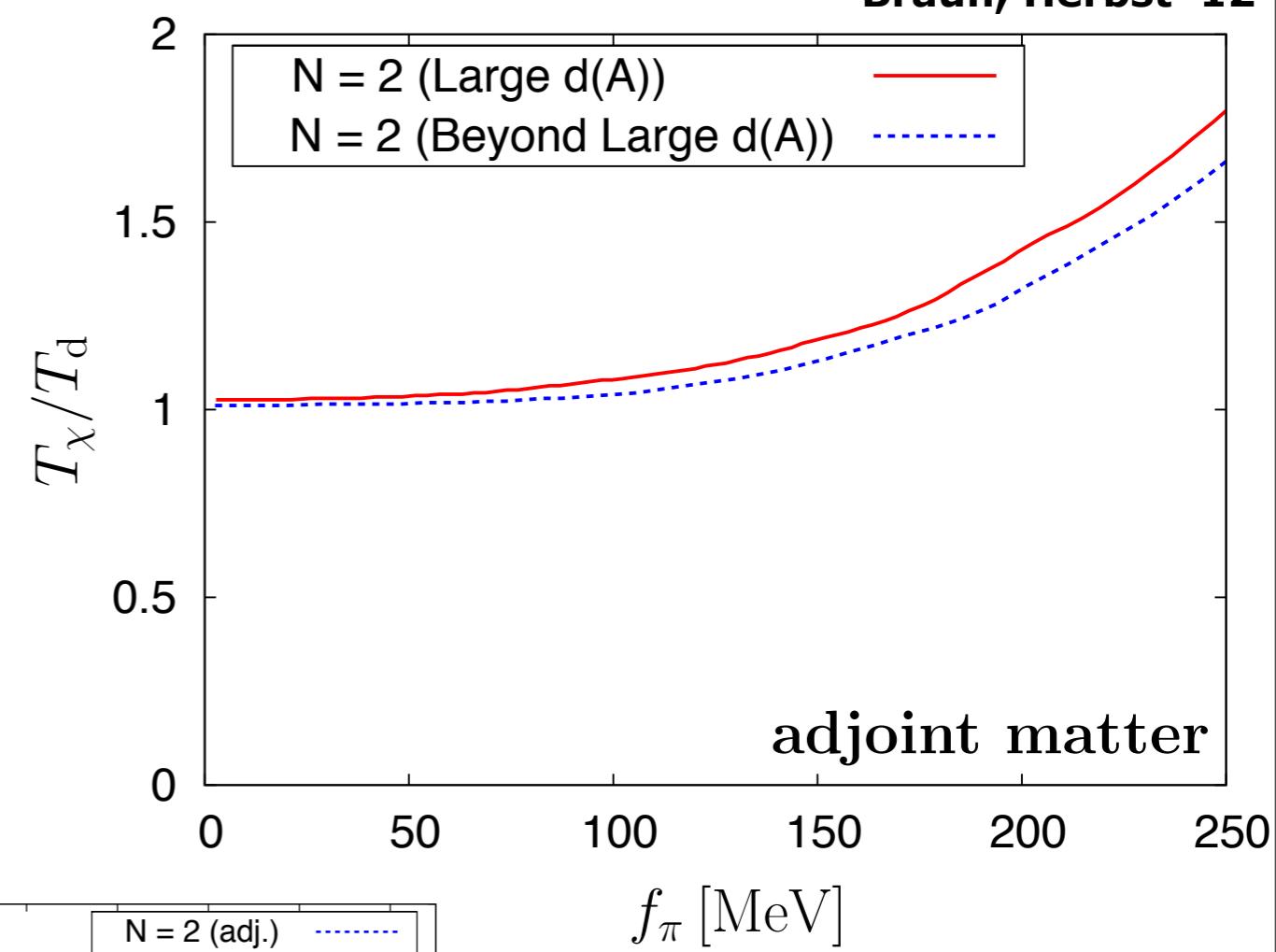
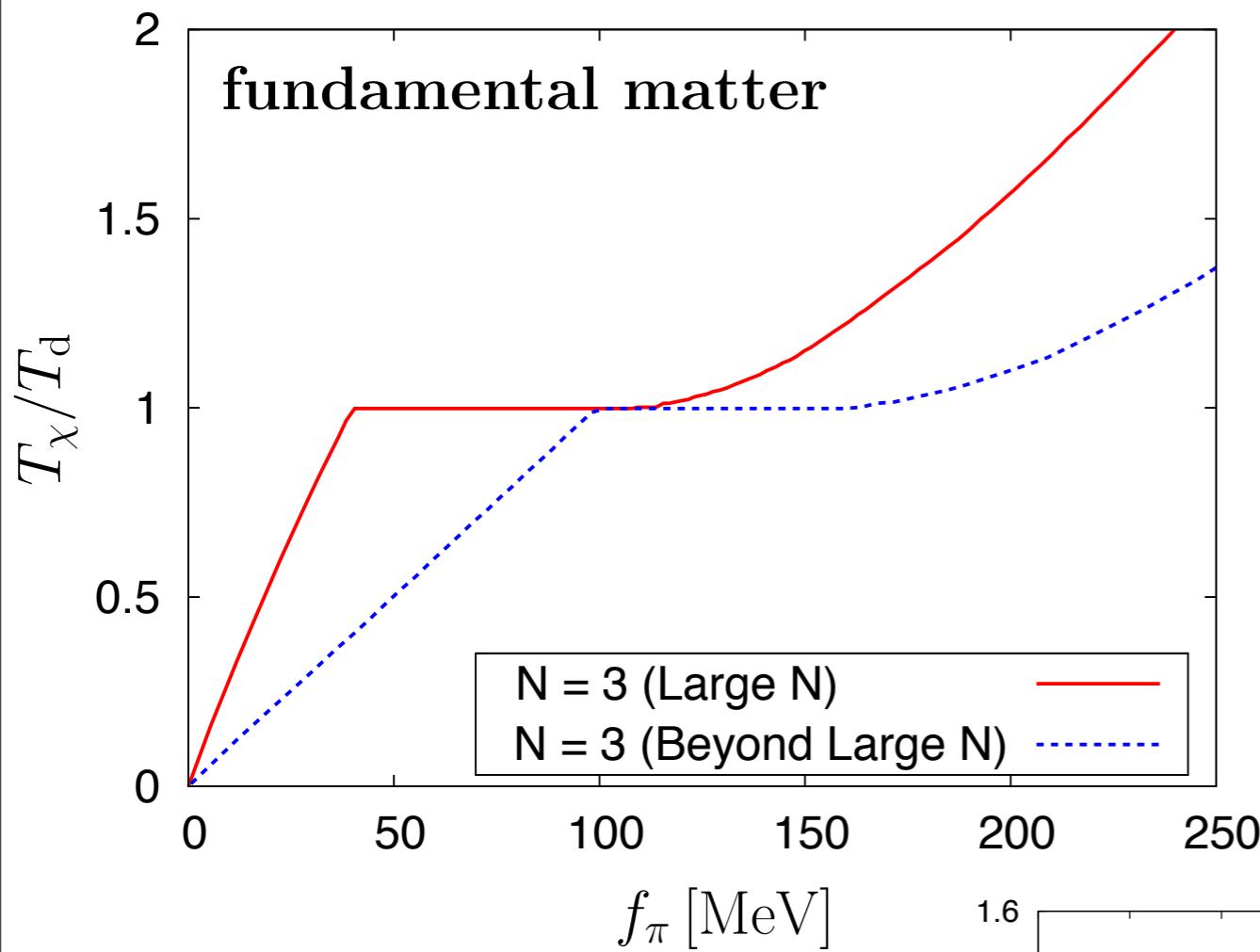
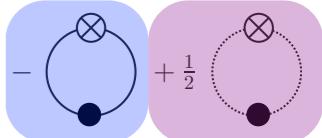
## Model results on the phase structure of QCD



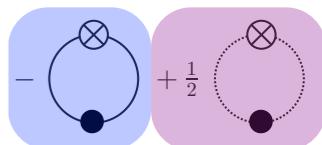
# Relation between chiral symmetry breaking & confinement



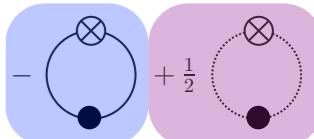
# Confinement & chiral symmetry



# Deconfinement & baryonic fluctuations



# Deconfinement & baryonic fluctuations



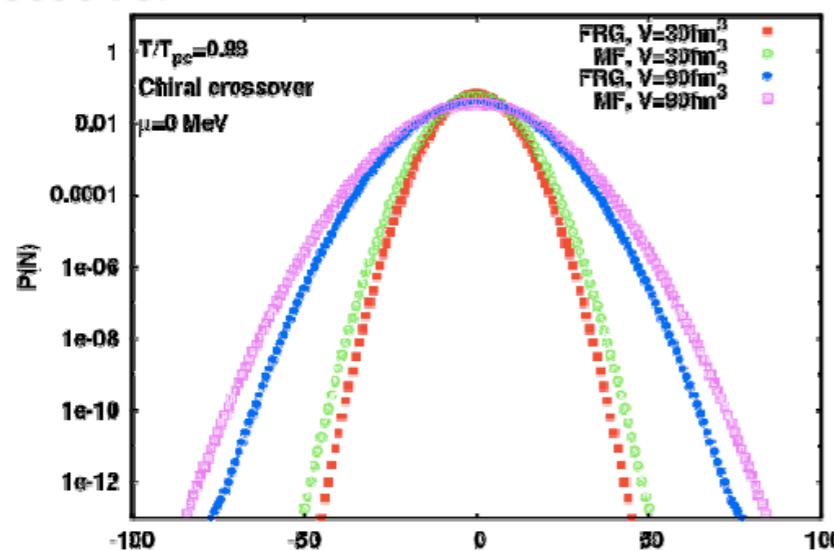
## Probability distribution P

Morita, Skokov, Friman, Redlich

$$P(N; T, \mu, V) = \frac{\mathcal{Z}(T, V, N) e^{\beta \mu N}}{\mathcal{Z}(T, V, \mu)}$$

P(N) in QM model

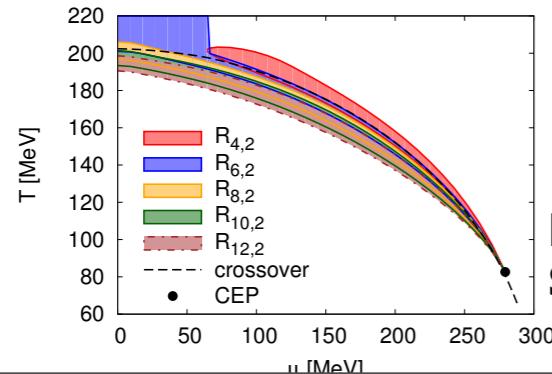
Crossover



MF case exhibits broader P(N)

Different tail behavior in FRG ( $V=90\text{fm}^3$ )

$$Z(T, V, \mu) = \sum_{N=-\infty}^{N=\infty} \mathcal{Z}(T, V, N) e^{\beta \mu N}$$

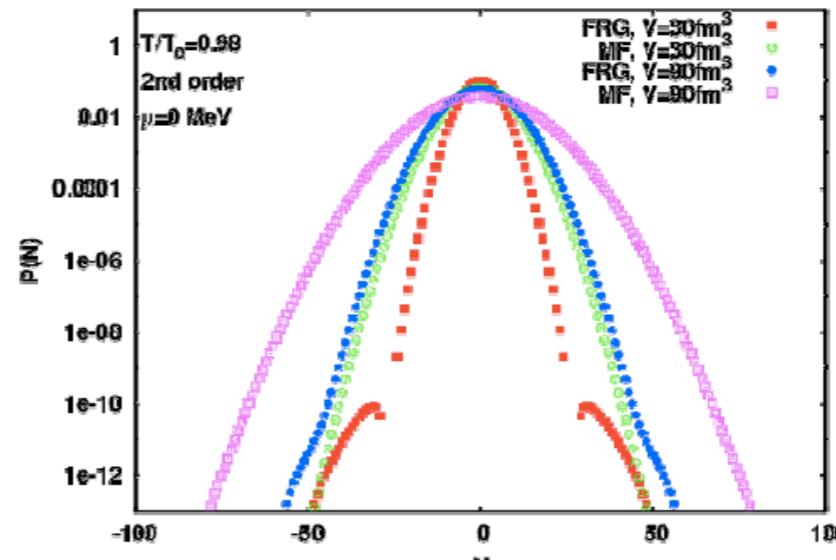


cumulants

higher cumulants  
Schaefer, Wagner '12

$P(N)$  in QM model

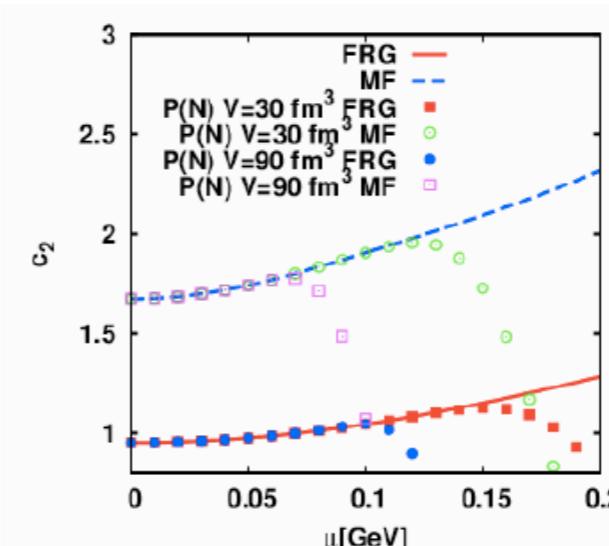
2<sup>nd</sup> order (chiral limit)



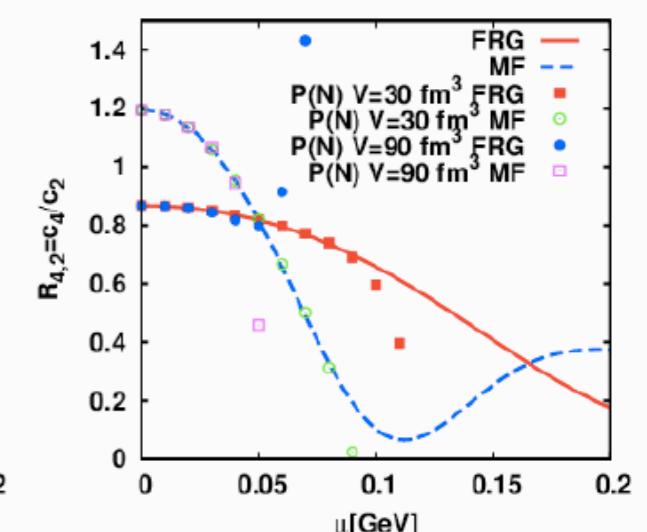
Enhanced tail behavior in FRG : consistent w/ Saddle point

Caveat : physical parameters are not the same as MF

see talk of K. Morita

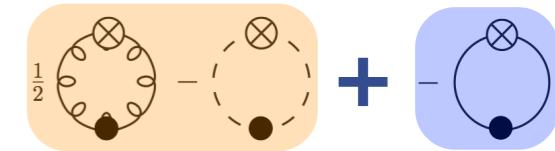


$$c_2 = \frac{1}{VT^3} \langle (\delta N)^2 \rangle$$

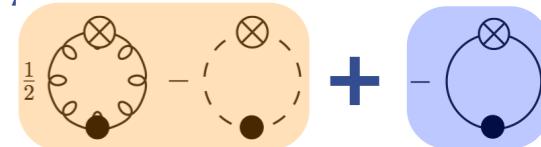


$$c_4 = \frac{1}{VT^3} [\langle (\delta N)^4 \rangle - 3 \langle (\delta N)^2 \rangle^2]$$

# Gauge independence of chiral symmetry breaking from four-fermi interactions

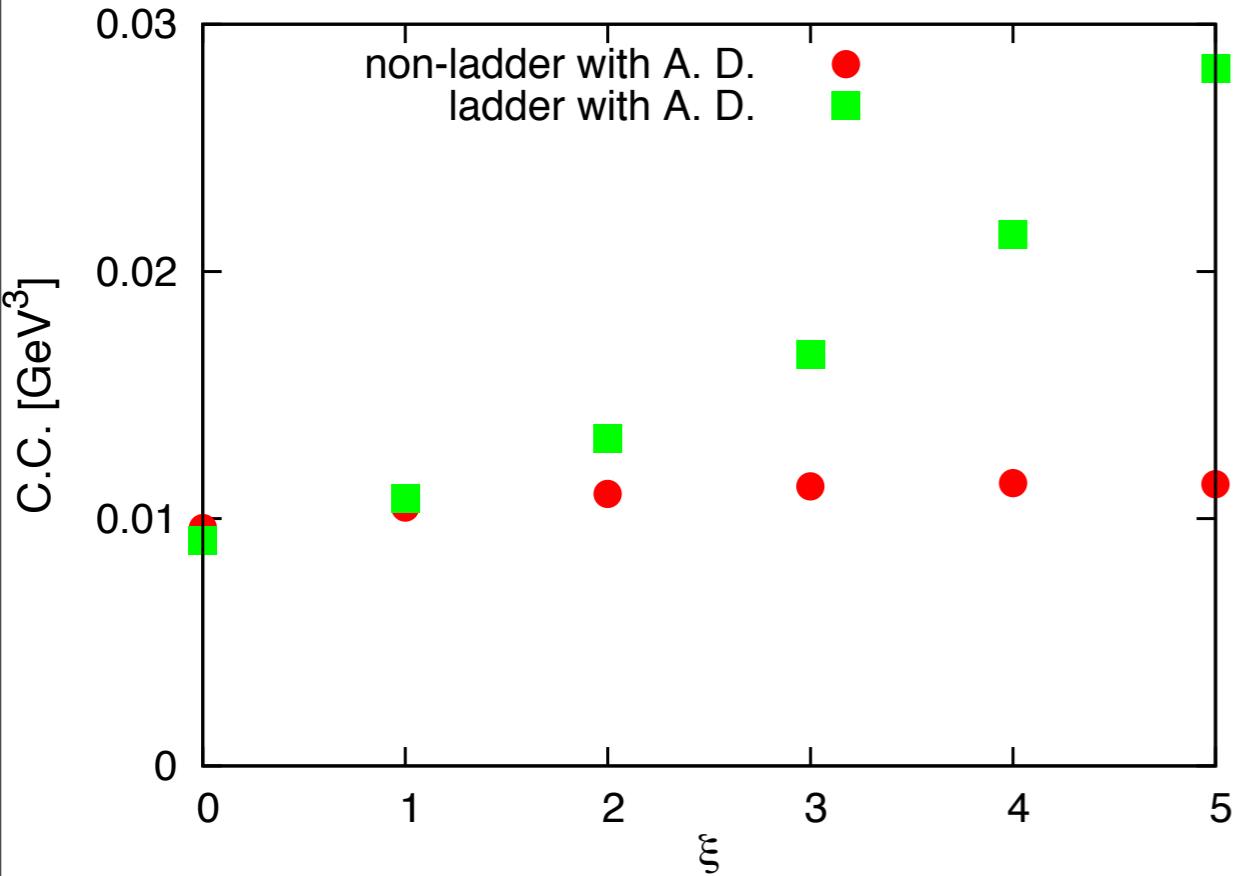


# Gauge-independence of chiral symmetry breaking

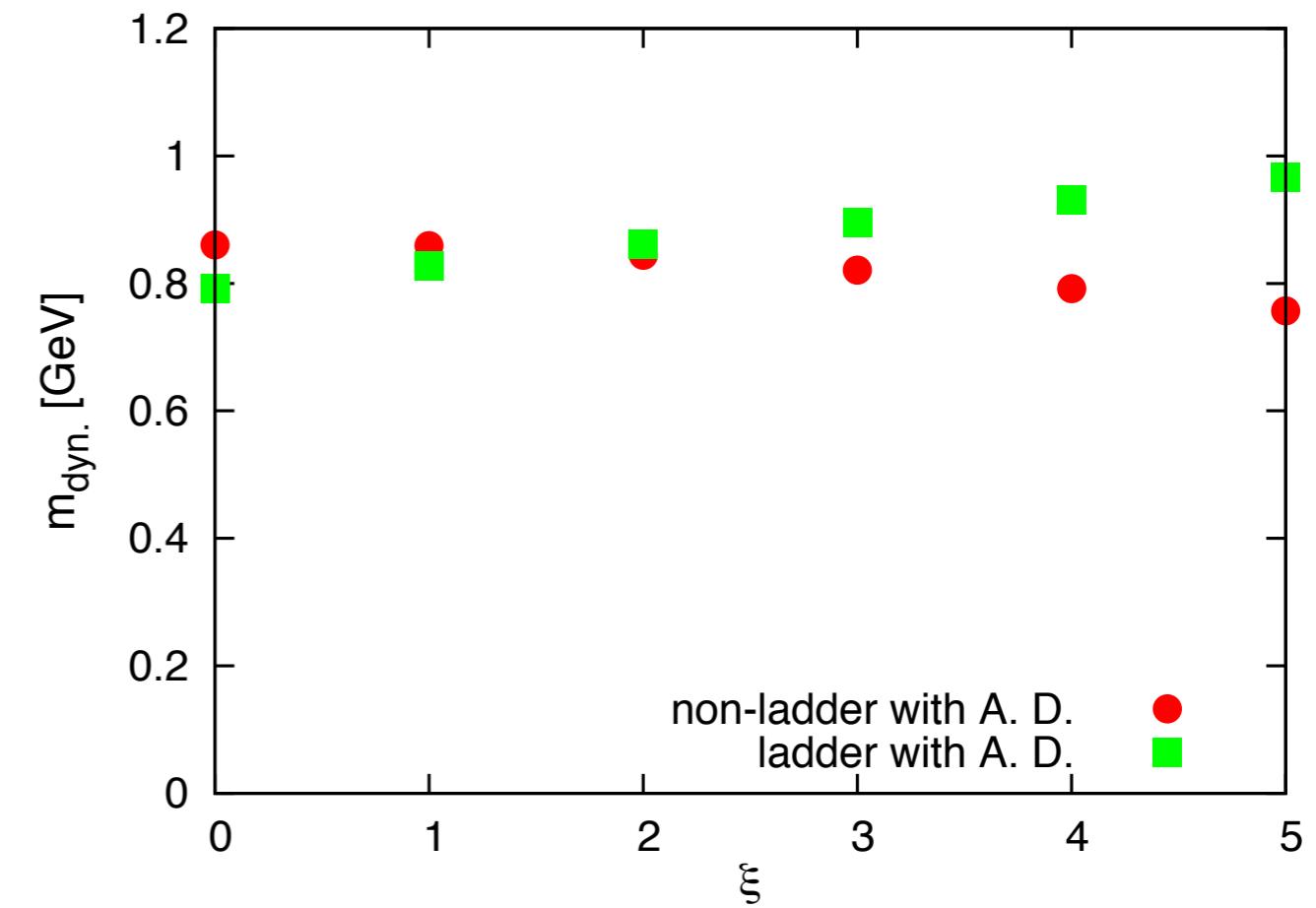


Aoki, Sato

chiral condensate



constituent mass

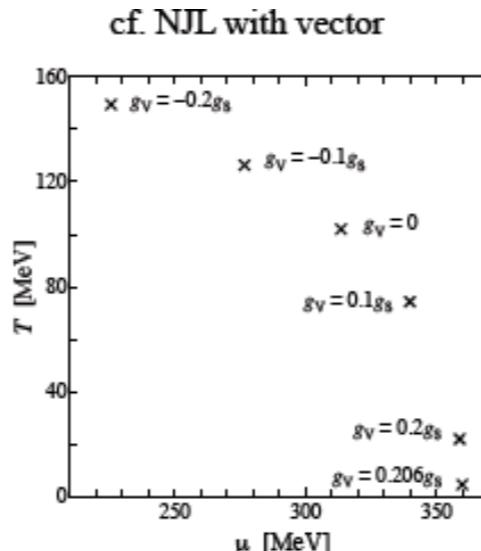
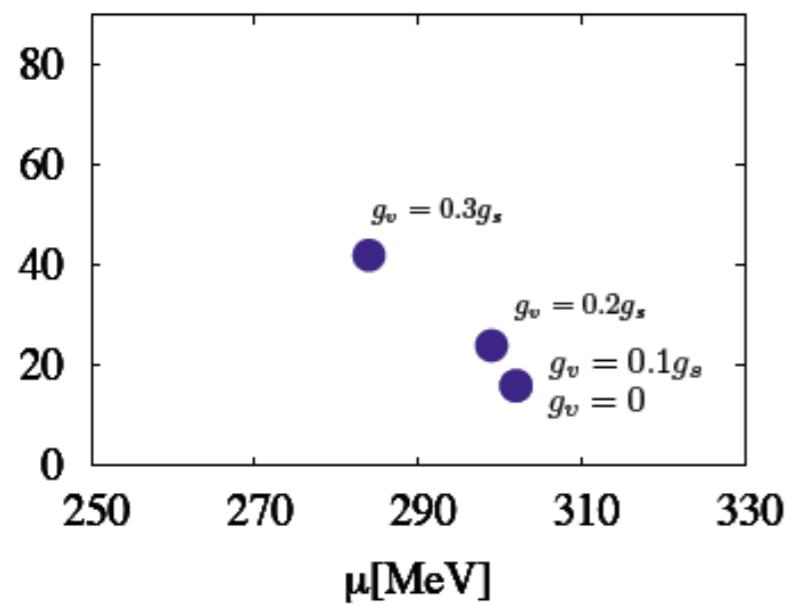
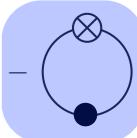


gauge-independence from beyond ladder resummation see poster of D. Sato

# Vector couplings



# Relevance of vector couplings

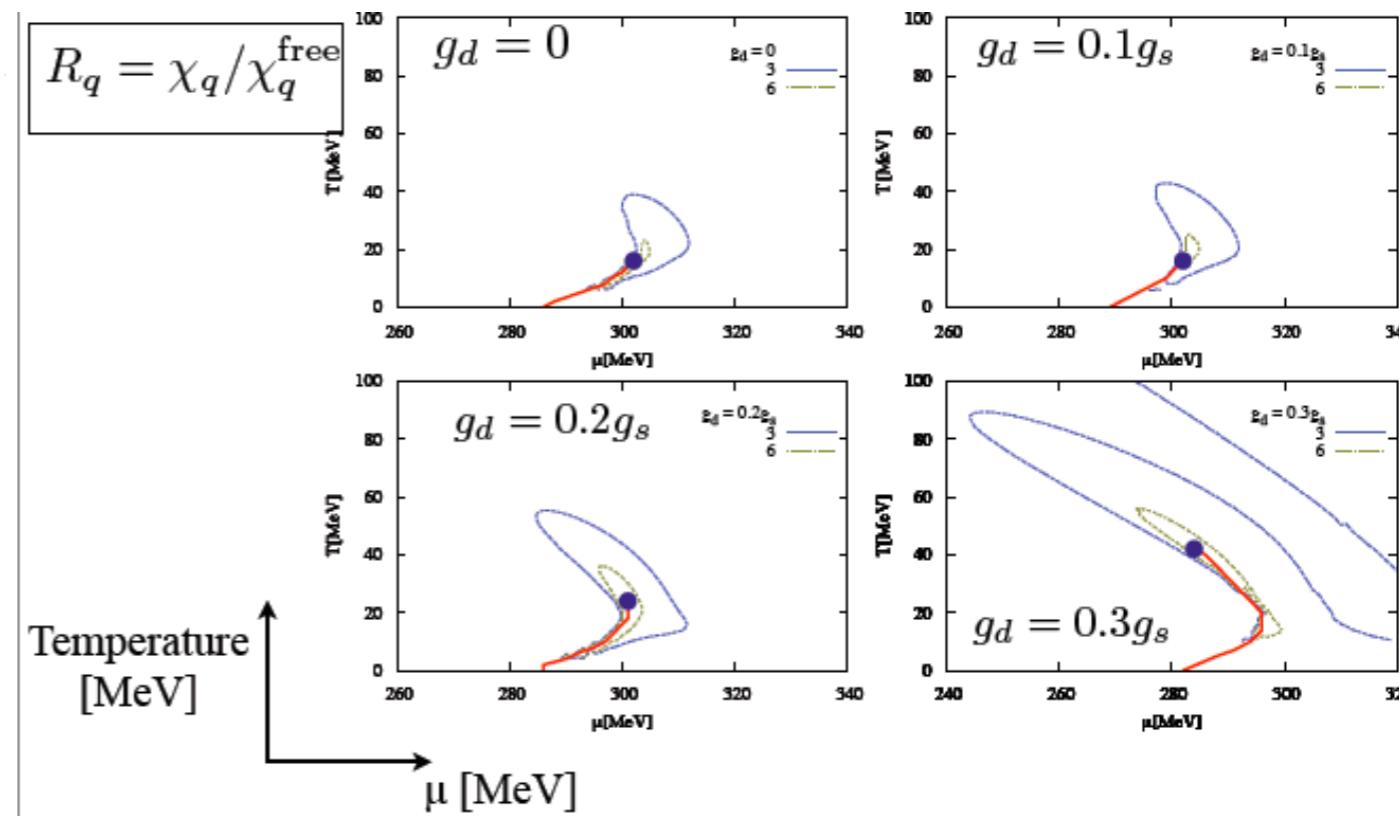


K. Fukushima (2008)

$$\mathcal{L} = \bar{\psi}[i\partial^\mu - g_s(\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi}) + g_d \varphi \gamma_0]\psi + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \frac{1}{2}(\partial_\mu \varphi)^2$$

$$- a(\sigma^2 + \vec{\pi}^2) - b(\sigma^2 + \vec{\pi}^2)^2 - \frac{m_\varphi^2}{2}\varphi^2 + c\sigma$$

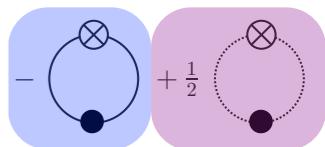
- a, b, c and  $g_s$  are fixed by vacuum physical value such as  $m_\pi$ ,  $f_\pi$ ,  $M\sigma \sim 600$  [MeV] and  $Mq \sim 300$  [MeV].
- We fix  $M\varphi$  and vary  $g_d$ .
- A ratio ( $g_d/M\varphi$ ) controls a strength of the mixing.



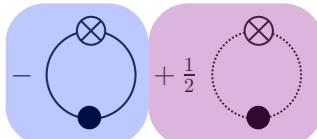
- The critical region is drastically expanded with  $g_d$ .

K. Kamikado, QGP meets cold atoms - Episode III

# Multi-scatterings



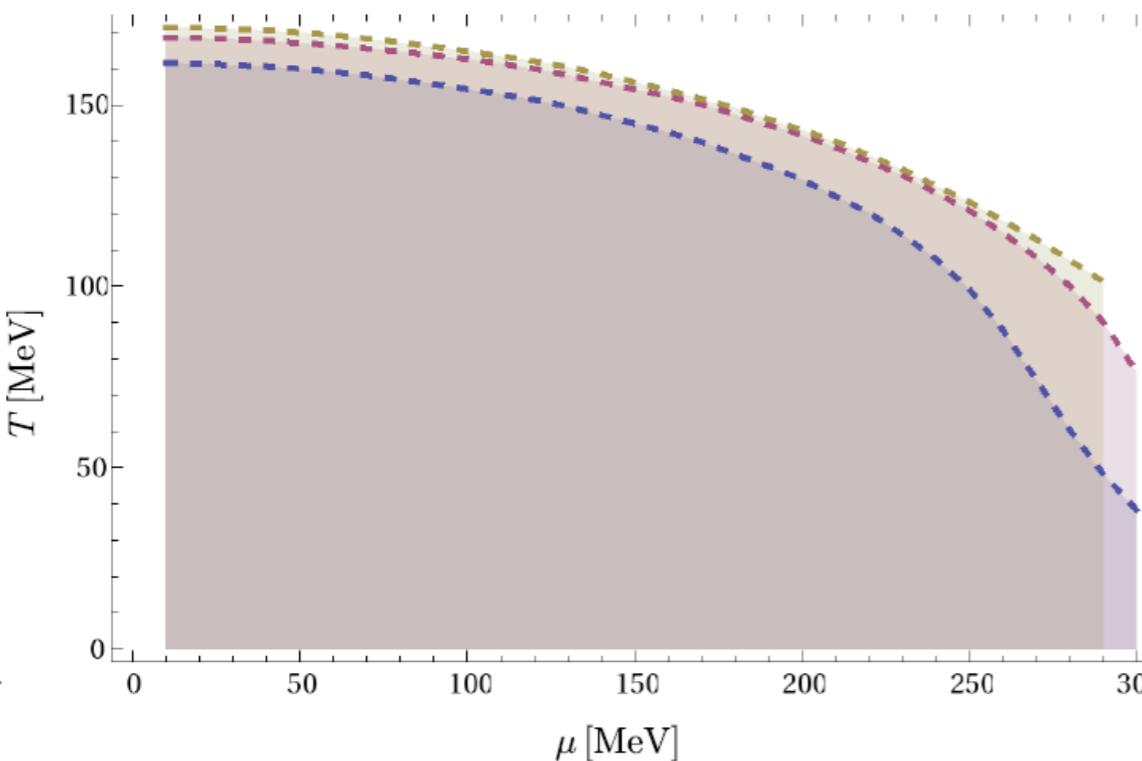
# Relevance of multi-scatterings



JMP, Rennecke, in prep

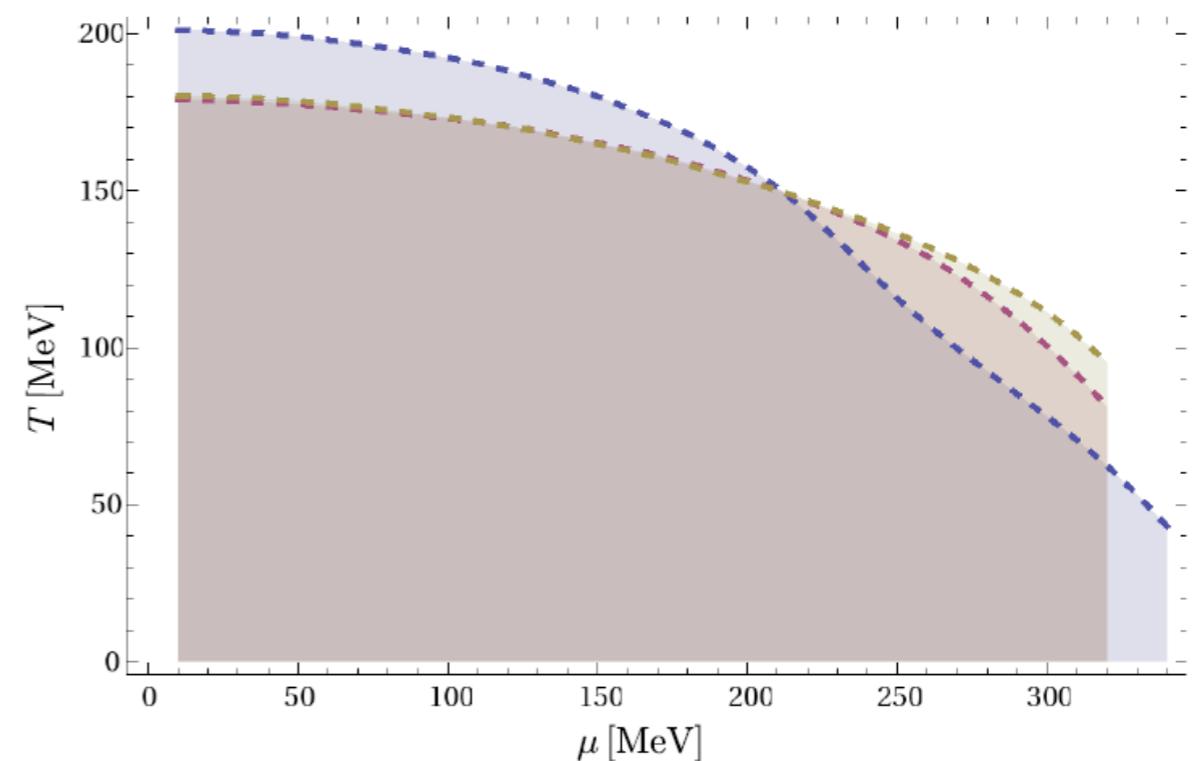
## Multi-meson-quark scatterings

- $h_k(\phi^2) \sim$



## Multi-meson scatterings

$$V_k(\phi^2) \sim$$

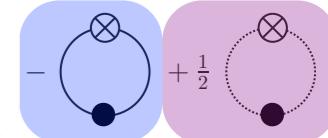


# Anomalous chiral symmetry breaking

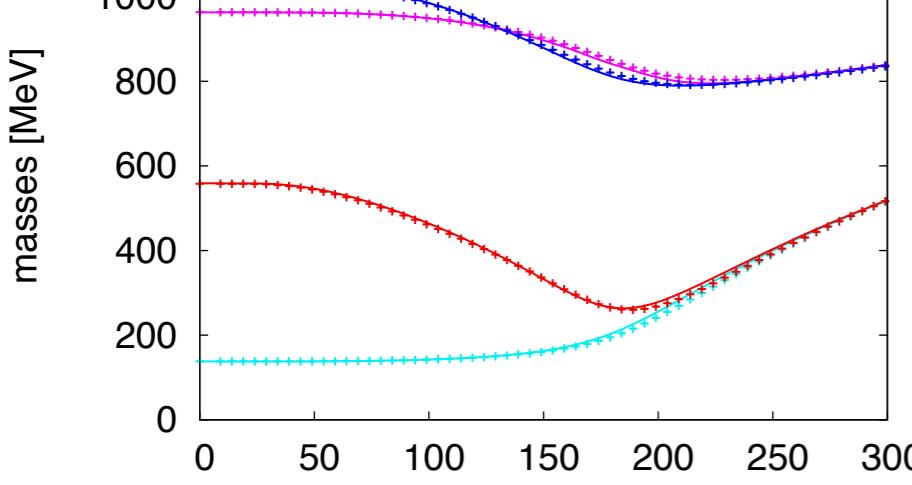


# Anomalous chiral symmetry breaking

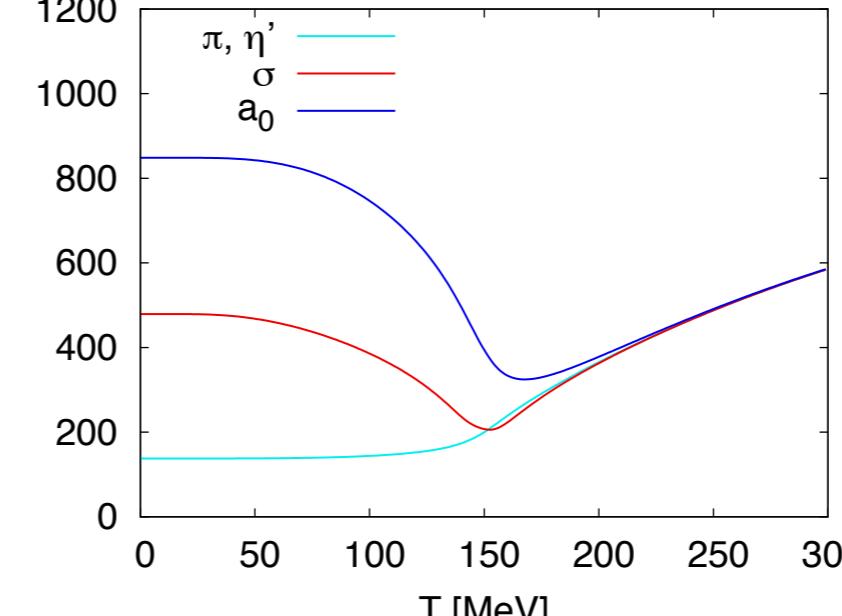
$$N_f = 2 + 1$$



**with 't Hooft determinant**

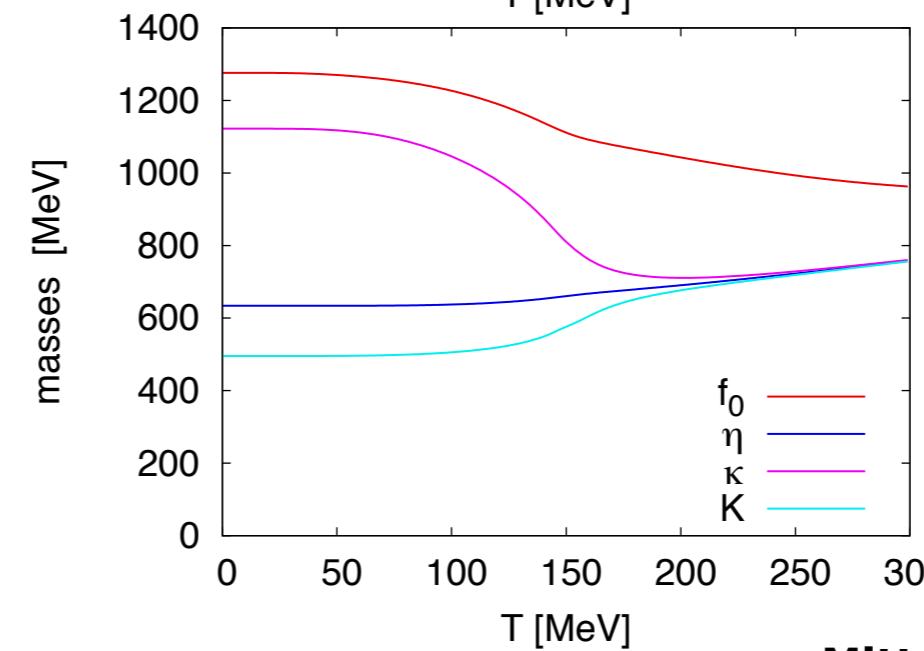
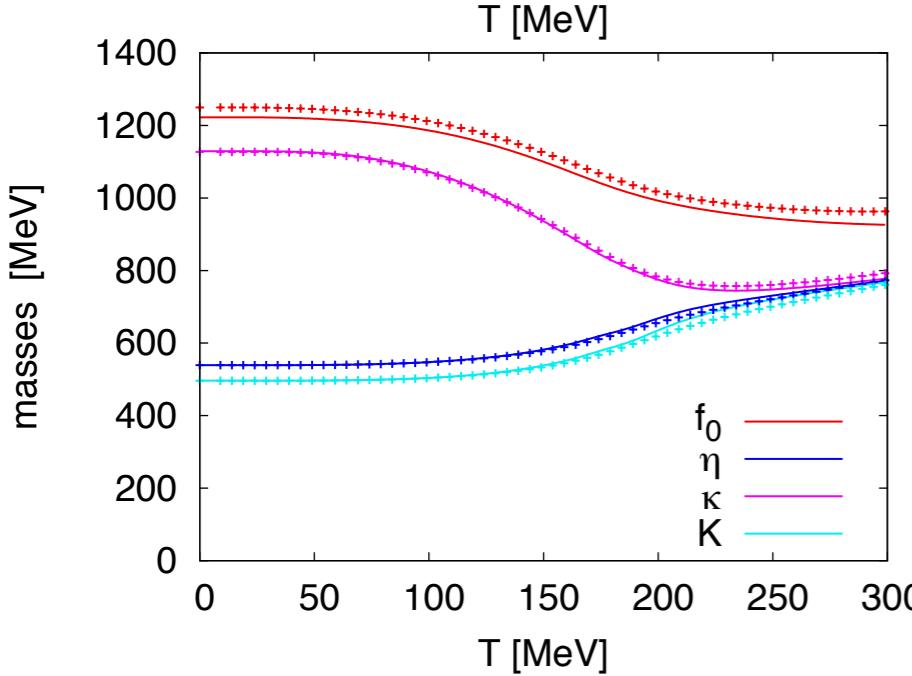


**without 't Hooft determinant**



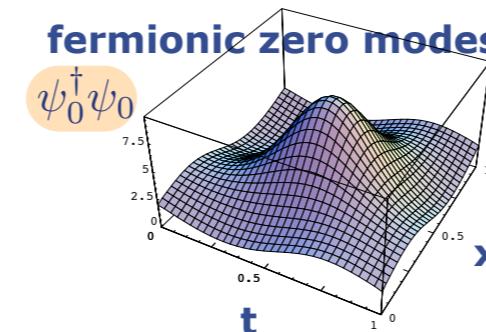
**Mitter, Schaefer, in prep**

**see talk of M. Mitter**



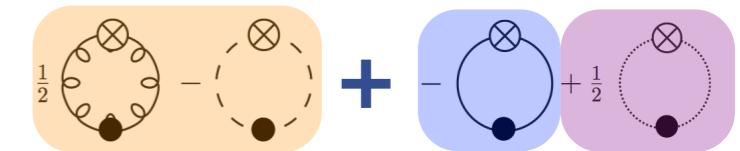
**N<sub>f</sub> = 2** Mitter, Schaefer, Strodthoff, von Smekal, in prep

**fermionic zero modes**

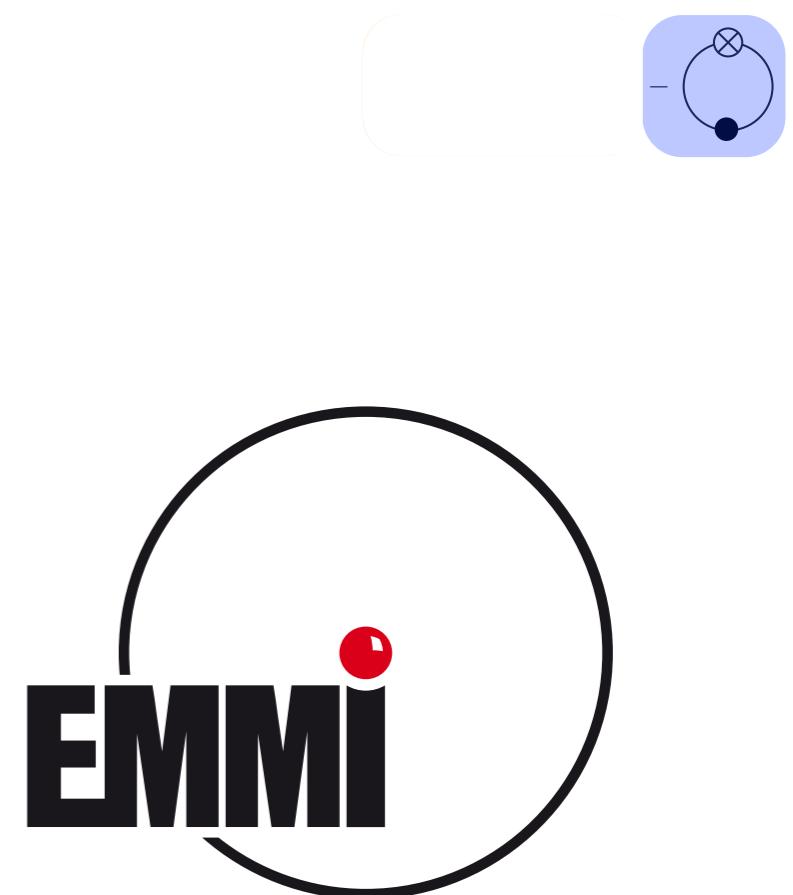


$$\Delta(k, \theta) \left( \det_{flav.} \bar{q}_L q_R + \det_{flav.} \bar{q}_R q_L \right)$$

$$\Delta(k, \theta) \propto (k^2 + c_k \Delta m_{\chi_{sb}}^2)^{-\frac{3}{2} N_f + 2} e^{-2\pi/\alpha_{s,k}}$$



## Strong magnetic fields

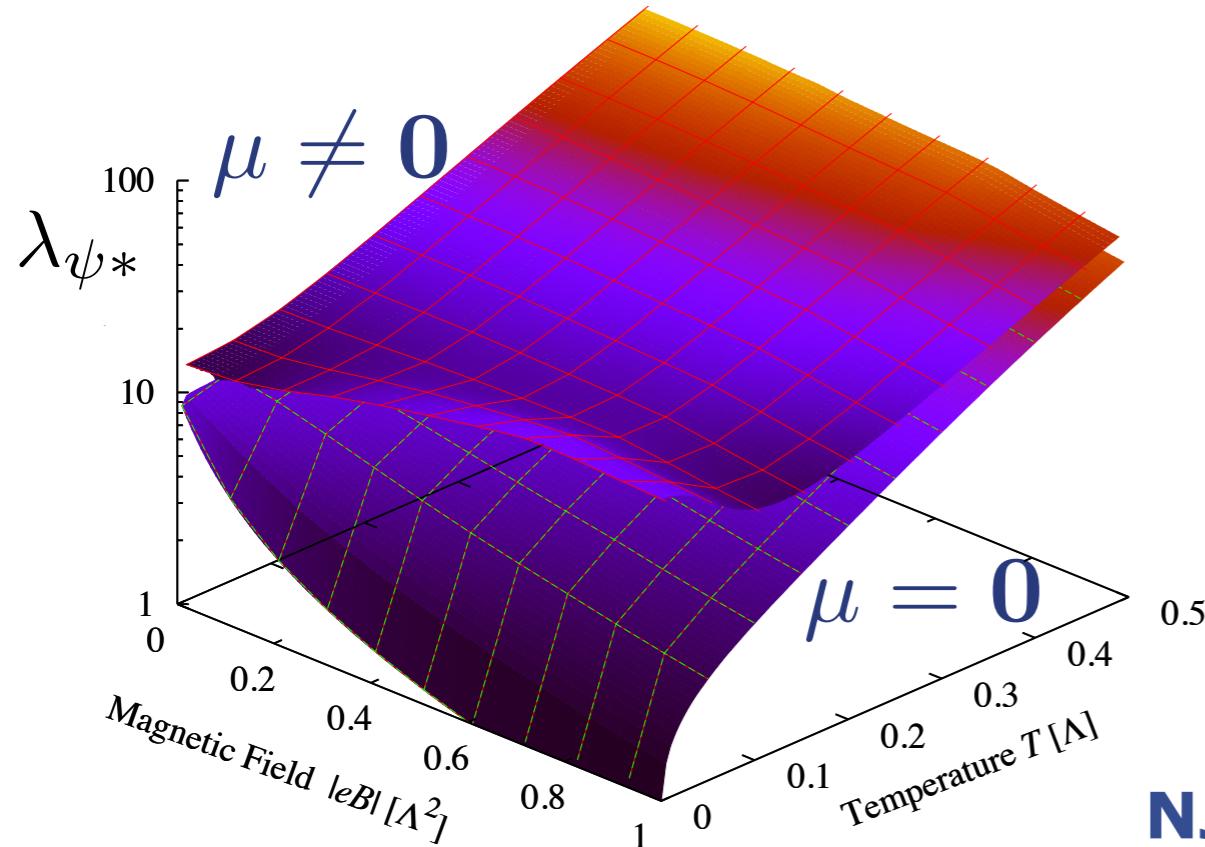


# Strong magnetic fields

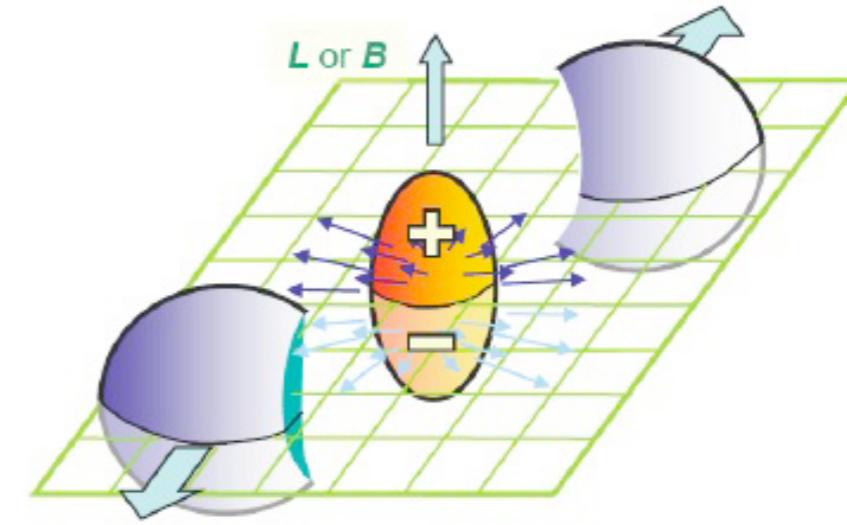
## chiral critical coupling



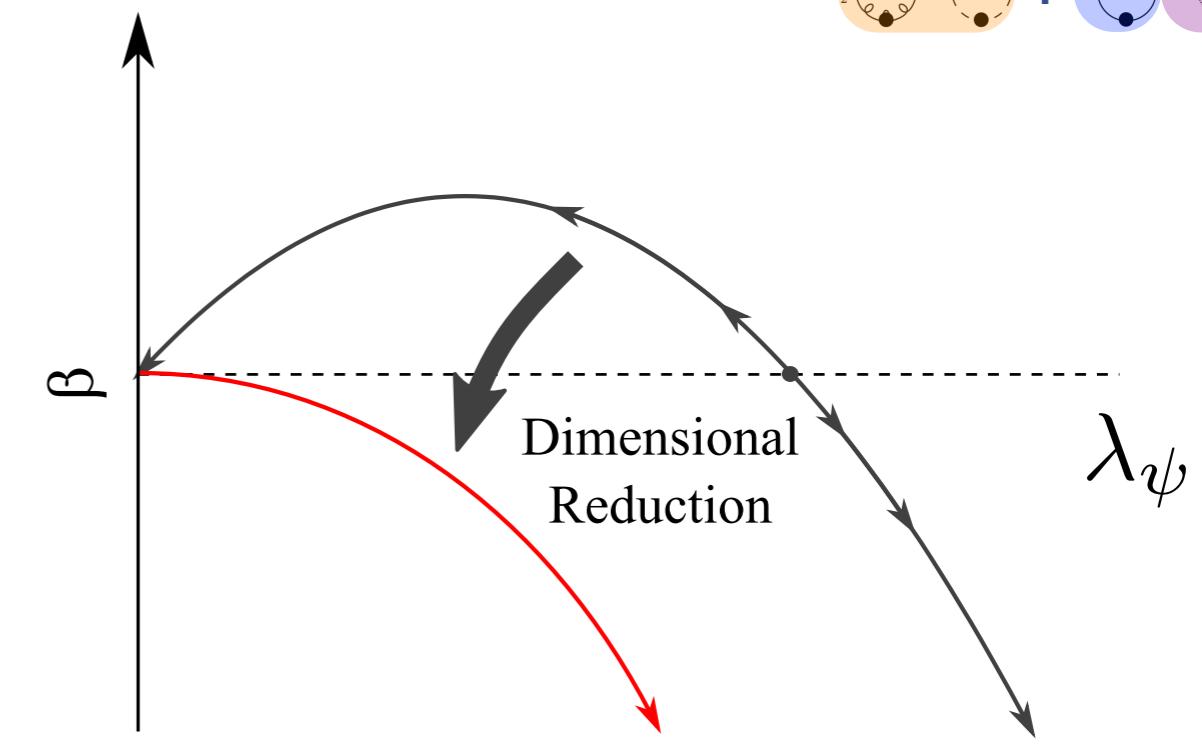
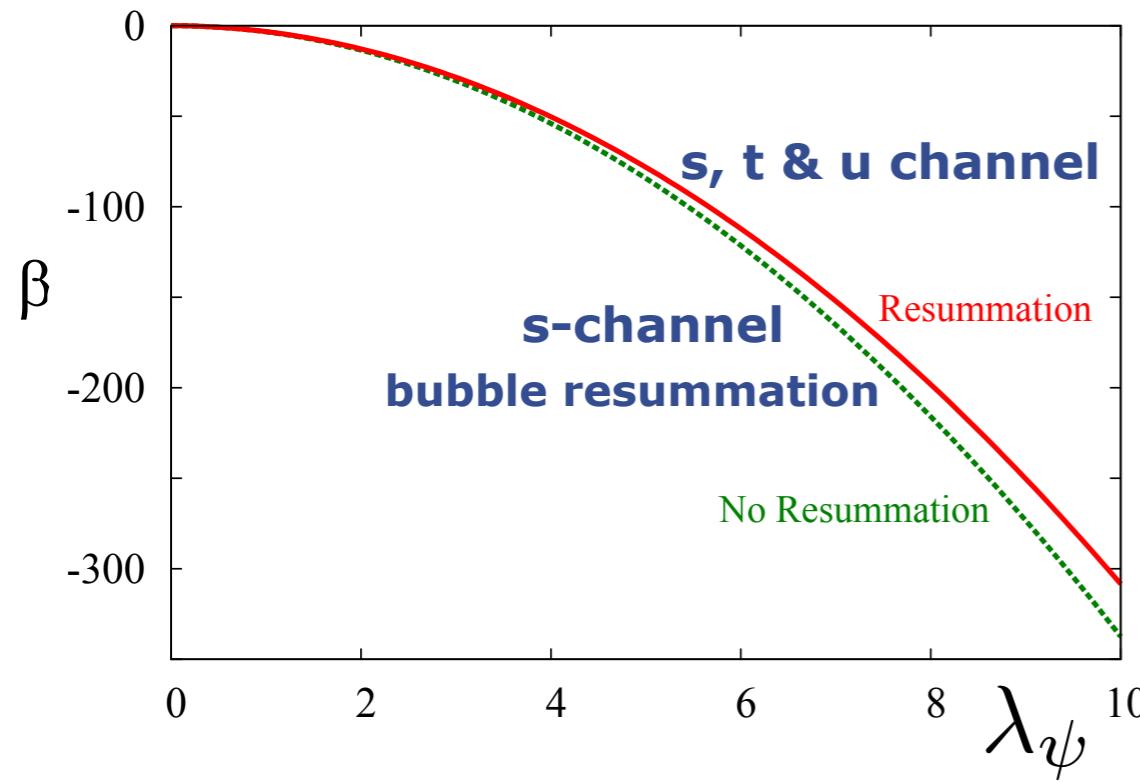
Fukushima, JMP '12



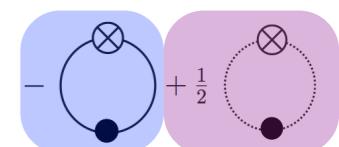
NJL-model



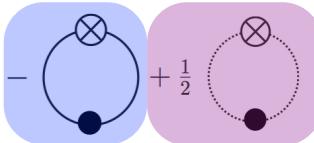
PQM Skokov '11



# Volume-dependence of the chiral phase structure

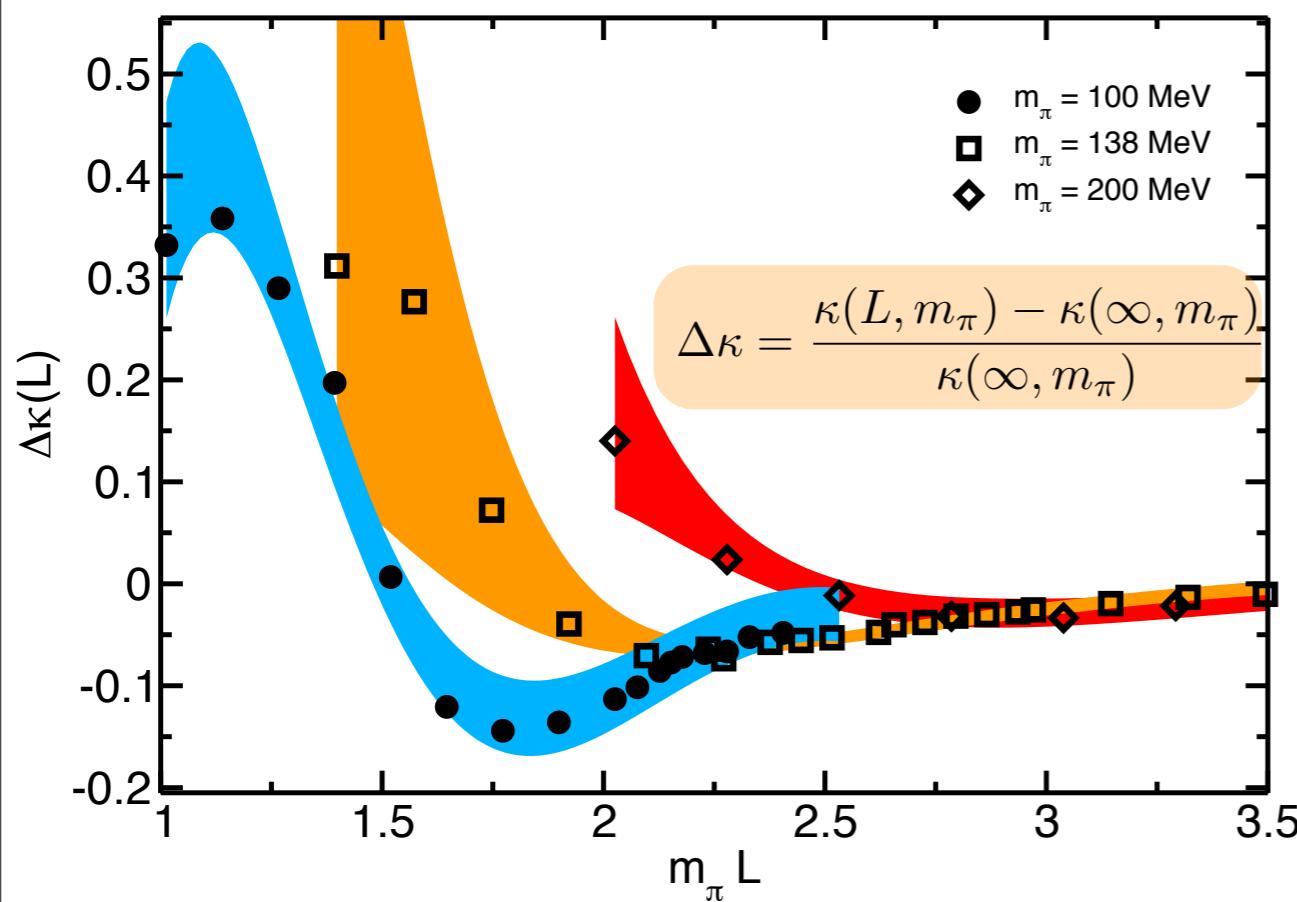


# Volume-dependence of the chiral phase structure



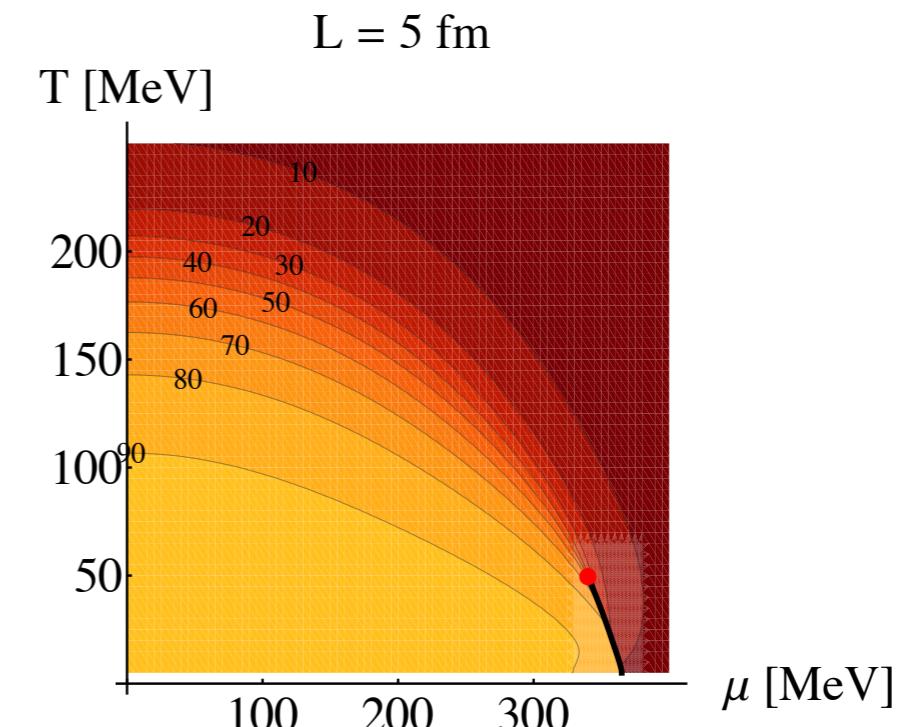
## Volume-dependence of the curvature $\kappa$

$$\frac{T_\chi(L, m_\pi, \mu)}{T_\chi(L, m_\pi, \mu = 0)} = 1 - \kappa \left( \frac{\mu}{(\pi T_\chi(L, m_\pi, 0))} \right)^2 + \dots$$

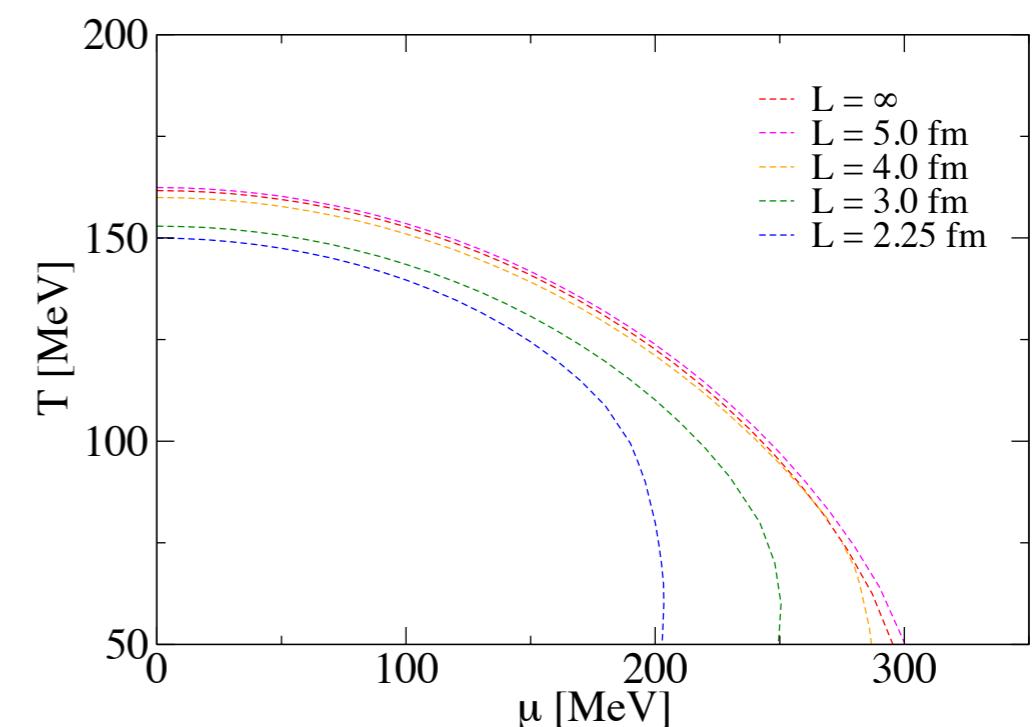


see talk of B. Klein

## Volume-dependence of the phase-structure



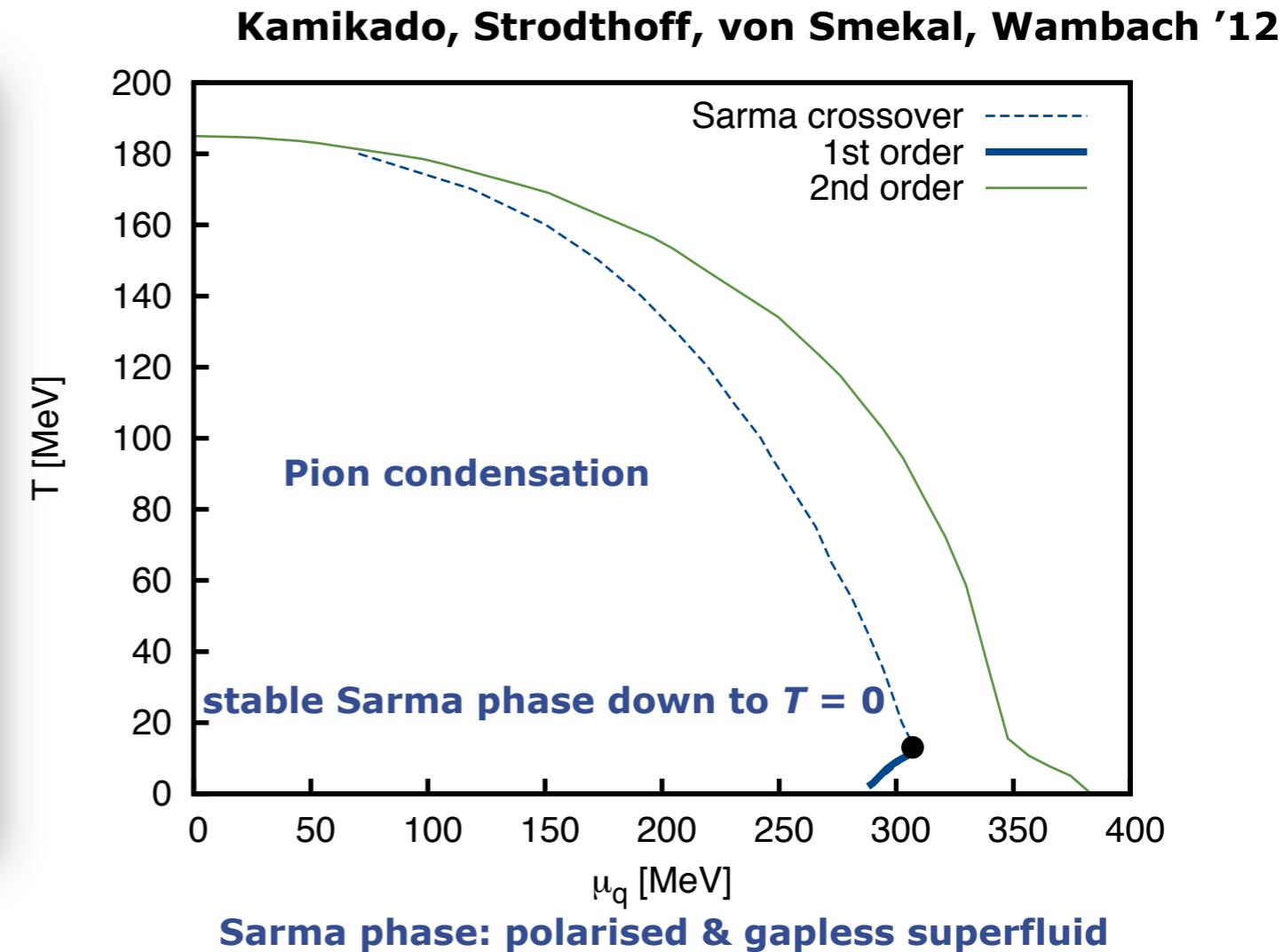
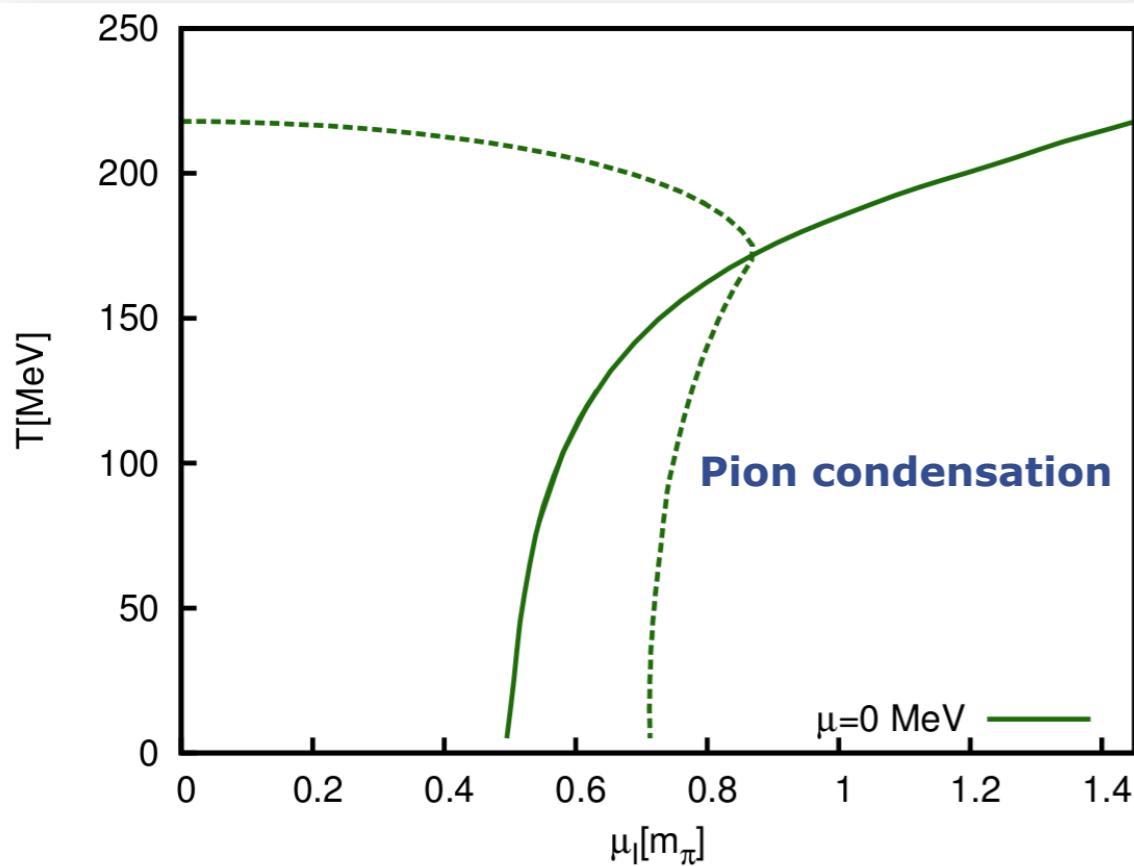
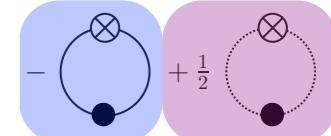
A. Tripolt, B.-J. Schaefer, J.Braun, B. Klein, in prep



# Isospin chemical potential

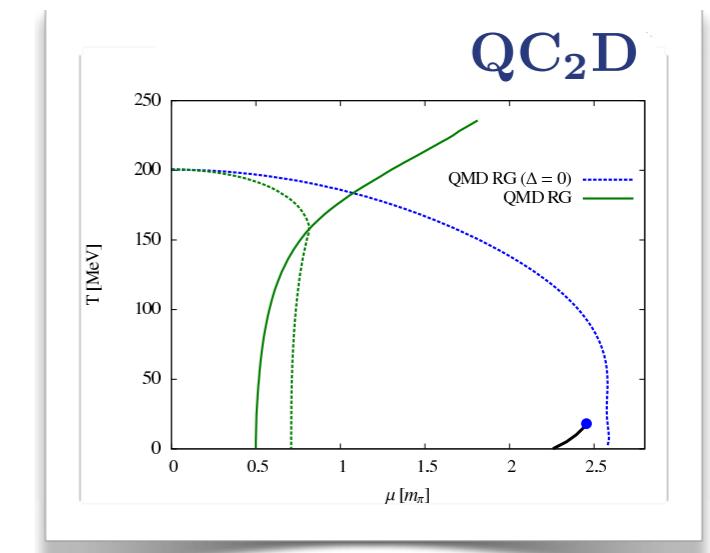


# Isospin chemical potential



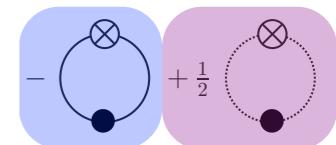
**see talk of N. Strodthoff**

**& poster of K. Kamikado**

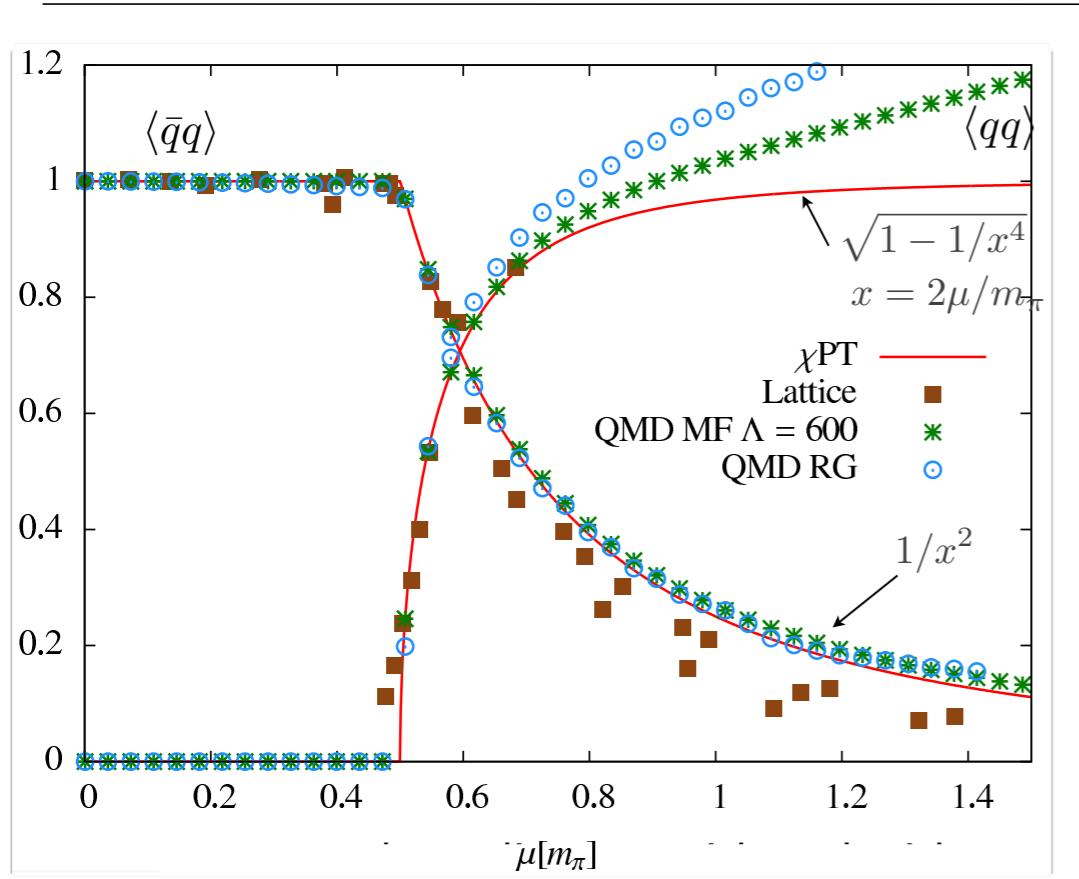
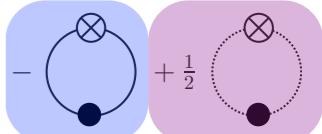


QC<sub>2</sub>D

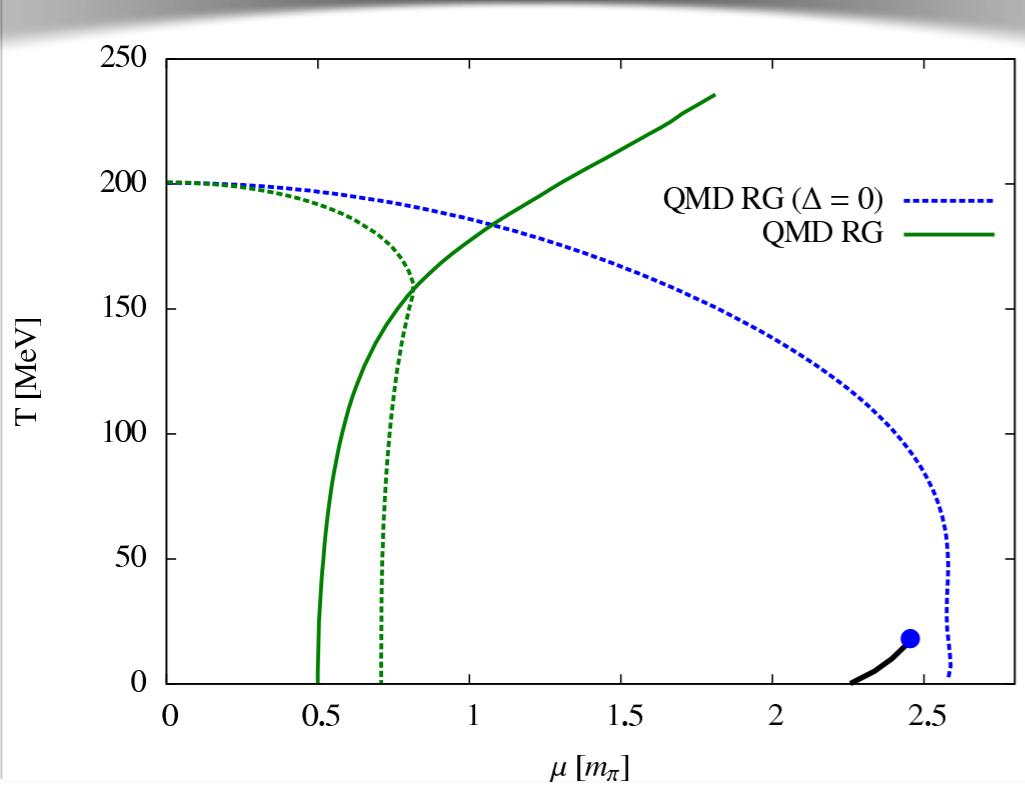
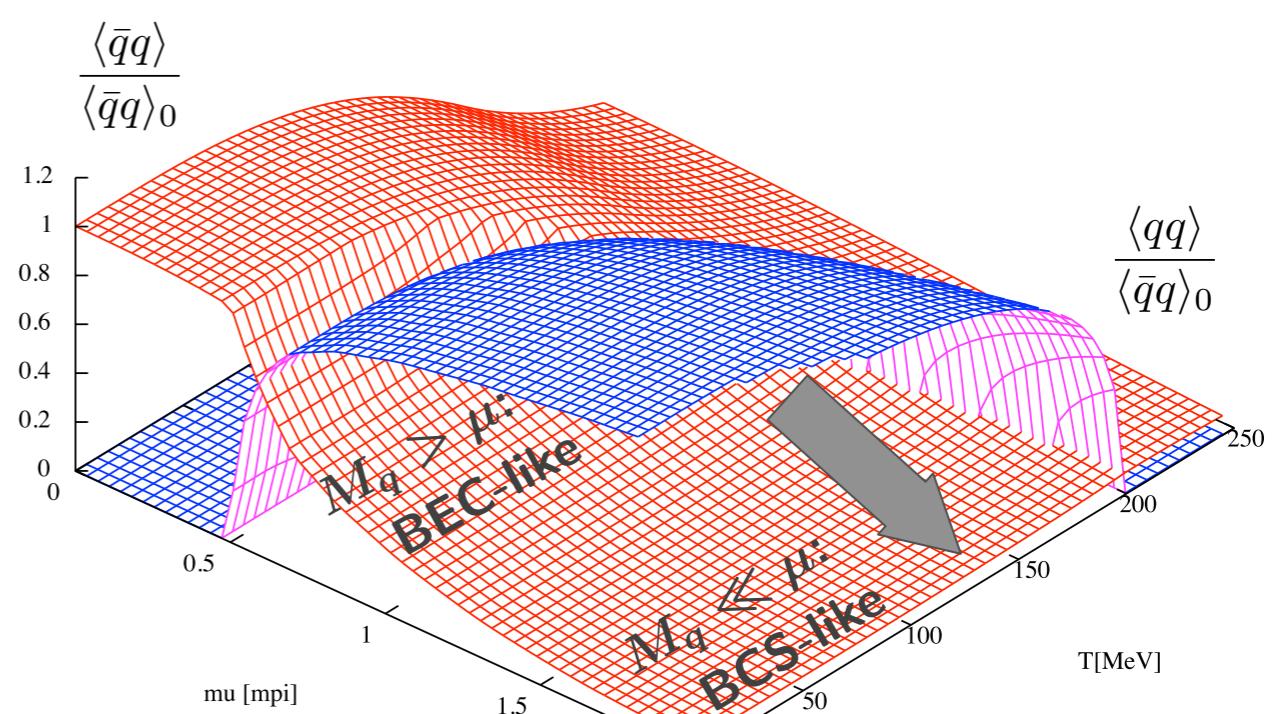
Baryons (Diquarks)



# Two-colour QCD

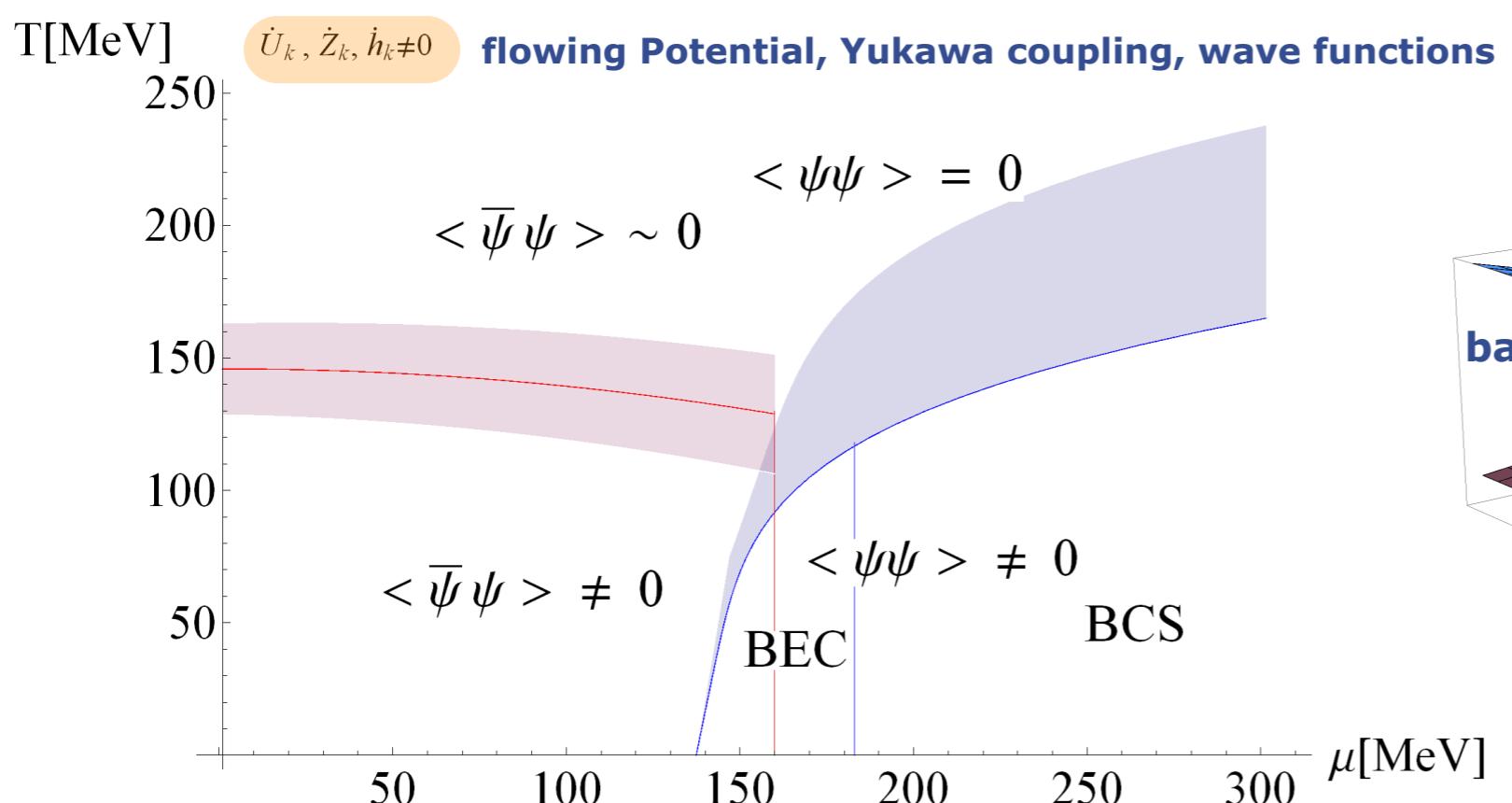
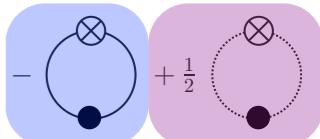


**Schaefer, Strodthoff, von Smekal '12**

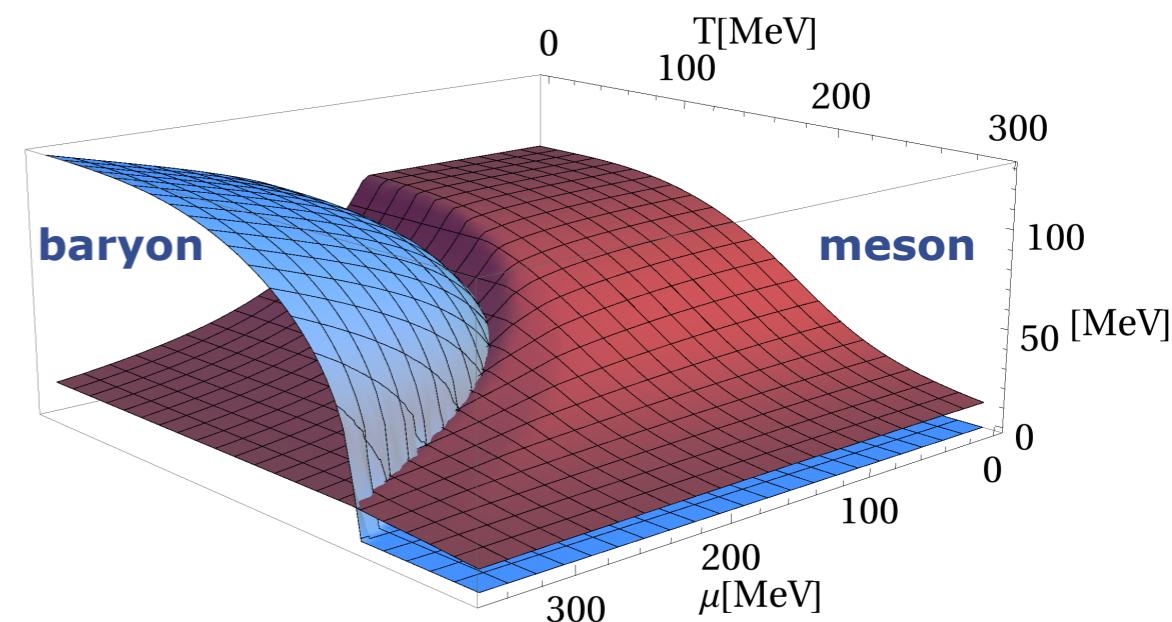


**see talk of B.-J. Schaefer**

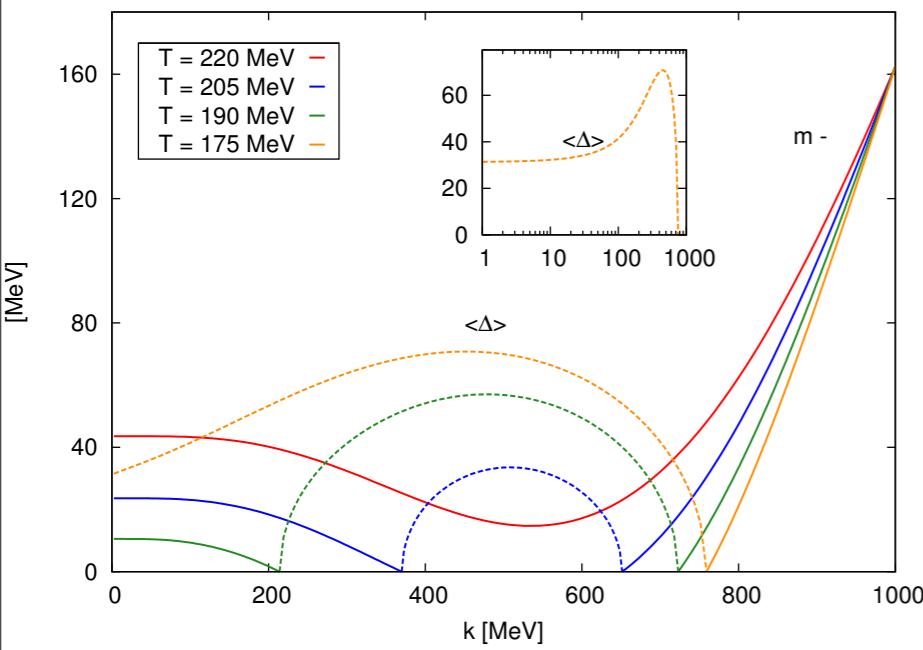
# Two-colour QCD



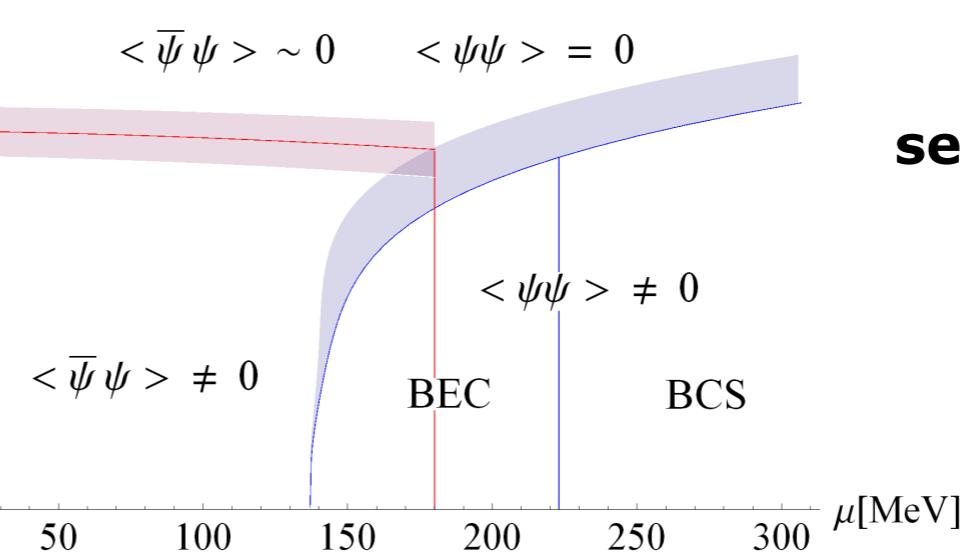
Khan, JMP, Rennecke, Scherer, in prep



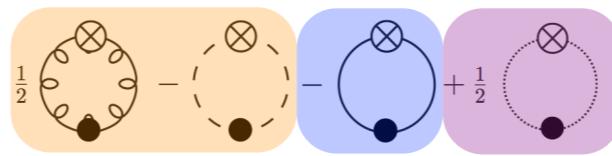
flowing mass/condensate



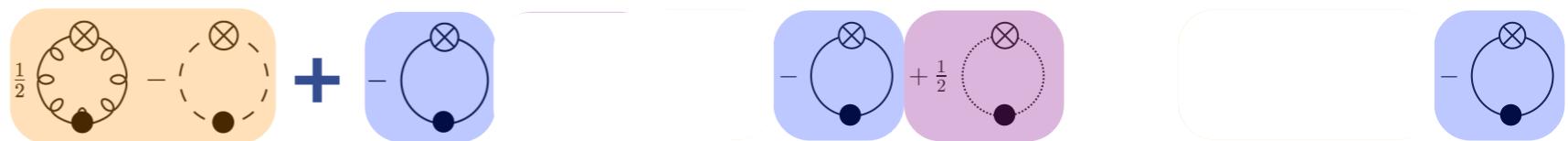
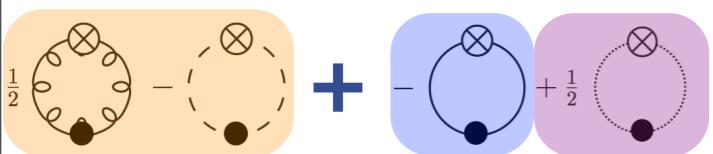
$T[\text{MeV}]$  flowing Potential



see poster of N. Khan

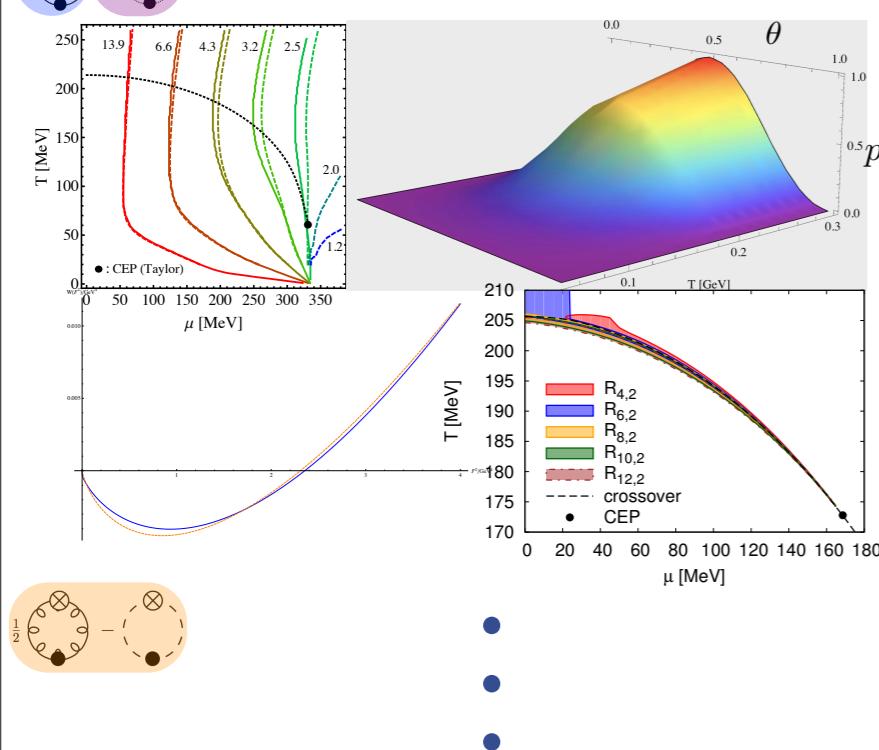


## towards the full phase diagram

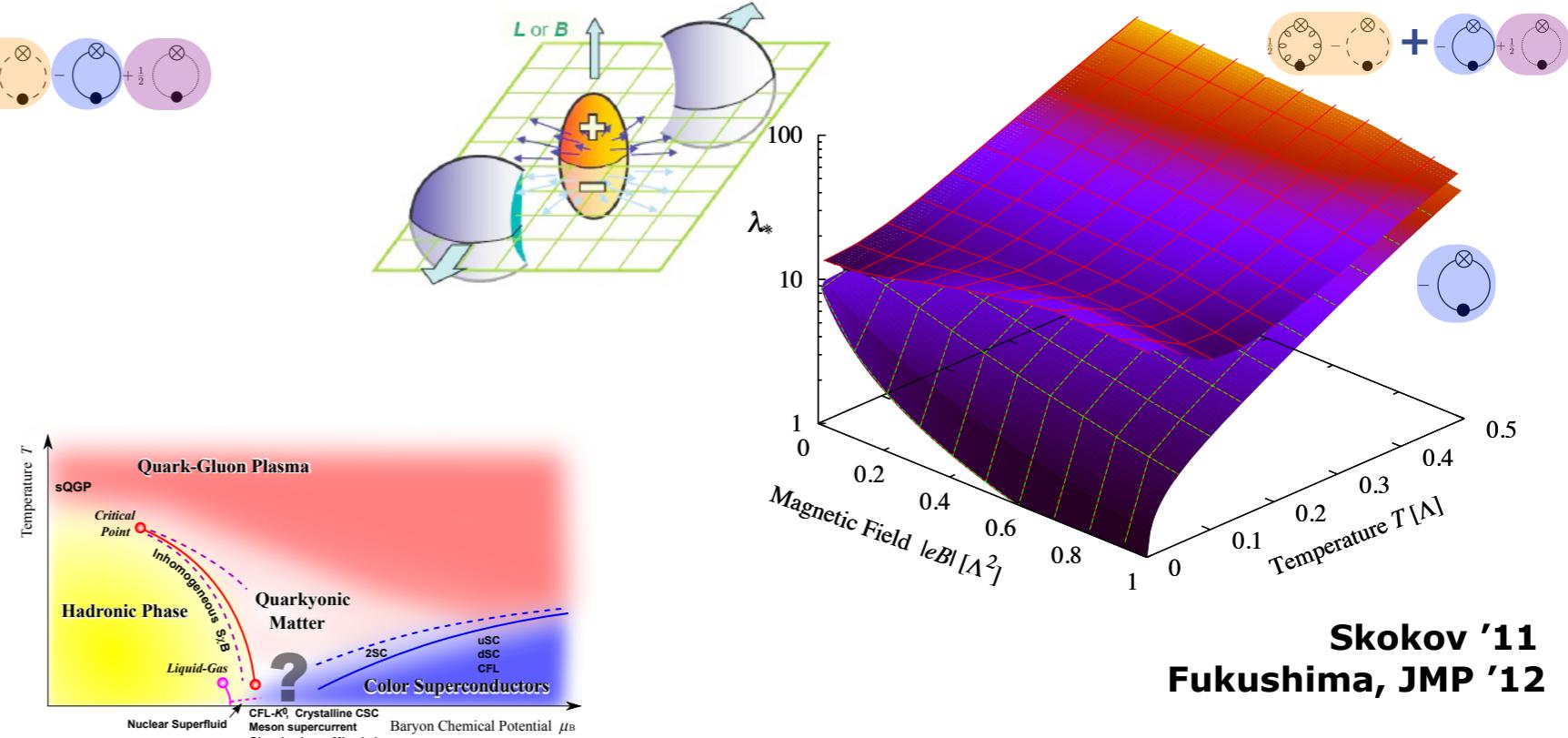


# towards the full phase diagram

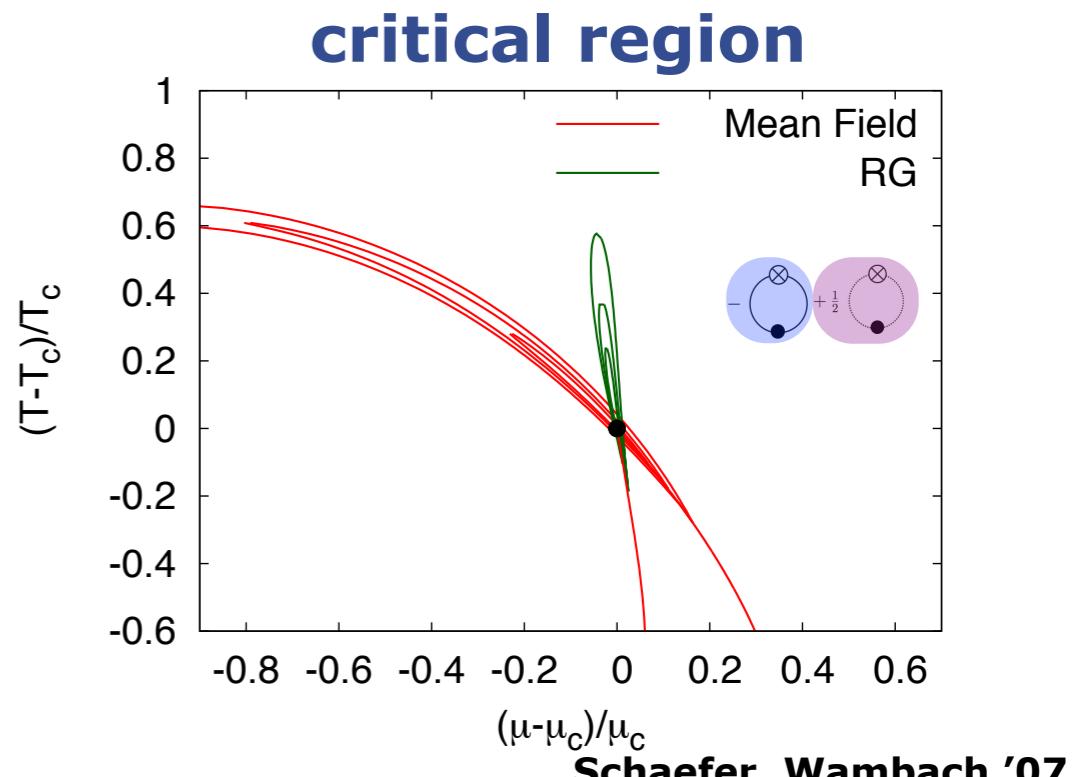
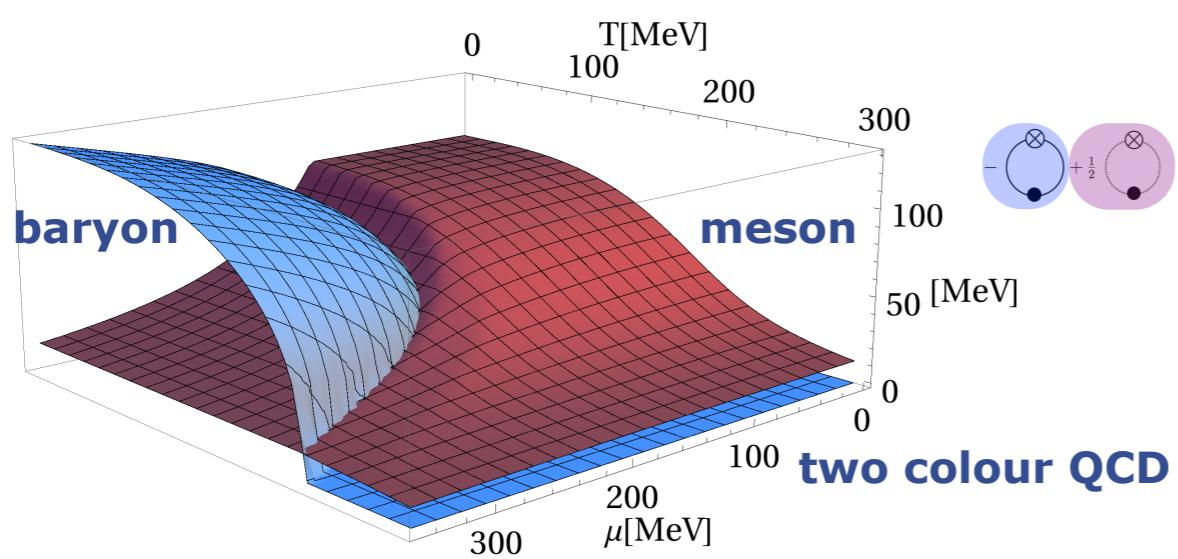
## phase-structure



## magnetic catalysis



## mesons & baryons



# **Summary & Outlook**

# Outline

- Motivation
- QCD
  - Asymptotic freedom and all that
  - confinement
  - chiral symmetry breaking
- Functional methods for QCD
  - FRG for QCD
  - Dynamical hadronisation
  - Gauge symmetry, gauge fixing and regularisation
  - Approximation scheme
- Results
  - Yang-Mills theory at zero and finite temperature
  - Many-flavour QCD
  - Phase diagram of QCD
- Summary & outlook

stay tuned