Stochastic quantisation of the Thirring model

and
the sign problem

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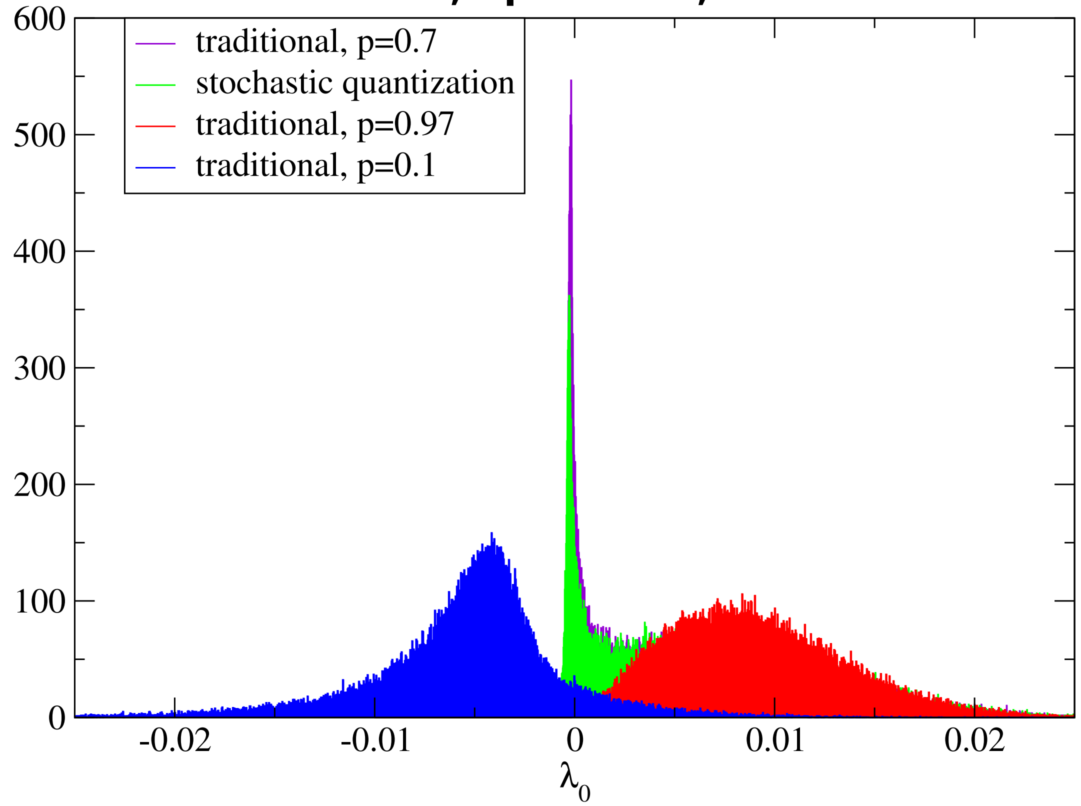
Regensburg, September 21th 2012

A laboratory for Langevin evolution

JMP, Spielmann, Stamatescu '09



Gribov problem/Neuberger problem

QCD @ finite density

Dynamics

Work with I.O. Stamatescu, C. Zielinski

thanx to

G. Aarts, C. Gattringer, S. Hands, V. Kasper, D. Sexty, E. Seiler

for discussions

Outline

- **Thirring model & stochastic quantisation**
- **Results**
- **Reparameterisations**
- **Outlook**

Thirring model & stochastic quantisation

see also talks of G. Aarts
E. Seiler
I.O. Stamatescu

Thirring model

Continuum version

Action

$$S[\psi,\bar{\psi}] = \int_x \bar{\psi}_f \left(\not{\partial} + m_f + \mu_f \gamma_0\right) \psi_f + \frac{g^2}{2N_f} \int_x \left(\bar{\psi}_f \gamma_\nu \psi_f\right)^2$$

$$f = 1, \ldots, N_f$$

Bosonised generating functional

$$Z = \int dA\, e^{-S_{\text{eff}}}$$

with

$$S_{\text{eff}} = N_f \beta \int_x A_\nu^2 - \sum_f \text{Tr} \log K_f$$

$$K_f = \not{\partial} + i \not{A} + m_f + \mu_f \gamma_0$$

$$\beta = \frac{1}{g^2}$$

$$\det K_f(\mu) = \left[\det K_f(-\mu^\star)\right]^\star$$

Thirring model

lattice version & Langevin equation

Staggered fermion matrix

$$K_f(x,y) = \frac{1}{2}\sum_{\nu=0}^{d-1}\varepsilon_\nu(x)\left[\left(1+iA_\nu(x)\right)e^{\mu_f\delta_{\nu 0}}\delta_{x+\hat{\nu},y} - \left(1-iA_\nu(y)\right)e^{-\mu_f\delta_{\nu 0}}\delta_{x-\hat{\nu},y}\right] + m_f\delta_{xy}$$

$$\varepsilon_\nu(x) = (-1)^{\sum_{i=0}^{\nu-1} x_i}$$

Langevin equation (continuum)

$$\frac{\partial}{\partial\Theta}A_\nu(x,\Theta) = -\frac{\delta S_{\mathrm{eff}}[A]}{\delta A_\nu(x,\Theta)} + \sqrt{2}\,\eta_\nu(x,\Theta)$$

Gaußian noise

$$\langle\eta_\nu(x,\Theta)\rangle = 0$$

$$\langle\eta_\nu(x,\Theta)\,\eta_\sigma(x',\Theta')\rangle = \delta_{\nu\sigma}\delta(x-x')\,\delta(\Theta-\Theta')$$

Thirring model

lattice version & Langevin equation

Staggered fermion matrix

$$K_f(x,y) = \frac{1}{2}\sum_{\nu=0}^{d-1}\varepsilon_\nu(x)\left[(1+iA_\nu(x))\,e^{\mu_f\delta_{\nu 0}}\delta_{x+\hat{\nu},y} - (1-iA_\nu(y))\,e^{-\mu_f\delta_{\nu 0}}\delta_{x-\hat{\nu},y}\right] + m_f\delta_{xy}$$

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$$\frac{\partial}{\partial\Theta}A_\nu(x,\Theta) = -\frac{\delta S_{\text{eff}}[A]}{\delta A_\nu(x,\Theta)} + \sqrt{2}\,\eta_\nu(x,\Theta)$$

complex noise

$$\langle \mathrm{Re}\,\eta_\nu(x,\Theta)\,\mathrm{Re}\,\eta_\sigma(x',\Theta')\rangle = \langle \mathrm{Im}\,\eta_\nu(x,\Theta)\,\mathrm{Im}\,\eta_\sigma(x',\Theta')\rangle = \delta_{\nu\sigma}\delta(x-x')\,\delta(\Theta-\Theta')$$

$$\langle \mathrm{Re}\,\eta_\nu(x,\Theta)\,\mathrm{Im}\,\eta_\sigma(x',\Theta')\rangle = 0$$

Thirring model

lattice version & Langevin equation

Staggered fermion matrix

$$K_f(x,y) = \frac{1}{2}\sum_{\nu=0}^{d-1} \varepsilon_\nu(x)\left[(1+iA_\nu(x))\, e^{\mu_f \delta_{\nu 0}}\delta_{x+\hat{\nu},y} - (1-iA_\nu(y))\, e^{-\mu_f\delta_{\nu 0}}\delta_{x-\hat{\nu},y}\right] + m_f\delta_{xy}$$

$$\varepsilon_\nu(x) = (-1)^{\sum_{i=0}^{\nu-1} x_i}$$

Langevin equation (lattice)

$$A_\nu(x,\Theta+\epsilon_L) = A_\nu(x,\Theta) + \epsilon_L D_\nu(x,\Theta) + \sqrt{2\epsilon_L}\,\eta_\nu(x,\Theta)$$

$$\epsilon_L = \frac{\delta}{\max_{x,\nu}|D_\nu(x,\Theta)|}$$

complex noise

$$\langle \mathrm{Re}\,\eta_\nu(x,\Theta)\,\mathrm{Re}\,\eta_\sigma(x',\Theta')\rangle = \langle \mathrm{Im}\,\eta_\nu(x,\Theta)\,\mathrm{Im}\,\eta_\sigma(x',\Theta')\rangle = \delta_{\nu\sigma}\delta(x-x')\,\delta(\Theta-\Theta')$$

$$\langle \mathrm{Re}\,\eta_\nu(x,\Theta)\,\mathrm{Im}\,\eta_\sigma(x',\Theta')\rangle = 0$$

Thirring model

Benchmarks

- **Hopping parameter expansion** $\kappa = \frac{1}{2m}$

$$\frac{\det K}{m^\Omega} = \prod_\ell \prod_{\{C_\ell\}} \left(1 - \kappa^\ell \gamma_{C_\ell} \mathbf{P}_{C_\ell}\right)$$

$$\gamma_{C_\ell} = (-)^{n_{C_\ell}} e^{\mu N_t n_{C_\ell}}$$

- **Heavy dense limit (symmetrised)**

$$\kappa \to 0 \qquad \mu \to \infty \qquad \kappa e^\mu \text{ fixed}$$

- **Phase-quenched theory**

- **Analytic solutions (0+1 dimensions)**

Thirring model

Numerics

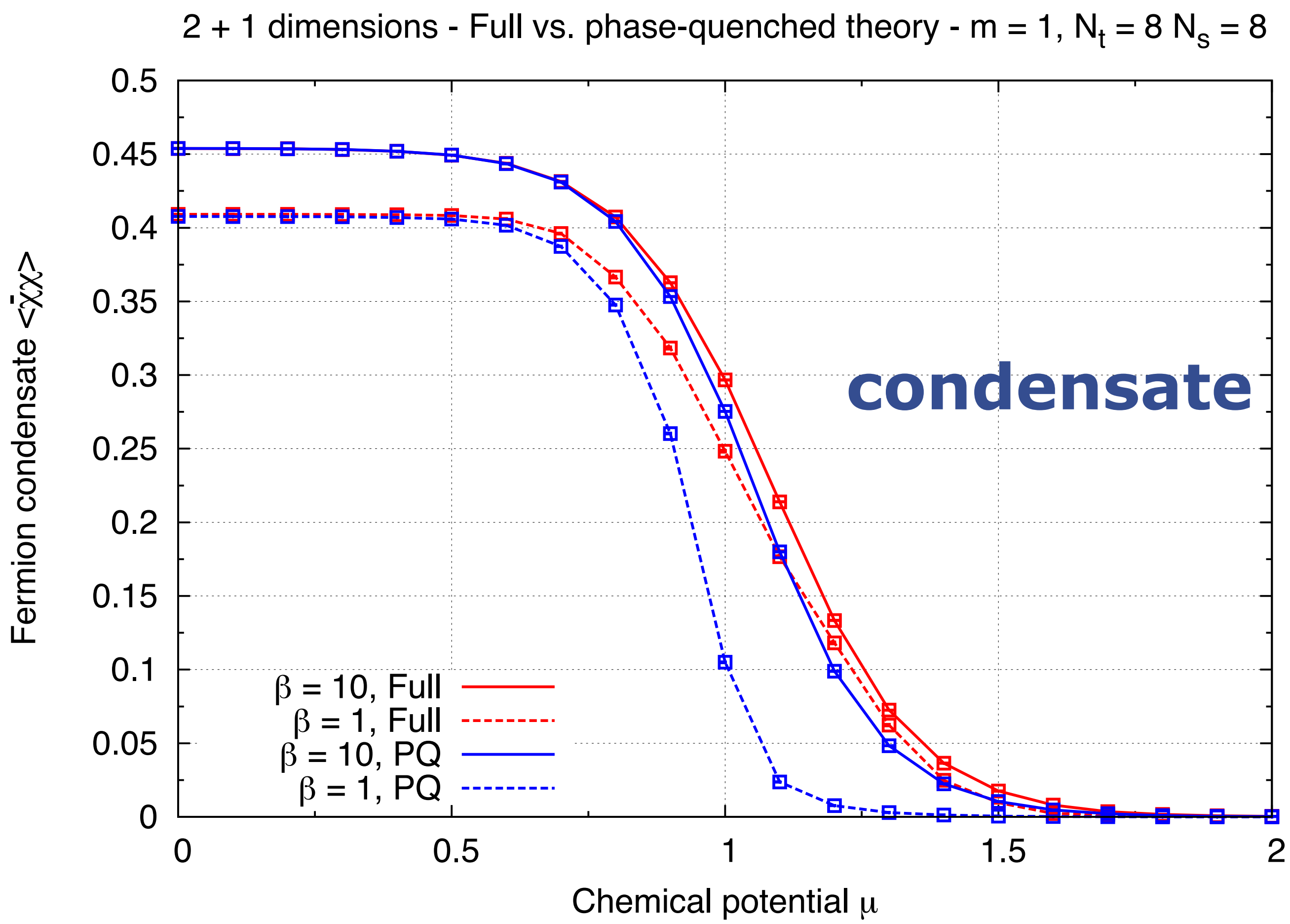
- **Hot start**
- **Thermalisation steps (~ 5000)**
- **Sampling ($\sim 10^4$)**
- **Error: bootstrap**
- **MC: brute force computation of the integrals**

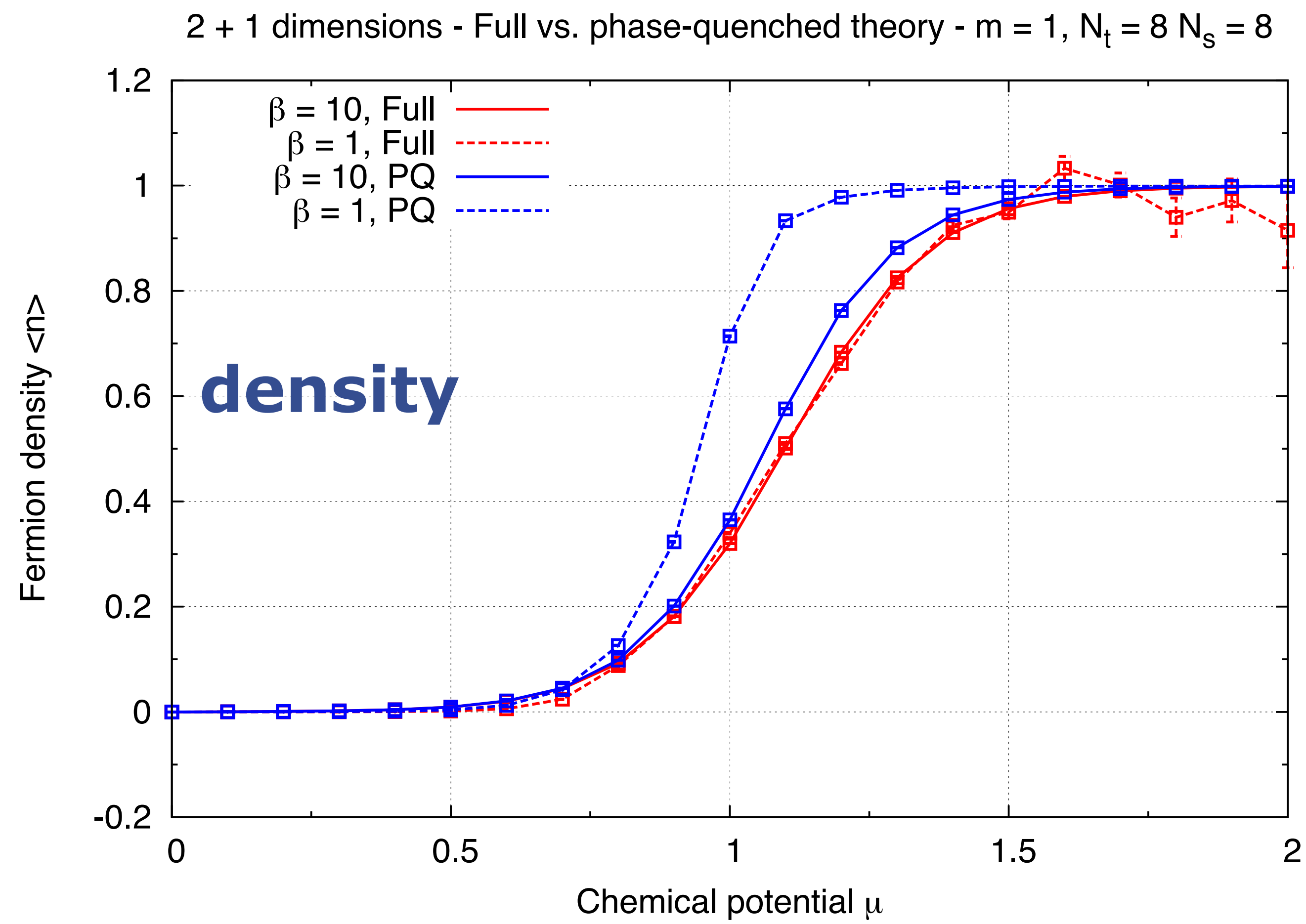
limited to small lattices

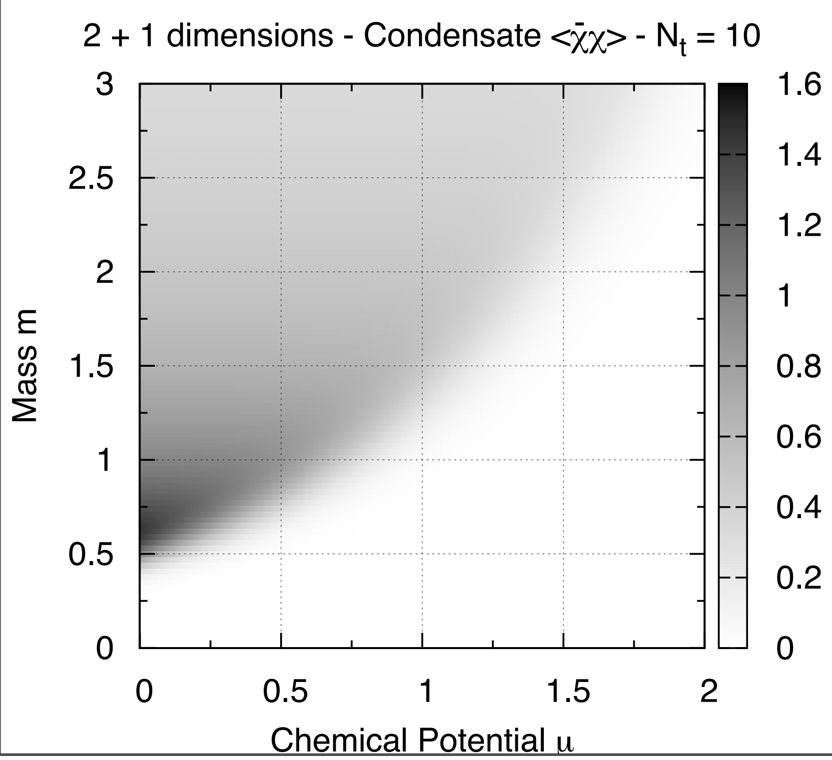
Results

Thirring model

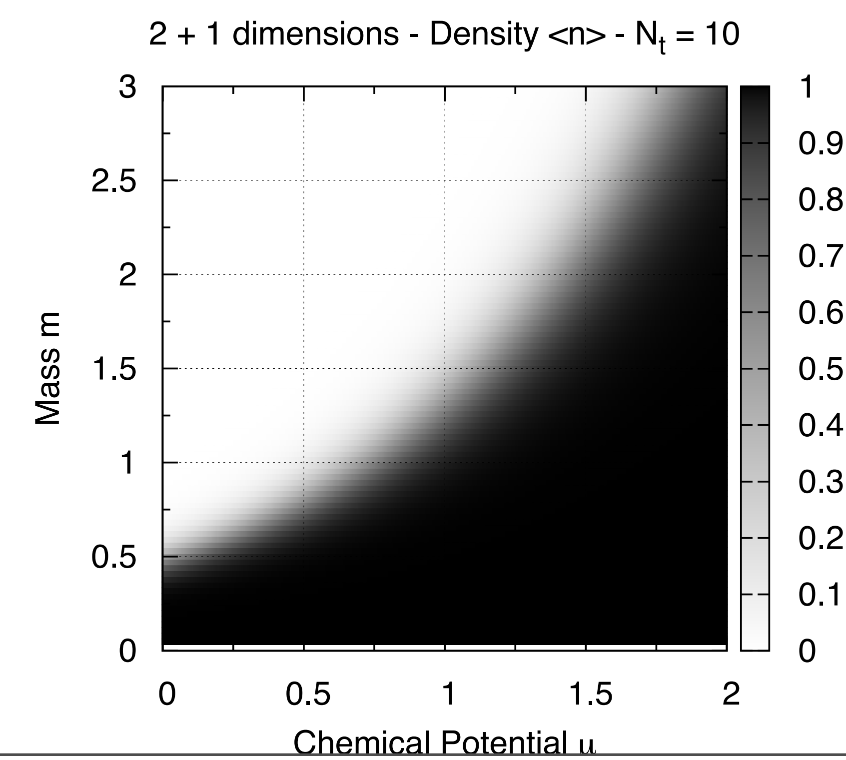
phase structure





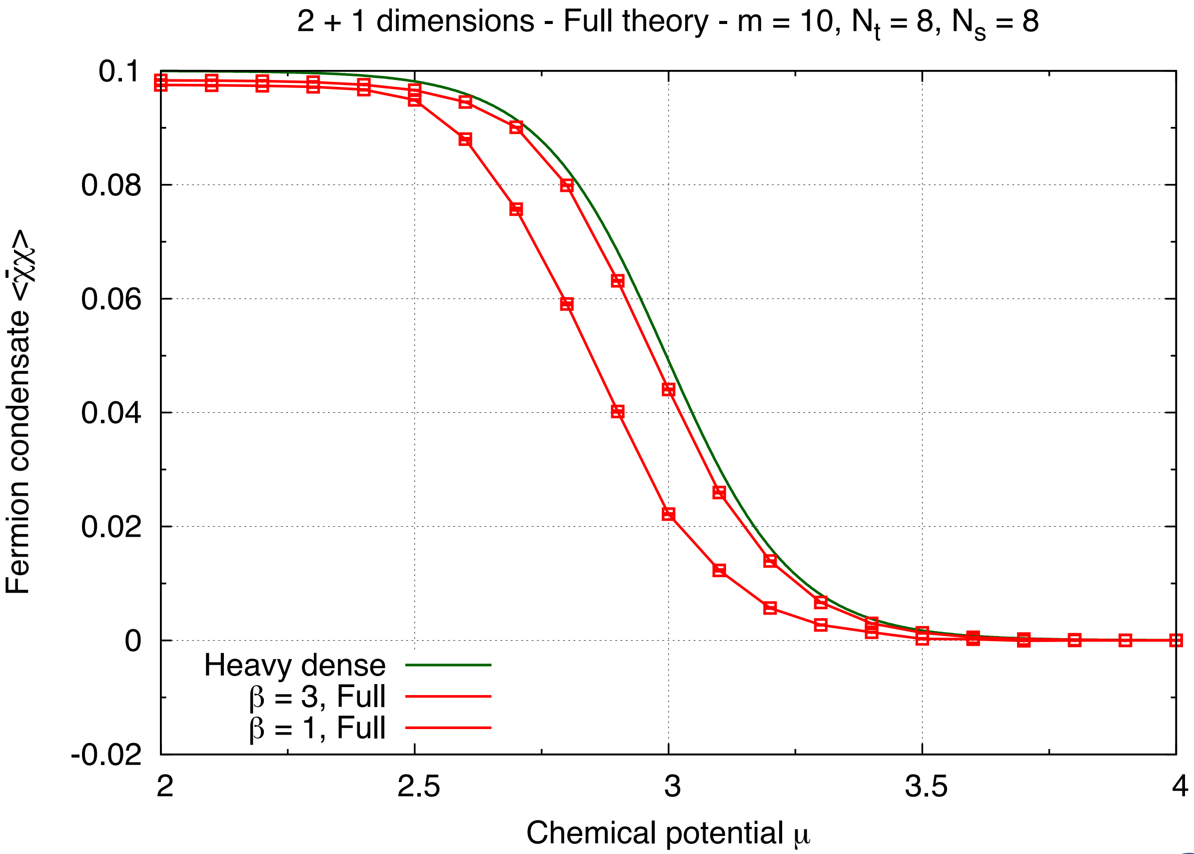


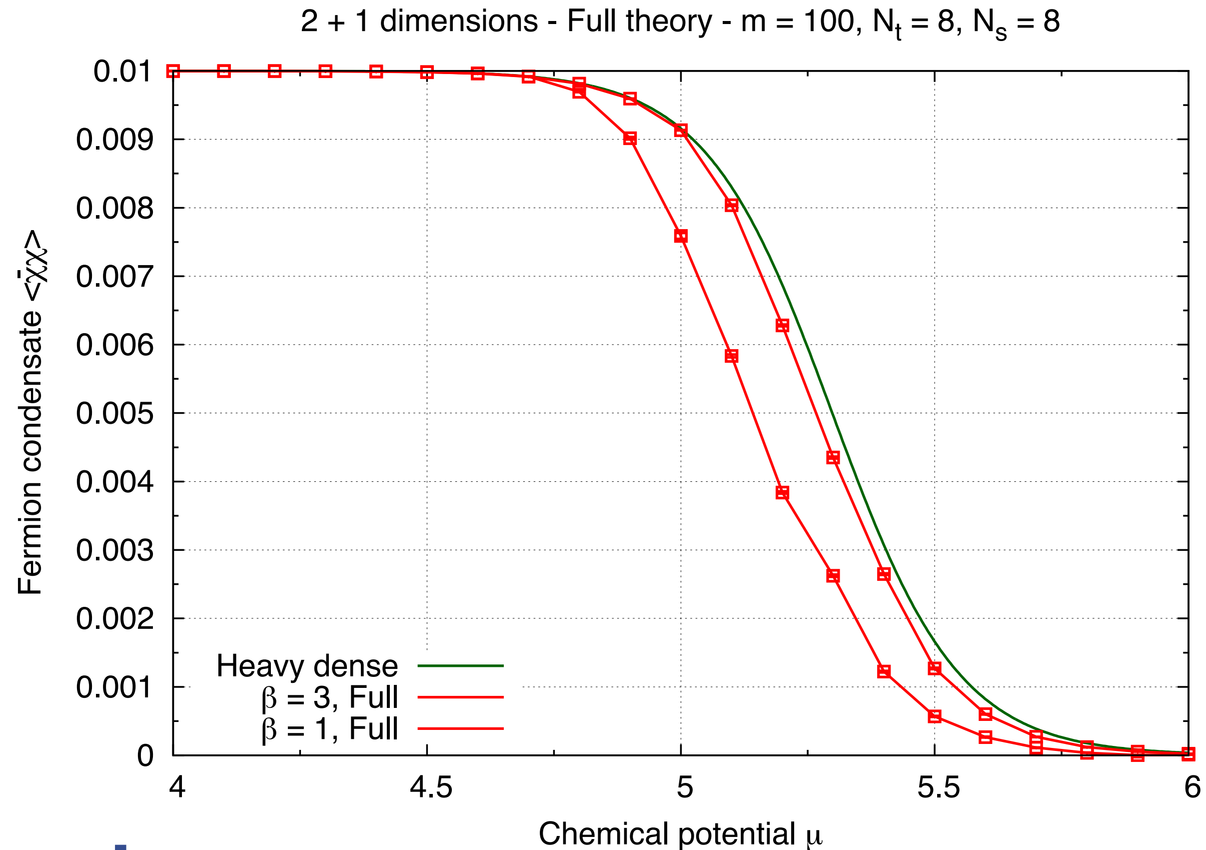
condensate **density**



Thirring model

full theory vs heavy dense limit





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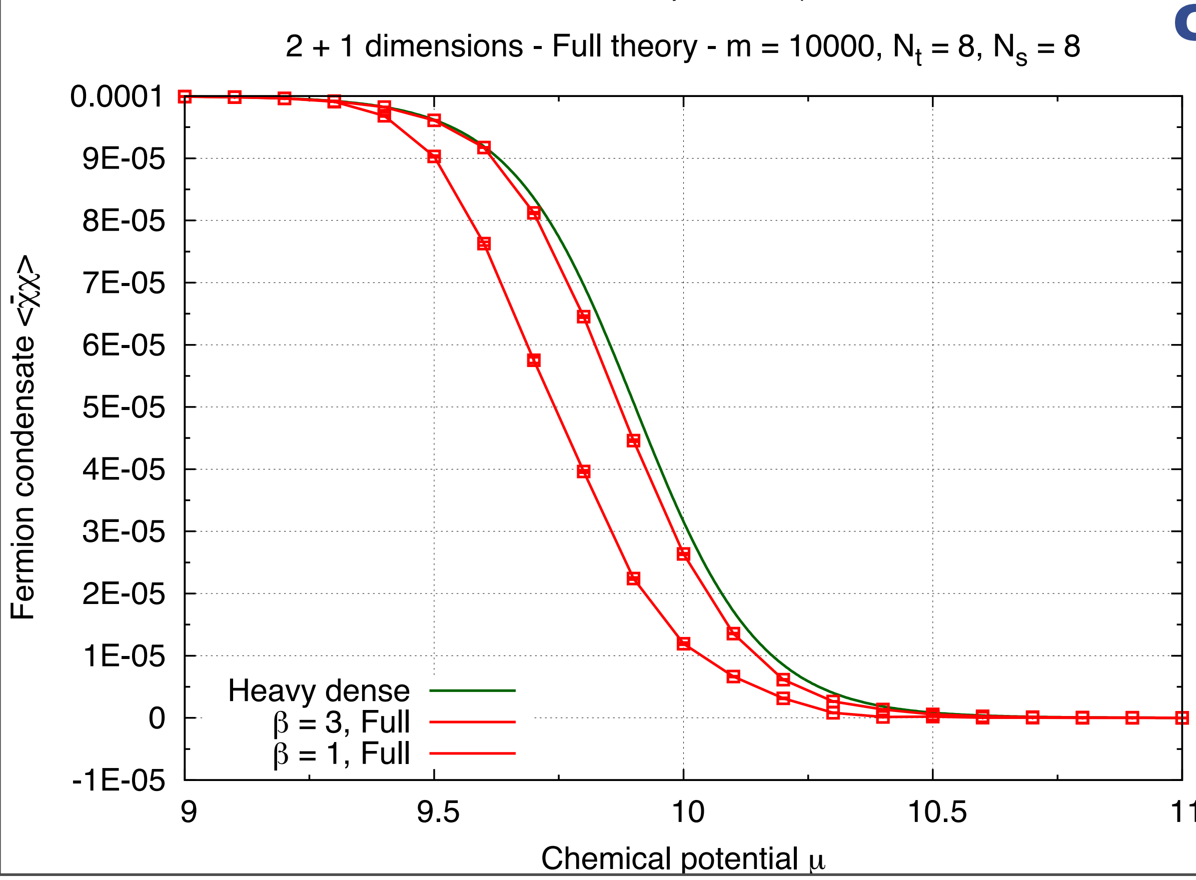
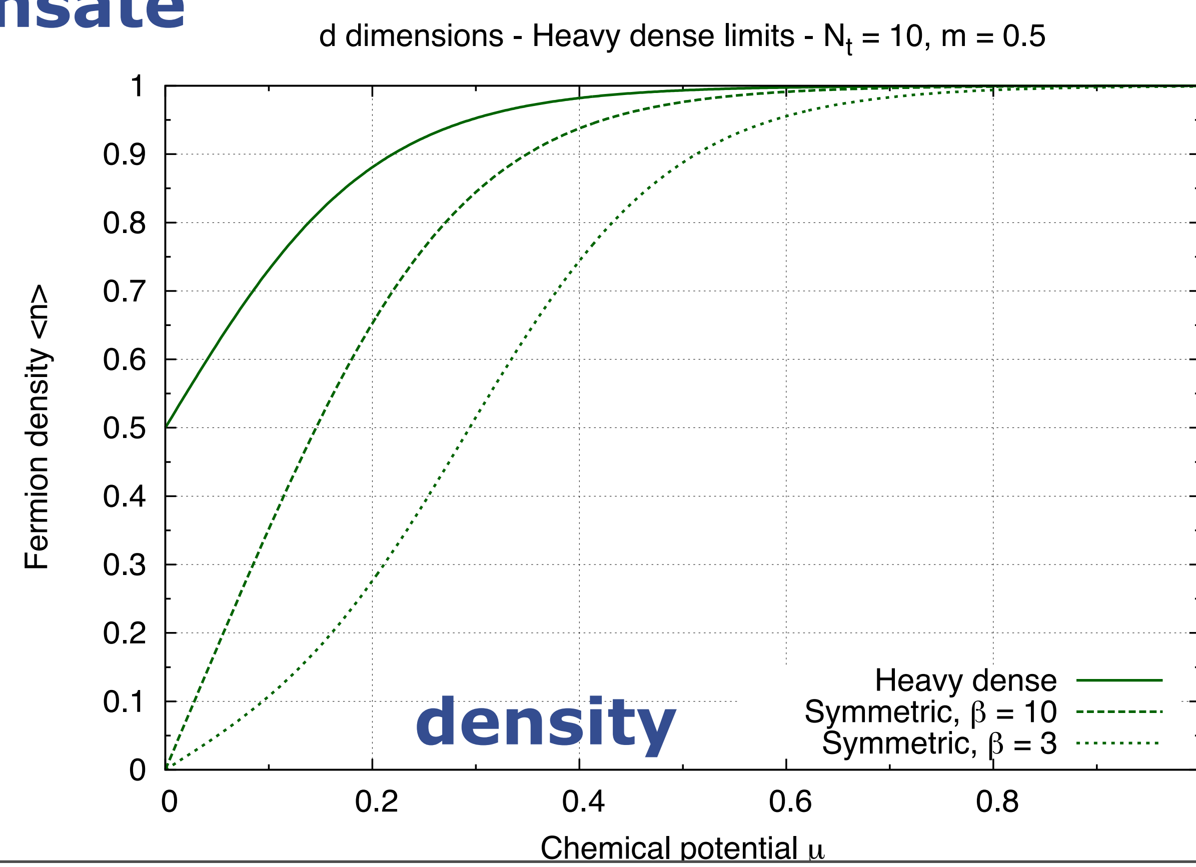
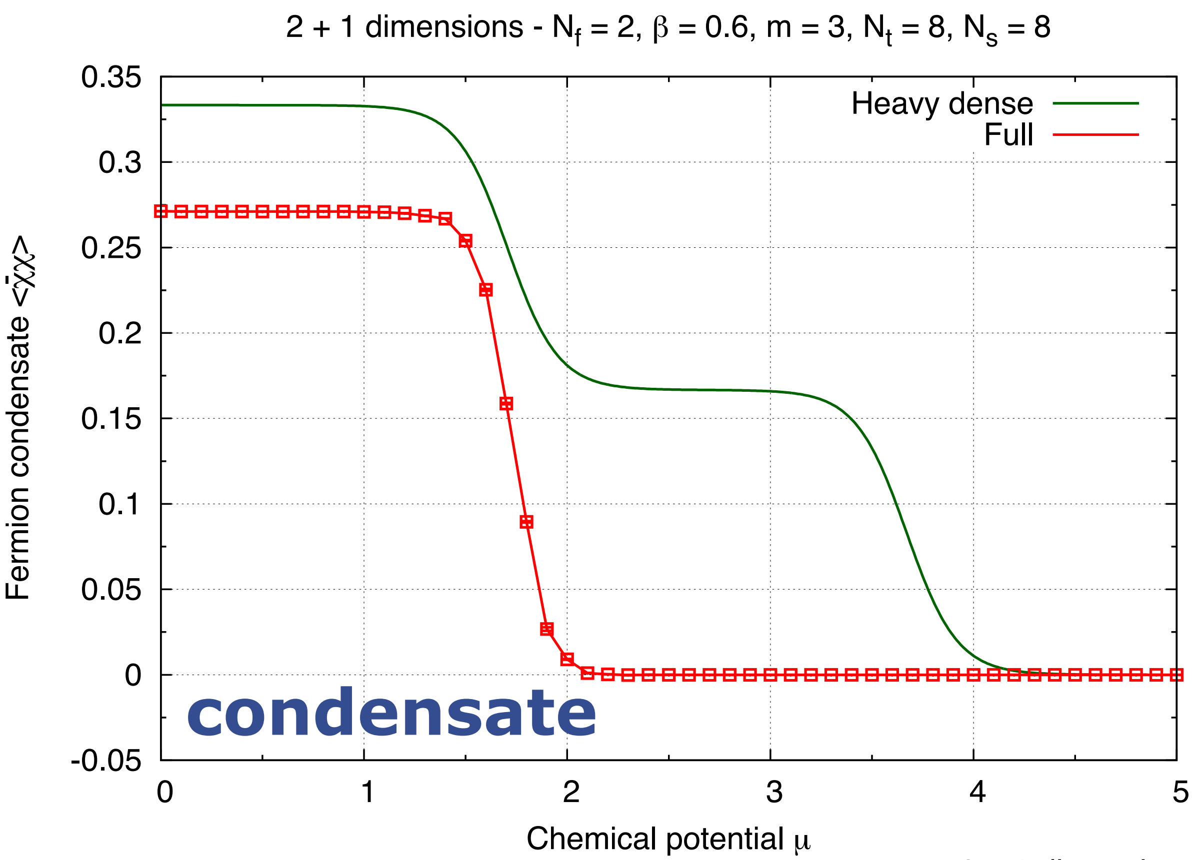
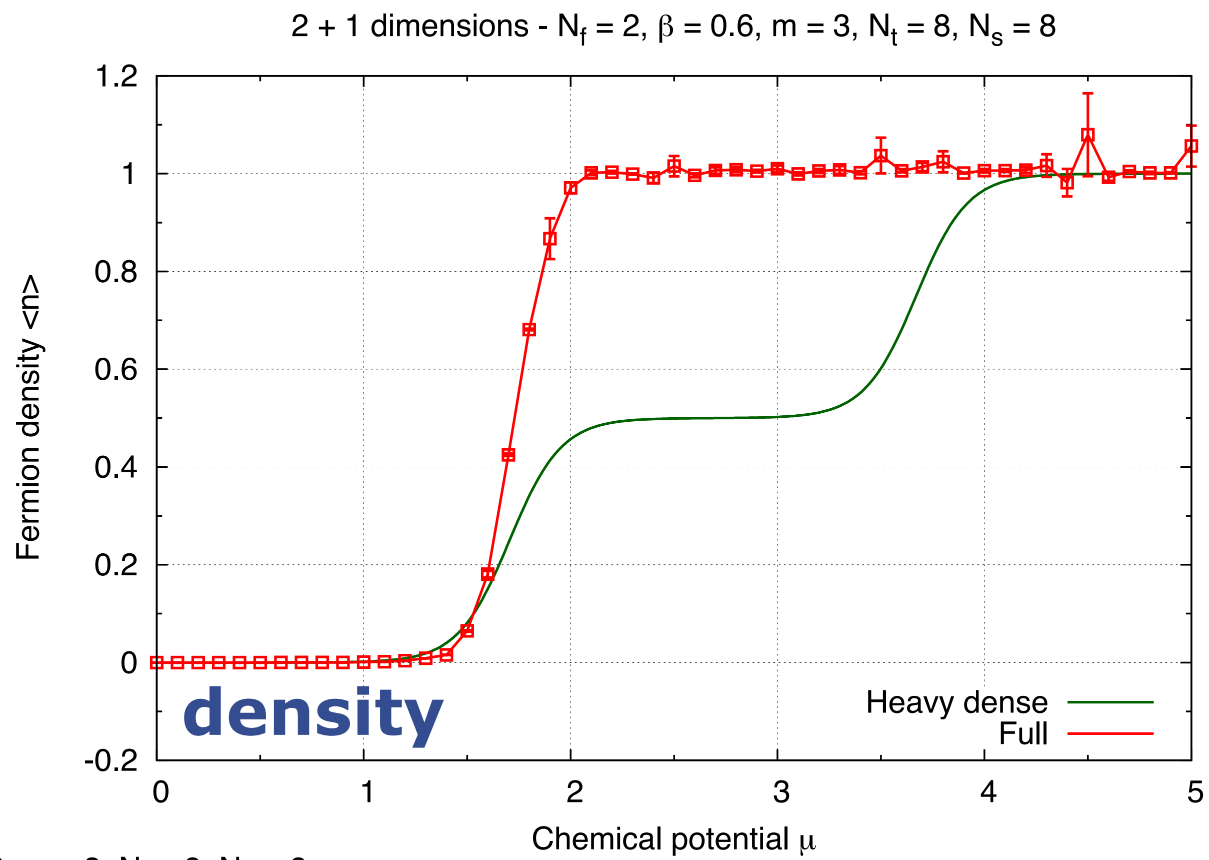


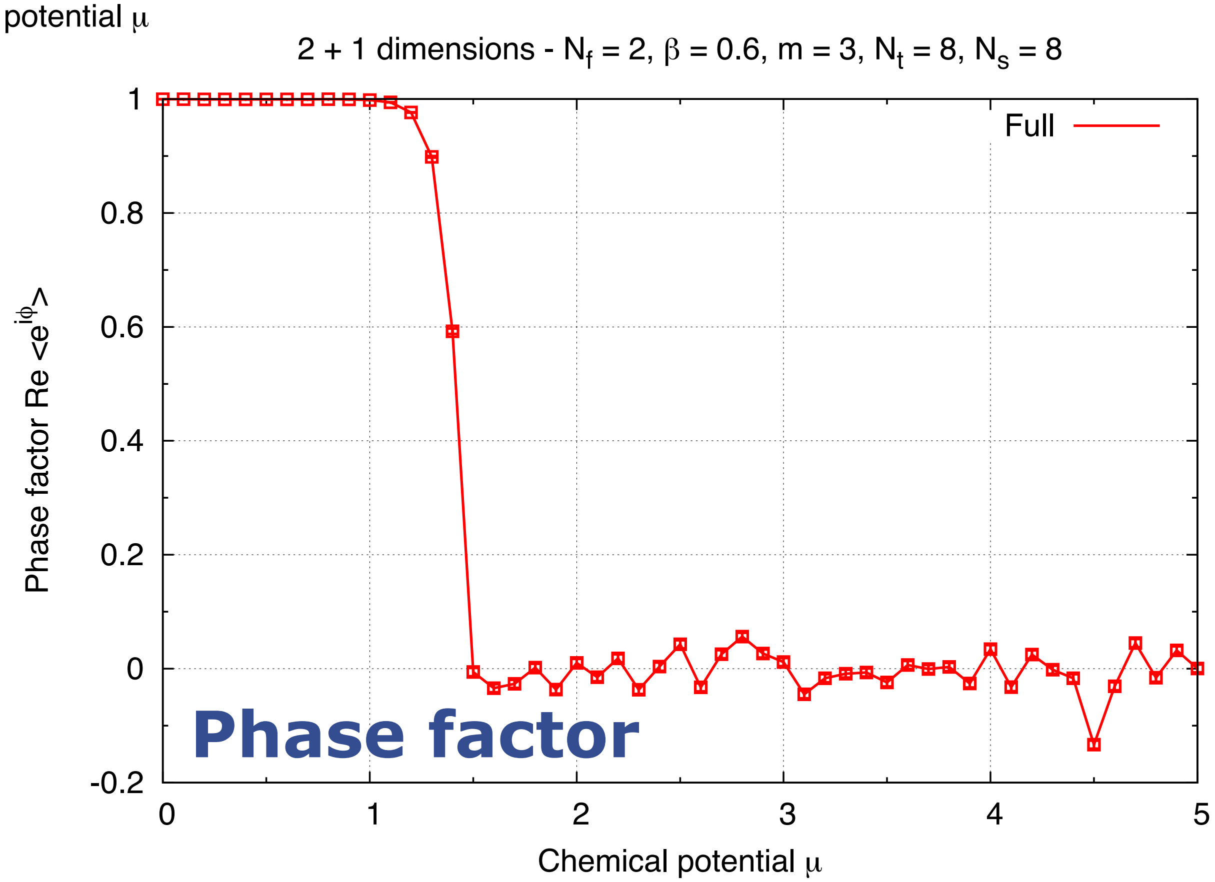


Thirring model

2 flavours



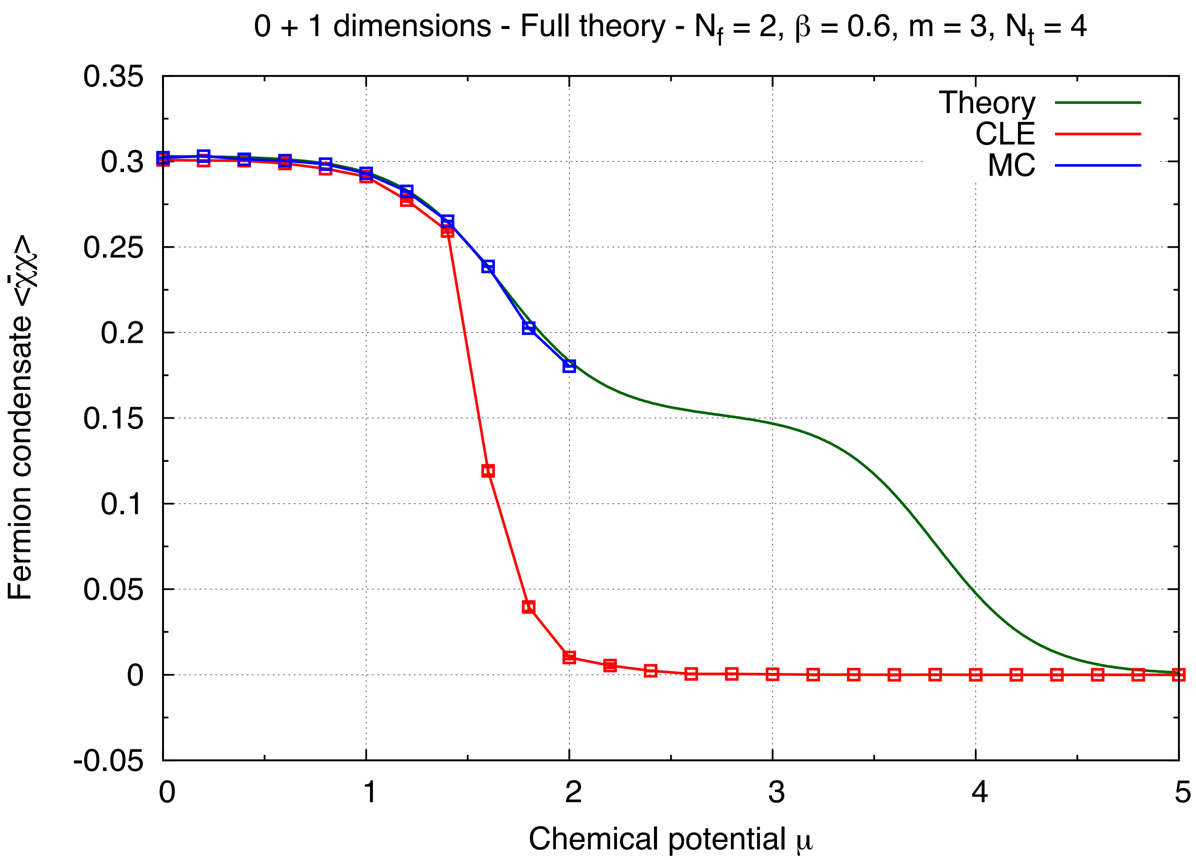


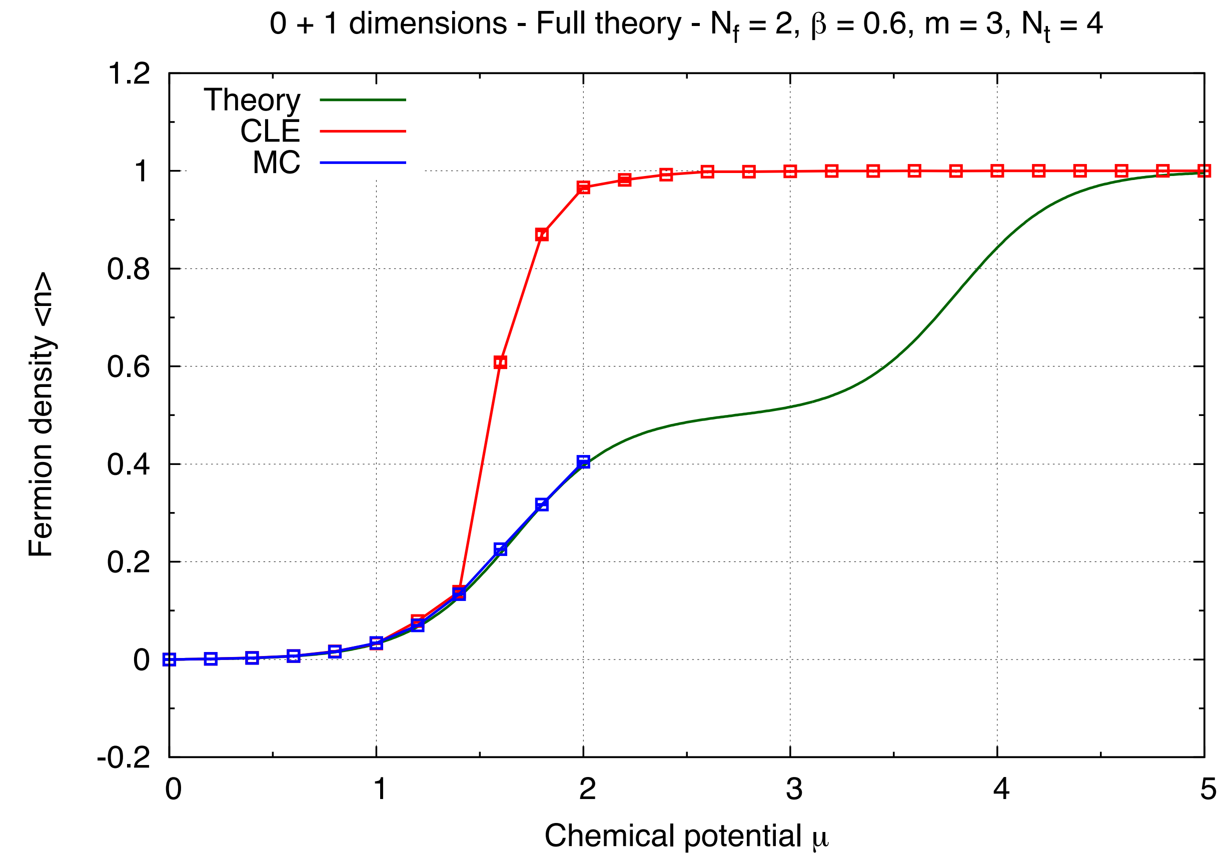


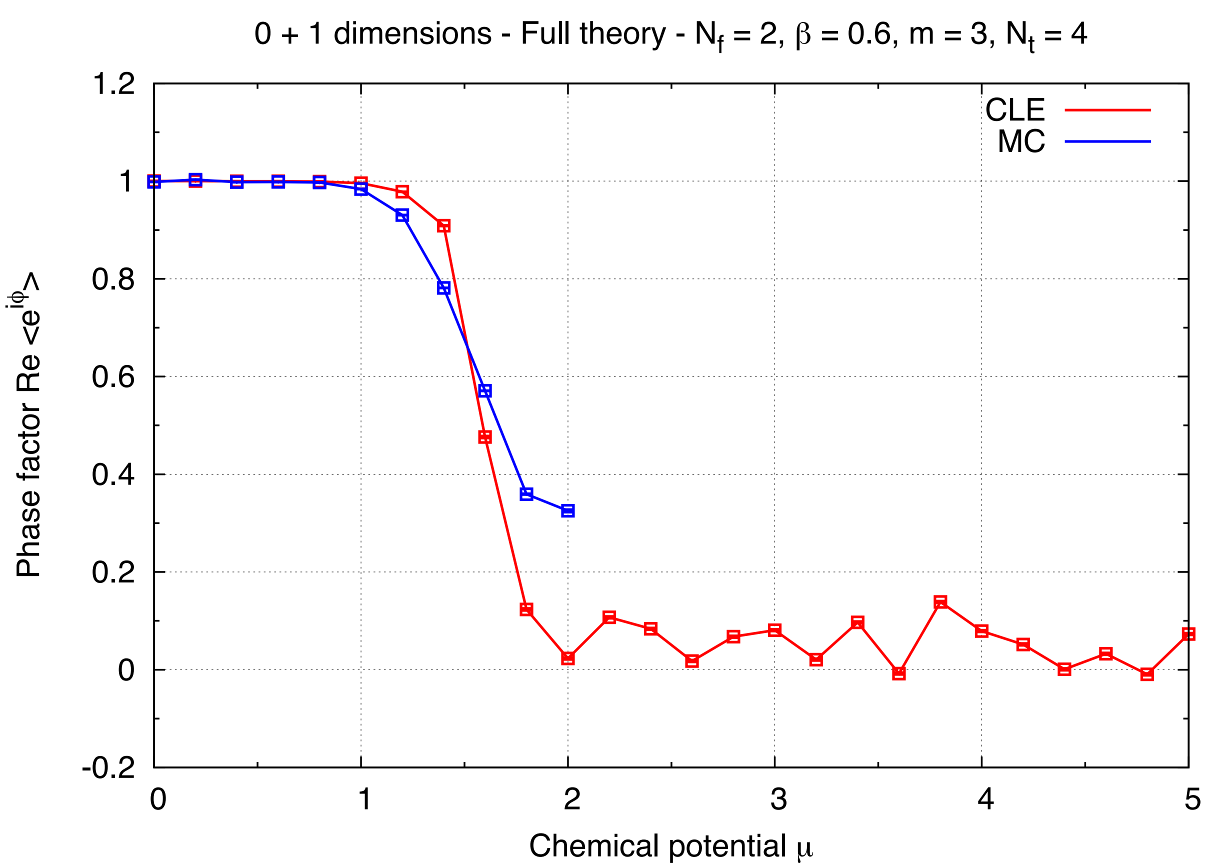
2 flavours

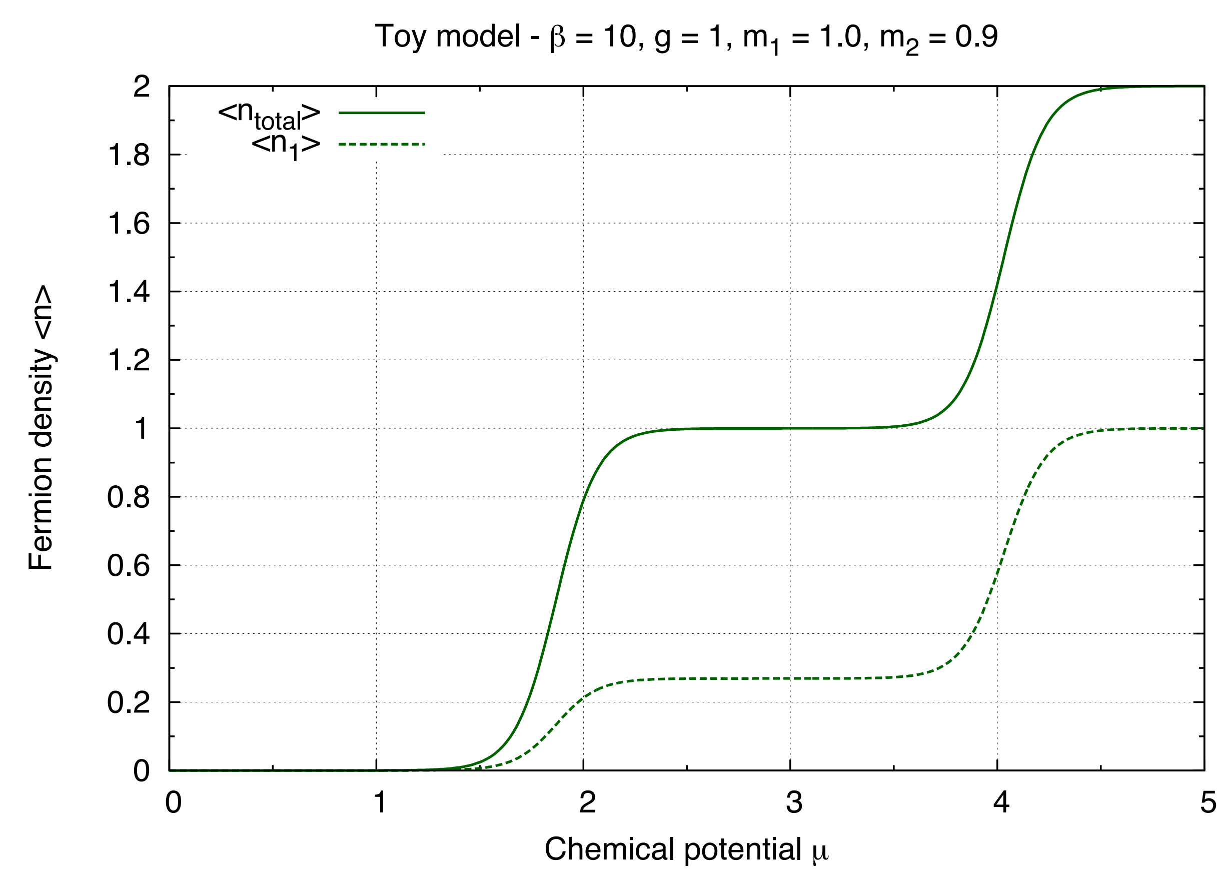
Thirring model

2 flavours, 0+1 dimensions



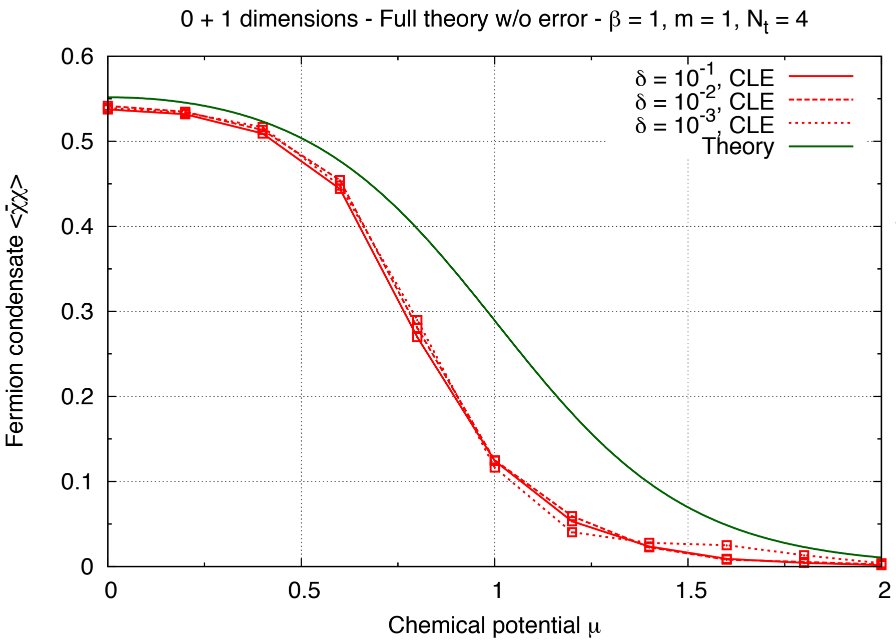


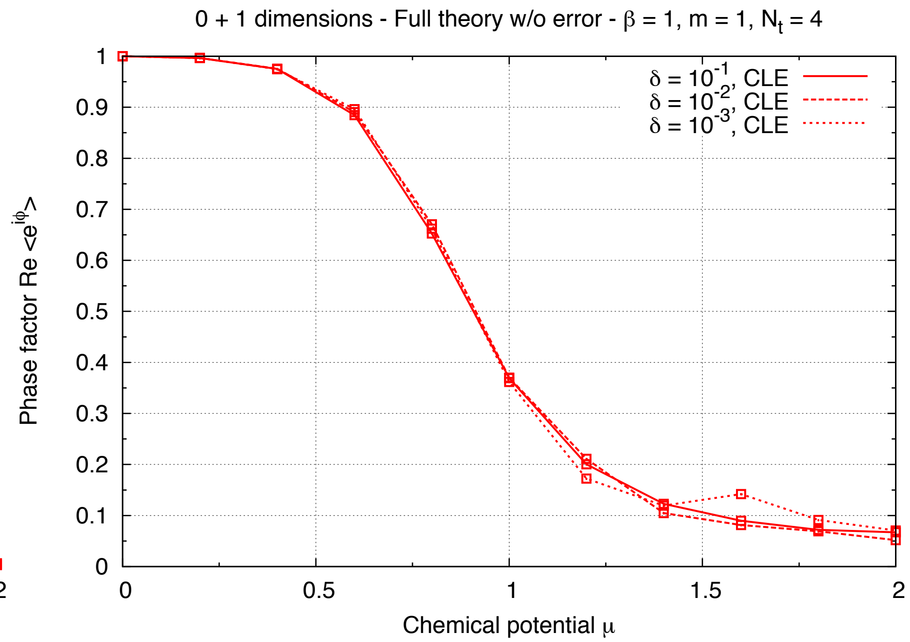


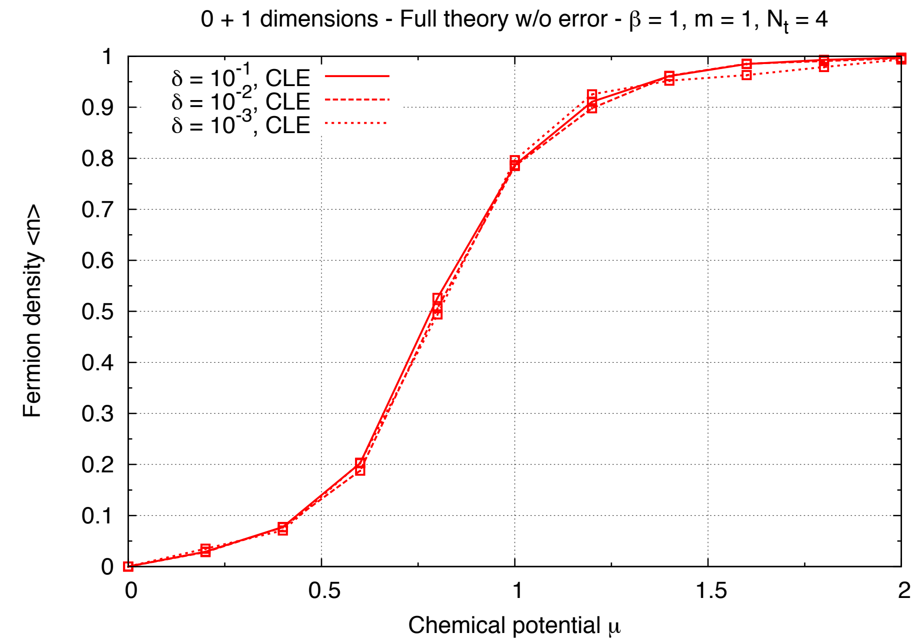


Thirring model

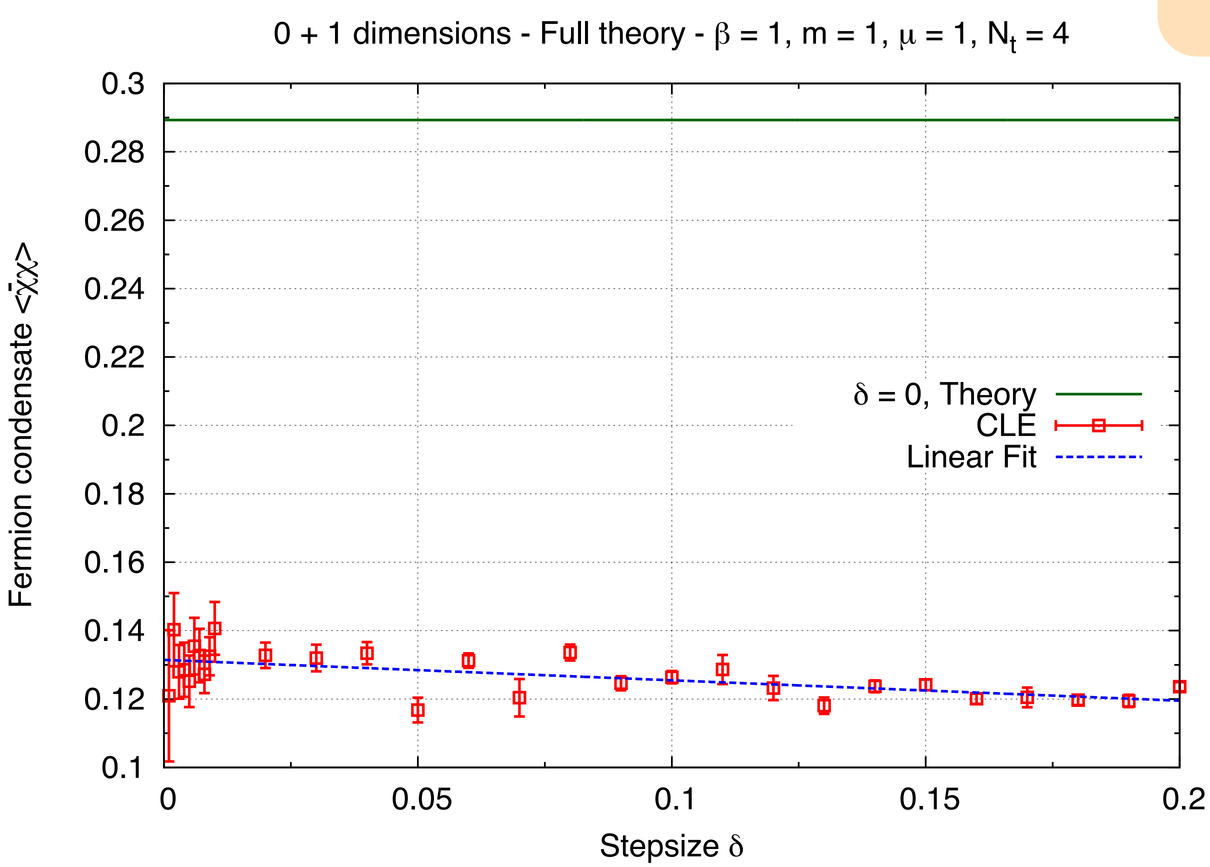
step size, 0+1 dimensions

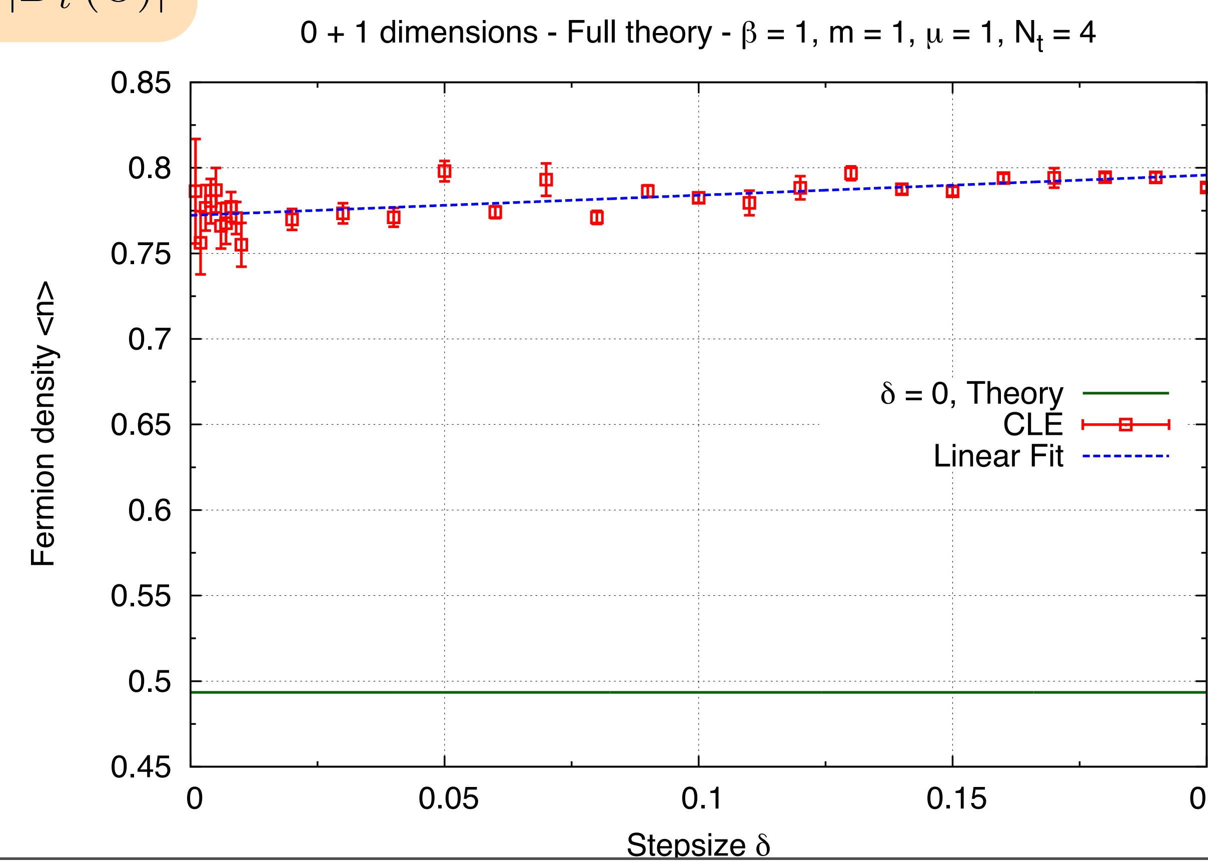






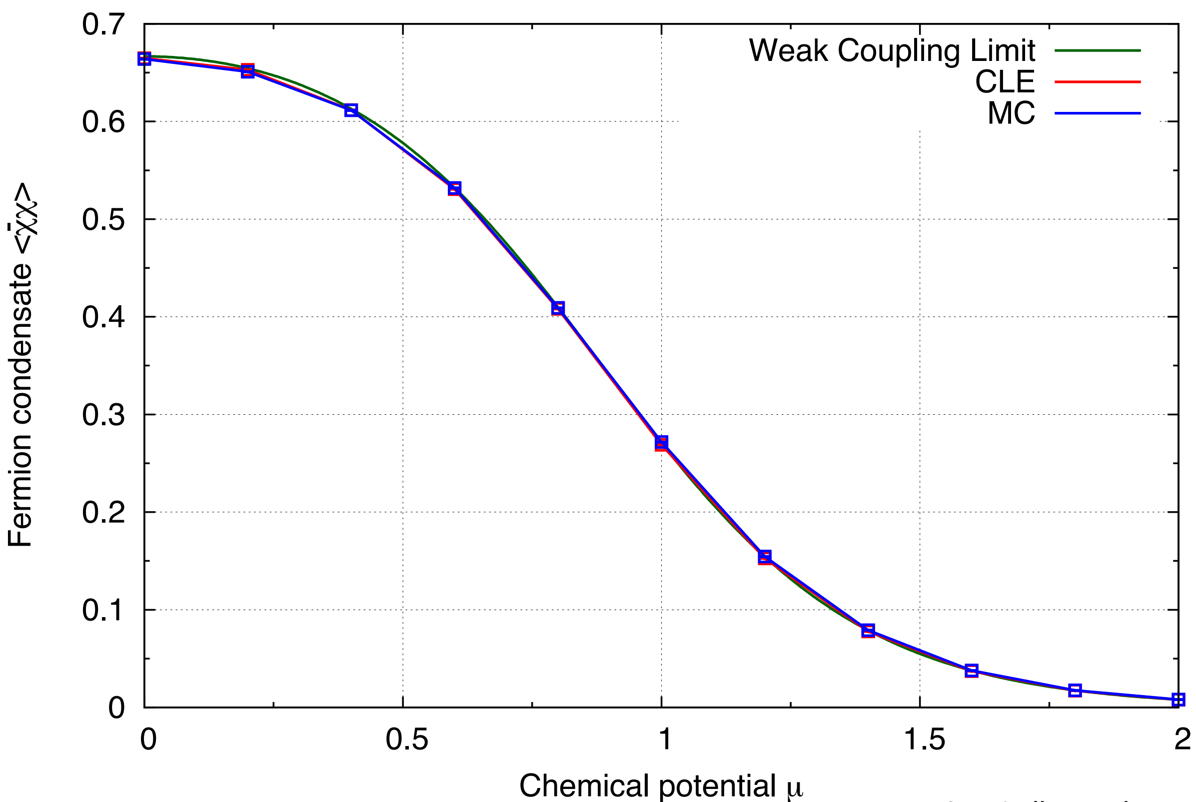
$$\epsilon_L\left(\Theta\right) = \frac{\delta}{\max_t\left|D_t\left(\Theta\right)\right|}$$

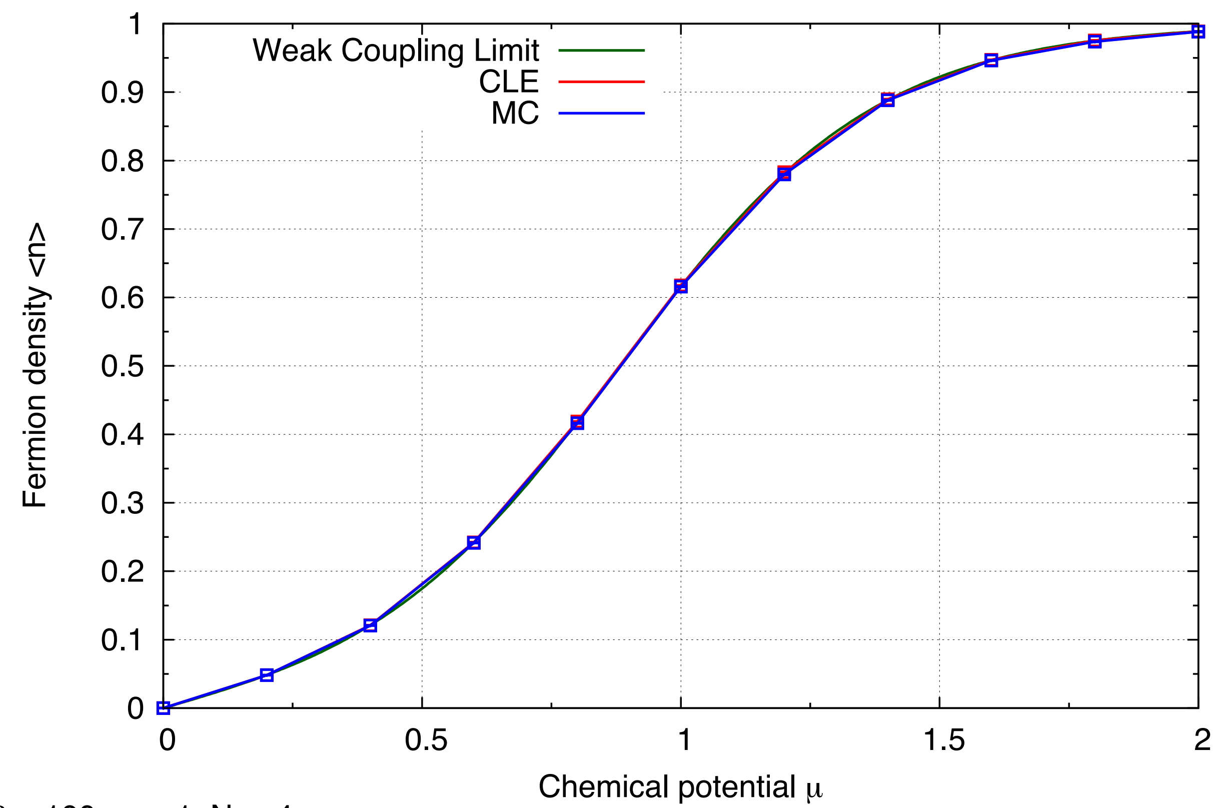


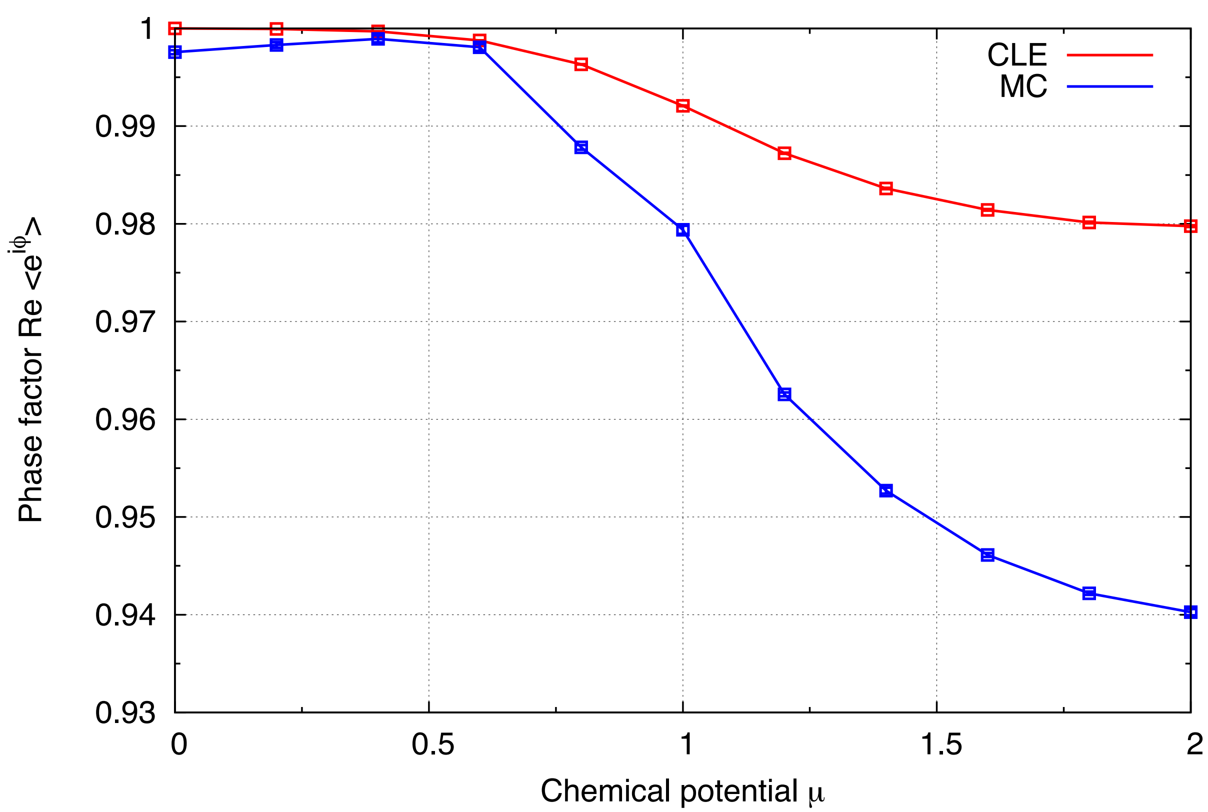


Thirring model

weak coupling, 0+1 dimensions



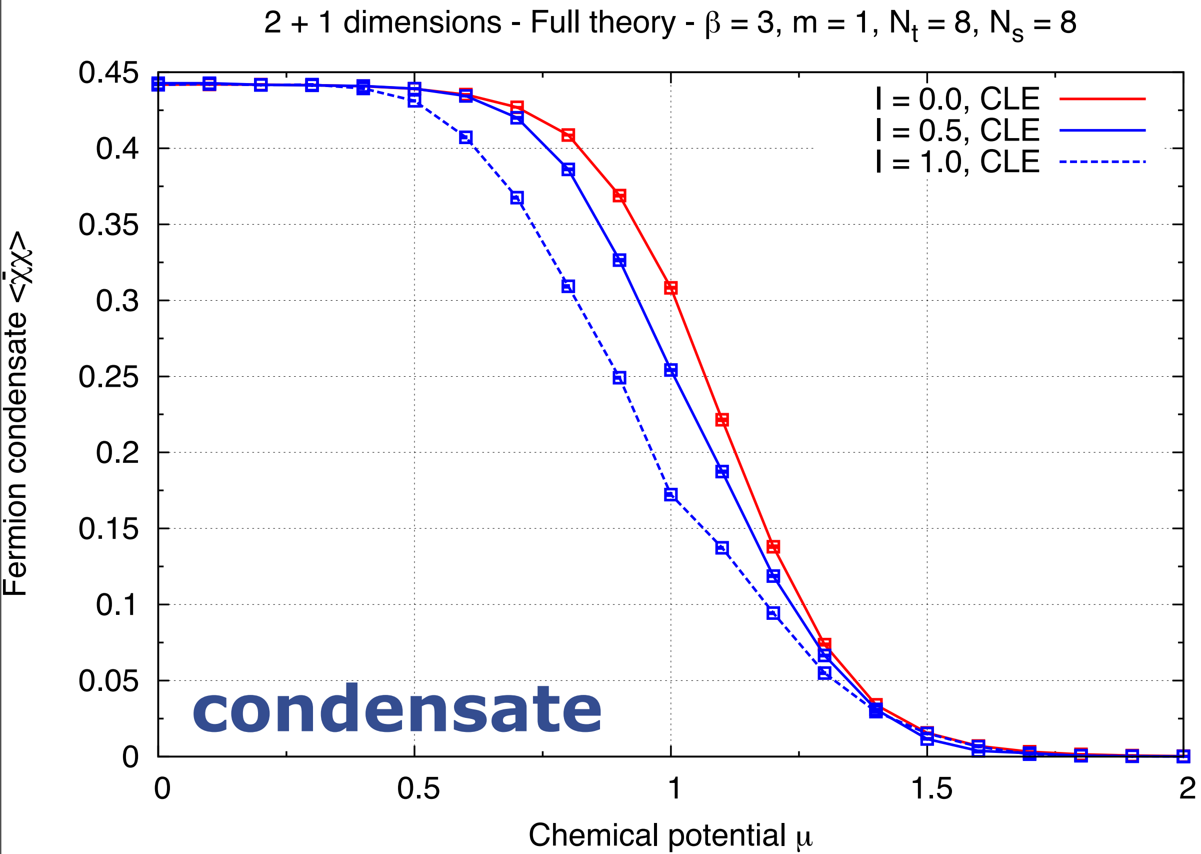


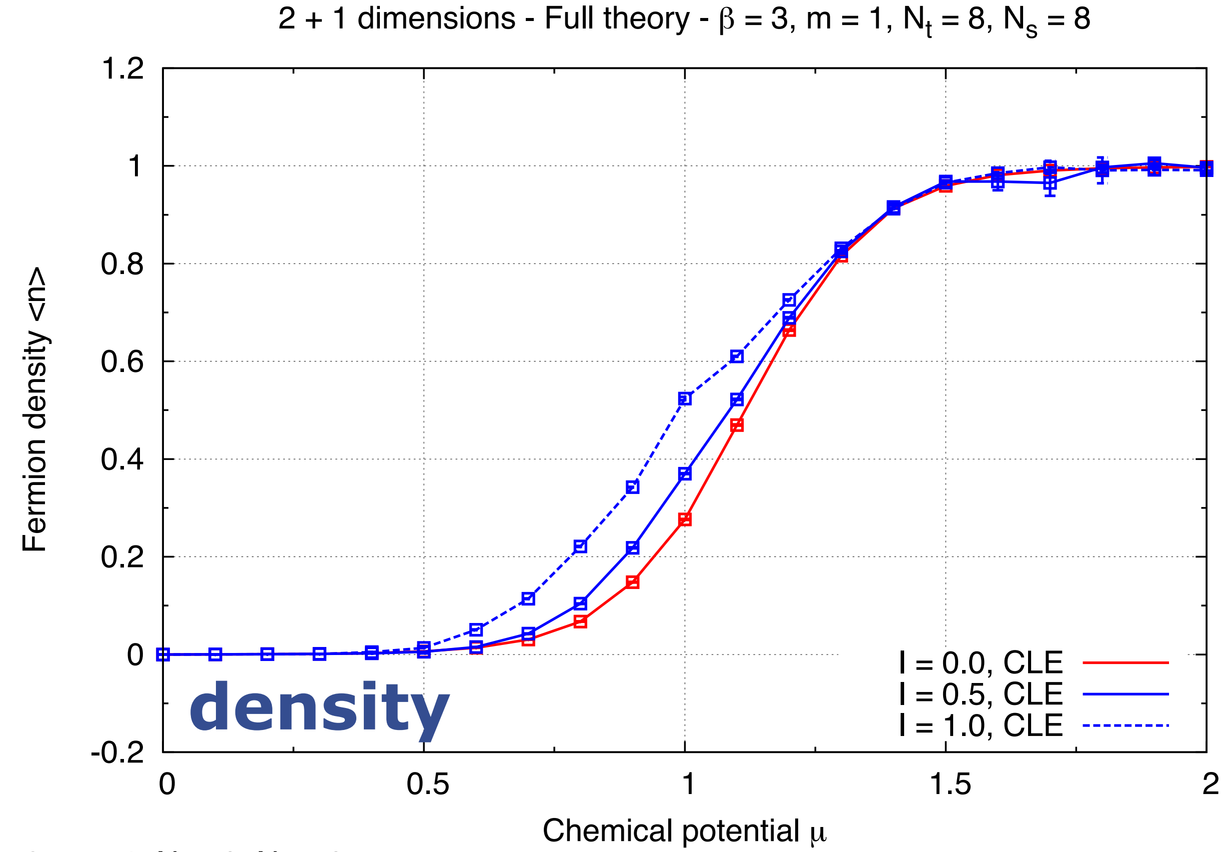


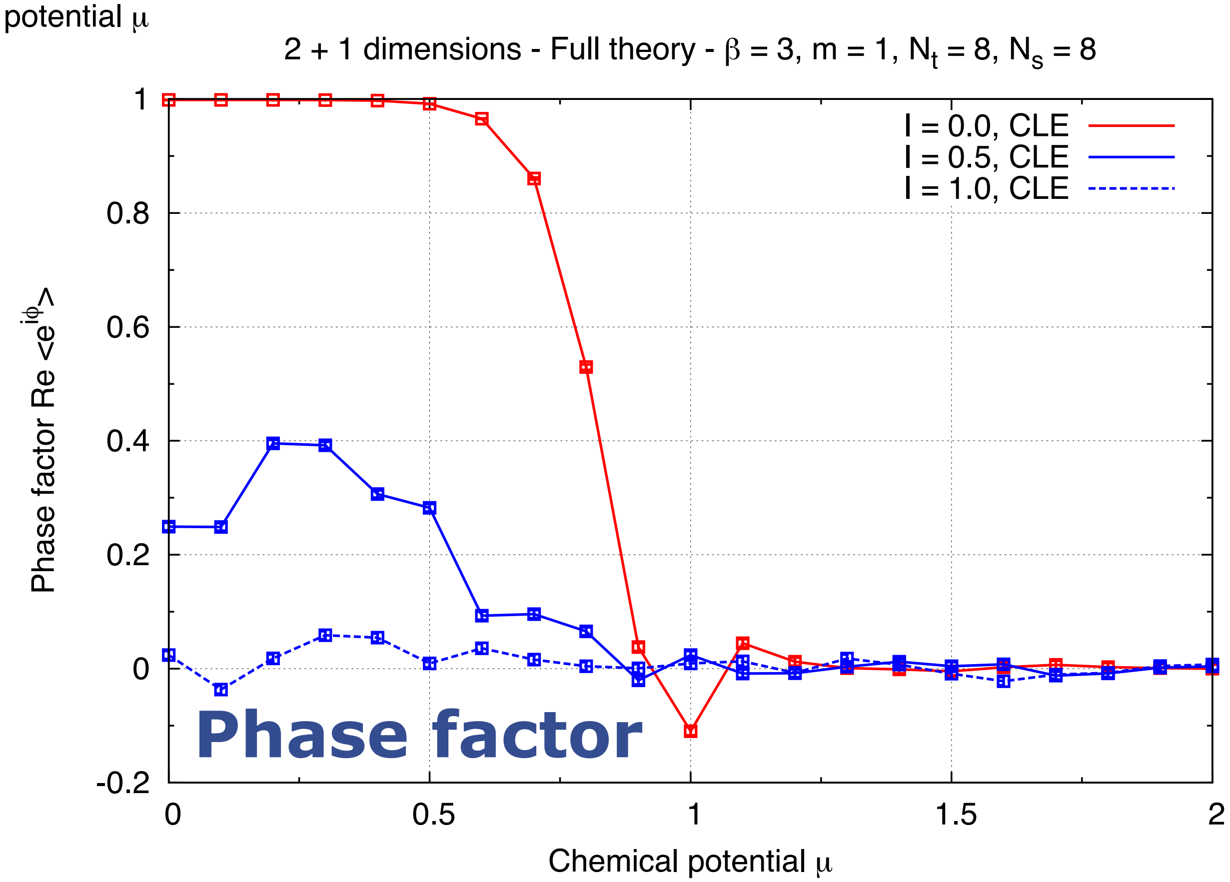
$\beta = 100$

Thirring model

Complex noise

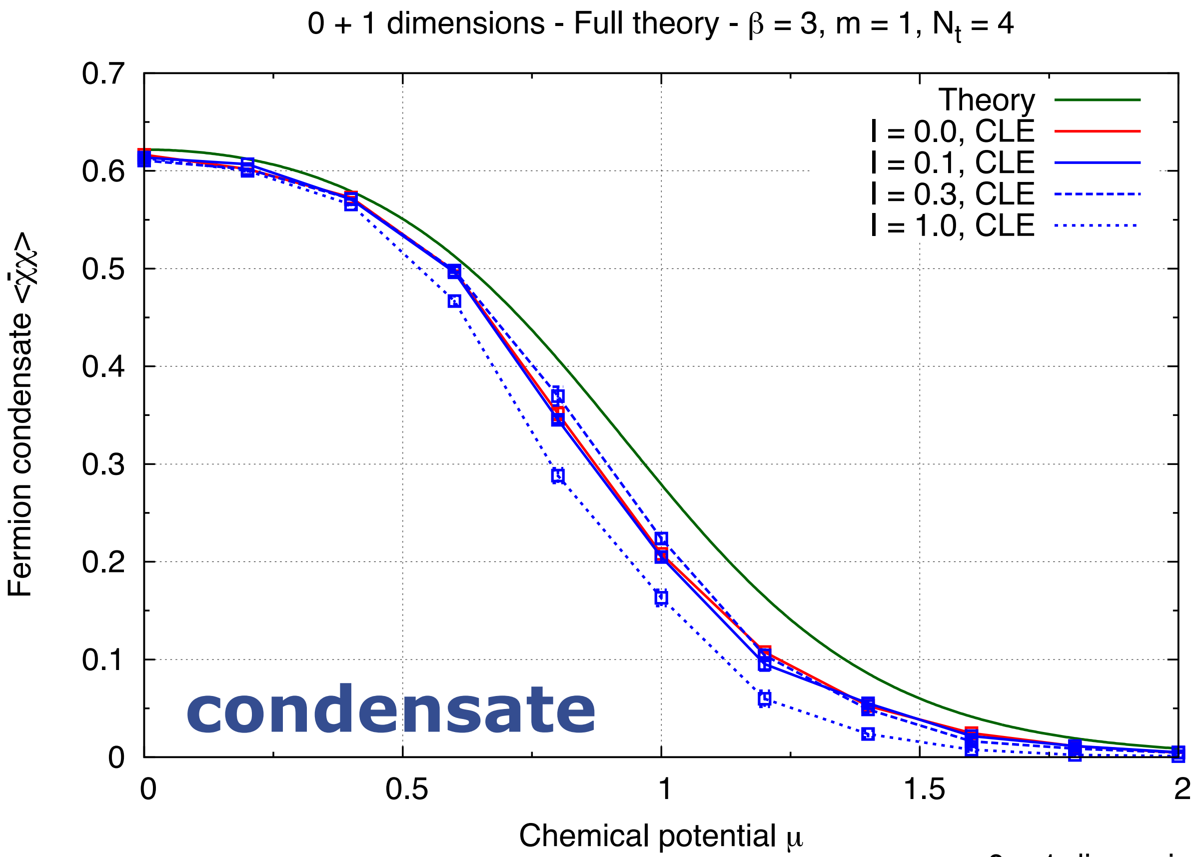


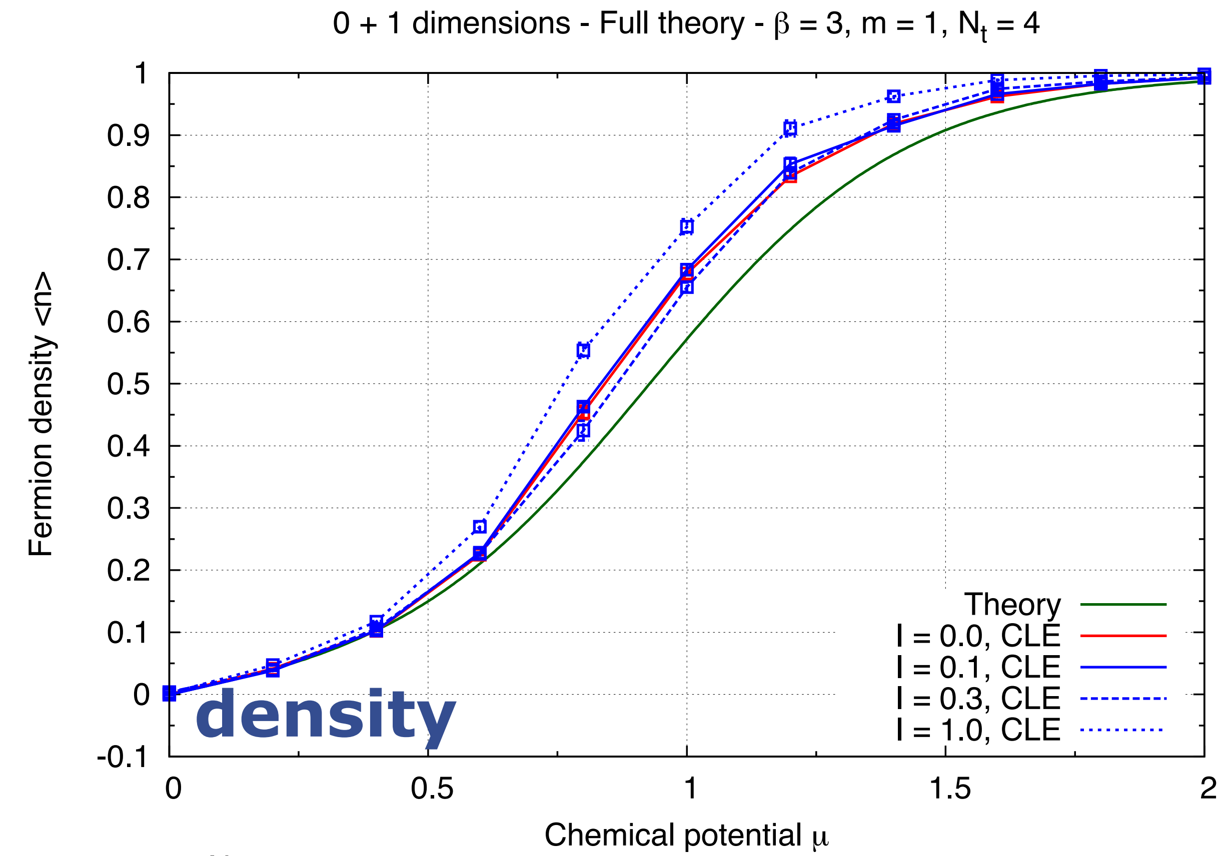


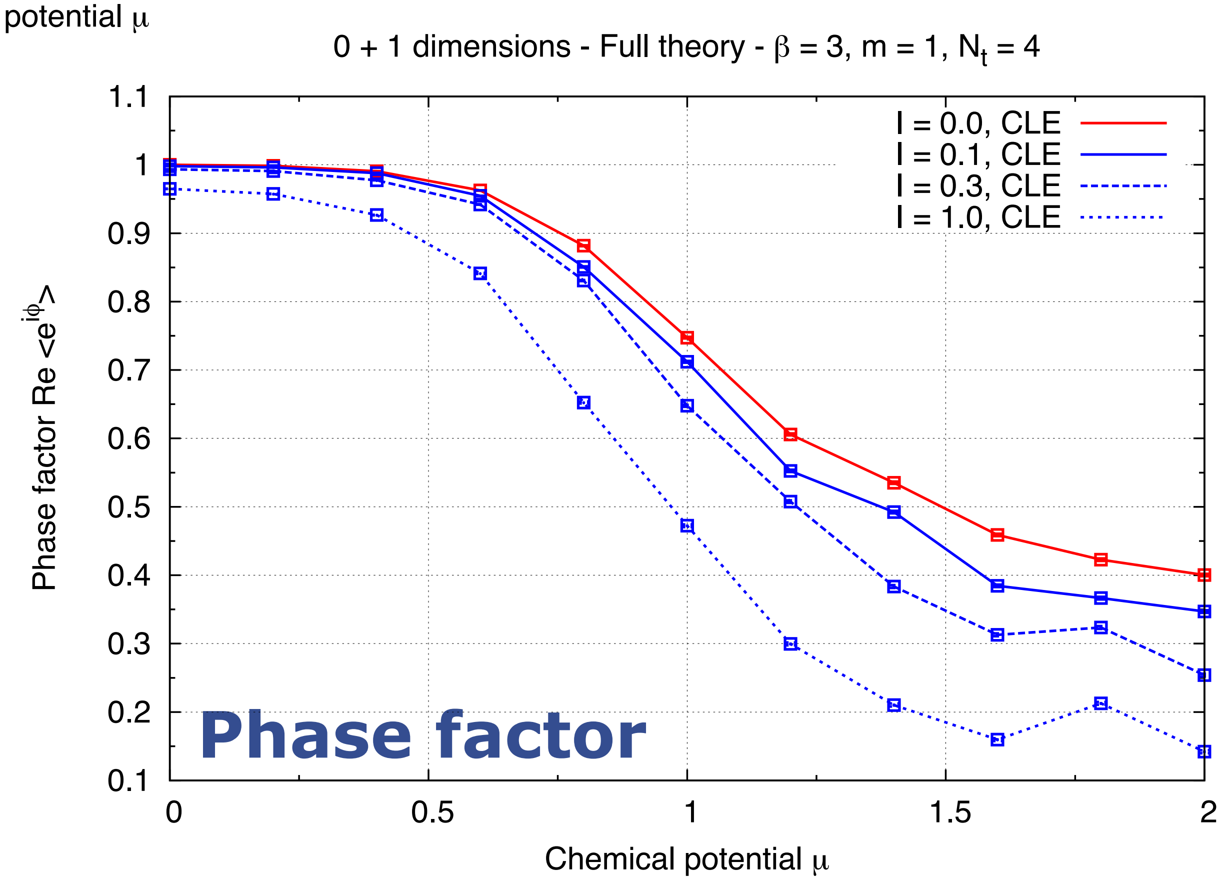


Thirring model

Complex noise, 0+1 dimensions

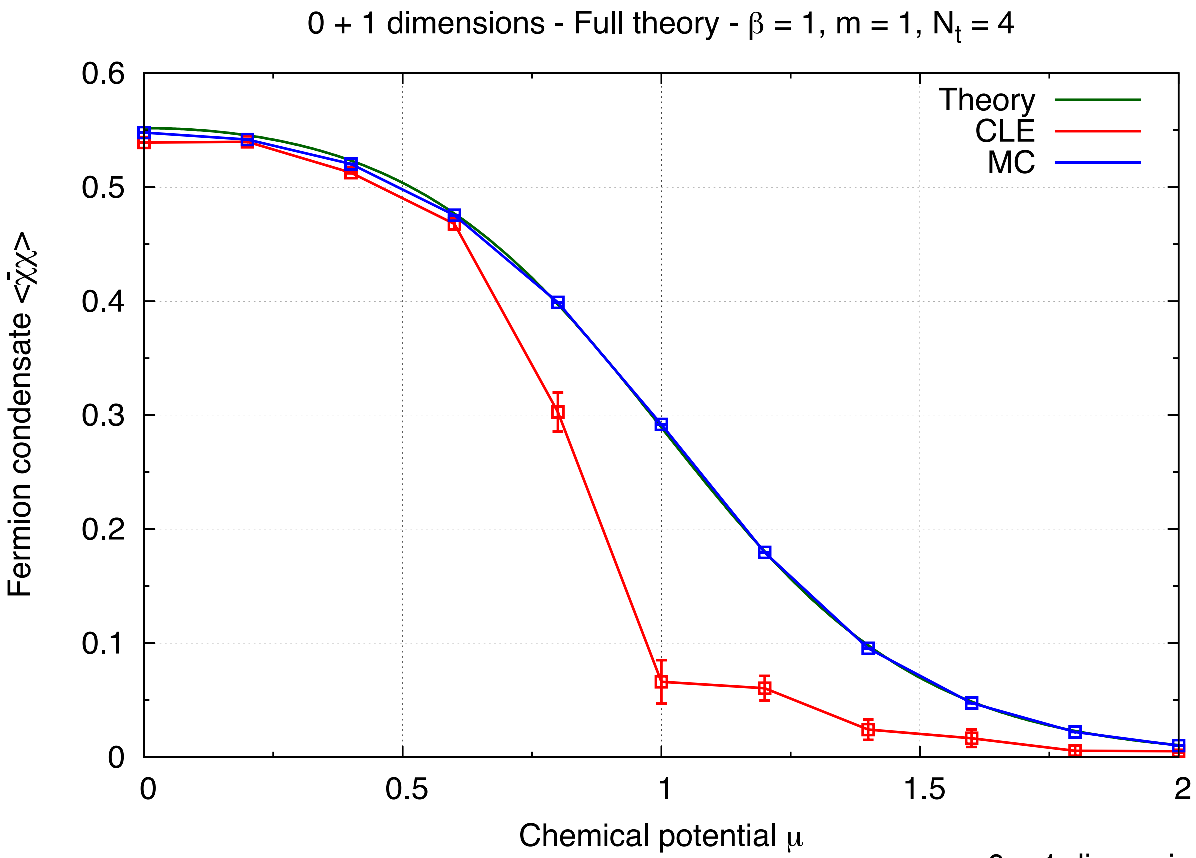


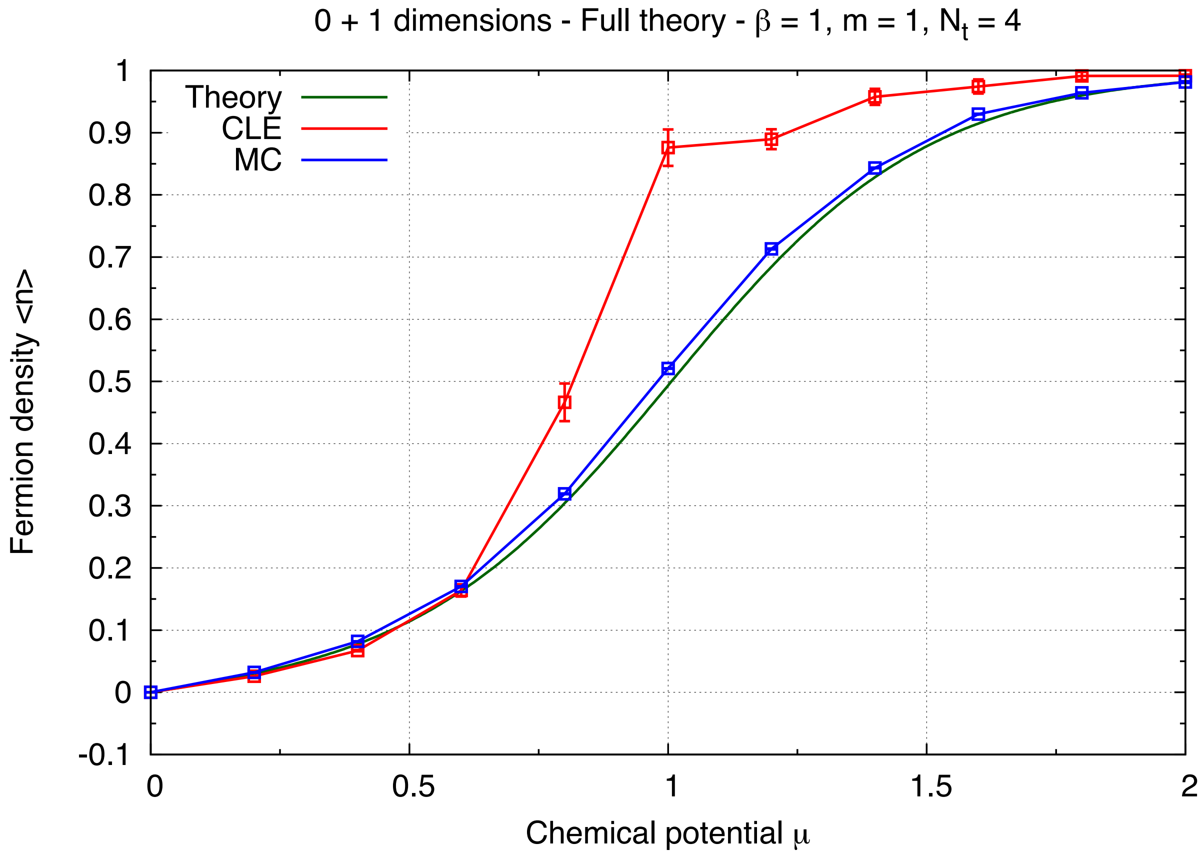




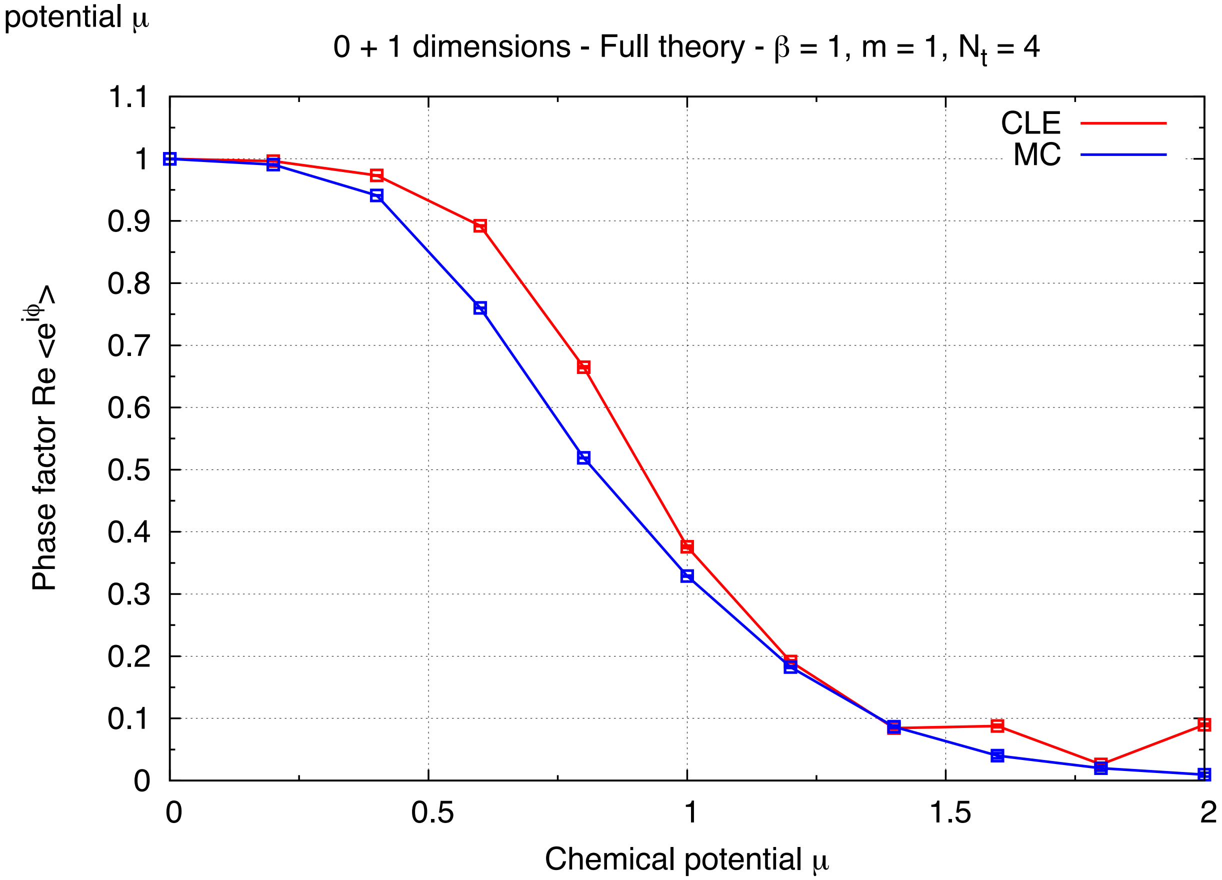
Thirring model

complex Langevin vs Monte Carlo, 0+1 dimensions



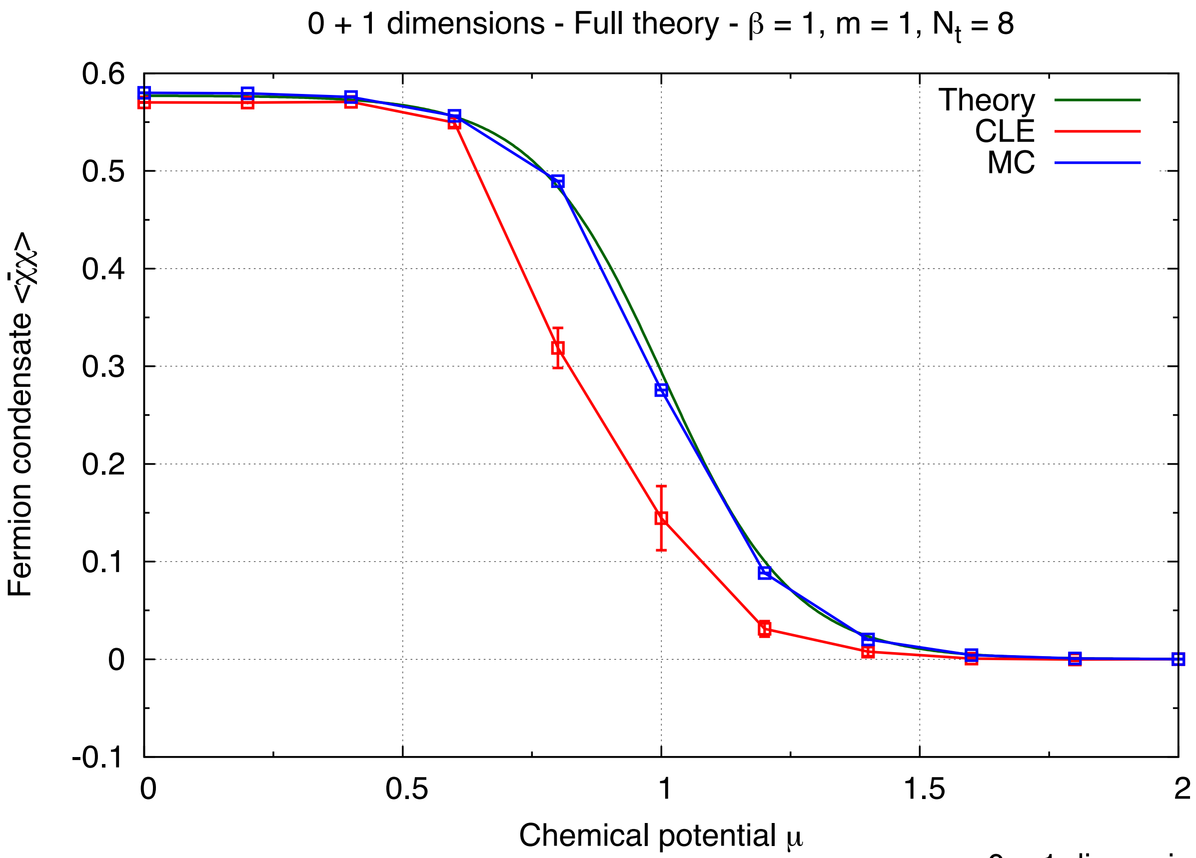


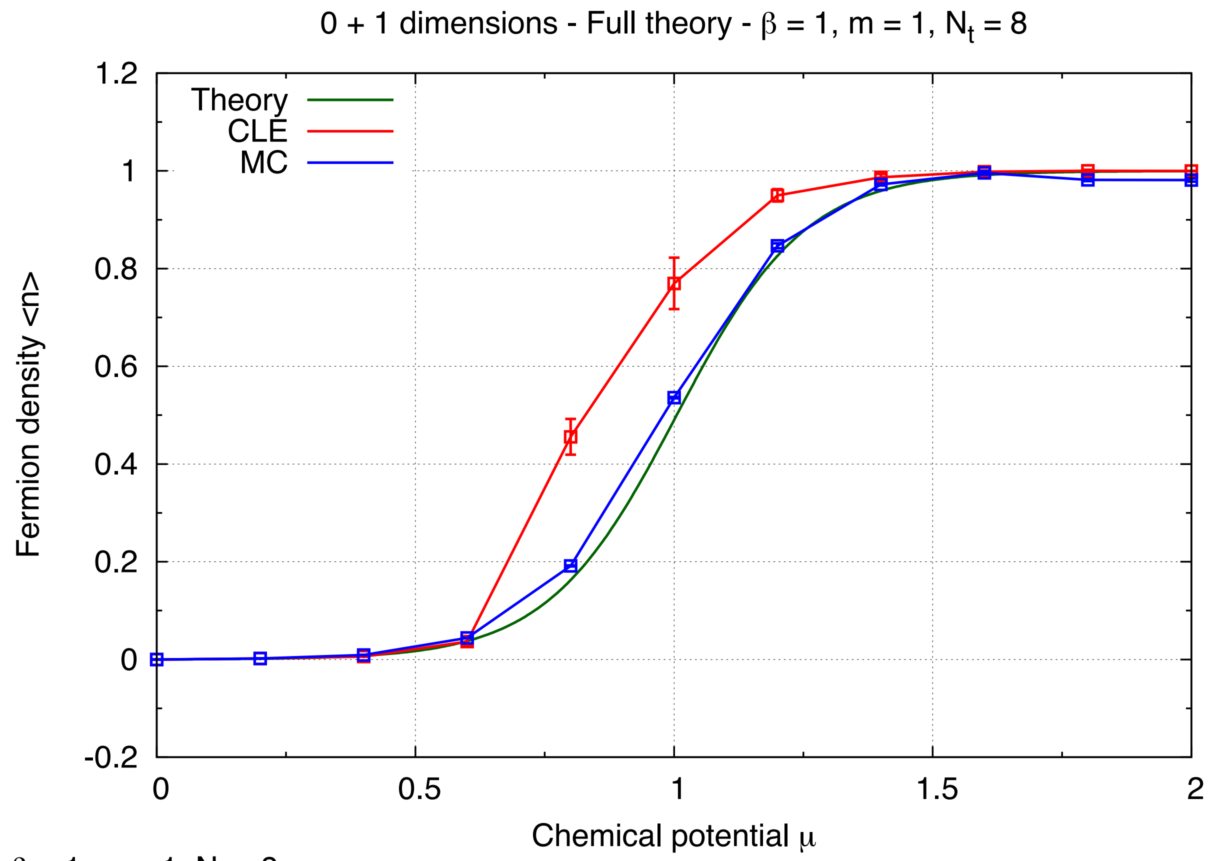
$$\mathbf{N_t = 4}$$



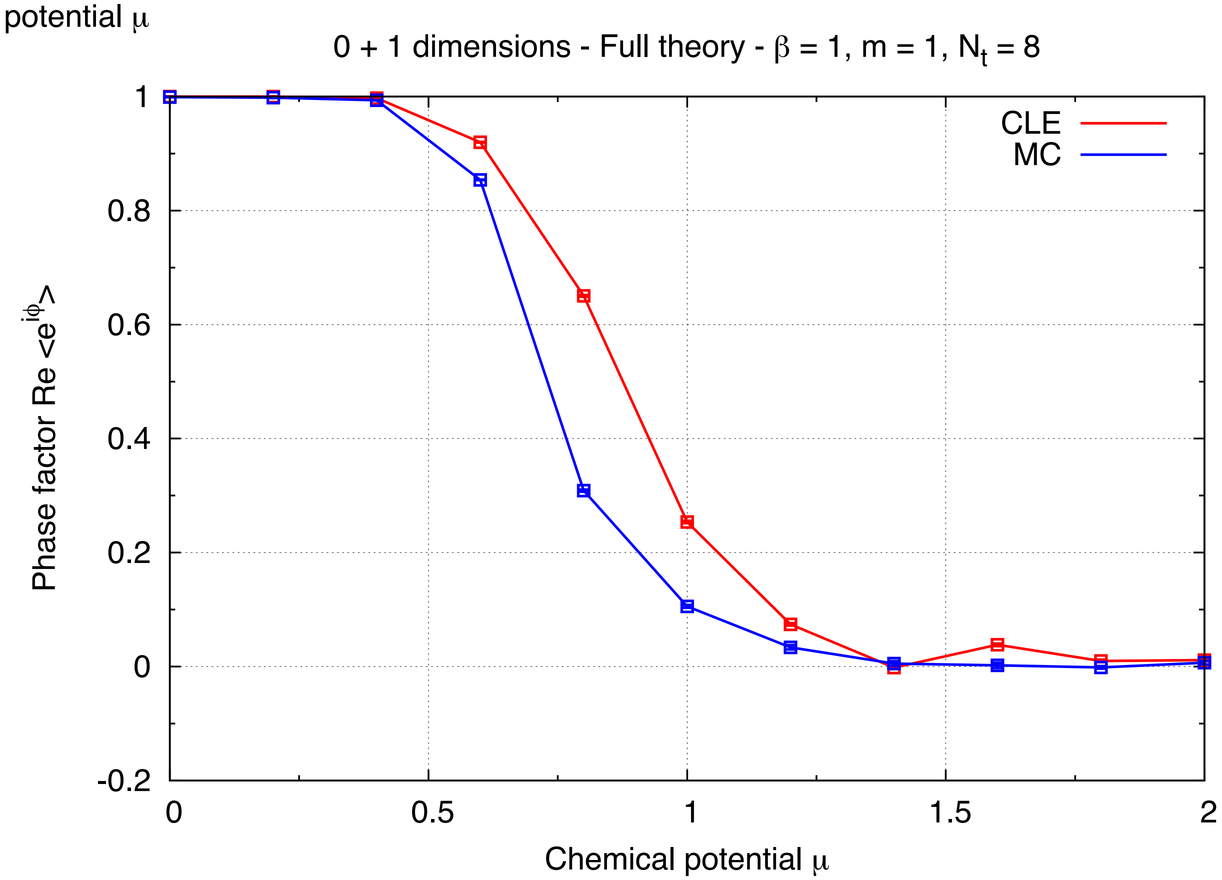
Thirring model

complex Langevin vs Monte Carlo, 0+1 dimensions



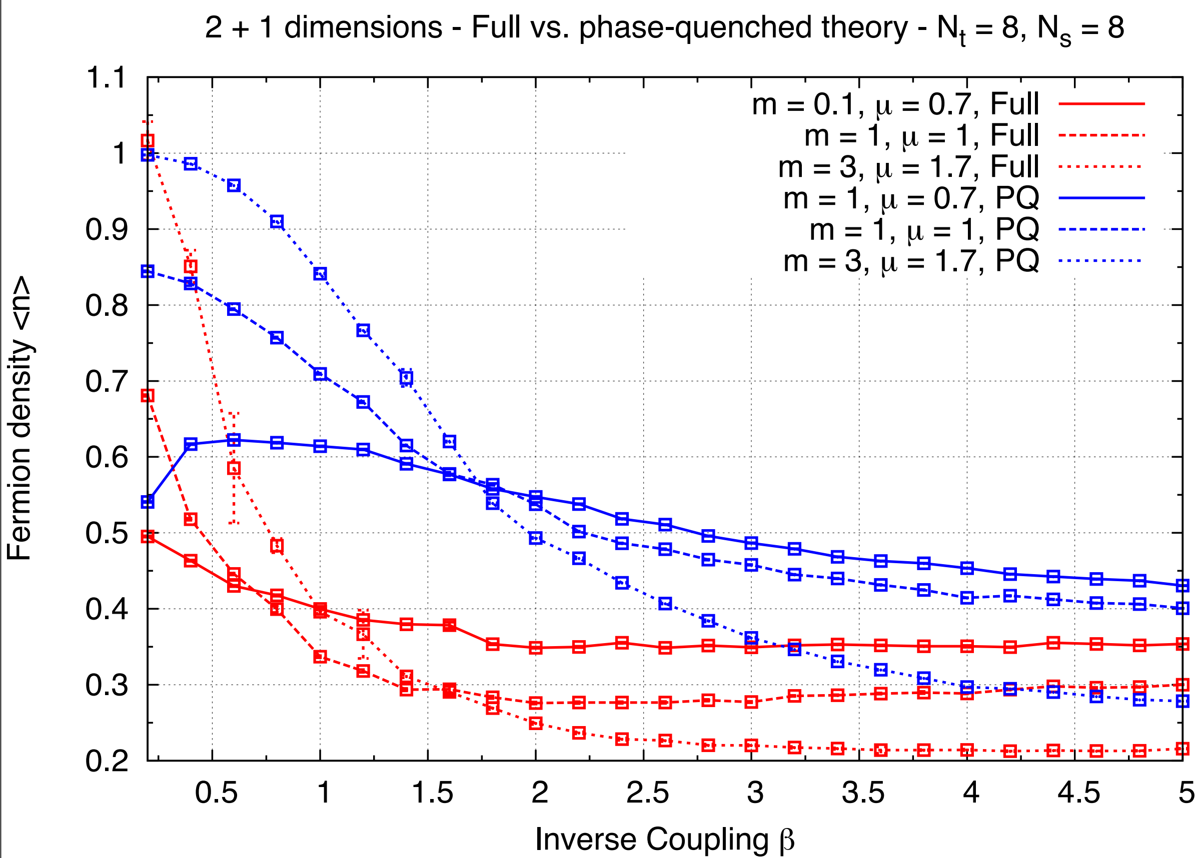


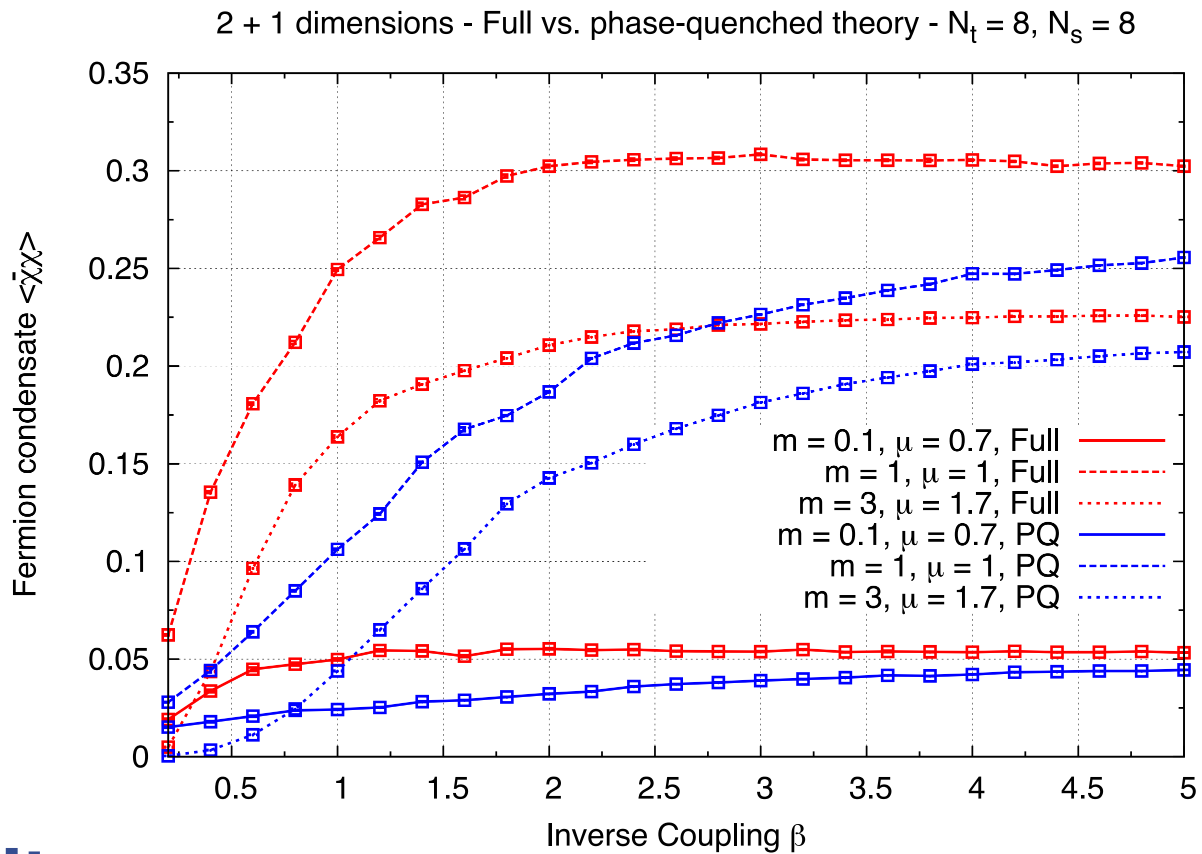
$\mathbf{N_t = 8}$



Thirring model

full theory vs phase quenched theory

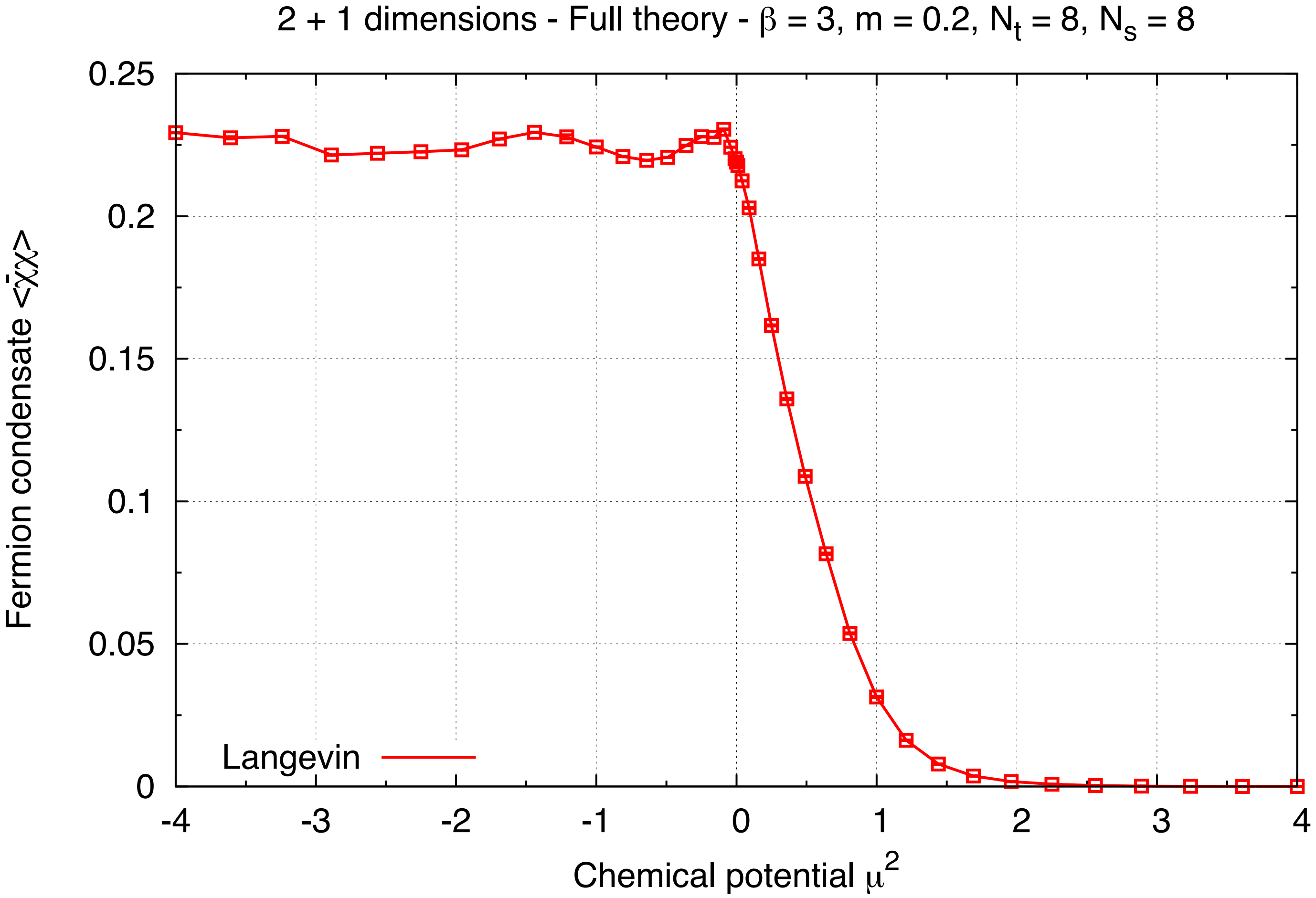




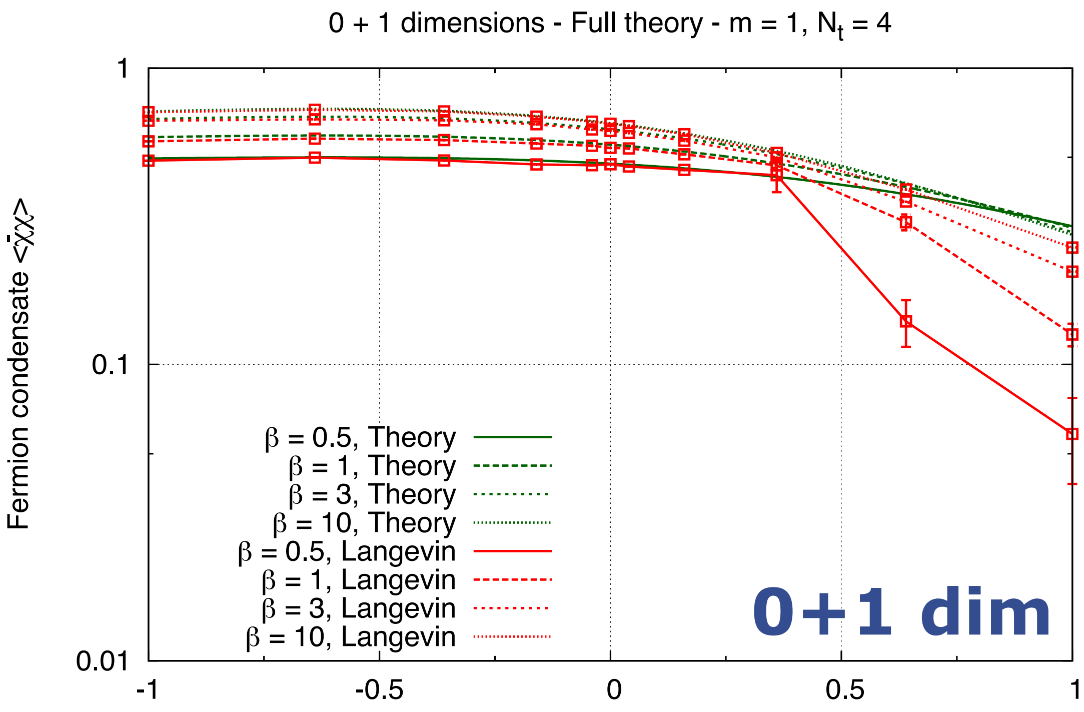
density

Thirring model

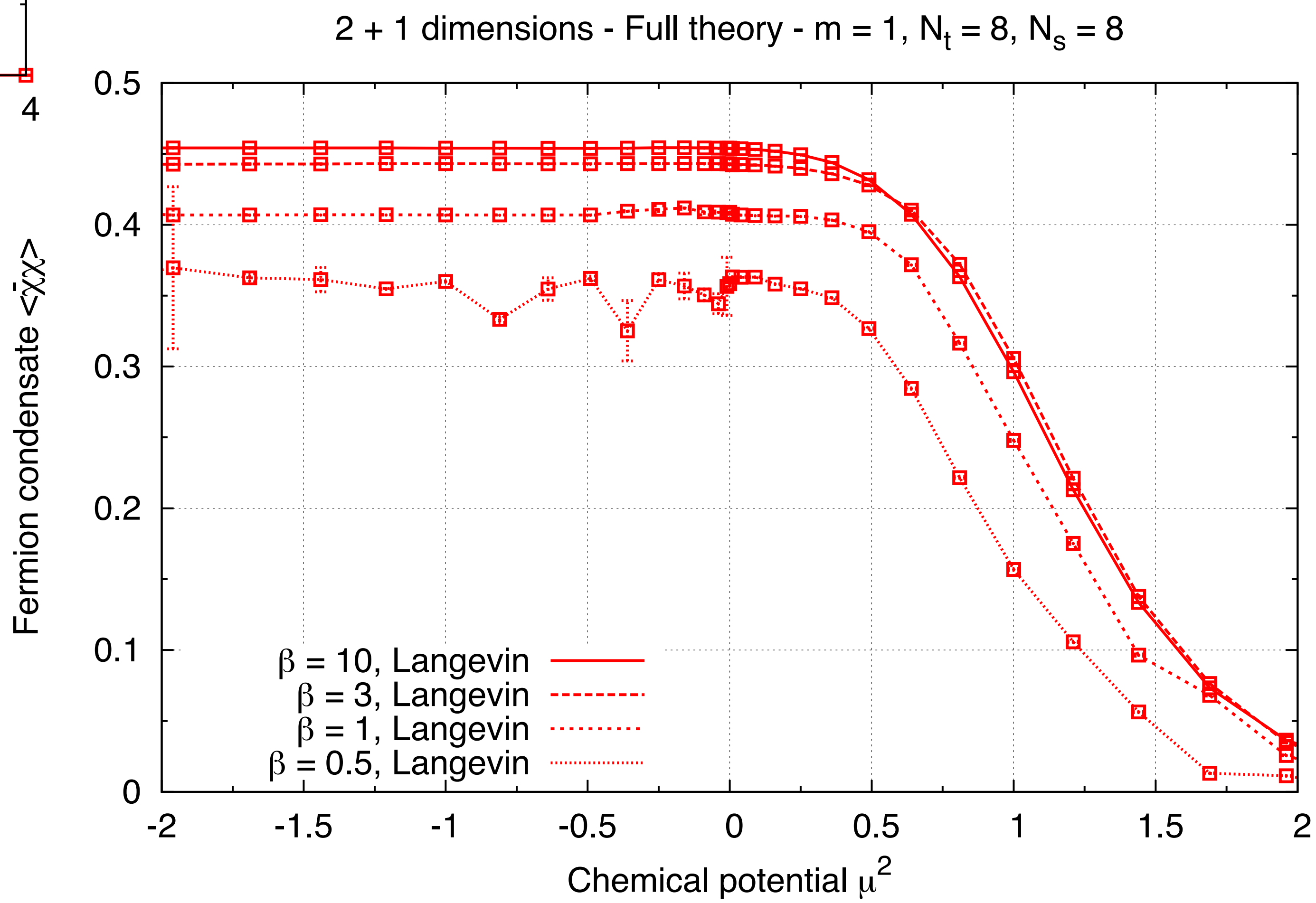
Real & imaginary chemical potential



condensate



0+1 dim

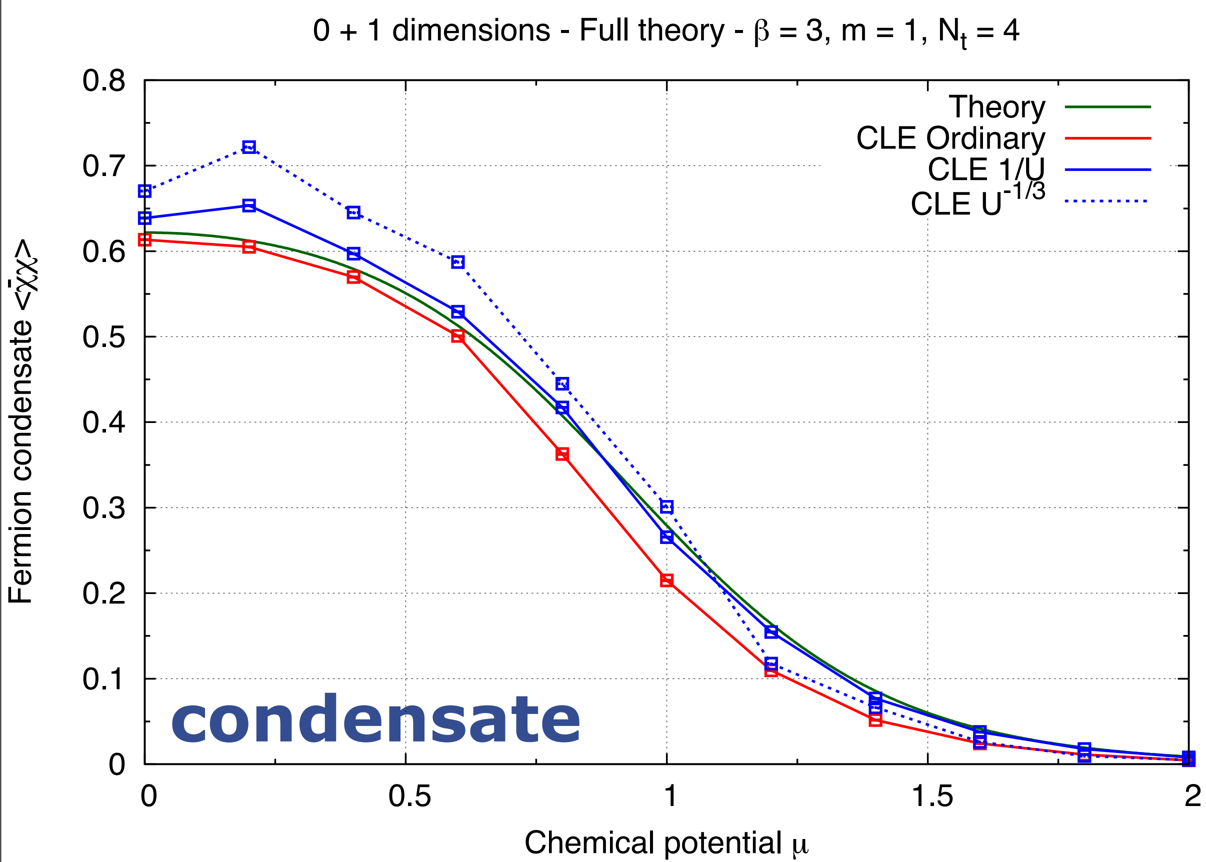


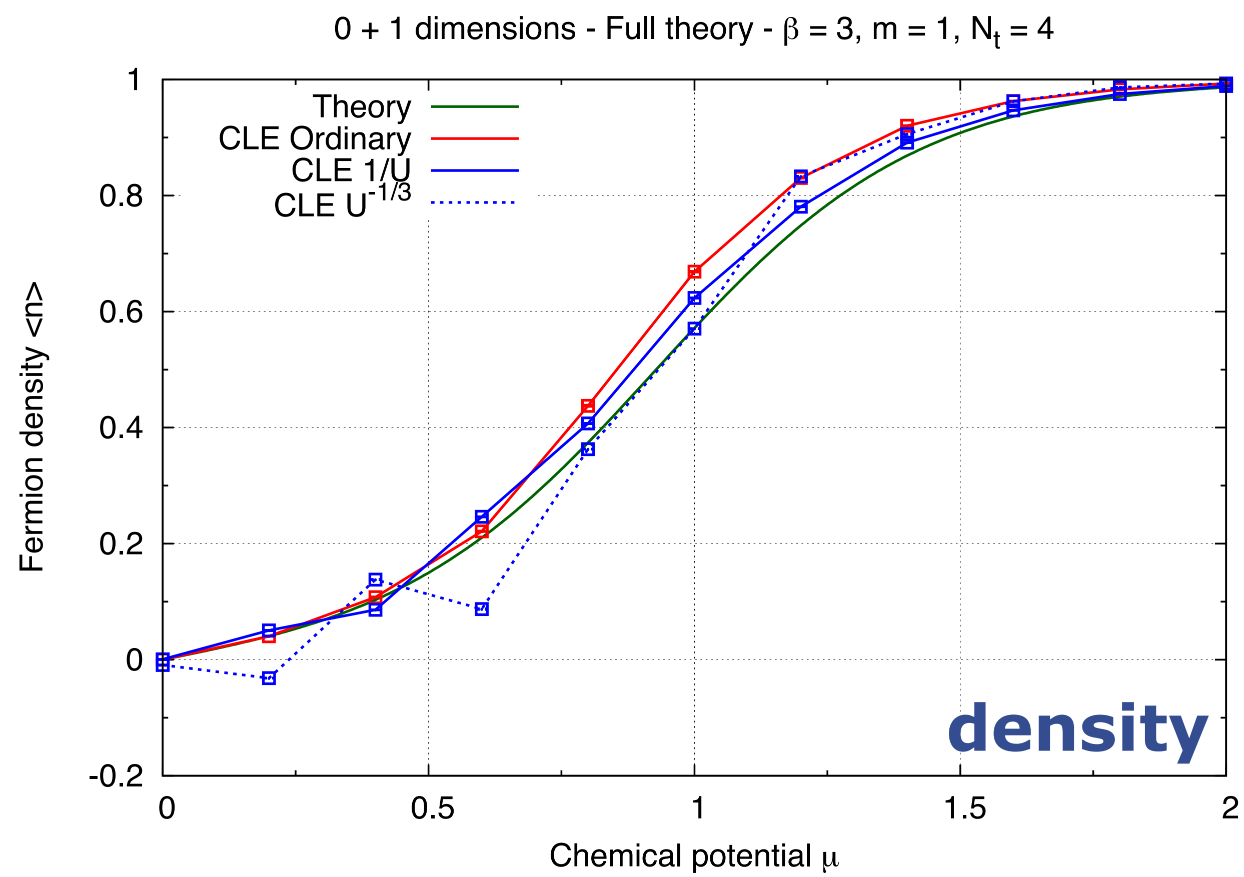
Reparameterisations

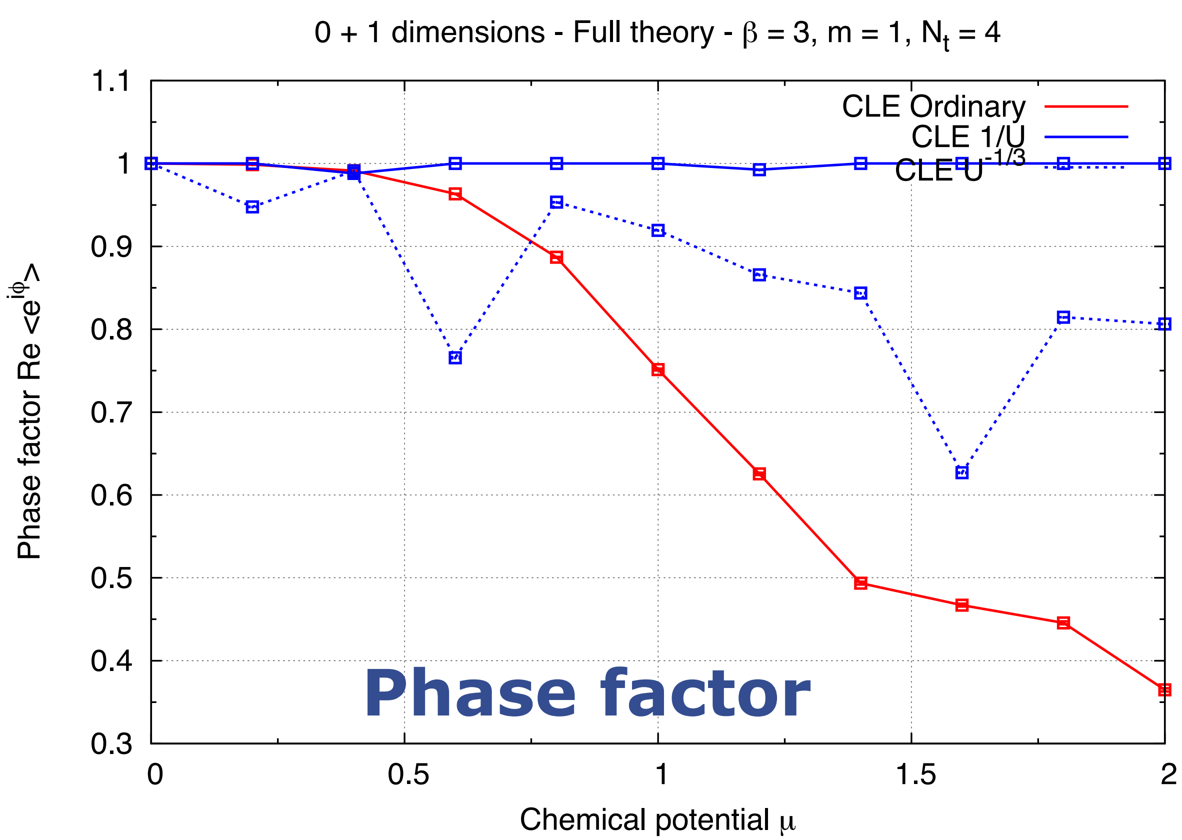
see also talks of G. Aarts
E. Seiler

Langevin evolution

0+1 dimensions







Summary & outlook

- **Thirring model**
  - **complex Langevin set-up**
  - **failure for large chemical potential/large couplings**

- **Outlook**
  - **reparameterisations** 

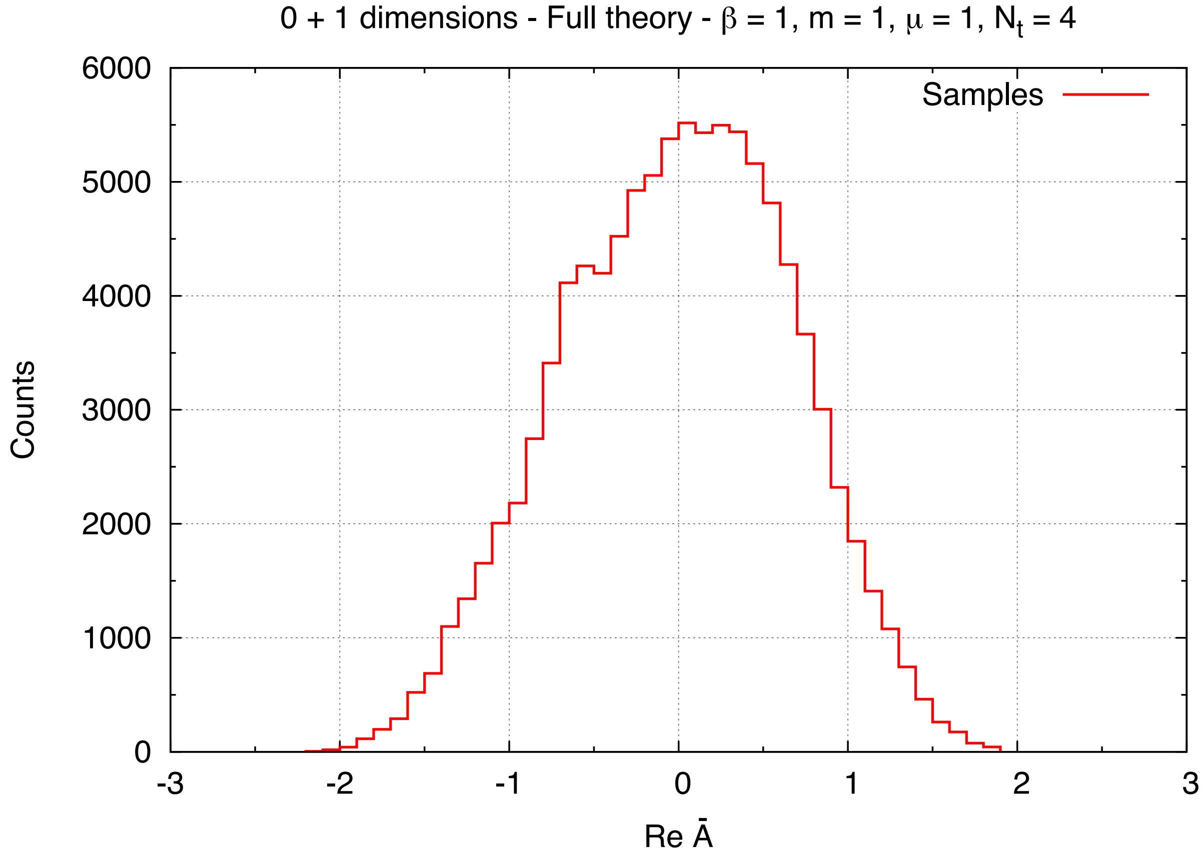
  - **non-Abelian models**

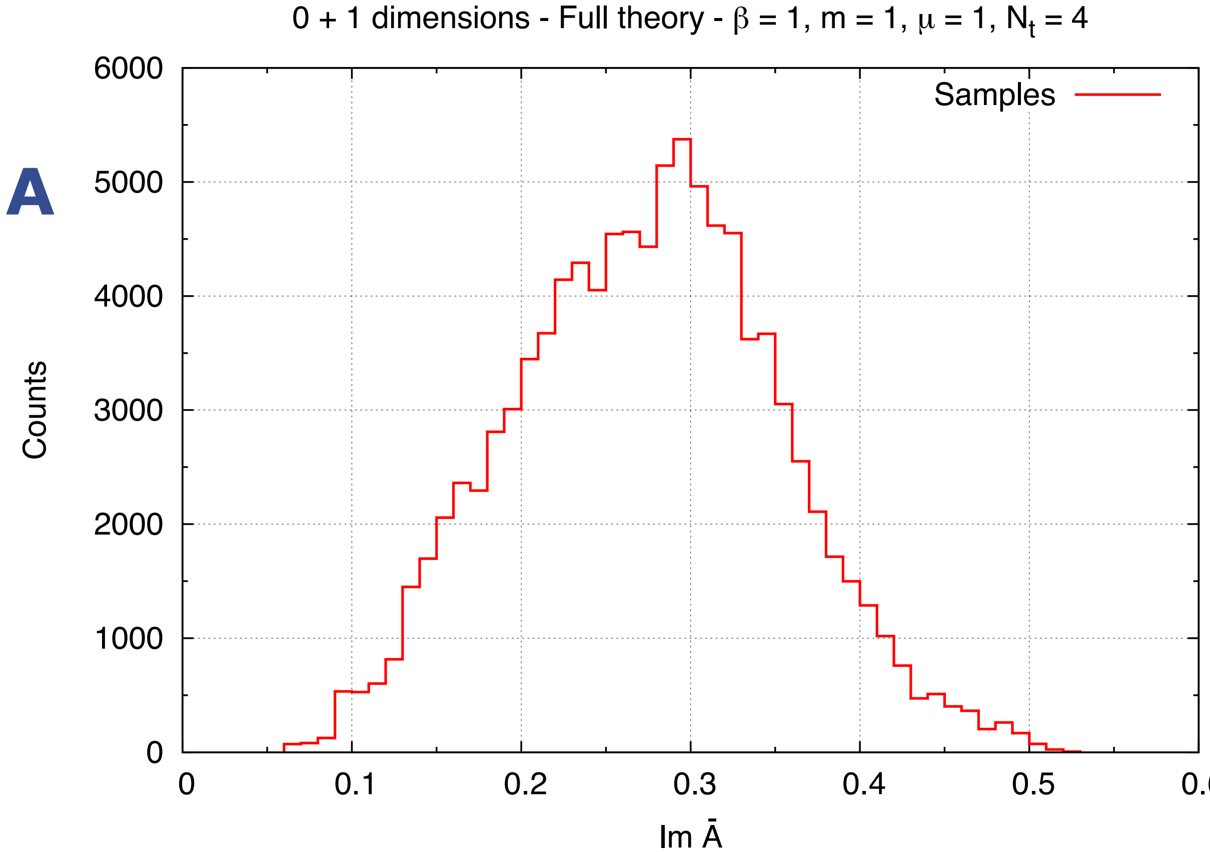
Additional material

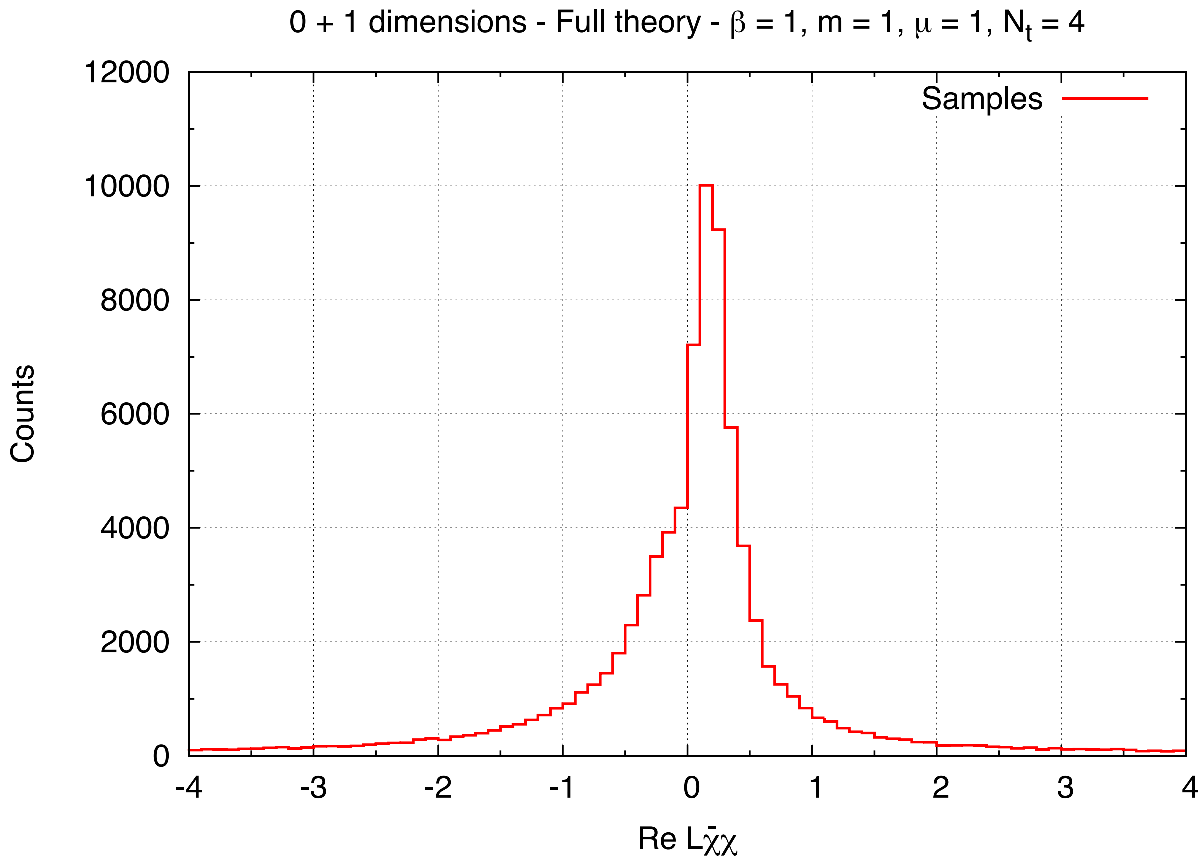
Thirring model

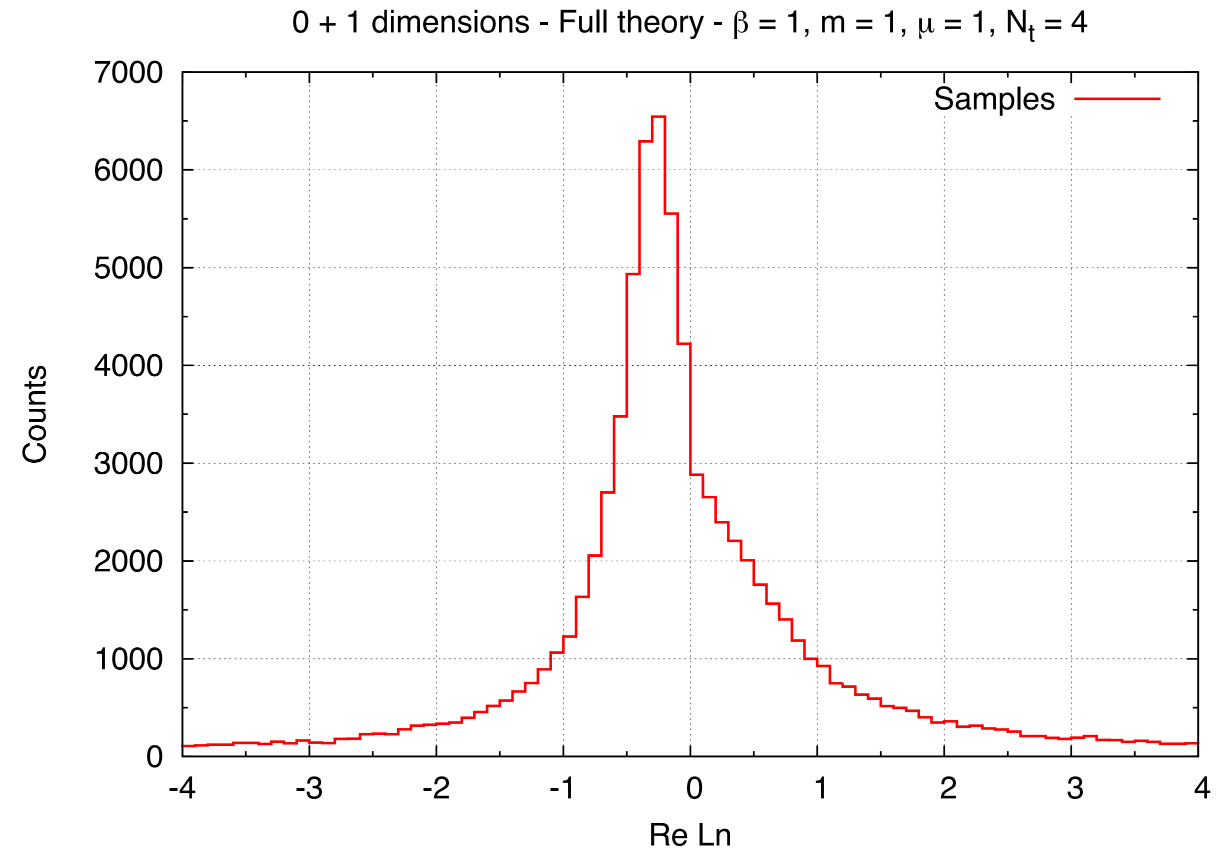
Consistency conditions

field A









Thirring model

Eigenvalues of K