On the QCD phase structure from functional methods

Jan M. Pawlowski

Universität Heidelberg & ExtreMe Matter Institute

RHIC-BES-Seminar, November 17th 2020

for the fQCD collaboration

GEFÖRDERT VOM
Bundesministerium für Bildung und Forschung

STRUCTURES CLUSTER OF EXCELLENCE

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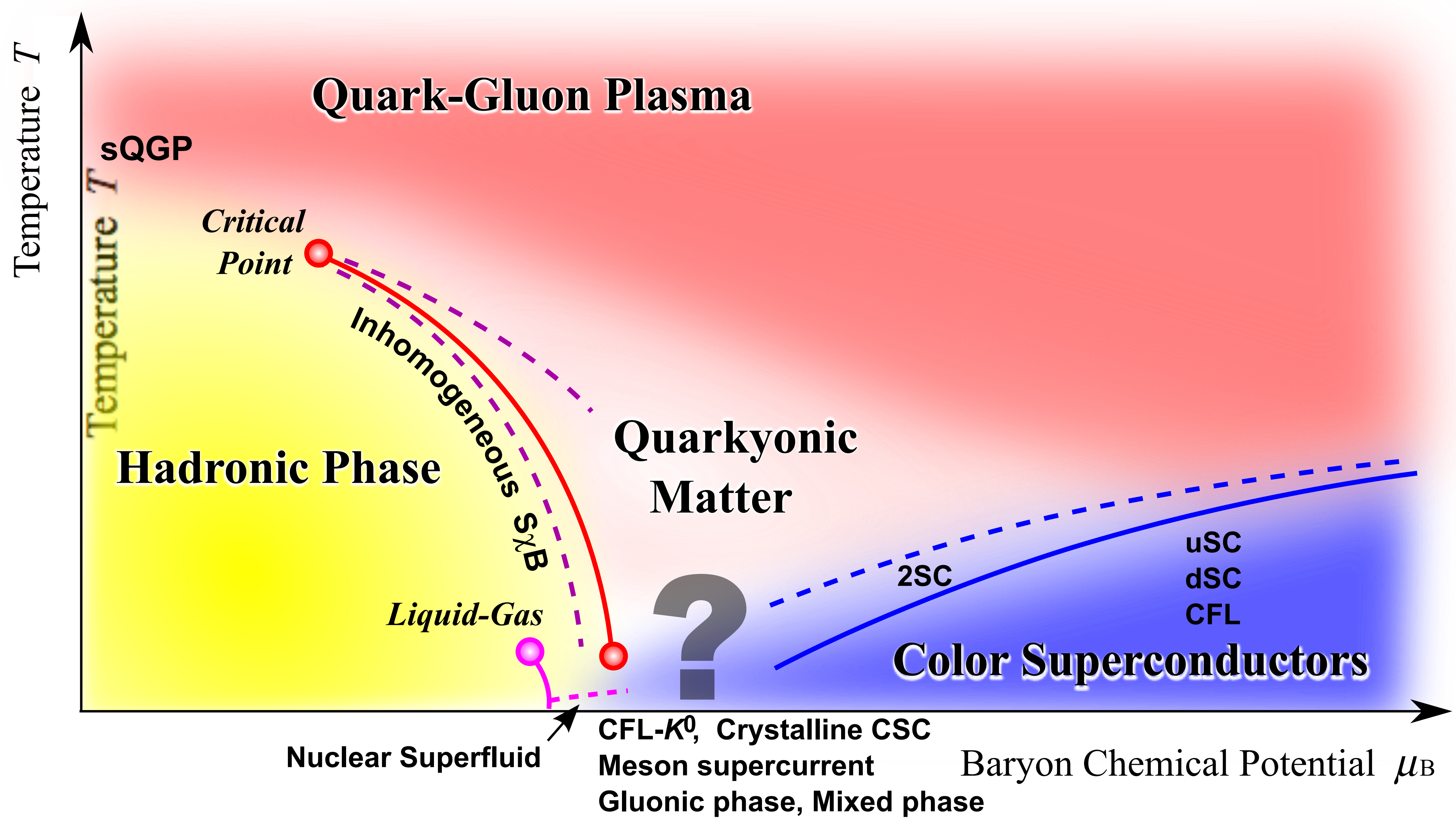
EMMI



fQCD collaboration

Braun, Chen, Fu, Huang, Ihssen, Horak, JMP, Rennecke, Rosenblüh, Schallmo, Schneider, Tan, Töpfel, Wen, Wink, Yin

Brookhaven, Dalian, Darmstadt, Heidelberg

Outline

- QCD from functional methods

Applications

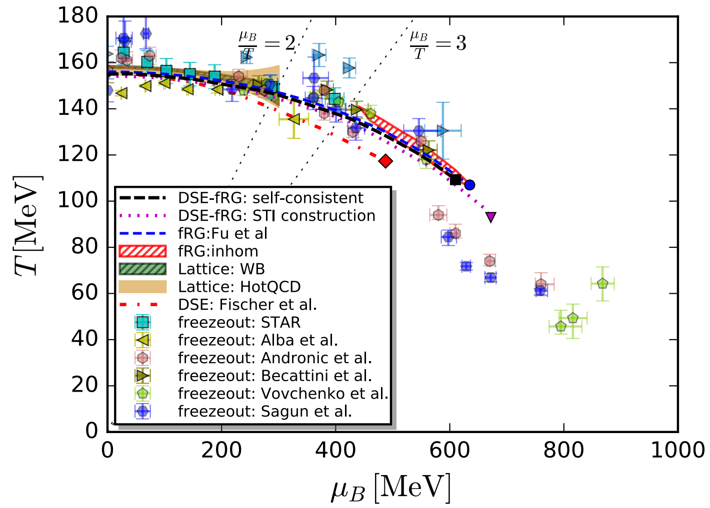
- QCD phase structure
- Fluctuations of conserved charges
- QCD-assisted transport
- Summary & outlook

Outline

● **QCD from functional methods**

Applications

QCD phase structure



● **QCD phase structure**

● **Fluctuations of conserved charges**

● **QCD-assisted transport**

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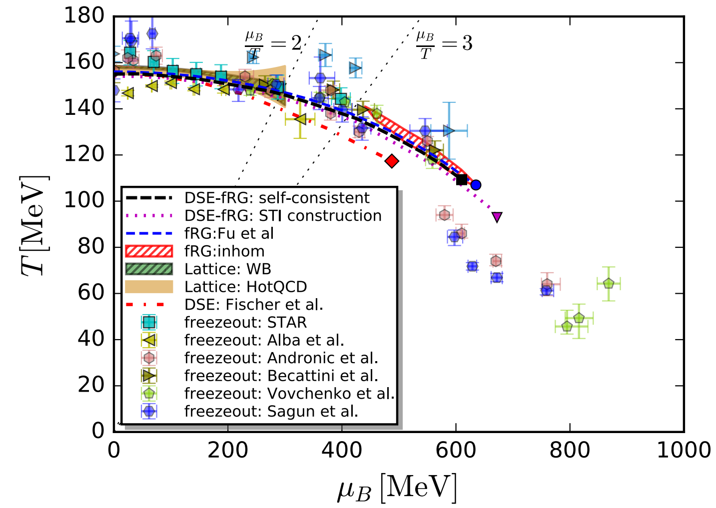
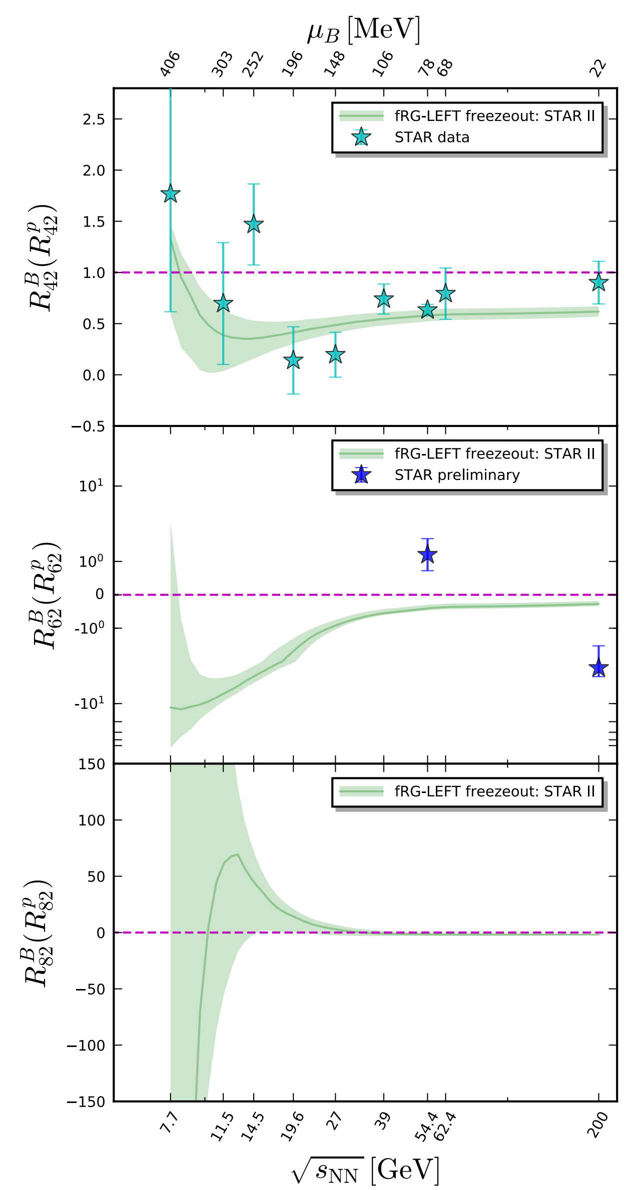
QCD phase structure



Hyper-fluctuations



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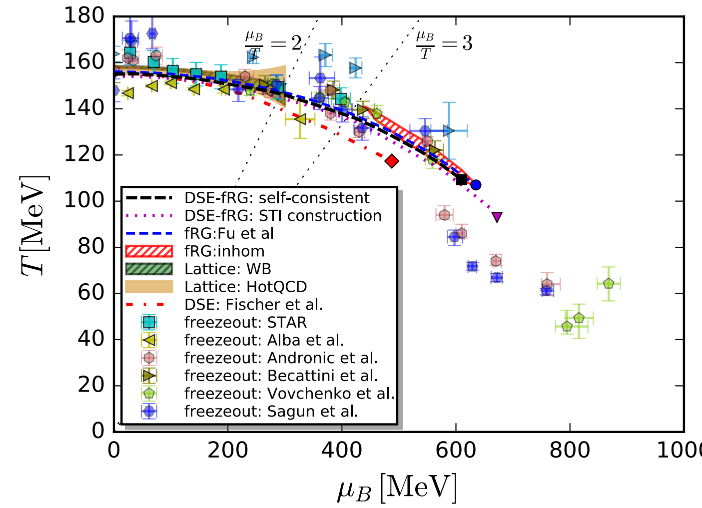
QCD phase structure



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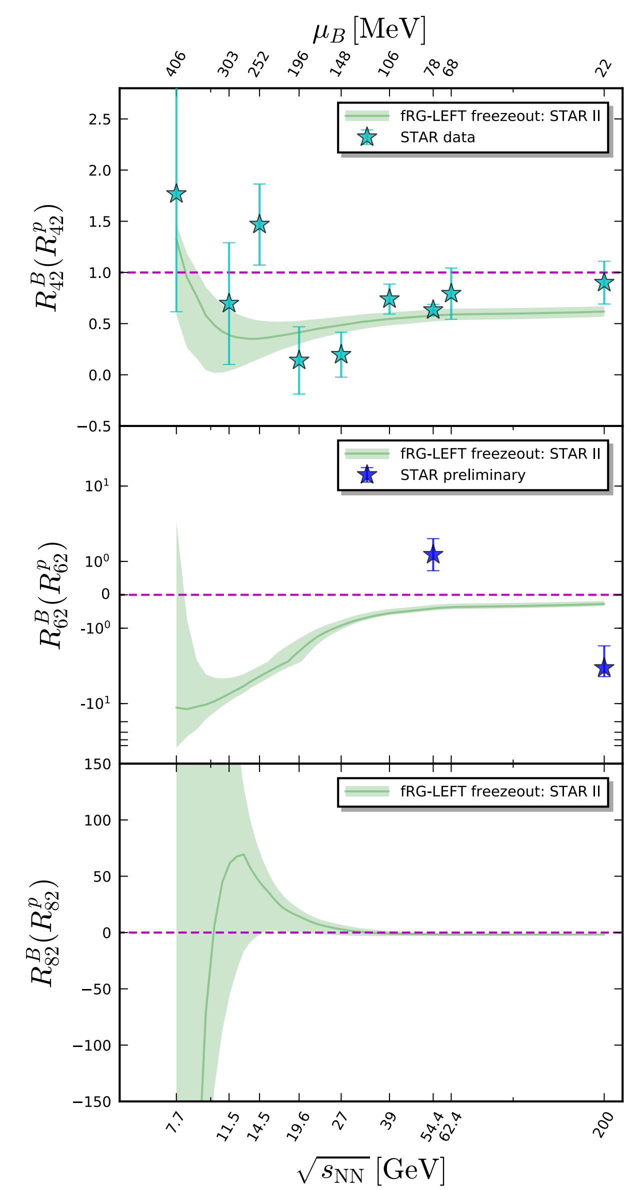
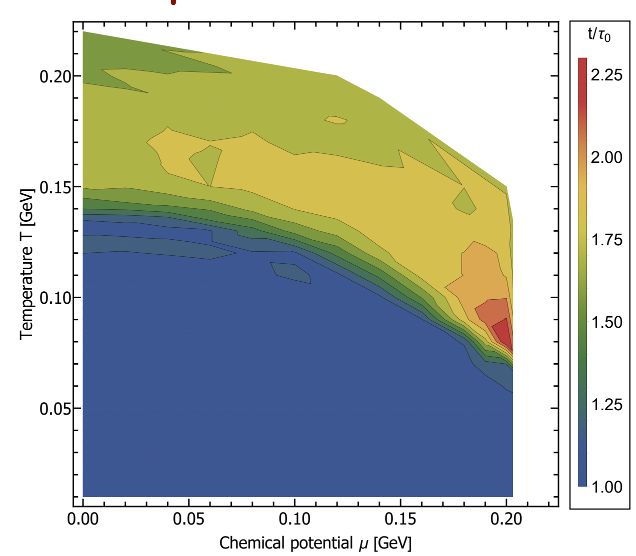


- Fluctuations of conserved charges

- QCD-assisted transport

Equilibration times



- Summary & outlook

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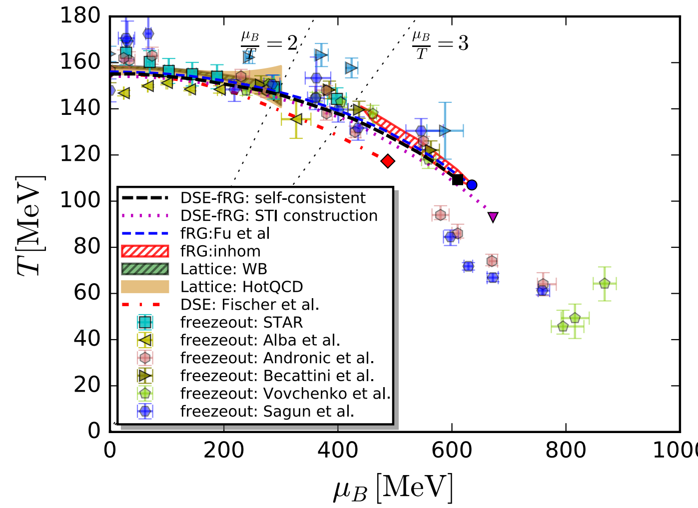
QCD phase structure



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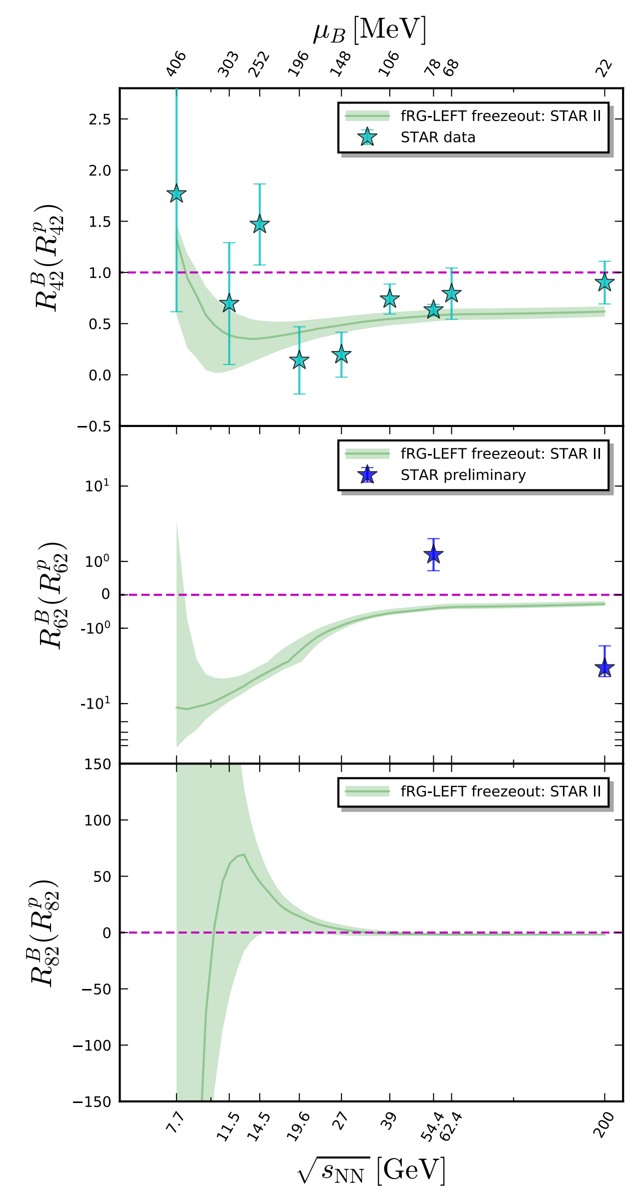

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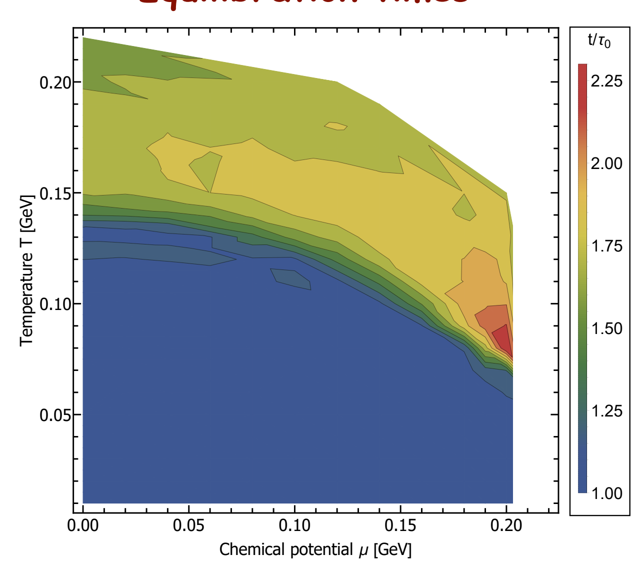
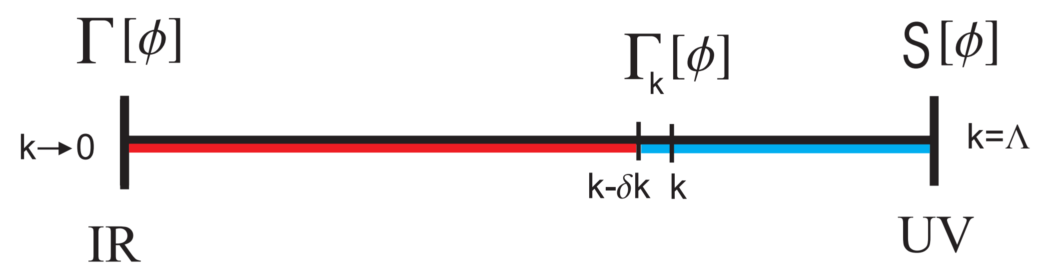


Functional Methods for QCD

FRG: JMP, NPA 931 (2014) 113
Dupuis et al, arXiv:2006.04853

DSE: Fischer, PPNP 105 (2019) 1

free energy at momentum scale k



ab initio

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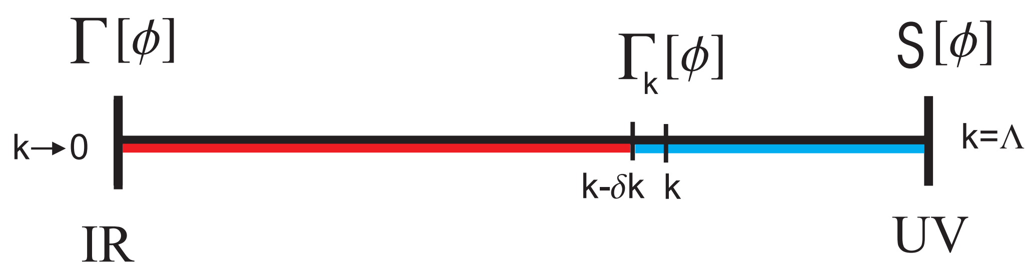

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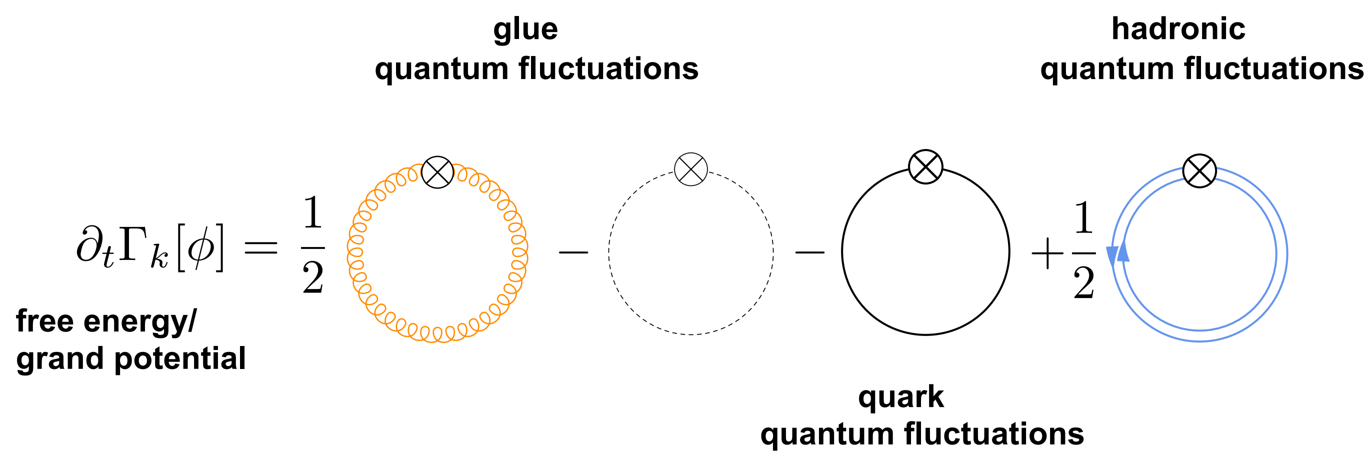
functional RG:



RG-scale k: $t = \ln k$

closed form

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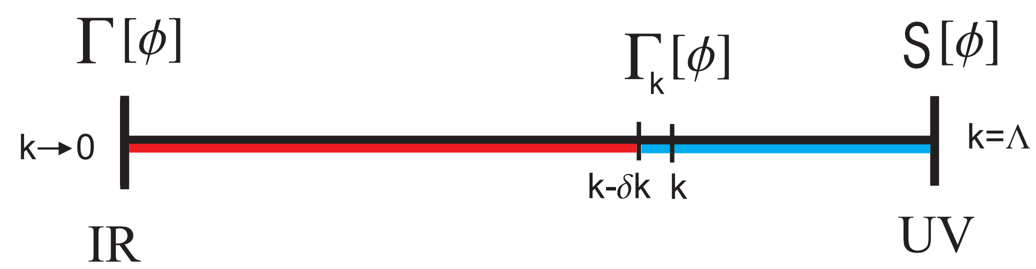
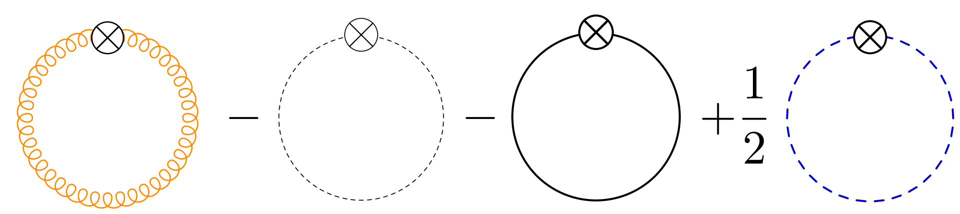


ab initio

glue quantum fluctuations **hadronic quantum fluctuations**

functional RG: $\partial_t \Gamma_k[\phi] = \frac{1}{2}$



free energy/ grand potential

quark quantum fluctuations

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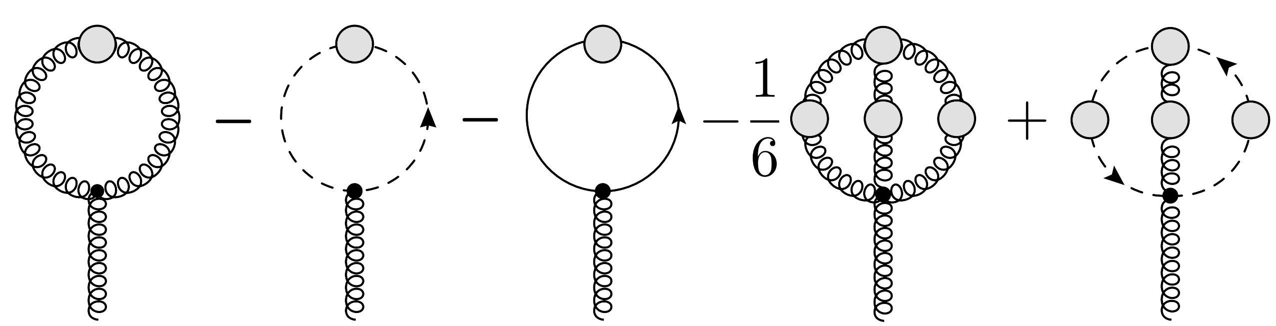
closed form

functional DSE : $\frac{\delta\,(\Gamma - S)}{\delta A_0} = \frac{1}{2}$



A_0 **: background field**

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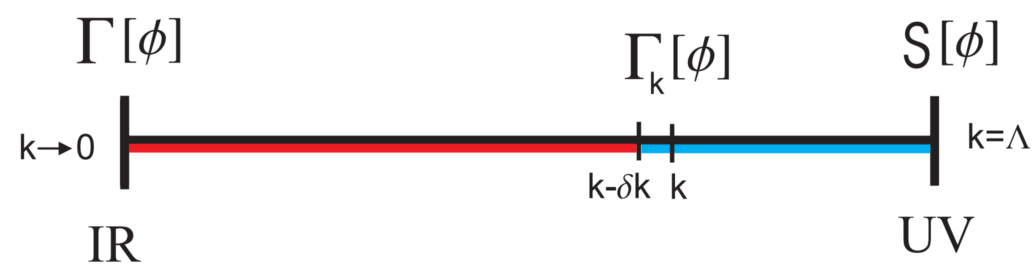

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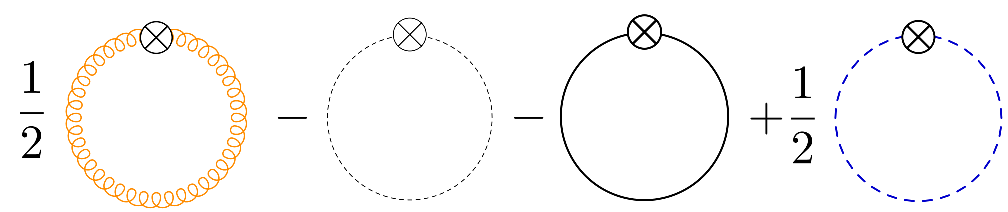

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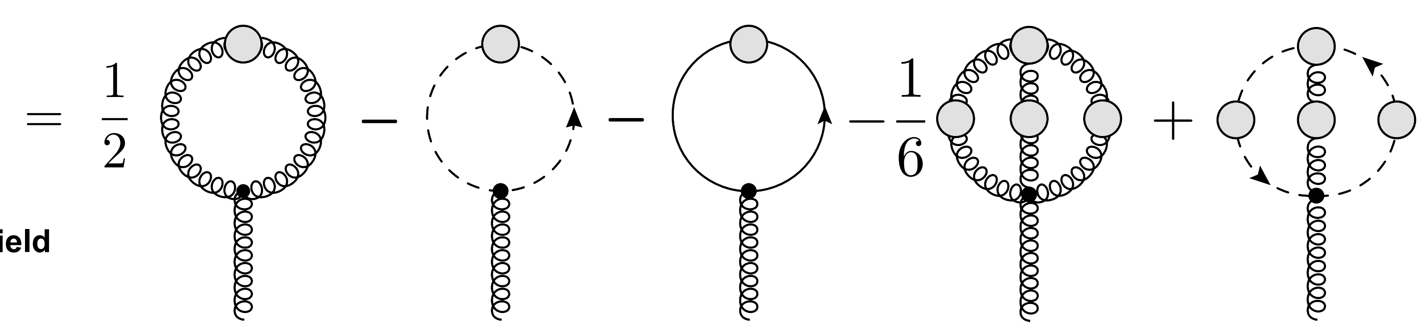

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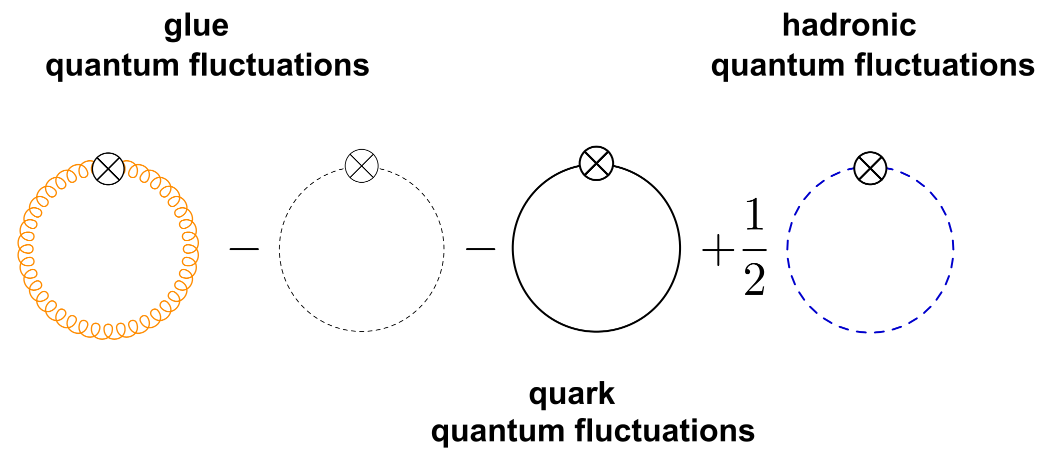

free energy/ grand potential

Correlation functions

Functional Methods for QCD

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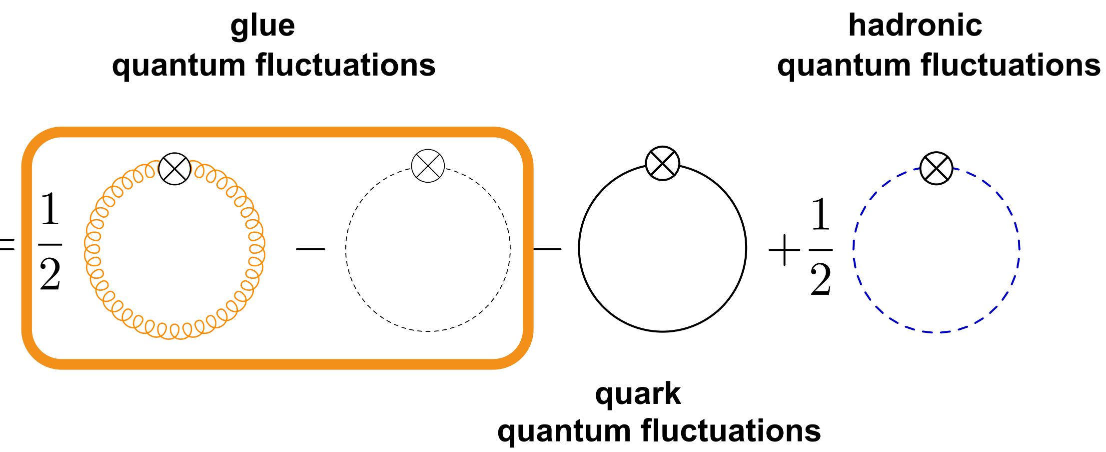

free energy/
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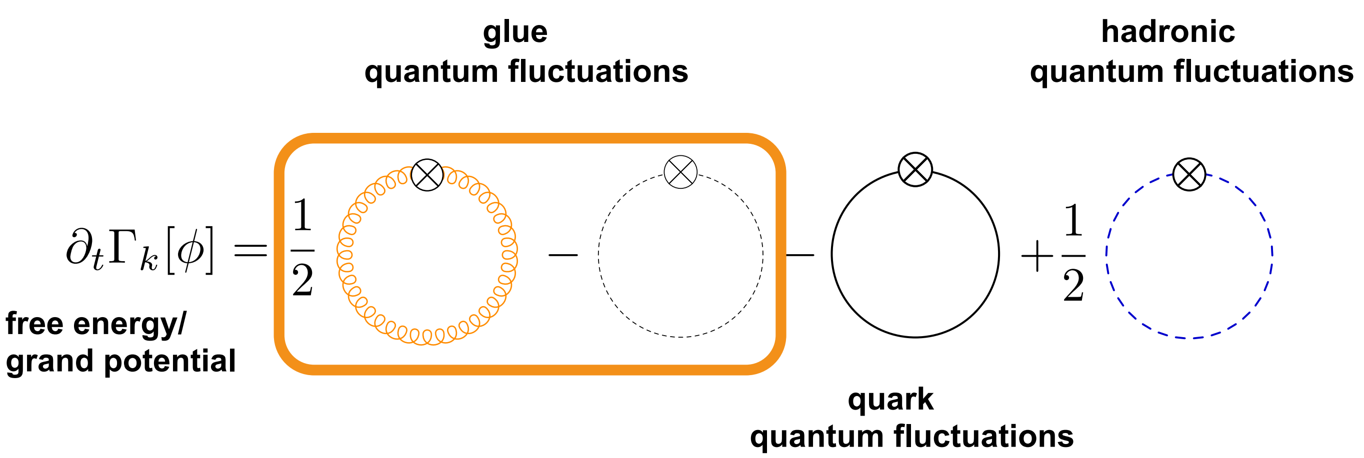
Correlation functions

gluon propagator

$$\langle A_\mu A_\nu \rangle (p)$$

Functional Methods for QCD

functional RG:



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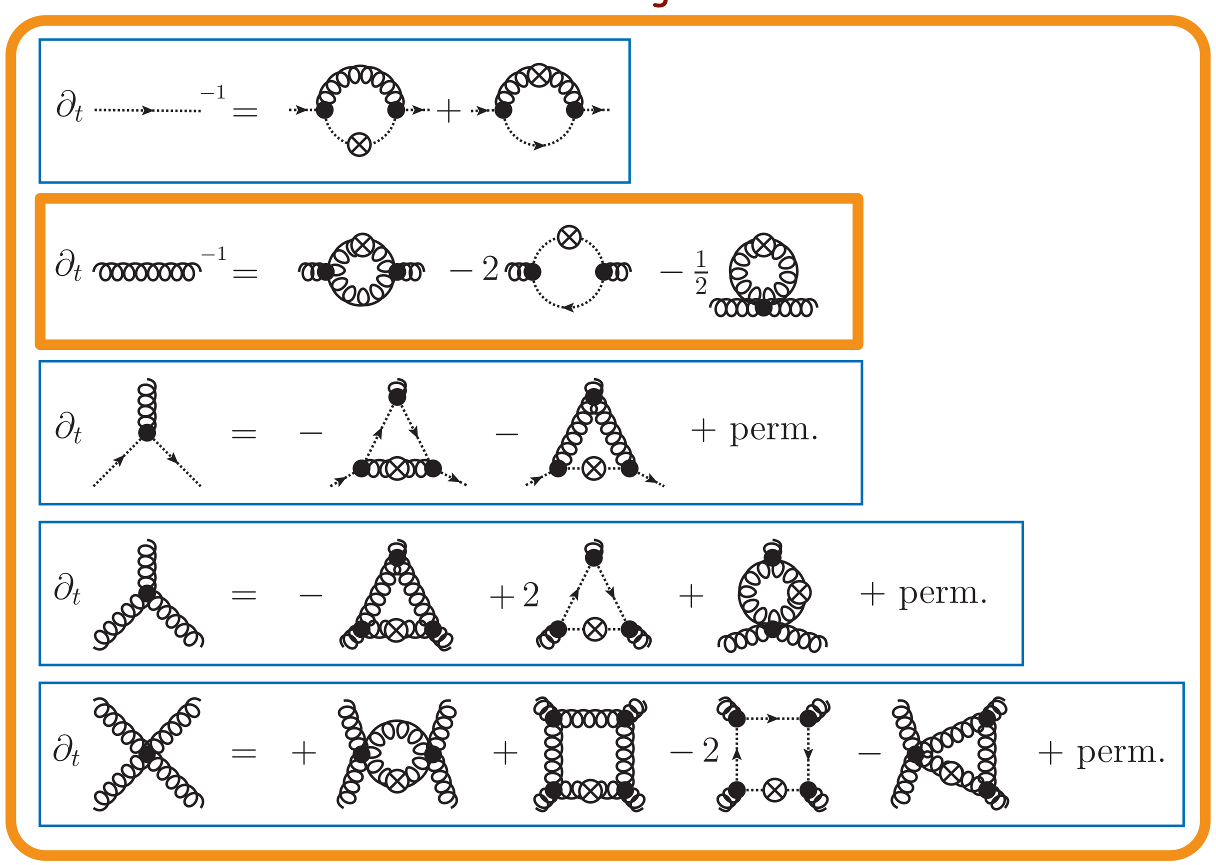
Pure glue



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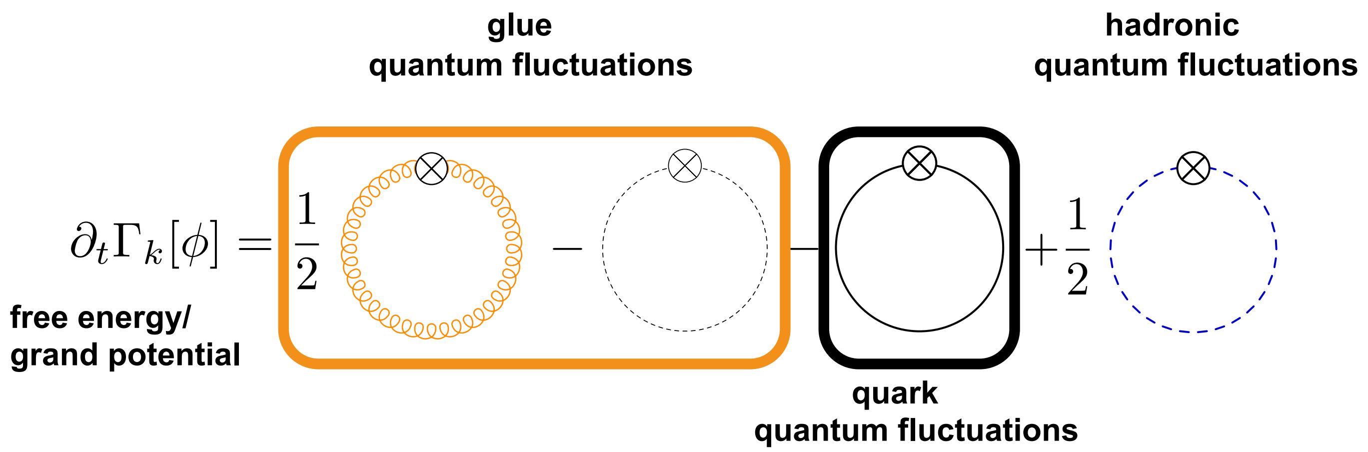

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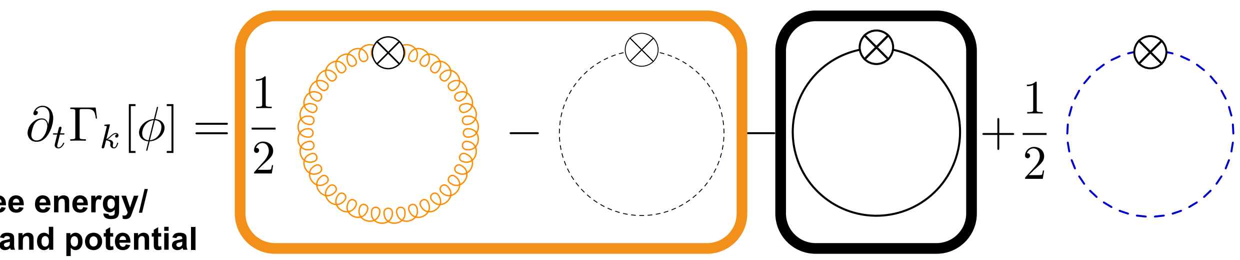

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quark-gluon vertex

$$\langle q\bar{q} A_\mu \rangle (p_1, p_2)$$

Eight tranverse tensor structures

Functional Methods for QCD

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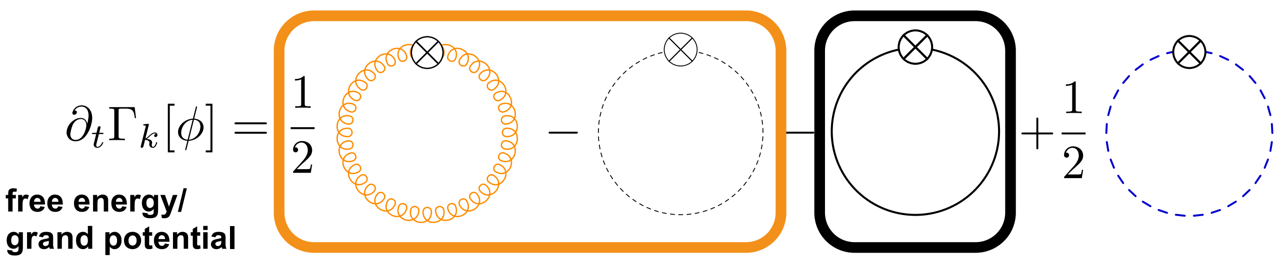

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$$\langle q \bar{q} q \bar{q} \rangle (p_1, p_2, p_3)$$

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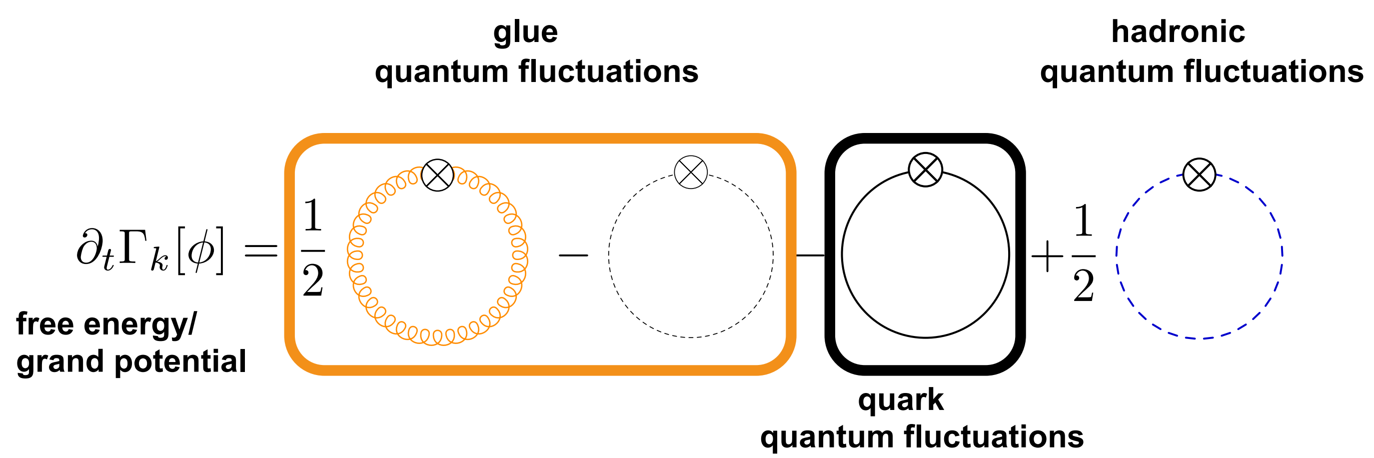

Correlation functions

gluon propagator	quark propagator	quark-gluon vertex	quark—anti-quark scattering
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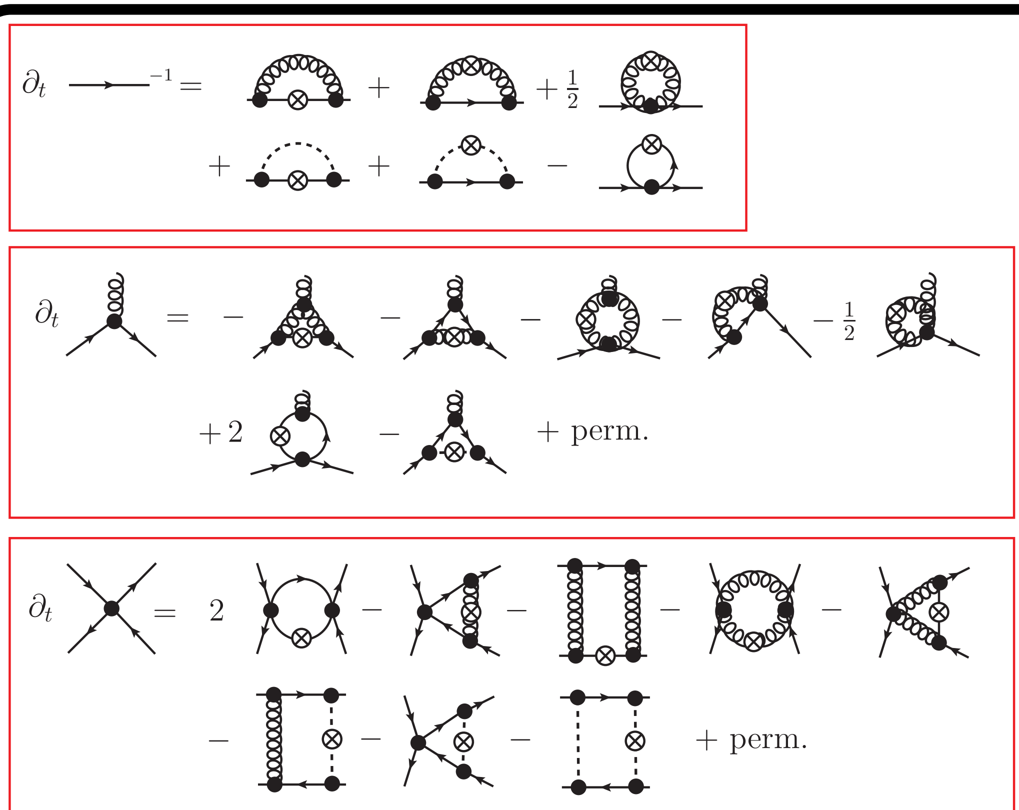

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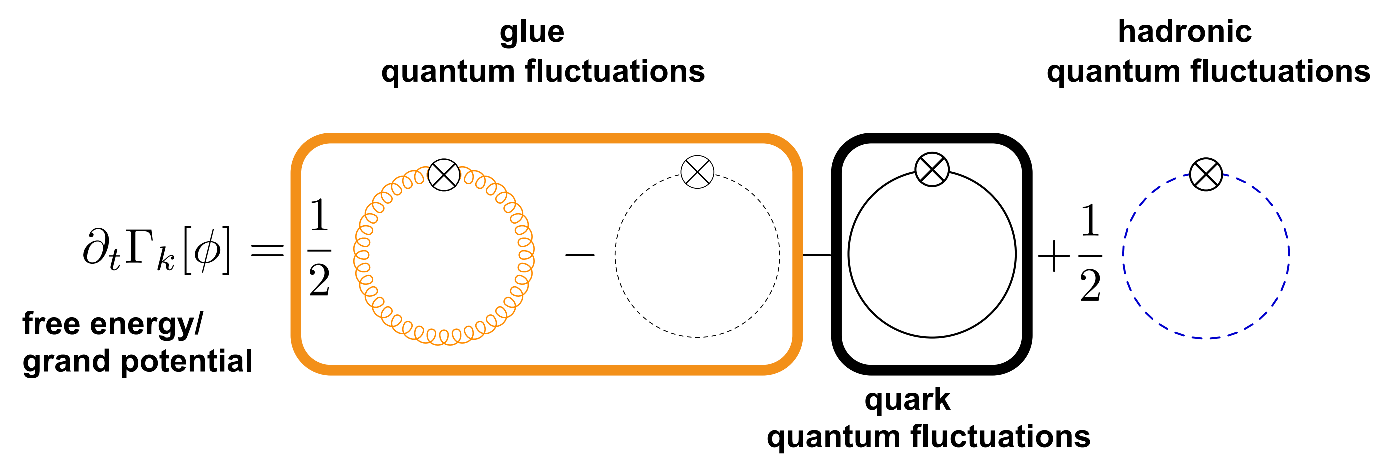

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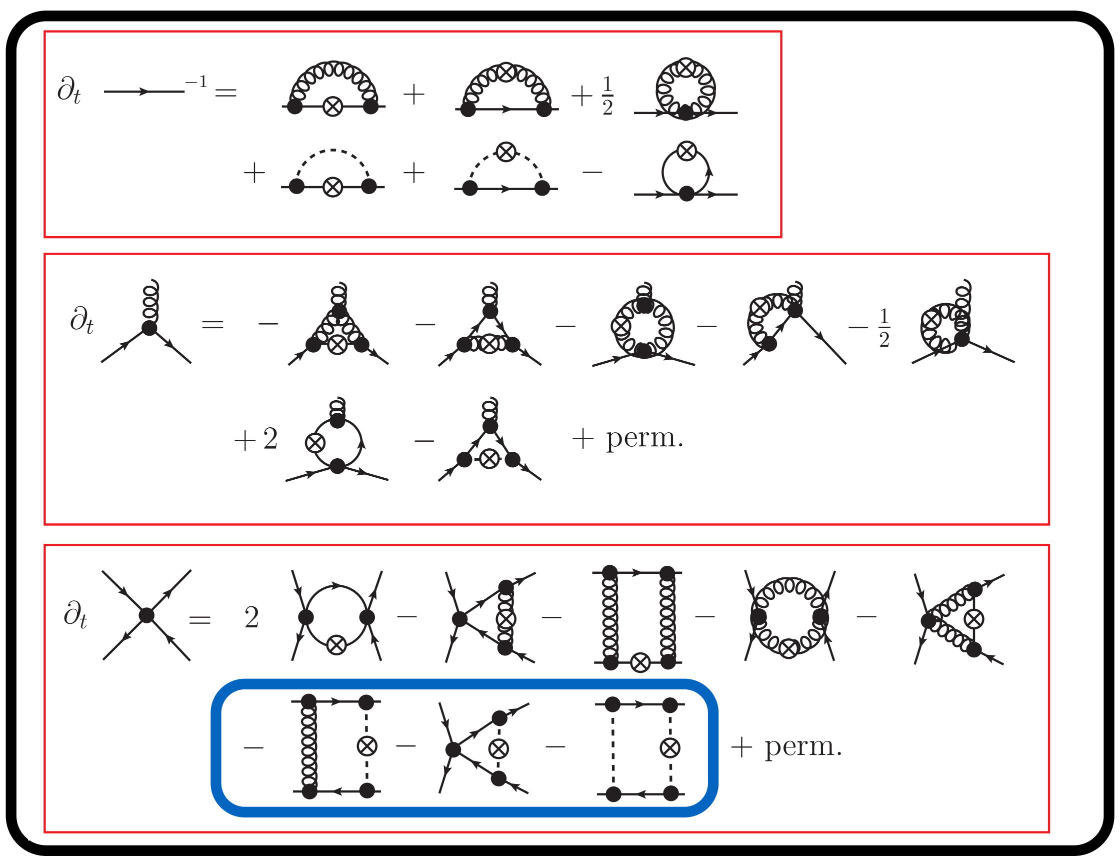

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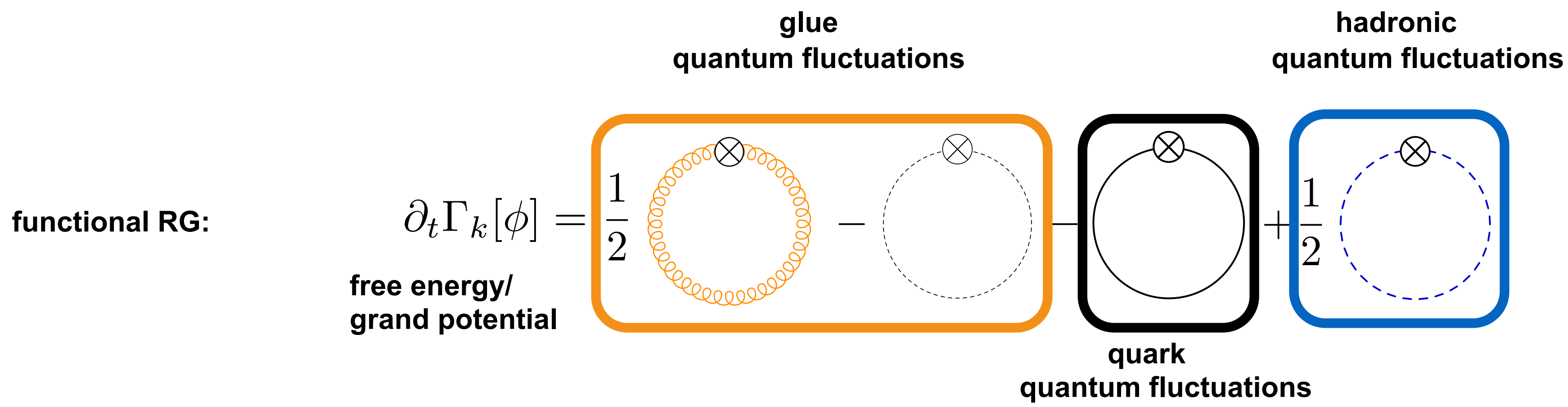

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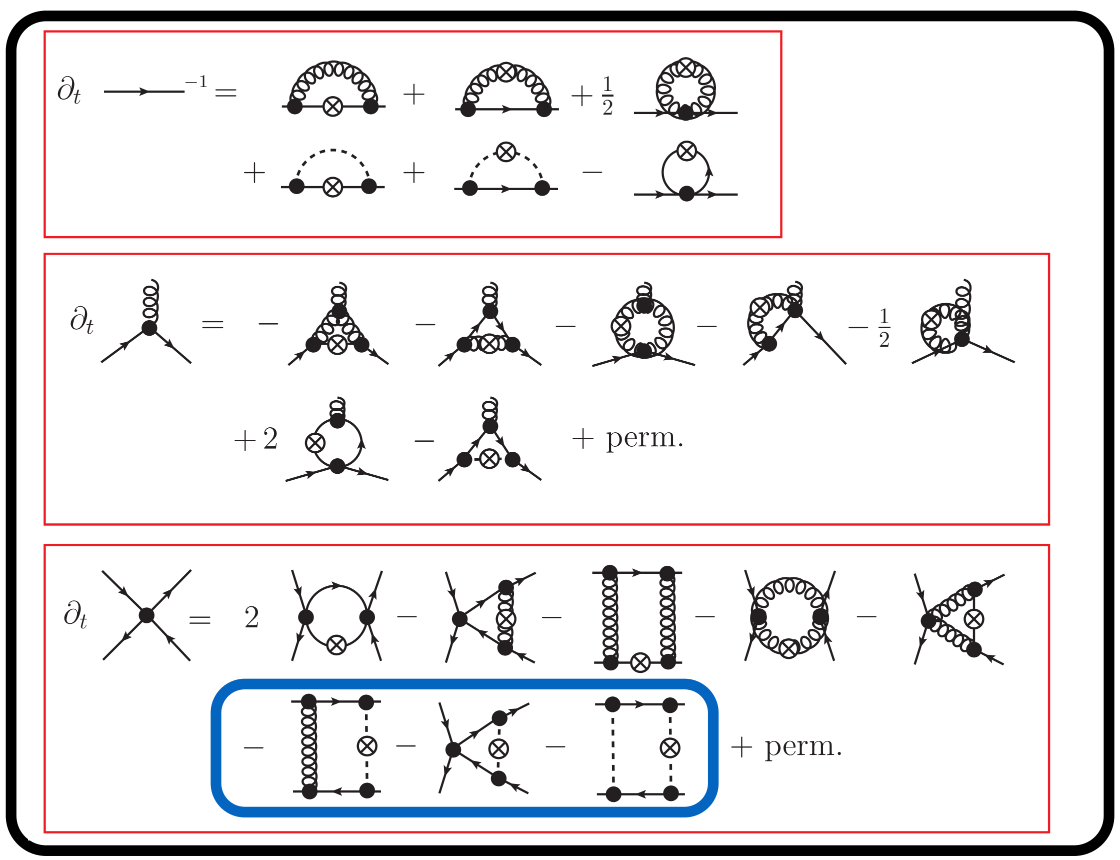
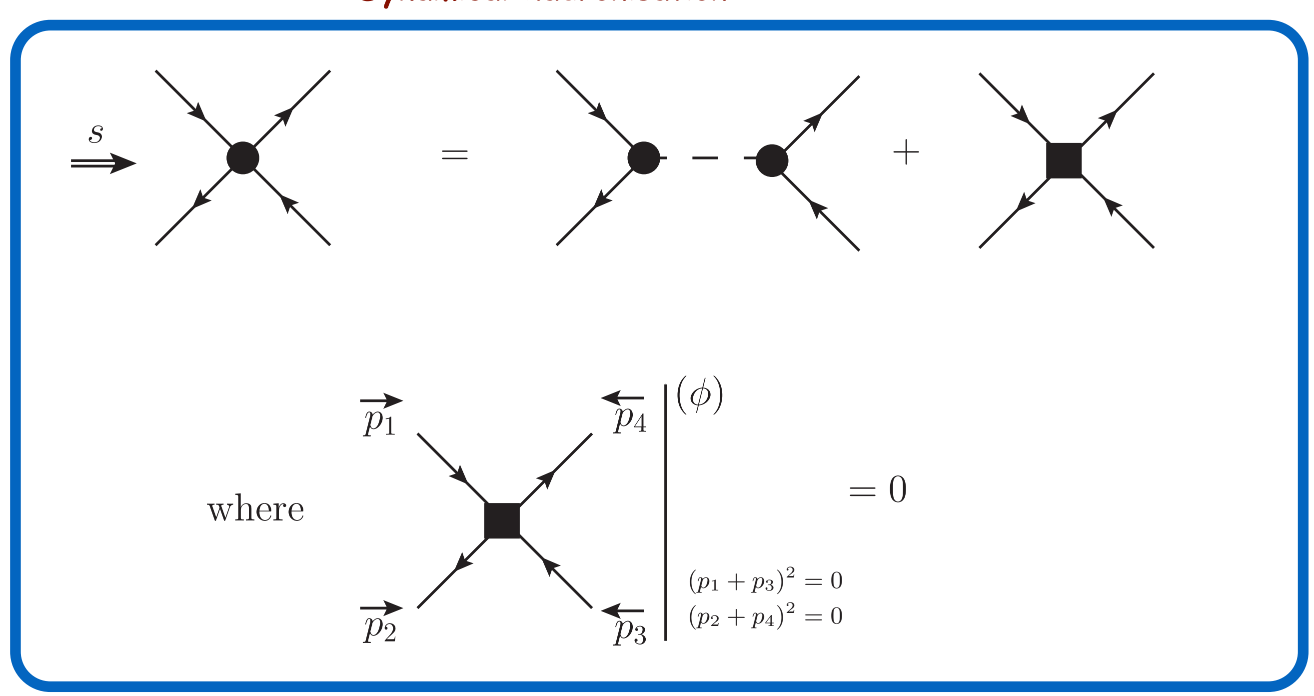
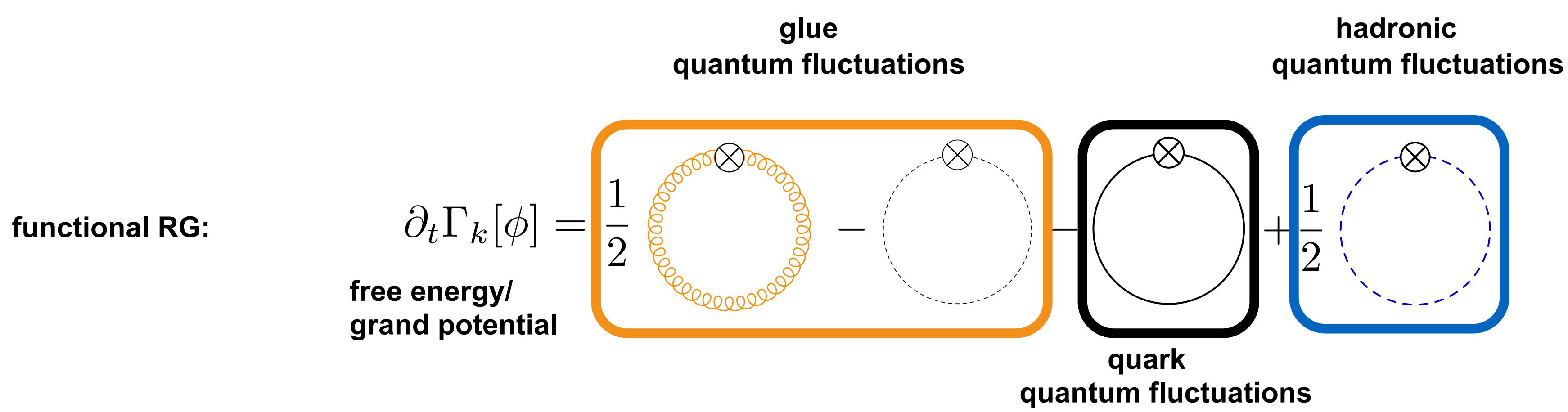


Dynamical hadronisation



Functional Methods for QCD



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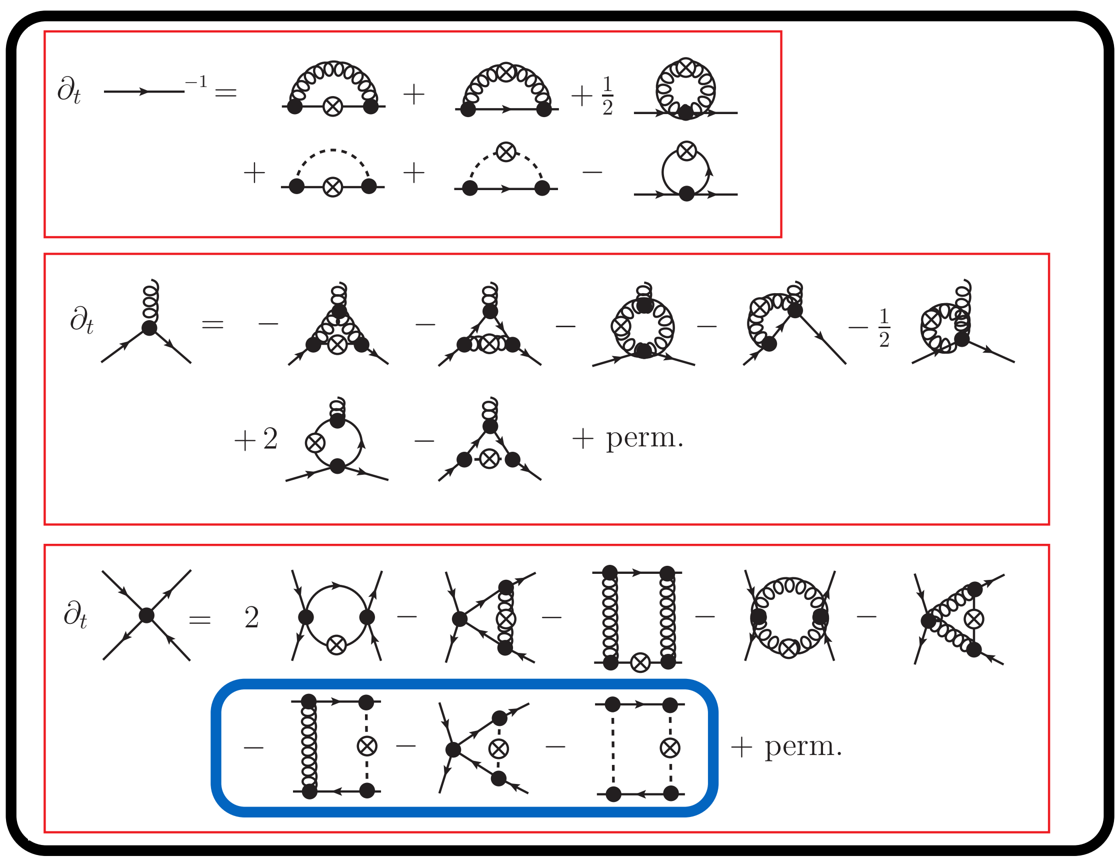

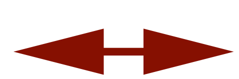

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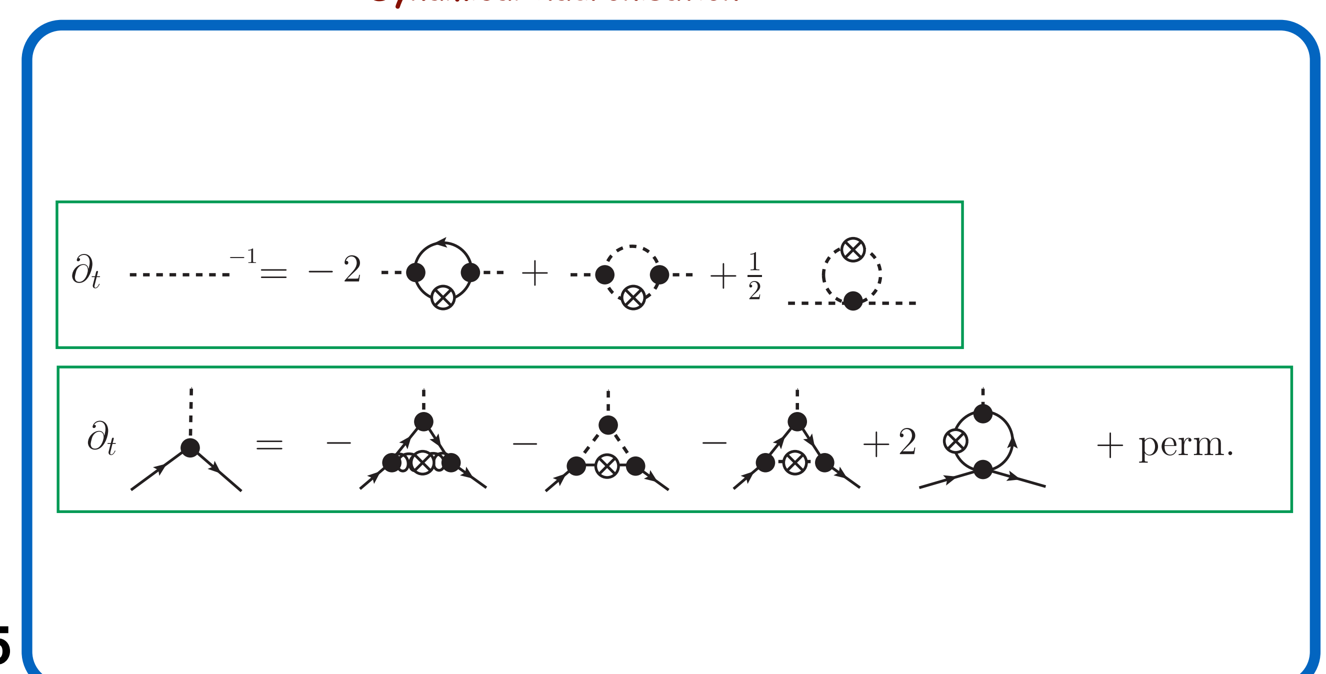

vacuum QCD with the fRG

1^{st} principles

Input: fundamental parameters of QCD at a large momentum scale: $\Lambda = 20\,\mathrm{GeV}$

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2-flavour QCD

(i) $\alpha_{s,\Lambda}$

(ii) $m_{u,\Lambda} = m_{d,\Lambda} = m_{l,\Lambda}(m_\pi)$ $\qquad m_\pi = 140\,\mathrm{MeV}$

vacuum QCD with the fRG

1^{st} principles

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2+1-flavour QCD

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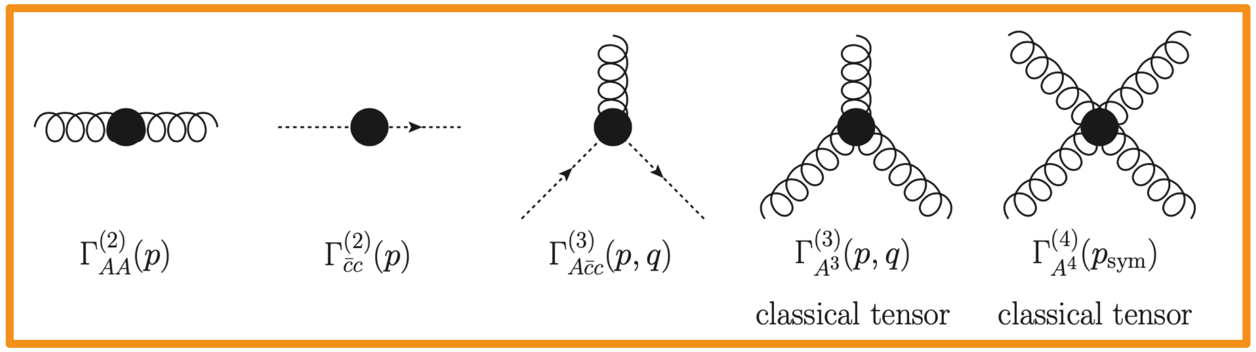
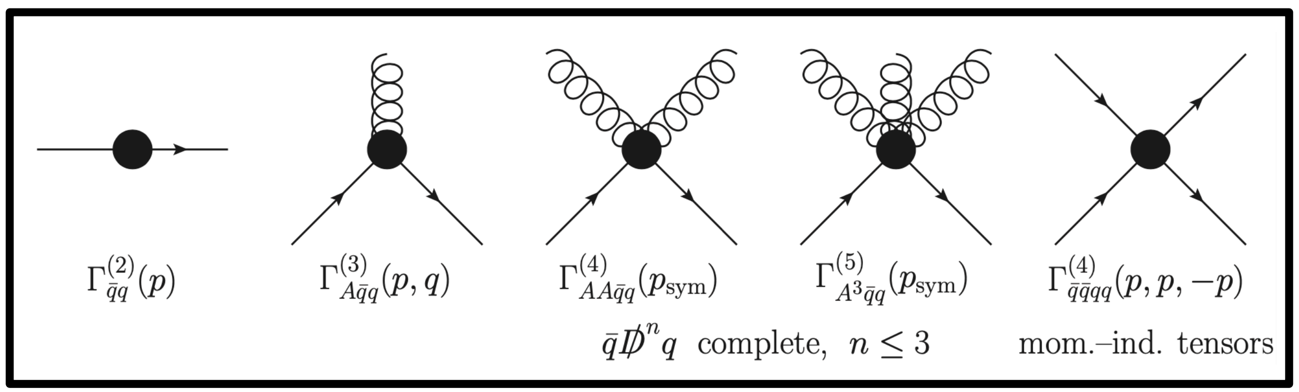
(iii) $\dfrac{m_{l,\Lambda}}{m_{s,\Lambda}} = 27$

vacuum QCD: current set of correlation functions

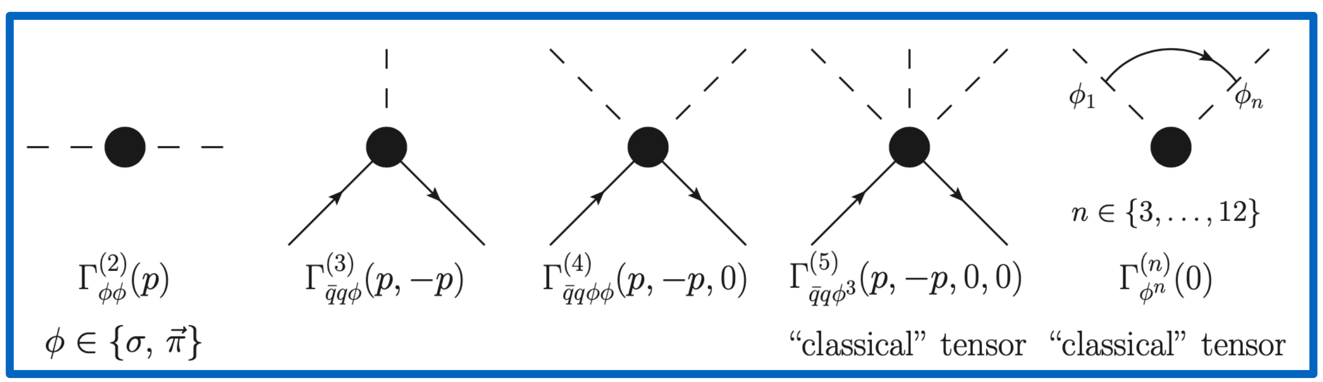


glue sector

FRG



quark-glue sector



quark-meson sector

Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006, PRD 97 (2018) 054015

Extension, work in progress:

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

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Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

vacuum QCD: current set of correlation functions

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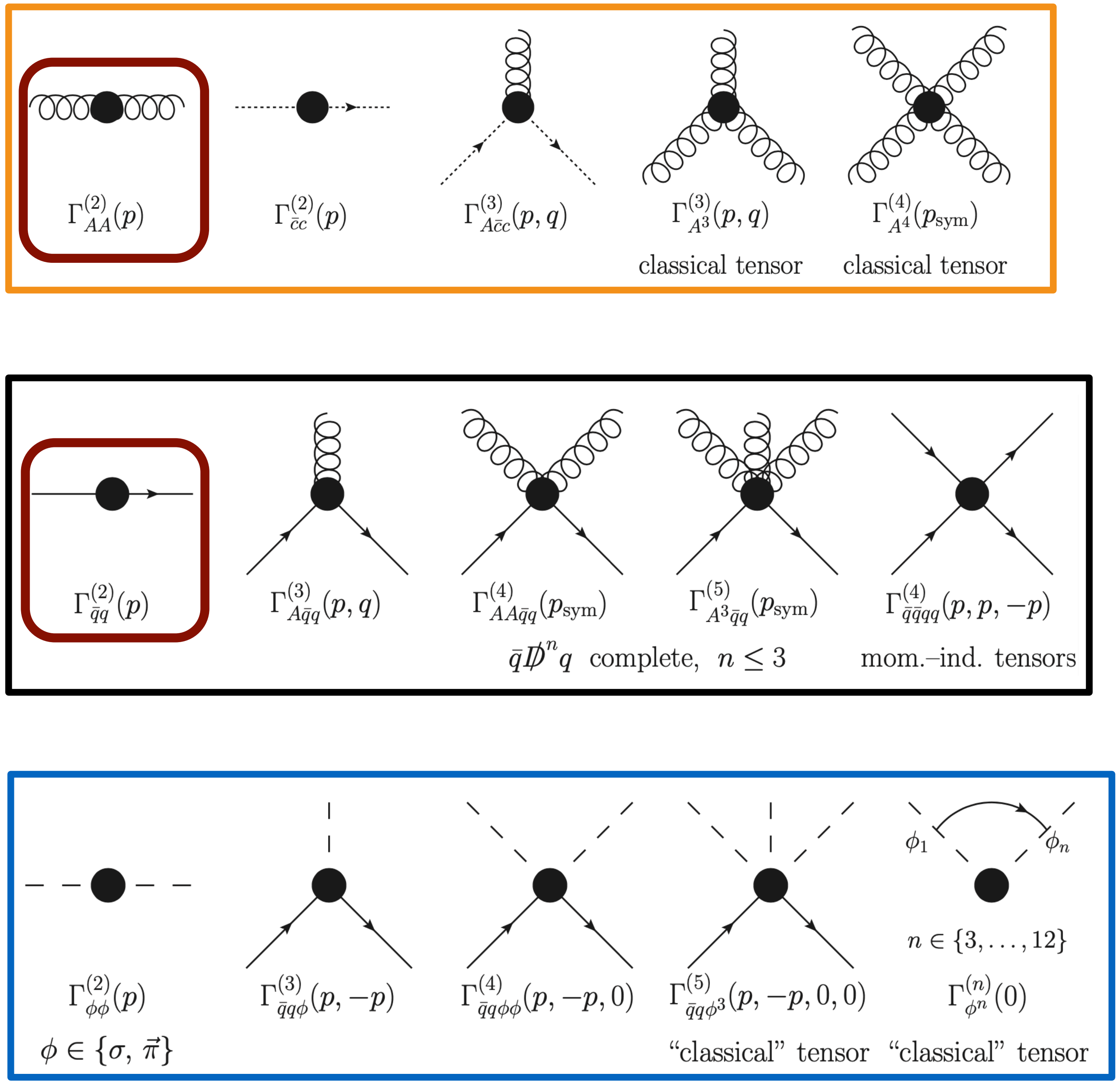

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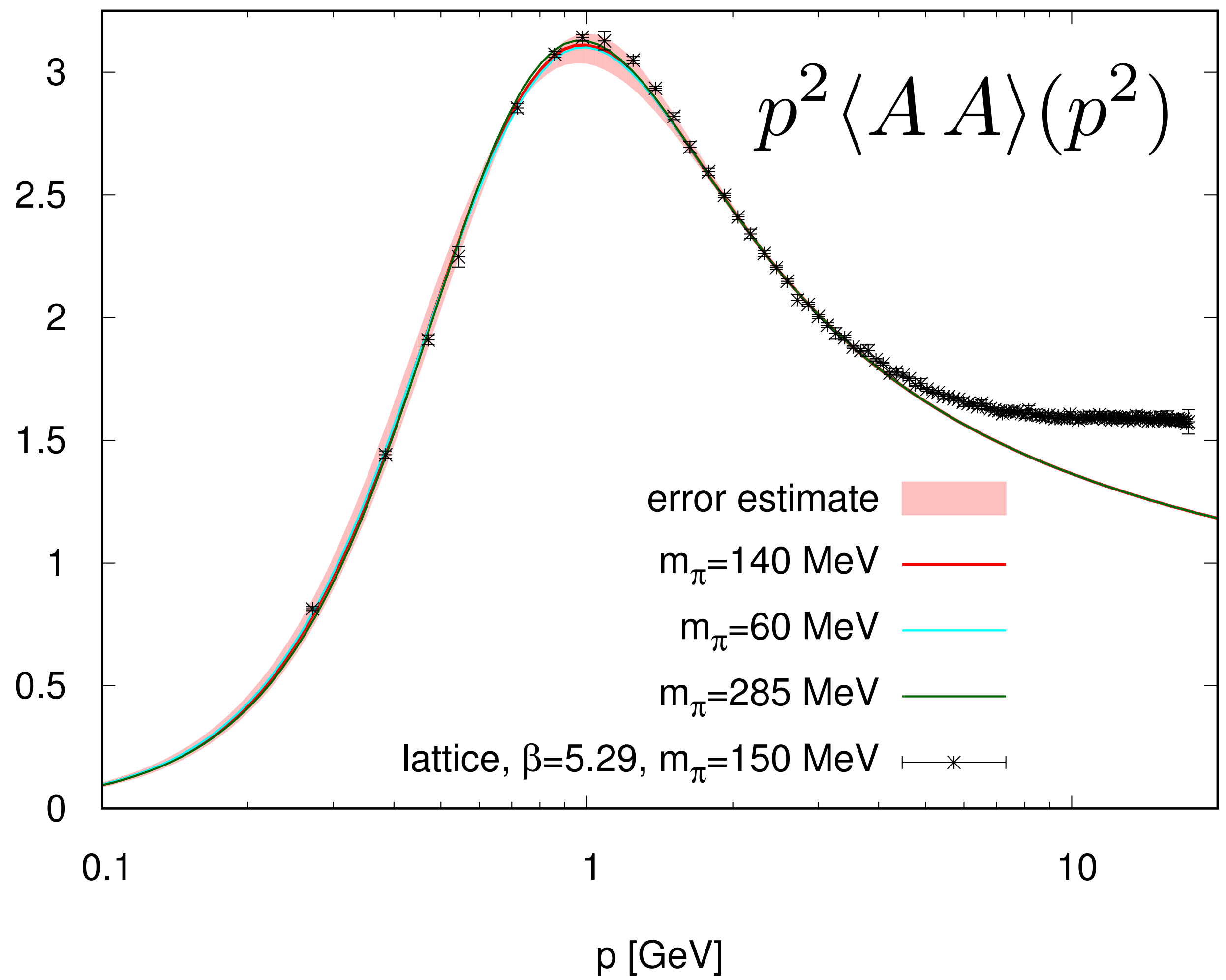
vacuum QCD: Euclidean propagators

Two-flavour QCD



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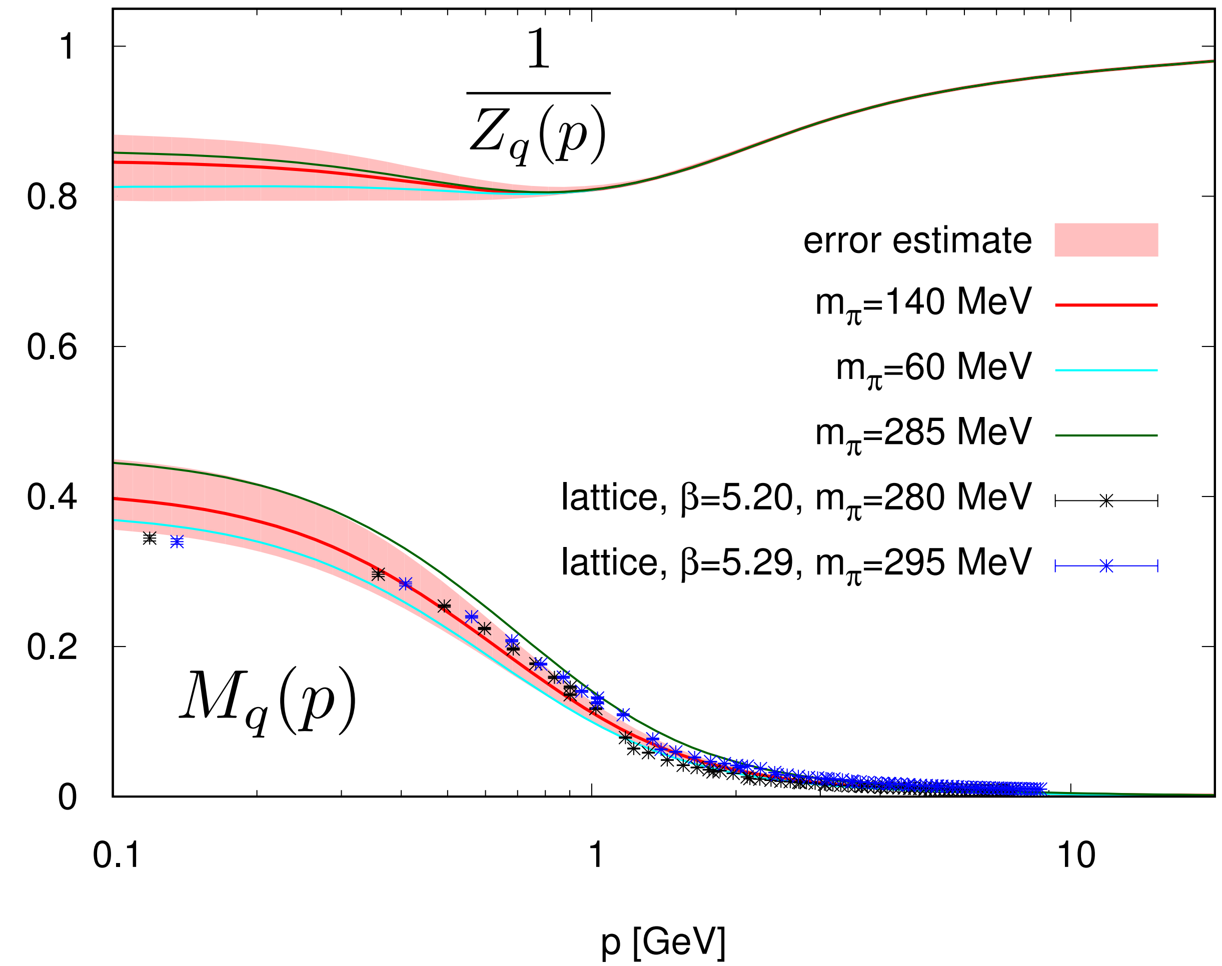

lattice, e.g.: Oliviera et al, Acta Phys.Polon.Supp. 9 (2016) 363
Sternbeck et al, PoS LATTICE2016 (2017)
A. Athenodorou et al, PLB 761 (2016) 444

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

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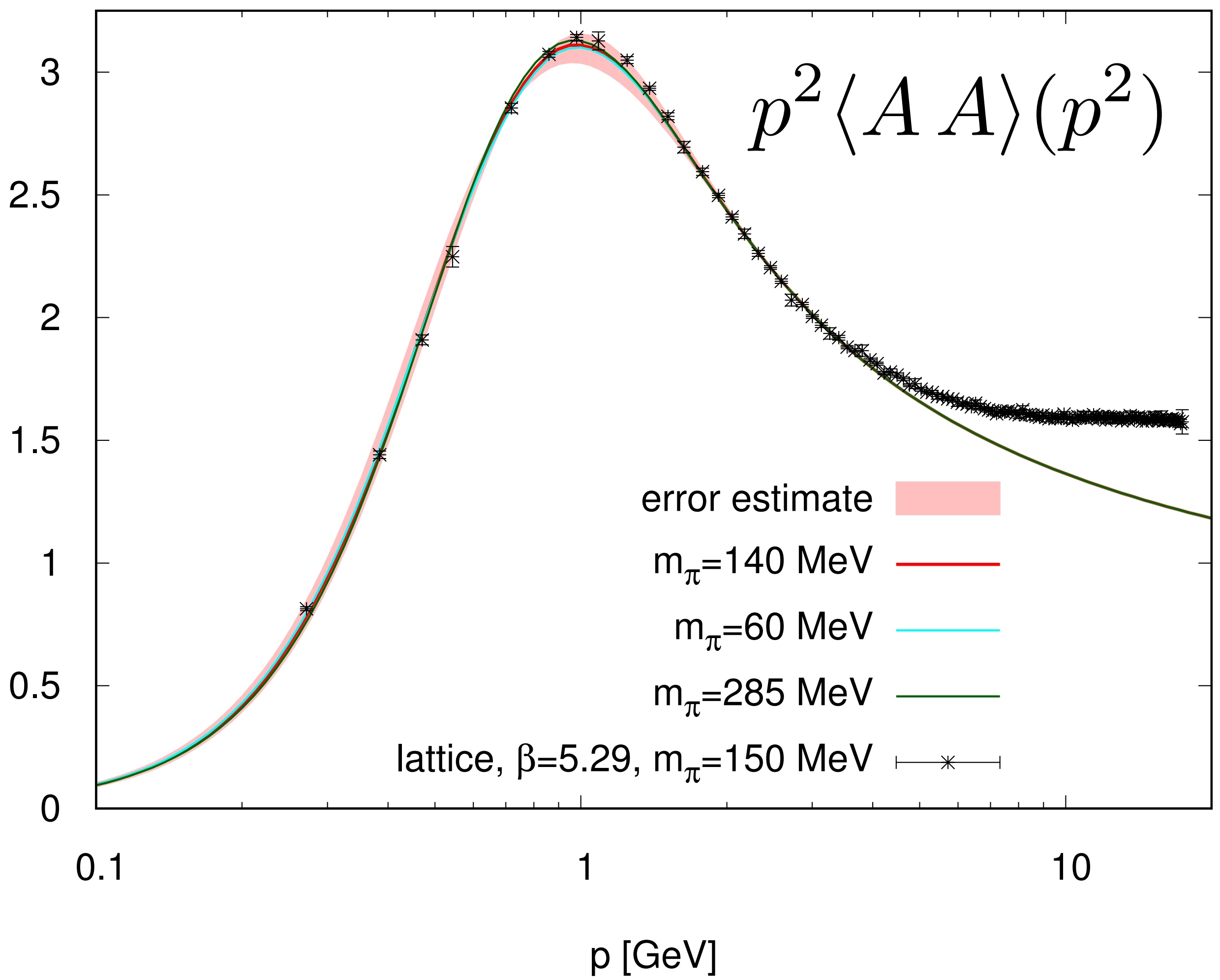

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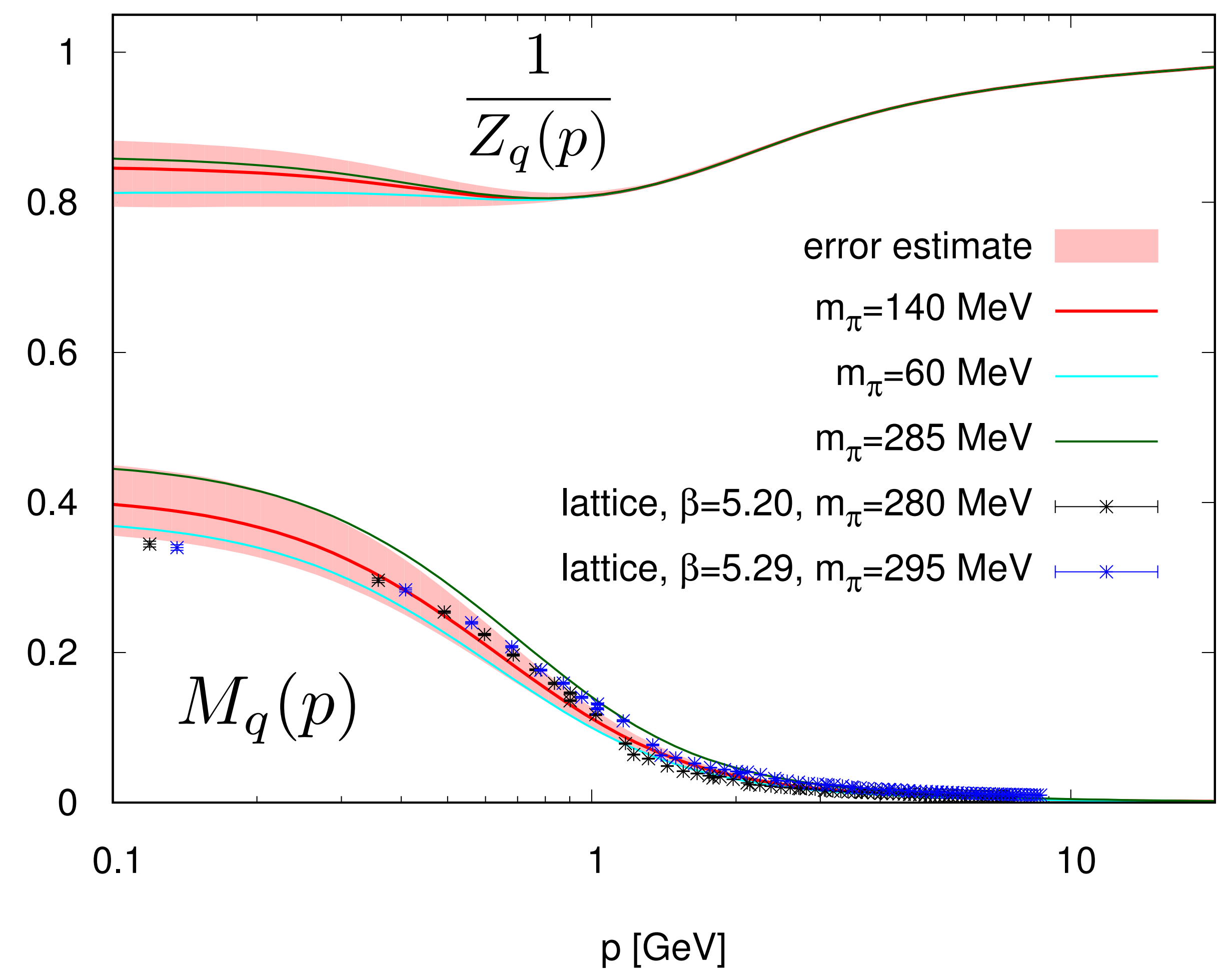

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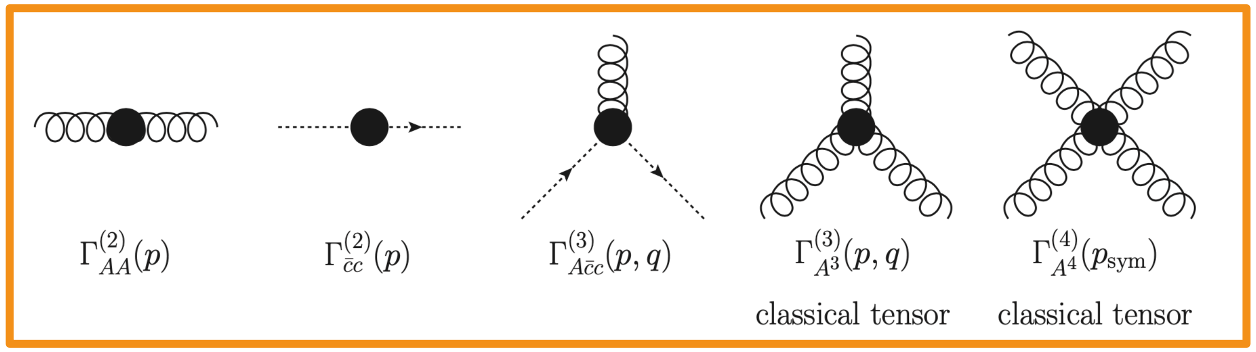
simple correlations

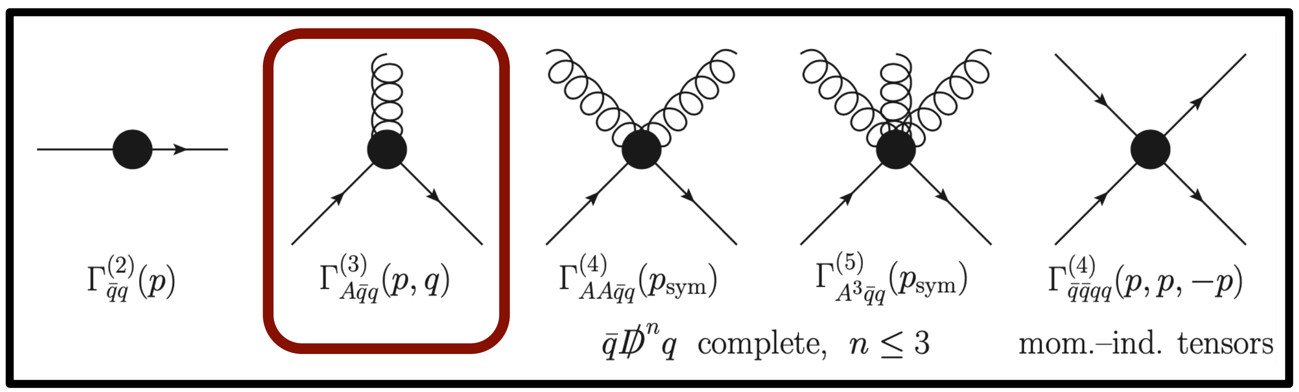
Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

vacuum QCD: current set of correlation functions

FRG

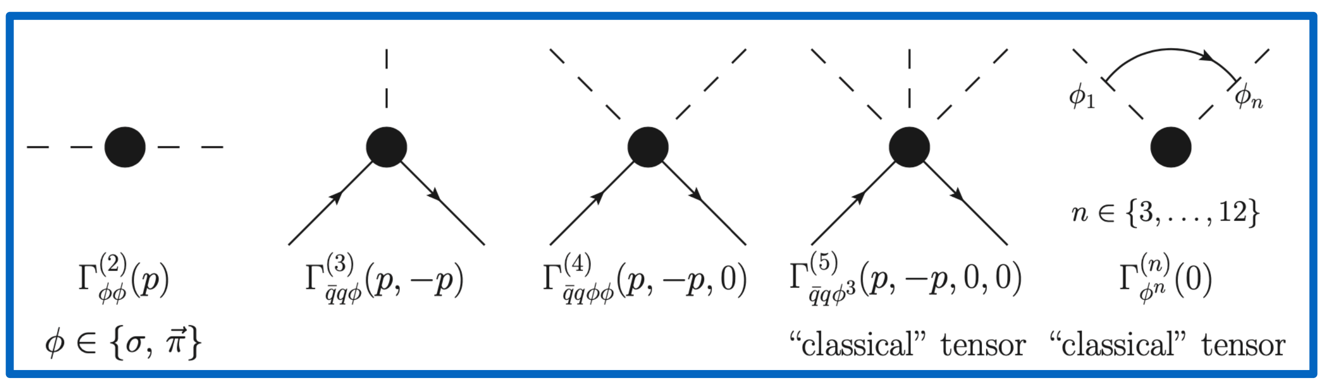


glue sector



quark-glue sector

Eight tranverse tensor structures



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Quark-gluon vertex

$$\left[\Gamma^{(3)}_{\bar{q}qA}\right]^a_\mu (p,q) = 1^{\text{flav}}_{2\times 2}\, T^a \sum_{i=1}^{8} \lambda_i(p,q) \left[\mathcal{T}^{(i)}_{\bar{q}qA}\right]_\mu (p,q)$$

covariant expansion scheme

$\bar{q}\not{D}q:$ $$\left[\mathcal{T}^{(1)}_{\bar{q}qA}\right]_\mu (p,q) = -i\,\gamma_\mu$$

$\bar{q}\not{D}^2 q:$ $$\left[\mathcal{T}^{(2)}_{\bar{q}qA}\right]_\mu (p,q) = (p-q)_\mu\, 1_{4\times 4}$$

$$\left[\mathcal{T}^{(3)}_{\bar{q}qA}\right]_\mu (p,q) = (\not{p}-\not{q})\gamma_\mu$$

$$\left[\mathcal{T}^{(4)}_{\bar{q}qA}\right]_\mu (p,q) = (\not{p}+\not{q})\gamma_\mu$$

$\bar{q}\not{D}^3 q:$ $$\left[\mathcal{T}^{(5)}_{\bar{q}qA}\right]_\mu (p,q) = i\,(\not{p}+\not{q})(p-q)_\mu$$

$$\left[\mathcal{T}^{(6)}_{\bar{q}qA}\right]_\mu (p,q) = i\,(\not{p}-\not{q})(p-q)_\mu$$

$$\left[\mathcal{T}^{(7)}_{\bar{q}qA}\right]_\mu (p,q) = \frac{i}{2}[\not{p},\not{q}]\gamma_\mu$$

Aiming at apparent convergence

quenched: **Mitter, JMP, Strodthoff, PRD 91 (2015) 054035**

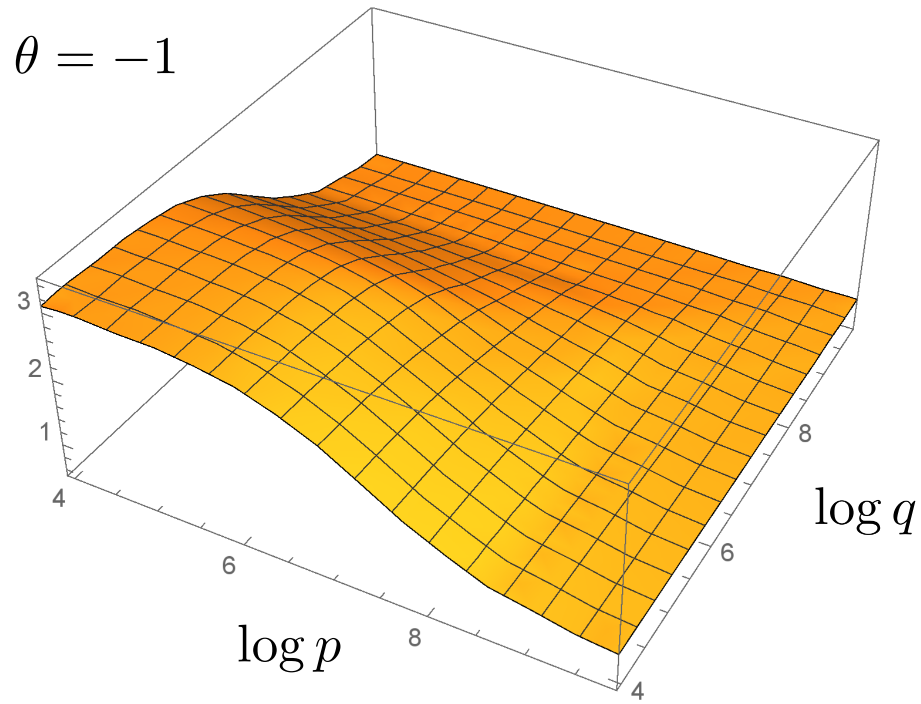
Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

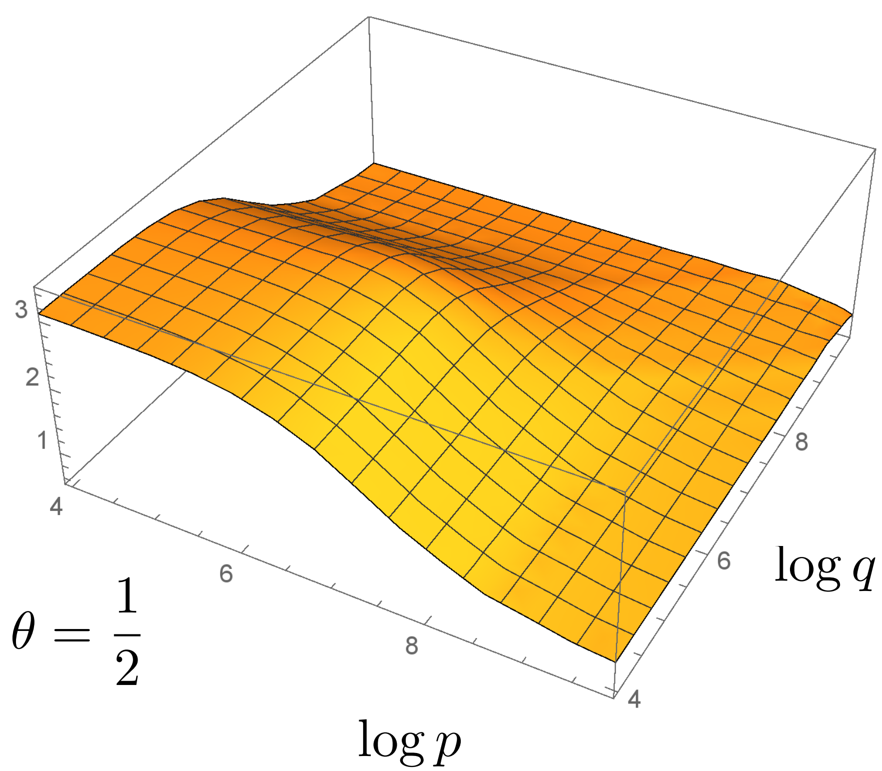
Quark-gluon vertex

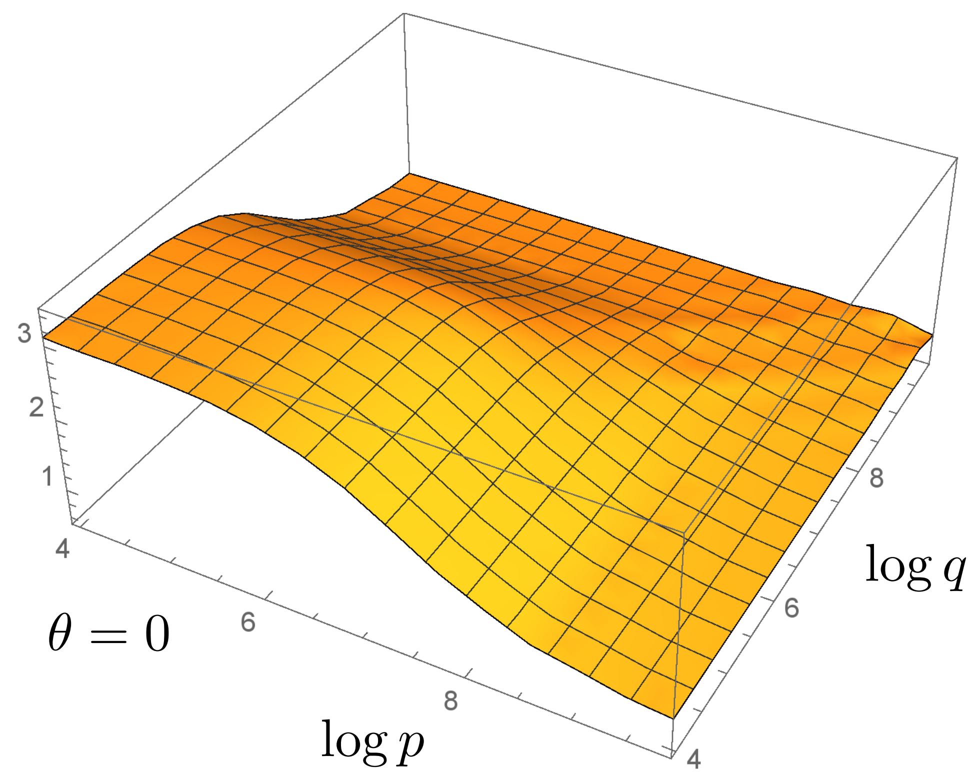
$$\theta = \frac{p \cdot q}{\sqrt{p^2\, q^2}}$$

p,q in MeV

$\lambda_1(p,q)$







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Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

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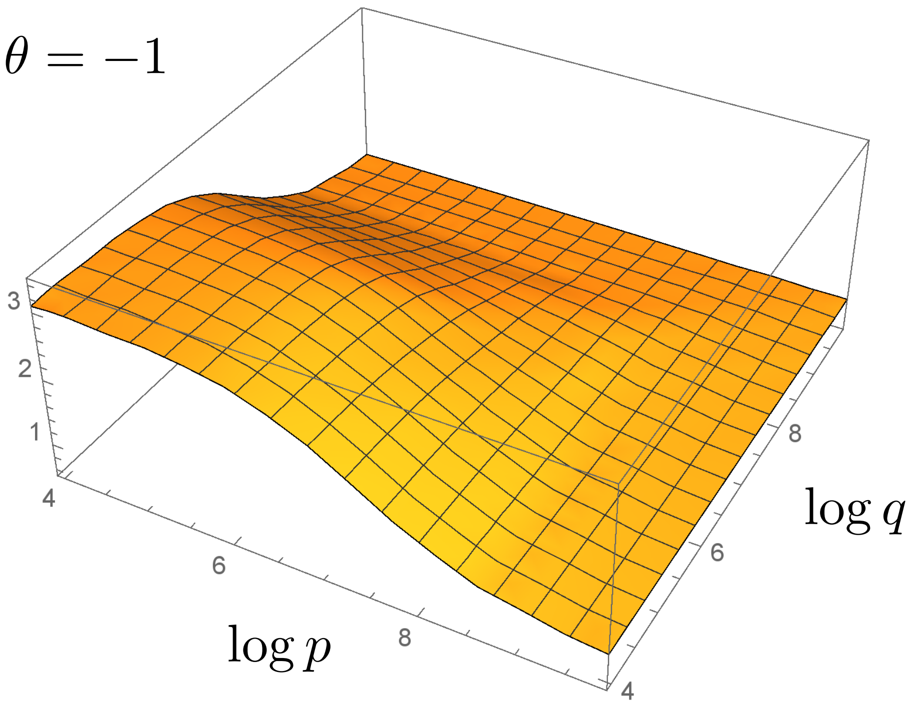
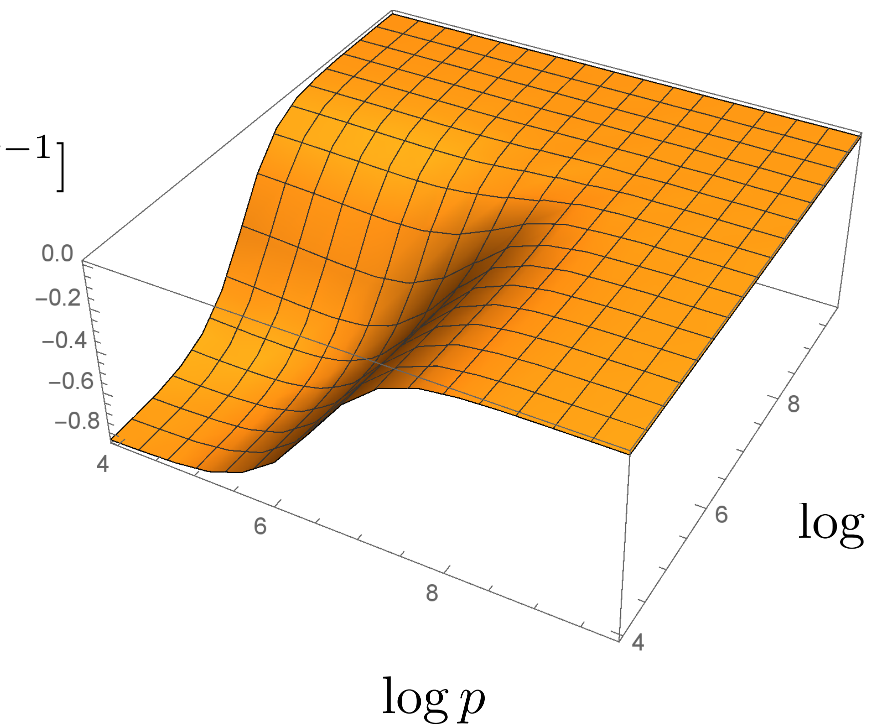


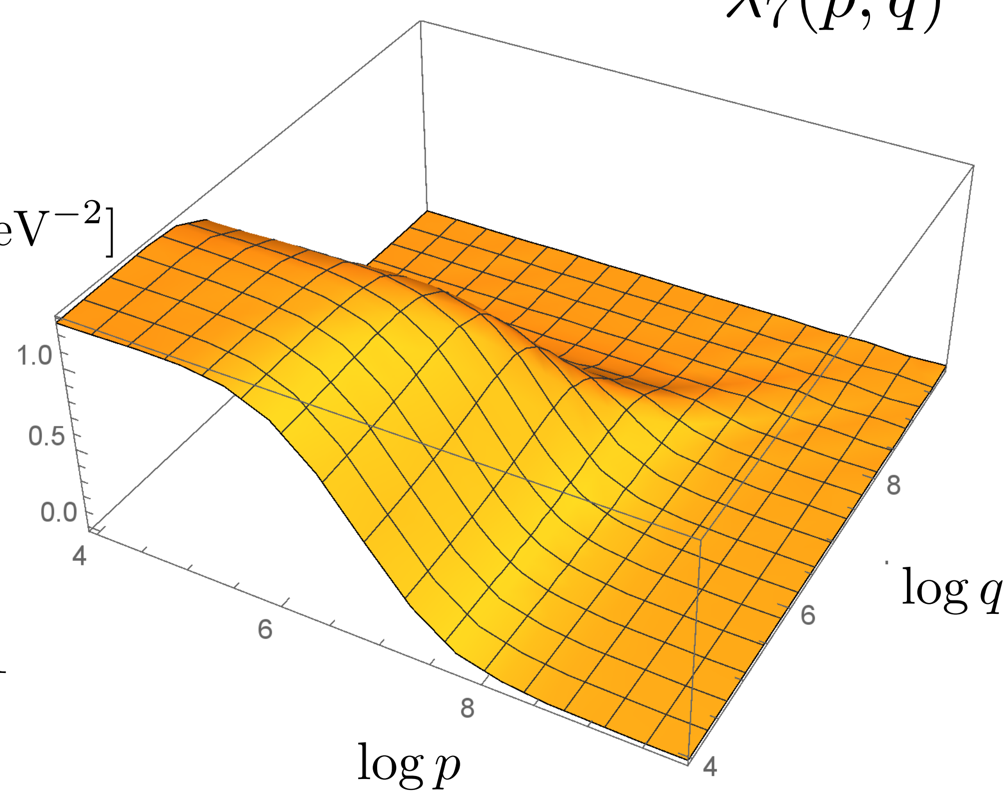
$\lambda_4(p, q)$

[GeV^{-1}]



$\lambda_7(p, q)$

[GeV^{-2}]



$\theta = -1$

Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

vacuum QCD: Quark-gluon vertex

Two-flavour QCD

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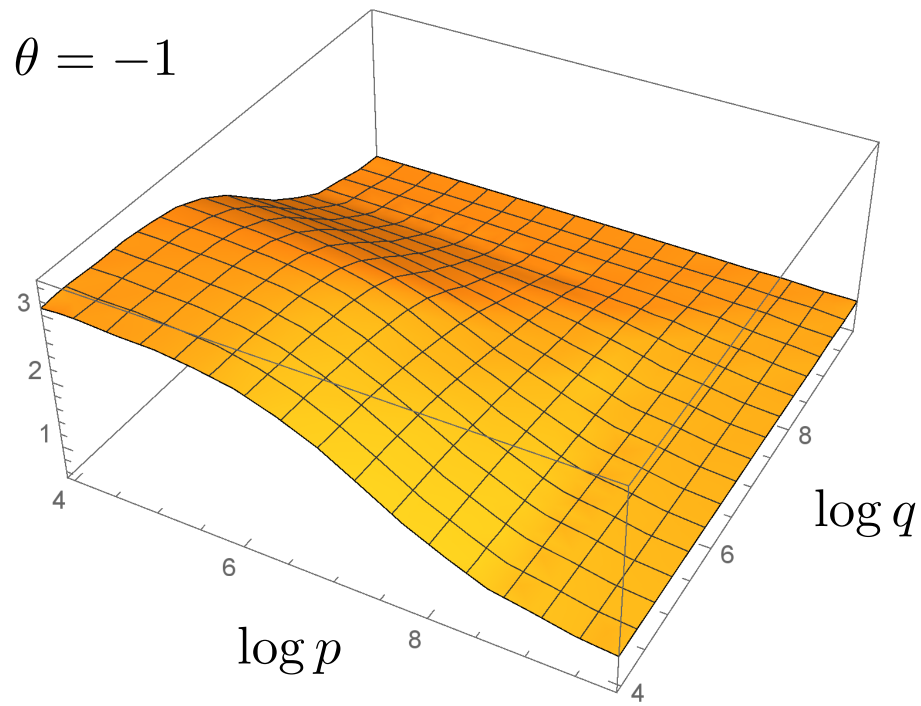
p,q in MeV



$\lambda_1(p, q)$

All (eight) tensor structures!

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

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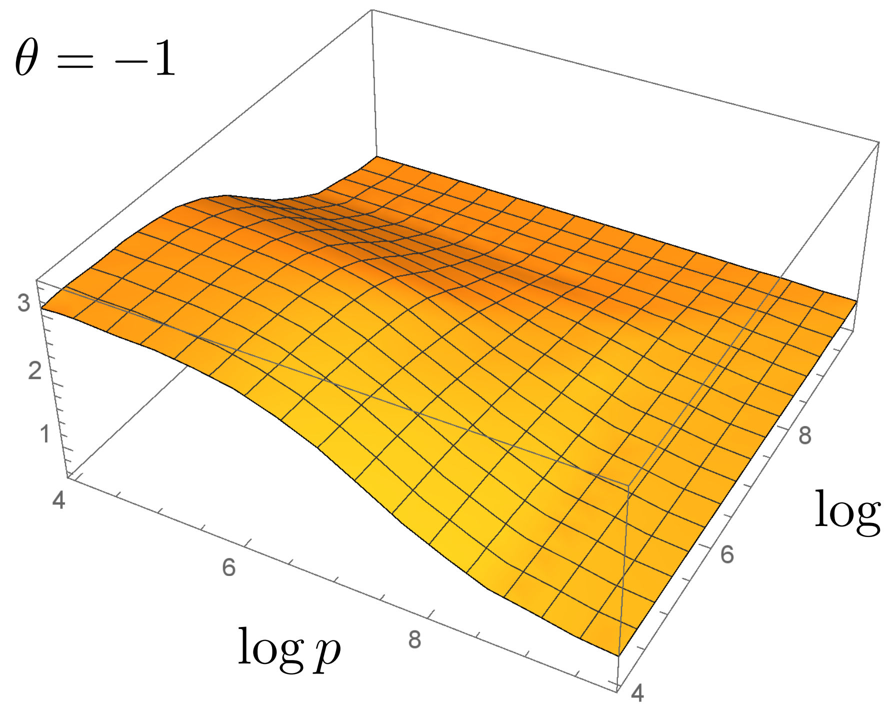

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Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

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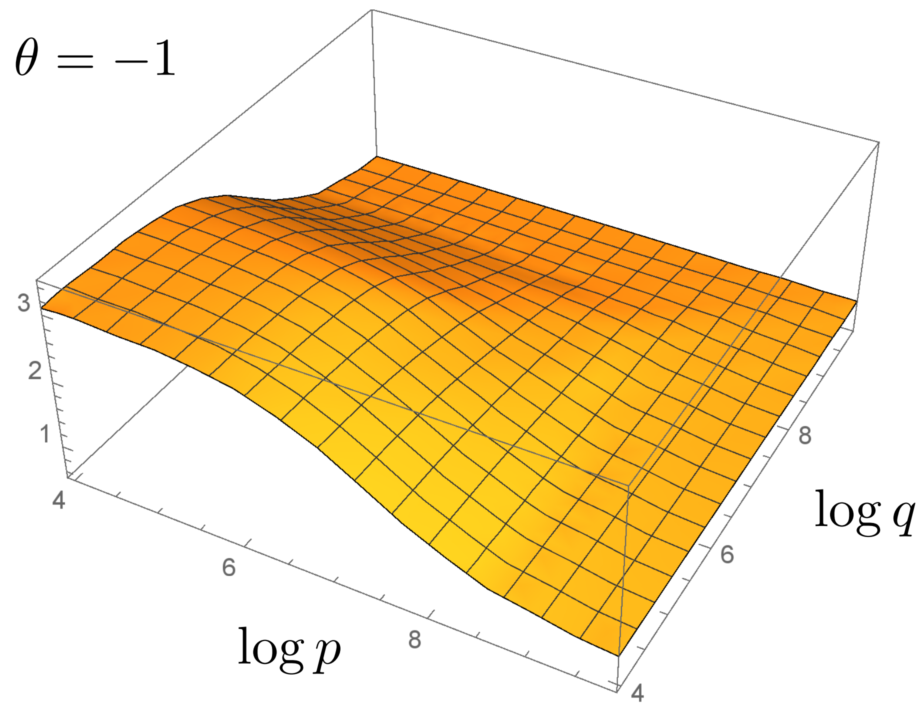

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All (eight) tensor structures!

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up-to-date 1st principles works:

FunMethods: Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

Williams, EPJ A51 (2015) 57
Sanchis-Alepuz, Williams, PLB 749 (2015) 592
Williams, Fischer, Heupel, PRD 93 (2016) 034026
Contant, Huber, Fischer, Welzbacher, Williams, APP.Supp. 11 (2018) 483

Aguilar, Binosi, Ibanez, Papavassiliou, PRD 89 (2014) 065027
Binosi, Chang, Papavassiliou, Qin, Roberts, PRD 95 (2017) 031501
Aguilar, Cardona, Ferreira, Papavassiliou, PRD 96 (2017) 014029
PRD 98 (2018) 014002

Pelaez, Tissier, Wschebor, PRD 92 (2015) 045012

Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016) 1

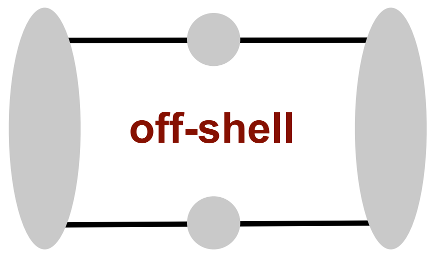
lattice, e.g.: Oliveira, Kizilersü, Silva, Skullerud, Sternbeck, Williams, APP Suppl. 9 (2016) 363

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

Three remarks on Functional Methods for QCD

- **off-shell representation of thermodynamic observables**

e.g. $\mathrm{Tr}\,\langle q(x)\bar{q}(x)\rangle$



pressure, trace anomaly, fluctuations, volume flucs., ...

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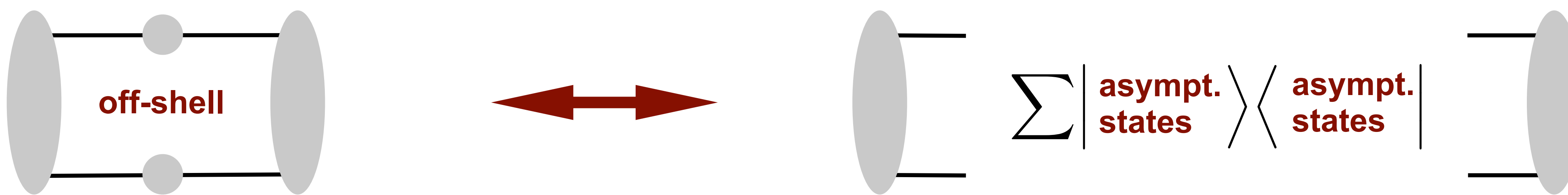

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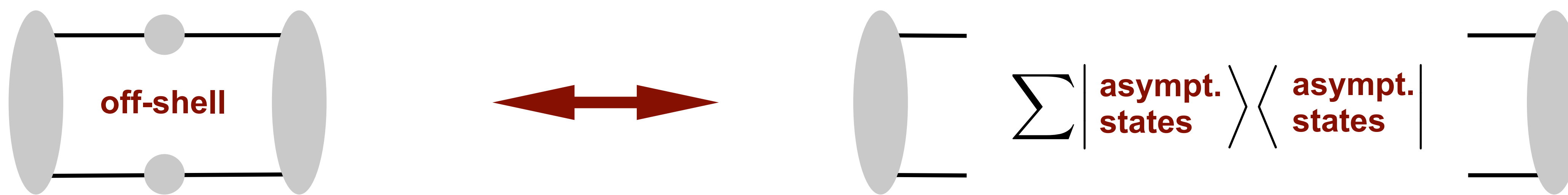

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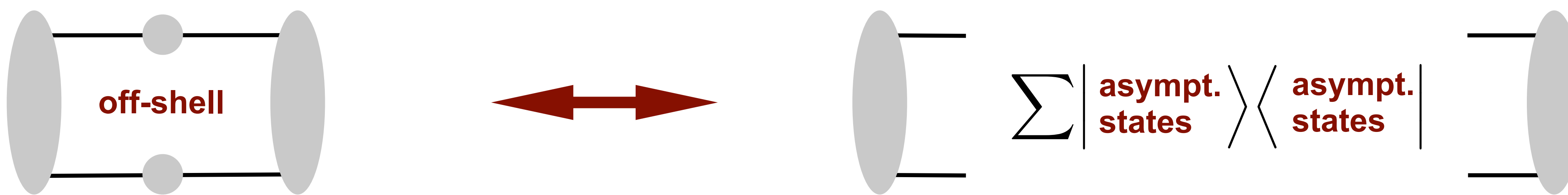

pressure, trace anomaly,
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- **gauge fixing = parameterisation**

$$\langle q(x_1)\cdots\bar{q}(x_{2n})\,A_\mu(y_1)\cdots A_\mu(y_m)\,h(z_1)\cdots h(z_l)\rangle$$

Consequences

I: simple correlations

Three remarks on Functional Methods for QCD

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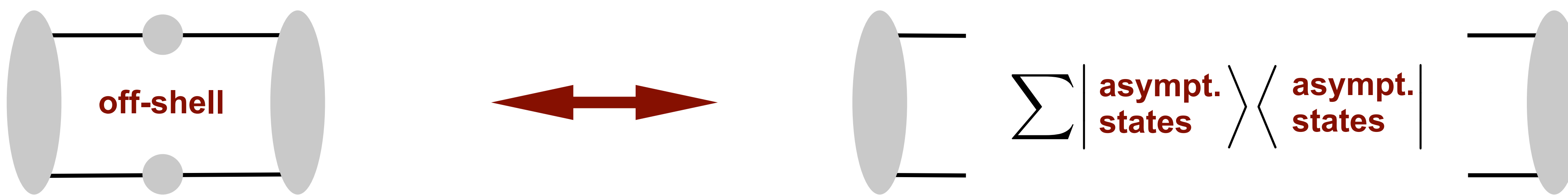

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'No free lunch theorem'

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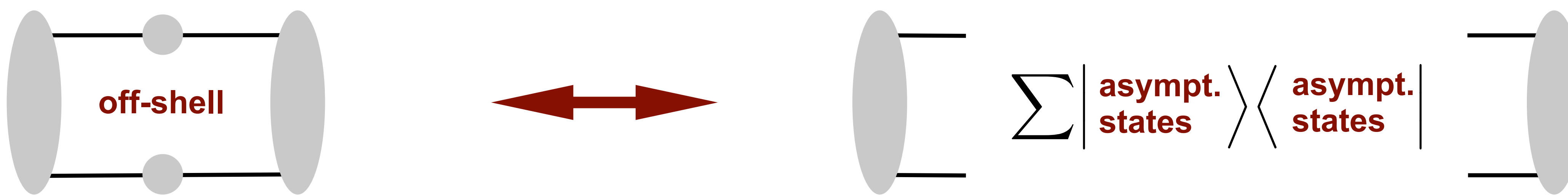

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Consequences

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'No free lunch theorem'

- **'Your mean field is not my mean field'**

$$\left.\frac{\delta S_{\rm cl}[\phi]}{\delta\phi}\right|_{\phi=\bar\phi}=0 \qquad\qquad \left.\frac{\delta\Gamma[\phi]}{\delta\phi}\right|_{\phi=\bar\phi_{\rm quant}}=0$$

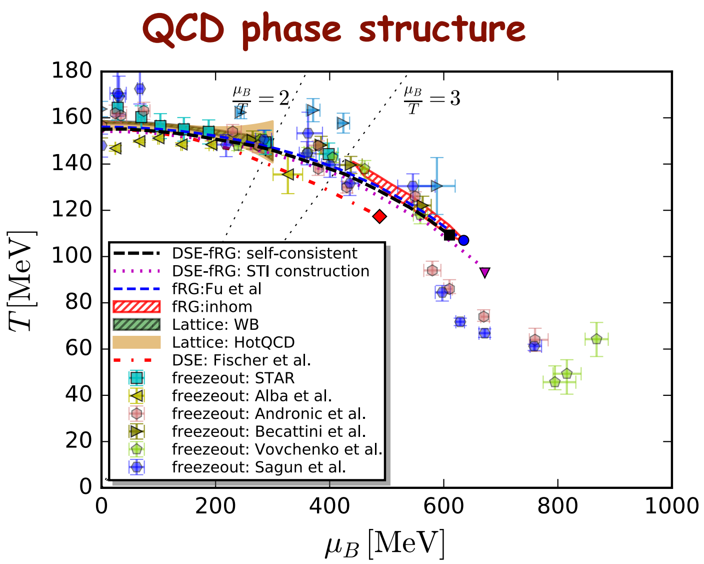
full quantum equation of motion

Outline

- QCD from functional methods

Applications

- **QCD phase structure**



- Fluctuations ob conserved charges

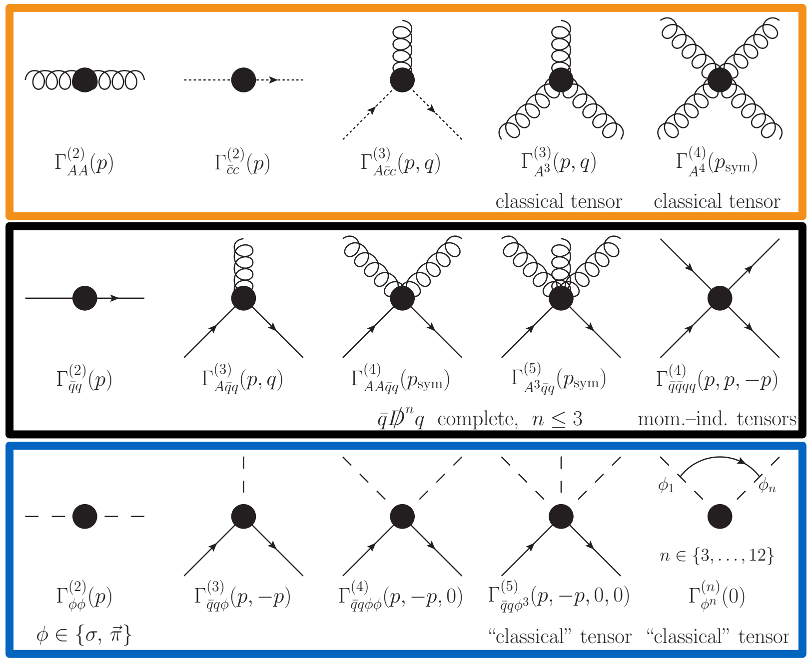
- QCD-assisted transport

- Summary & outlook

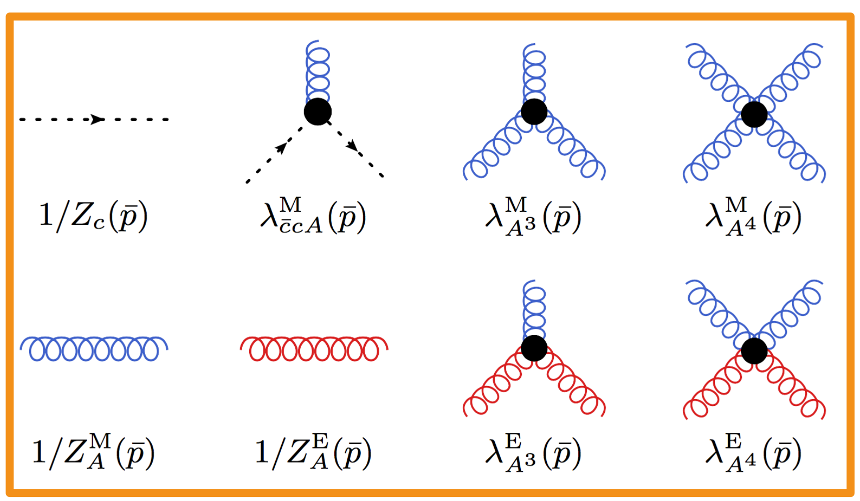
QCD at finite density

Input

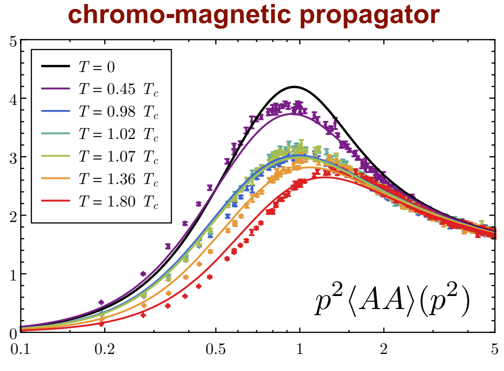
two flavour vacuum QCD



finite T Yang-Mills



chromo-magnetic propagator



Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 97 (2018) 054015

$$\Gamma^{(n)} = \left[\Gamma^{(n)}\right]_{\textbf{Input}} + \Delta\Gamma^{(n)}$$

Fu, JMP, Rennecke, PRD 101, (2020) 054032

QCD at finite density

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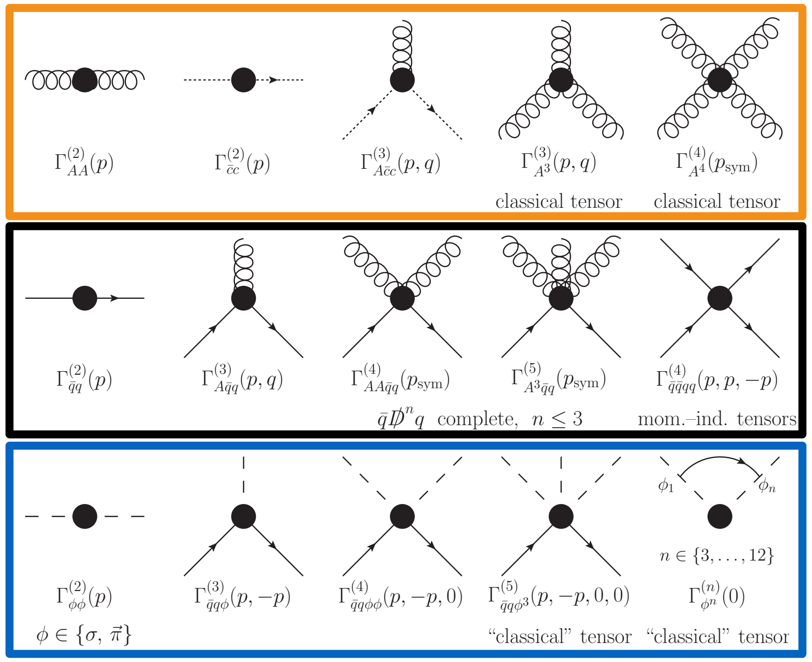
two flavour vacuum QCD



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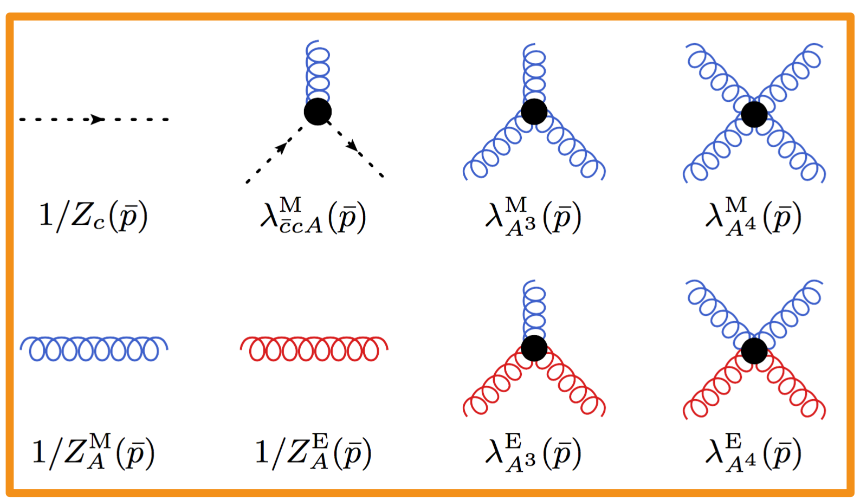

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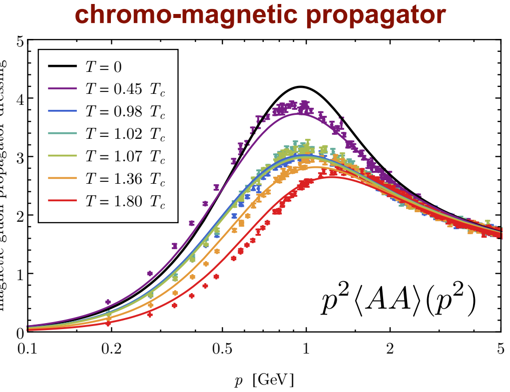

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 97 (2018) 054015

Output

$$\partial_t \Delta\Gamma^{(n)} = \left[\partial_t \Gamma^{(n)}\right]_{\text{Input}} + \Delta\text{Flow}^{(n)}\left[\left\{\left[\Gamma^{(m)}\right]_{\text{Input}}\right\}, \left\{\Delta\Gamma^{(m)}\right\}\right]$$

vacuum: Braun, Fister, Pawlowski, Rennecke, PRD 94, 034016 (2016)

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Fu, JMP, Rennecke, PRD 101, (2020) 054032

QCD at finite density

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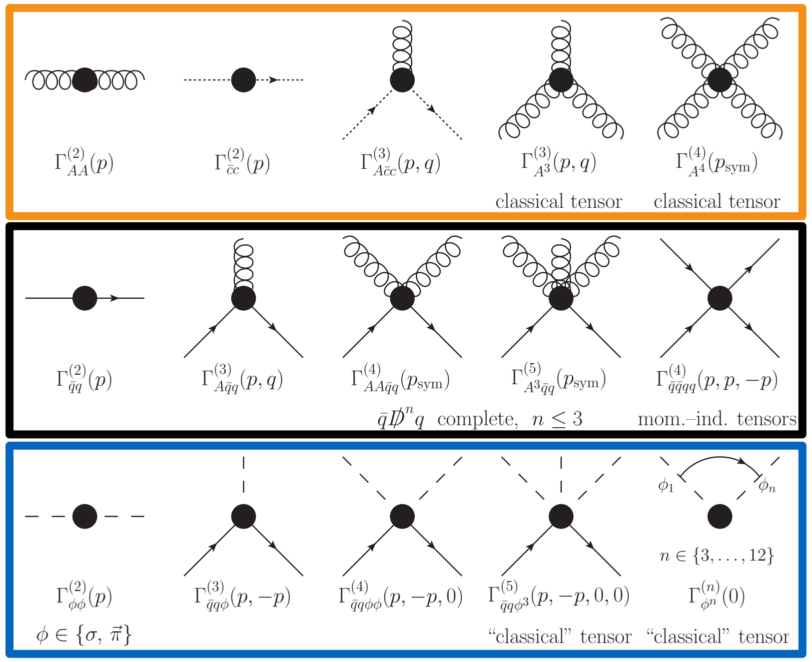

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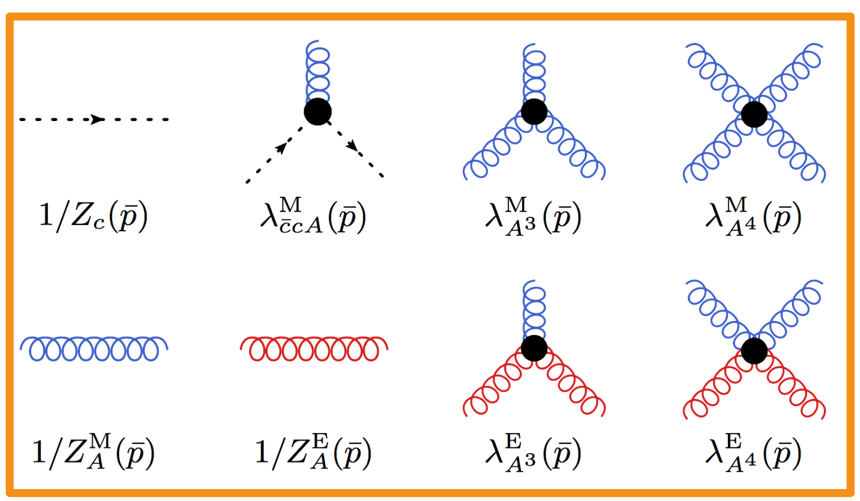

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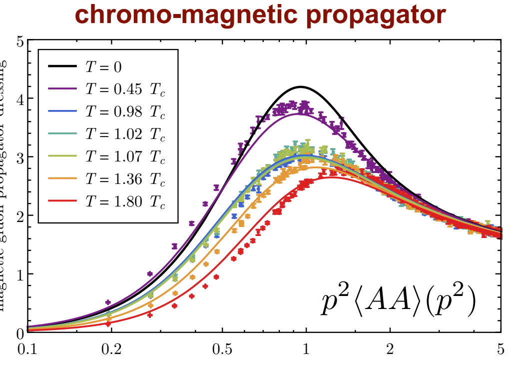

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 97 (2018) 054015

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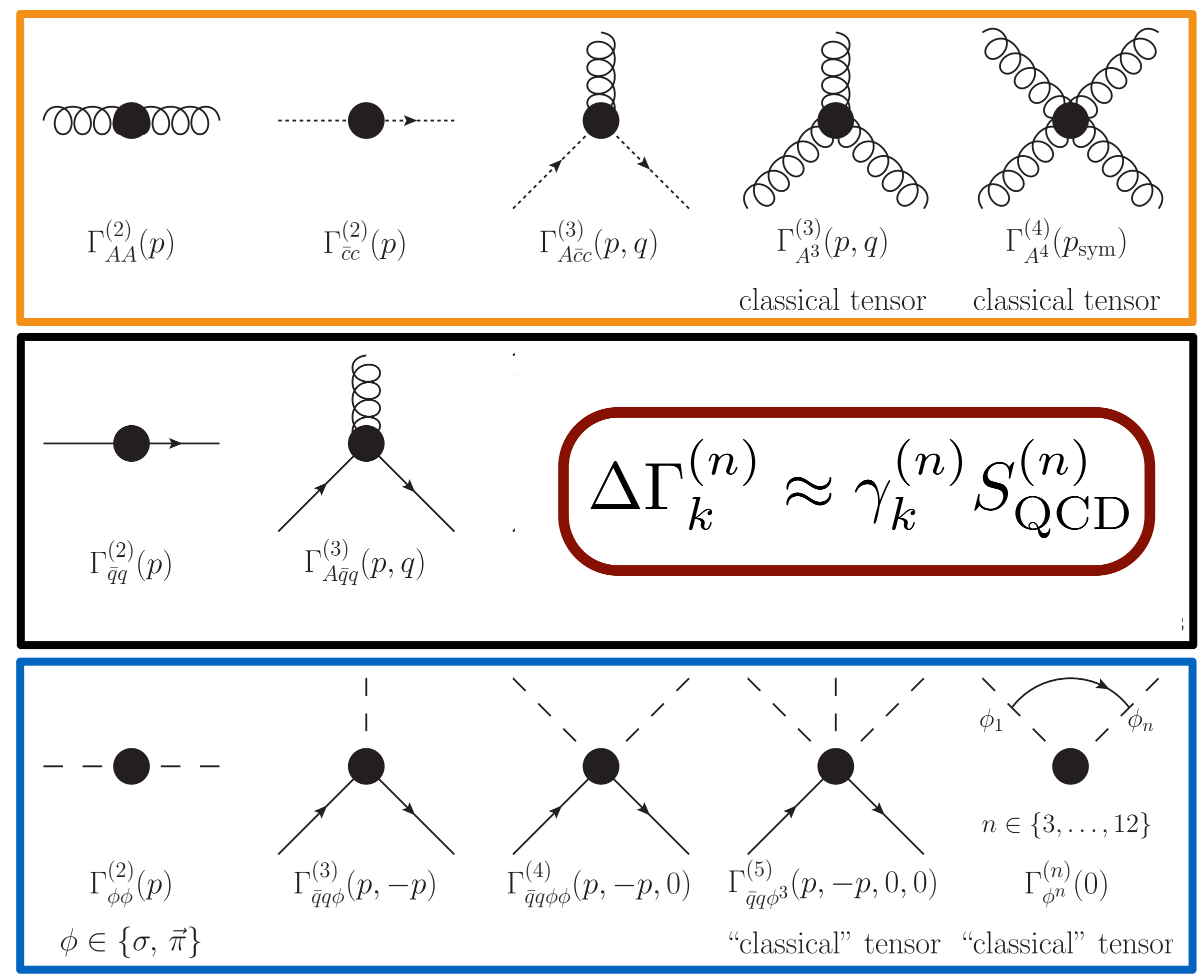
2+1 flavour QCD at finite T & mu

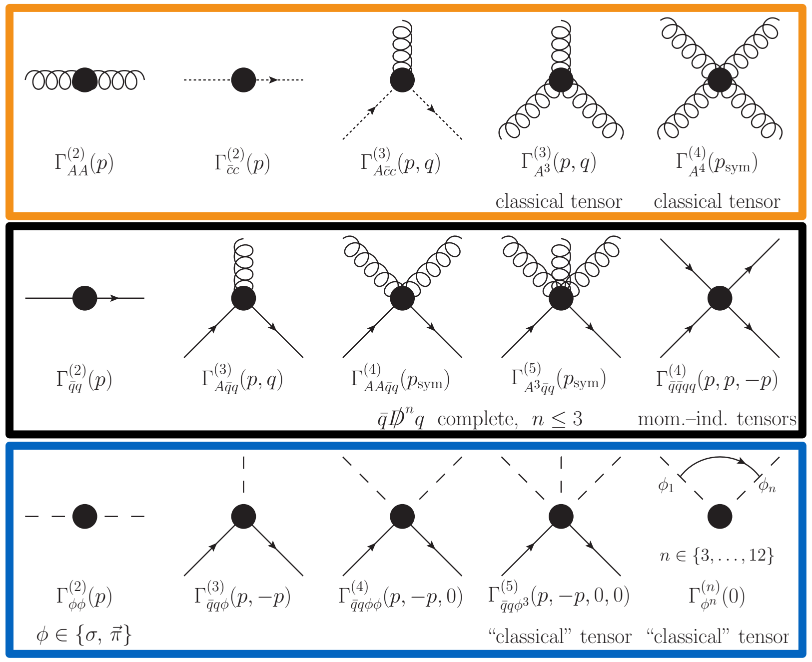


Fu, JMP, Rennecke, PRD 101, (2020) 054032

QCD at finite density

Input

two flavour vacuum QCD



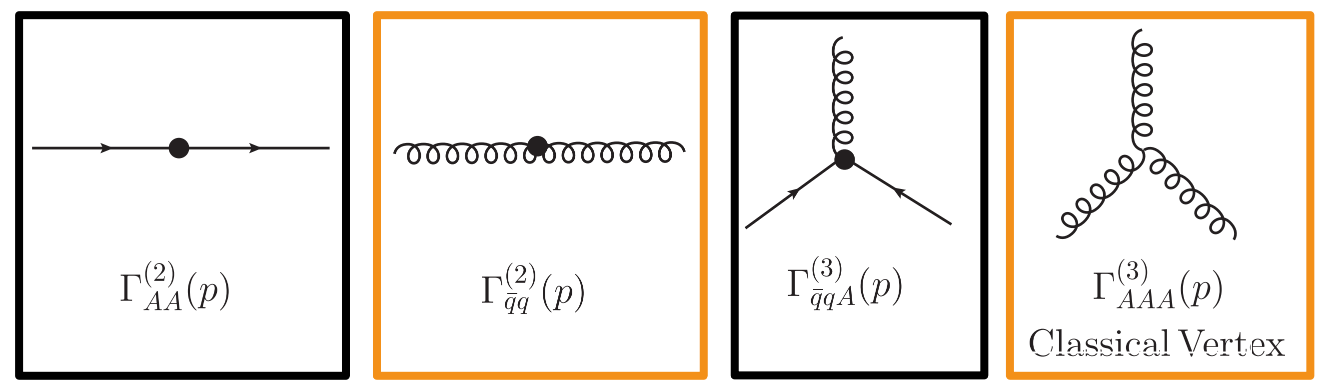
Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

Output

$$\Delta\Gamma^{(n)} = \left[\Gamma^{(n)}\right]_{\text{Input}} + \Delta\text{DSE}^{(n)}\left[\left\{\left[\Gamma^{(m)}\right]_{\text{Input}}\right\}, \left\{\Delta\Gamma^{(m)}\right\}\right]$$

fRG-assisted DSE

2+1 flavour QCD at finite T & mu



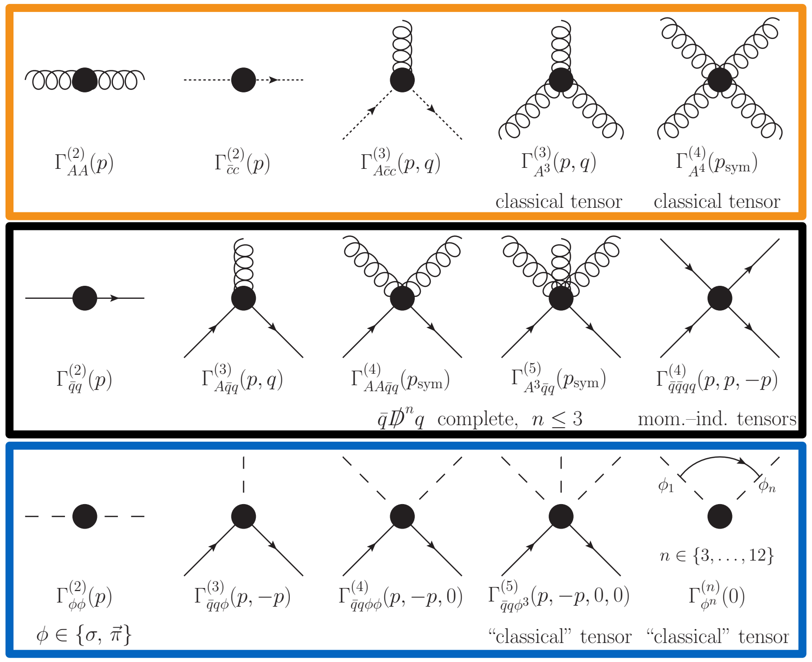
New: all eight tensor structures

Gao, JMP, PRD 102, (2020) 034027
arXiv:2010.137005

QCD at finite density

Input

two flavour vacuum QCD



Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

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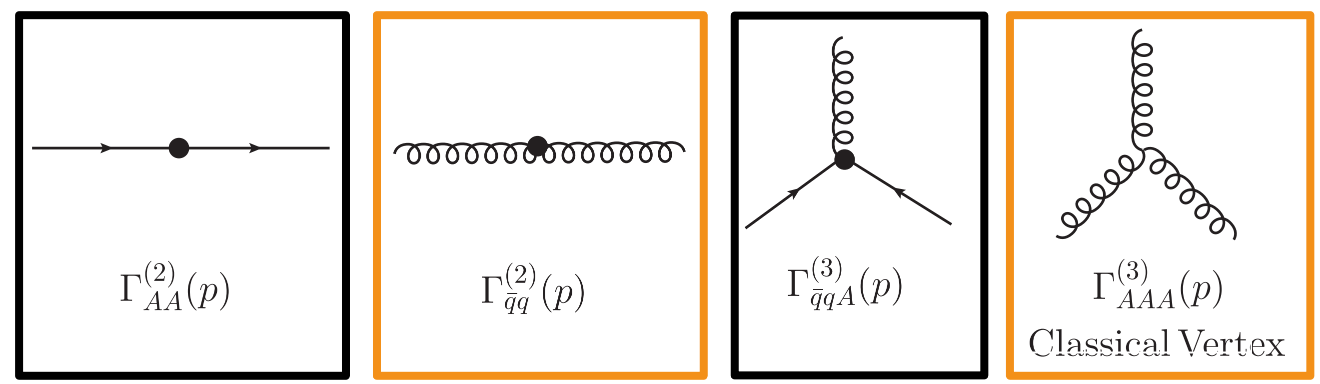
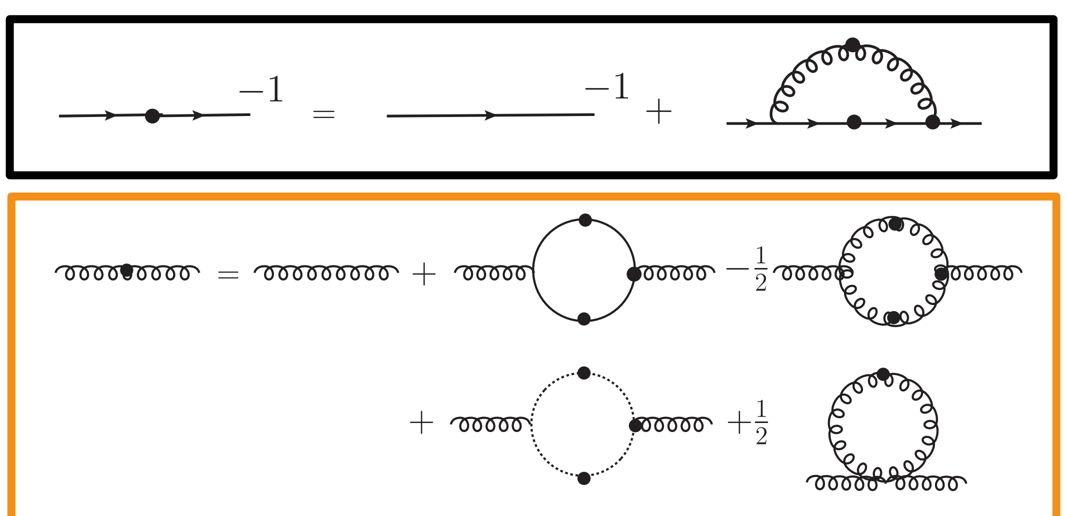
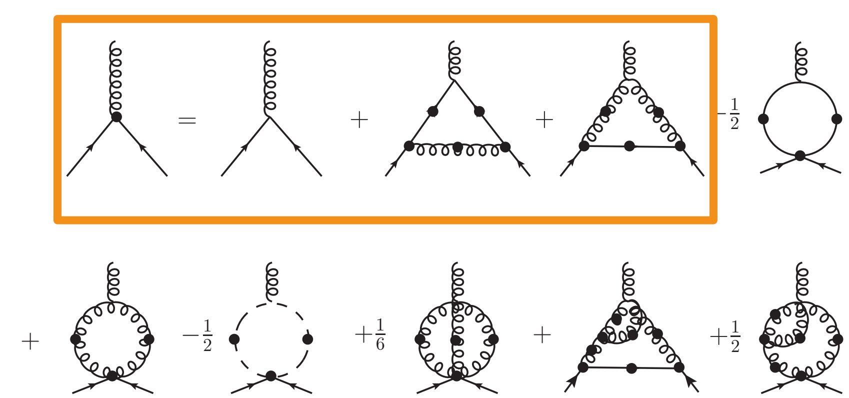


New: all eight tensor structures

System of DSEs





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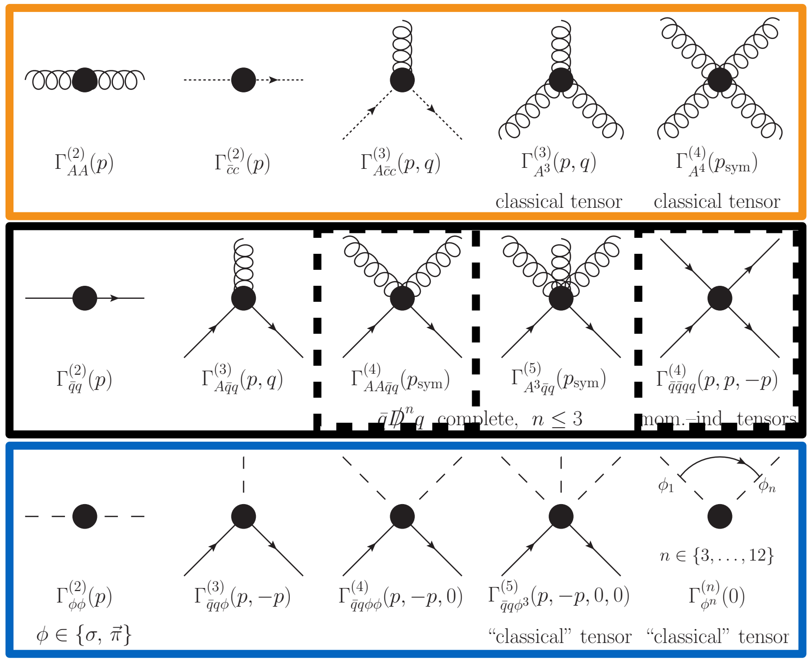
QCD at finite density

DSE

Input

two flavour vacuum QCD



Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

Output

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fRG-assisted DSE

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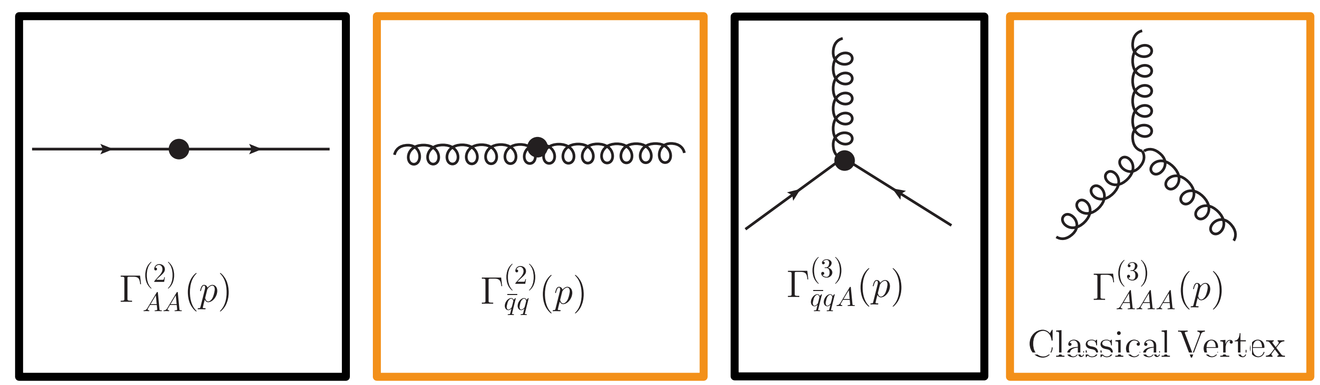

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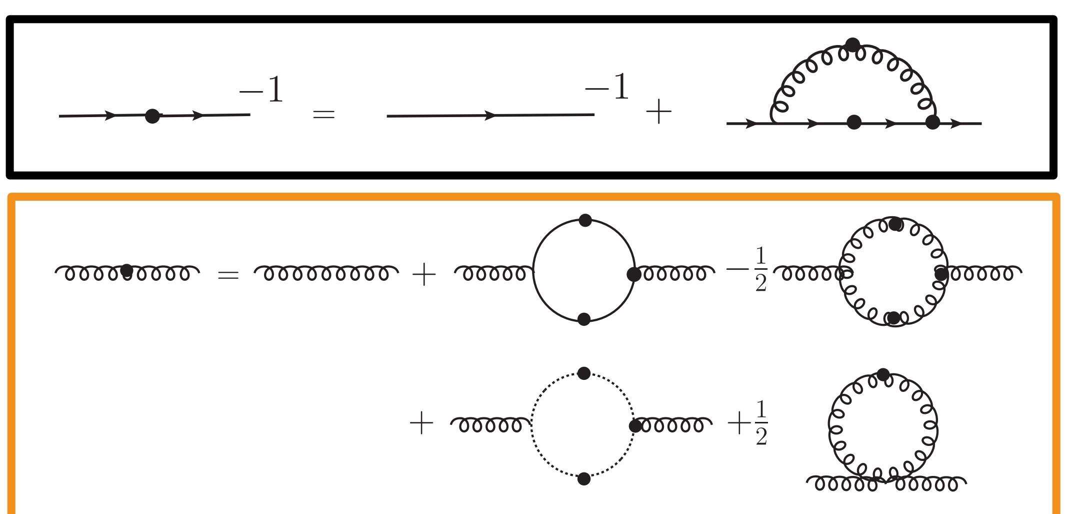
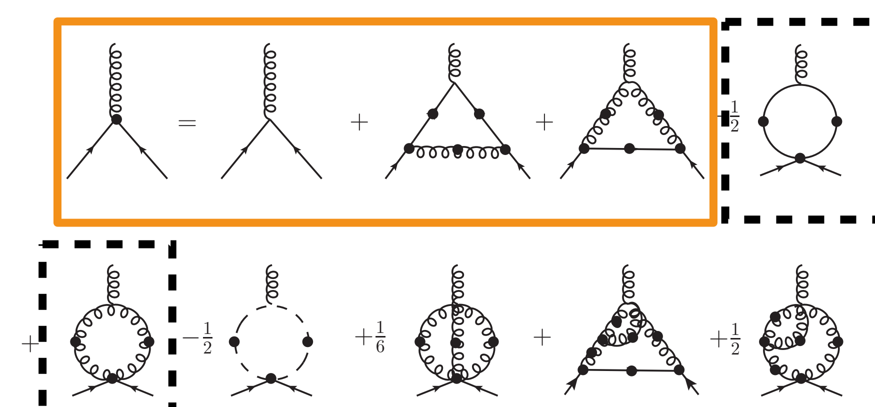




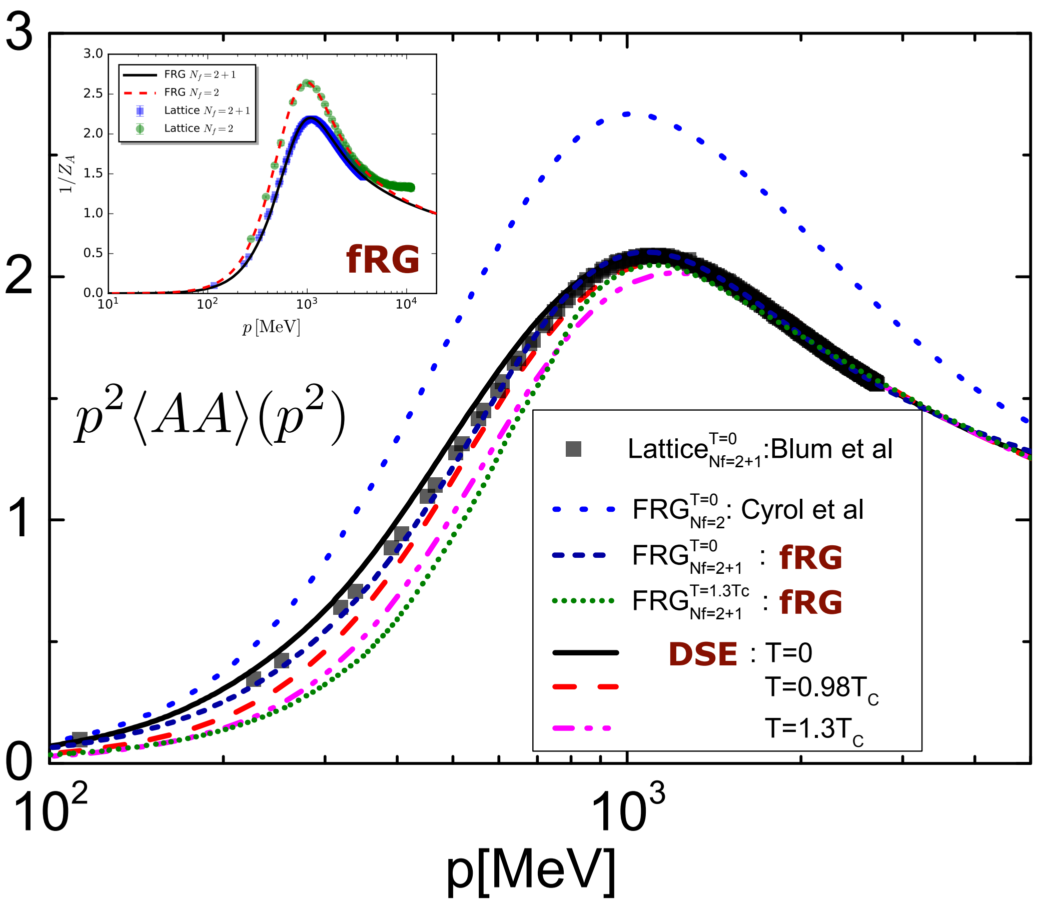
Next step

Gao, JMP, PRD 102, (2020) 034027
arXiv:2010.137005

QCD at finite density

Nf=2+1 Gluon and quark benchmark results in the vacuum and at finite T

vacuum



$$\vec{p}^2\langle AA\rangle(\vec{p}^2)$$

fRG: Fu, JMP, Rennecke, PRD 101, (2020) 054032

DSE: Gao, JMP, PRD 102, (2020) 034027
arXiv:2010.137005

DSE: vacuum & finite T

Fischer, Luecker, PLB 718 (2013) 1036
Fischer, Luecker,Welzbacher, PRD 90 (2014) 034022
Isserstedt, Buballa, Fischer, Gunkel, PRD 100 (2019) 074011

lattice: Nf=2: Sternbeck, Maltman, Müller-Preussker, von Smekal, PoS LATTICE2012, 243 (2012)

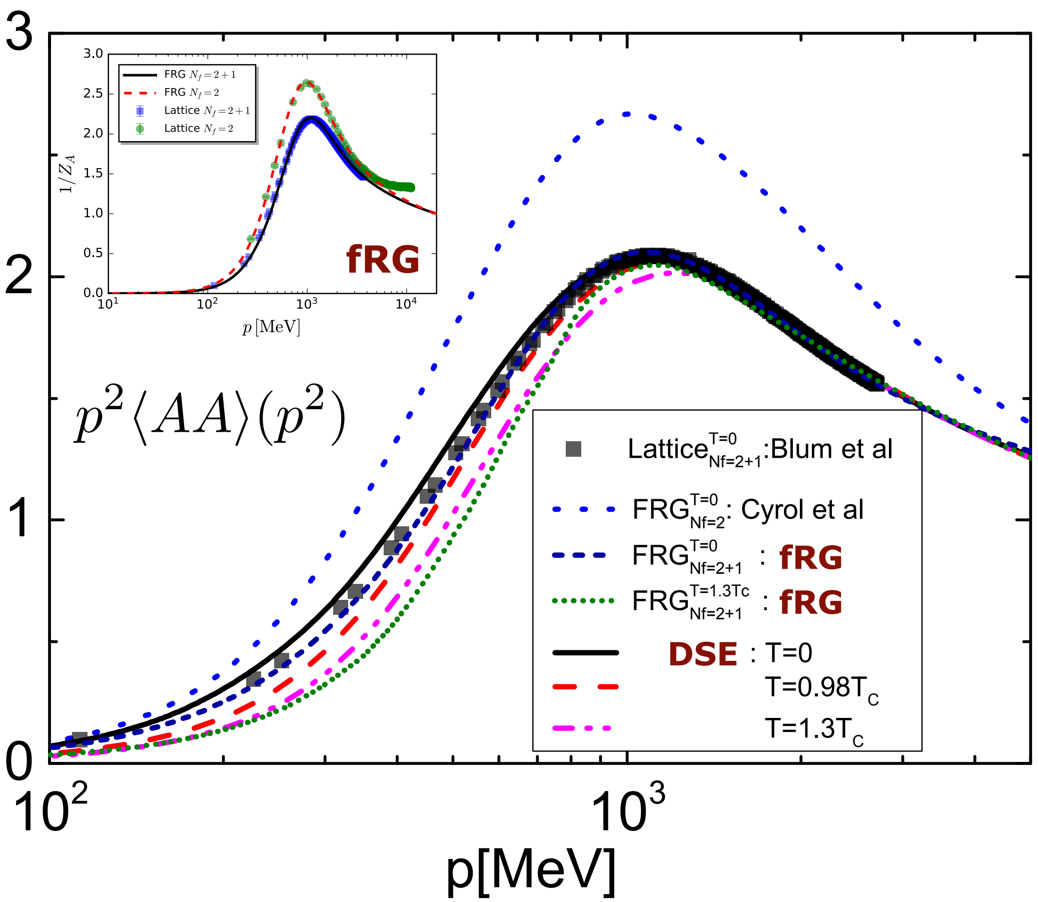
Nf=2+1: Aguilar, De Soto, Ferreira, Papavassiliou, Rodriguez-Quintero, Zafeiropoulos, EPJC 80 (2020) 2, 154,
Boucaud, De Soto, Raya, Rodriguez-Quintero, Zafeiropoulos, PRD 98, 114515 (2018)

Finite T: Ilgenfritz, JMP, Rothkopf, Trunin, EPJ C78, 127 (201) (Nf=2+1+1)

QCD at finite density

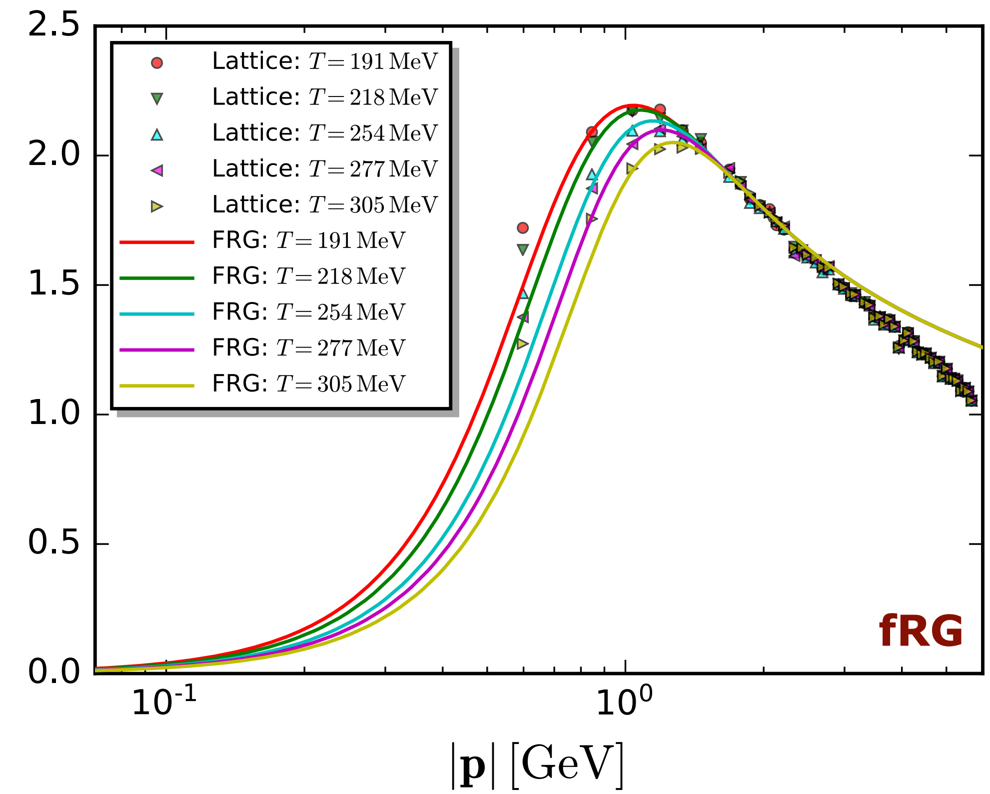
Nf=2+1 Gluon and quark benchmark results in the vacuum and at finite T

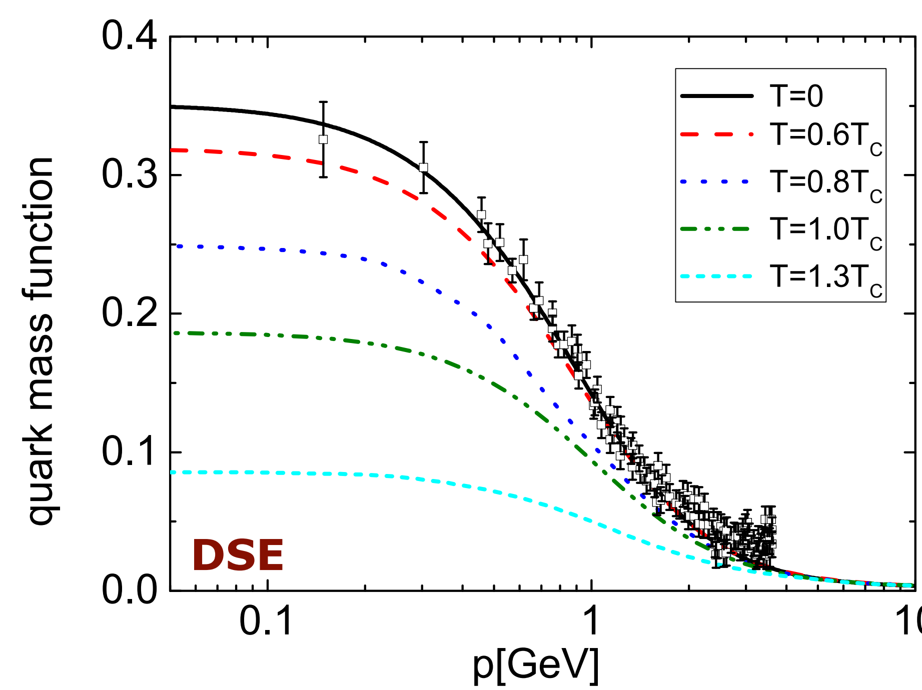
vacuum



$\vec{p}^2\langle AA\rangle(\vec{p}^2)$

finite T





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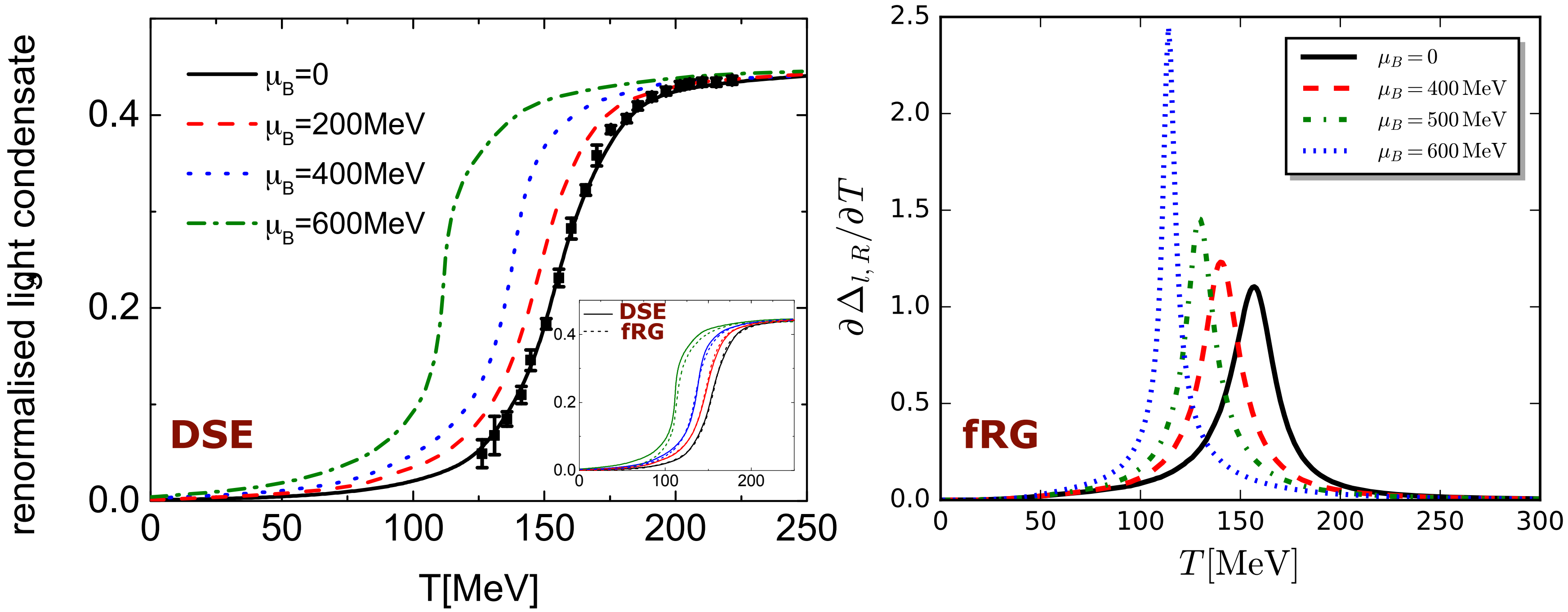
Boucaud, De Soto, Raya, Rodriguez-Quintero, Zafeiropoulos, PRD 98, 114515 (2018)

Finite T: Ilgenfritz, JMP, Rothkopf, Trunin, EPJ C78, 127 (201) (Nf=2+1+1)

QCD at finite density

Chiral order parameter benchmark results at finite T

renormalised condensate



$$\Delta_{l,R}(T, \mu_B) \simeq \Delta_l(T, \mu_B) - \Delta_l(0,0)$$

$$\Delta_q(T, \mu_B) = \frac{T}{\mathcal{V}} \, m_q^0 \int_x \langle \bar{q}(x) q(x) \rangle$$

lattice: S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, C. Ratti, and K. K. Szabo, JHEP 09, 073 (2010)

DSE: quark condensates

See also

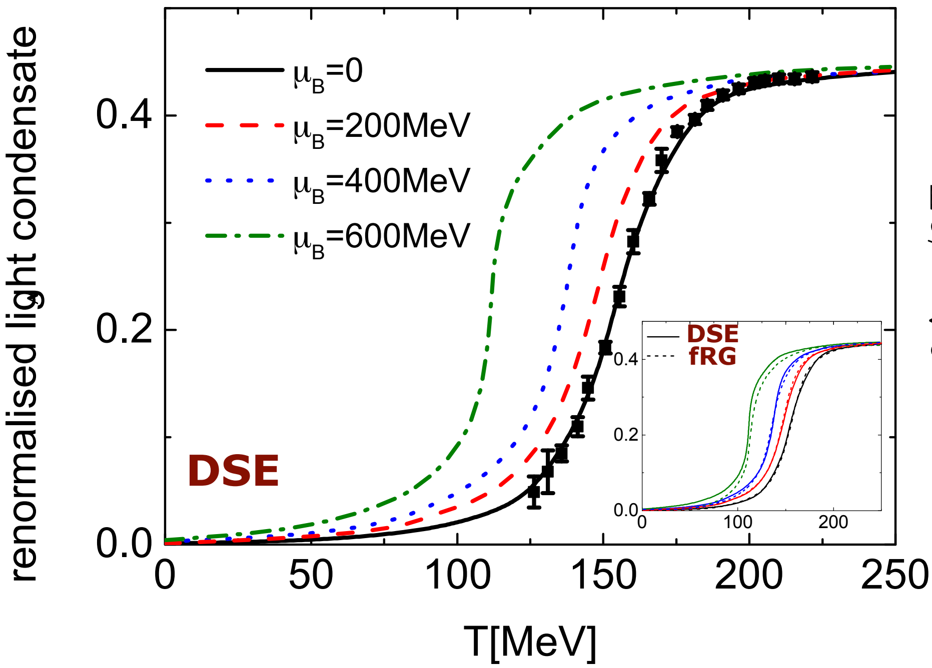
Fischer, Luecker, PLB 718 (2013) 1036
Fischer, Luecker,Welzbacher, PRD 90 (2014) 034022
Isserstedt, Buballa, Fischer, Gunkel, PRD 100 (2019) 074011

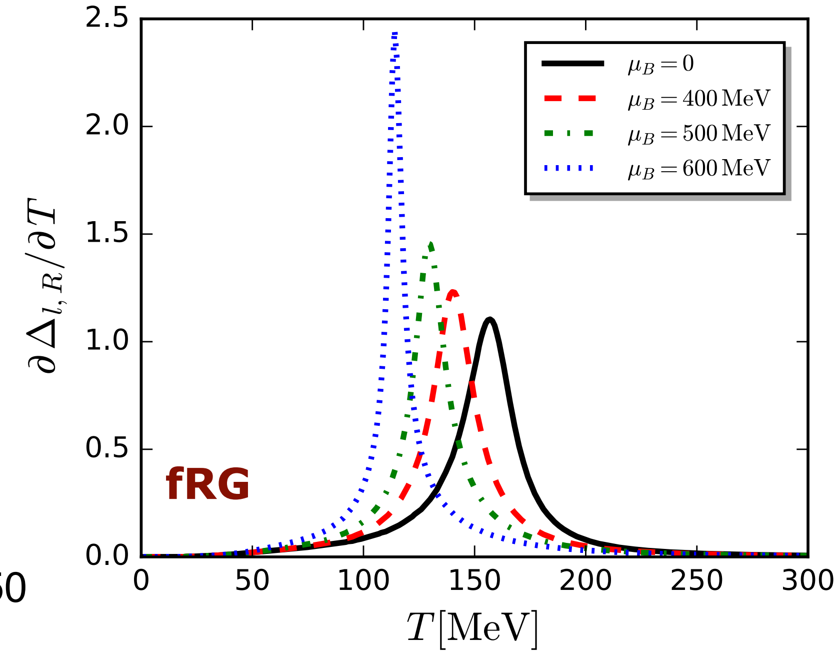
fRG: **Fu, JMP, Rennecke, PRD 101, (2020) 054032**
DSE: **Gao, JMP, arXiv:2010.137005**

QCD at finite density

Chiral order parameter benchmark results at finite T

renormalised condensate



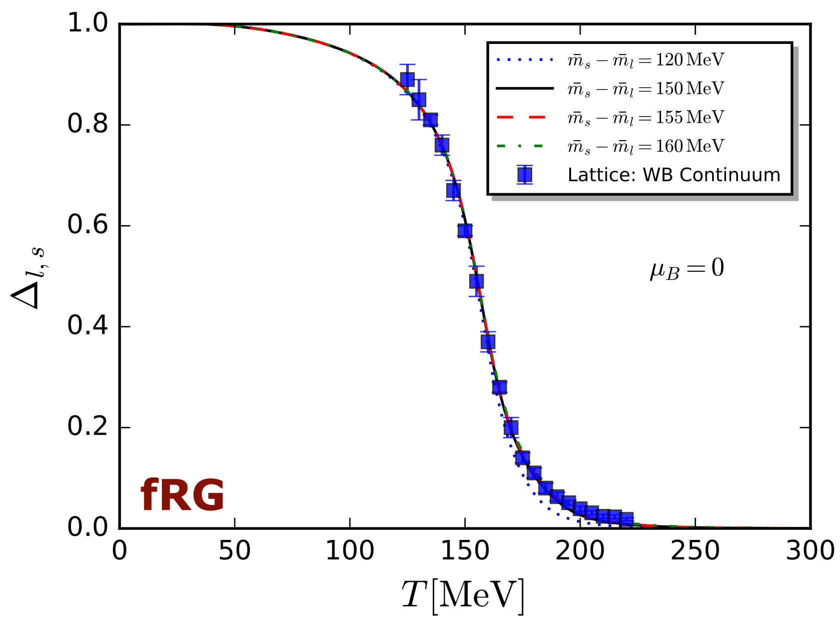


$$\Delta_{l,R}(T,\mu_B) \simeq \Delta_l(T,\mu_B) - \Delta_l(0,0)$$

$$\Delta_q(T,\mu_B) = \frac{T}{\mathcal{V}}\, m_q^0 \int_x \langle \bar{q}(x) q(x) \rangle$$

lattice: S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, C. Ratti, and K. K. Szabo, JHEP 09, 073 (2010)

reduced condensate

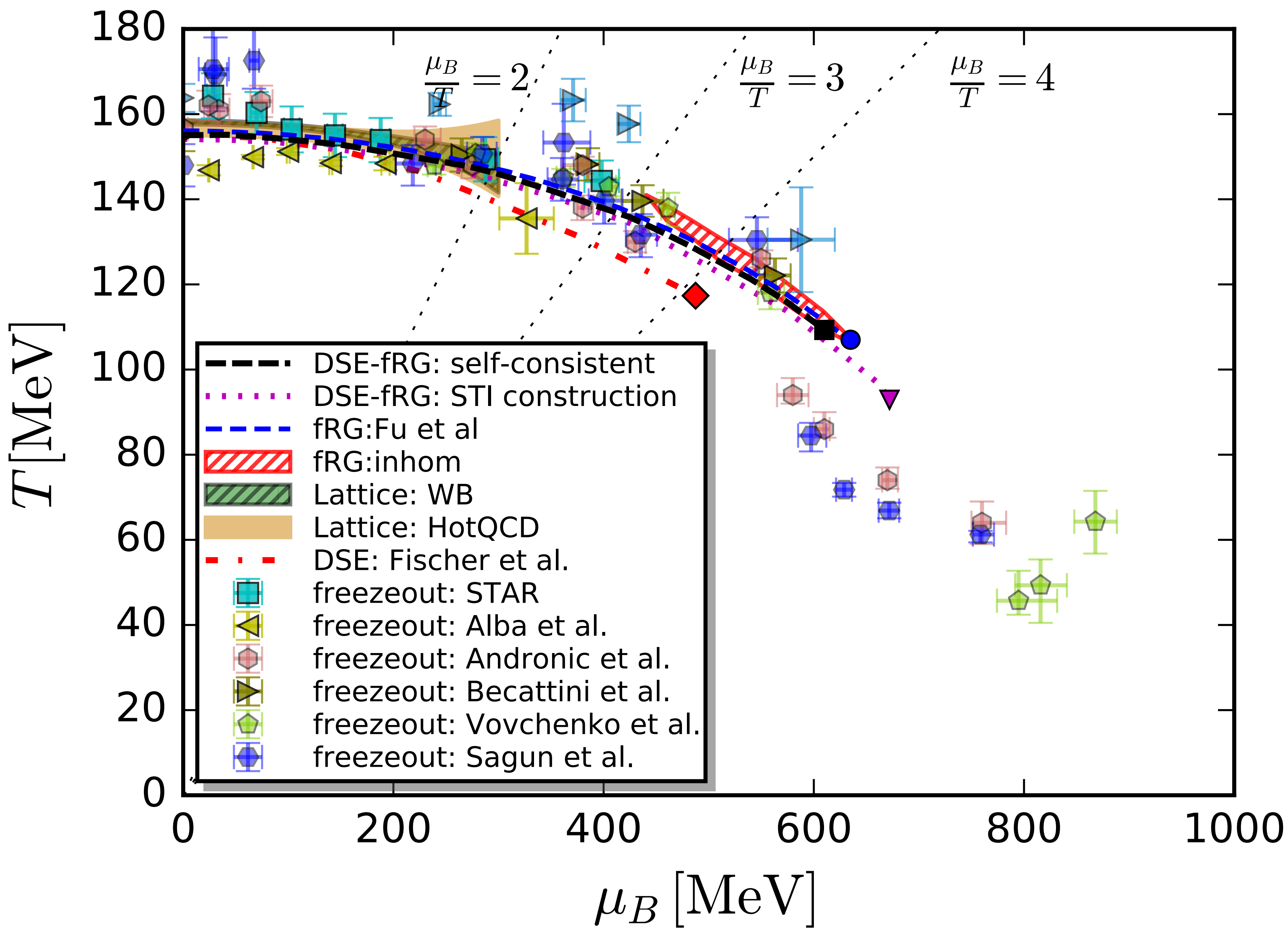


$$\Delta_{l,s}(T,\mu_B) = \frac{\Delta_l(T,\mu_B) - \left(\frac{m_l^0}{m_s^0}\right)^2 \Delta_s(T,\mu_B)}{\Delta_l(0,0) - \left(\frac{m_l^0}{m_s^0}\right)^2 \Delta_s(0,0)}$$

fRG: Fu, JMP, Rennecke, PRD 101, (2020) 054032

DSE: Gao, JMP, arXiv:2010.137005

QCD phase structure



See also

Fischer, Luecker,Welzbacher, PRD 90 (2014) 034022
Isserstedt, Buballa, Fischer, Gunkel, PRD 100 (2019) 074011

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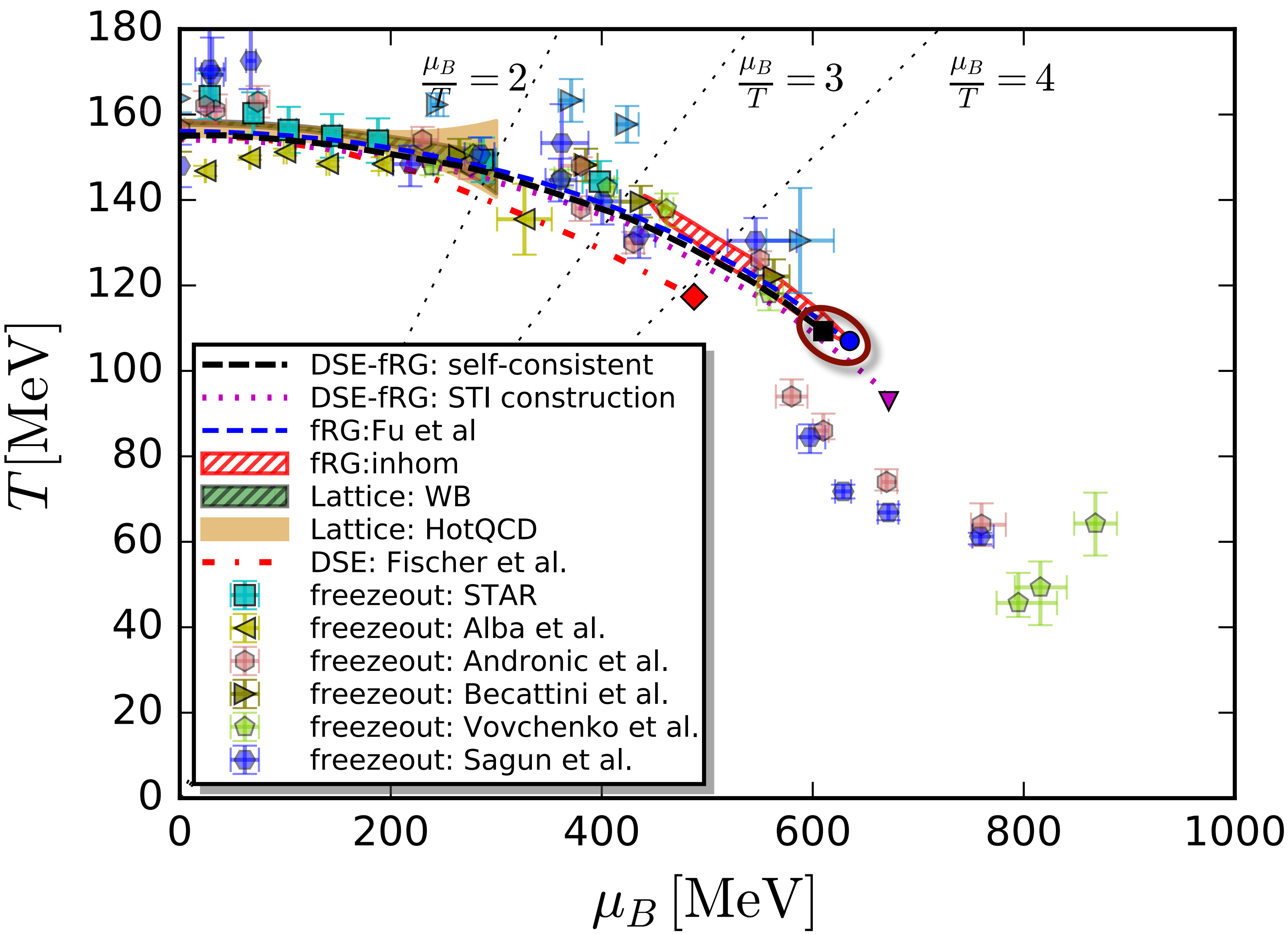
QCD phase structure



CEP fRG-DSE

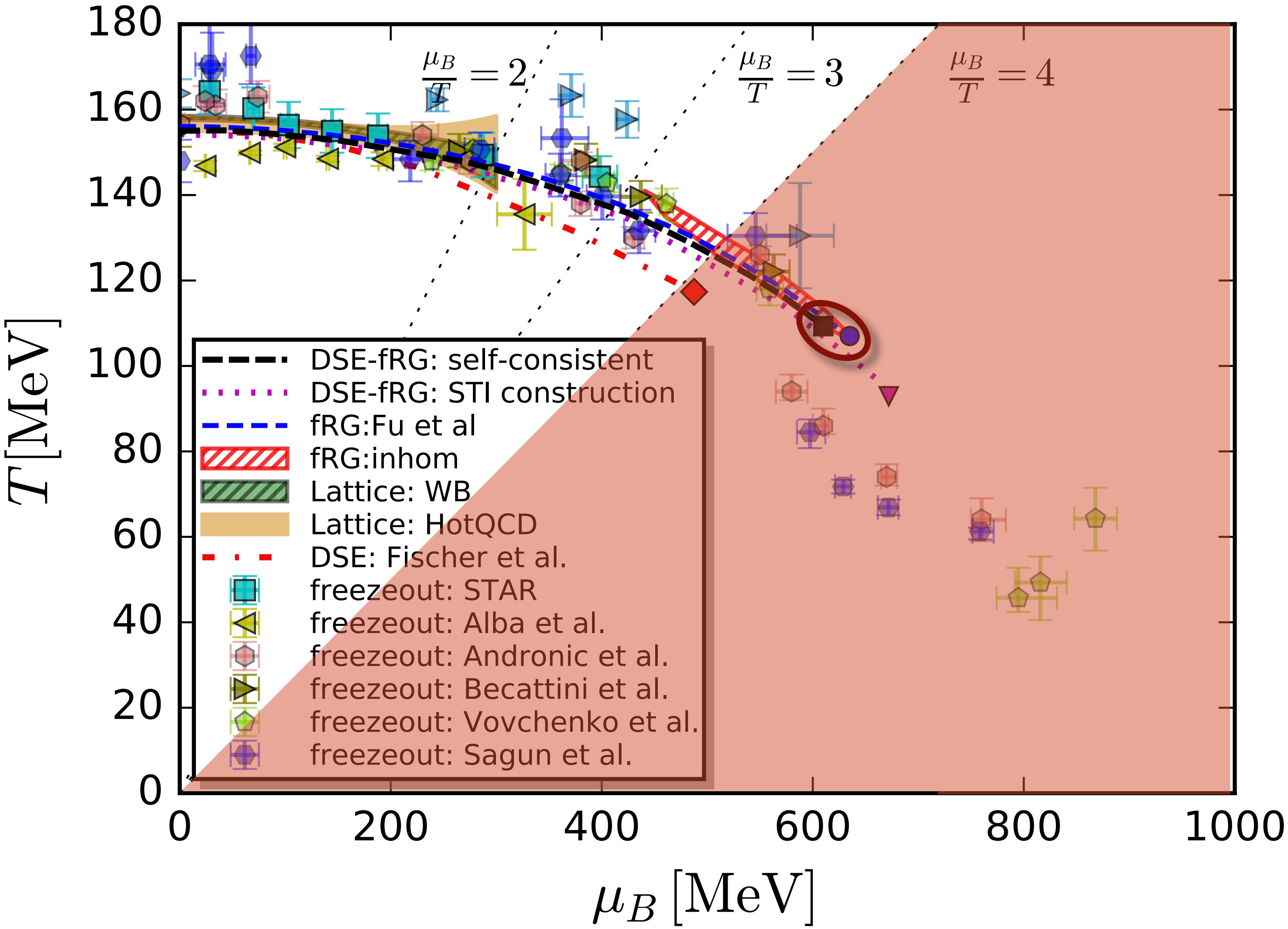
$$(T,\mu_B)_{\rm CEP} = (107, 635)\,\mathrm{MeV}$$

$$(T,\mu_B)_{\rm CEP} = (109, 610)\,\mathrm{MeV}$$

fRG: Fu, JMP, Rennecke, PRD 101, (2020) 054032

DSE: Gao, JMP, arXiv:2010.137005

QCD phase structure



curvature fRG-DSE

$$\kappa_{\mathrm{FRG}} = 0.0142(2)$$

$$\kappa_{\mathrm{DSE}} = 0.0147(5)$$

curvature lattice

$$\kappa_{\mathrm{WB}} = 0.0149(21)$$

WB, PLB 751 (2015) 559

$$\kappa_{\mathrm{hotQCD}} = 0.015(4)$$

hotQCD, PLB 795 (2019) 15

area beyond quantitative reliability bound

$$\mu_{\mathrm{B}}/\mathrm{T} \gtrsim 4$$

CEP fRG-DSE

$$(T, \mu_B)_{\mathrm{CEP}} = (107, 635)\,\mathrm{MeV}$$

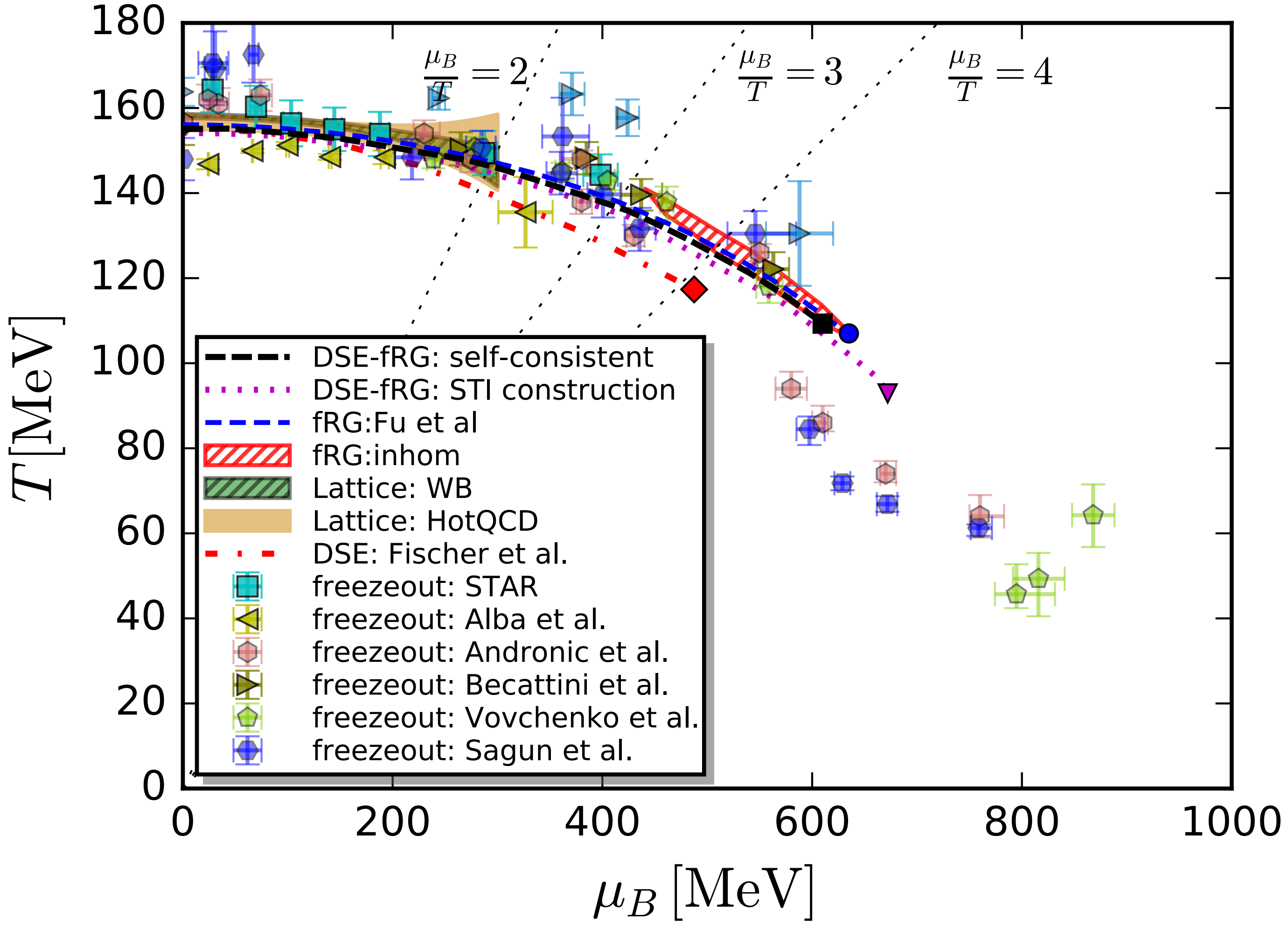
$$(T, \mu_B)_{\mathrm{CEP}} = (109, 610)\,\mathrm{MeV}$$

fRG: Fu, JMP, Rennecke, PRD 101, (2020) 054032

DSE: Gao, JMP, arXiv:2010.137005

QCD phase structure

Reliability considerations



fRG: Fu, JMP, Rennecke, PRD 101, (2020) 054032

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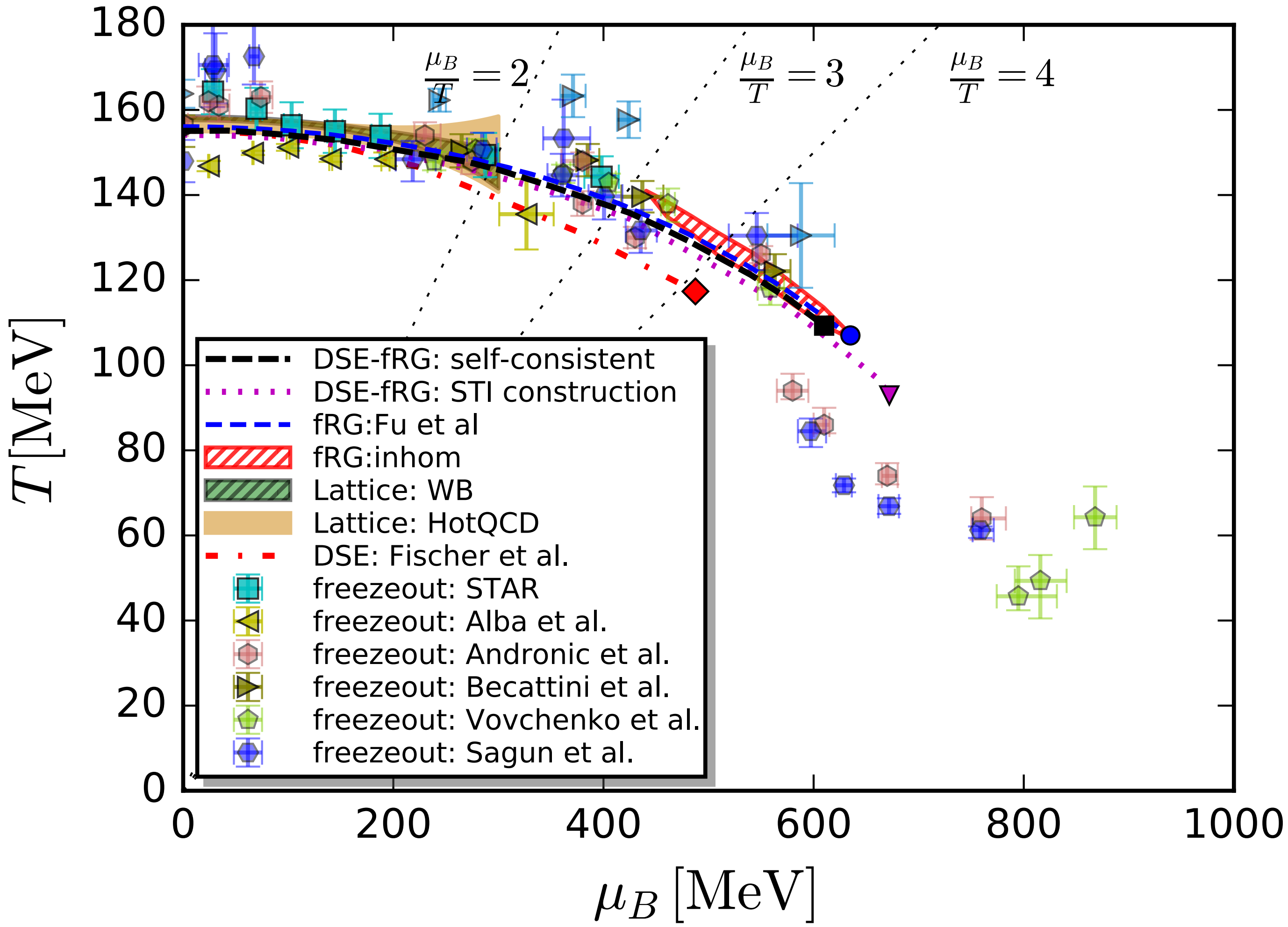
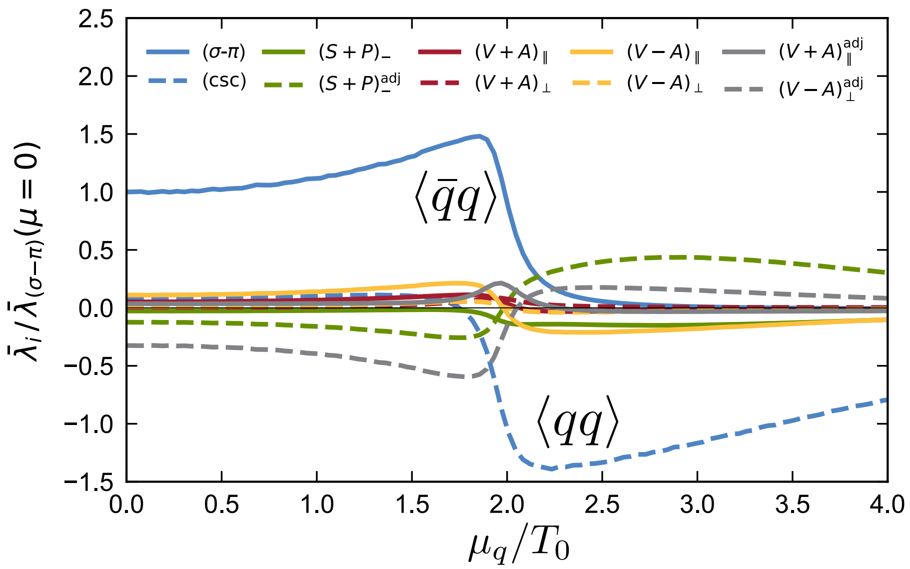
QCD phase structure

Reliability considerations



Dominant channels I (fRG)

Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004

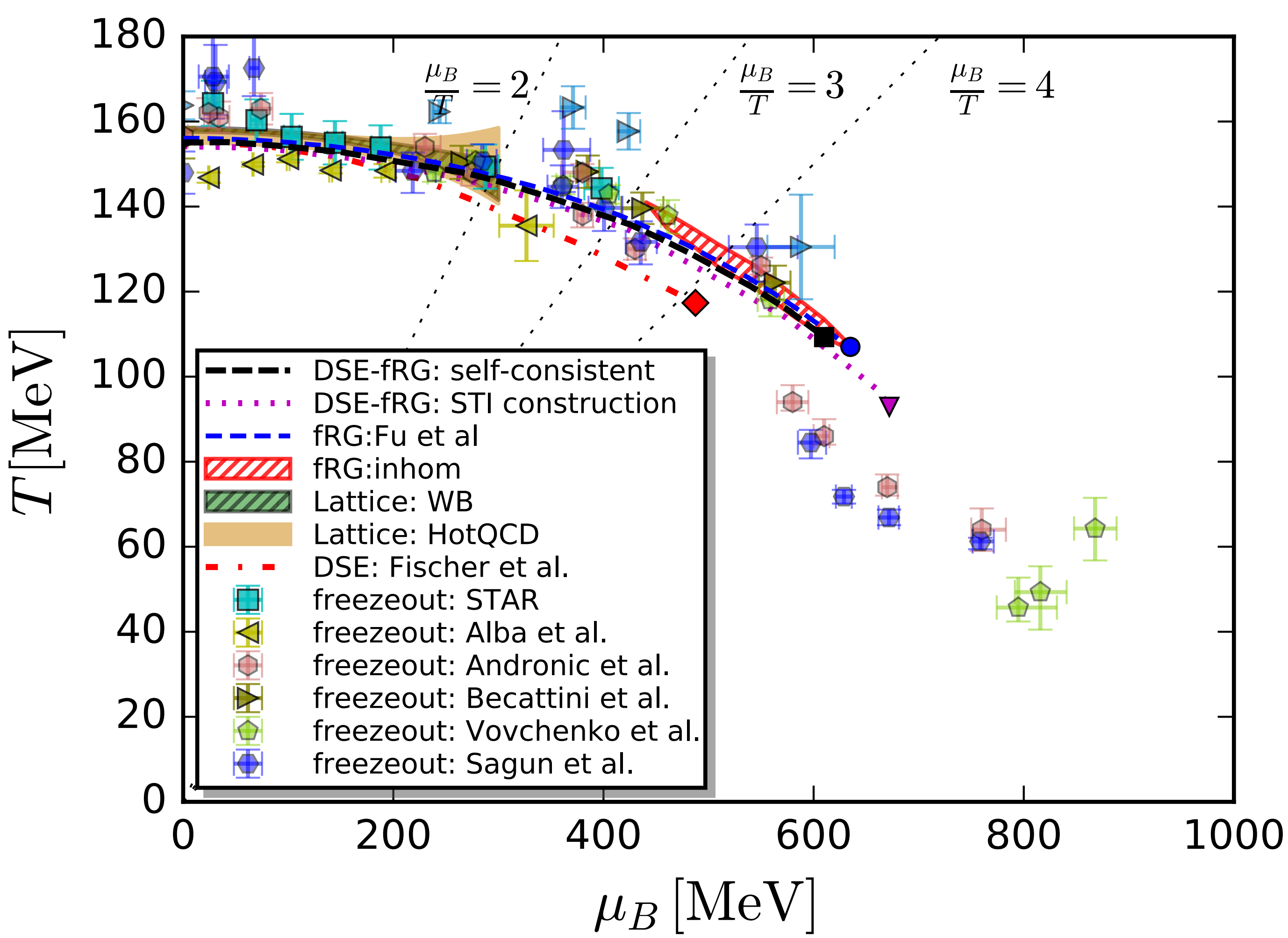


fRG: Fu, JMP, Rennecke, PRD 101, (2020) 054032

DSE: Gao, JMP, arXiv:2010.137005

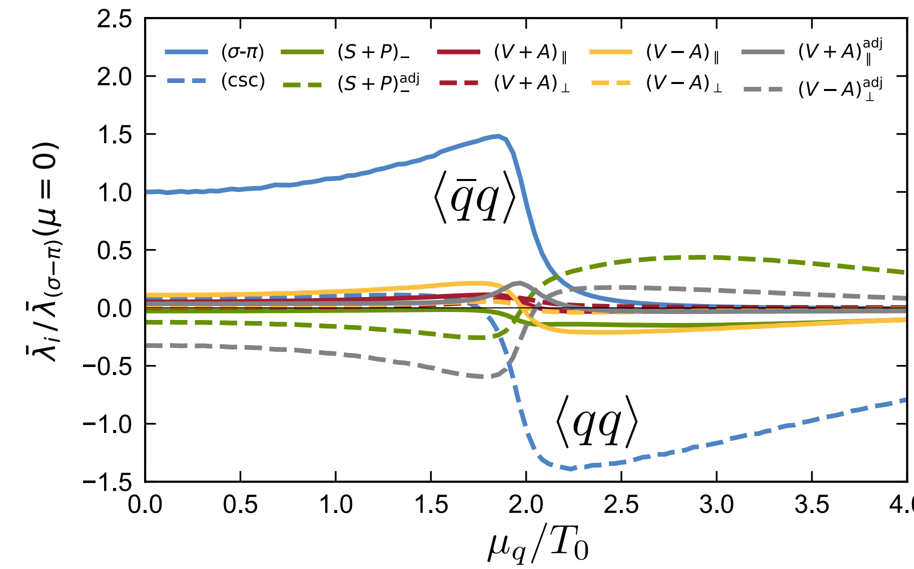
QCD phase structure

Reliability considerations



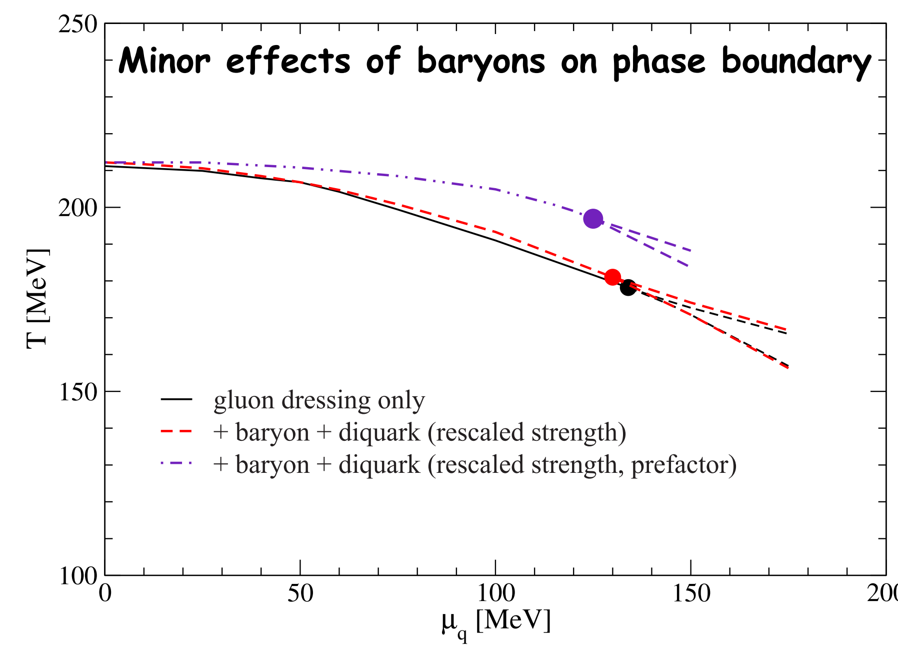
Dominant channels I (fRG)

Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004



Dominant channels II (DSE)

Eichmann, Fischer, Welzbacher, PRD 93 (2016) 034013



fRG: **Fu, JMP, Rennecke, PRD 101, (2020) 054032**

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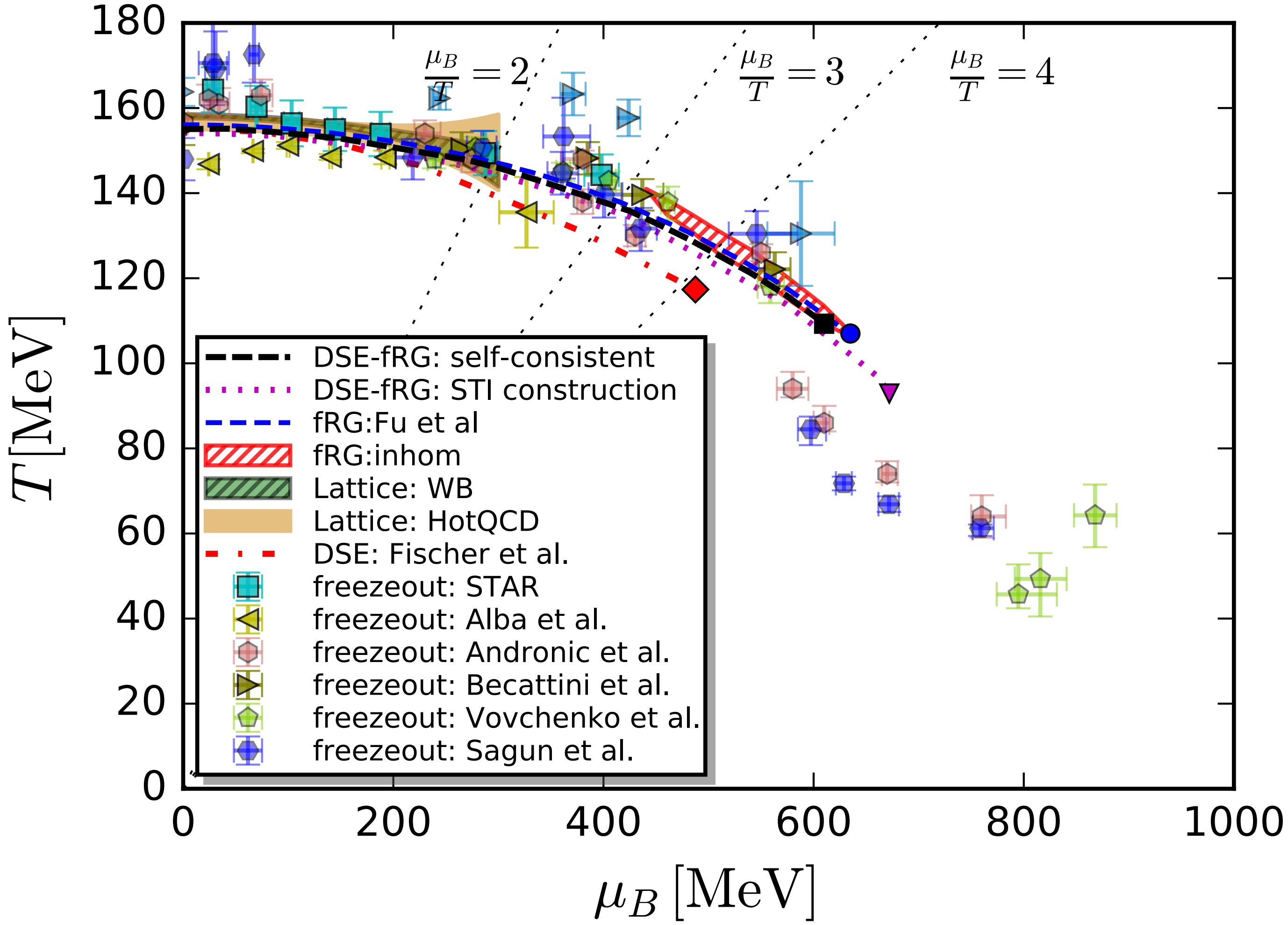
QCD phase structure

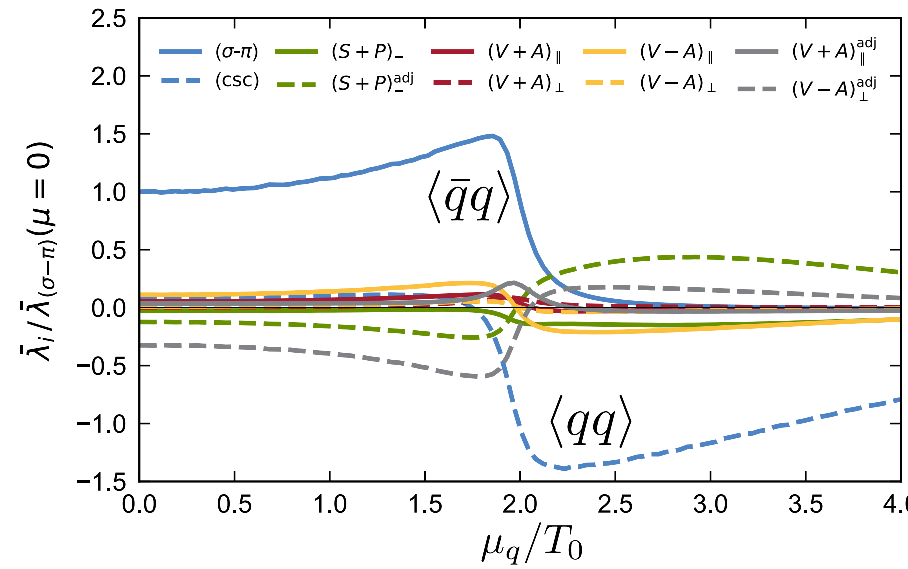
Reliability considerations



I+II → Fierz-complete computation

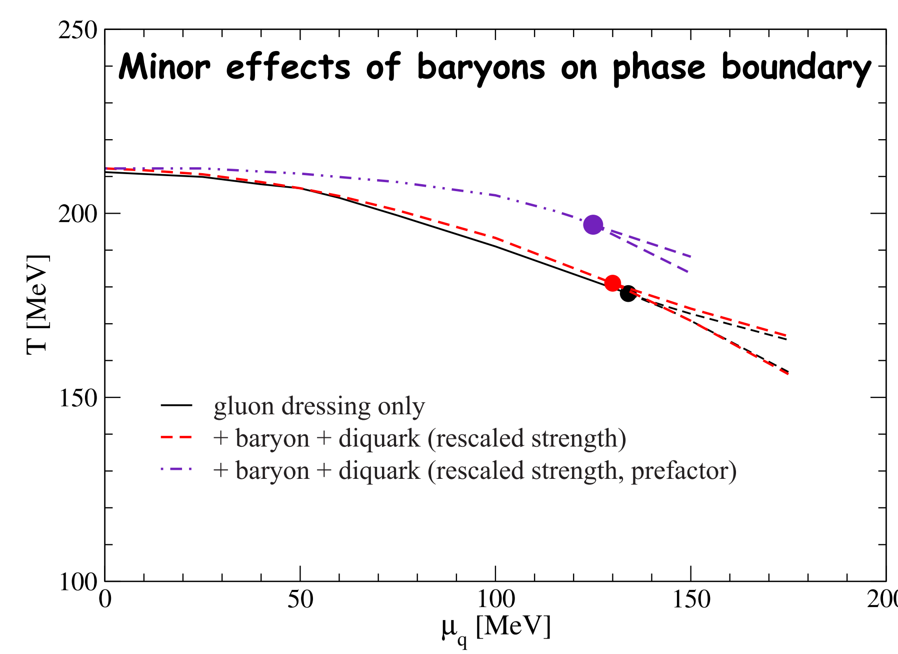
Dominant channels I (fRG)

Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004



Dominant channels II (DSE)

Eichmann, Fischer, Welzbacher, PRD 93 (2016) 034013



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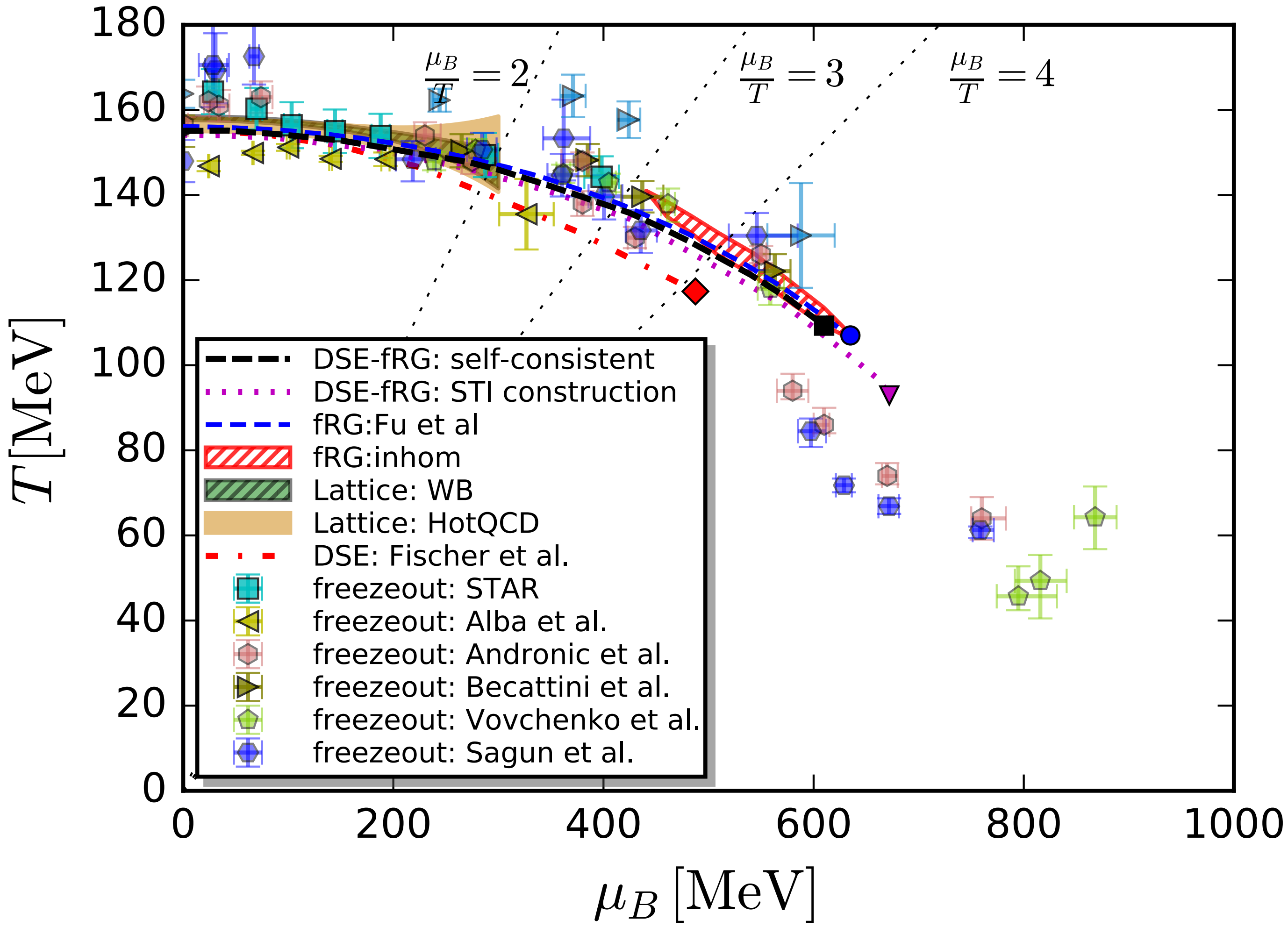
QCD phase structure

Reliability considerations



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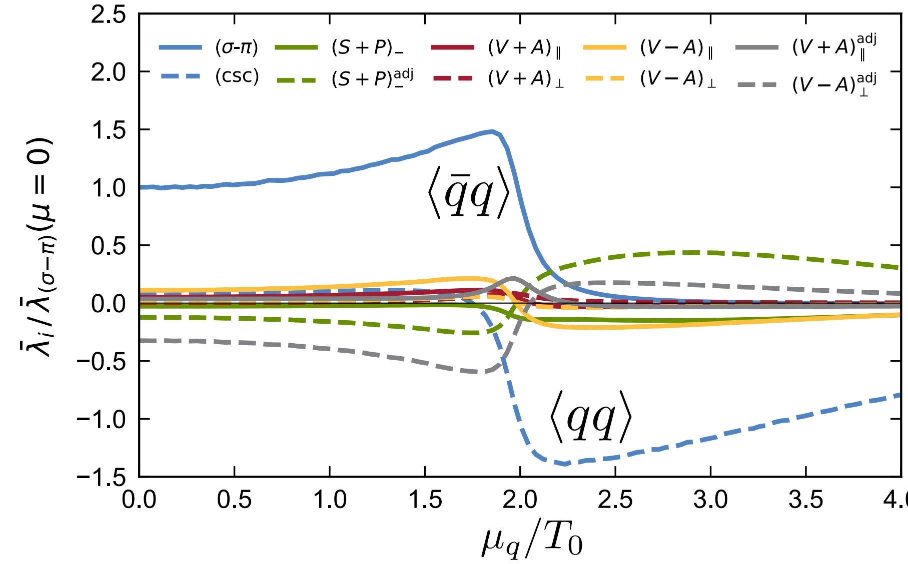
Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004



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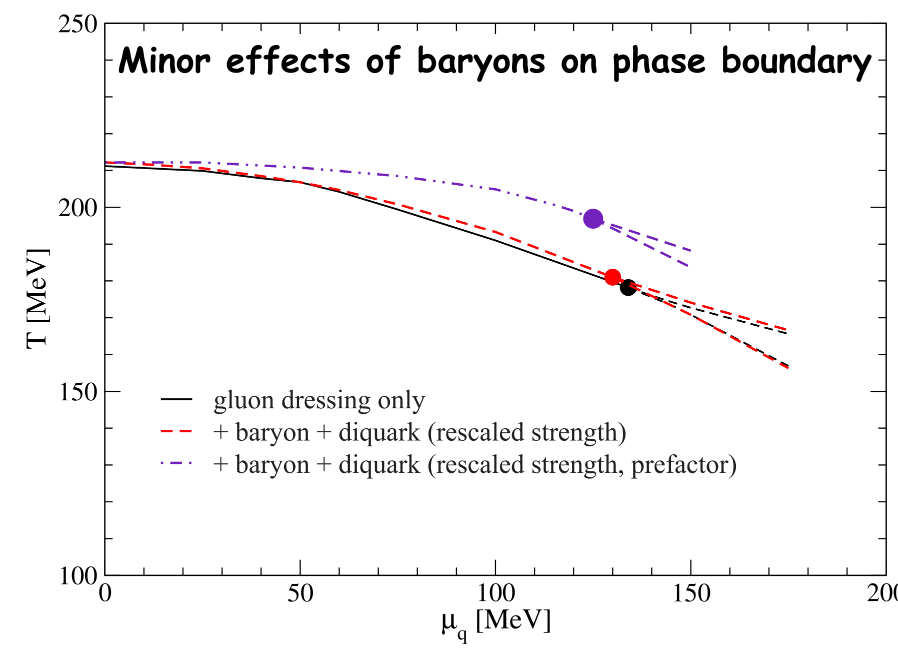


'Inhomogeneous' phase

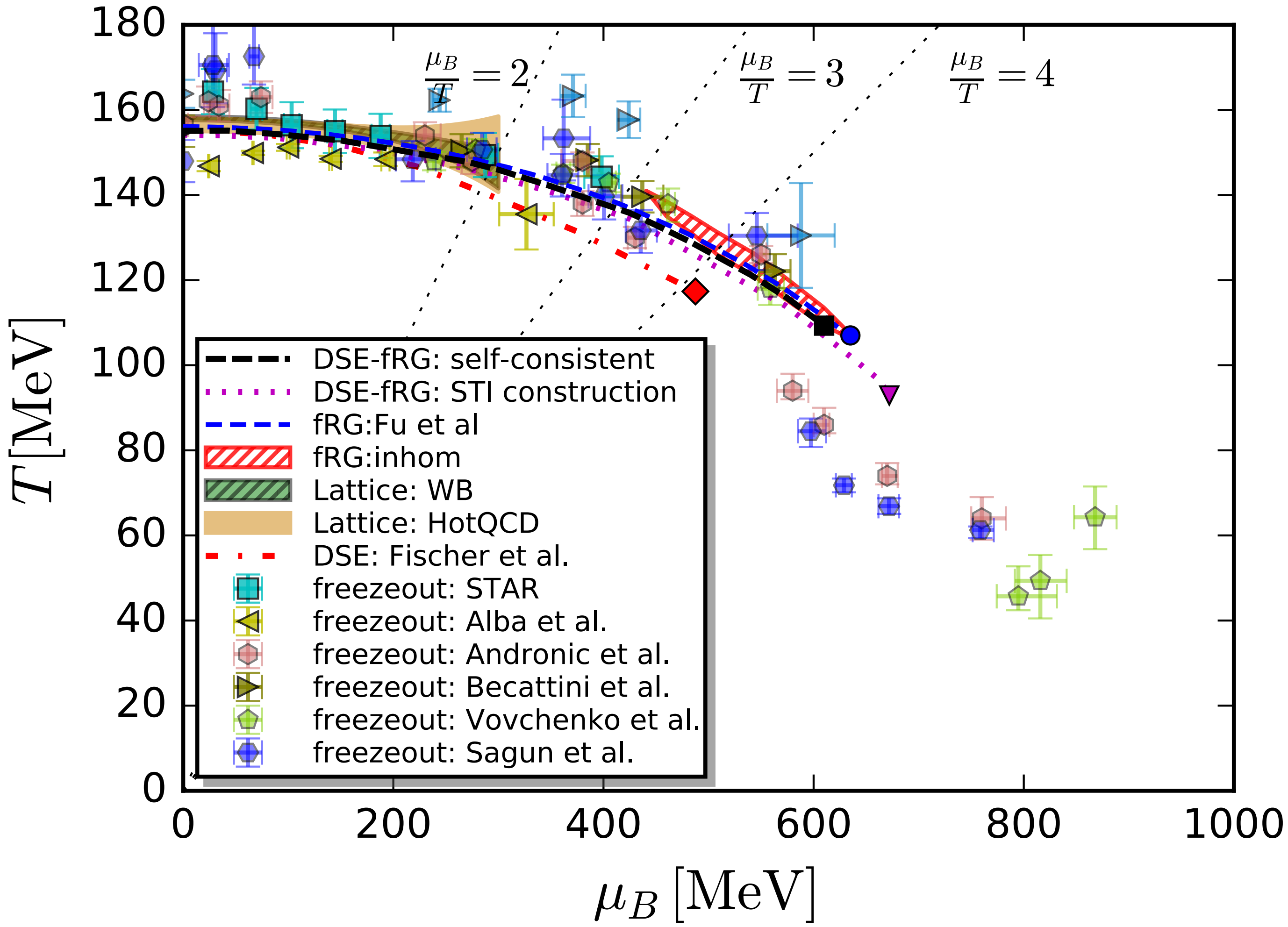
 : Pion dispersion has minimum at non-vanishing spatial momentum

fRG: Fu, JMP, Rennecke, PRD 101, (2020) 054032

DSE: Gao, JMP, arXiv:2010.137005

QCD phase structure

Reliability considerations



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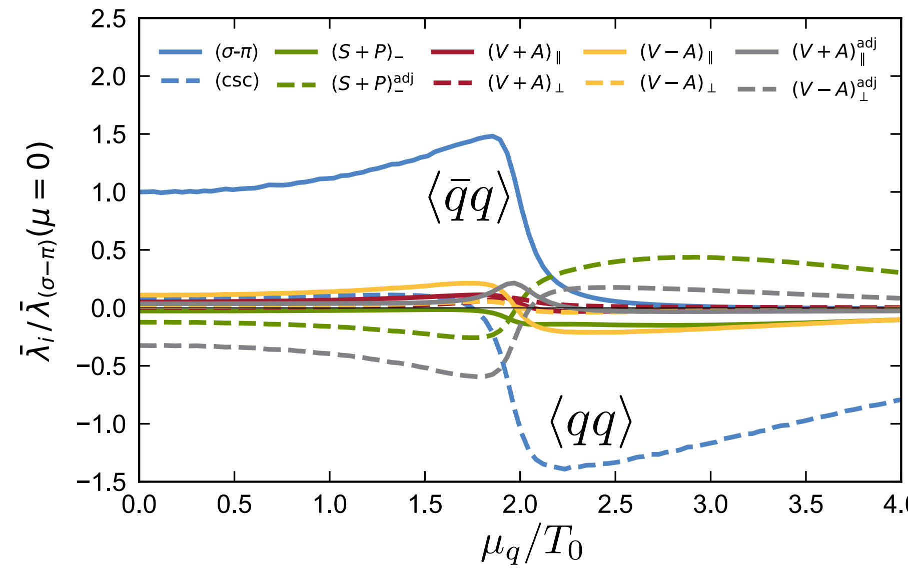
Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004



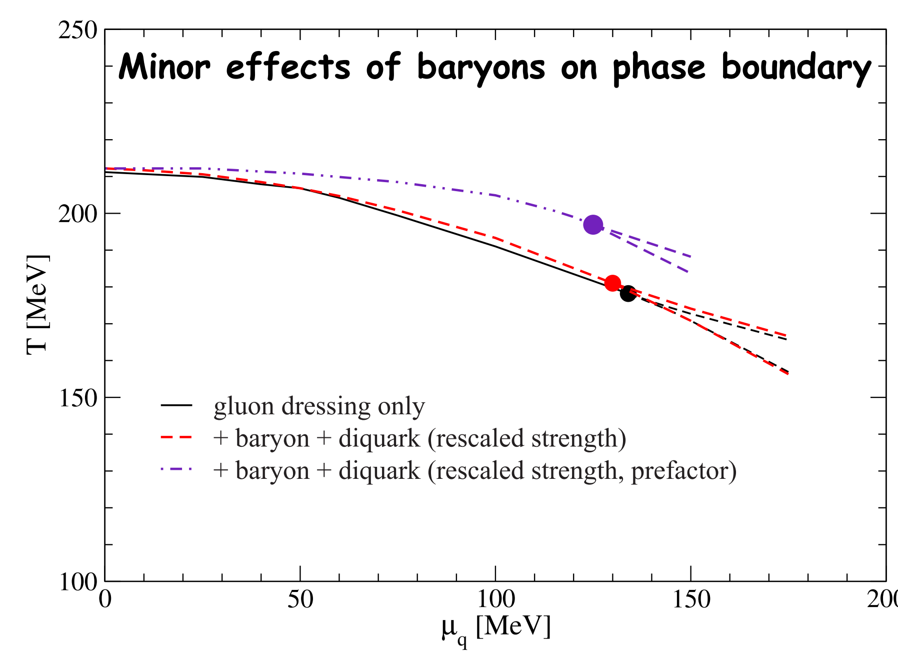
'Inhomogeneous' phase

: Pion dispersion has minimum at non-vanishing spatial momentum

→ Non-trivial background

Dominant channels II (DSE)

Eichmann, Fischer, Welzbacher, PRD 93 (2016) 034013

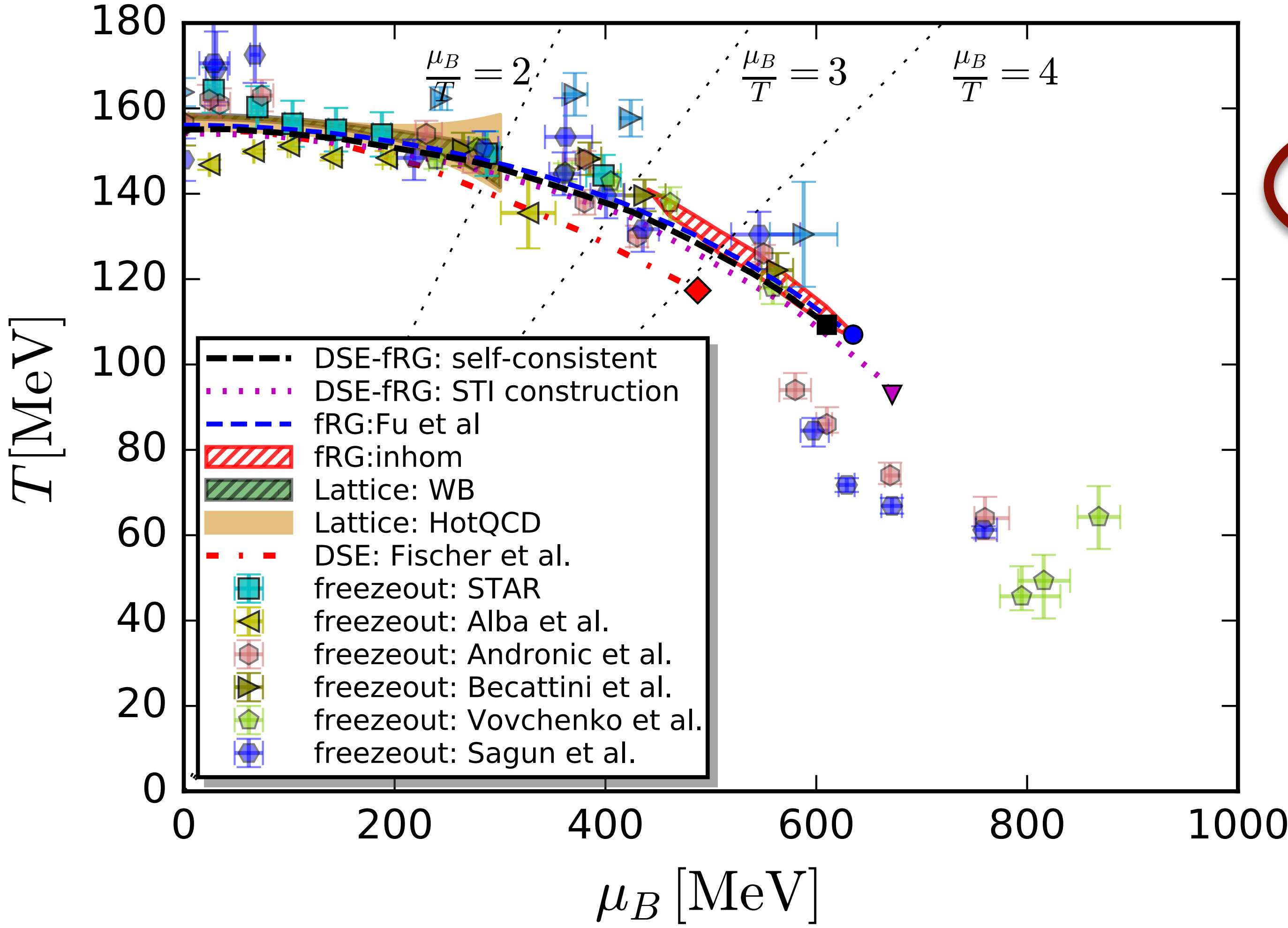


fRG: Fu, JMP, Rennecke, PRD 101, (2020) 054032

DSE: Gao, JMP, arXiv:2010.137005

QCD phase structure

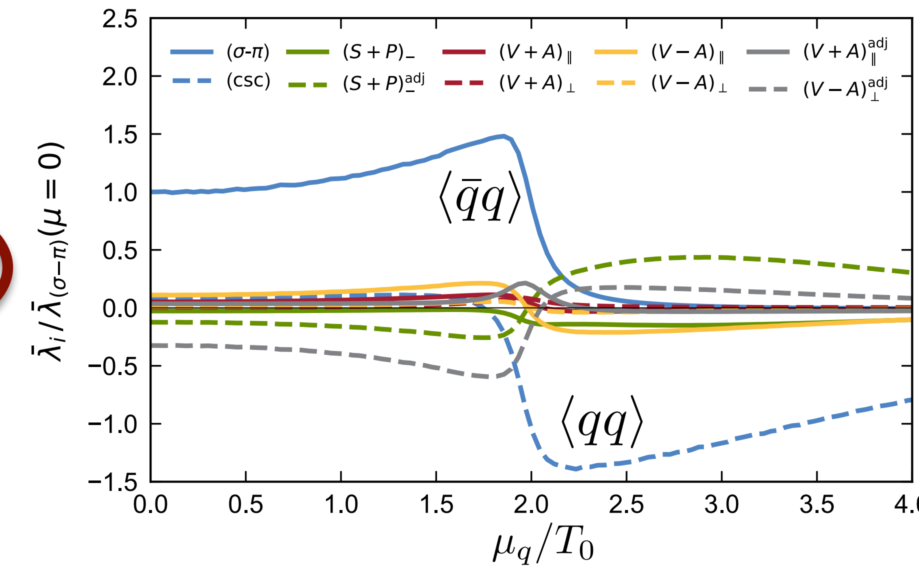
Reliability considerations



$\mu_\mathbf{B}/\mathbf{T} \lesssim 4$

Dominant channels I (fRG)

Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004



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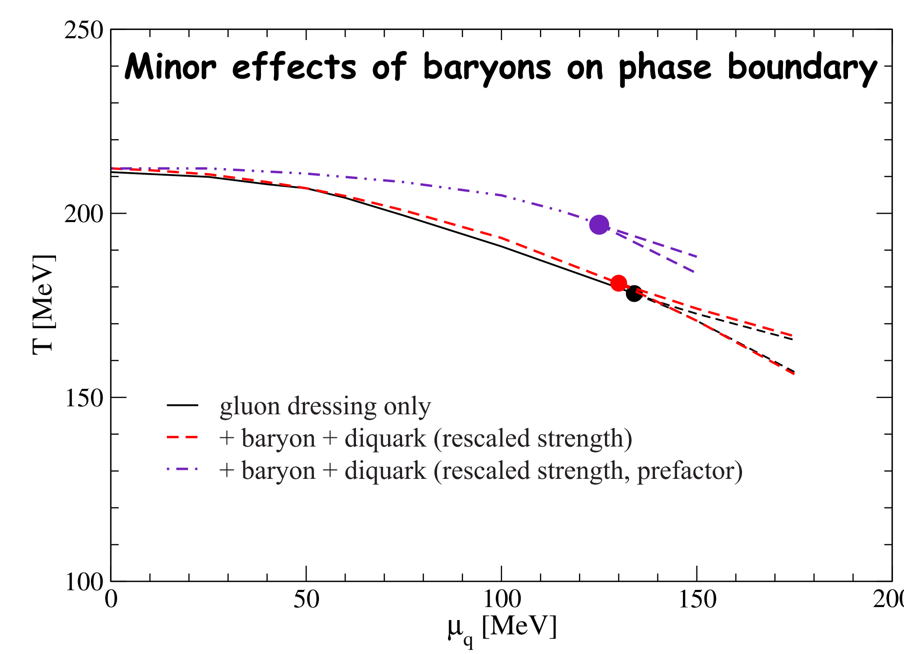
Eichmann, Fischer, Welzbacher, PRD 93 (2016) 034013



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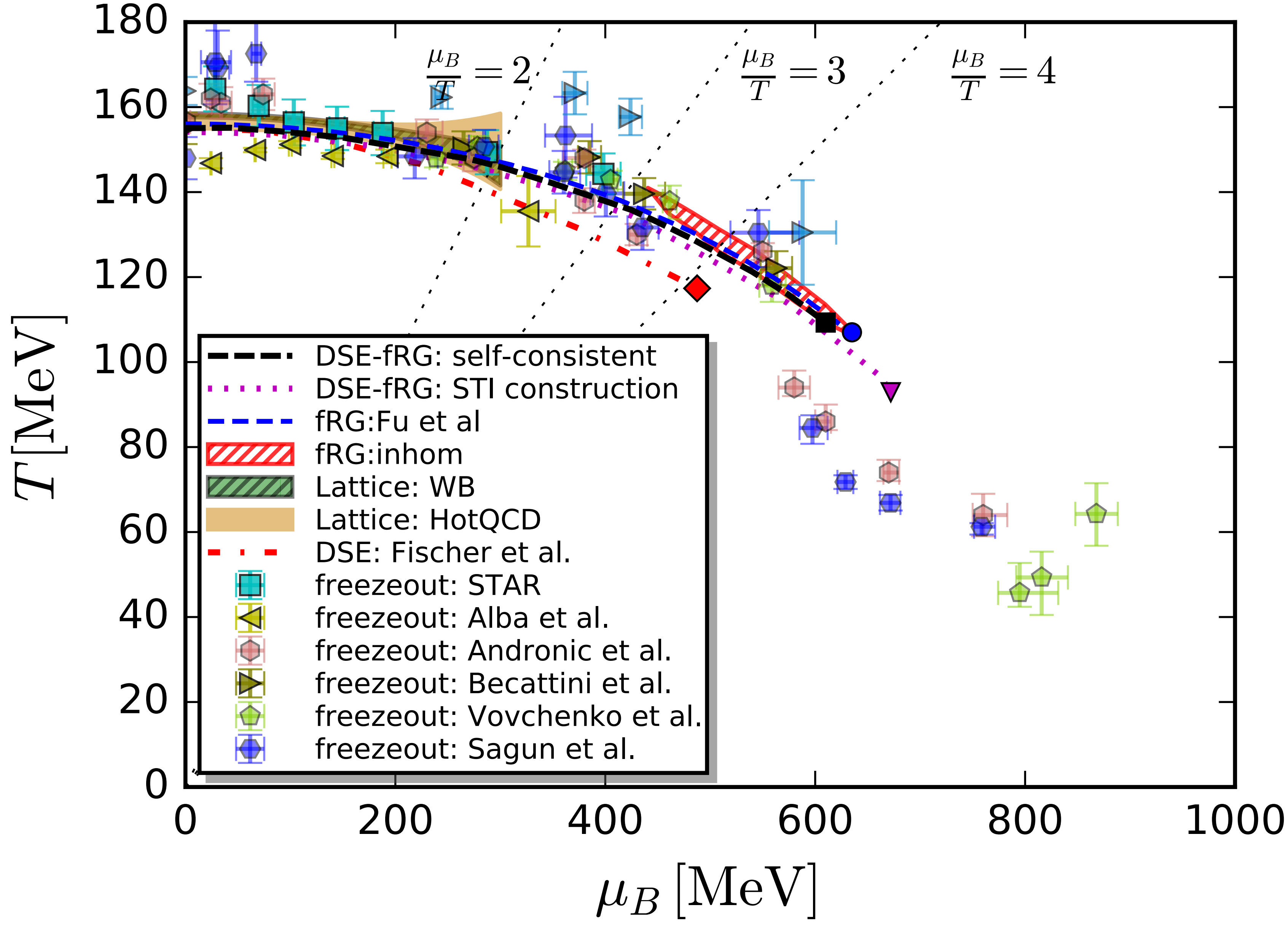
QCD phase structure

Estimate for CEP



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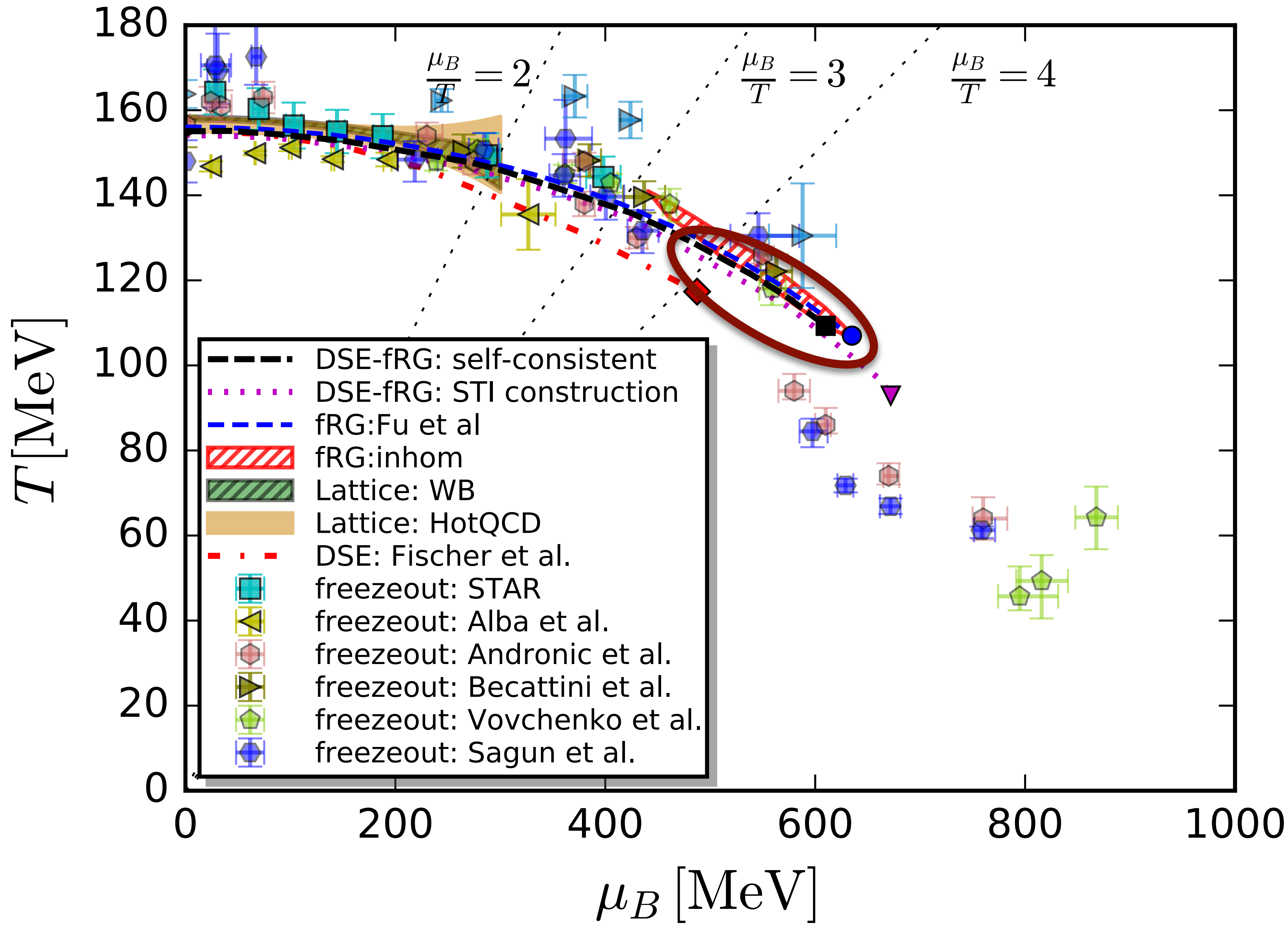
QCD phase structure

Estimate for CEP



CEP-estimate fRG-DSE

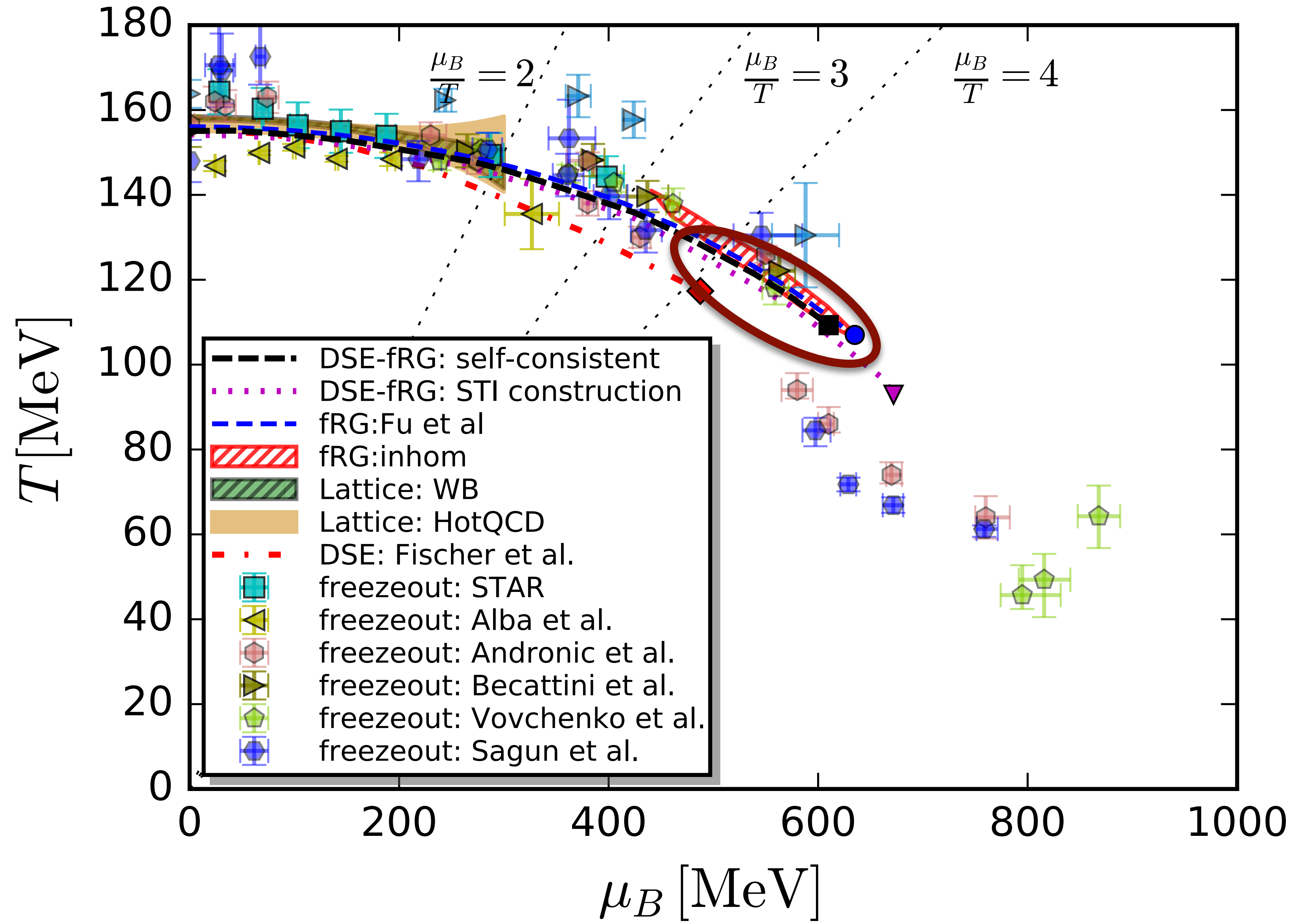
$$(135\,,\;450\,)\,\mathrm{MeV} \lesssim (T_{\mathrm{CEP}}\;,\;\mu_{B_{\mathrm{CEP}}}) \lesssim (100\,,\;650)\,\mathrm{MeV}$$

fRG: Fu, JMP, Rennecke, PRD 101, (2020) 054032

DSE: Gao, JMP, arXiv:2010.137005

QCD phase structure

Estimate for CEP



Stay tuned!

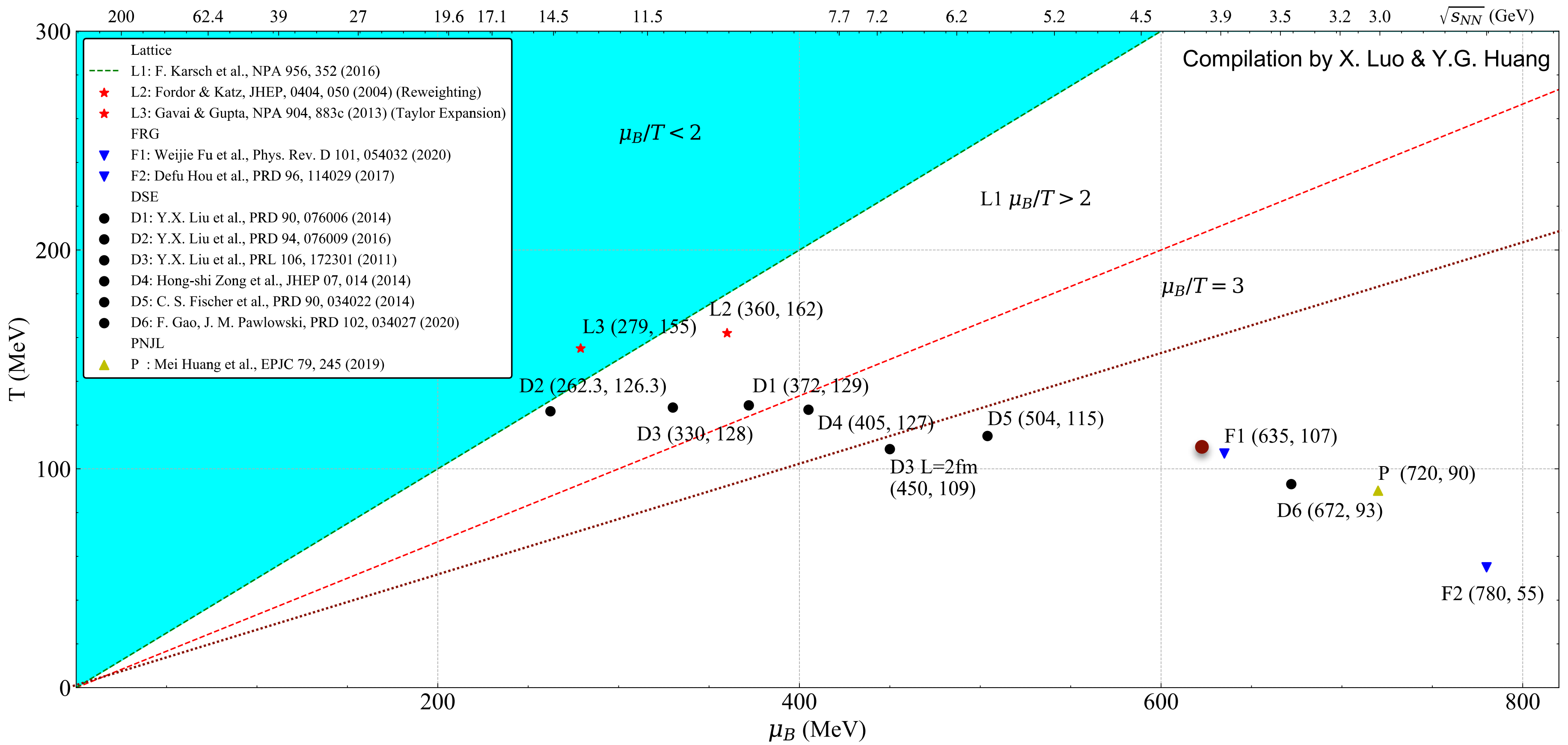
CEP-estimate fRG-DSE

$$(135\,,\ 450\,)\,\mathrm{MeV} \lesssim (T_{\rm CEP}\,,\ \mu_{B_{\rm CEP}}) \lesssim (100\,,\ 650)\,\mathrm{MeV}$$

fRG: Fu, JMP, Rennecke, PRD 101, (2020) 054032
DSE: Gao, JMP, arXiv:2010.137005

Location of CP : Theoretical Prediction

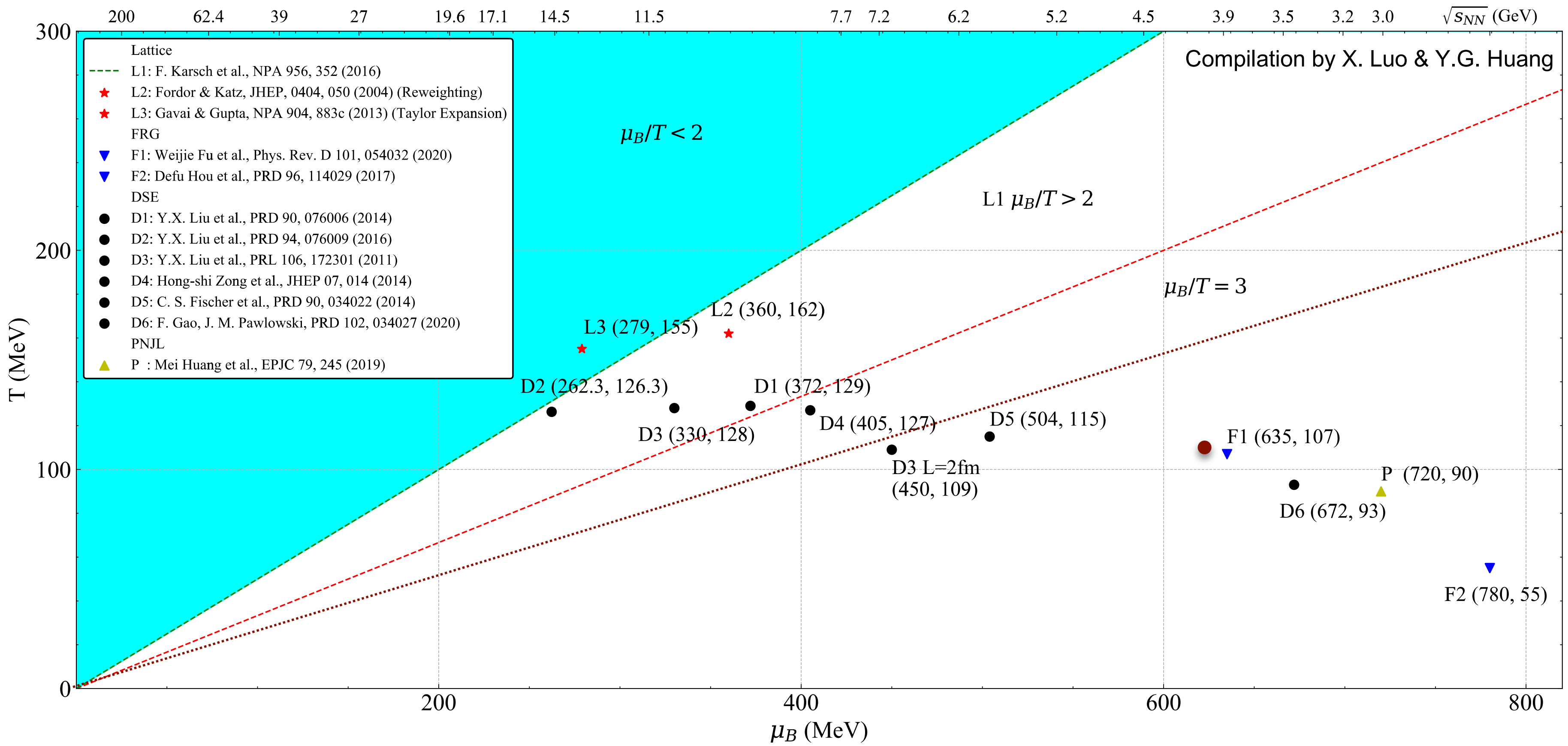
Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)



Large uncertainties for the estimation of CP location.

Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)



Large uncertainties for the estimation of CP location.

Disclaimer

Most functional computations (LEFT or QCD) have not been set-up for CEP-predictions!

Location of CP : Theoretical Prediction

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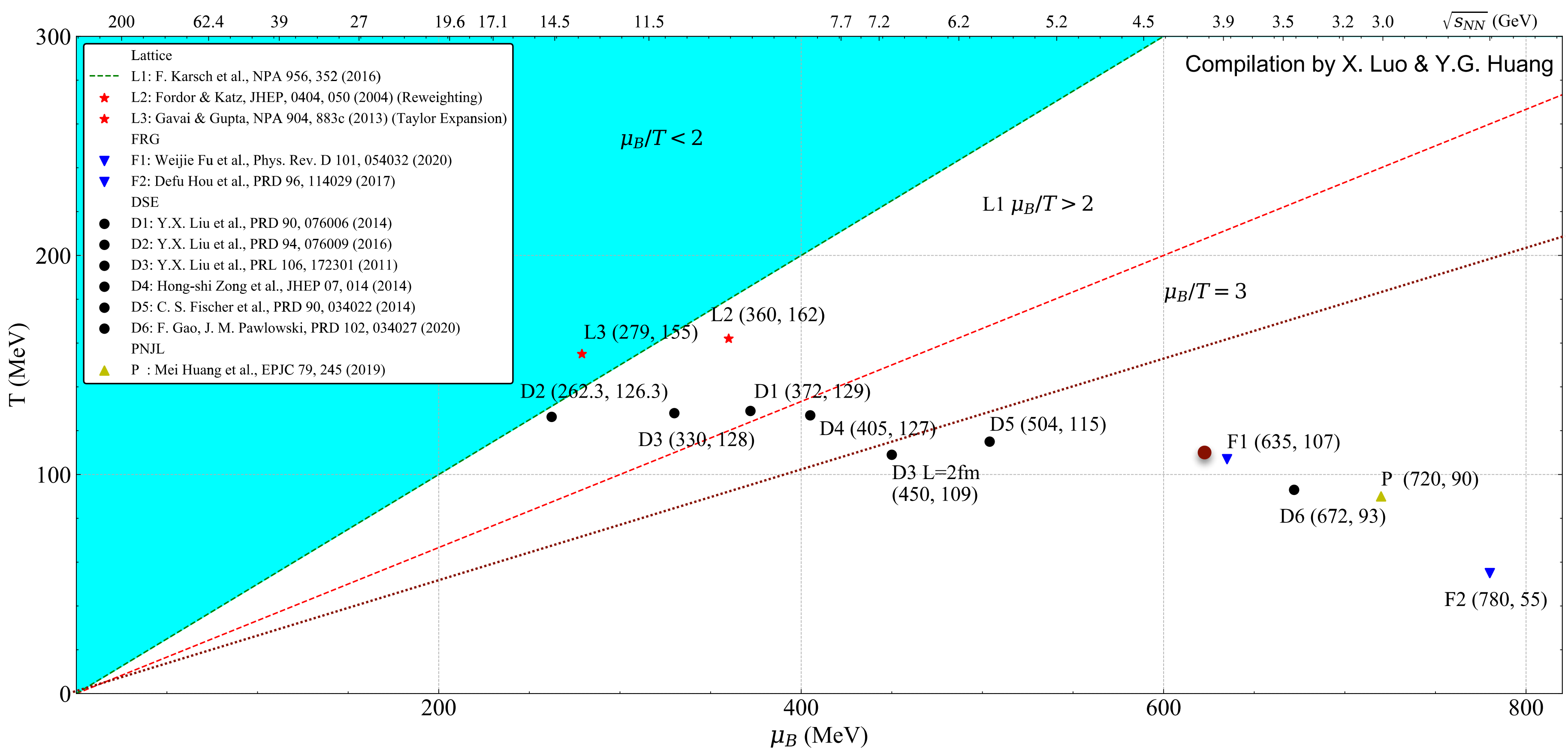

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Lack of predictive power for CEP-predictions is no quality measure!

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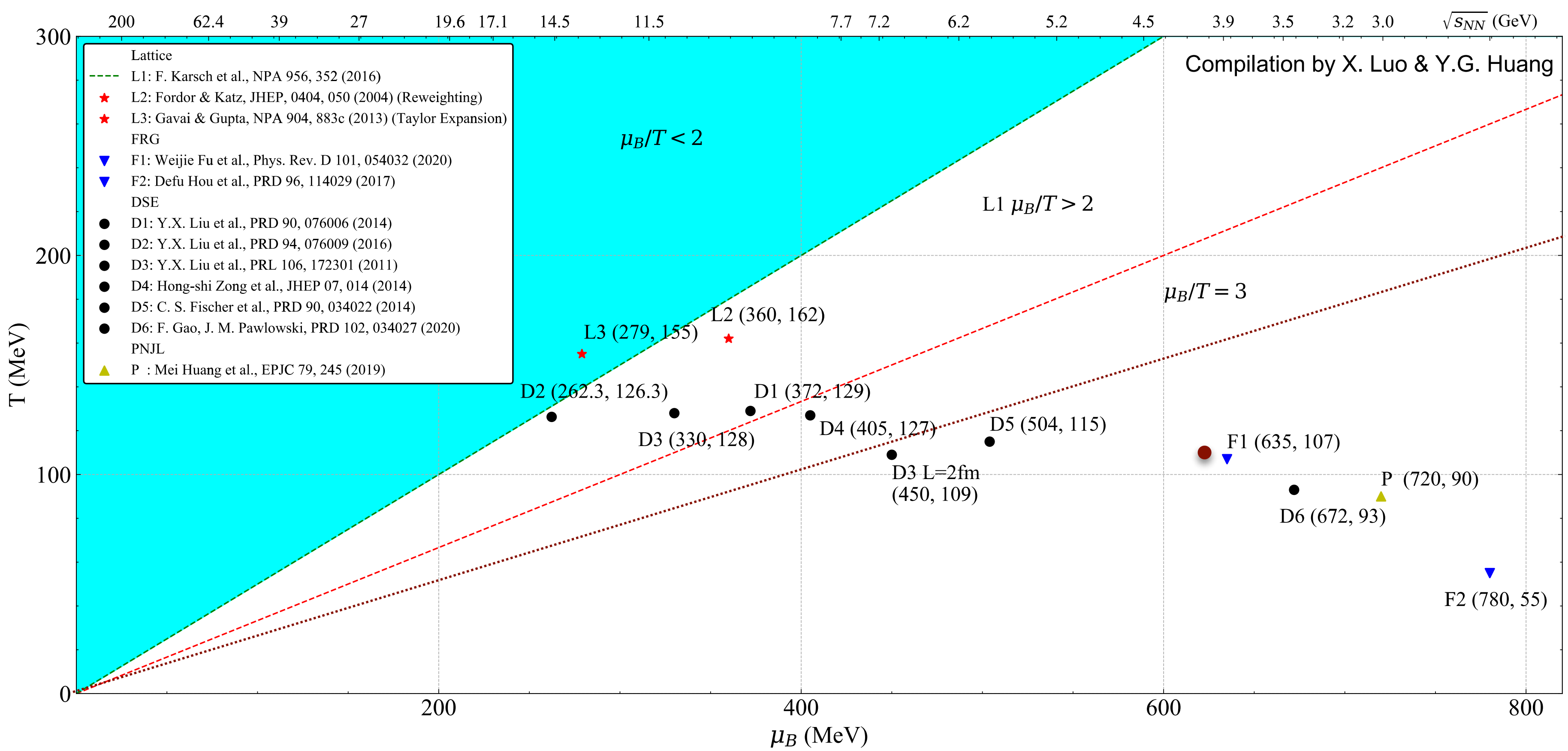

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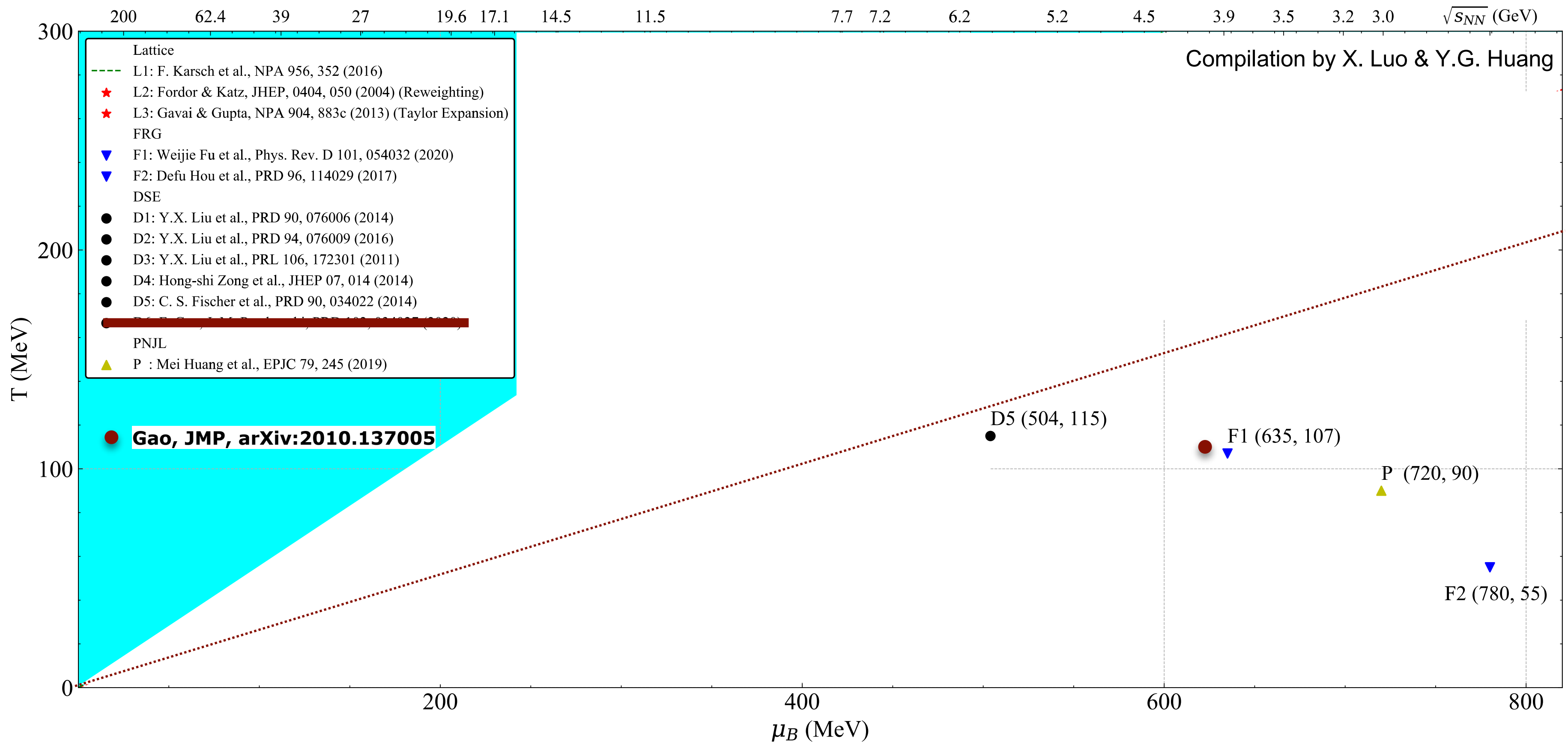
Remove for CEP-predictions:

Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)



Large uncertainties for the estimation of CP location.

Disclaimer

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Lack of predictive power for CEP-predictions is no quality measure!

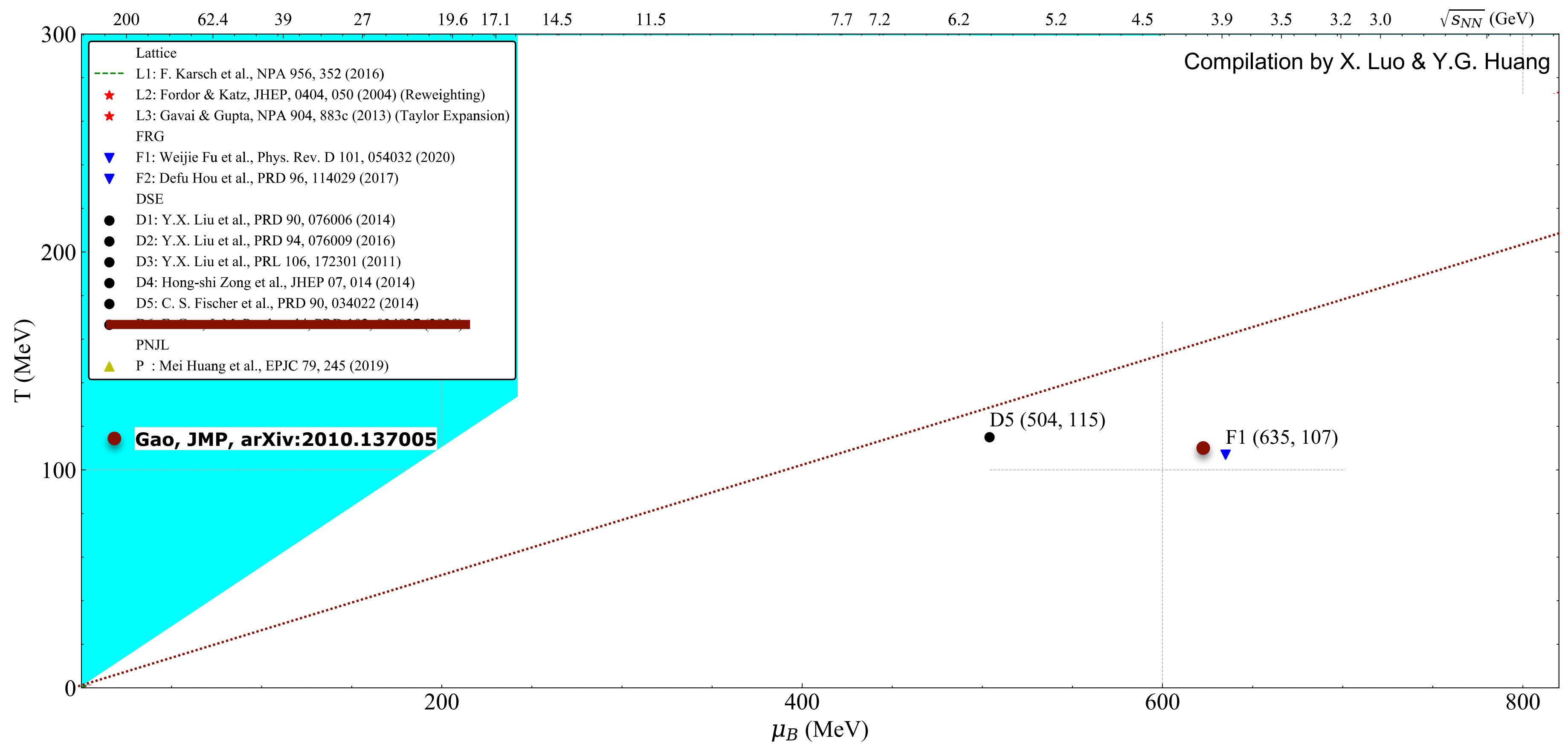
Remove for CEP-predictions:

(i) 'old' CEPs: lattice, Functional QCD approaches, LEFTS (updated computations available)

(ii) LEFTs & Functional Results (qualitative approximations) that miss lattice benchmarks at μ_B=0

Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)



Large uncertainties for the estimation of CP location.

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Remove for CEP-predictions:

(i) 'old' CEPs: lattice, Functional QCD approaches, LEFTS (updated computations available)

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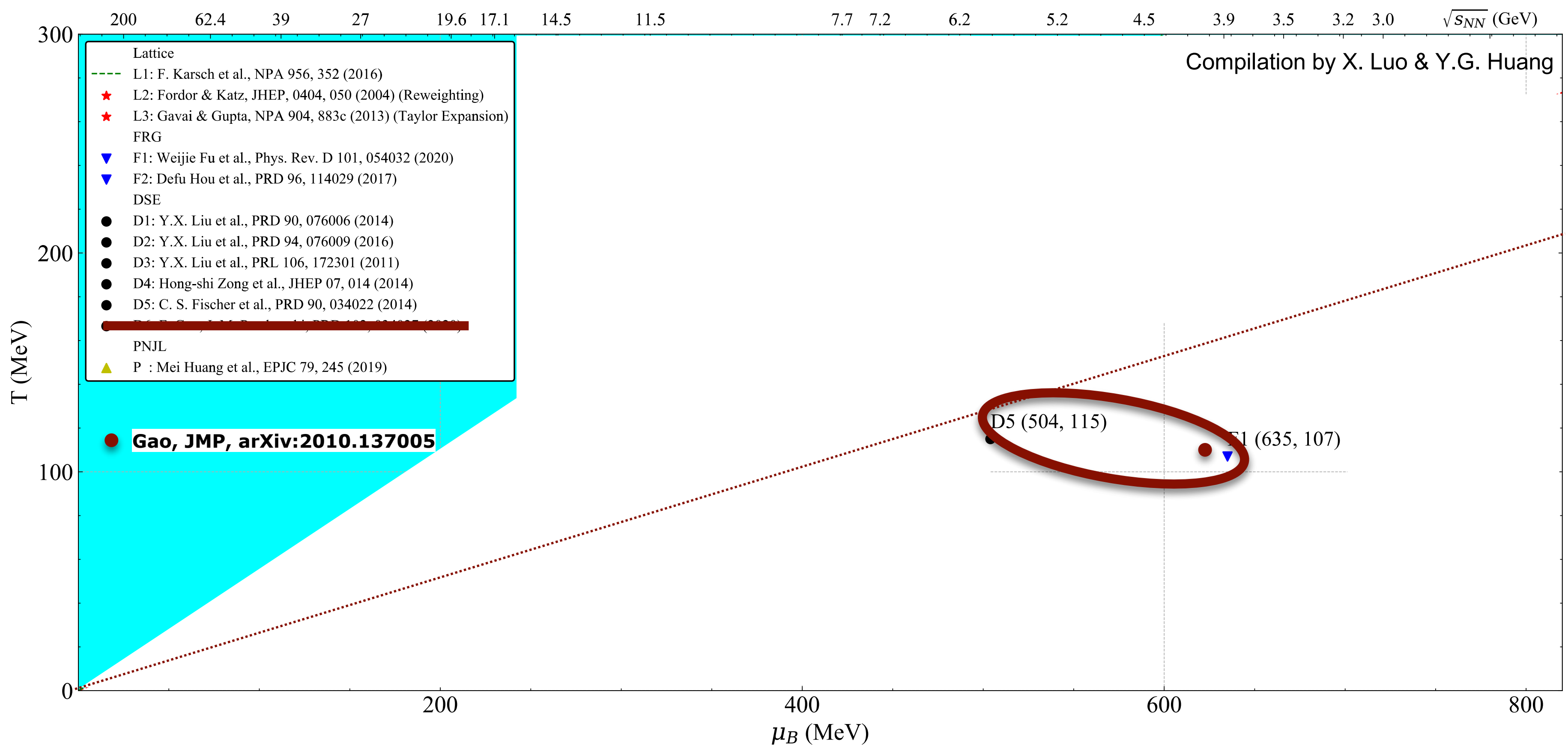
(iii) LEFTs with CEPs at large density (missing quark-gluon back reaction)

Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)



Still uncertainties for the estimation of CP location.

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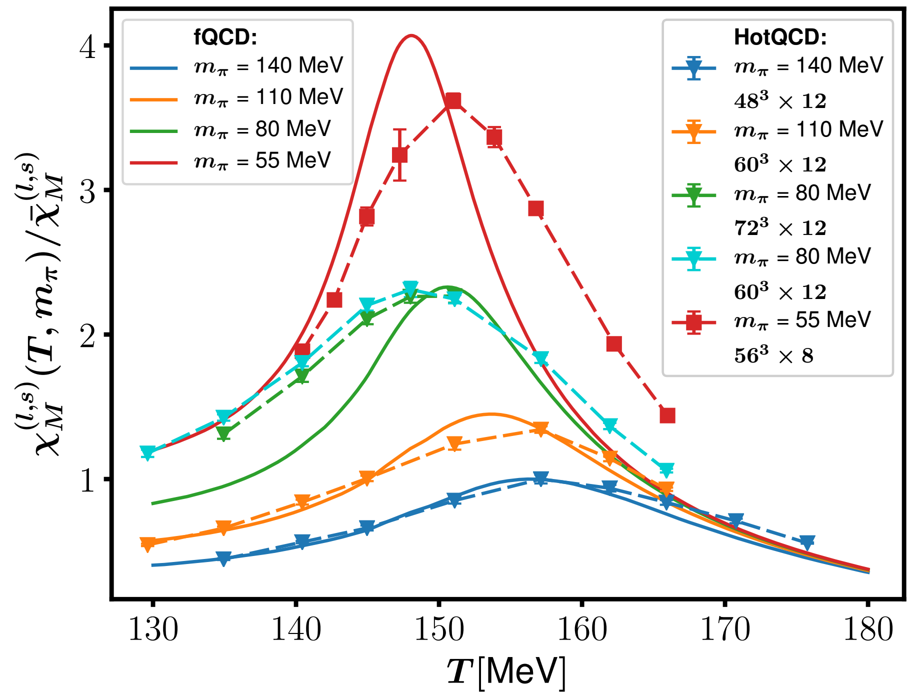
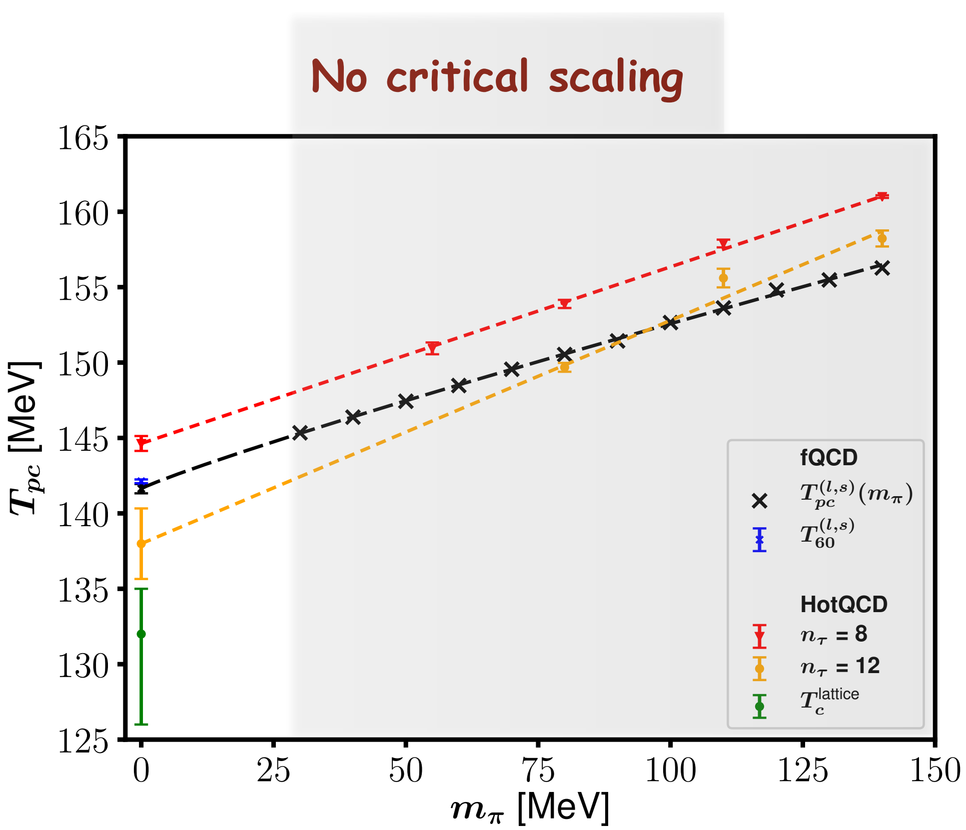
(iii) LEFTs with CEPs at large density (missing quark-gluon back reaction)

Some applications (fQCD)

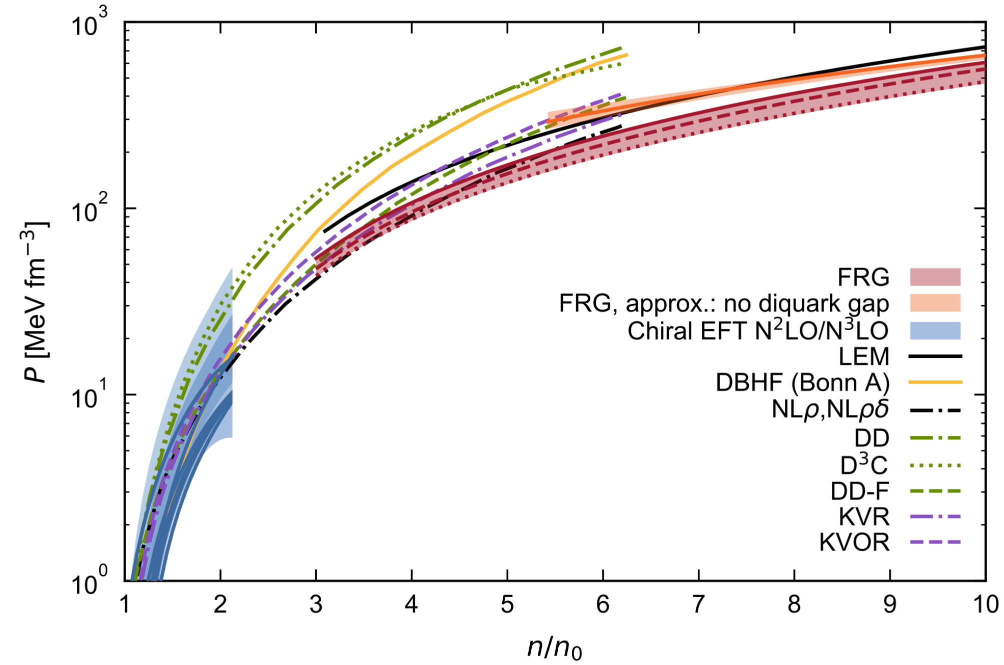
Magnetic EoS





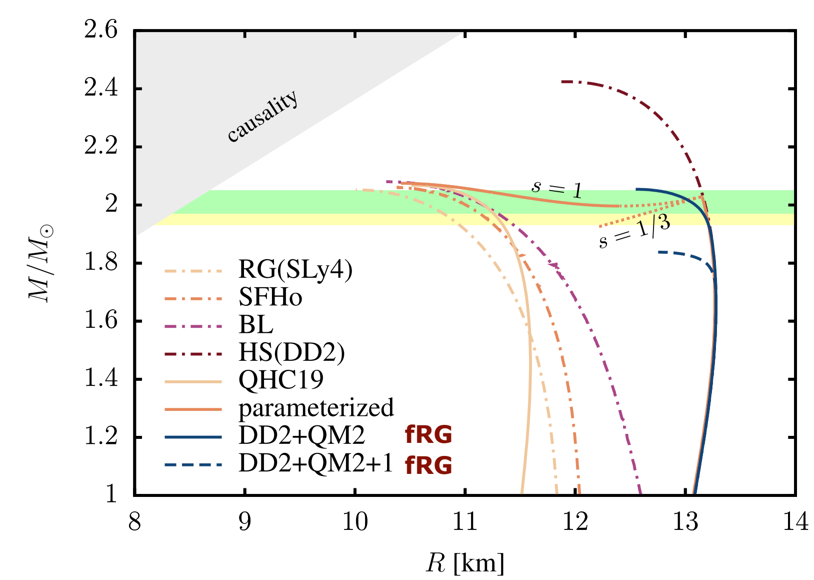
Braun, Fu, JMP, Rennecke, Rosenblüh, Yin, PRD 102 (2020) 056010

EoS of symmetric nuclear matter



Leonhardt, Pospiech, Schallmo, Braun, Drischler, Hebeler, Schwenk, PRL 125 (2020) 142502

Recent fRG work on EoS in cold and dense matter



Otto, Oertel, Schaefer, PRD 101 (2020) 10, 103021; 2007.07394

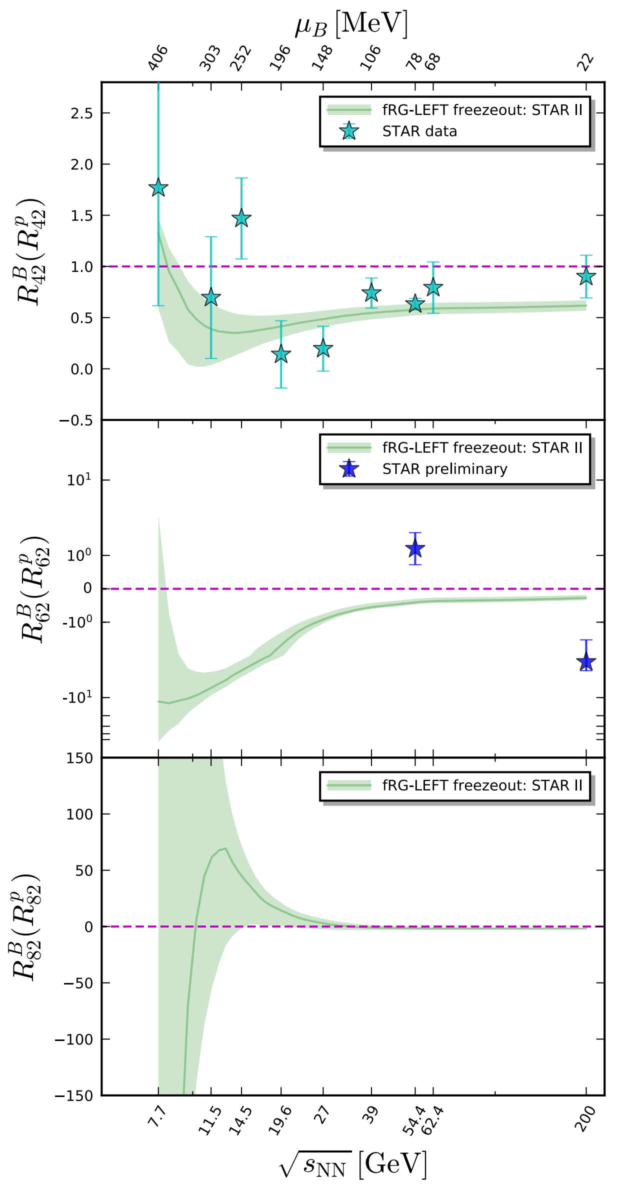
Outline

- QCD from functional methods

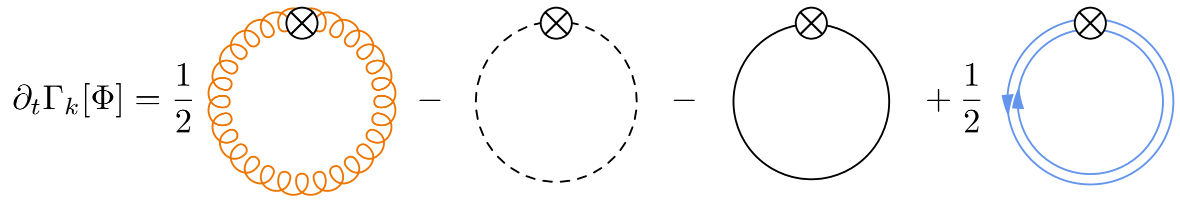
Applications

- QCD phase structure
- Fluctuations of conserved charges
- QCD-assisted transport
- Summary & outlook

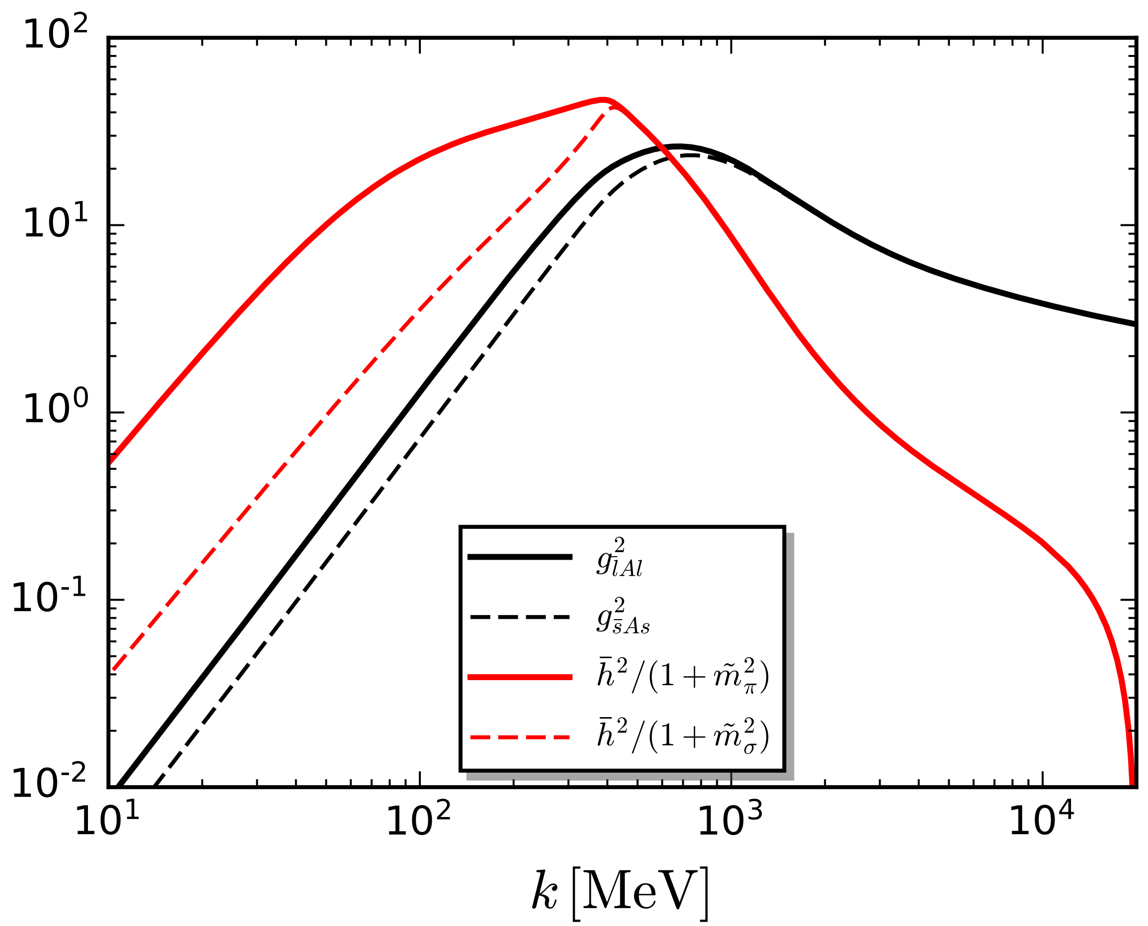
Hyper-fluctuations



On the unreasonable effectiveness of low energy effective theories



Sequential decoupling of gluon, quark, sigma, pion fluctuations



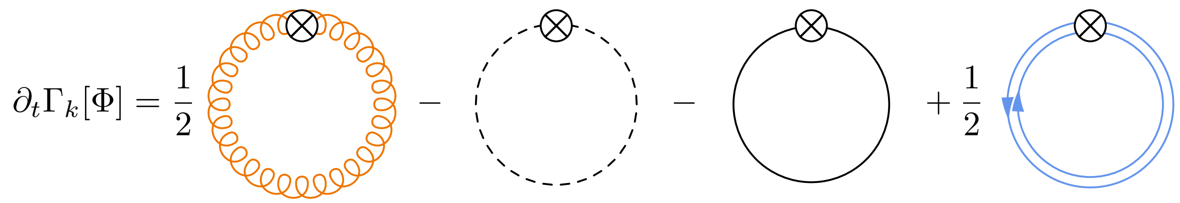
Fu, JMP, Rennecke, PRD 101, (2020) 054032

Based on:

Braun, Fister, Haas, JMP, Rennecke, PRD 94 (2016) 034016

Rennecke, PRD 92 (2015) 076012

On the unreasonable effectiveness of low energy effective theories



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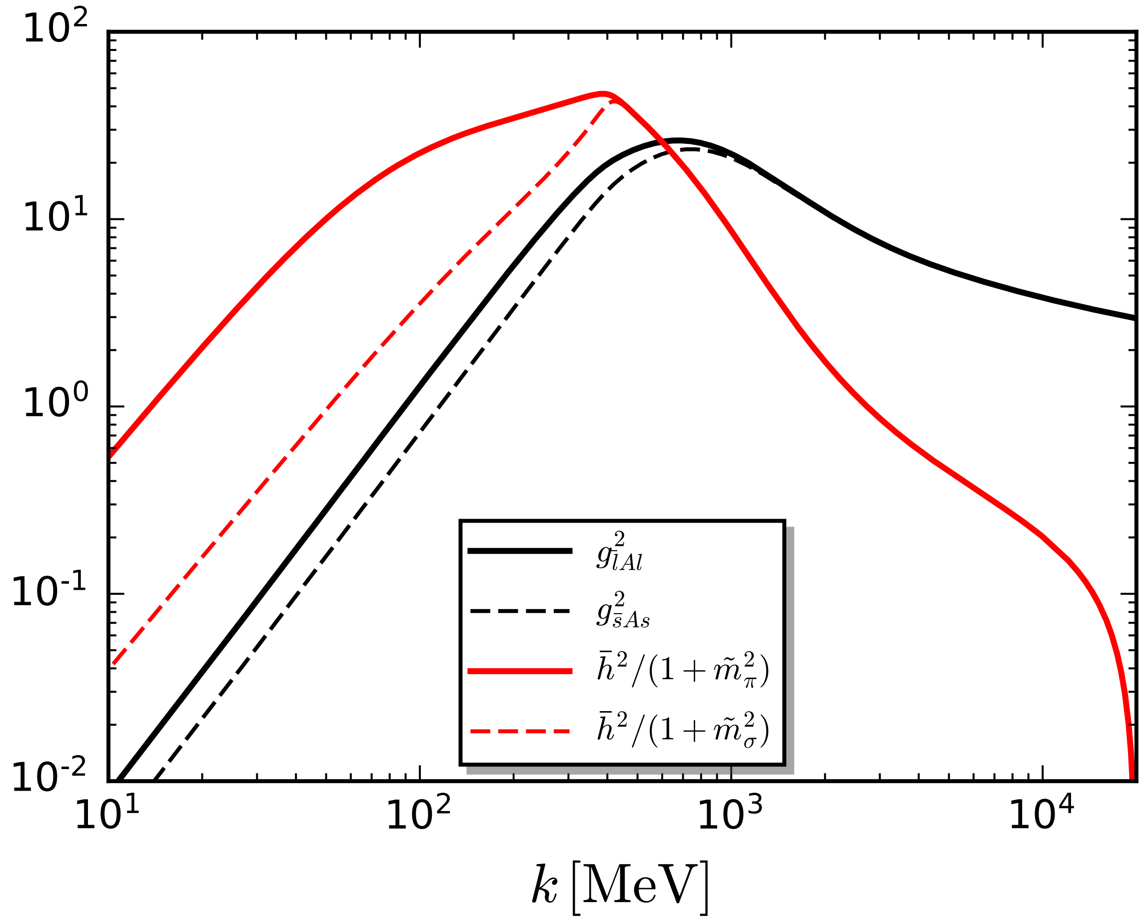
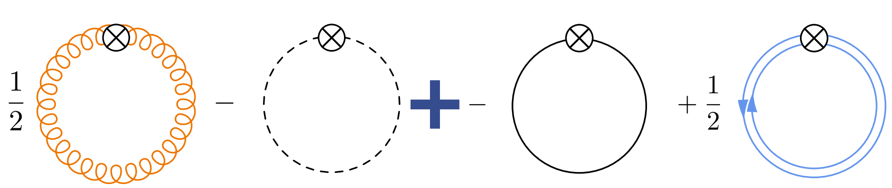
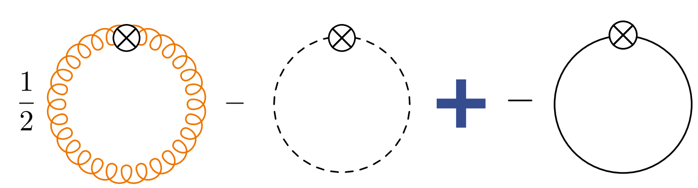


Fu, JMP, Rennecke, PRD 101, (2020) 054032

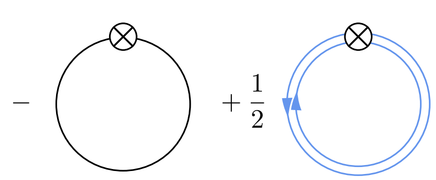
PQM-model



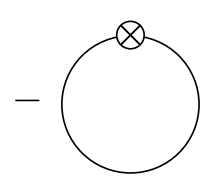
PNJL-model



QM-model



NJL-model

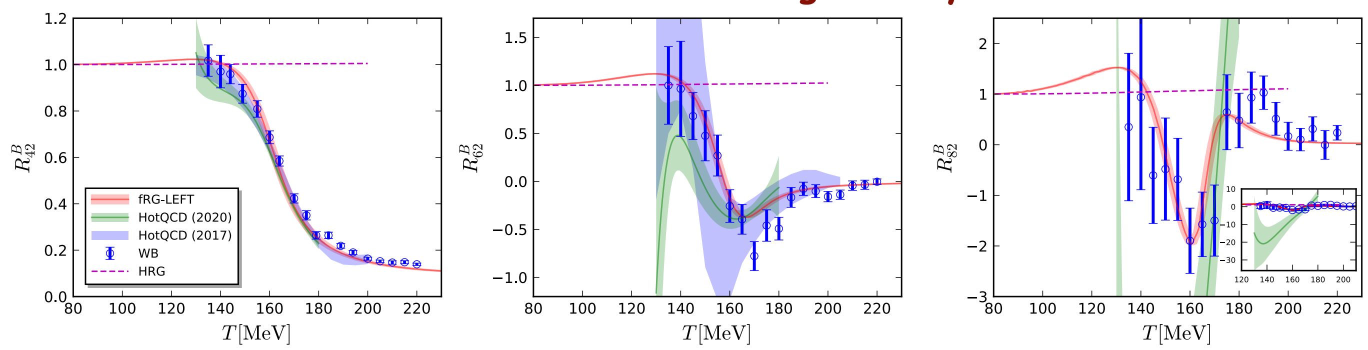


QCD-assisted low energy effective theories

Fluctuations of conserved charges

Fluctuation of conserved charges

Benchmark at vanishing density



QCD-assisted LEFT

Builds on

Fu, JMP, PRD 93 (2016) 091501
Fu, JMP, Schaefer, Rennecke, PRD 94 (2016) 116020

Strangeness

Fu, JMP, Rennecke, SciPost Phys. Core 2, 002 (2020)
PRD 100 (2019) 11, 111501
Wen, Huang, Fu, PRD 99 (2019) 094019

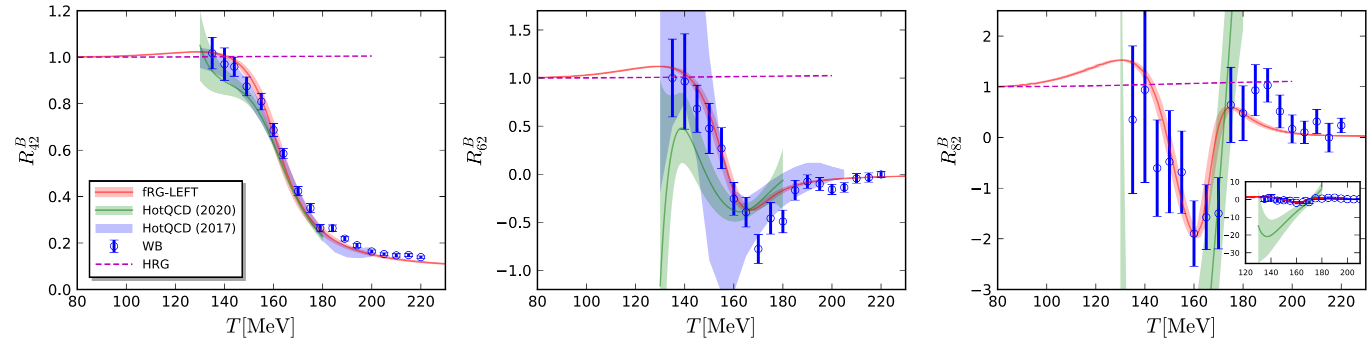
Fu, Luo, JMP, Rennecke, Wen, Yin, in preparation

Fluctuations of conserved charges

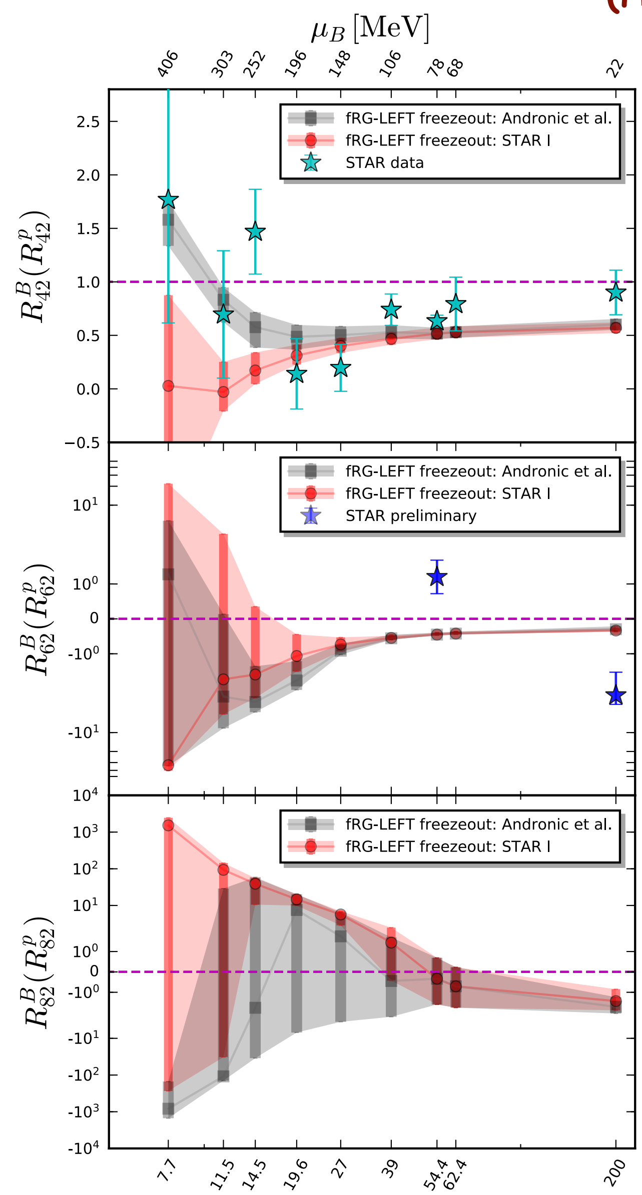
Fluctuation of conserved charges

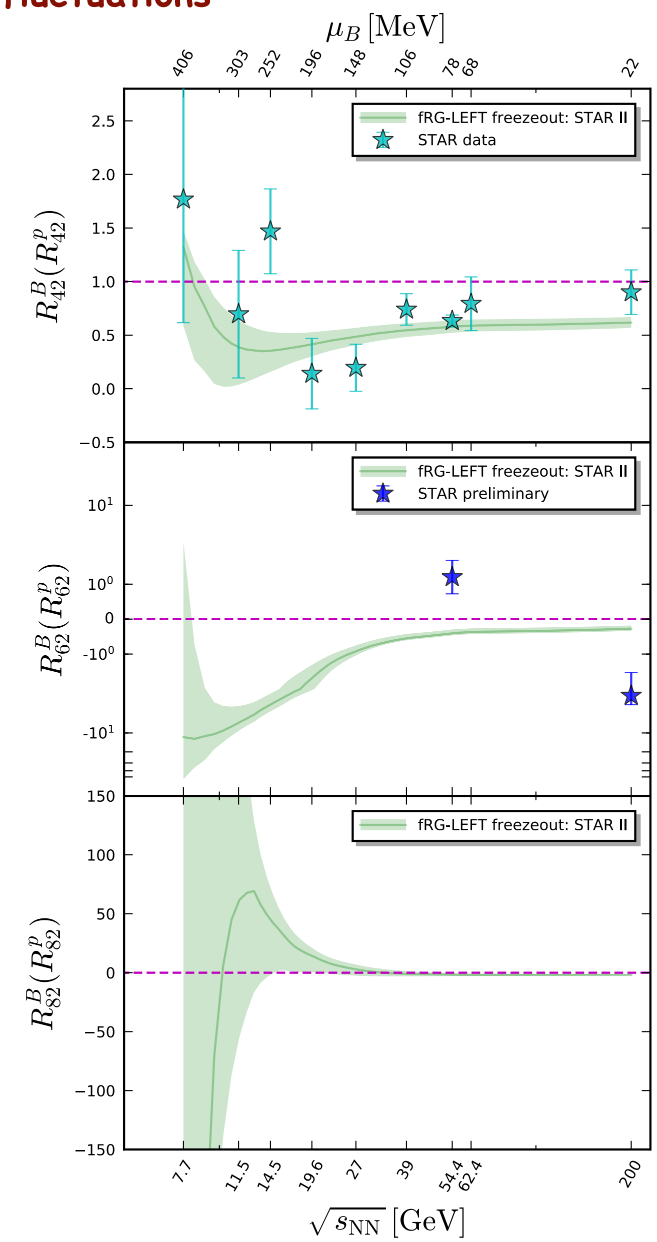
Benchmark at vanishing density

QCD-assisted LEFT

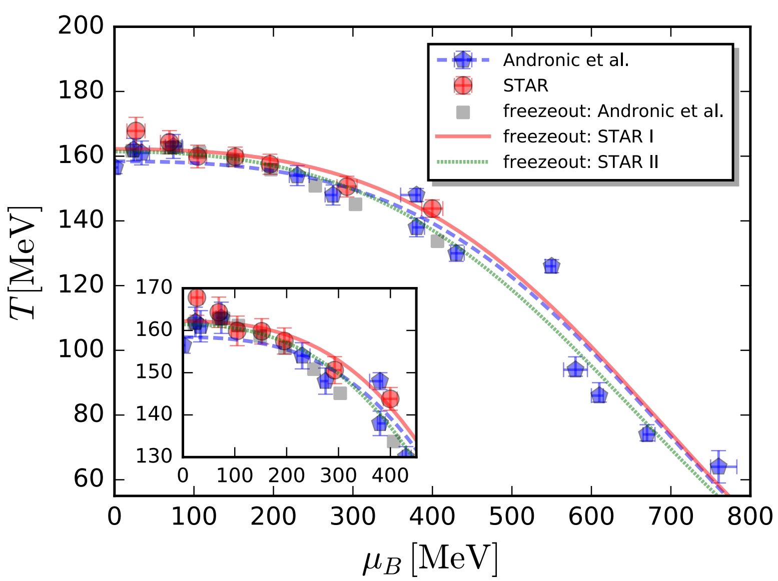


(Hyper-) fluctuations





Freezeout curve



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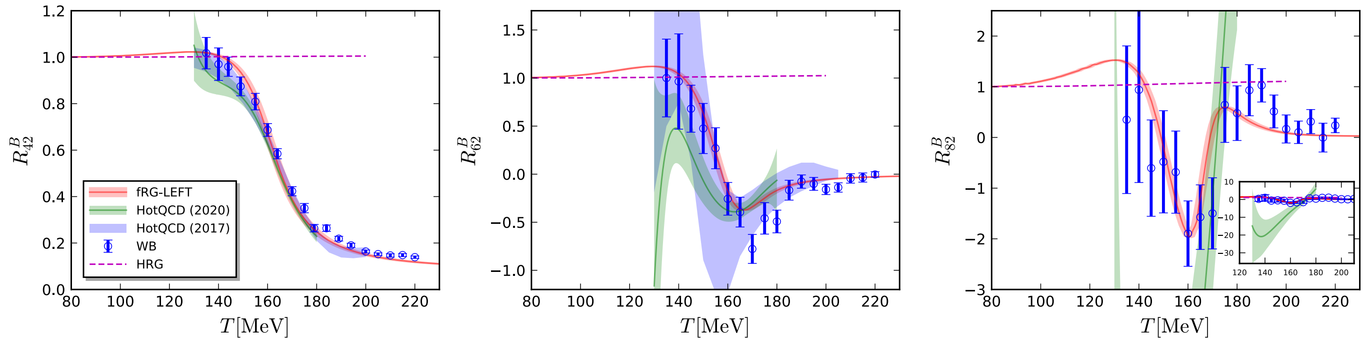
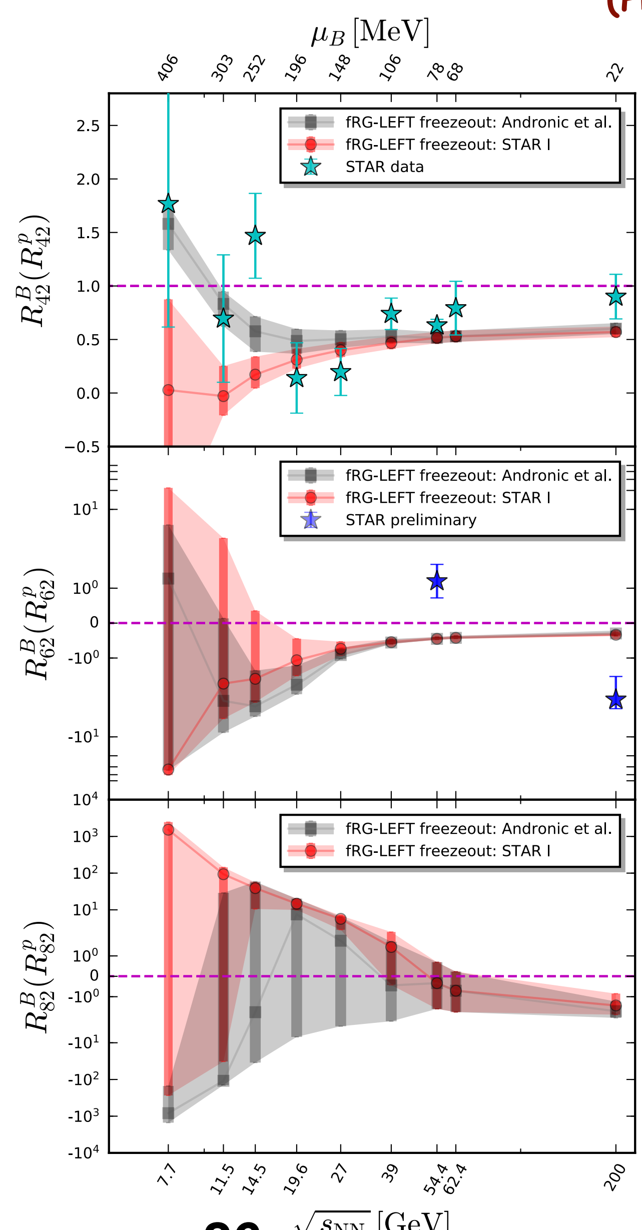
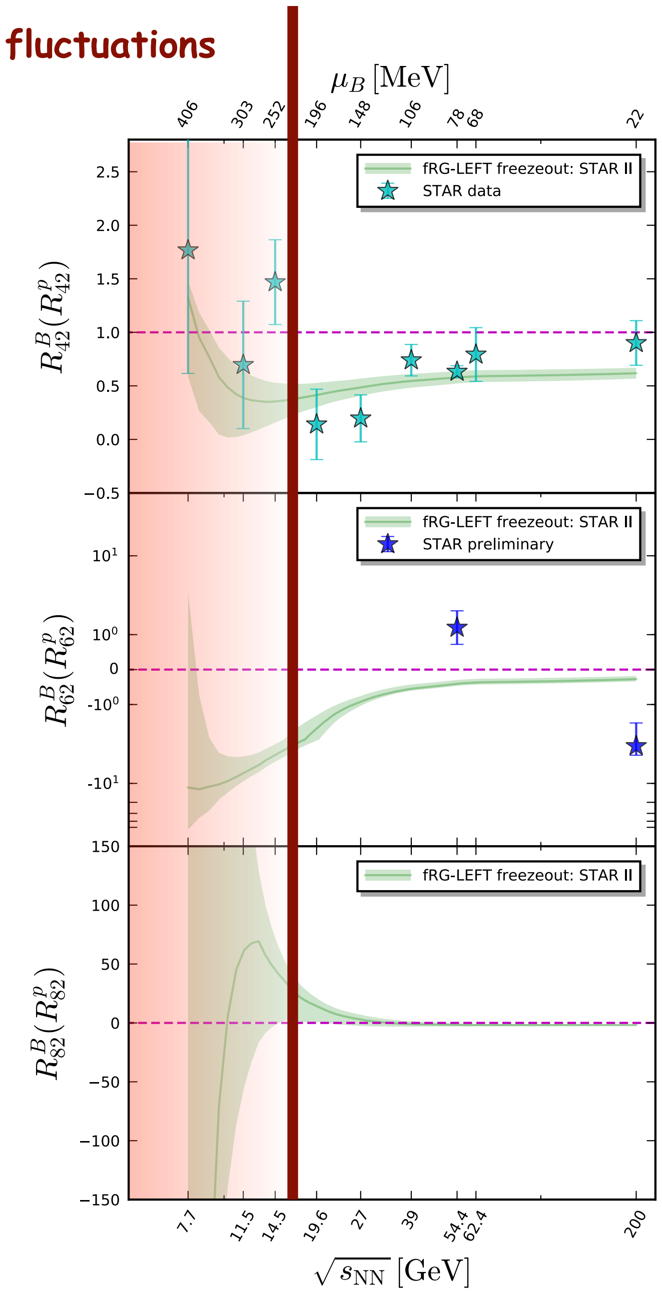


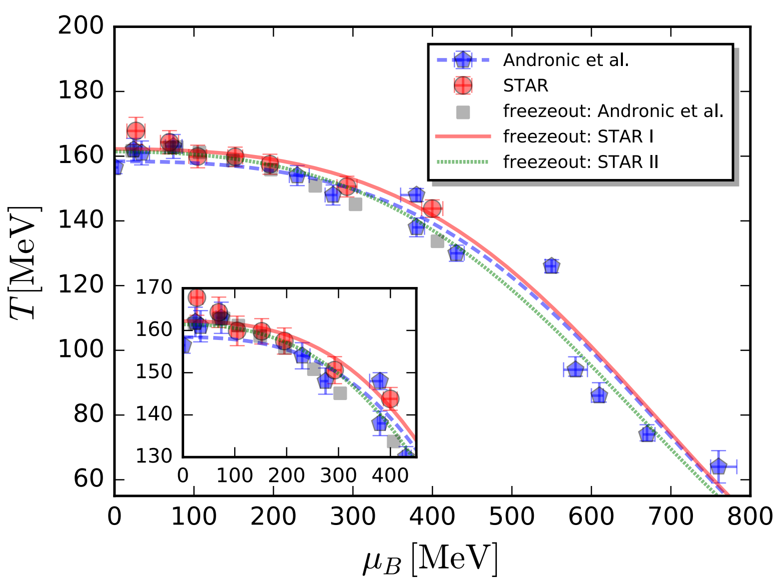
QCD-assisted LEFT

(Hyper-) fluctuations





Freezeout curve



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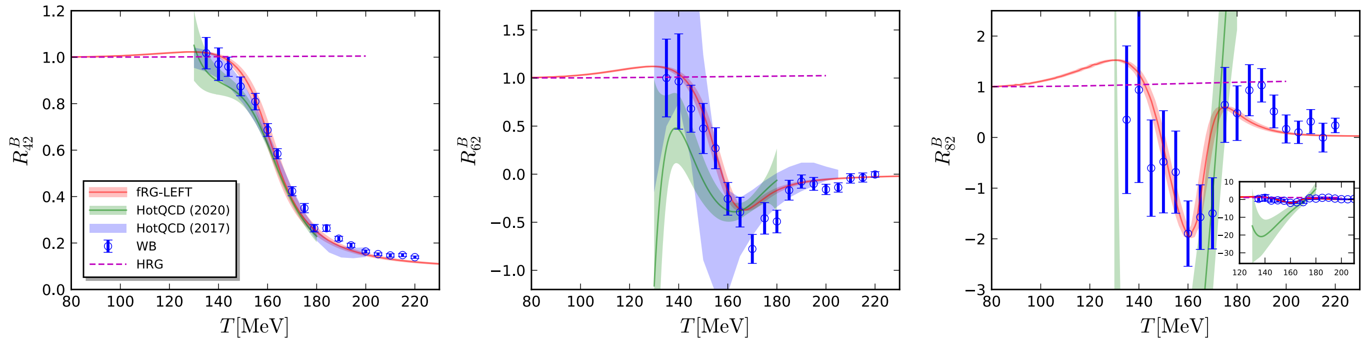

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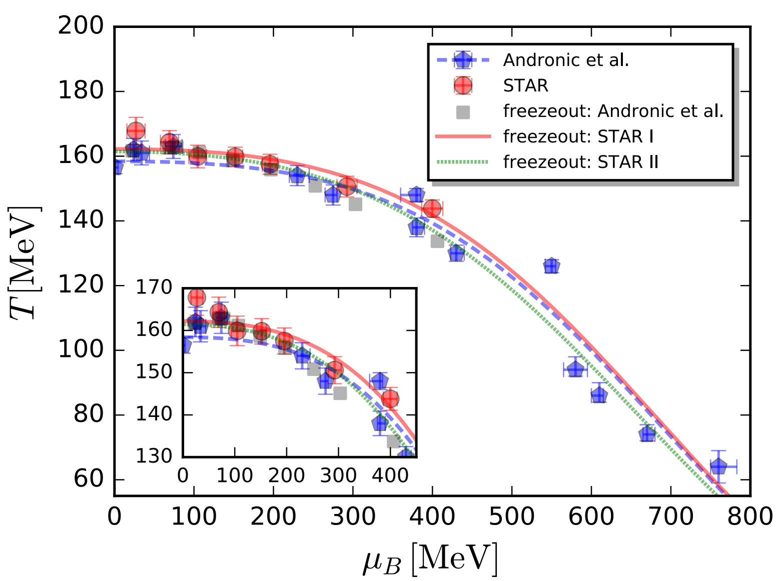
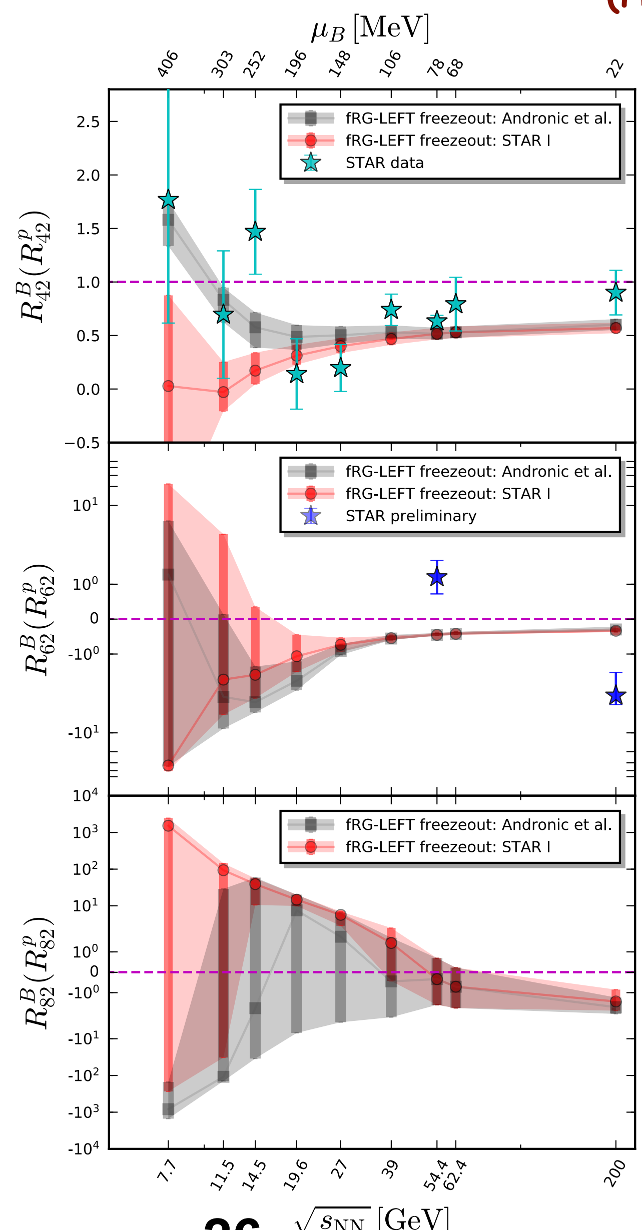
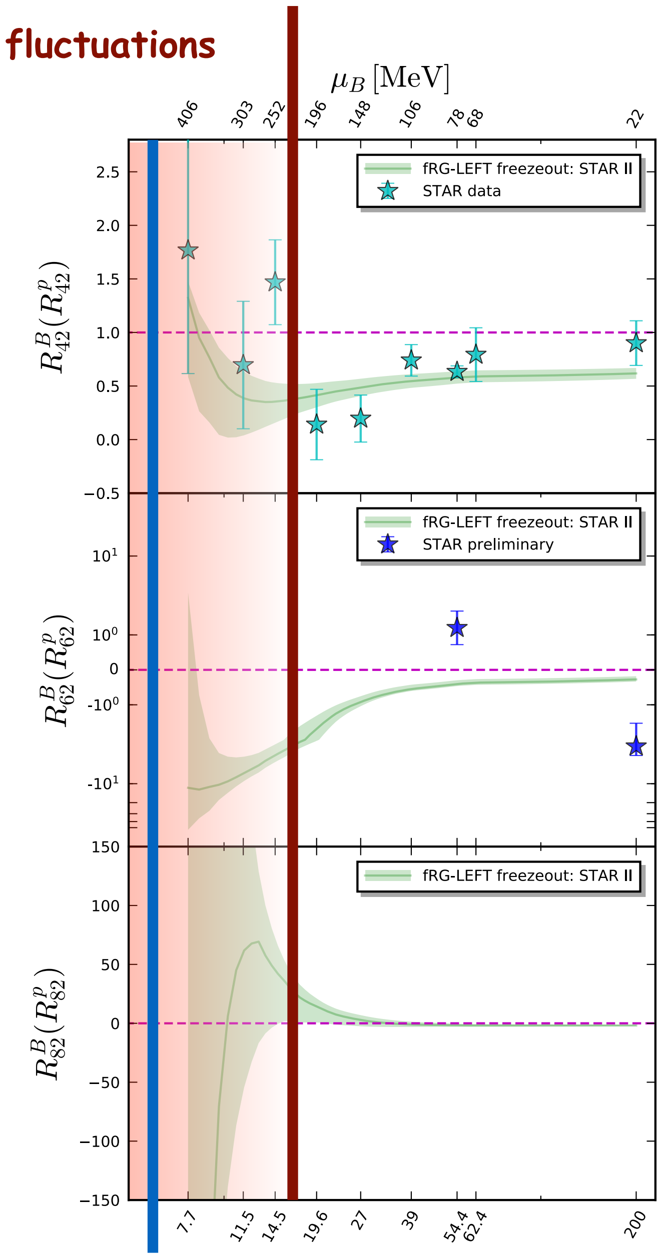


(Hyper-) fluctuations



QCD-assisted LEFT



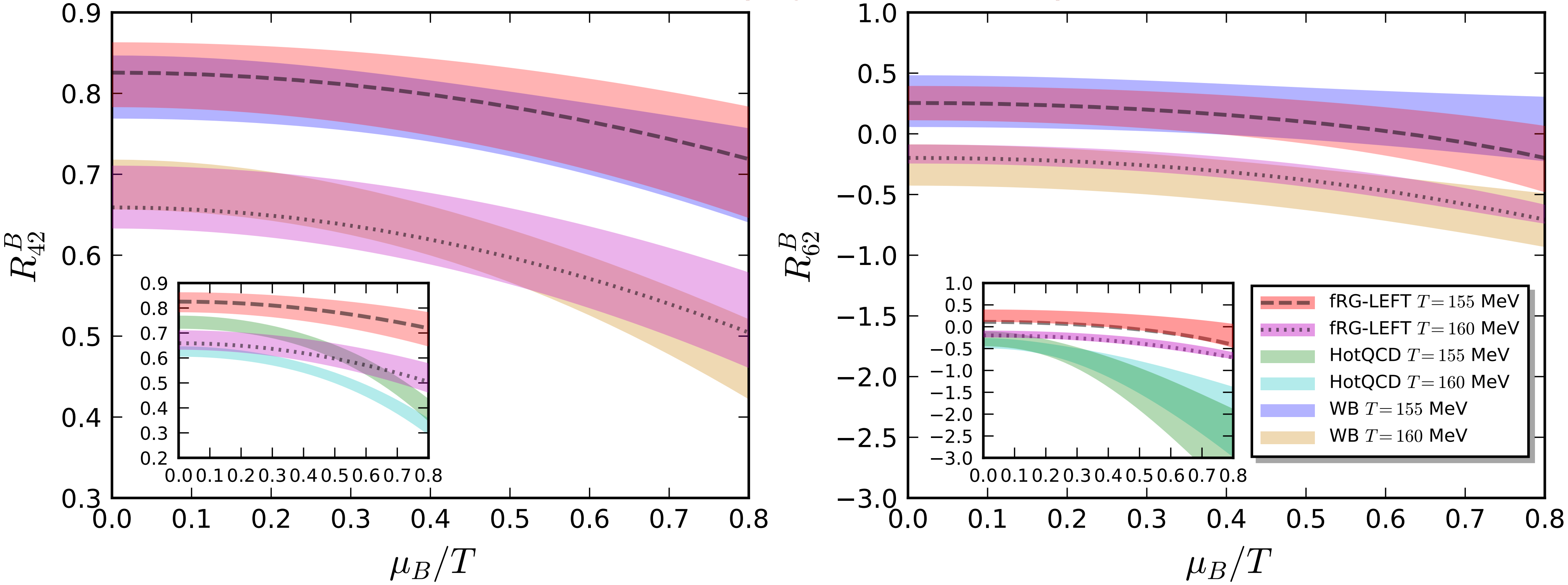
Fu, Luo, JMP, Rennecke, Wen, Yin, in preparation

Fun QCD, in progress

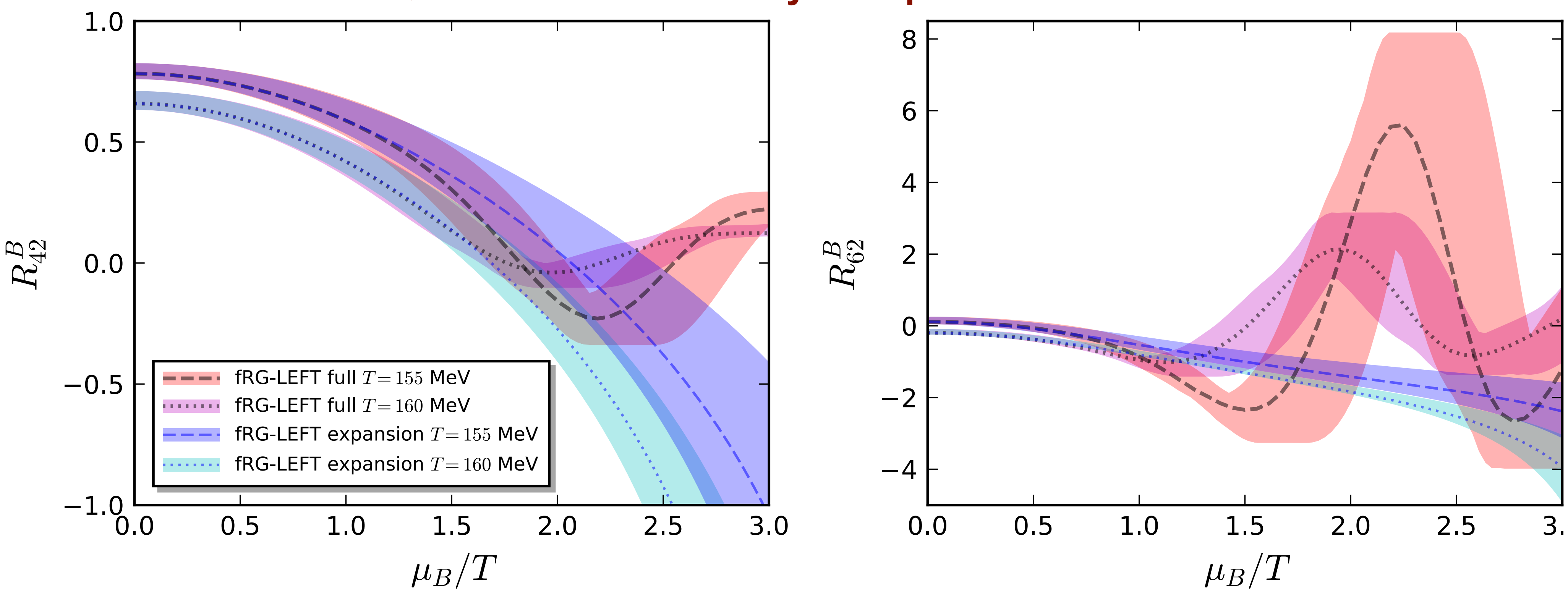
Fluctuations of conserved charges

Fluctuation of conserved charges

QCD-assisted LEFT vs lattice results (Taylor expansion) at small chemical potential



QCD-assisted LEFT: Taylor expansion vs full results



Fu, Luo, JMP, Rennecke, Wen, Yin, in preparation

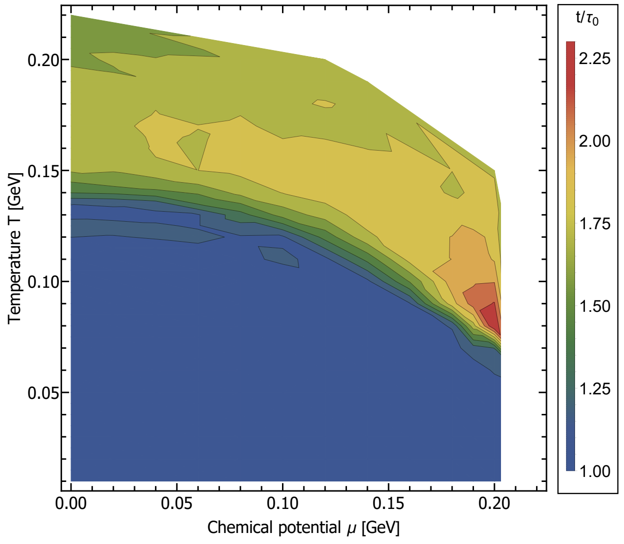
Outline

- QCD from functional methods

Applications

- QCD phase structure
- Fluctuations of conserved charges
- **QCD-assisted transport**
- Summary & outlook

Equilibration times



Transport approach to QCD

Blum, Jiang, Mitter, Nahrgang, JMP, Rennecke, Wink

Time evolution of the critical (scalar) σ -mode

$$\frac{\delta\Gamma}{\delta\sigma} = \xi$$

quantum equation of motion noise field

Extension of mean-field version

Nahrgang, Leupold, Herold, Bleicher PRC84 (2011)

see also

Stephanov, Rajagopal, Shuryak PRL 81 (1998) 4816

Mukherjee, Venugopalan, Yin PRC 92 (2015) 034912

Herold, Nahrgang, Yan, Kobdaj PRC 93 93 (2016) 021902

Nahrgang, Bluhm, Schäfer, Bass PRD 99 (2019) 116015

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Time evolution of the critical (scalar) σ -mode

$$\frac{\delta\Gamma}{\delta\sigma} = \xi$$

quantum equation of motion noise field

Input from equilibrium low energy effective action of QCD

$$\mathrm{Re}\,\Gamma_\sigma^{(2)}(\omega,\vec{p}) \qquad \mathrm{Im}\,\Gamma_\sigma^{(2)}(\omega,\vec{p}) \qquad U(\sigma)$$

kinetic term

diffusion term $\eta\,\partial_t\sigma$

effective potential

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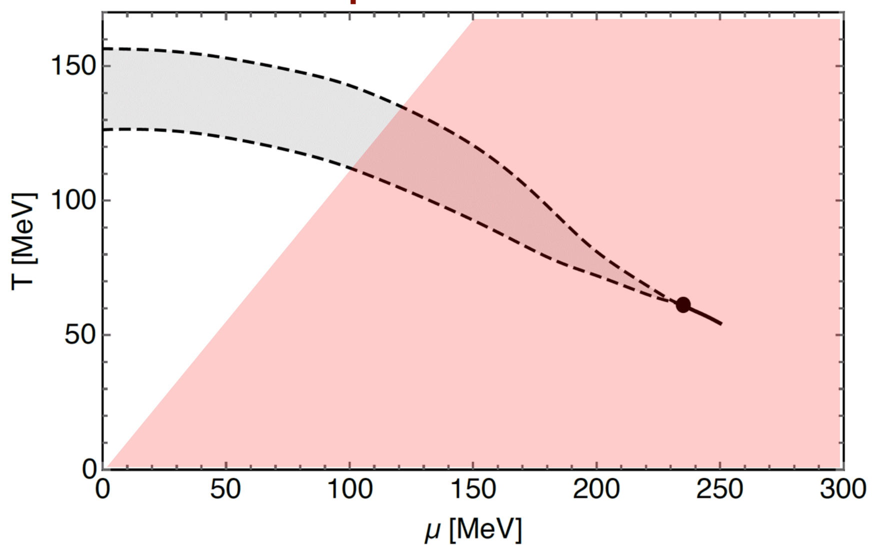
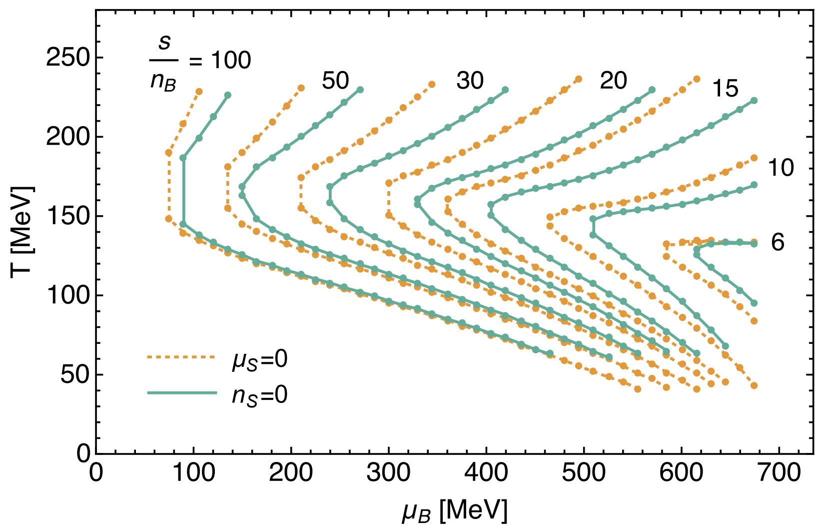
Phase structure of low energy QCD

2+1 flavour quark-meson model



Schaefer, Rennecke, PRD 96 (2017) 016009

strangeness neutrality & strangeness fluctuations



Fu, JMP, Rennecke, SciPost Core 002 (2020), PRD 100 (2019) 111501

$N_f = 2$: Nakano, Schaefer, Stokic, Friman, Redlich, PLB 682 (2010) 401

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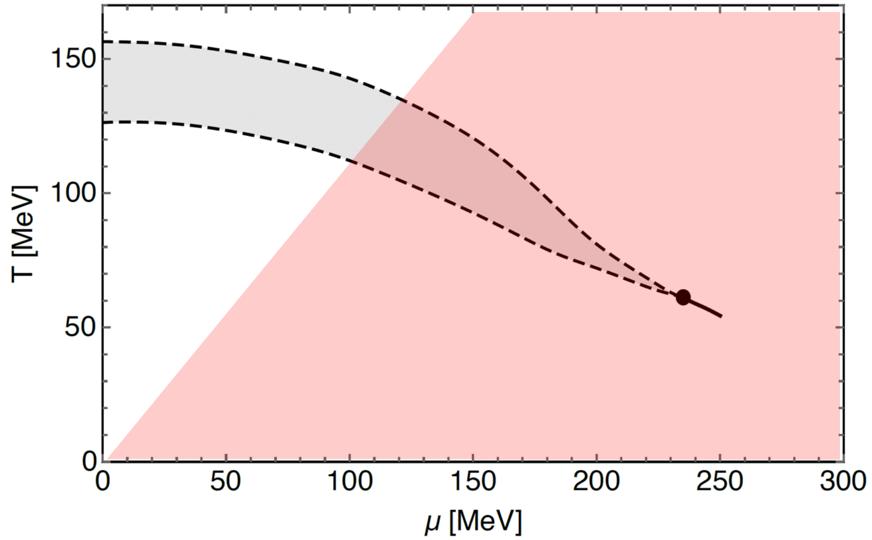
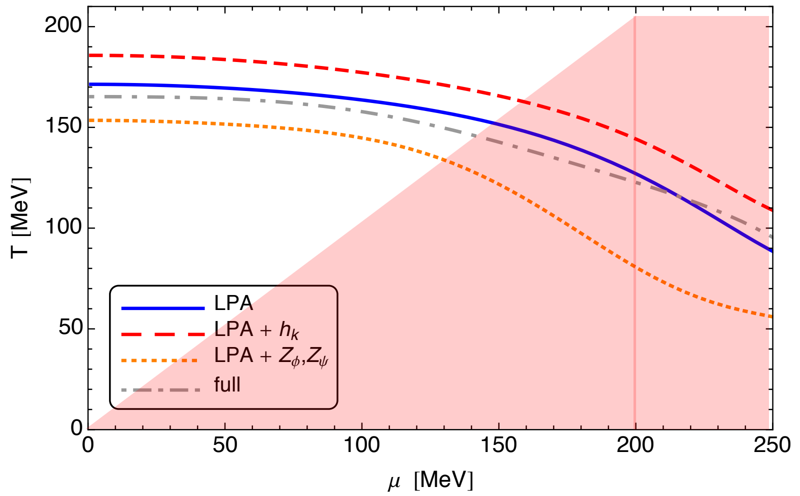


Schaefer, Rennecke, PRD 96 (2017) 016009

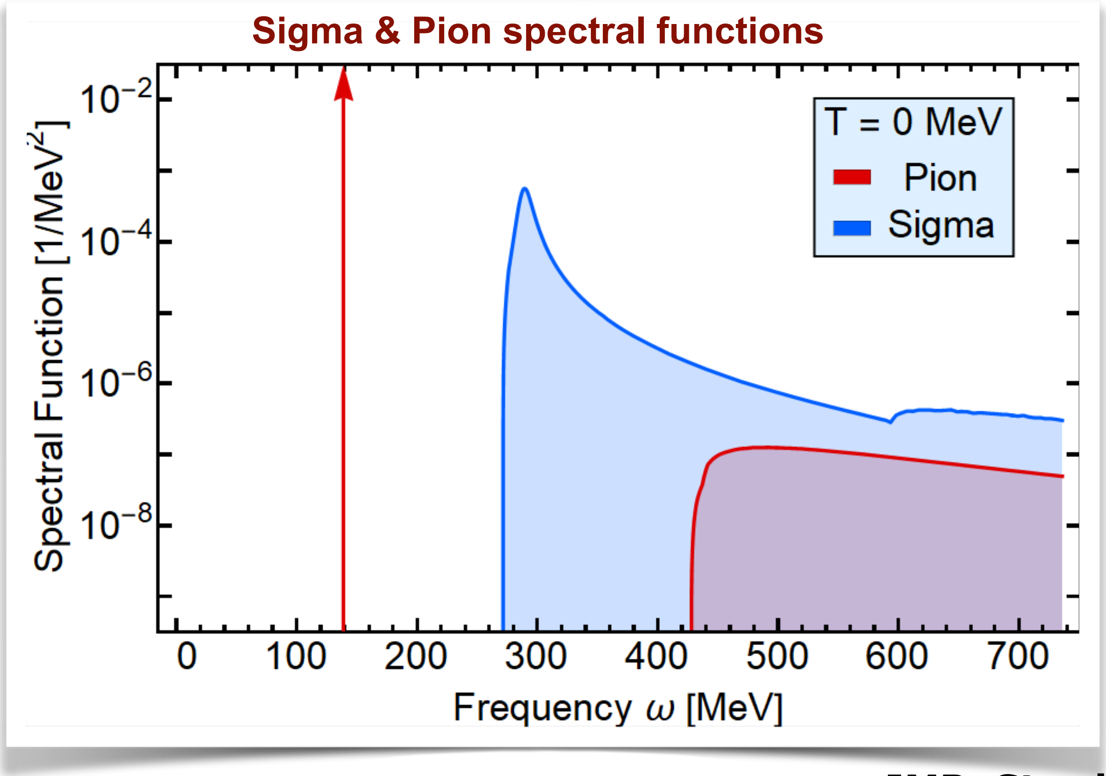
Comparison of truncations (2 flavours)



JMP, Rennecke, PRD 90 (2014) 076002

Pion & sigma spectral functions

Show case in linear sigma model



JMP, Strodthoff, Wink, PRD 98 (2018) 074008

Real-time FRG computations, e.g.

Flörchinger JHEP 1205 (2012) 021

Kamikado, Strodthoff, von Smekal, Wambach, EPJC 74 (2014) 2806

JMP, Strodthoff, PRD 92 (2015) 094009

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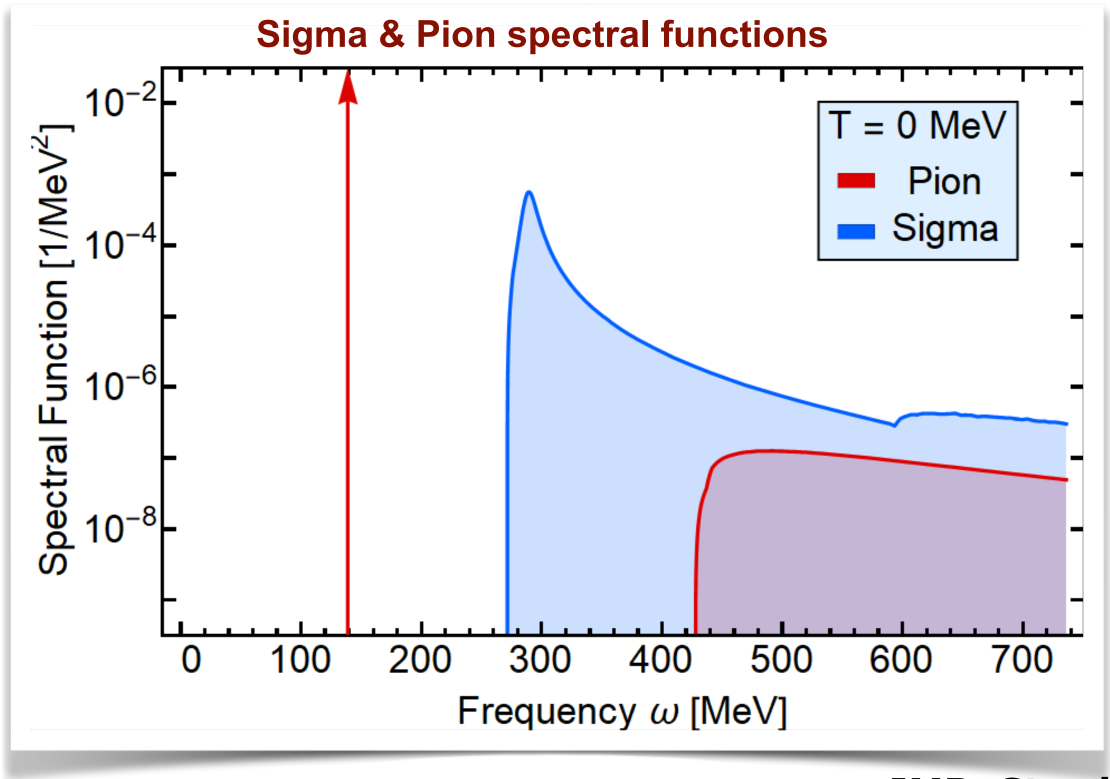

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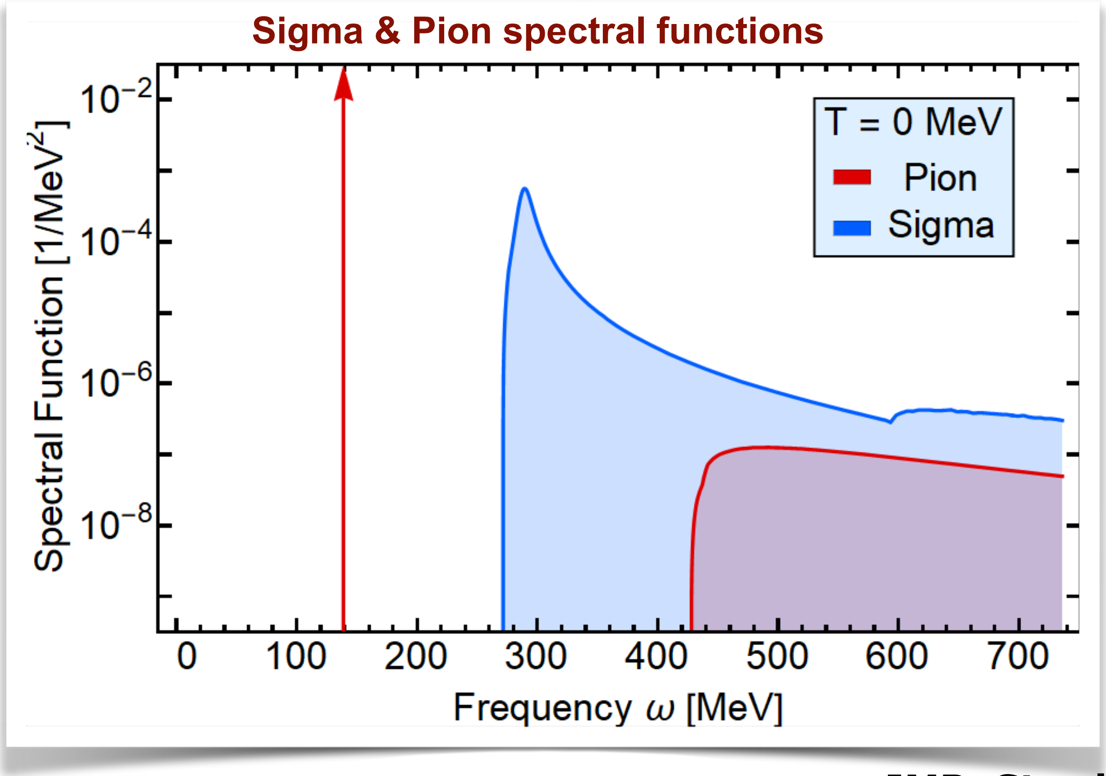

JMP, Strodthoff, Wink, PRD 98 (2018) 074008

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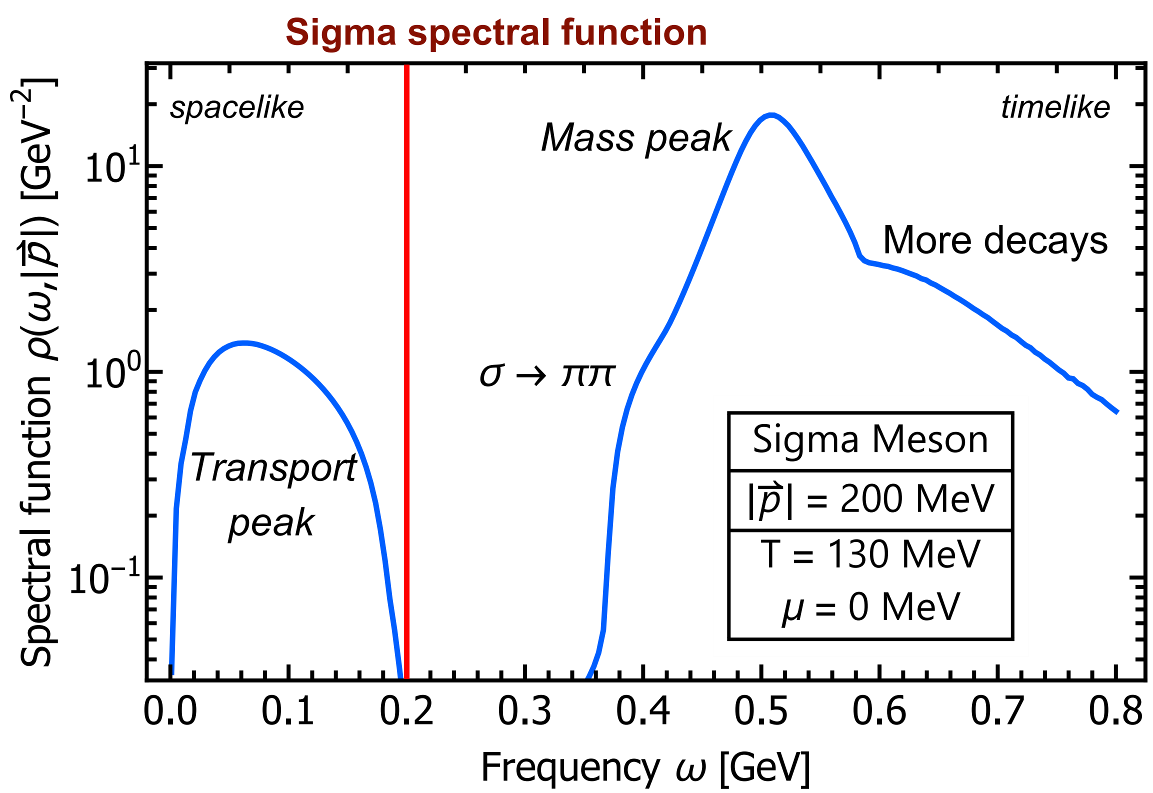
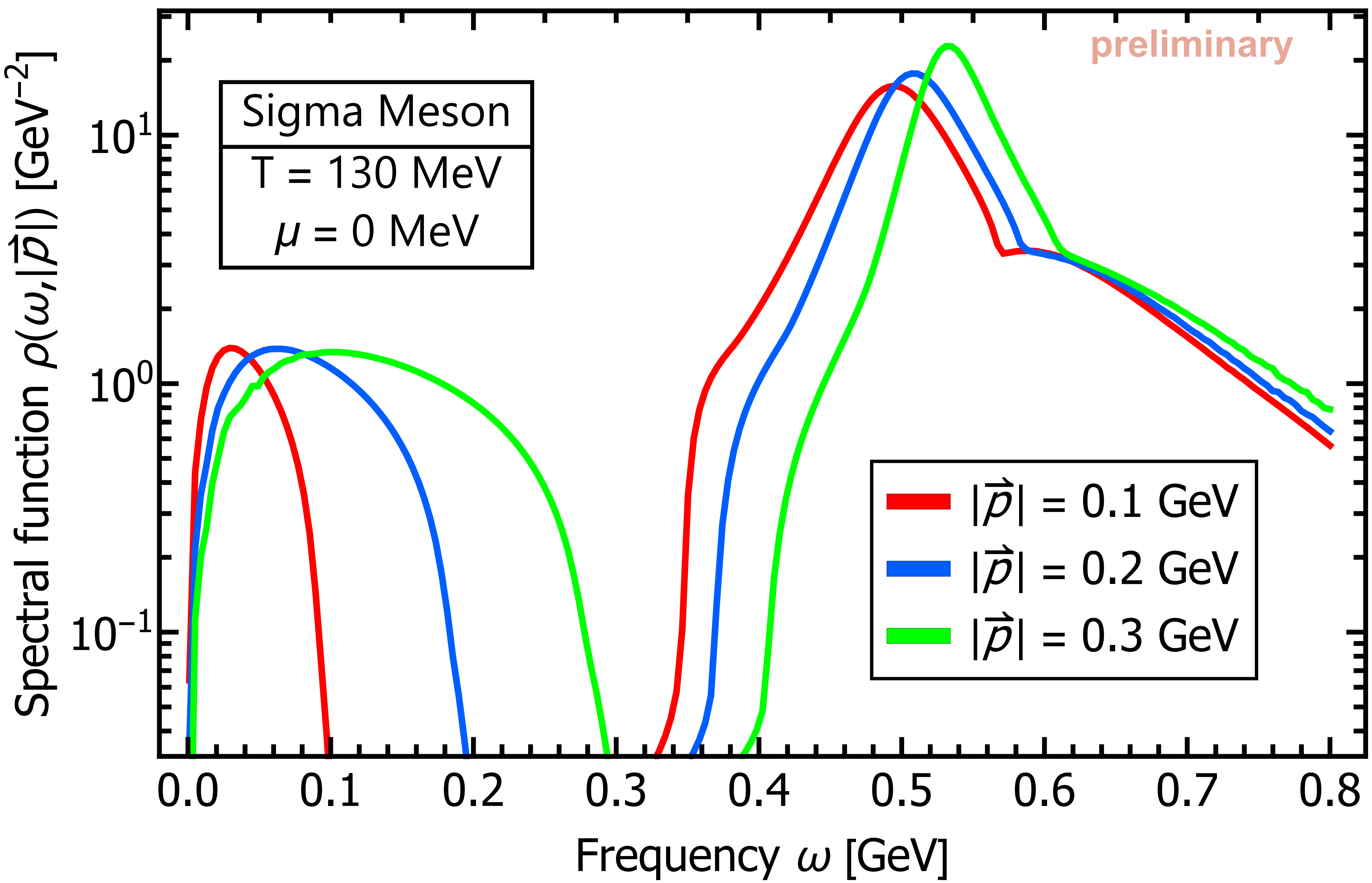


preliminary

JMP, Rennecke, Wink, in prep

Pion & sigma spectral functions

2+1 flavour quark-meson model sigma spectral function

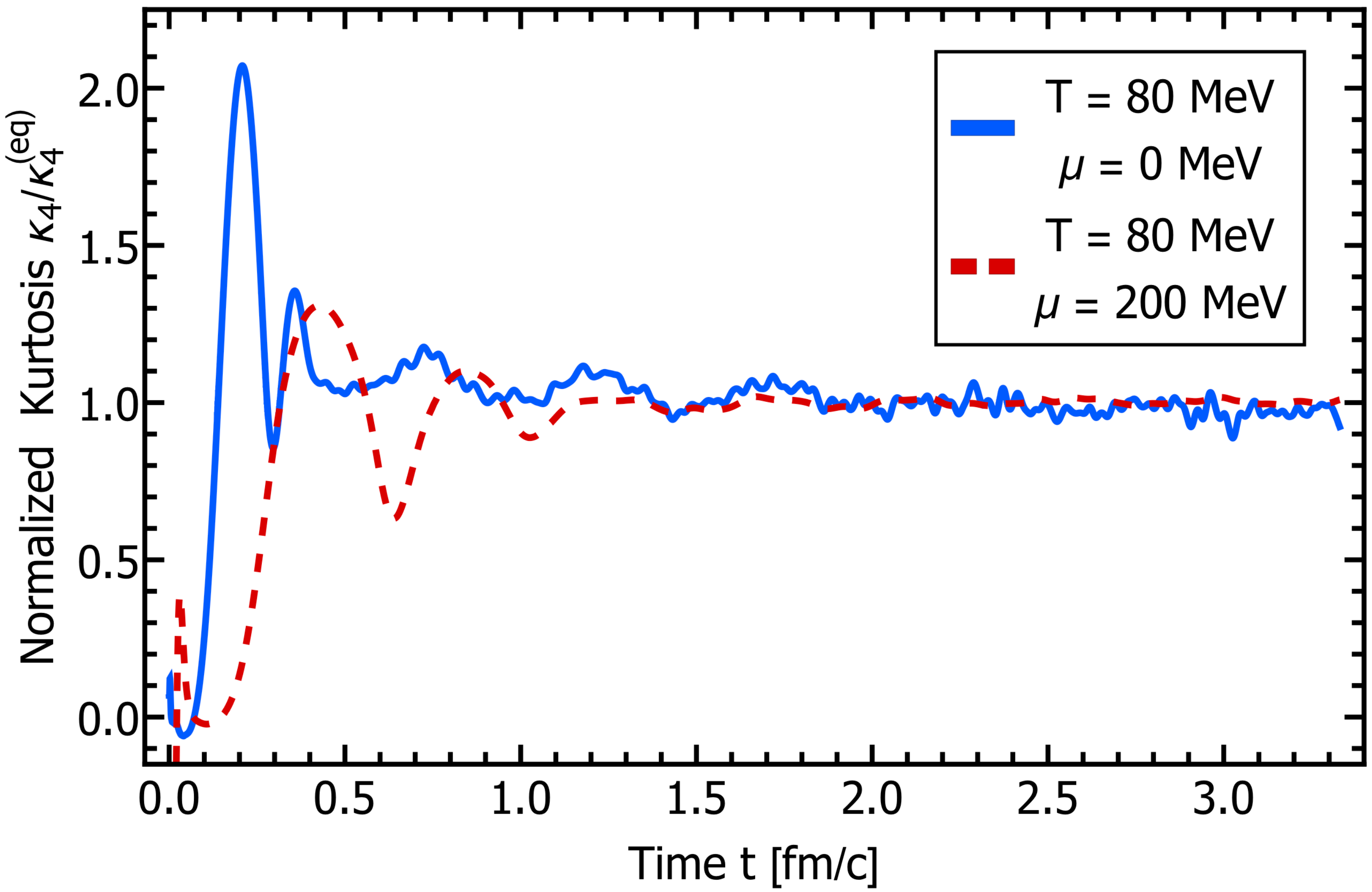


JMP, Rennecke, Wink, in prep

Time evolution of cumulants

Blum, Jiang, Nahrgang, JMP, Rennecke, Wink, NPA 982 (2019) 871

Time evolution of sigma-kurtosis



Time evolution of the critical (scalar) σ- mode

noise field

$$\frac{\delta\Gamma}{\delta\sigma} = \xi$$

quantum equation of motion

nth central moment of the sigma field: χ_n

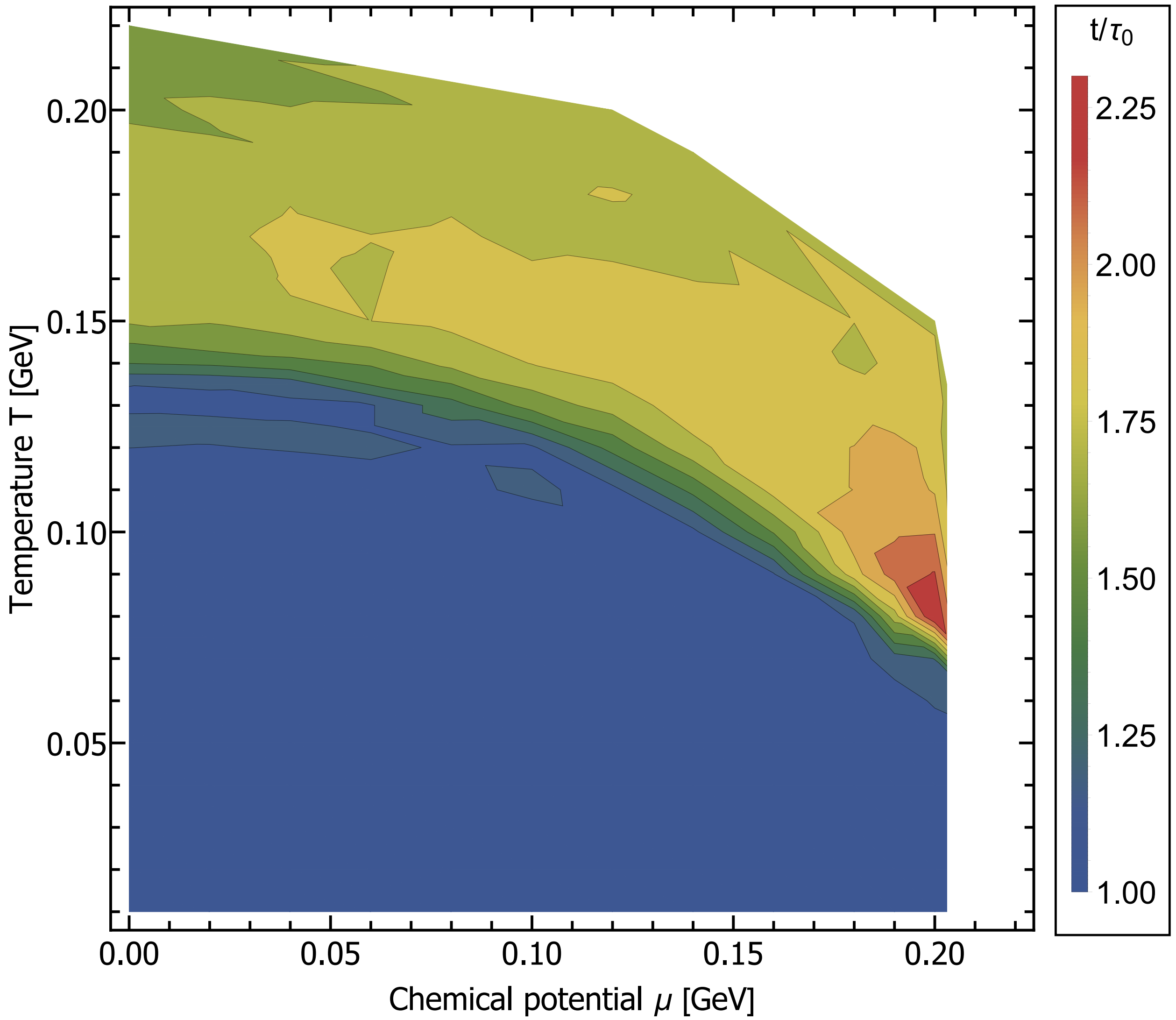
$$\chi_2 = \langle(\sigma - \langle\sigma\rangle)^2\rangle$$

kurtosis: $\kappa = \frac{\chi_4}{\chi_2^2} - 3$

Equilibration time phase structure

Blum, Jiang, Nahrgang, JMP, Rennecke, Wink, NPA 982 (2019) 871

Equilibration time of sigma-kurtosis



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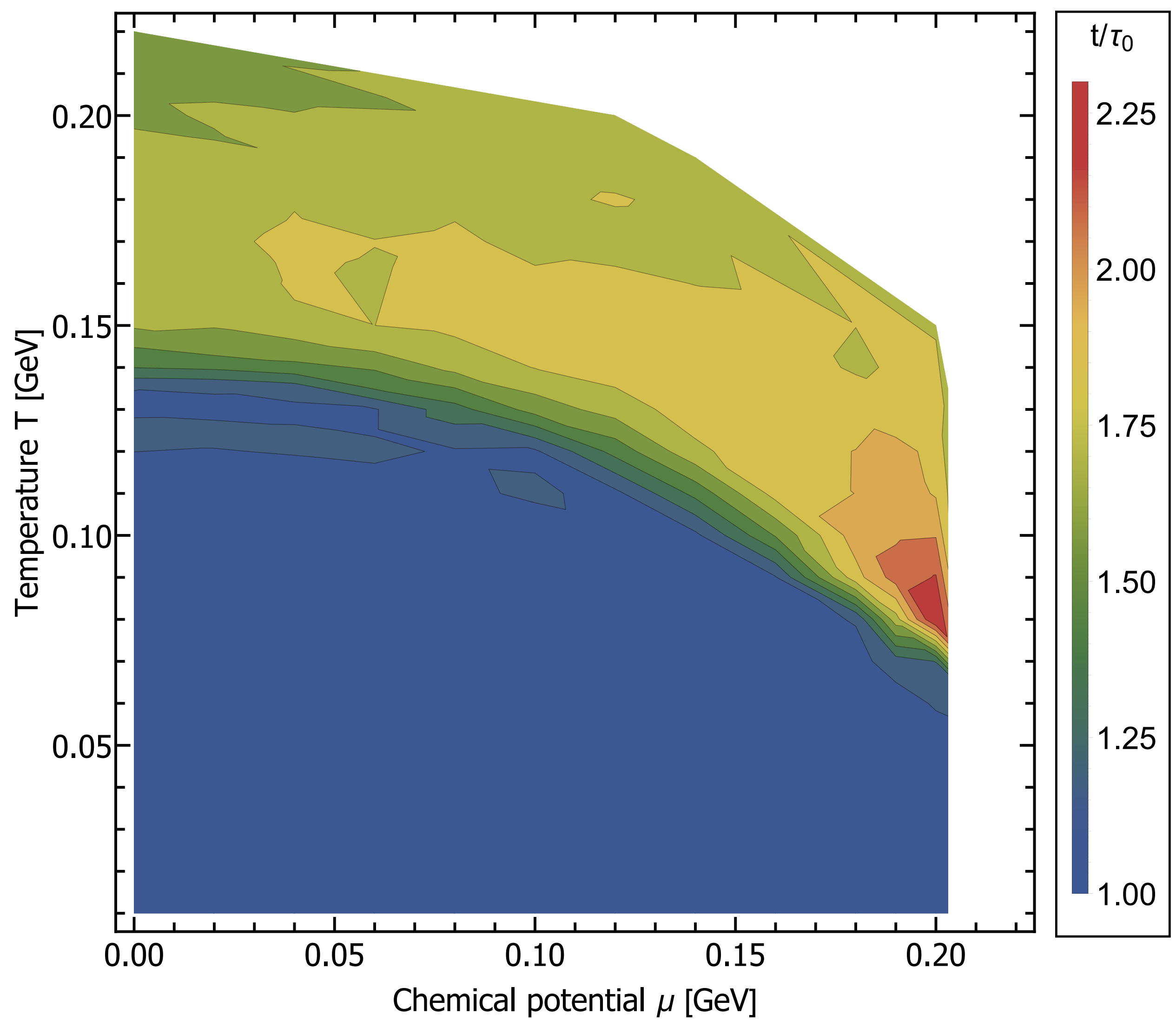
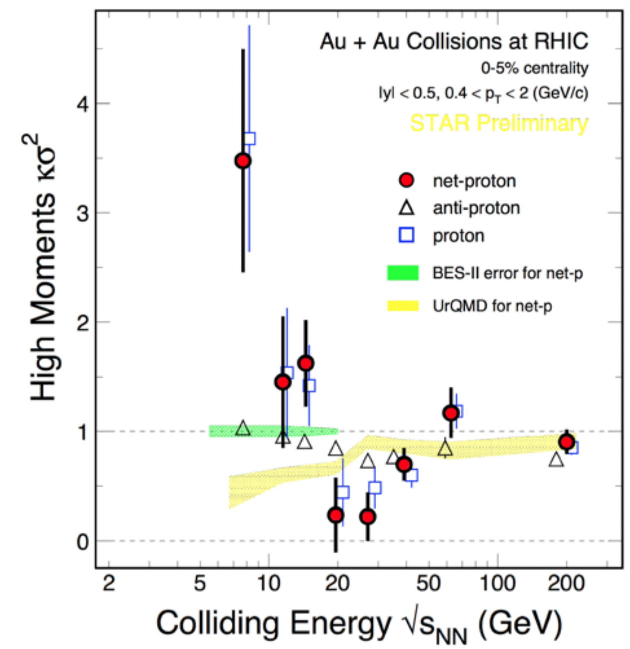


nth central moment of the sigma field: χ_n

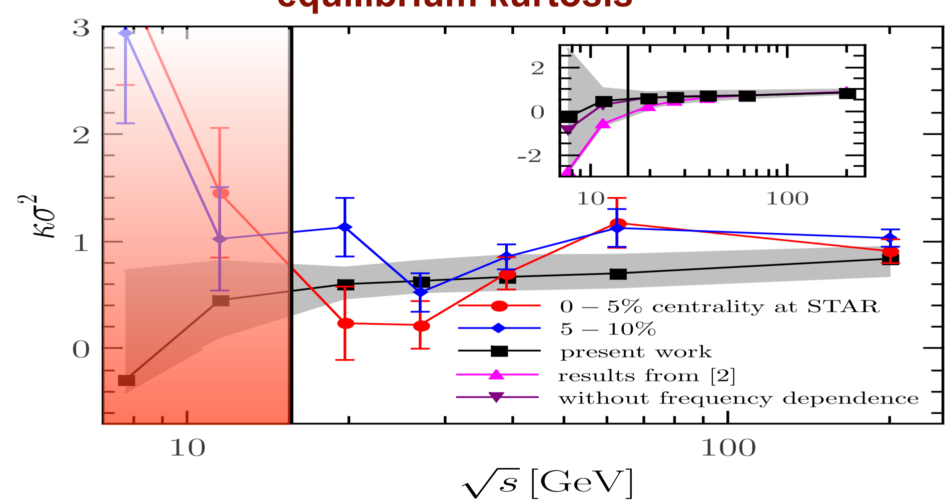
variance: $\chi_2 = \langle (\sigma - \langle \sigma \rangle)^2 \rangle$

kurtosis of baryon number fluctuations



Luo, Cu, NST 28 (2017)

equilibrium kurtosis



Fu, JMP, Schaefer, Rennecke, PRD 94 (2016) 116020

kurtosis: $\kappa = \dfrac{\chi_4}{\chi_2^2} - 3$

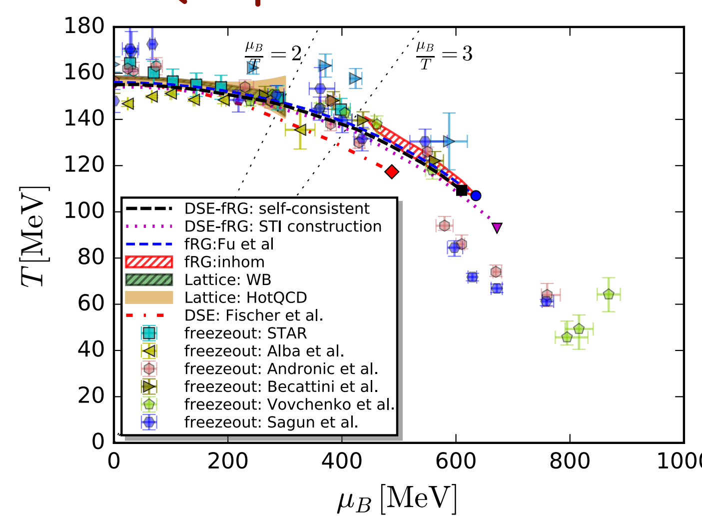
Outline

- **QCD from functional methods**

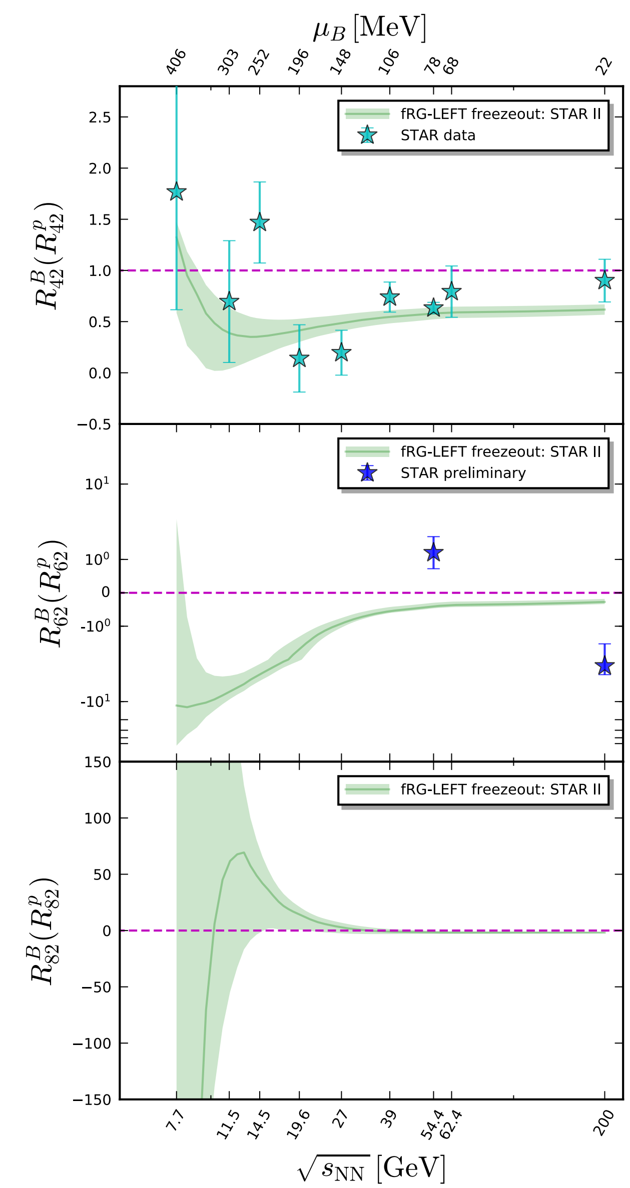
Applications

- **QCD phase structure**
- **Fluctuations of conserved charges**
- **QCD-assisted transport**
- **Summary & outlook**

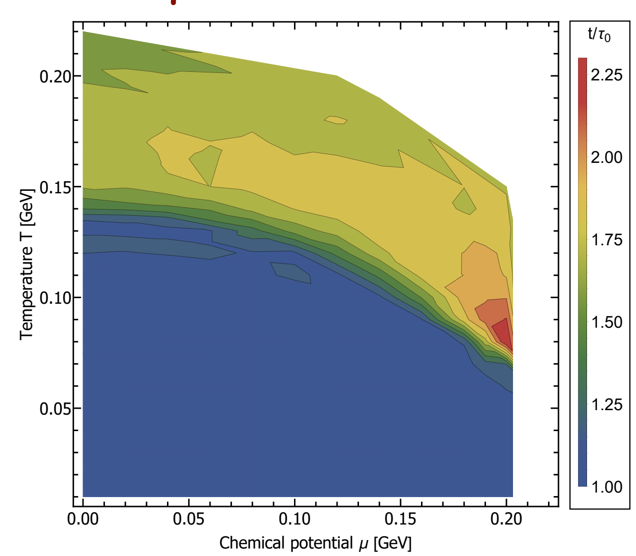
QCD phase structure



Hyper-fluctuations



Equilibration times



Summary & Outlook

- **Towards apparent convergence in functional approaches to QCD**
- **Results & predictive power for the phase structure of QCD**
  - **Observables: quark condensates, fluctuations of conserved charges**
- **Towards quantitative precision at high densities**
  - **Systematic improvements under way for** $\mu_\mathbf{B}/\mathbf{T} \gtrsim 4$

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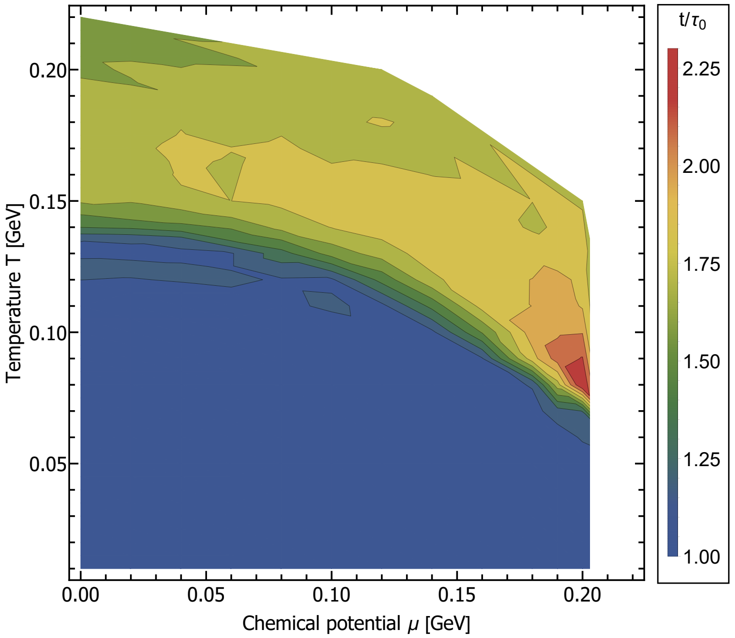

Blum, Jiang, Nahrgang, JMP, Rennecke, Wink, NPA 982 (2019) 871

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  - Observables: quark condensates, fluctuations of conserved charges
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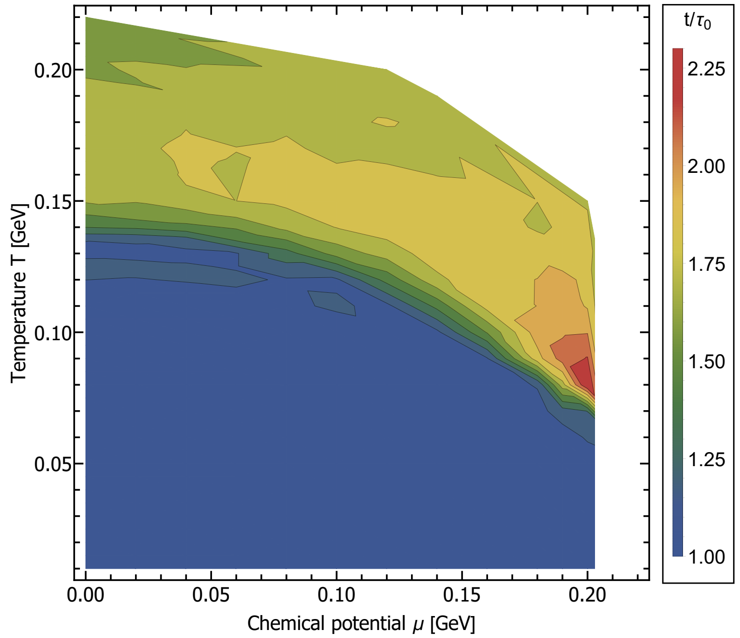
- Transport, hydro, and critical region
  - Real-time correlation functions
  - Transport at finite mu & T
  - Transport coefficients



Blum, Jiang, Nahrgang, JMP, Rennecke, Wink, NPA 982 (2019) 871