Complex flows for complex problems

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Exploring new topics with functional renormalisation, Bad Honnef 2023

GEFÖRDERT VOM
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STRUCTURES CLUSTER OF EXCELLENCE

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EMMI

Outline

- Complexity

- Flows for complex systems

- Flows for complex actions

- Flows at complex frequencies aka real time flows

- Summary & outlook

Complexity

Complex actions

Finite density

spin & mass imbalance

Complex couplings & fields

Complex systems

Emergent degrees of freedom

Non-trivial ground state

Complex phase structure

Competing order

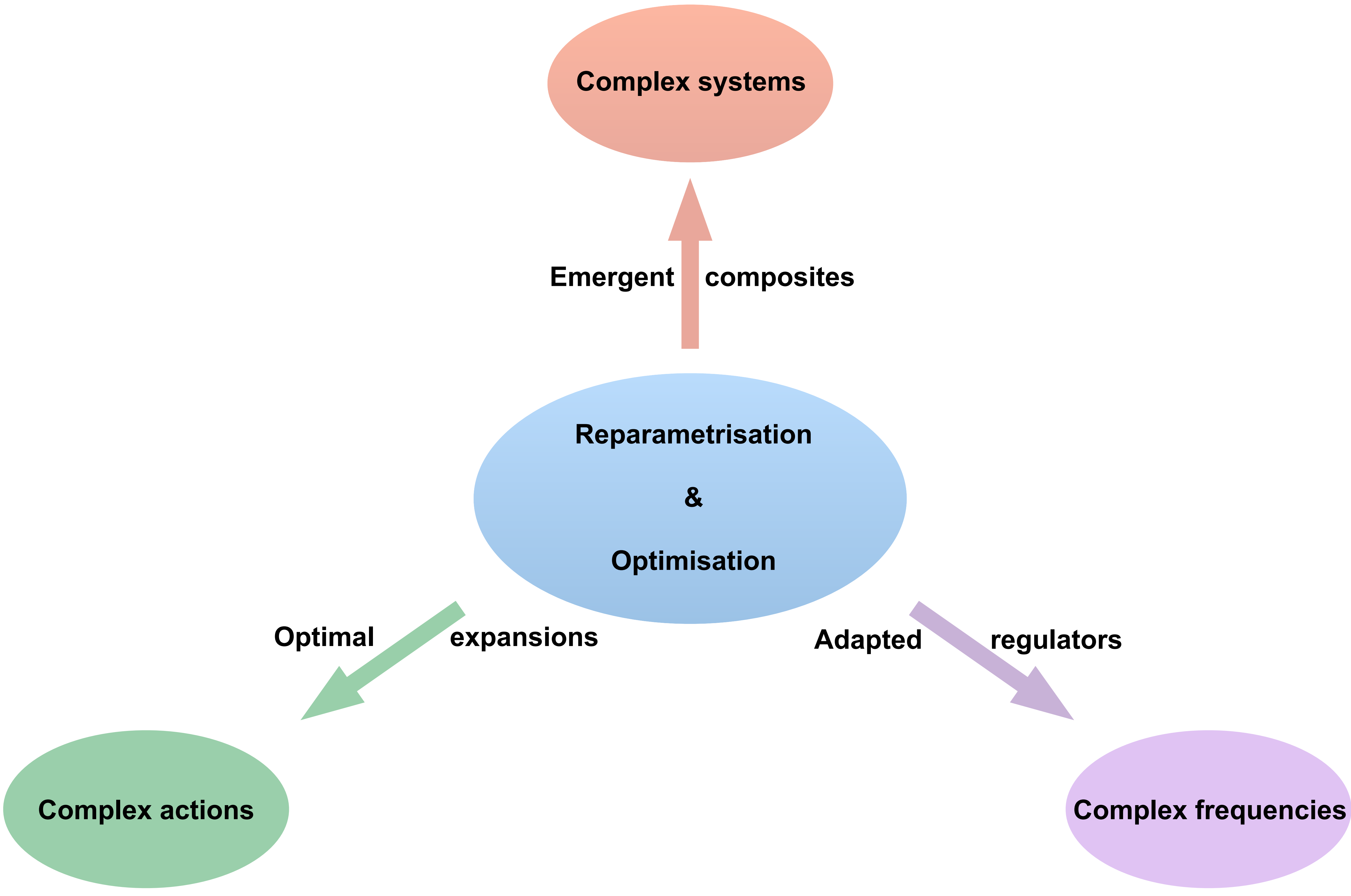
Complex frequencies

Resonances & bound states

Spectral properties of QFT

Real-time
&
non-equilibrium evolution

Complexity & the functional RG



Complexity & the functional RG

Classical action

Current term

$$\int \mathcal{D}\hat\varphi\, e^{-S[\hat\varphi]-\frac{1}{2}\int \hat\varphi\, R_k\, \hat\varphi+\int J\,\hat\varphi}$$

Fundamental field $\hat\varphi$

Cutoff term: **choice of regulator**

Cutoff scale k

Reparametrisation

&

Optimisation

Complexity & the functional RG

Classical action

Current term: choice of composite $\hat{\phi}_k[\hat{\varphi}]$

$$\int \mathcal{D}\hat{\varphi}\, e^{-S[\hat{\varphi}]-\frac{1}{2}\int \hat{\phi}\, R_k\, \hat{\phi}+\int J\, \hat{\phi}}$$

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Spatial momentum

Frequency

Spectral value

Time

Field amplitude

⋮

Reparametrisation

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Reparametrisation

&

Optimisation

Choice of RG functional

Wilsonian effective action

1PI effective action

General flow functional

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Flows for complex systems

choice of composite $\hat{\phi}_k[\hat{\varphi}]$

RG functionals

choice of regulator

General flows for the Wilsonian effective action

$$\partial_t P[\phi] + \frac{\delta}{\delta\phi(x)}\Big(\Psi[\phi]\, P[\phi]\Big) = 0$$

Wegner, J. Phys. C 7 (1974) 2098

Measure: $P[\phi] = e^{-S_{\rm eff}[\phi]}$

Flow of field parametrisation: $\Psi[\phi] \simeq \langle \partial_t \hat{\phi}_k \rangle$

Flows for complex systems

choice of composite $\hat{\phi}_k[\hat{\varphi}]$

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Applications

Gauge theories & gravity: T. R. Morris, Nucl. Phys. B 573, 97 (2000) ←

⋮

Review: O. J. Rosten, Phys. Rept. 511, 177 (2012)

Gradient fRG flows: Sonoda and H. Suzuki, PTEP 2021, 023B05 (2021)
Haruna, JMP, Yamada, in prep

Machine Learning & optimal transport: Cotler, Rezchikov, (2022), arXiv:2202.11737 [hep-th]

⋮

Flows for complex systems

choice of composite $\hat{\phi}_k[\hat{\varphi}]$

RG functionals

choice of regulator

General flows for the 1PI effective action

$$\left(\partial_t + \int_x \dot{\phi}\frac{\delta}{\delta\phi}\right)\Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\, G_k[\phi]\,\partial_t R_k + \mathrm{Tr}\, G_k[\phi]\,\frac{\delta\dot{\phi}}{\delta\phi} R_k$$

JMP, AoP 322 (2007) 2831

Flow of field parametrisation: $\dot{\phi} = \langle \partial_t \hat{\phi}_k \rangle$

Propagator of the composite: $G_k[\phi](x,y) = \langle \hat{\phi}(x)\hat{\phi}(y)\rangle_c[\phi]$

$$= \frac{1}{\Gamma_k^{(2)}[\phi] + R_k}$$

Flows for complex systems

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General flow functional $\mathcal{O}_k$

$$\partial_t\mathcal{O}_k[\phi] \simeq -\frac{1}{2}\mathrm{Tr}\left[G_k[\phi]\,\partial_t R_k\,G_k[\phi]\frac{\delta^2}{\delta\phi^2}\right]\mathcal{O}_k[\phi]$$

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JMP, AoP 322 (2007) 2831

Example: $\mathcal{O}_k = G_k + \phi^2$

$$\partial_t G_k \simeq -\frac{1}{2}\mathrm{Tr}\left[G_k\,\partial_t R_k\, G_k\right] G_k^{(2)} - \frac{1}{2}\mathrm{Tr}\left[G_k\,\partial_t R_k\, G_k\right]$$

Talk of Stefan Flörchinger

Flows for complex systems

choice of composite $\hat{\phi}_k[\hat{\varphi}]$

RG functionals

choice of regulator

General flows for the Wilsonian effective action

$$\partial_t P[\phi] + \frac{\delta}{\delta \phi(x)} \Big(\Psi[\phi]\, P[\phi] \Big) = 0$$

General flows for the 1PI effective action

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Numerics for complex systems

Talks

Nicolas Wink

General flow functional $\mathcal{O}_k$

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Posters

Nicolas Hendricks

Adrian Königstein

Franz Sattler

Niklas Zorbach

1PI Flows for complex systems

(Bi-)linear emergent composites

Gies, Wetterich, PRD 65 (2002) 065001

Zero modes: Fu, JMP, Rennecke, PRD 101, (2020) 054032

Wetterich, Z. Phys. C 72, 139 (1996)

Flörchinger, Wetterich, PLB 680, 371 (2009)

Applications and further developments mostly in QCD

Since ~one decade common practice/tool in QCD on a ‘daily basis’

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Applications and further developments mostly in QCD

Phase fields in the Luttinger liquid

Talk Nicolas Dupuis

Composite Higgs & BSM physics

Poster Álvaro Pastor Gutiérrez

fermionic composites in the Hubbard model

Talk Walter Metzner

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JMP, AoP 322 (2007) 283

Essential RG: Baldazzi, Ben Alì Zinati, Falls, SciPost Phys 13 (2022) 4, 085

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Applications

Goldstonisation: **Lamprecht, JMP, Master work Lamprecht (2007)**

$$(\phi_1, \phi_2) \xrightarrow{\dot{\phi}_k} \sqrt{\phi_1^2 + \phi_2^2}\; e^{i\,\theta}$$

Isaule, Birse, Walet, PRD 98 (2018) 144502
AP 412 (2020) 168006

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$$(\phi_1,\phi_2) \xrightarrow{\dot\phi_k} \sqrt{\phi_1^2+\phi_2^2}\; e^{i\,\theta}$$

Flowing fields & optimised RG flows

Talk Friederike Ihssen

Expansion about ground state & optimisation

Essential RG: Baldazzi et al, SciPost Phys 13 (2022) 4, 085
Baldazzi, Falls, *Universe* 7 (2021) 8, 294

Remove inessential operators

$$\Gamma^{(2)}[\phi] \overset{\dot\phi}{=} p^2 + m_\phi^2(\phi)$$

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Flows for complex actions

Complex couplings & fields

Lee-Yang zeros

$$\hat\varphi \in \mathbb{R}$$

Simple example in d dimensions: $$Z[J] \simeq \int \mathcal{D}\hat\varphi \, e^{-S[\hat\varphi] - \frac{1}{2}\int \hat\varphi \, R_k \, \hat\varphi + \int J \, \hat\varphi}$$

$$J \in \mathbb{C}$$

Ihssen, JMP, 2207.10057

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$\hat{\varphi} \in \mathbb{R}$

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Complex effective action/potential

General flows for the Wilsonian effective action

- **Polchinski flow**
  kernel classical propagator
- **Wegner flow**
  kernel full propagator at minimum

$$\partial_t P[\phi] + \frac{\delta}{\delta\phi(x)}\Big(\Psi[\phi]\, P[\phi]\Big) = 0$$

General flows for the 1PI effective action

- **Complex Legendre transform, no optimisation**

$$\left(\partial_t + \int_x \dot{\phi}\frac{\delta}{\delta\phi}\right)\Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\, G_k[\phi]\,\partial_t R_k + \mathrm{Tr}\, G_k[\phi]\,\frac{\delta\dot{\phi}}{\delta\phi}R_k$$

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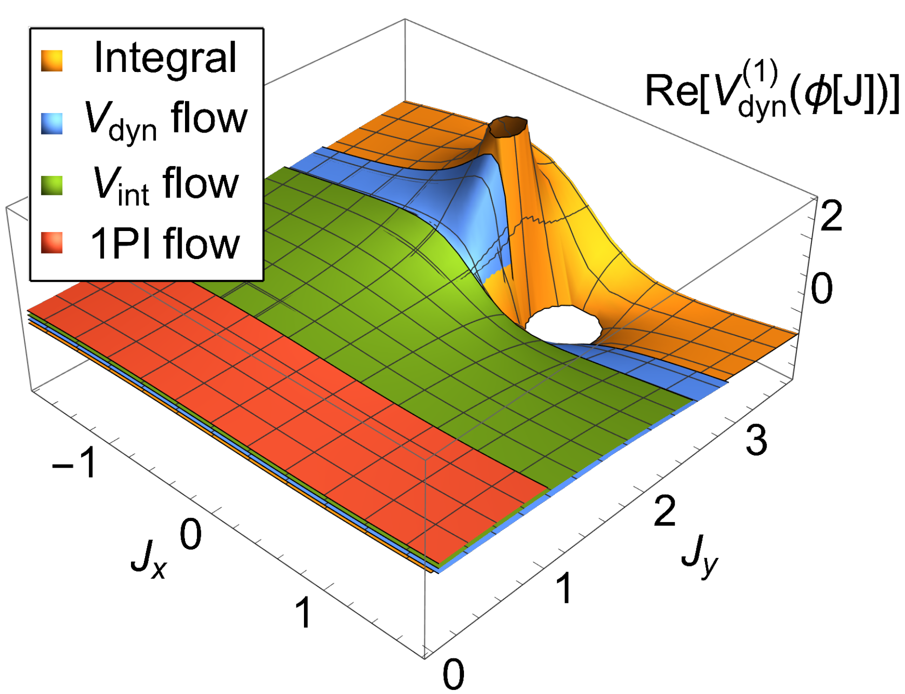

General flows for the 1PI effective action

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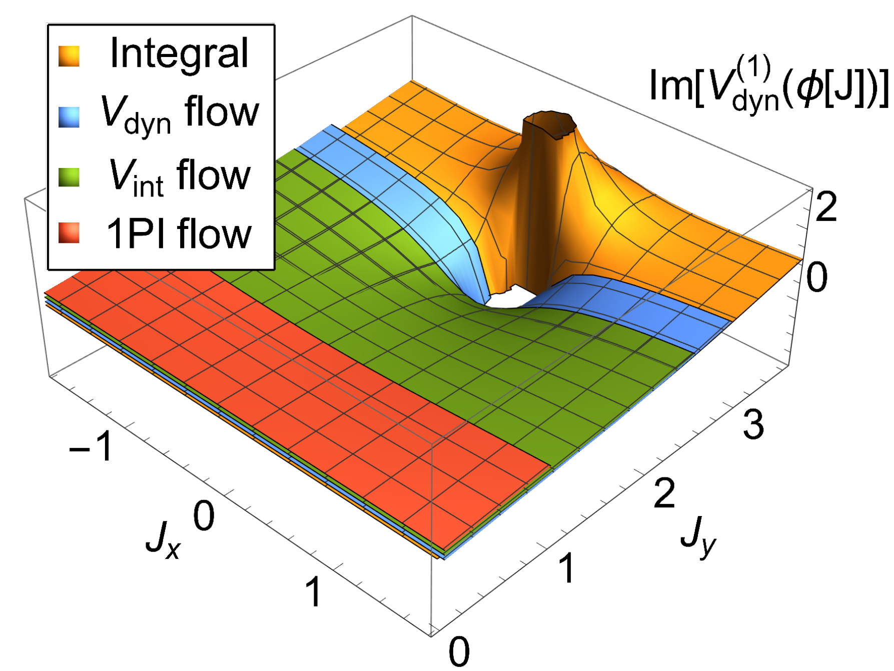


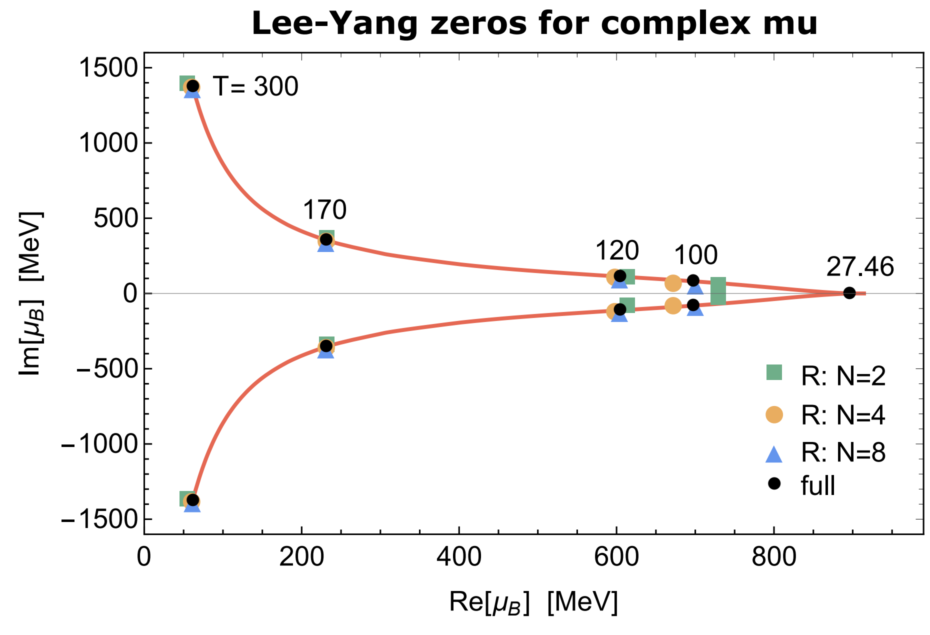
Ihssen, JMP, 2207.10057

Flows for complex actions

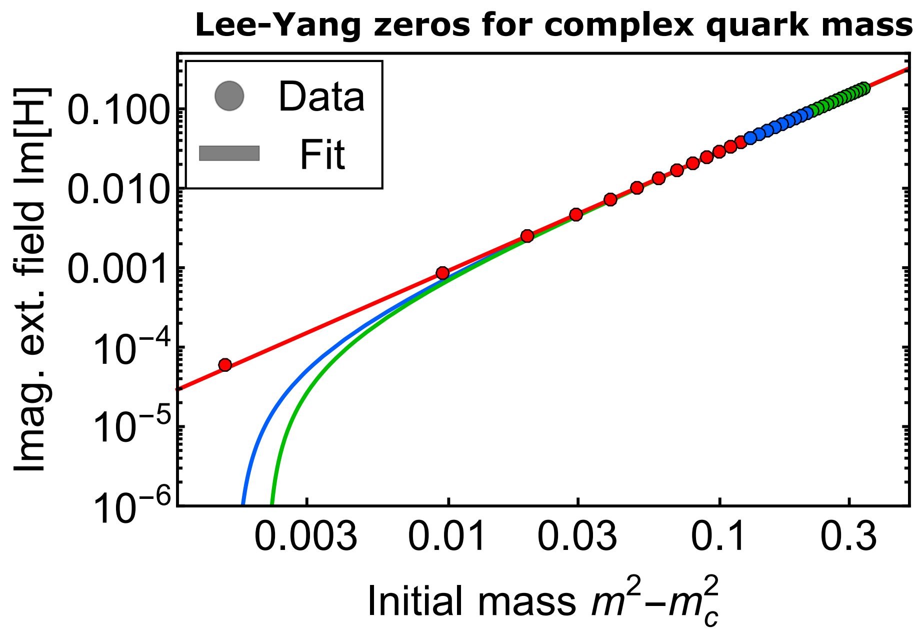
Complex couplings & fields

Lee-Yang zeros

Connelly, Johnson, Rennecke, Skokov, PRL 125 (2020) 191602



Mukherjee, Rennecke, Skokov, PRD 105 (2022) 014026



Ihssen, JMP, 2207.10057

Scaling at Lee-Yang-zeros: Talk Gregory Johnson

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Real time flows

Spectral representation

$$G_k(p) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \, \frac{\lambda \, \rho_k(\lambda, \vec{p})}{\lambda^2 + p_0^2}$$

Non-equilibrium: $\rho \simeq i \, \langle [\phi(x), \phi(y)] \rangle$ $F \simeq \langle \{\phi(x), \phi(y)\} \rangle$

Braun, Chen, Fu, Geißel, Horak, Huang, Ihssen, JMP, Reichert, Rennecke, Tan, Töpfel, Wessely, Wink, arXiv:2206.10232

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fQCD, Reichert, arXiv:2206.10232

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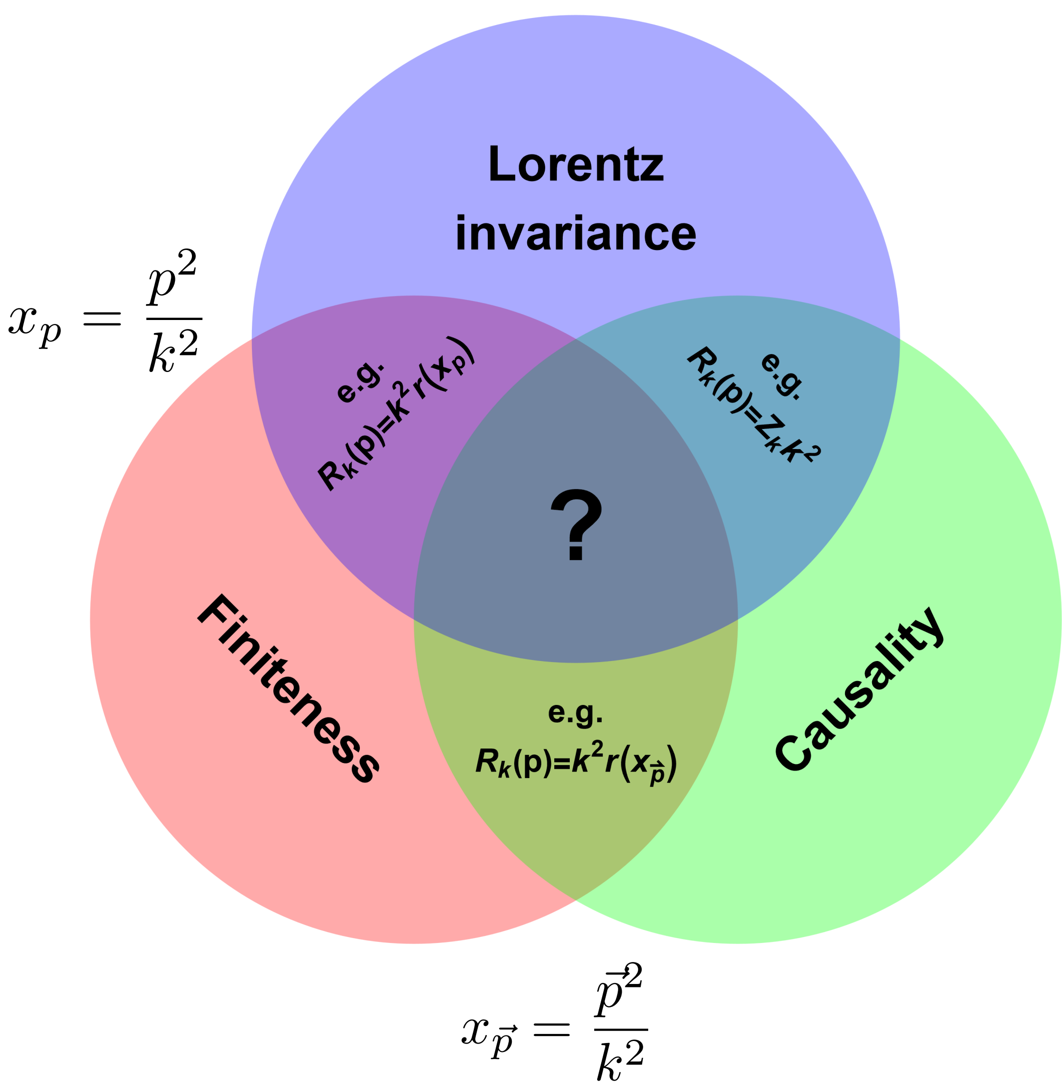
Symmetry preserving regulators



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Real time flows

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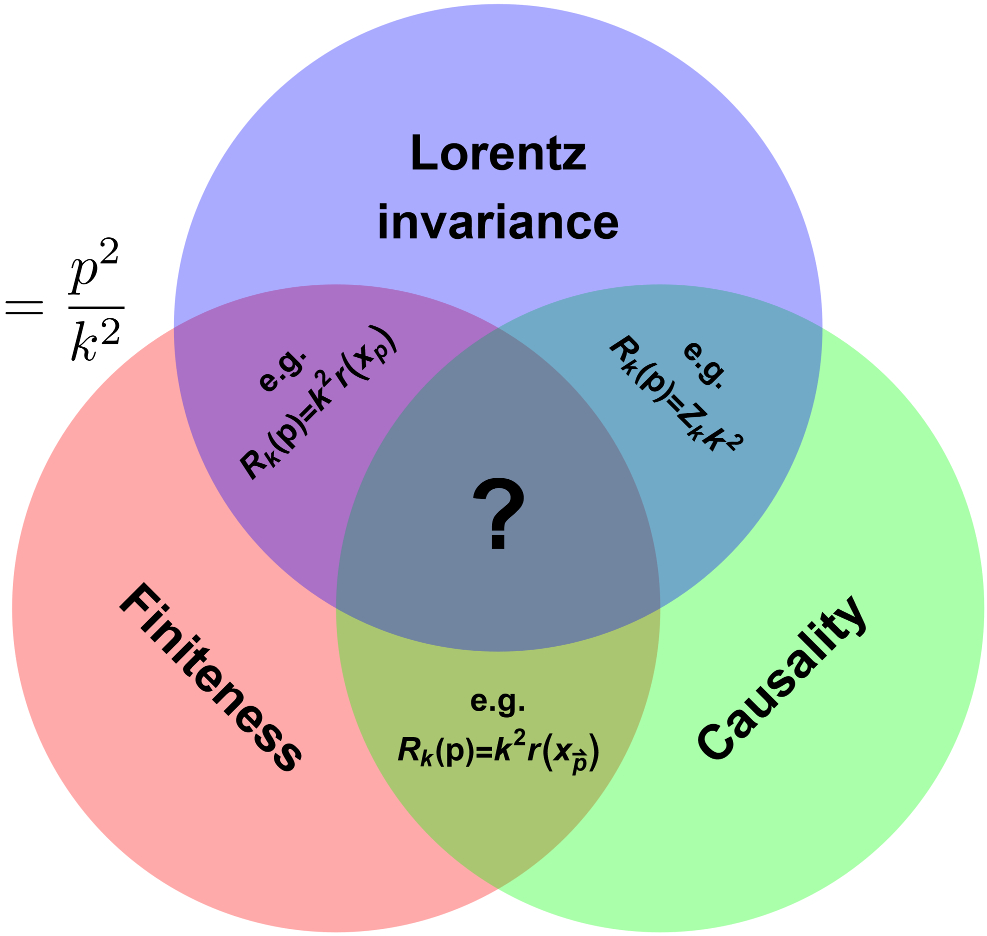
Symmetry preserving regulators

$$x_p = \frac{p^2}{k^2}$$



$$x_{\vec{p}} = \frac{\vec{p}^{\,2}}{k^2}$$

Lorentz invariance & Causality

Callan-Symanzik regulator

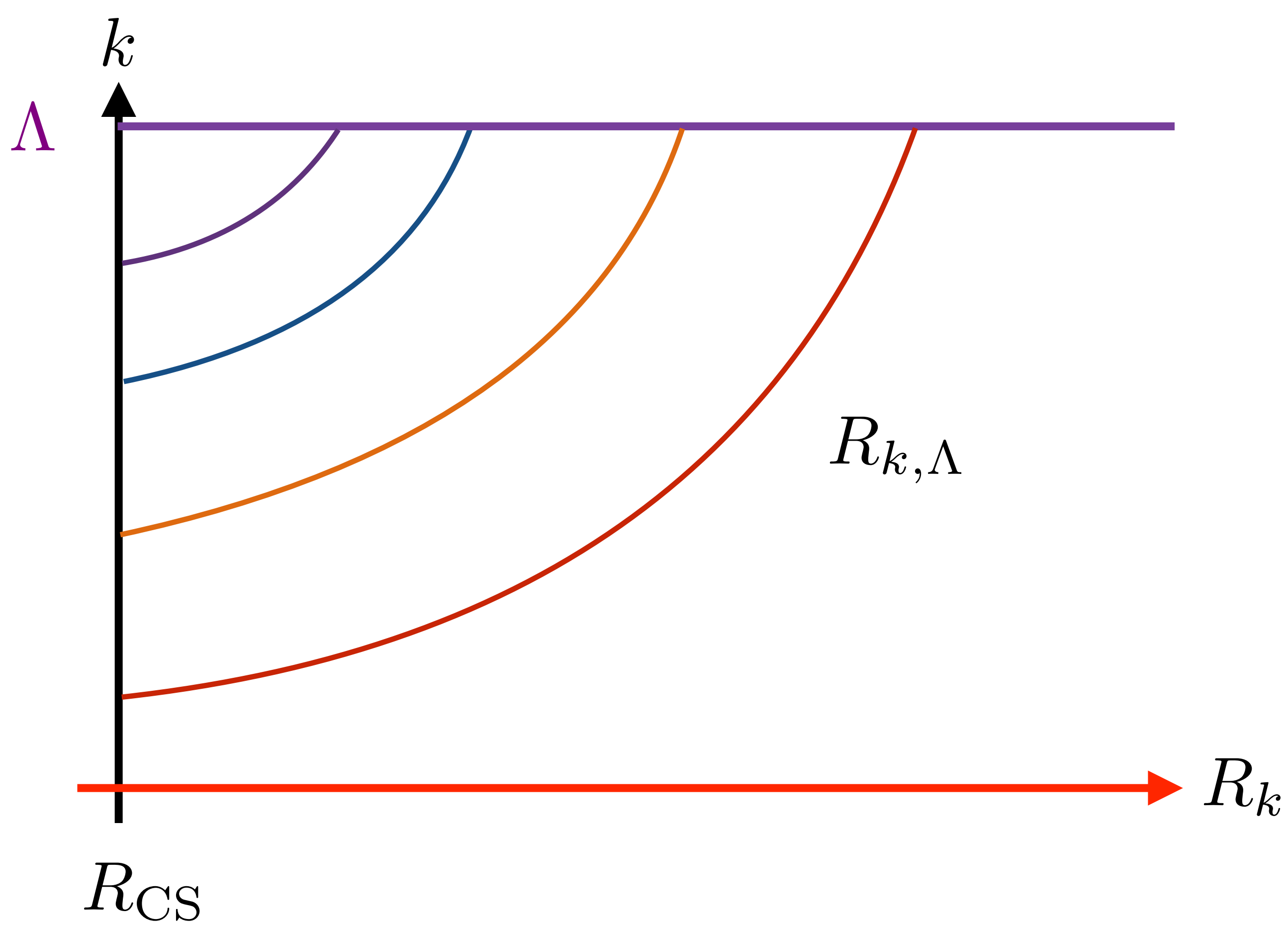
$$R_{\mathrm{CS}} = Z_\phi k^2$$

Finiteness lost

fQCD, Reichert, arXiv:2206.10232

Renormalised renormalisation group flows

Idea



Composite UV-IR fRG flow

$$\left(\partial_t\big|_\Lambda + \mathcal{D}_k\,\partial_{t_\Lambda}\right)\Gamma_{k,\Lambda} = \frac{1}{2}\mathrm{Tr}\,G^\Phi_{k,\Lambda}\left(\partial_t\big|_\Lambda R^\Phi_{k,\Lambda} + \mathcal{D}_k\,\partial_{t_\Lambda}R^\Phi_{k,\Lambda}\right)$$

fQCD, Reichert, arXiv:2206.10232

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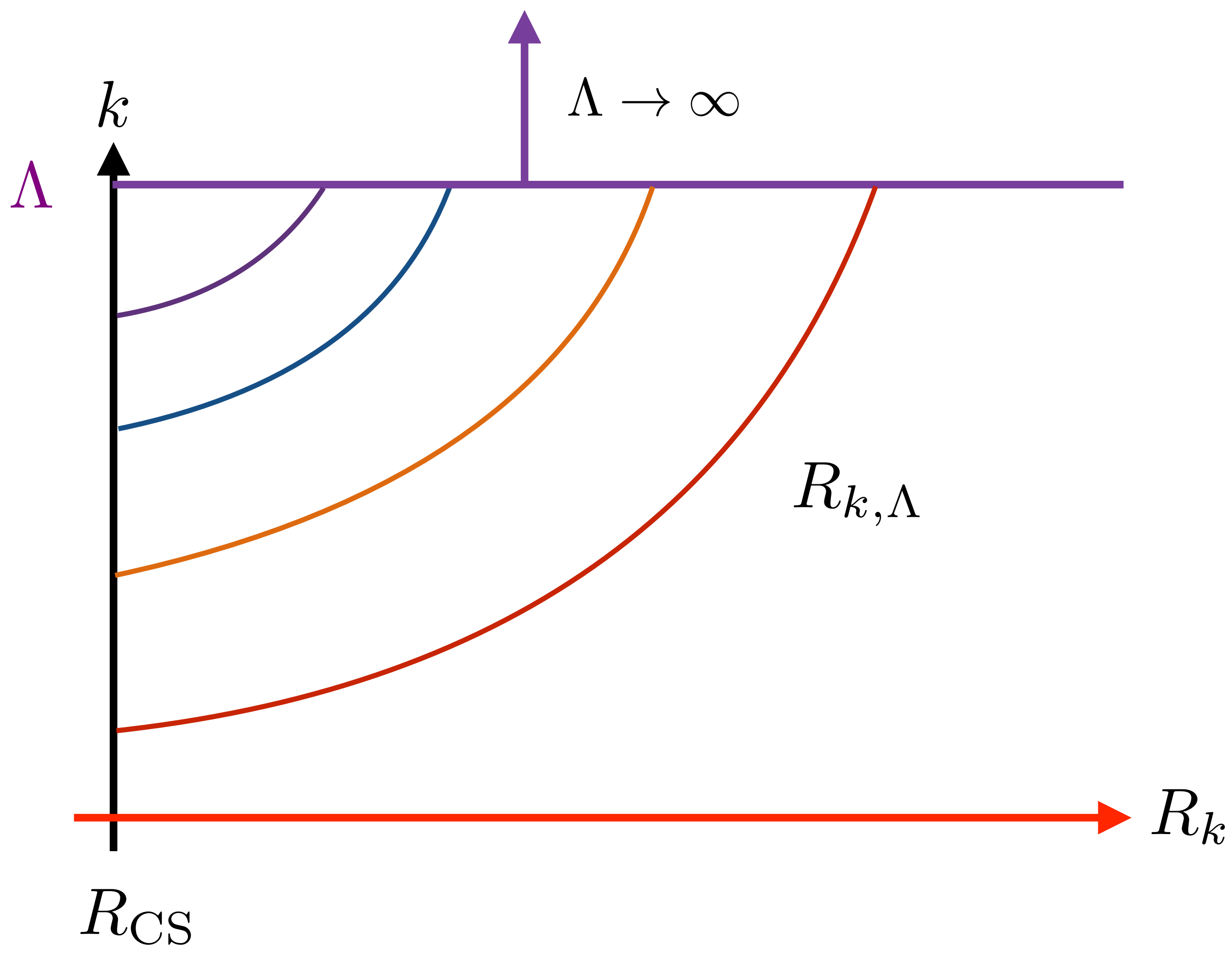

Limes $\Lambda \to \infty$

$$\mathcal{D}_k\, \partial_{t_\Lambda} \Gamma_{k,\Lambda} - \frac{1}{2} \mathrm{Tr}\, G^{\phi}_{k,\Lambda}\, \mathcal{D}_k\, \partial_{t_\Lambda} R^{\phi}_{k,\Lambda} \to \partial_t S_{\rm cl}[\phi]$$

Local

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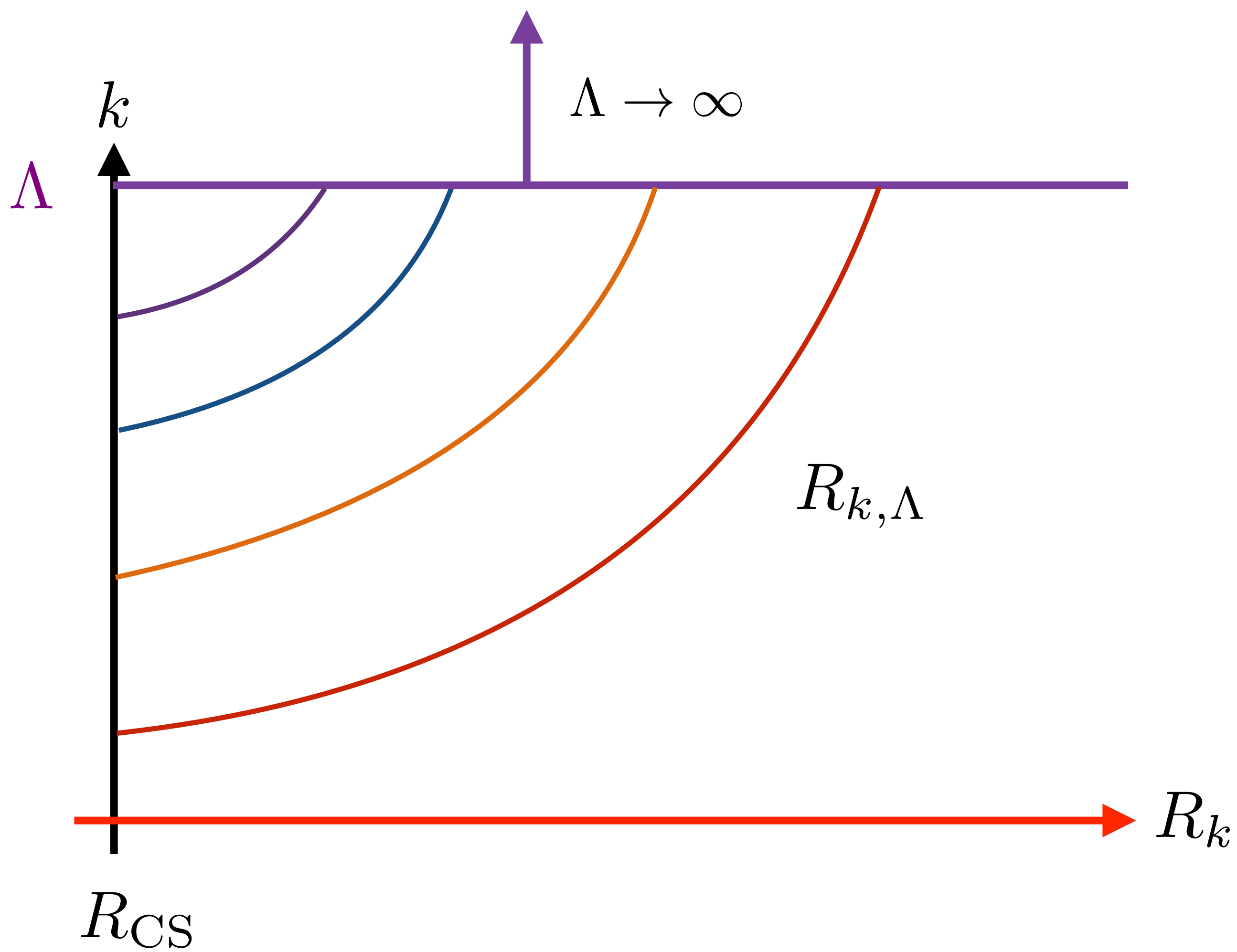

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Local

fRG flow with flowing renormalisation

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\, G_\phi[\phi]\, \partial_t R^\phi - \partial_t S_{\rm ct}[\phi]$$

fQCD, Reichert, arXiv:2206.10232

Renormalised renormalisation group flows

Poster Jonas Wessely

Spectral CS-fRG flow with flowing renormalisation

$$\partial_t \Gamma_k[\phi] = \mathrm{Tr}\, G_\phi[\phi]\, k^2 - \partial_t S_{\mathrm{ct}}[\phi]$$

$$G_k[\phi](p) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi}\, \frac{\lambda\, \rho_k[\phi](\lambda, \vec{p})}{\lambda^2 + p_0^2}$$

fQCD, Reichert, arXiv:2206.10232

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Flowing on-shell renormalisation

$$\lim_{\Lambda \to \infty} \Gamma^{(2)}_{k,\Lambda}[\bar{\phi}](p)\Big|_{p_0^2 = -\mu^2(k)} = -k^2$$

Controlled UV-limit

fQCD, Reichert, arXiv:2206.10232

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Controlled UV-limit | **Controlled UV-limit**

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Controlled UV-limit **Controlled UV-limit**

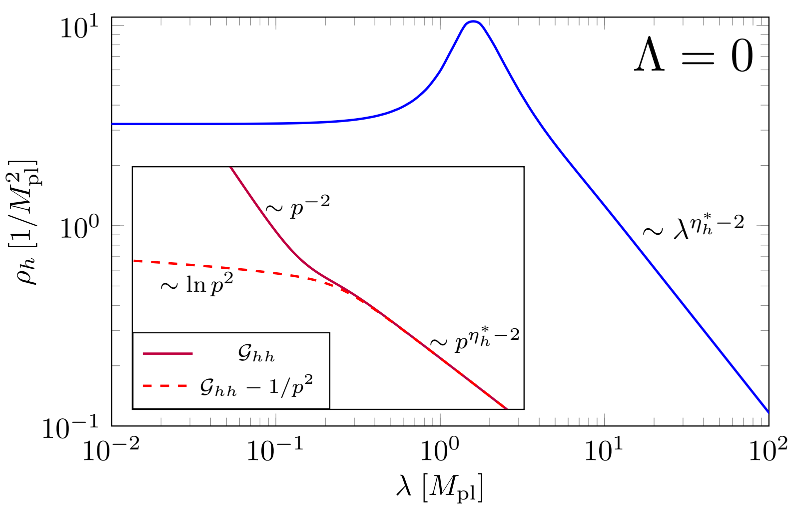
Similarly for wave function and coupling renormalisation

fQCD, Reichert, arXiv:2206.10232

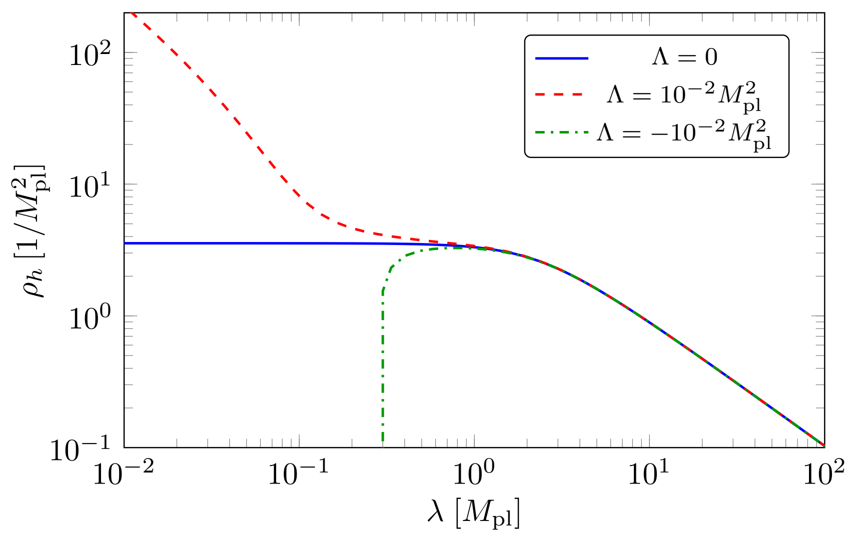
Renormalised renormalisation group flows

Spectral properties of Lorentzian asymptotically safe gravity

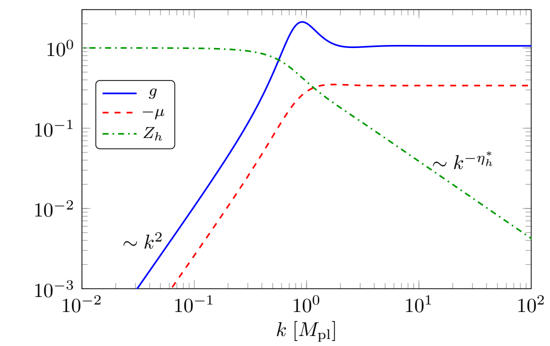
Graviton spectral function



Dependence on Λ



Asymptotically safe couplings

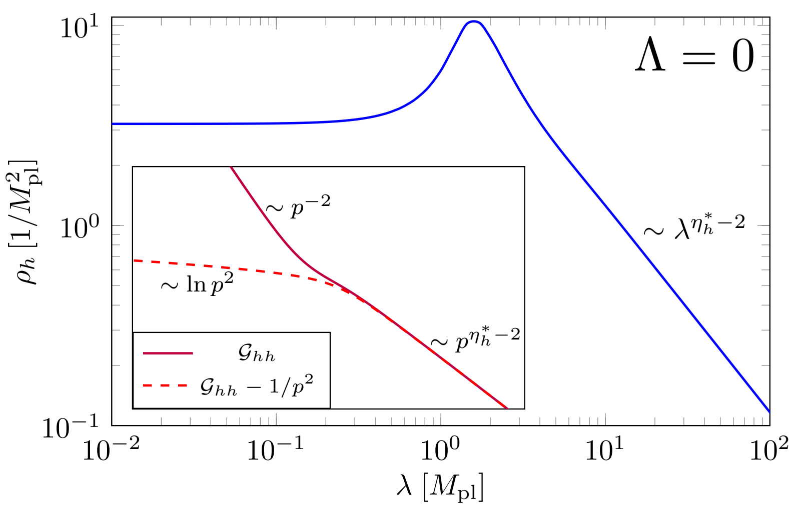


Fehre, Litim, JMP, Reichert, PRL 130 (2023) 081501

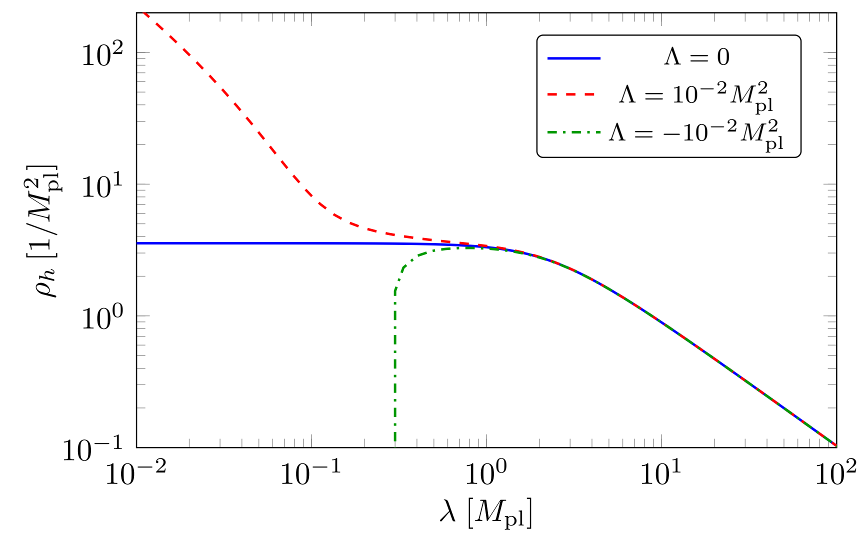
Renormalised renormalisation group flows

Spectral properties of Lorentzian asymptotically safe gravity

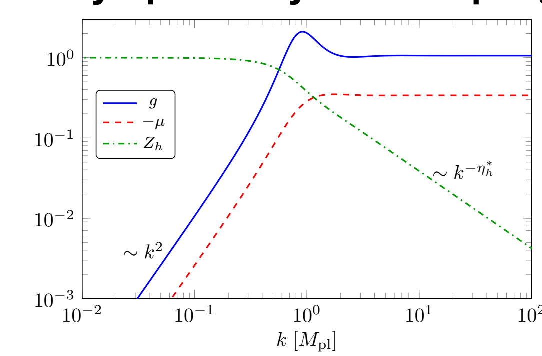
Graviton spectral function



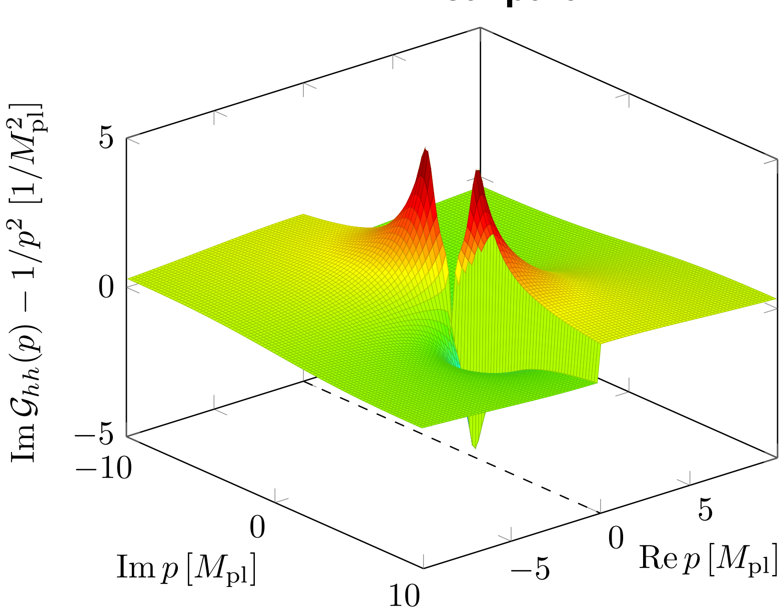
Dependence on Λ



Asymptotically safe couplings

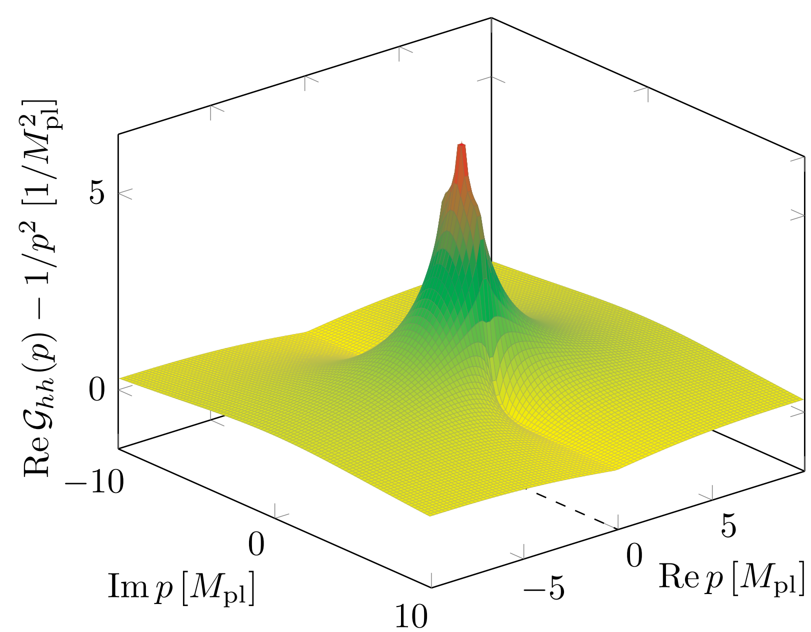


Real part



$$\langle h_{TT}(p) h_{TT}(-p) \rangle$$

Imaginary part



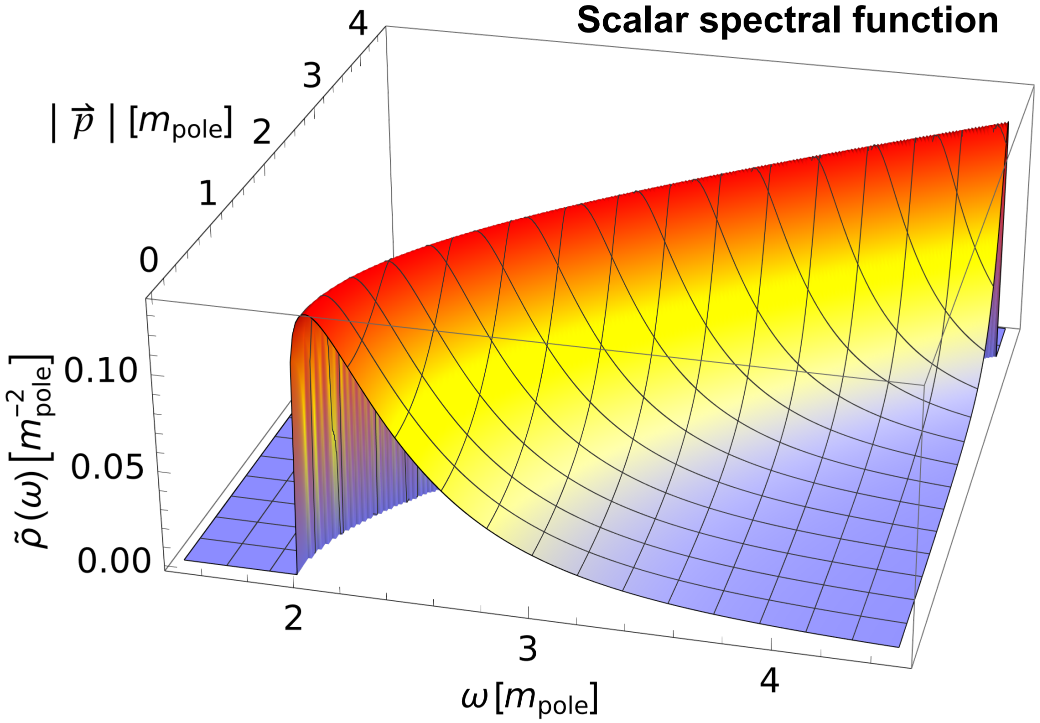
Fehre, Litim, JMP, Reichert, PRL 130 (2023) 081501

Renormalised renormalisation group flows

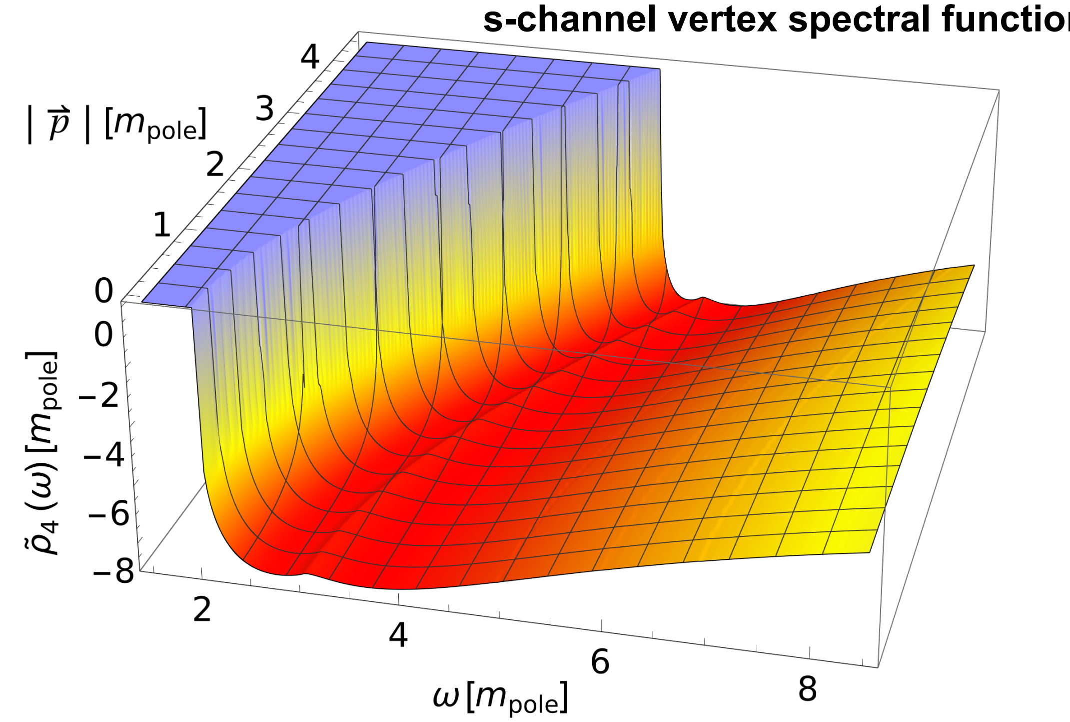
Poster Jonas Wessely

Spectral flows in the scalar theory

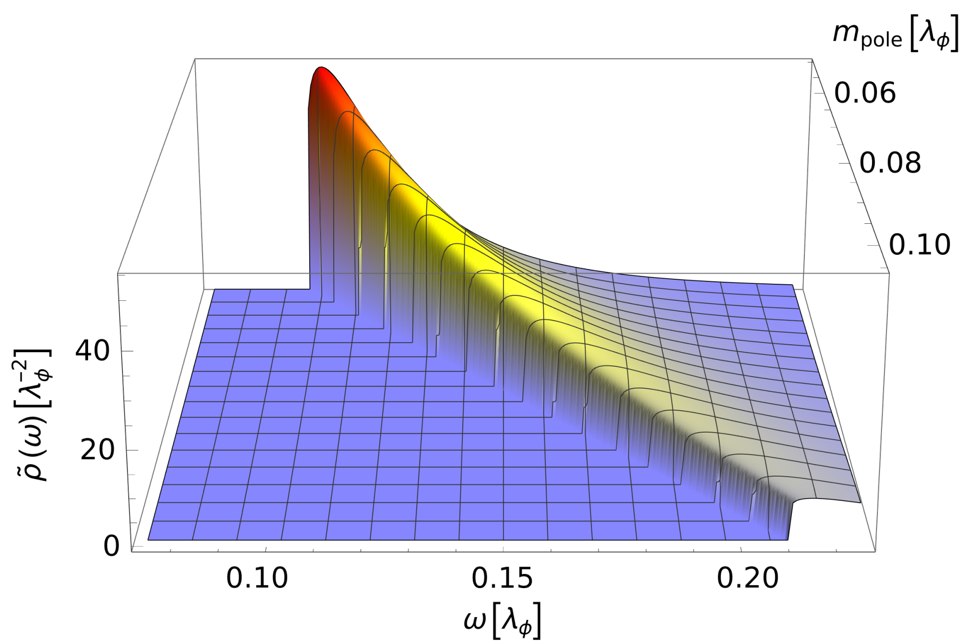
Scalar spectral function

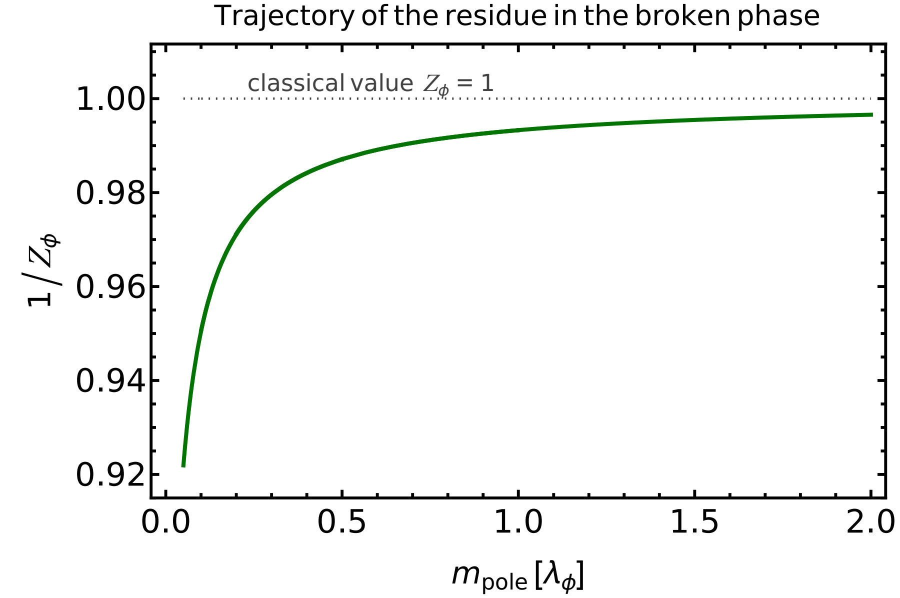


s-channel vertex spectral function



Mass dependence of scalar spectral function





Horak, Ihssen, JMP, Wessely, Wink, arXiv:2303.16719

Renormalised renormalisation group flows

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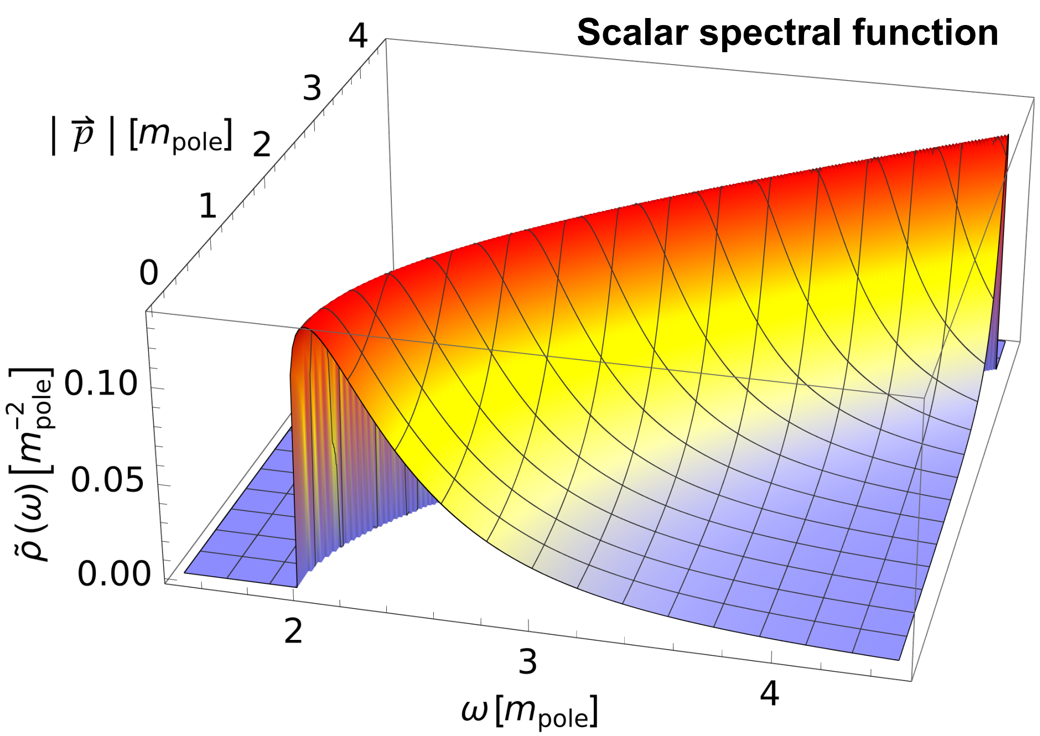
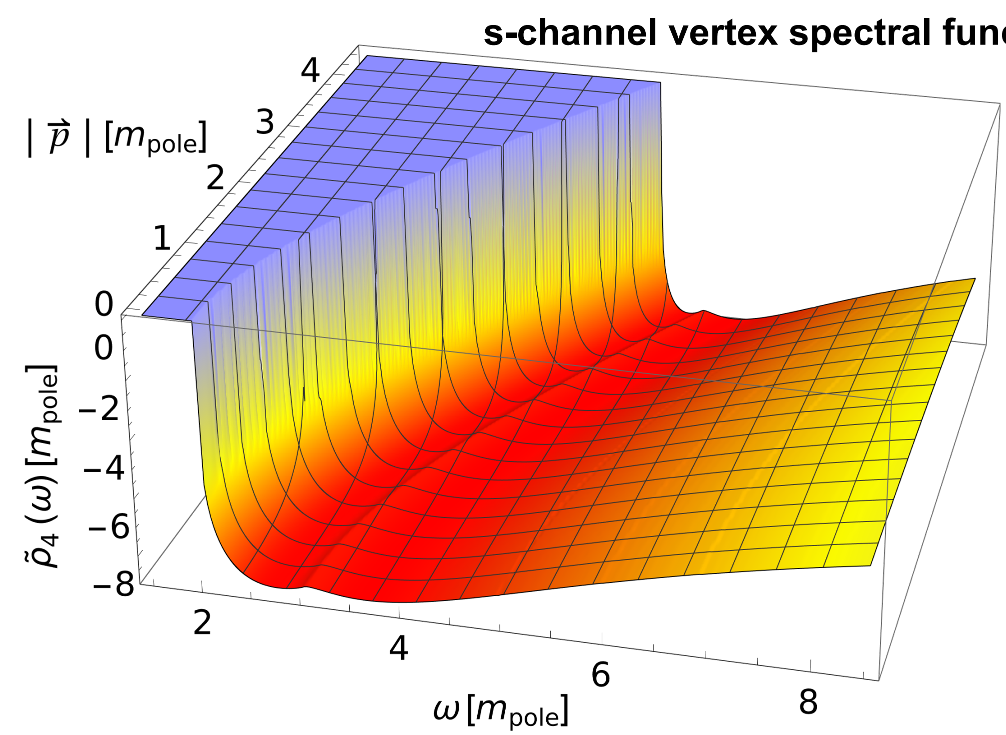
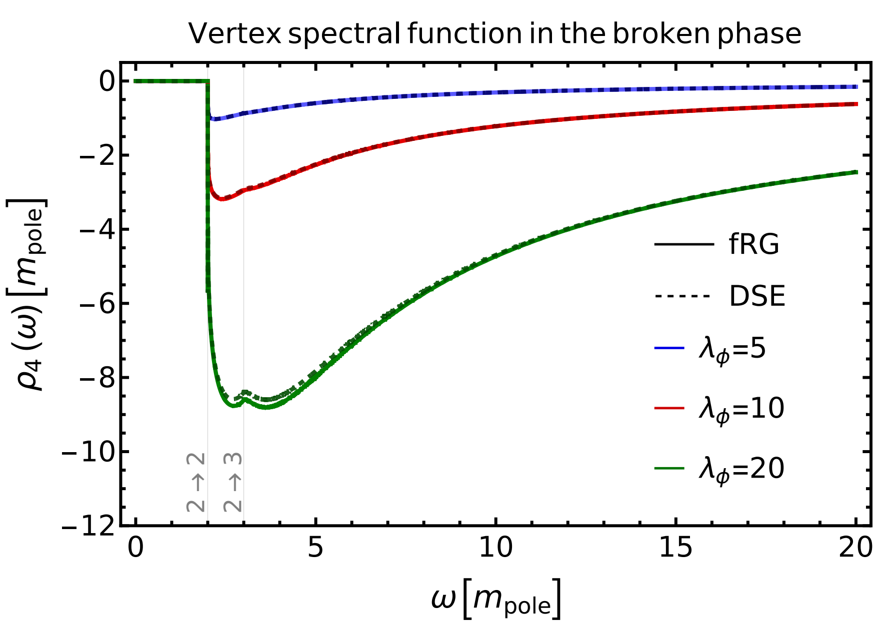
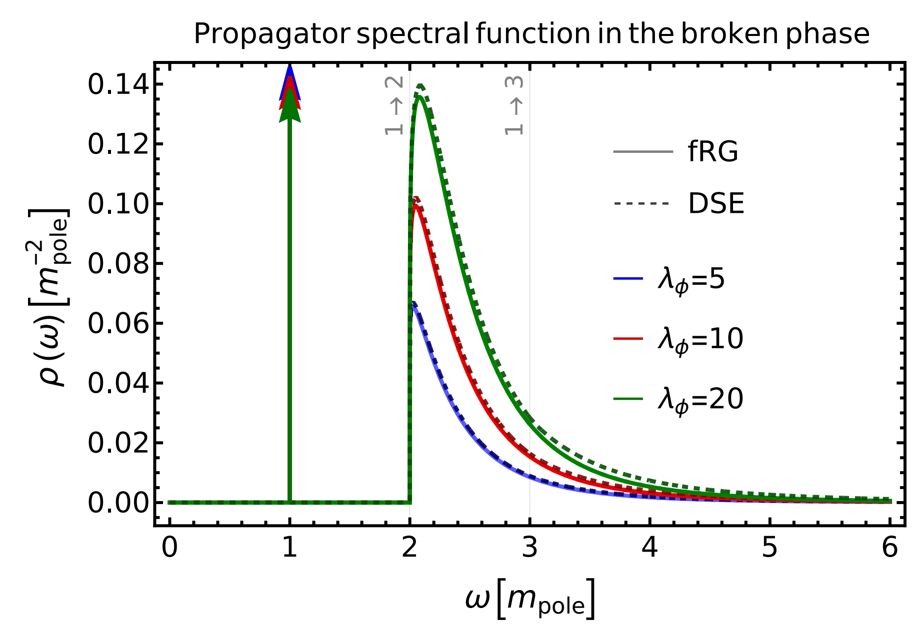




Benchmark test with spectral DSE results





DSE: Horak, JMP, Wink, PRD 102 (2020) 125016

Horak, Ihssen, JMP, Wessely, Wink, arXiv:2303.16719

Outline

- Complexity
- Flows for complex systems
- Flows for complex actions
- Flows at complex frequencies aka real time flows
- Summary & outlook

Complexity

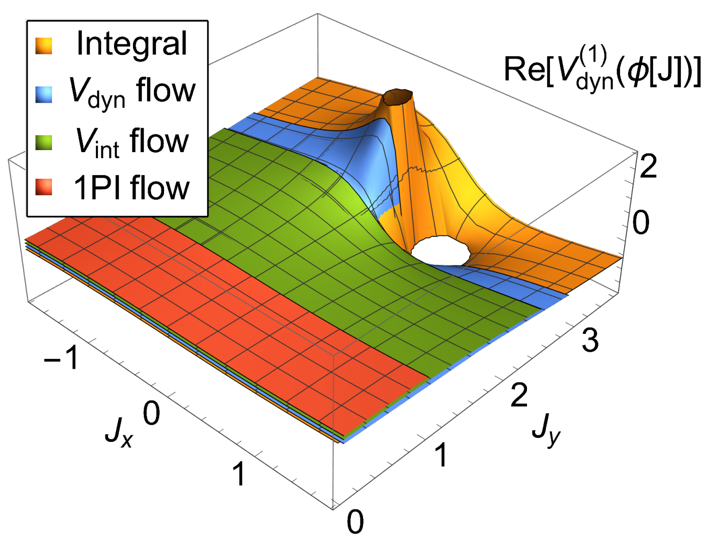
Complex actions

Complex systems

Complex frequencies

Generalised flows for general functionals

Complex potential



Ihssen, JMP, 2207.10057

Talk

Gregory Johnson

Complexity

Complex actions

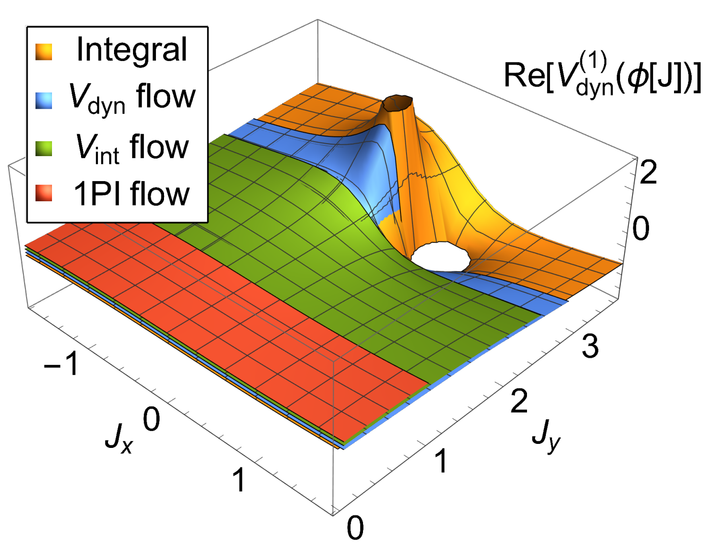
Complex systems

Complex frequencies

Generalised flows for general functionals

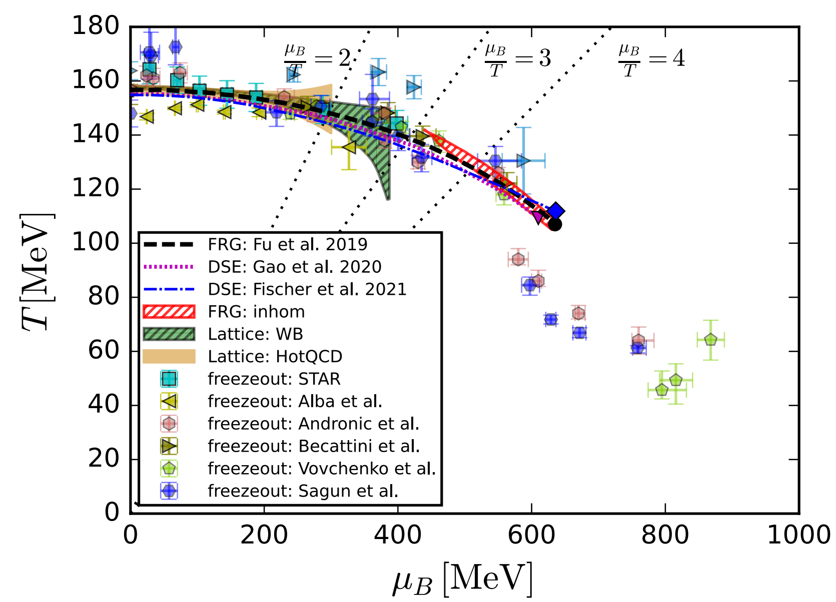
$$\left(\partial_t + \int_x \dot\phi \frac{\delta}{\delta\phi}\right)\Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\, G_k[\phi]\,\partial_t R_k + \mathrm{Tr}\, G_k[\phi]\,\frac{\delta\dot\phi}{\delta\phi} R_k$$

Complex potential



Ihssen, JMP, 2207.10057

Phase structure of QCD



Fu, JMP, Rennecke, PRD 101, (2020) 054032

Talk

Gregory Johnson

Talks/Poster

Nicolas Dupuis

Friederike Ihssen

Walter Metzner

Álvaro Pastor Gutiérrez

Complexity

Complex actions

Complex systems

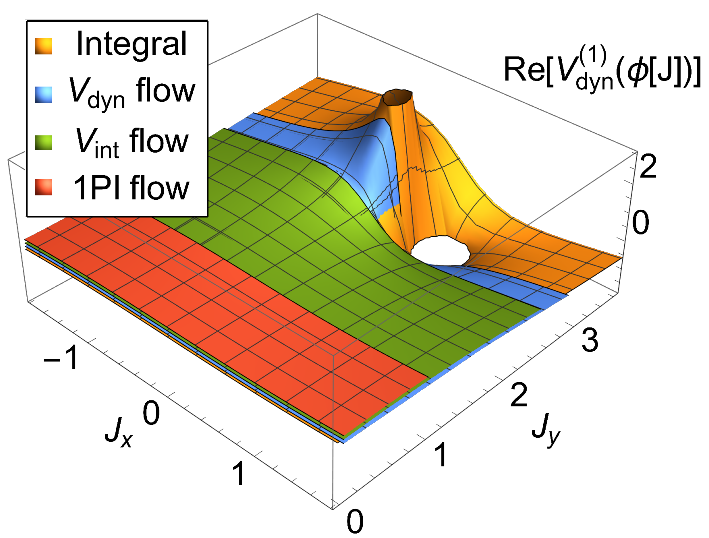
Complex frequencies

Generalised flows for general functionals

$$\left(\partial_t + \int_x \dot{\phi}\,\frac{\delta}{\delta\phi}\right)\Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\,G_k[\phi]\,\partial_t R_k + \mathrm{Tr}\,G_k[\phi]\,\frac{\delta\dot{\phi}}{\delta\phi}R_k$$

$$\partial_t\Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\,G_\phi[\phi]\,\partial_t R^\phi - \partial_t S_{\rm ct}[\phi]$$

Complex potential



Ihssen, JMP, 2207.10057

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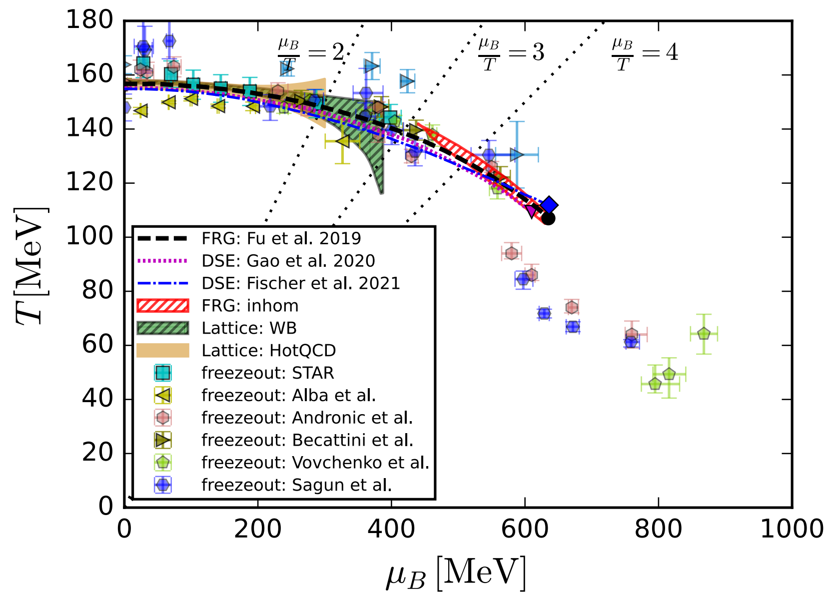
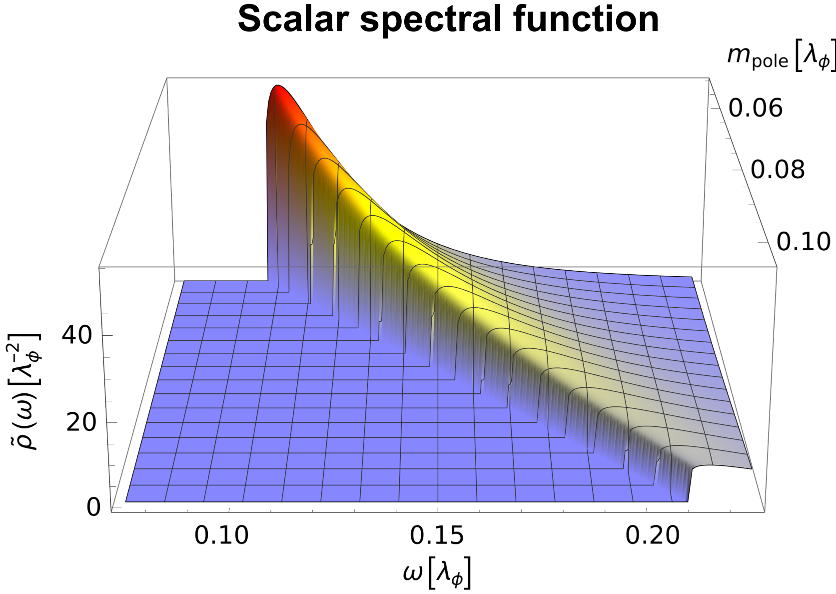


Fu, JMP, Rennecke, PRD 101, (2020) 054032

Scalar spectral function



Horak, Ihssen, JMP, Wessely, Wink, arXiv:2303.16719

Talk
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Laura Batini
Marcel Horstmann
Aleksandr Mikheev
Viktoria Noel
Jonas Wessely