New Insights on the Phase Structure with Functional QCD

Jan M. Pawlowski

Universität Heidelberg & ExtreMe Matter Institute

GSI Darmstadt, April 13th 2025

fqcd-collaboration.github.io

Plenary talk QM 2025 Rennecke

STRUCTURES CLUSTER OF EXCELLENCE

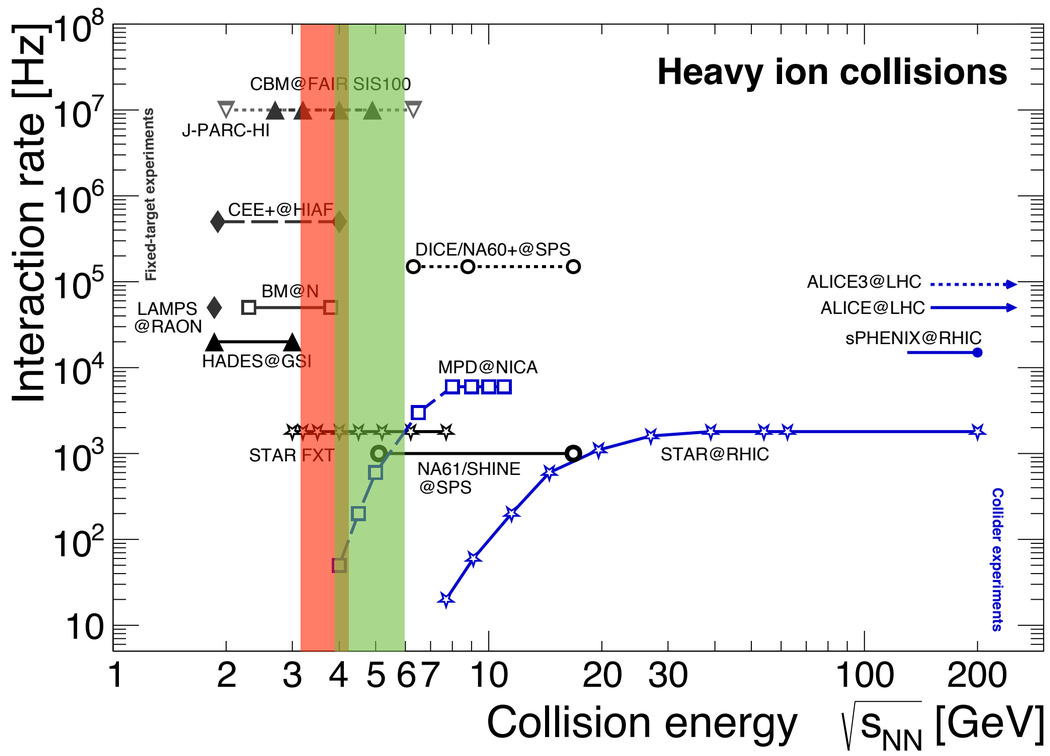
UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

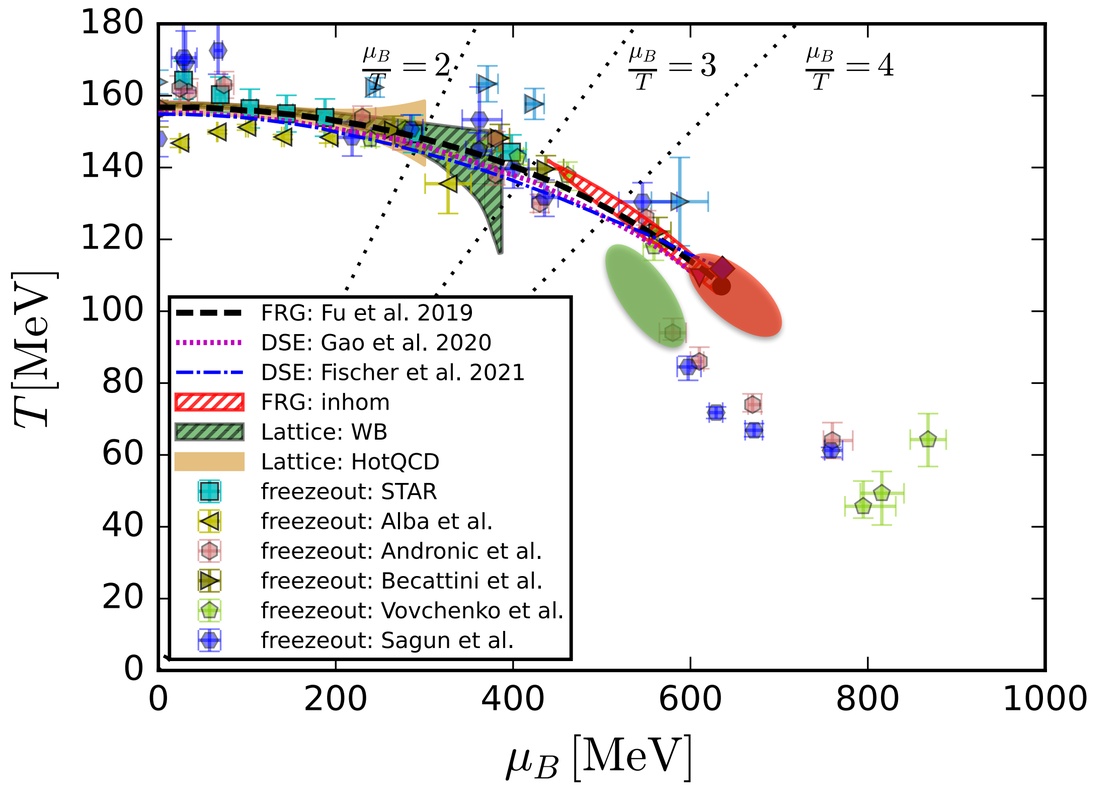
Experimental & Theoretical Landscape

experimental landscape

theoretical landscape

Onset of new physics (CEP)





Peak of kurtosis at the freeze-out line

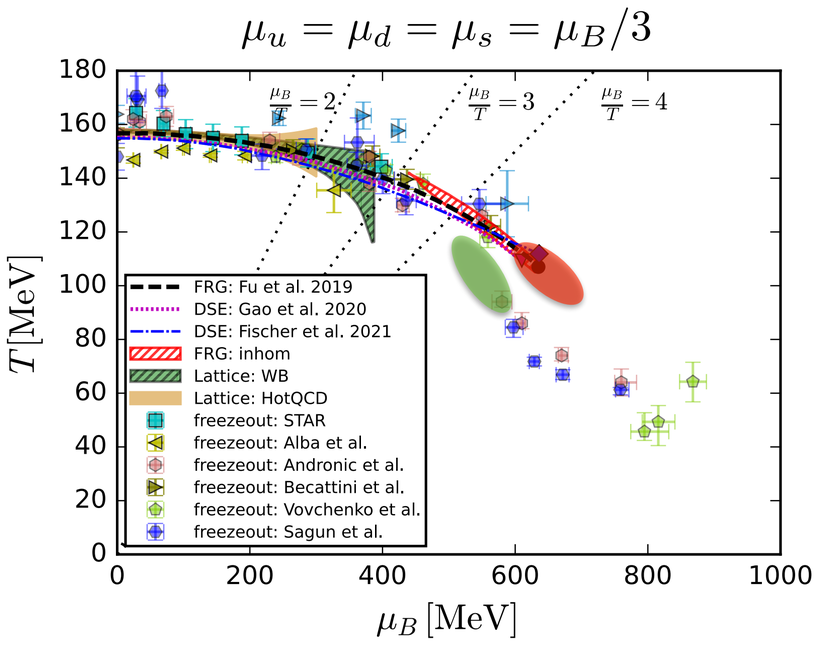
Galatyuk, A982 (2019) update 2025; CBM, EPJA 53 3 (2017) 60

Fu, JMP, Rennecke, PRD 101 (2020) 054032

Gao, JMP, PLB 820 (2021) 136584

Gunkel, Fischer, PRD 104 (2021) 054022

Phase structure of QCD and the CEP



Functional QCD: CEP estimate

fRG: **Fu, JMP, Rennecke, PRD 101 (2020) 054032**

DSE: **Gao, JMP, PLB 820 (2021) 136584**
Gunkel, Fischer, PRD 104 (2021) 054022

$$(\mu_B\,,\,T)_{\rm CEP} \sim (600-650\,,\,105-115)\ {\rm MeV}$$

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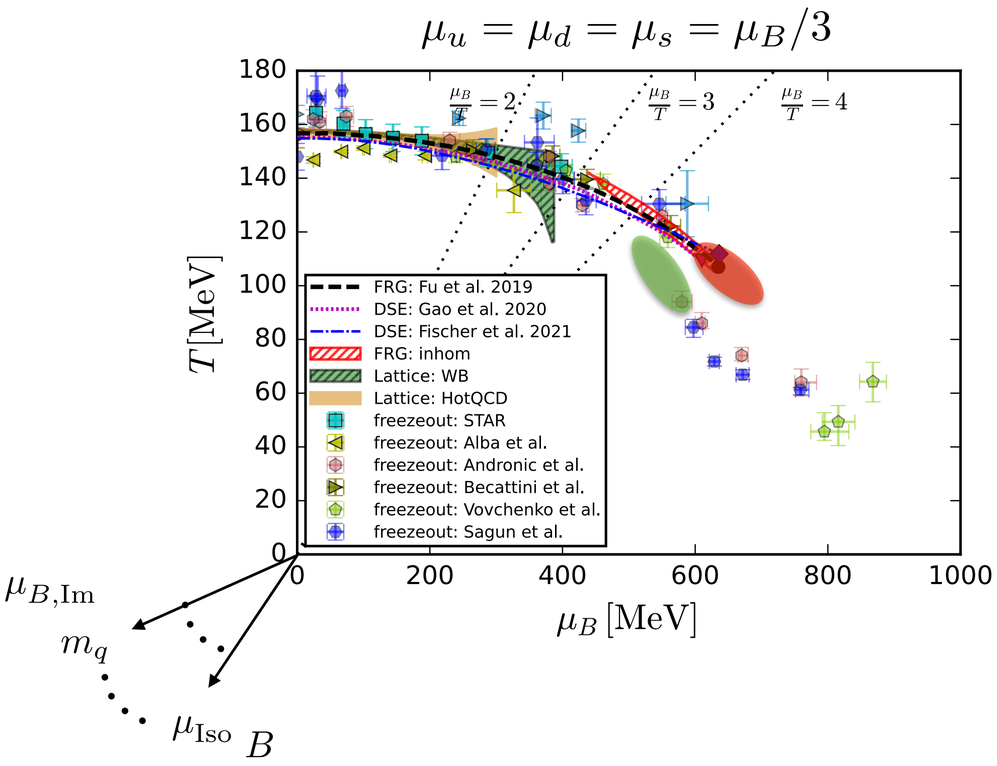

Collect all possible information/structure for physics understanding & extrapolations

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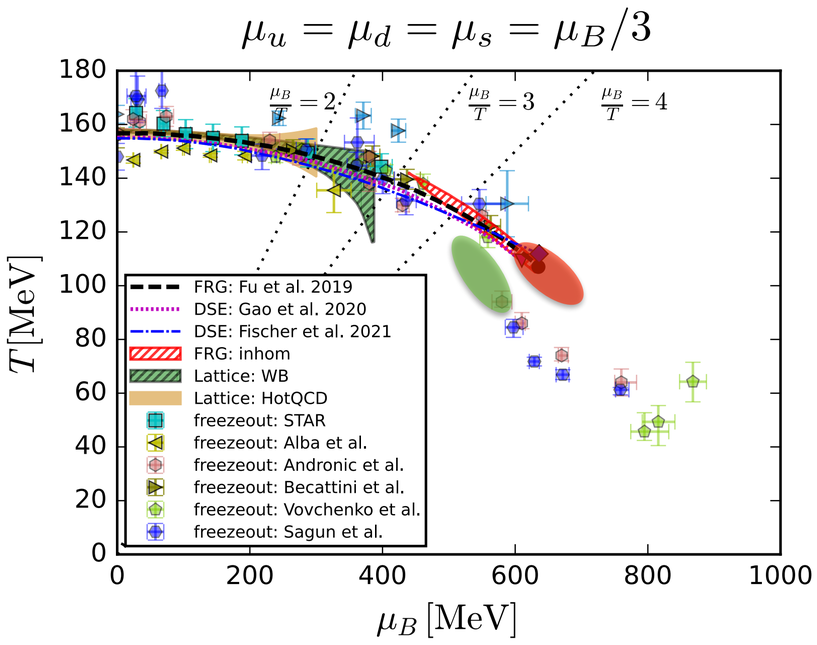

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Estimates & predictions

Requires computations in 1st principles QCD at

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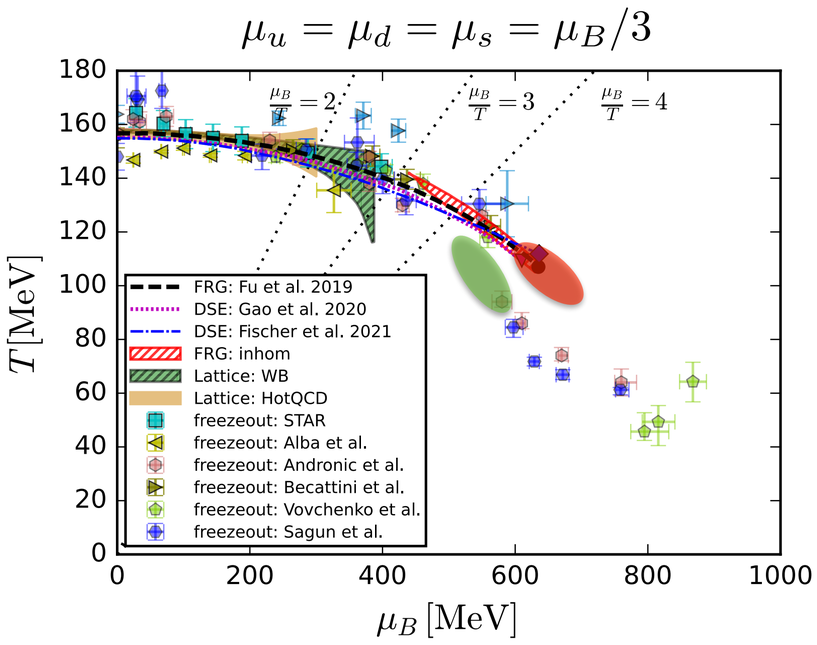

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Extrapolations for Pheno

Requires a discussion of the explicit & implicit assumptions

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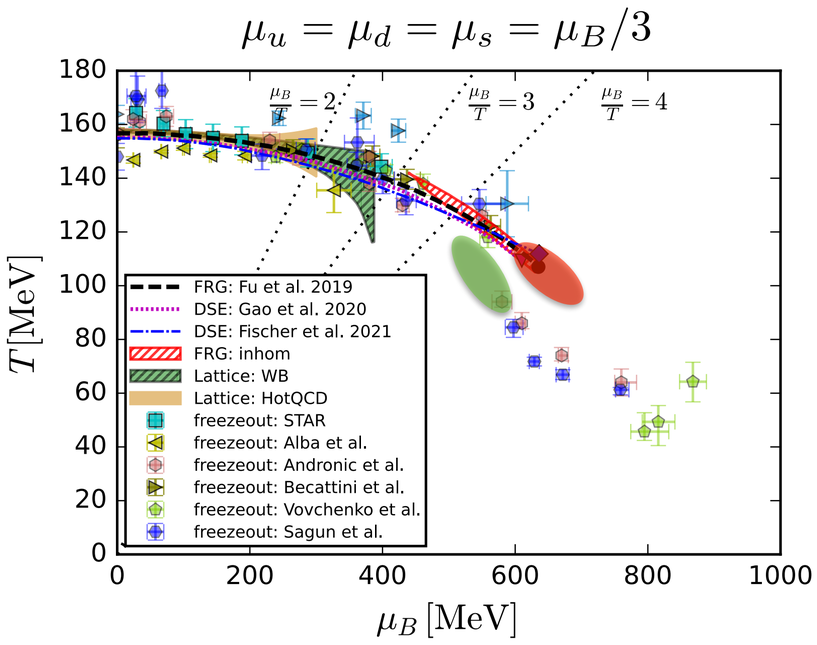

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Lattice extrapolations

low energy effective theories:
QM, NJL, PQM, PNJL, …, Holography

Functional approaches are the only first principle approaches *to date*

that allow for *direct* computations at finite density with

$$\frac{\mu_B}{T} \gtrsim 3$$

Supplementary Material:

Phase structure from functional QCD: how to

Phase structure from functional QCD: Predictions & estimates

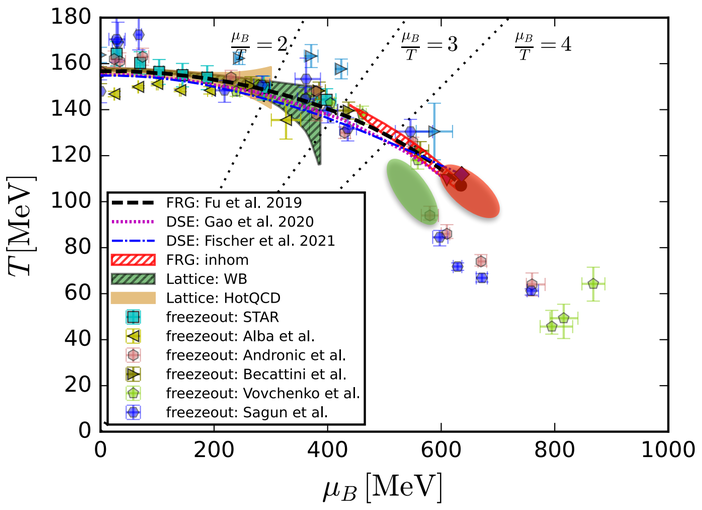
Ripples of the CEP

Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 104 (2021) 9

Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

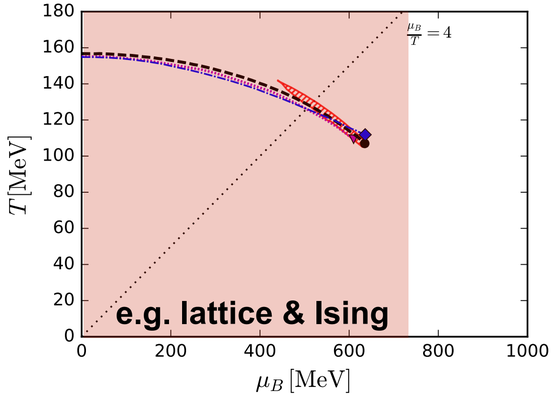
Lu, Gao, Liu, JMP, arXiv:2504.05099

Experimental signatures of the critical end point



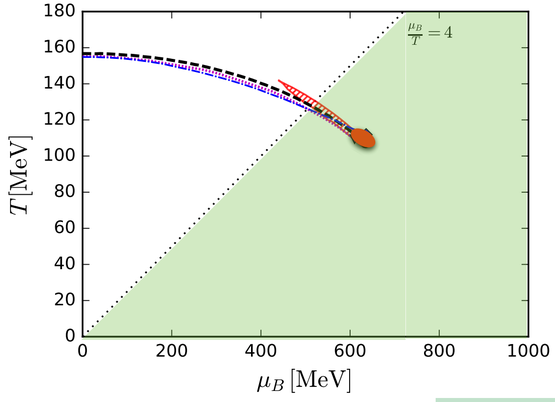
Scenario I

Scaling



Scenario II

Minimal scaling and new phases



Moat, …

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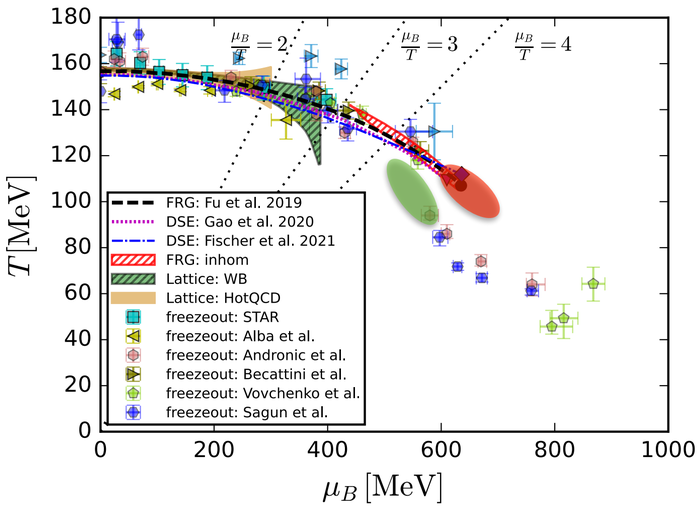

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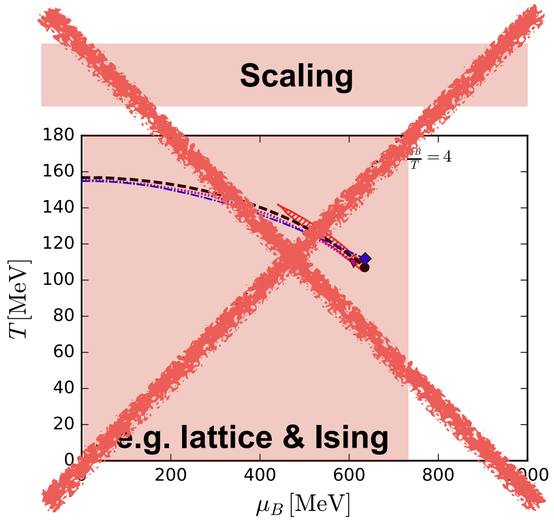

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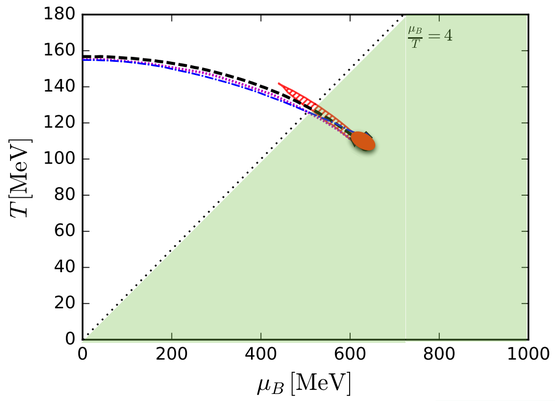


Moat, …

Supplementary Material:

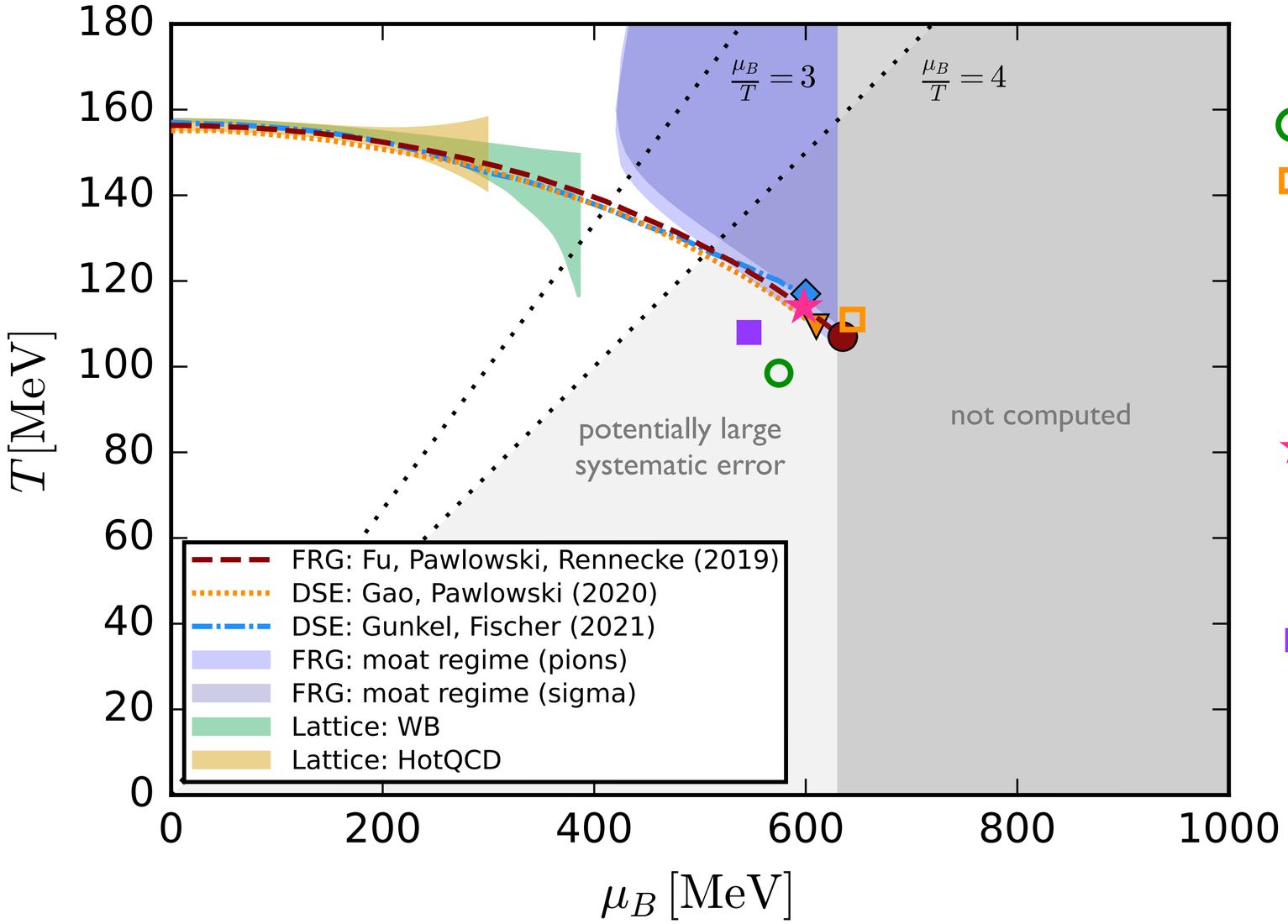
Predictions, estimates & extrapolations and how to use them

Chiral dynamics & soft modes

QCD PHASE DIAGRAM & THE CEP

Modified from Fabian Rennecke's plenary talk @ QM 2025

FRG & DSE results corroborated by subsequent "extrapolations" of lattice data



Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

using Yang-Lee edge singularities:

- ○ conformal Padé [Basar, 2312.06952]
- □ multi-point Padé [Clarke et al., 2405.10196]
  $N_\tau = 6, 8$ results + continuum estimate [Schmidt, 2504.00629]

Alexander Adam's talk, Thu 9:40

using thermodynamics:

- ★ constant entropy density [Shah et al. 2410.16206]

Hitansh Shah's talk, Wed 9:40

model-based extrapolation (tiny selection):

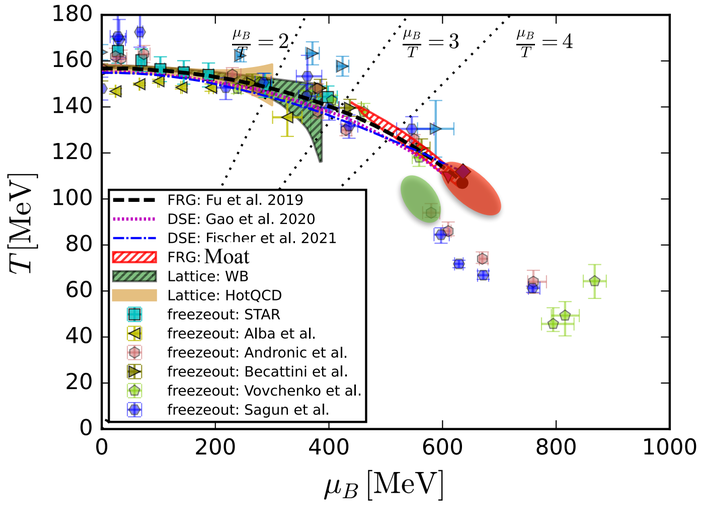
- ■ holography [Hippert et al., 2309.00579]
  (agrees with [Cai et al., 2201.02004])

CEP location well constrained by now. And it's **in FAIR range!**

$\sqrt{s_{NN}} \approx 3.6 - 4.1\,\mathrm{GeV}$

QCD-assisted low energy effective theory

chiral phase structure

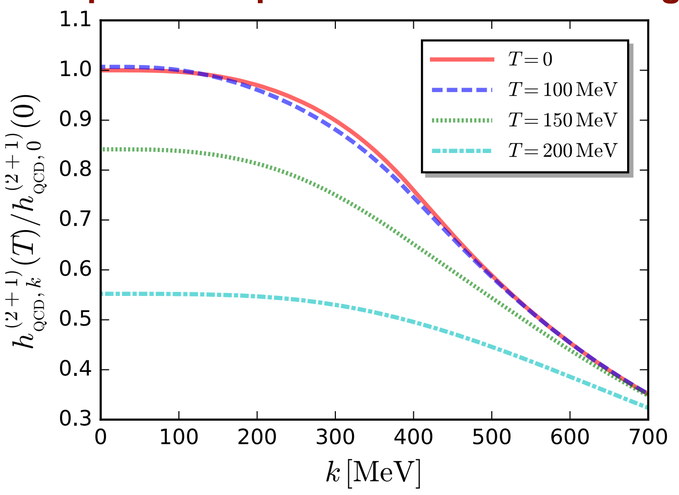


Direct functional QCD input

Fu, JMP, Rennecke, PRD 101 (2020) 054032

+

quark-antiquark — meson scattering



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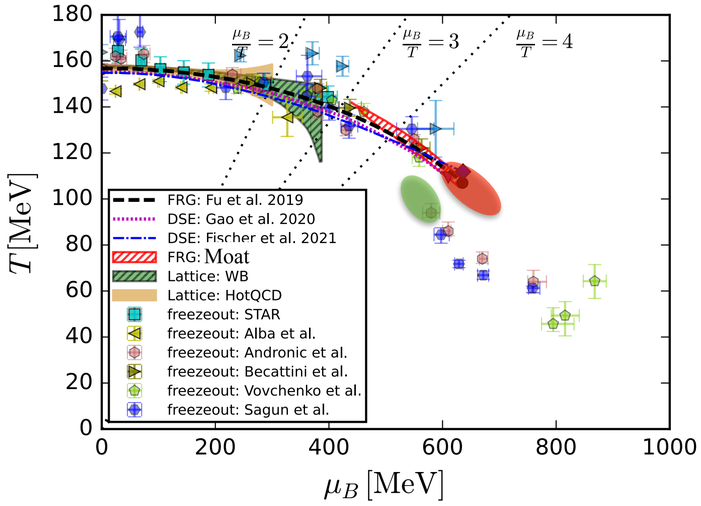

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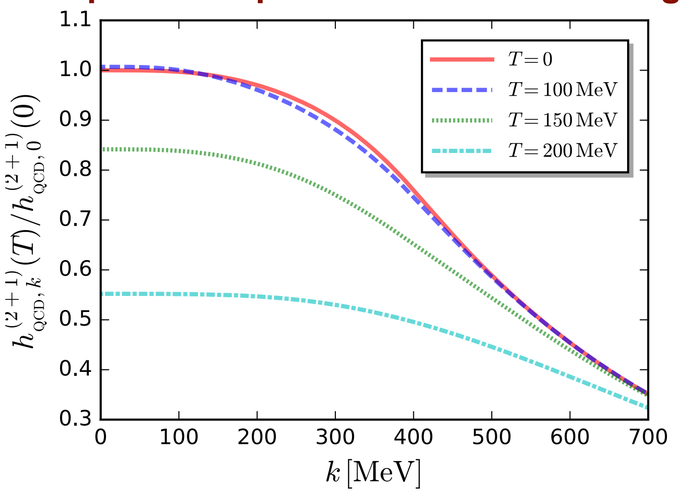

low energy quantum, thermal & density fluctuations via fRG (QCD-assisted PQM model)

Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

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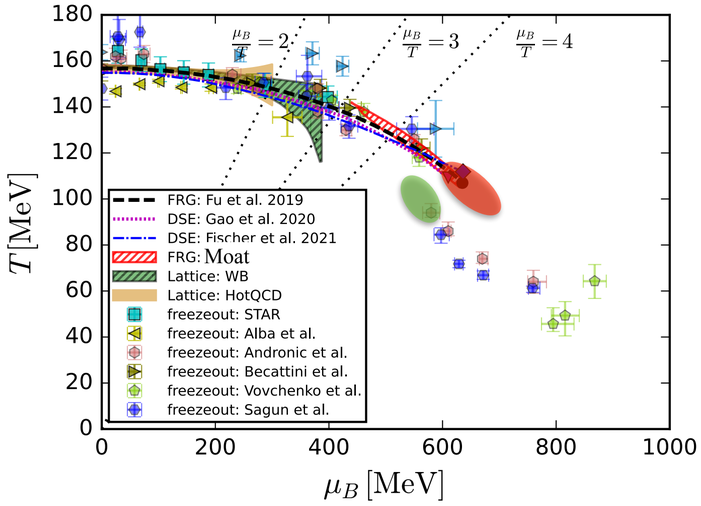

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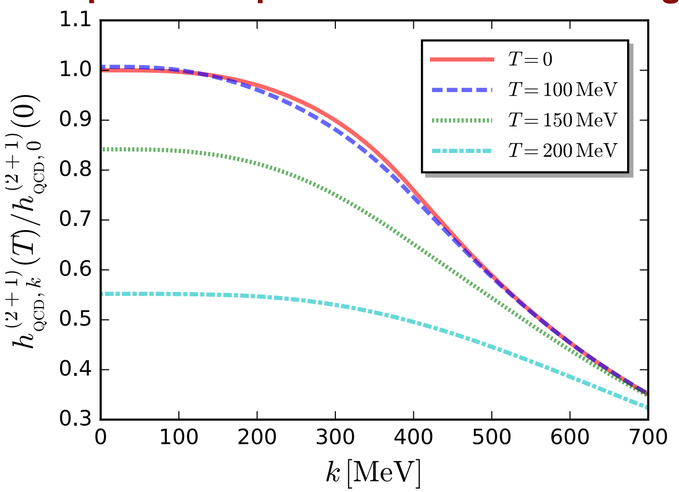

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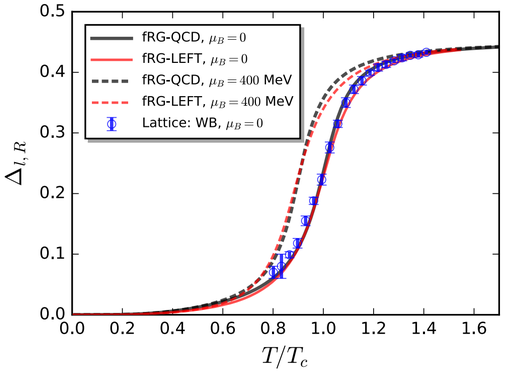
renormalised chiral condensate

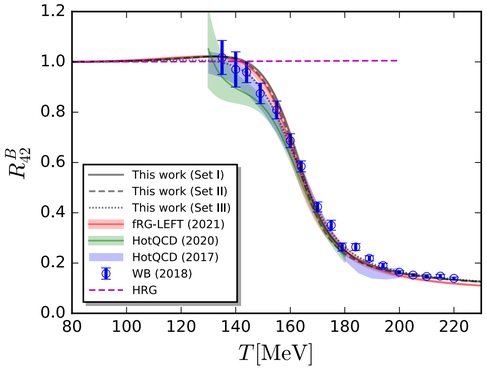


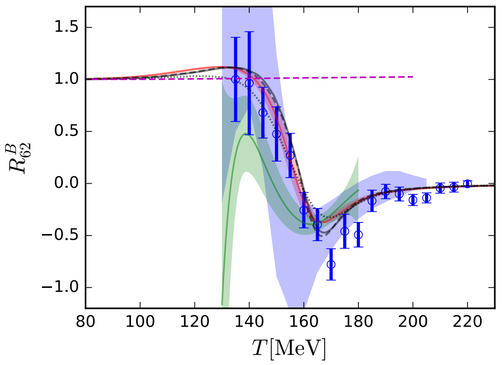
Benchmarks with lattice and fQCD

at

vanishing density and fQCD at finite density

baryon number fluctuations

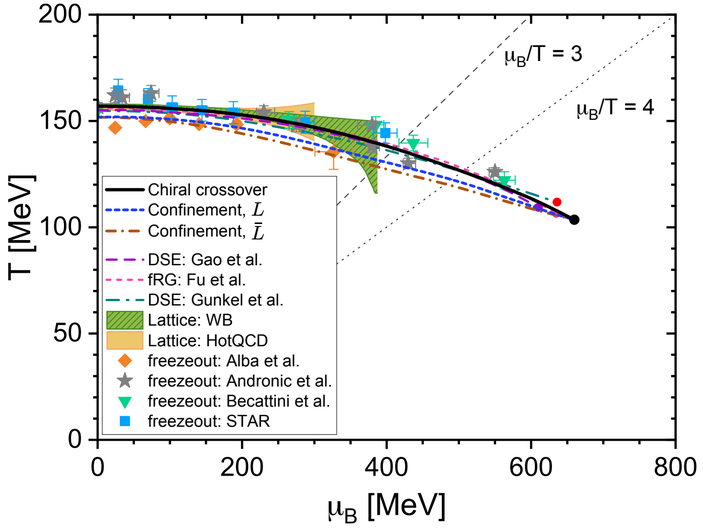


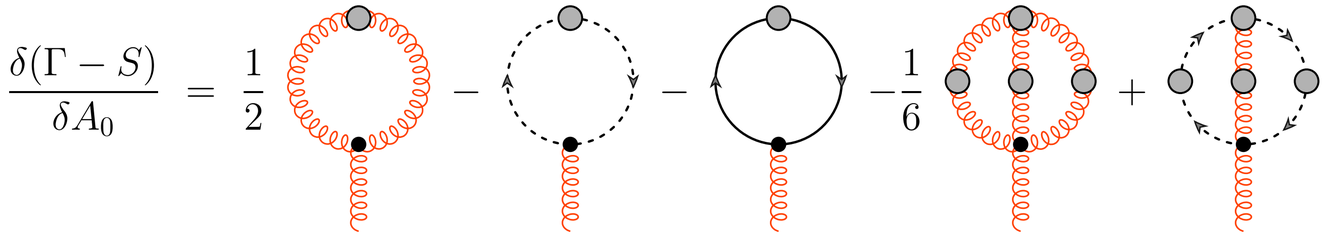


Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

Functional QCD

chiral & 'confinement' phase structure





DSE

L: 'cousin' of the Polyakov loop

Braun, Gies, JMP, PLB 684 (2010) 262
Fister, JMP, PRD 88 (2013) 045010
Herbst, Lücker, JMP, arXiv:1510.03830

Explorative DSE studies EoS & fluctuations of the net-baryon charge

Isserstedt, Buballa, Fischer, Gunkel, PRD 100 (2019) 074011
Isserstedt, Fischer, Steinert, PRD 103 (2021) 054012

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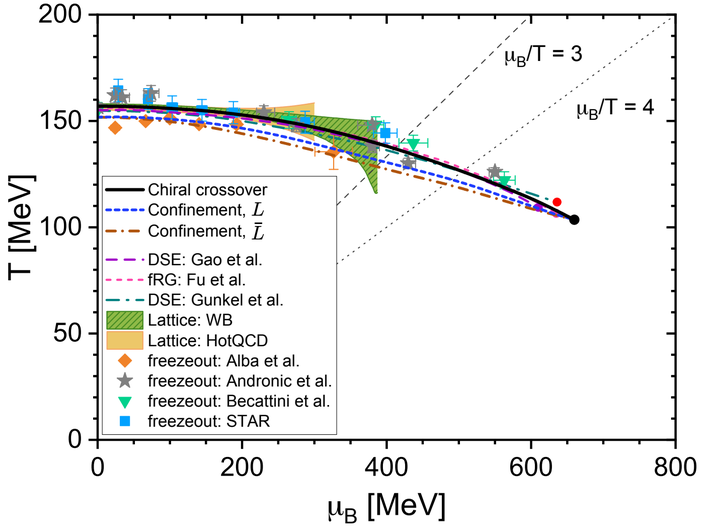
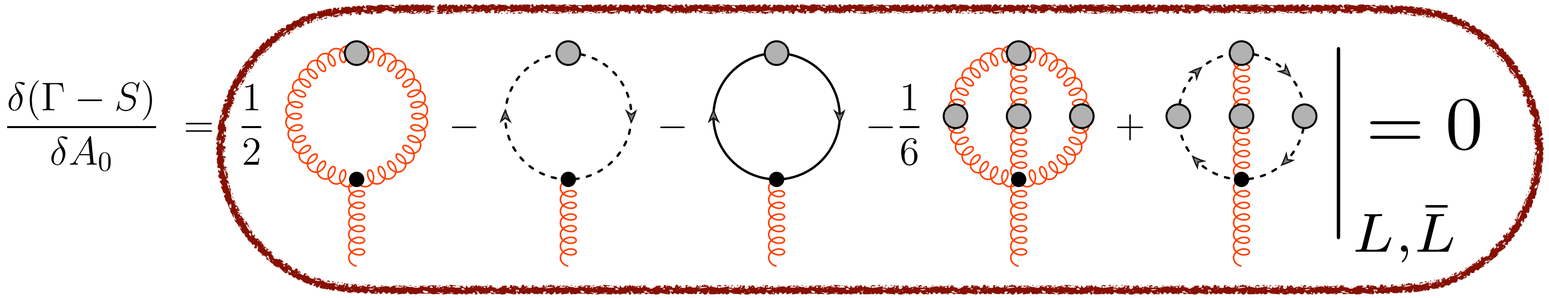


On-shell



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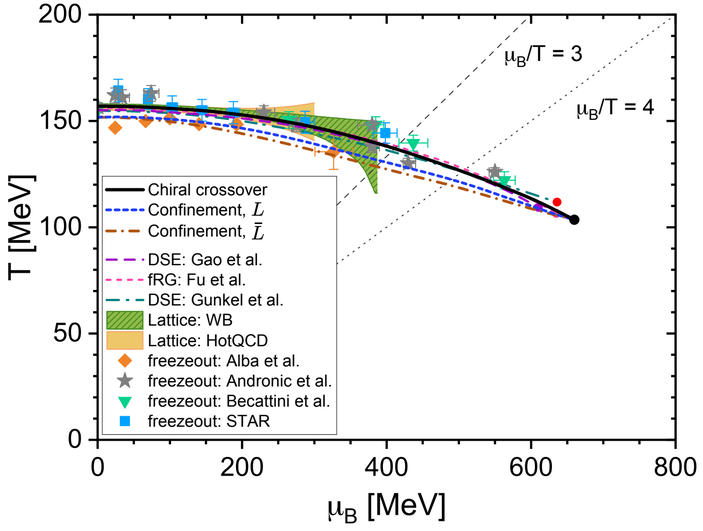
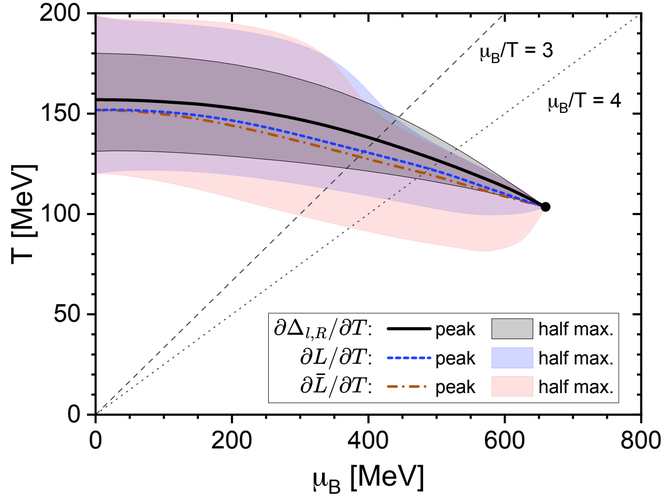


Crossovers with width



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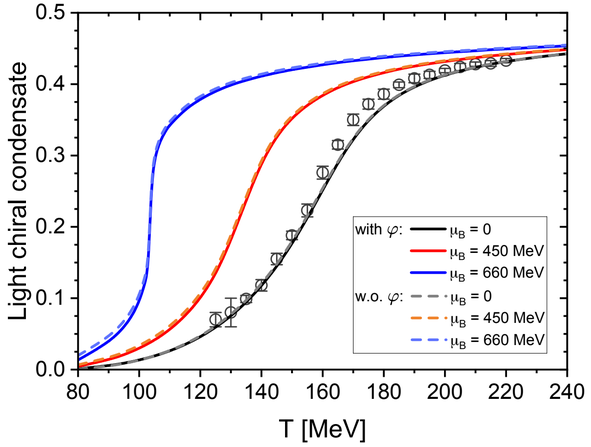

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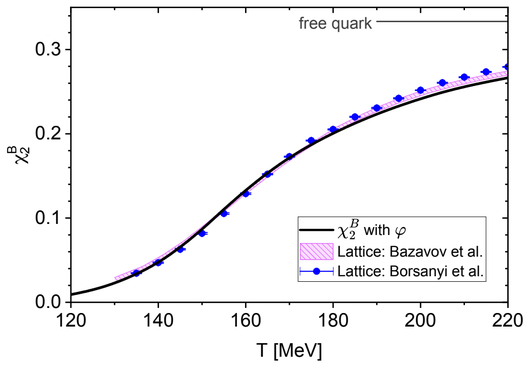

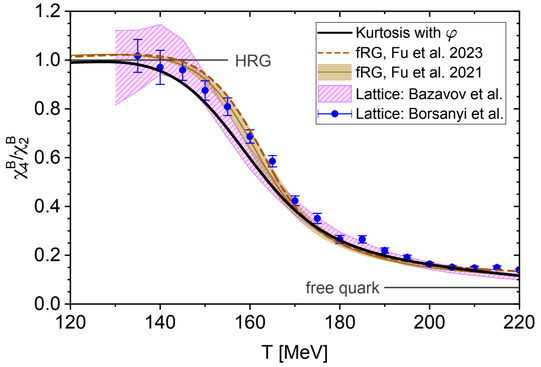

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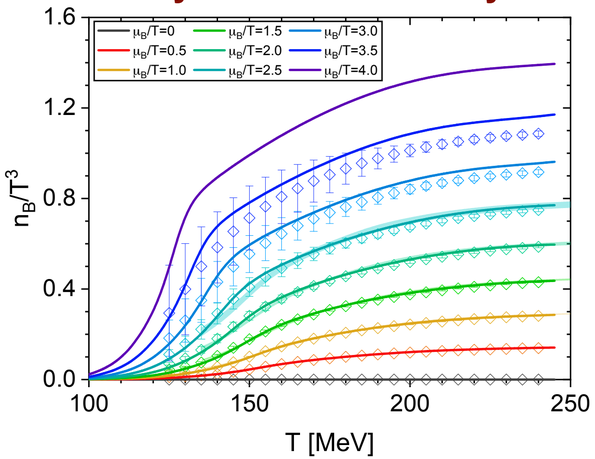
Functional QCD

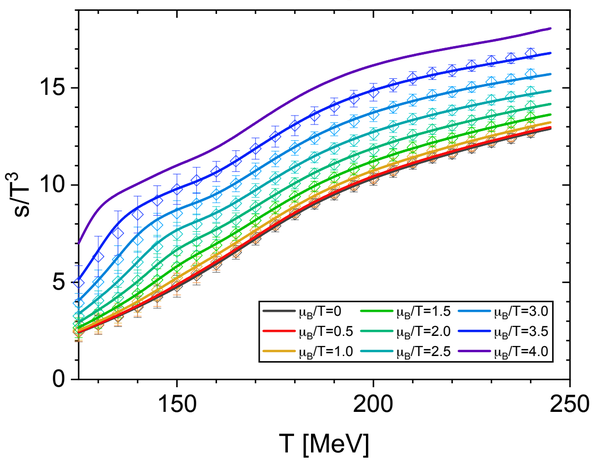
Observables

Thermodynamics

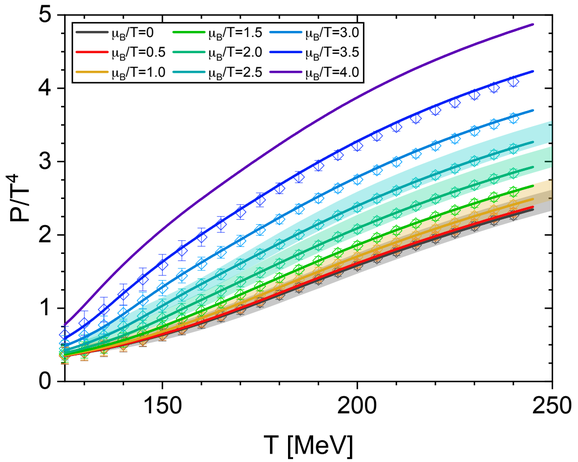
baryon number density



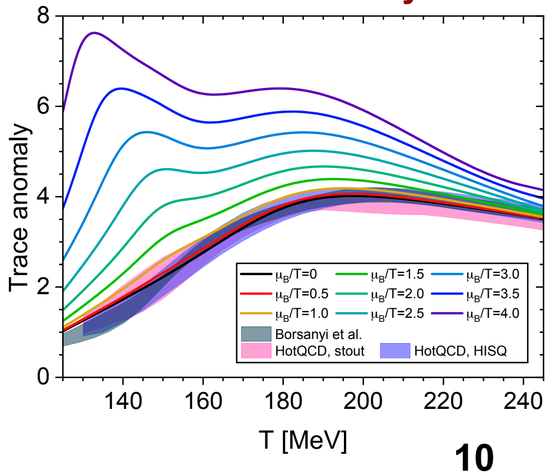
entropy



Pressure



trace anomaly



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Functional QCD

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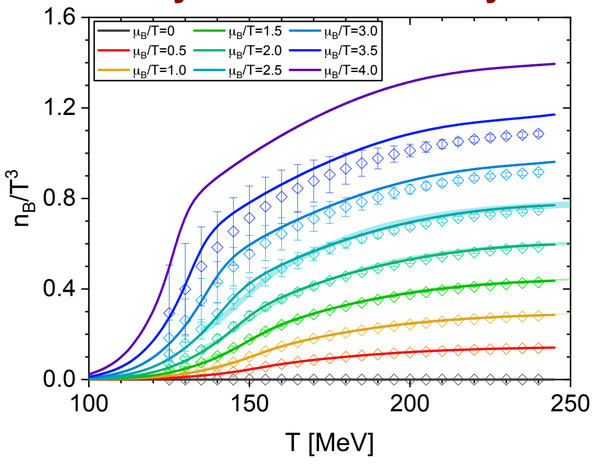
Fluctuations of net-baryon charge

baryon number density



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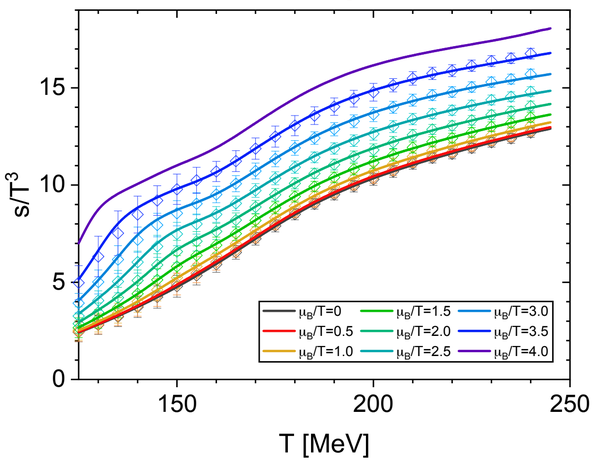
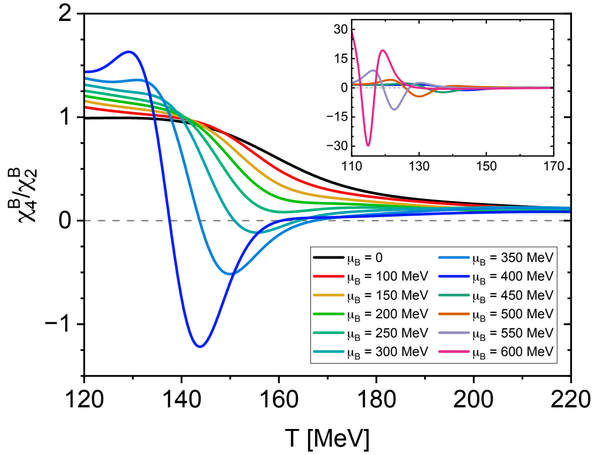


kurtosis



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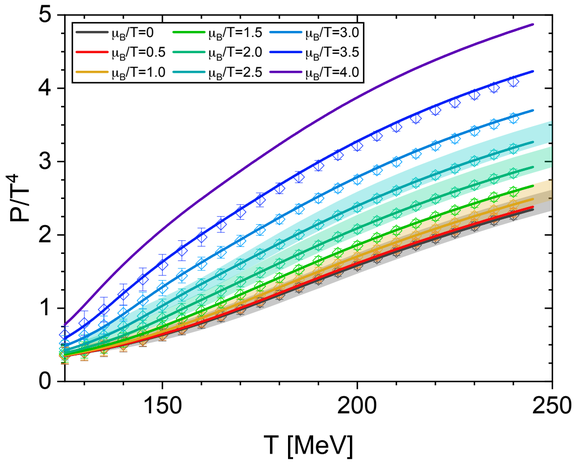

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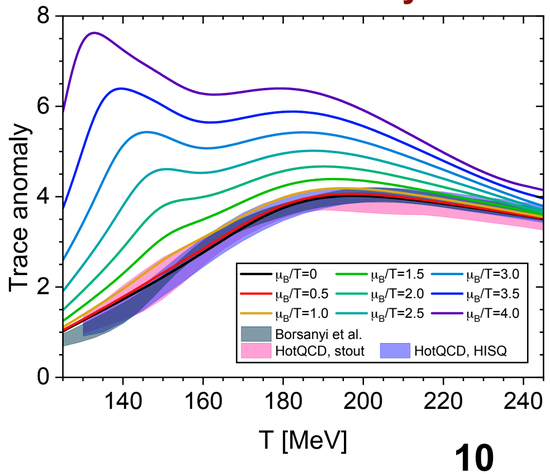
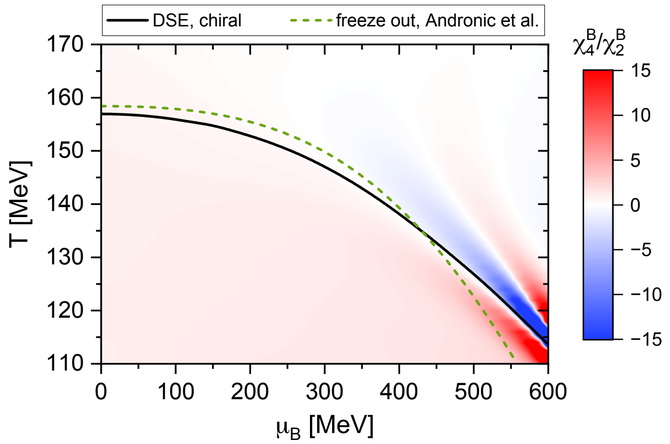
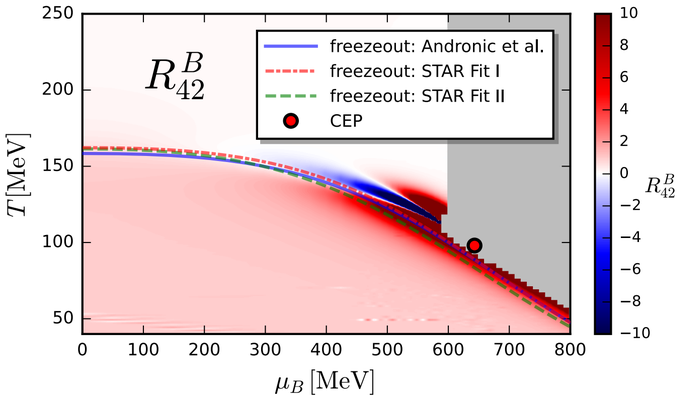


heat map kurtosis



Lu, Gao, Liu, JMP, arXiv: 2504.05099

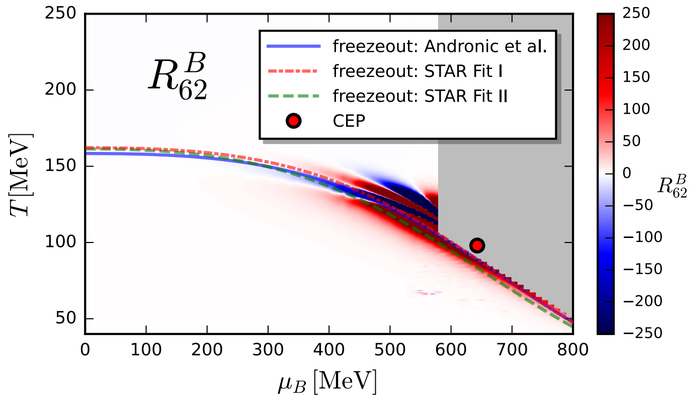
Ripples of the critical point



baryon number fluctuations in the phase structure

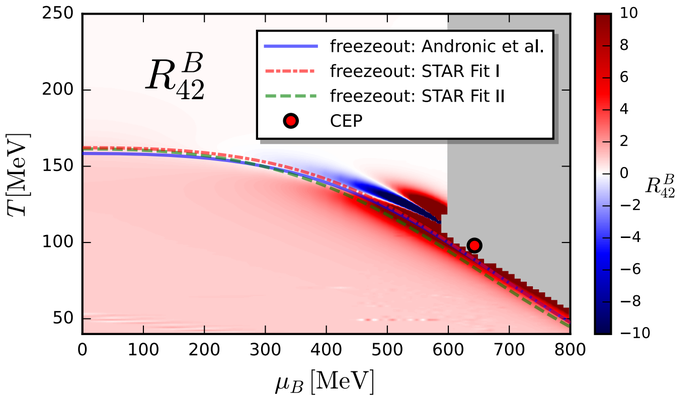
$$(T_{\mathrm{CEP}}, \mu_{B_{\mathrm{CEP}}}) = (98, 643)\,\mathrm{MeV}$$

QCD-assisted LEFT



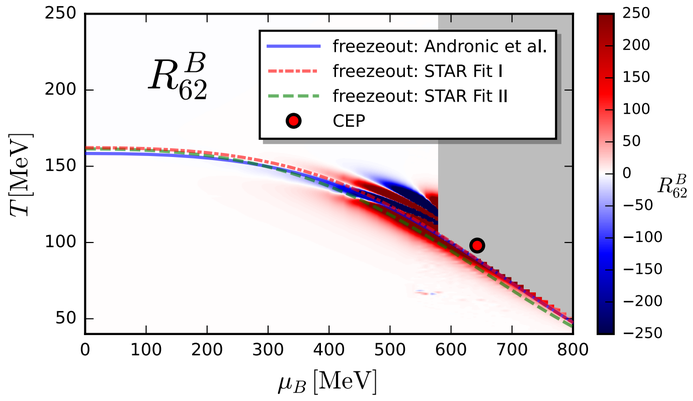
Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

Ripples of the critical point



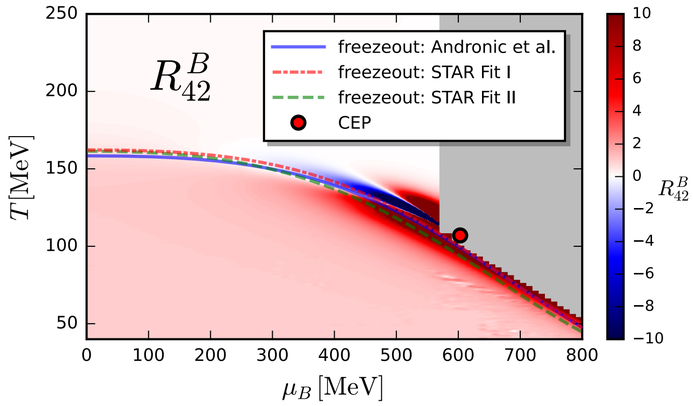
barvon number fluctuations in the phase structure

$$(T_{\rm CEP}, \mu_{B_{\rm CEP}}) = (98, 643)\,{\rm MeV}$$

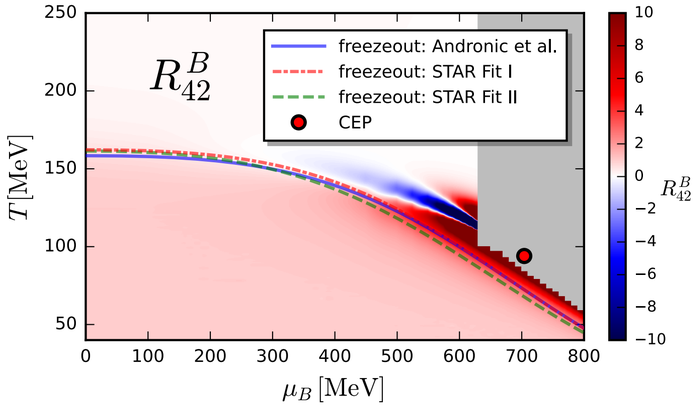


QCD-assisted LEFT

Variations of the CEP in the allowed estimate regime



$$(T_{\rm CEP}, \mu_{B_{\rm CEP}}) = (108, 604)\,{\rm MeV}$$

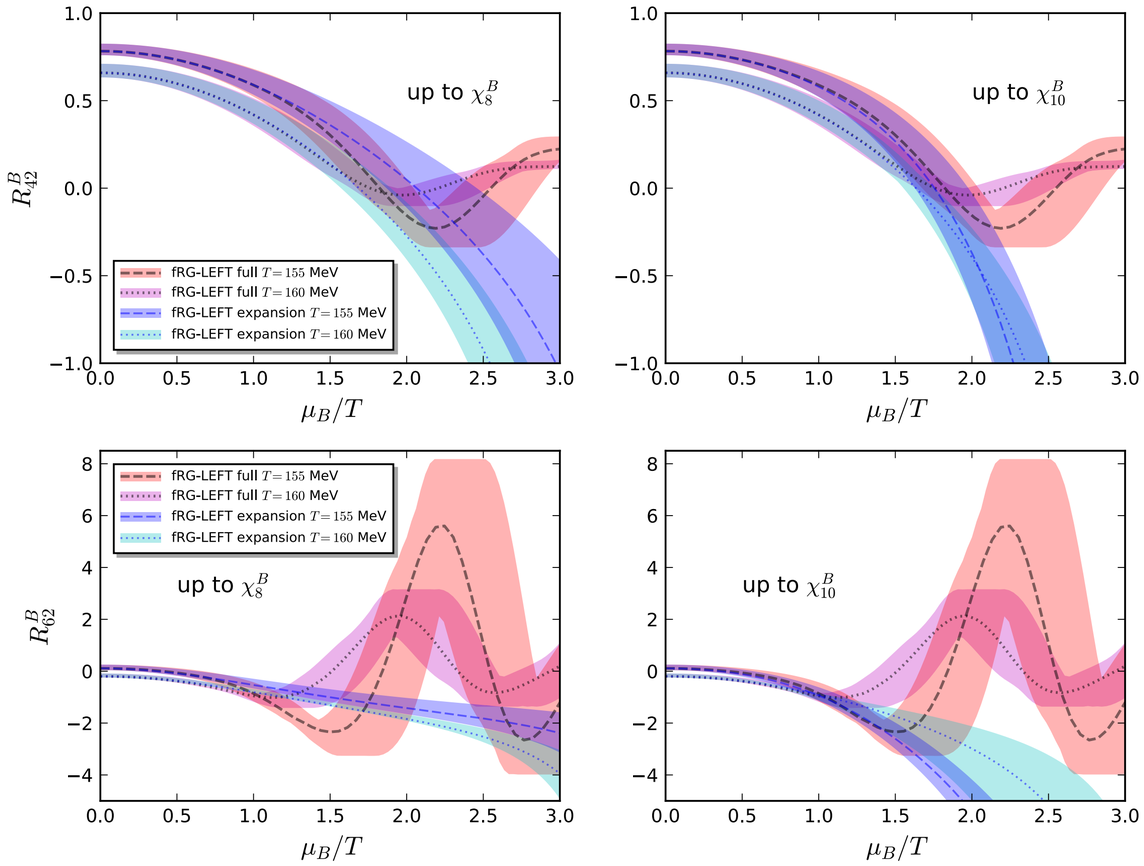


$$(T_{\rm CEP}, \mu_{B_{\rm CEP}}) = (94, 704)\,{\rm MeV}$$

Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

Fluctuations of conserved charges

Validity regime of extrapolations



Great opportunity for a combined analysis of high density QCD (Exp. data + lattice QCD + functional QCD)

Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 104 (2021) 9

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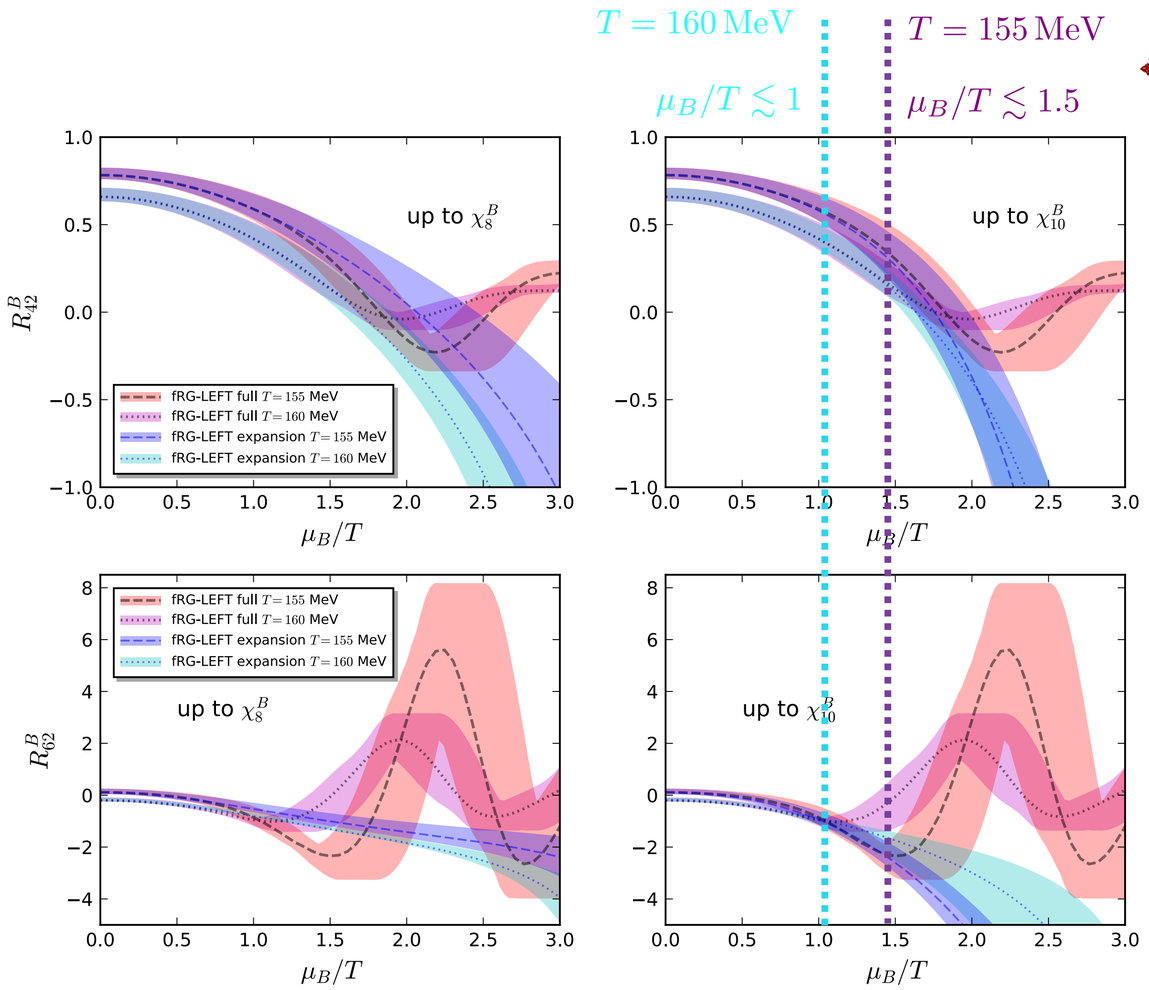


Validity of Taylor expansion

Great opportunity for a combined analysis of high density QCD (Exp. data + lattice QCD + functional QCD)

Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 104 (2021) 9

Ripples of the critical point

Canonical corrections via subensemble acceptance

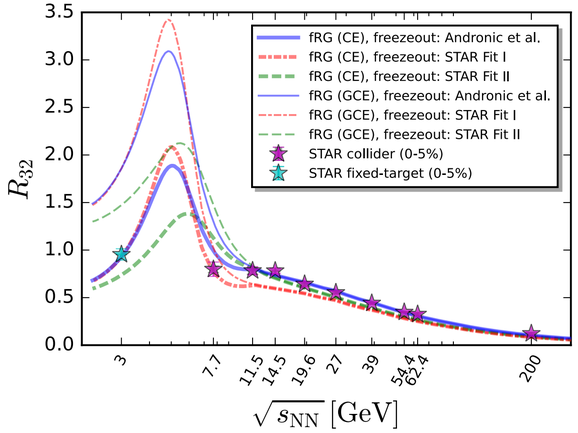
Vovchenko, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

fixing the subensemble volume

subensemble volume **system volume**

$$V_1 = \alpha V$$

$$\bar{R}^B_{32} = (1 - 2\alpha) R^B_{32}$$



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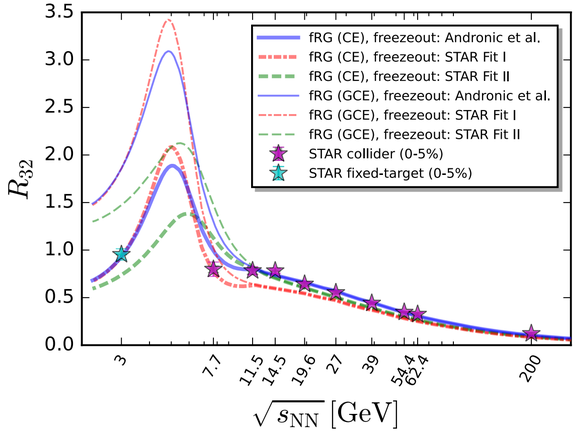

qualitative adjustment

$$\alpha(\bar{s}) = a\left(1 - \sqrt{\bar{s}}\right)\theta\left(1 - \bar{s}\right)$$

$$a = 0.33 \qquad \sqrt{\bar{s}} = \frac{\sqrt{s_{\rm NN}}}{11.9\,\mathrm{GeV}}$$

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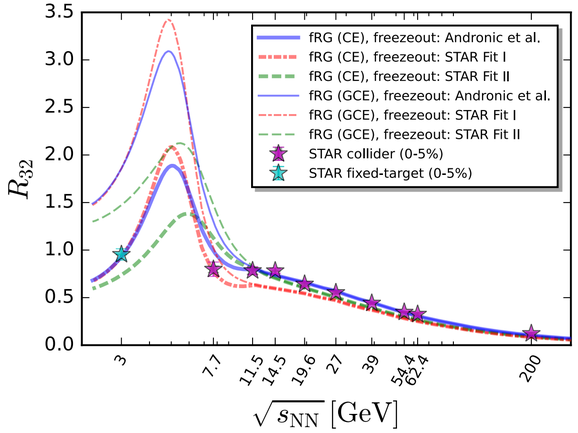

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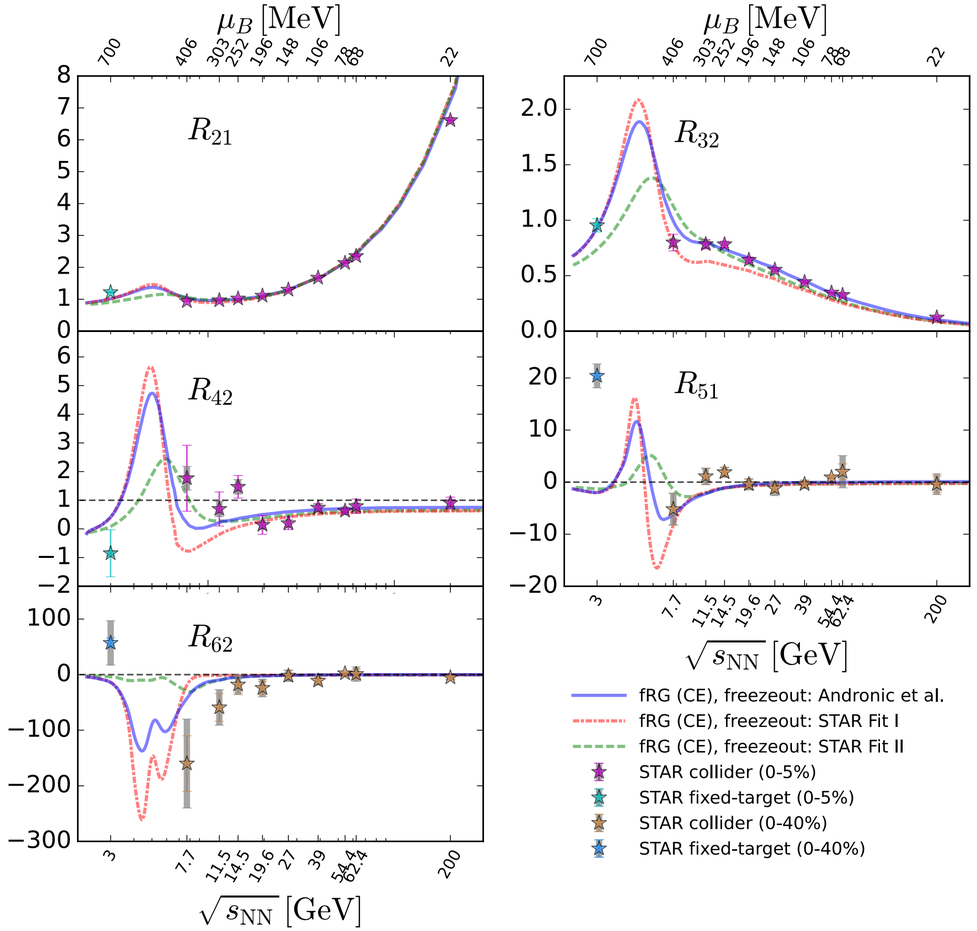
baryon & proton number fluctuations



Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

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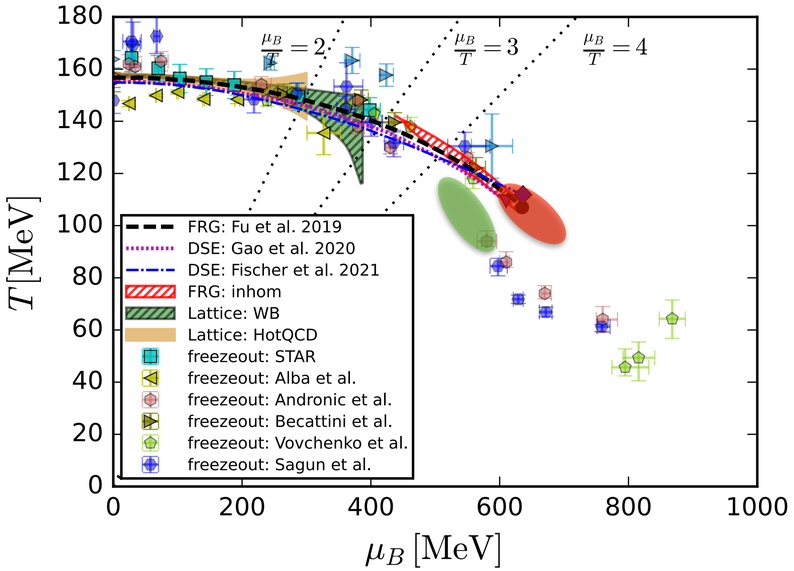
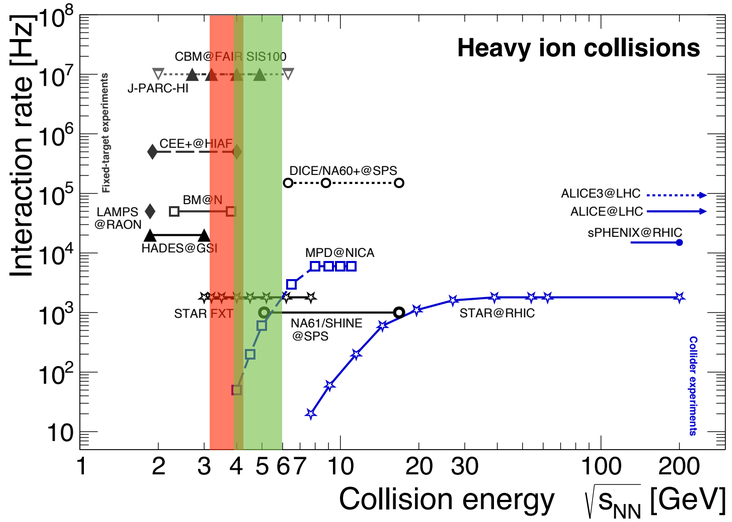
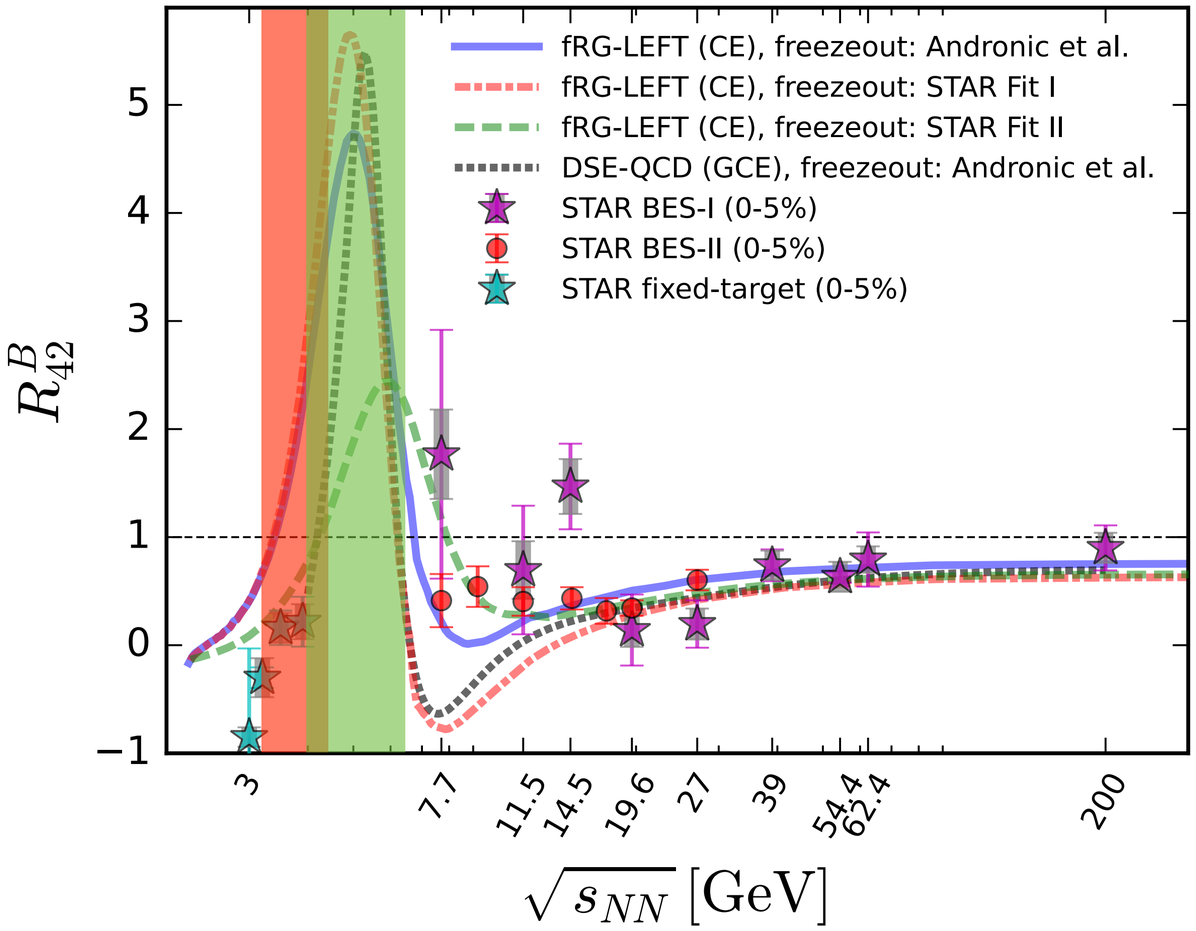




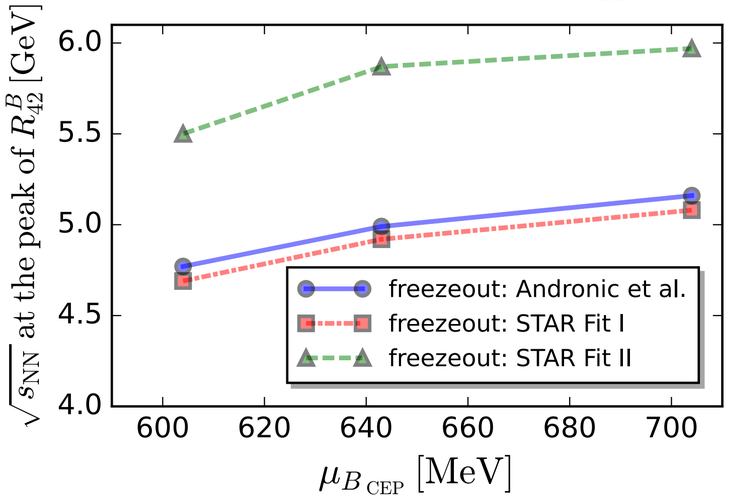
net-baryon fluctuations in QCD vs net-proton fluctuations at STAR



Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

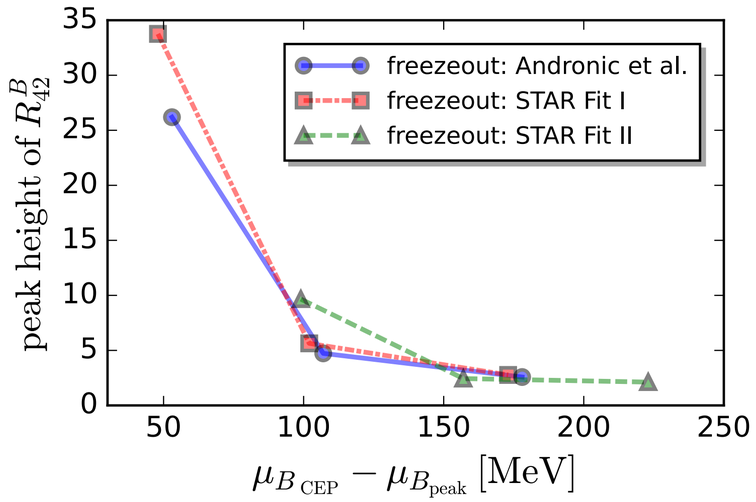
Ripples of the critical point

Position of the peak of R^B_{42}



Reconstructing the CEP

Height of the peak of R^B_{42}



Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 111, L031502

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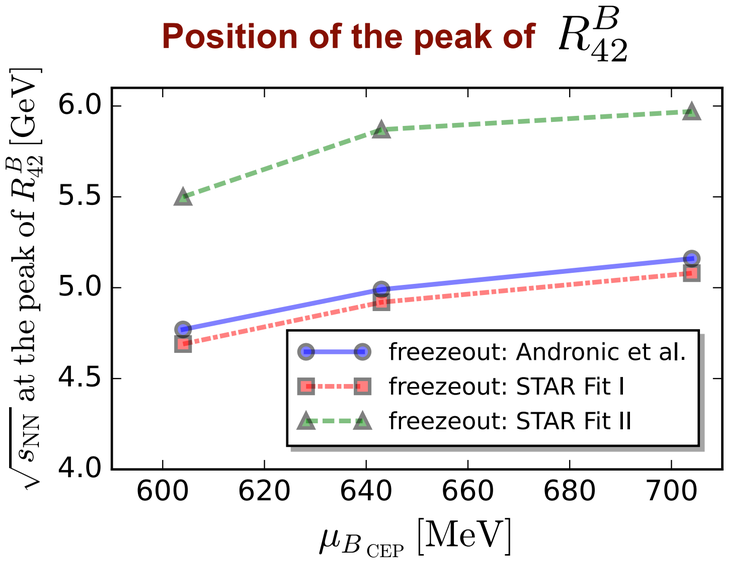
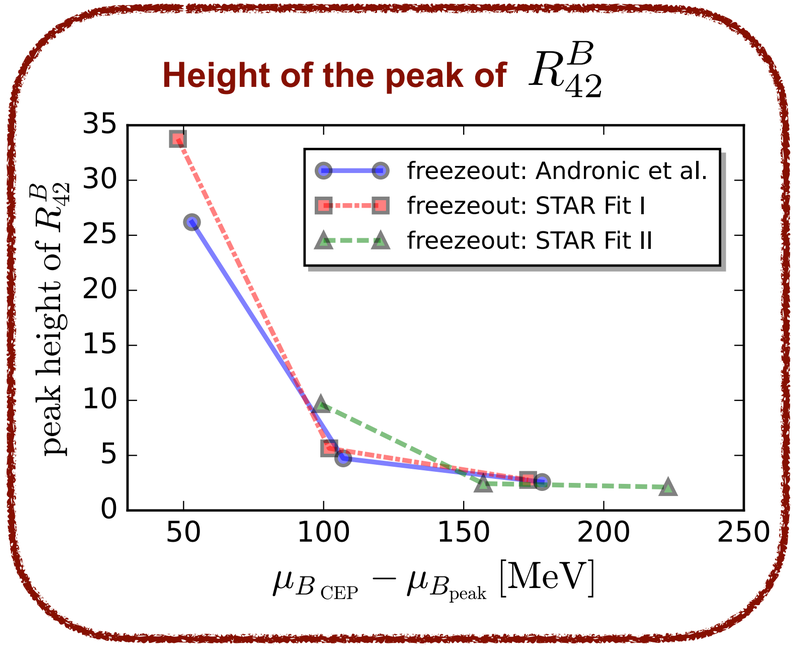


Reconstructing the CEP



Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 111, L031502

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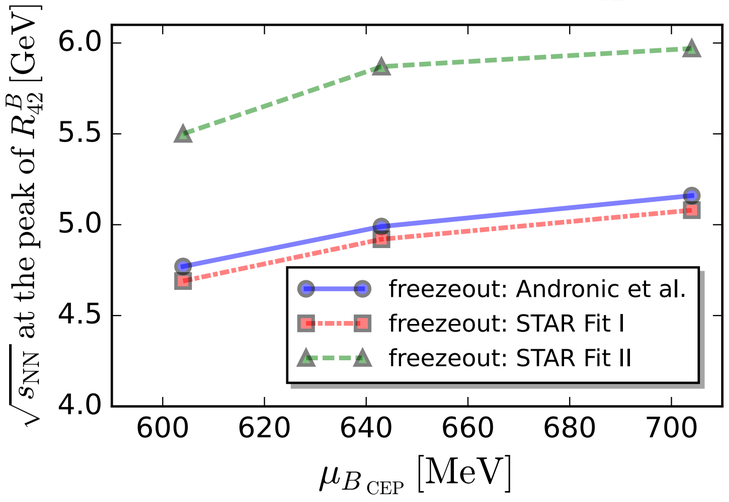

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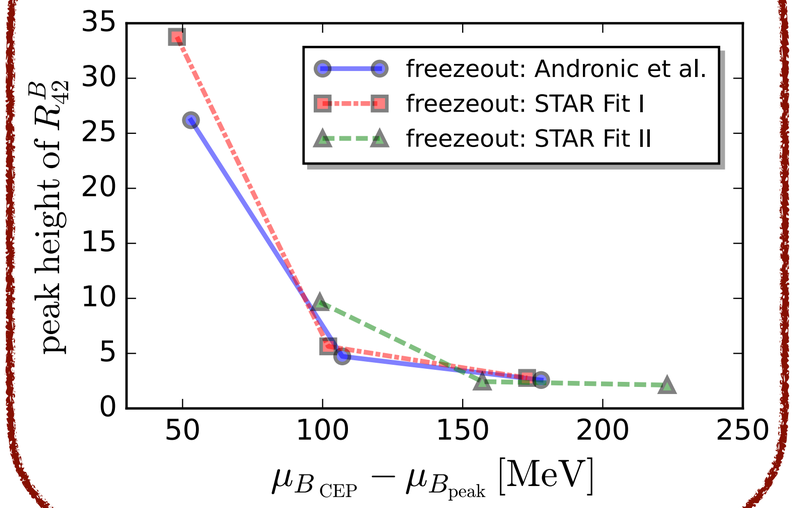


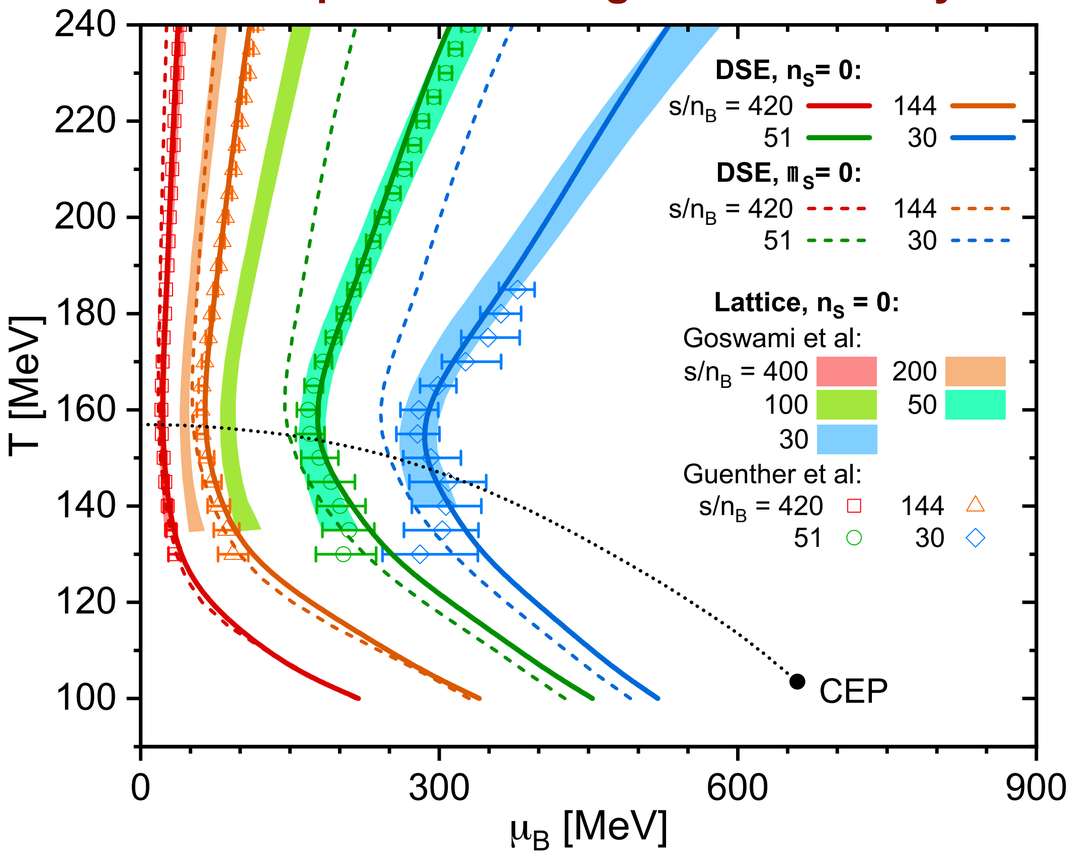
Unfolding the high density regime with new phases & physics

Great opportunity for a combined high precision analysis of high density QCD (Exp. data + lattice QCD + functional QCD)

Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 111, L031502

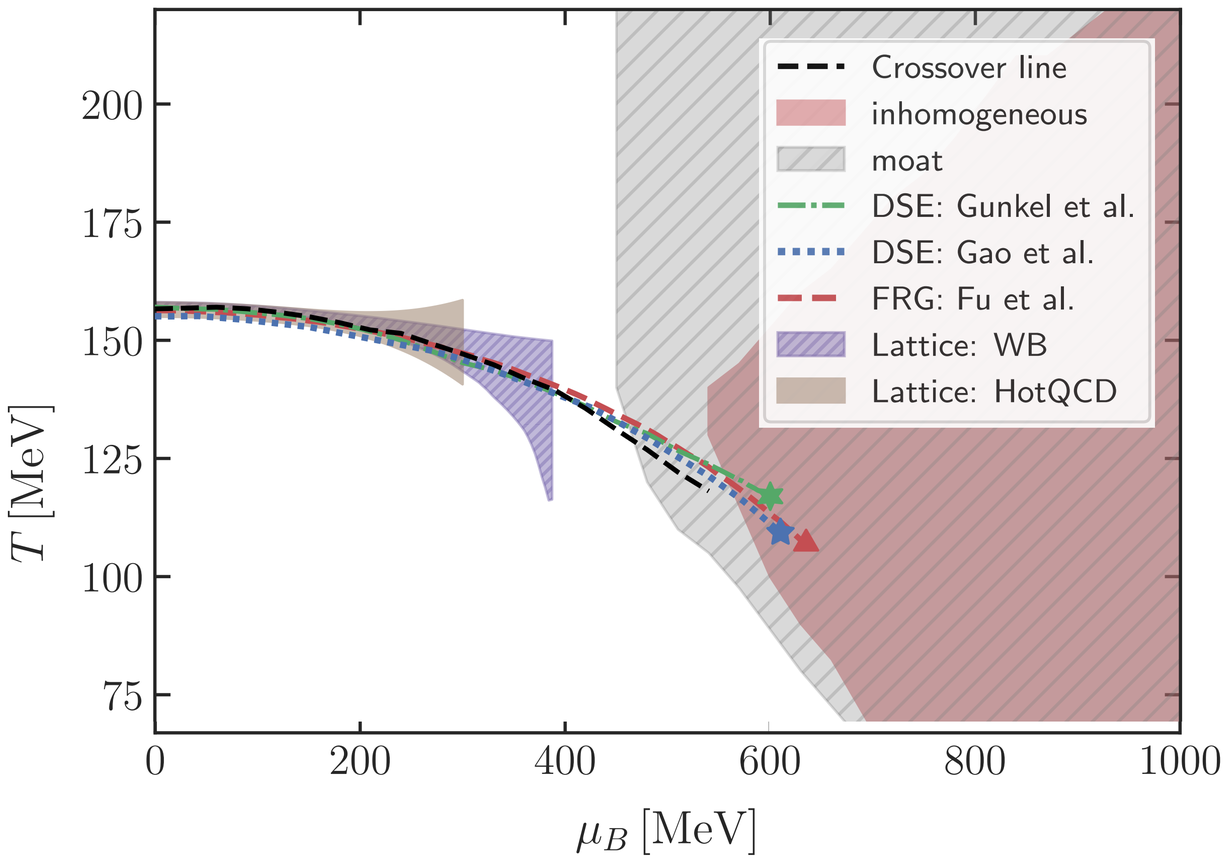
Ripples of the critical point: Sneak preview

isentropes with strangeness neutrality



Lu, Gao et al, in preparation

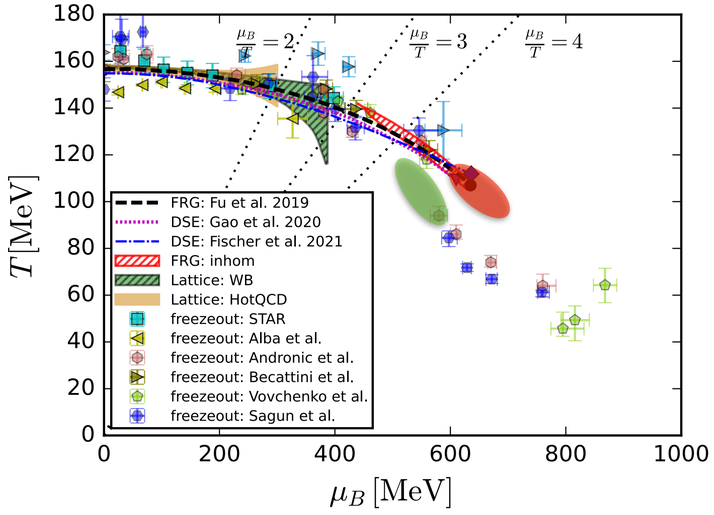
resolution of the moat regime in QCD



Sattler et al, in preparation

Summary

- **Functional QCD provides direct 1st principle results for the phase structure at finite density**
  - **Predictions:** $\frac{\mu_B}{T} \lesssim 4$
  - **Estimates:** $4 \lesssim \frac{\mu_B}{T} \lesssim 8$



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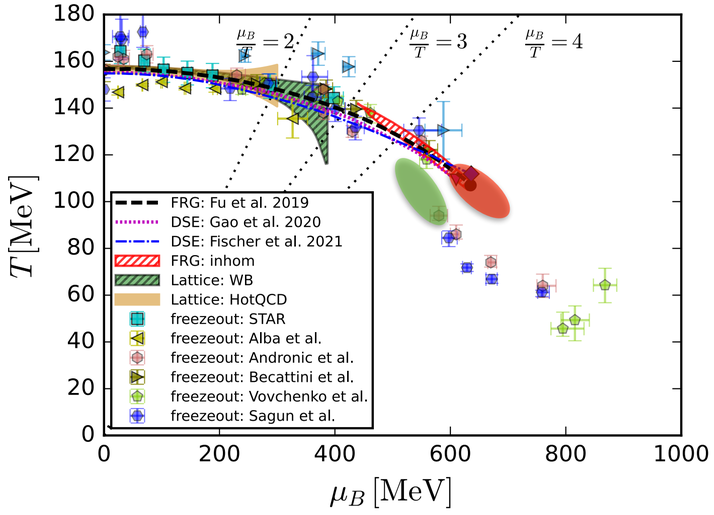
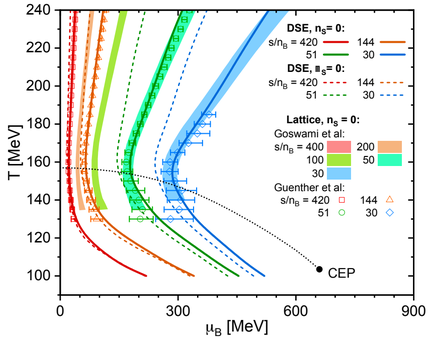
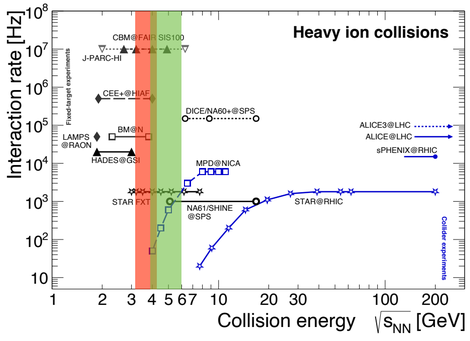
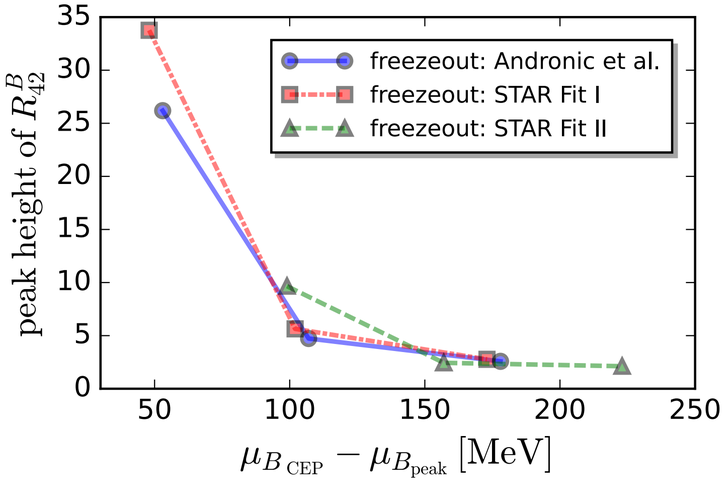


- **Ripples of new phases aka of the critical end point**
  - **Explanation for the convergence of CEP locations of extrapolation approaches**
  - **Results & observables: EoS, fluctuations of conserved charges, tiny critical regime & onset of new phases; moat regime, ….**
  - **Peak height of kurtosis carries the location of the CEP**



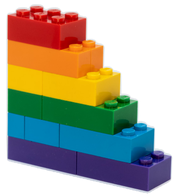


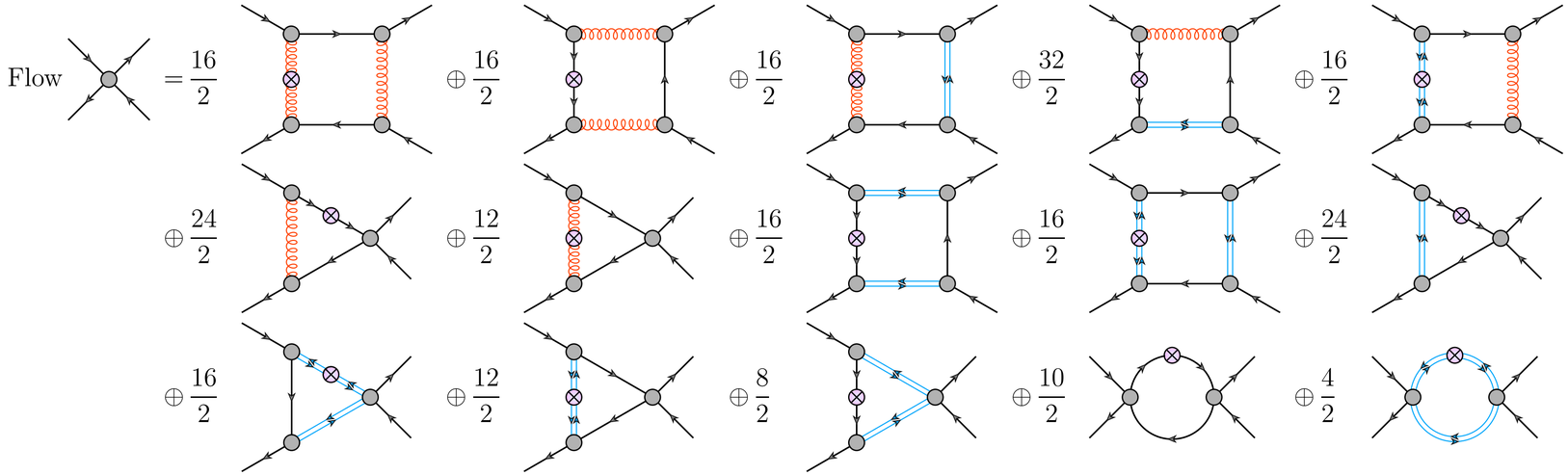


Supplementary material

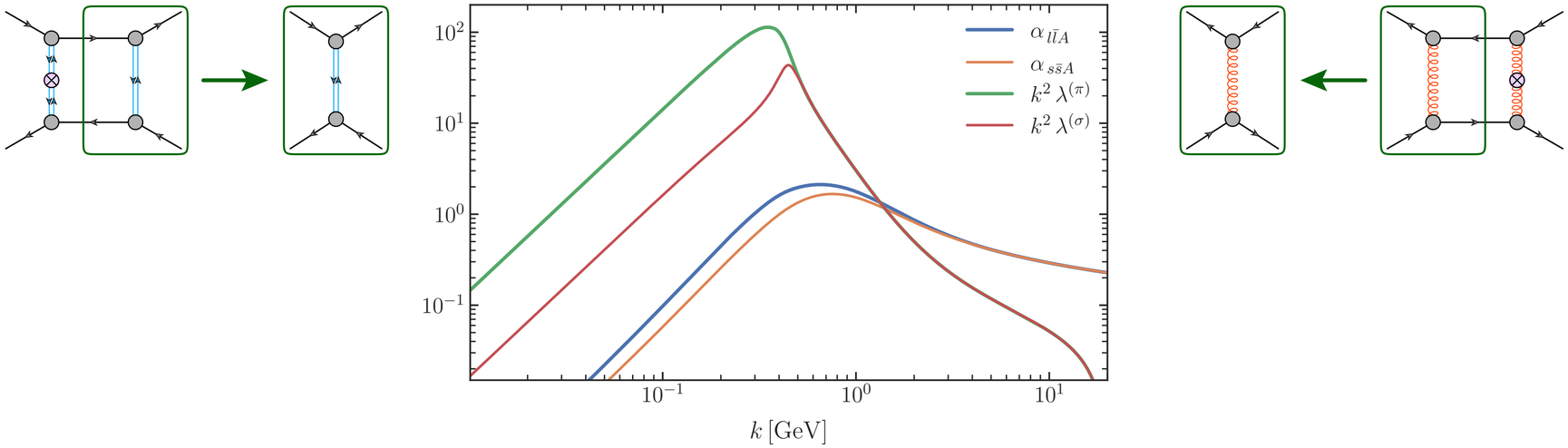
Phase structure from functional QCD: how to

How to: systematic error estimates & the LEGO® principle



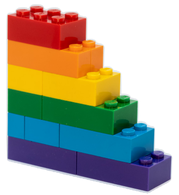


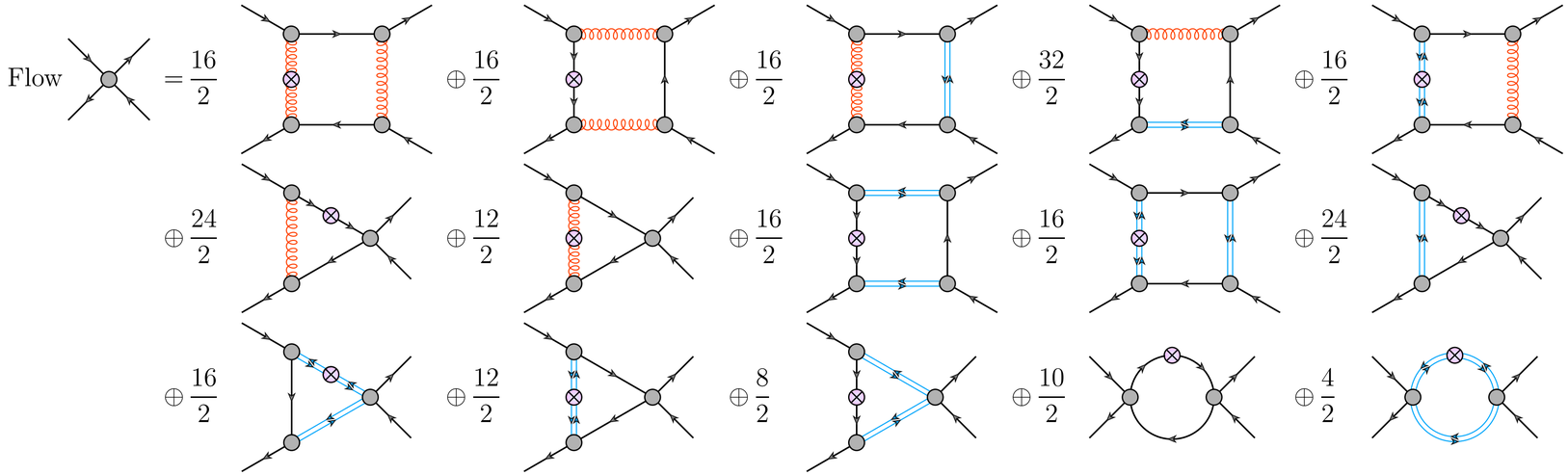
Example: 4-quark scattering vertex



Ihssen, JMP, Sattler, Wink, arXiv:2408.08413

How to: systematic error estimates & the LEGO® principle





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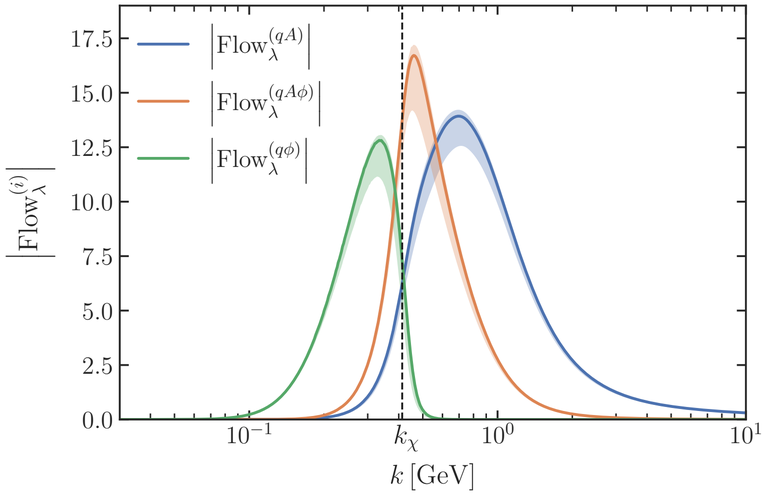

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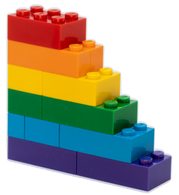

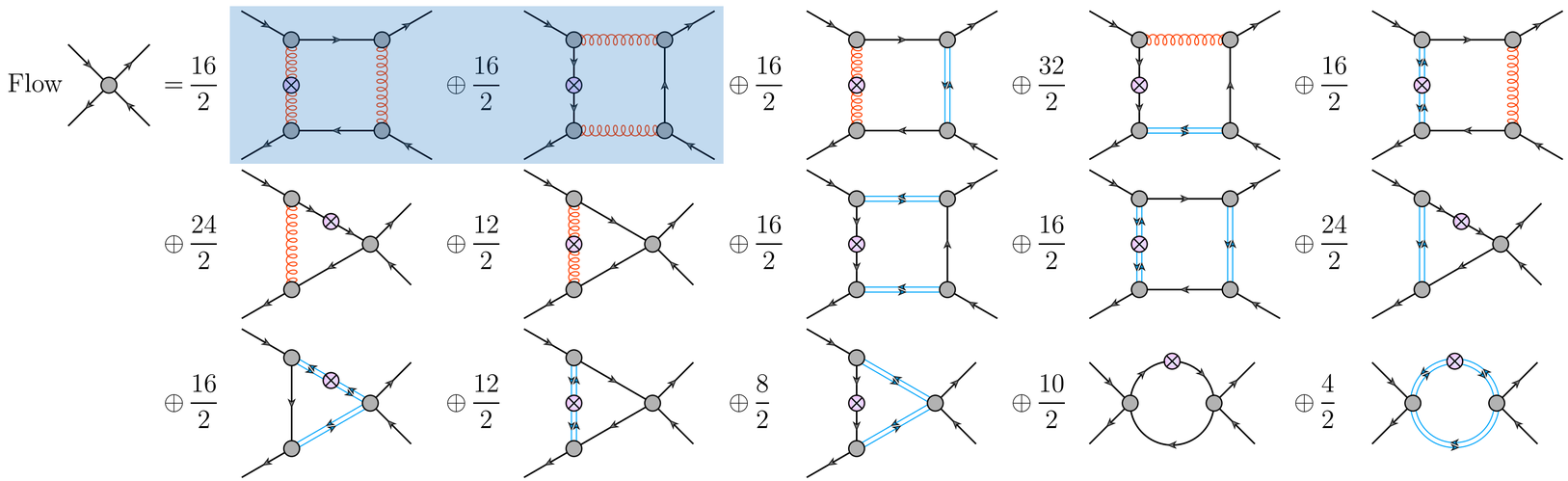

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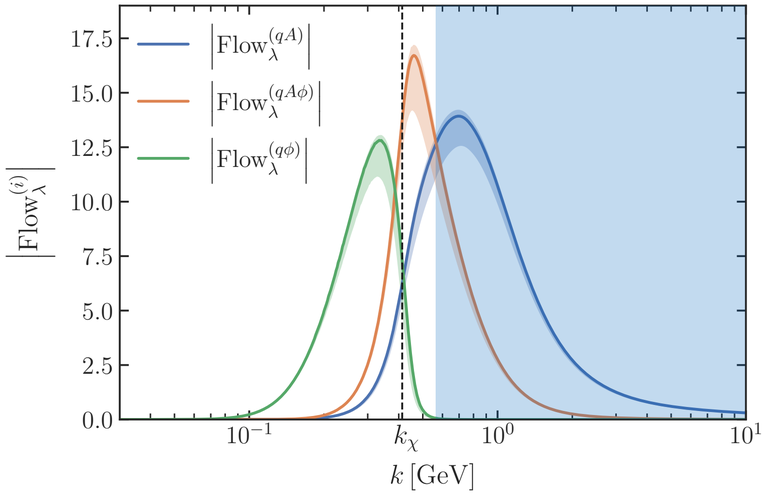

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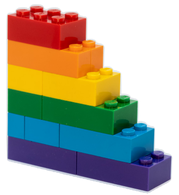

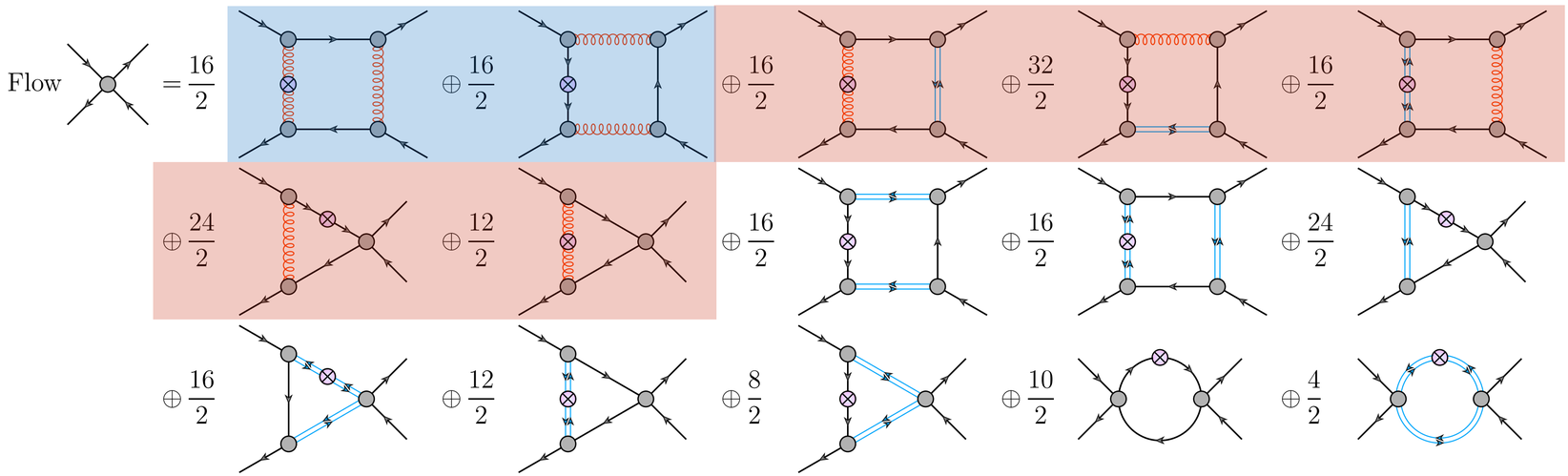

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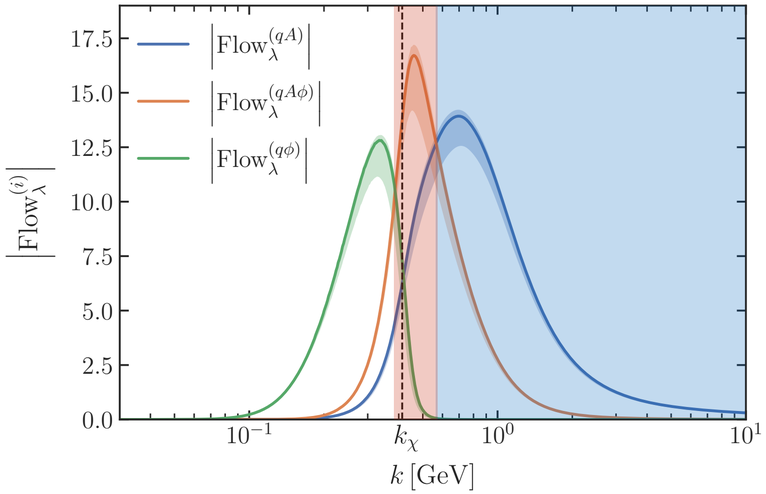

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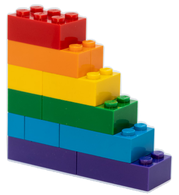

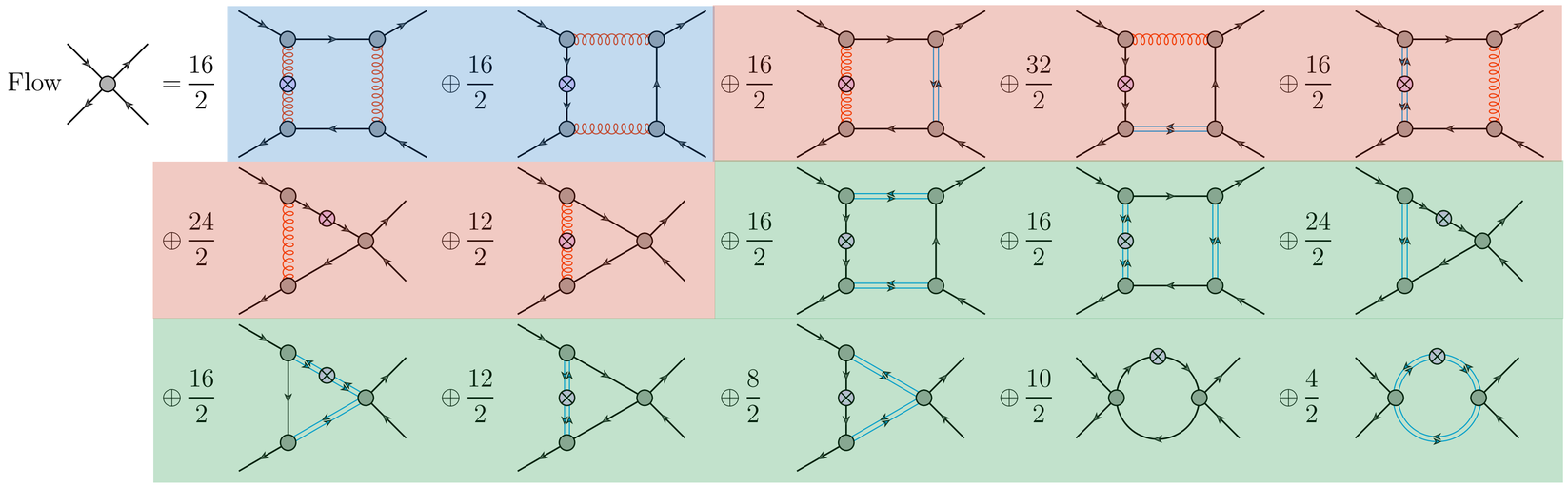

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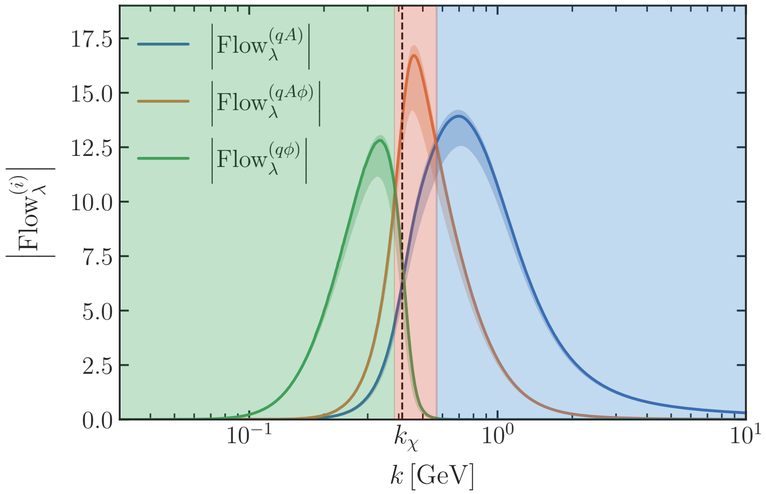

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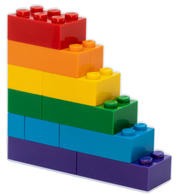
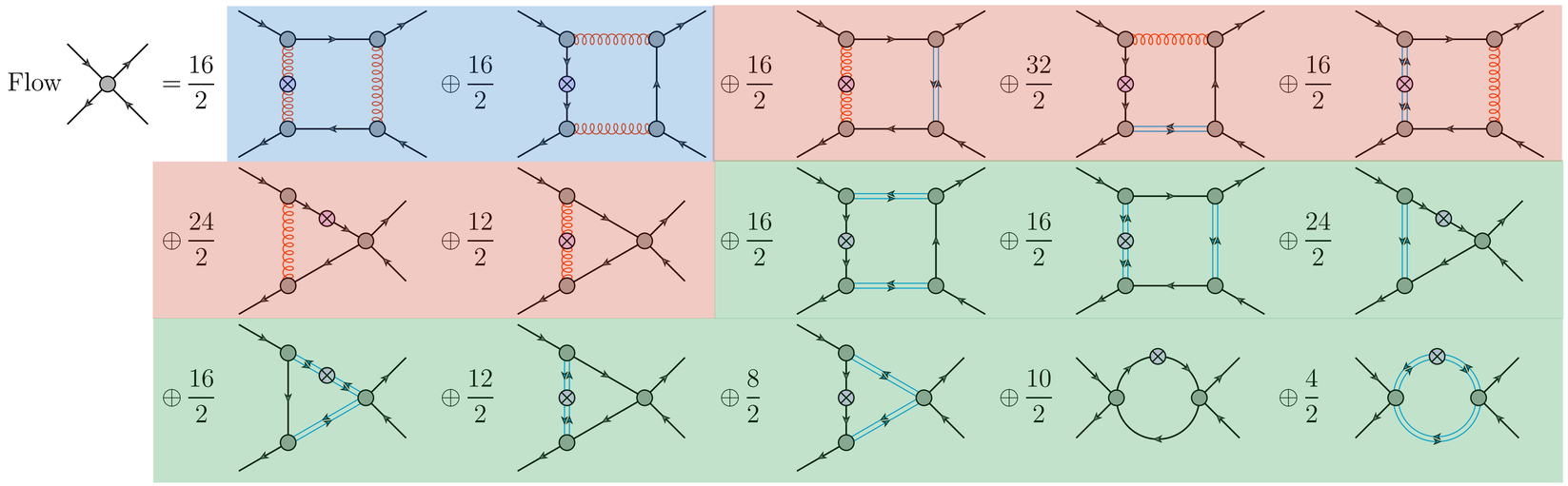
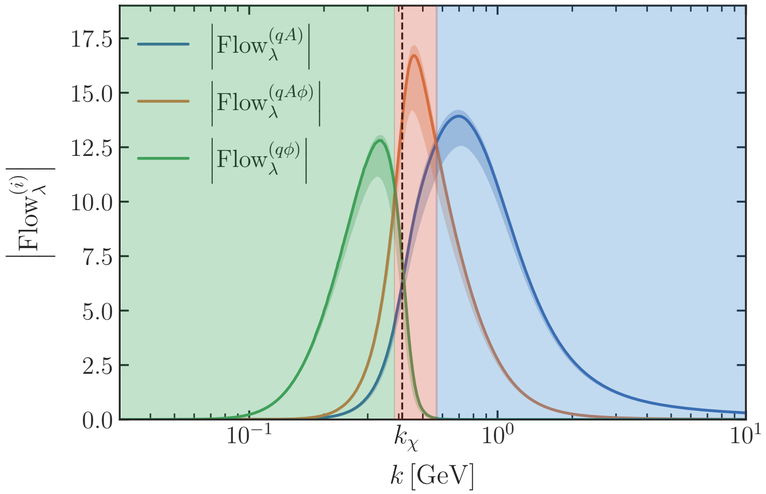


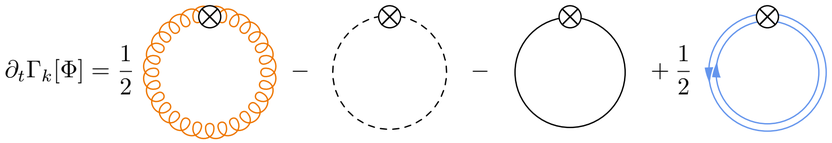




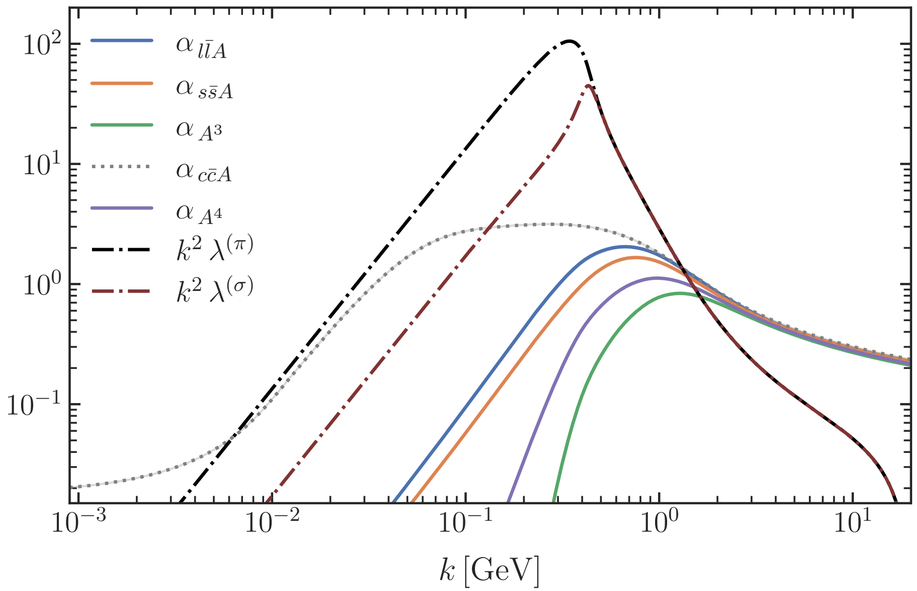
Access and combined use of error estimates from functional QCD & LEFTs

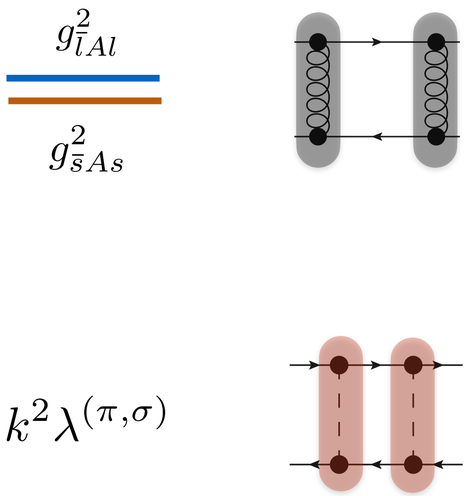
Ihssen, JMP, Sattler, Wink, arXiv:2408.08413

fQCD



Sequential decoupling of gluon, quark, sigma, pion fluctuations

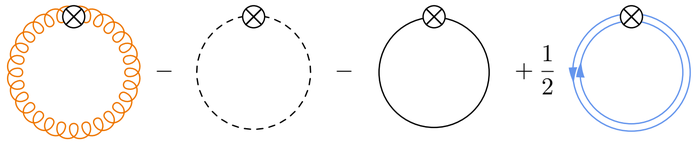




Vacuum: Ihssen, JMP, Sattler, Wink, arXiv:2408.08413

Phase structure: Fu, JMP, Rennecke, PRD 101, (2020) 054032

fQCD

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2}$$ 

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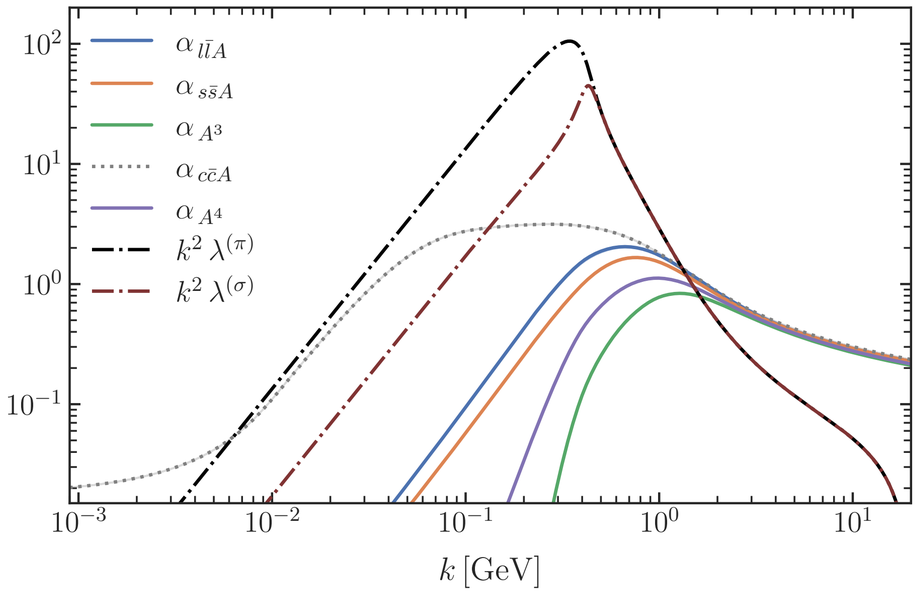
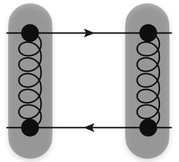
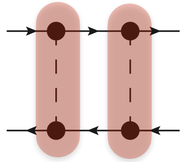


$g^2_{\bar{l}Al}$

$g^2_{\bar{s}As}$



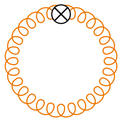
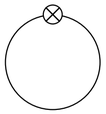
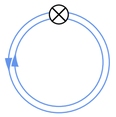
$k^2\lambda^{(\pi,\sigma)}$



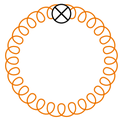
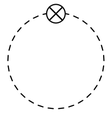
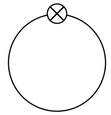
Vacuum: Ihssen, JMP, Sattler, Wink, arXiv:2408.08413

Phase structure: Fu, JMP, Rennecke, PRD 101, (2020) 054032

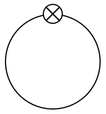
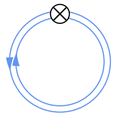
PQM-model

$$\frac{1}{2}$$  $-$  $+$ $-$  $+\frac{1}{2}$ 

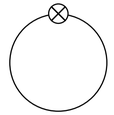
PNJL-model

$$\frac{1}{2}$$  $-$  $+$ $-$ 

QM-model

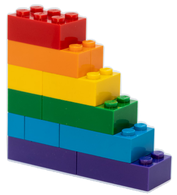
$-$  $+\frac{1}{2}$ 

NJL-model

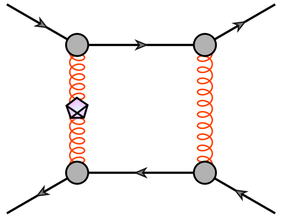
$-$ 

QCD-assisted low energy effective theories

How to: systematic error estimates & the LEGO® principle



Example: Disect quark-gluon box



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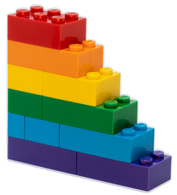
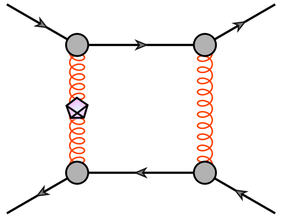
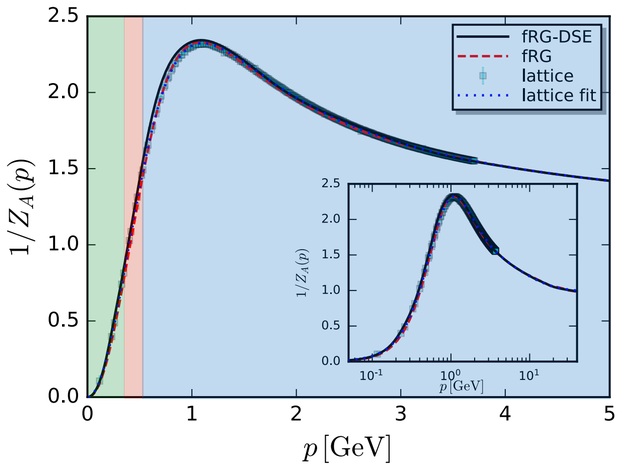
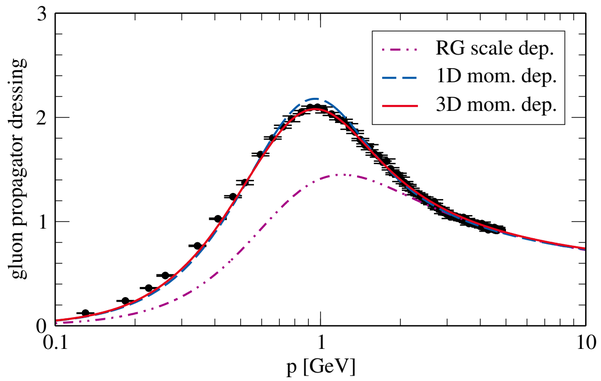


Example: Disect quark-gluon box



gluon two-point correlator





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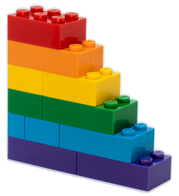

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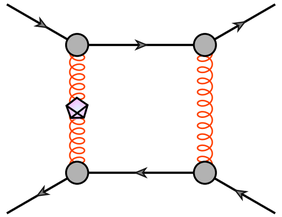

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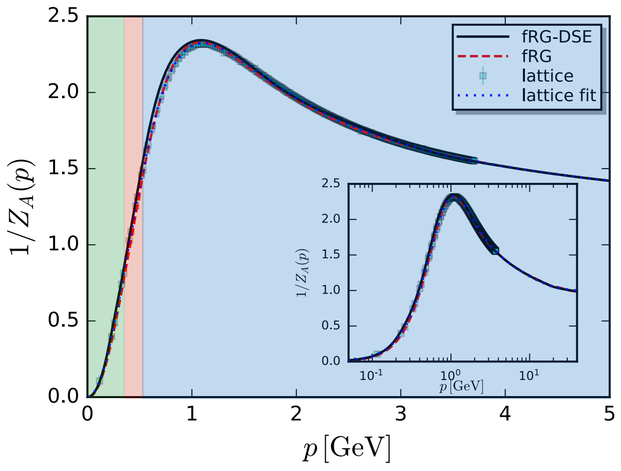
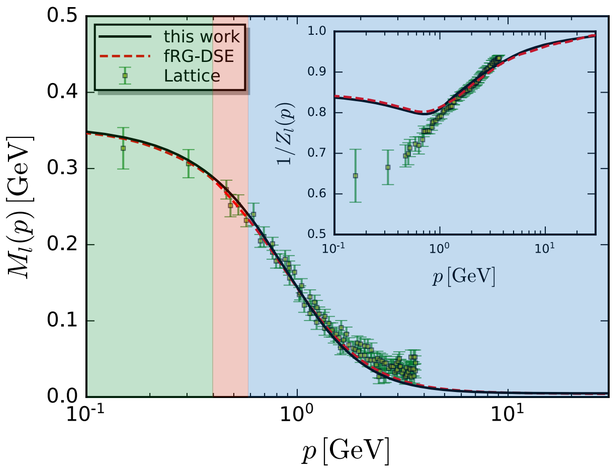
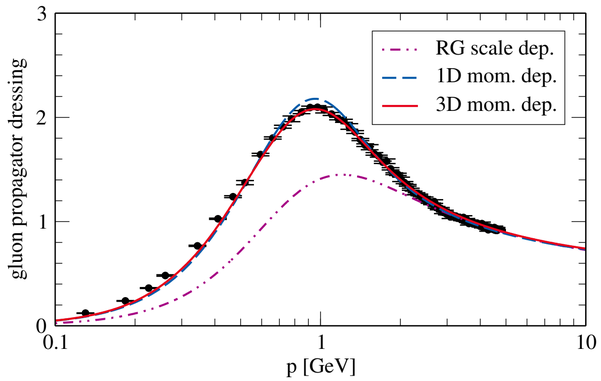


quark two-point correlator





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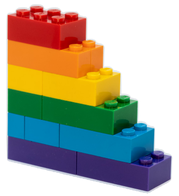
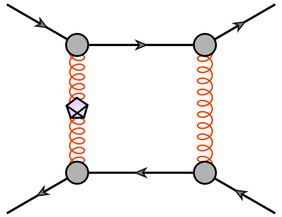
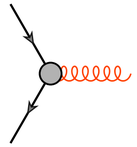


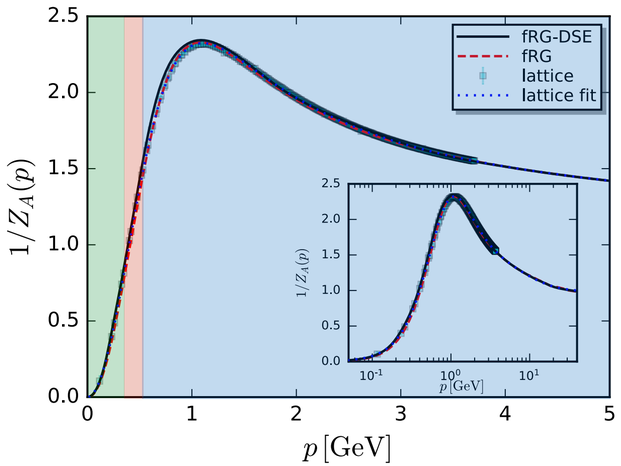
Example: Disect quark-gluon box



gluon two-point correlator

quark-gluon scattering





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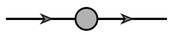
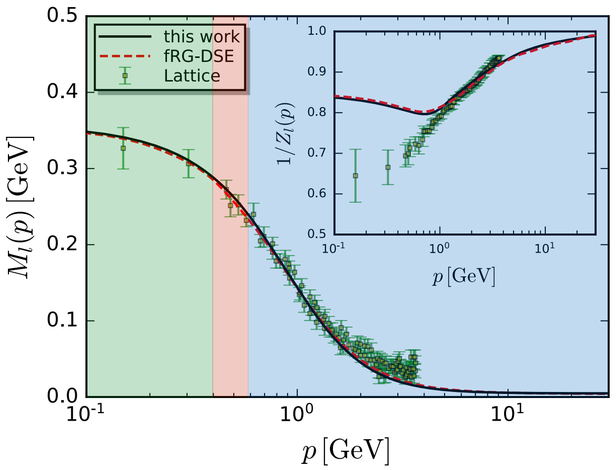
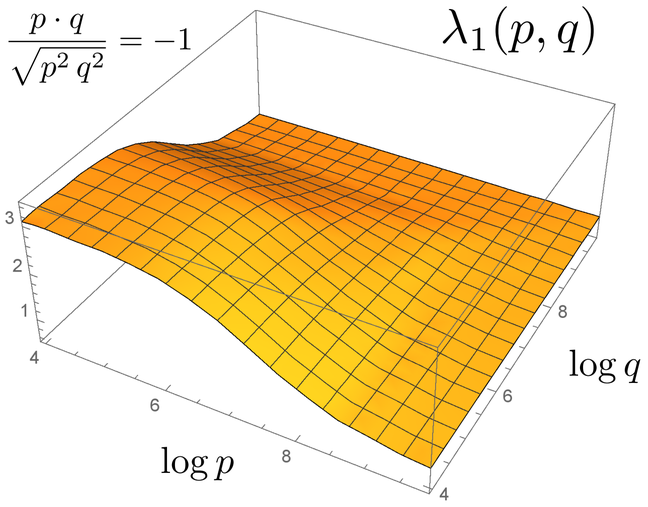
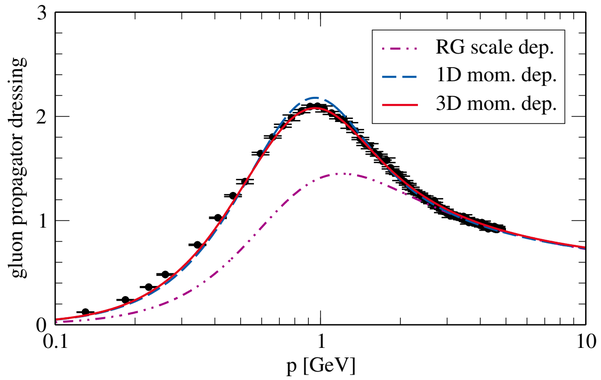








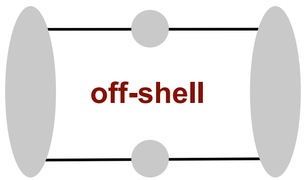
$$\lambda_1(p,q), \cdots, \lambda_8(p,q)$$

Relevant: $\lambda_1\,,\ \lambda_4\,,\ \lambda_7$

Three remarks on Functional Methods for QCD

- **off-shell representation of thermodynamic observables**

e.g. $\mathrm{Tr}\,\langle q(x)\bar{q}(x)\rangle$



pressure, trace anomaly,
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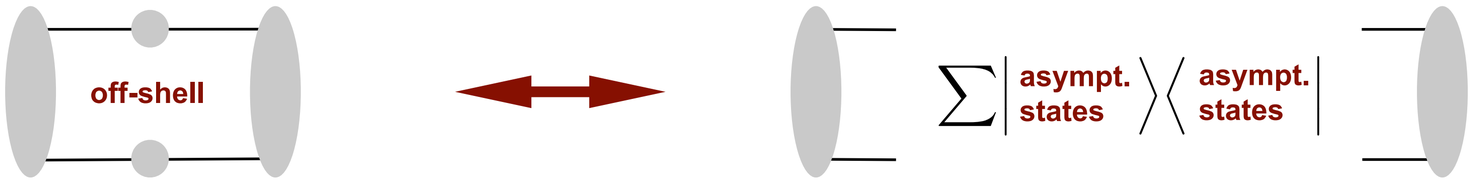

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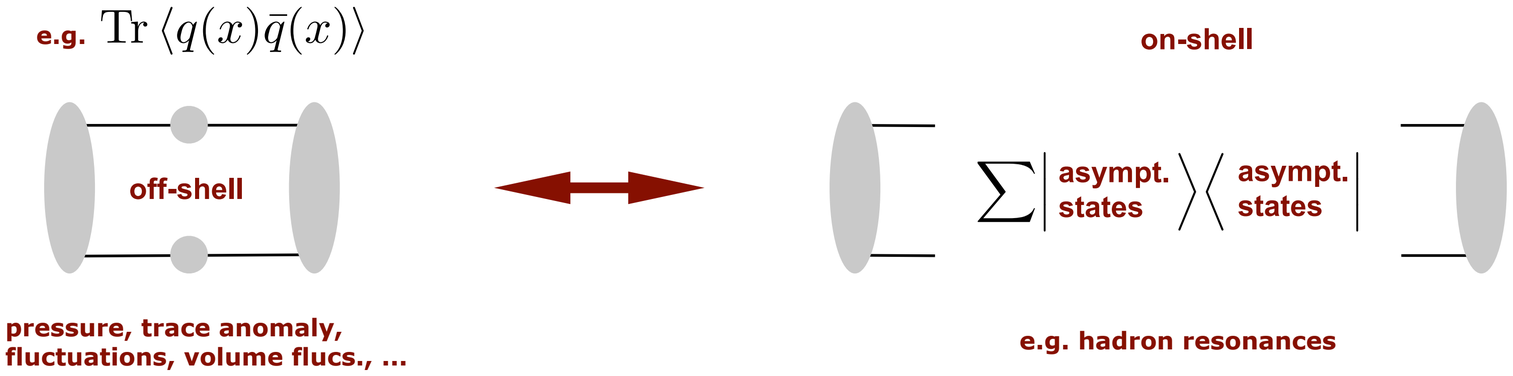

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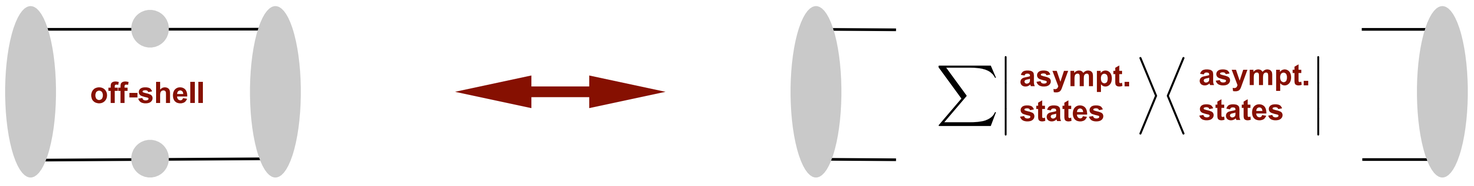

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- **gauge fixing = parameterisation**

$$\langle q(x_1)\cdots\bar{q}(x_{2n})\,A_\mu(y_1)\cdots A_\mu(y_m)\,h(z_1)\cdots h(z_l)\rangle$$

Consequences

I: simple correlations

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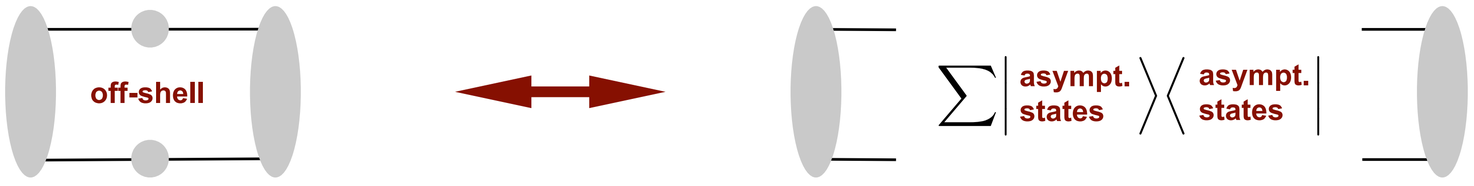

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II: Difficult access to some observables

'No free lunch theorem'

Three remarks on Functional Methods for QCD

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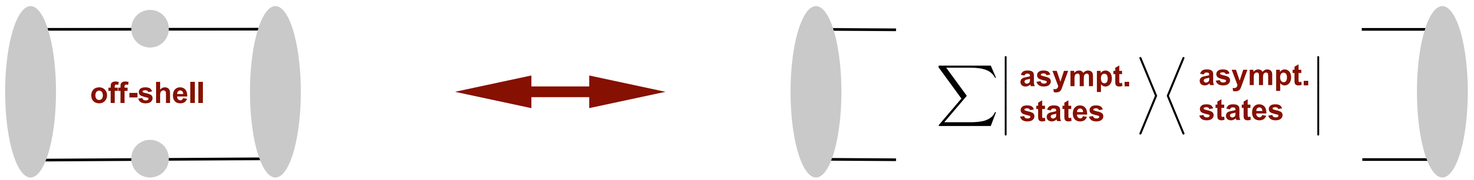
on-shell



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Consequences

I: simple correlations

II: Difficult access to some observables

'No free lunch theorem'

- **'Your mean field is not my mean field'**

$$\left.\frac{\delta S_{\mathrm{cl}}[\phi]}{\delta\phi}\right|_{\phi=\bar{\phi}} = 0$$

$$\left.\frac{\delta\Gamma[\phi]}{\delta\phi}\right|_{\phi=\bar{\phi}_{\mathrm{quant}}} = 0$$

full quantum equation of motion

How to: direct computations and the minimal point of view

Those are my interpretations,
and if you don't like them….
well, I have others

- **Self-consistent truncations to functional relations define analytic functions in μ_{B} , eg:**

$$\partial_t \Big\langle q(x)\bar{q}(y)\Big\rangle(\mu_B) = \mathrm{Loop}\left[\Big\langle q(x)\bar{q}(y)\Big\rangle(\mu_B),\ \Big\langle q(x)A_\mu(y)\bar{q}(z)\Big\rangle(\mu_B), \cdots ;\ \mu_B\right]$$

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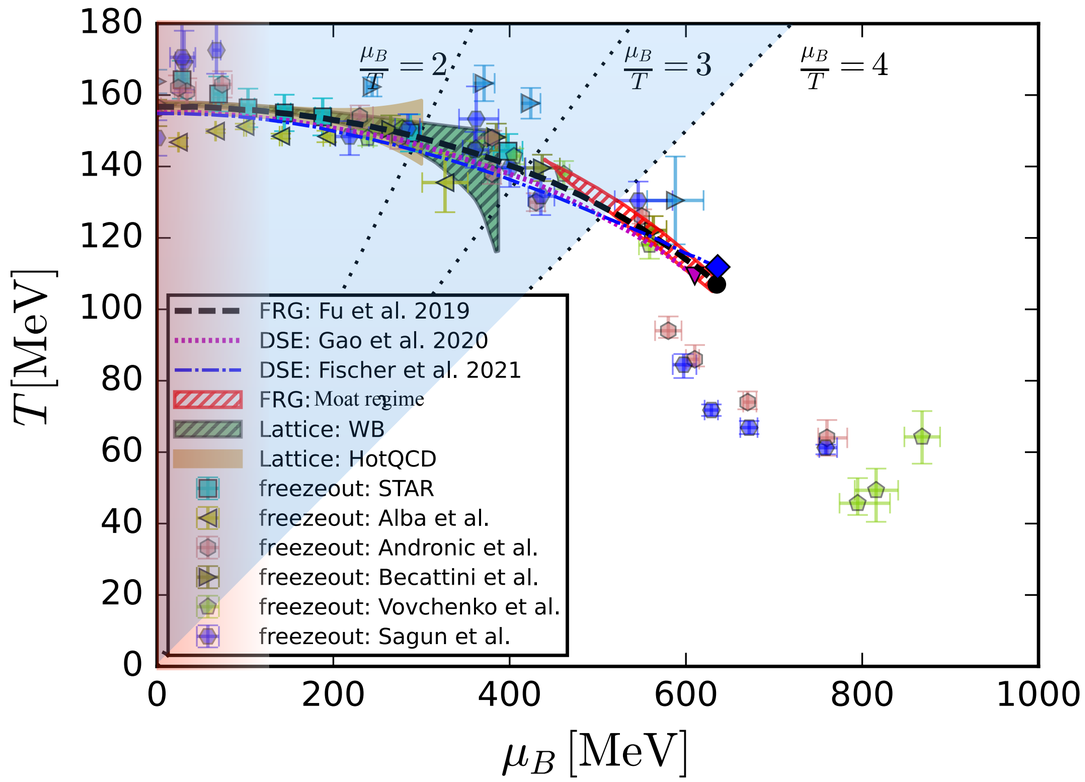
- Consequences for functional QCD predictions at finite density



By now the best truncations to functional QCD pass lattice benchmark tests at vanishing and small chemical potential

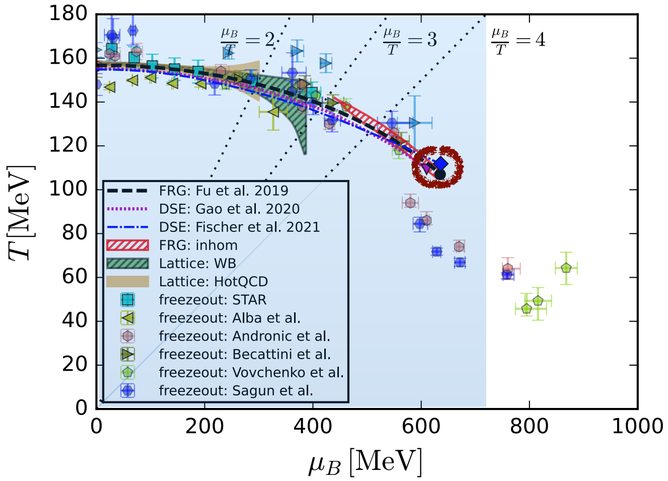
Regime of quantitative reliability of current best truncation

Unique: QCD-based analytic continuations that satisfy the lattice benchmarks at small chemical potential.

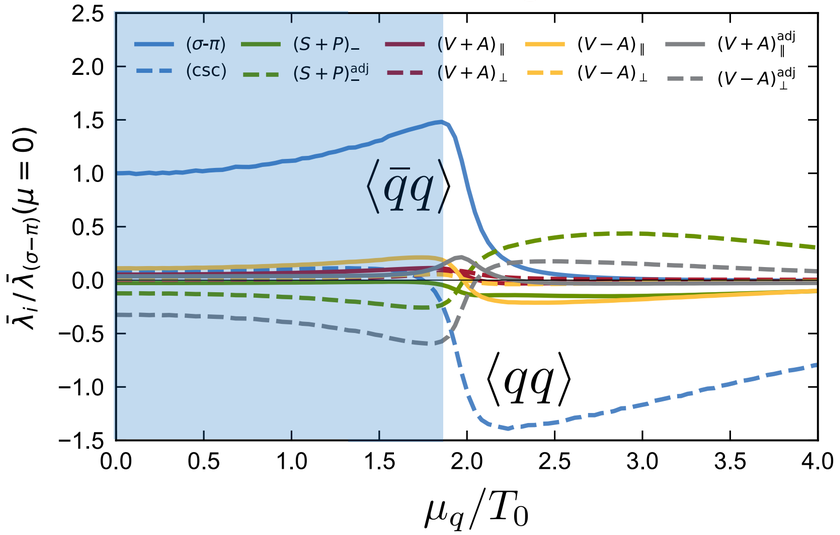
Great opportunity for a combined analysis of high density QCD (Exp. data + lattice QCD + functional QCD + LEFTs)

Phase structure from functional QCD: Predictions & estimates

Predictions & estimates



Four-quark scattering channels



Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004

Dominance of scalar-pseudoscalar fluctuations

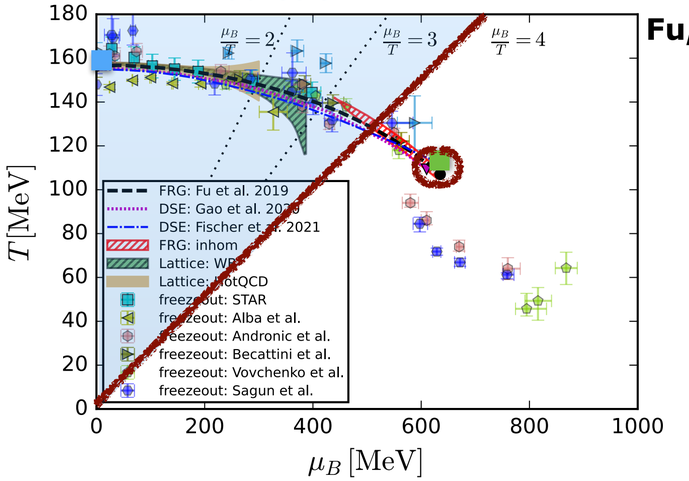
Pions & sigma mode

Predictions & estimates

Moat regime

Pisarski, Rennecke, PRL 127 (2021) 152302

Prediction

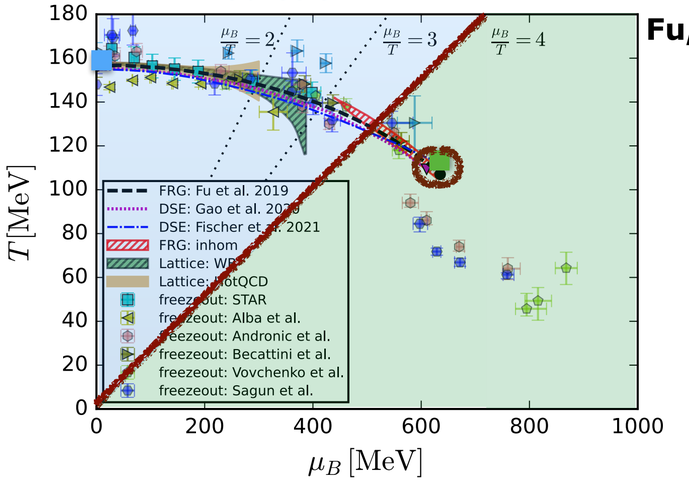


Fu, JMP, Rennecke, PRD 101 (2020) 054032

Regime of quantitative reliability
of
current best truncation

Predictions & estimates

Prediction



Fu, JMP, Rennecke, PRD 101 (2020) 054032

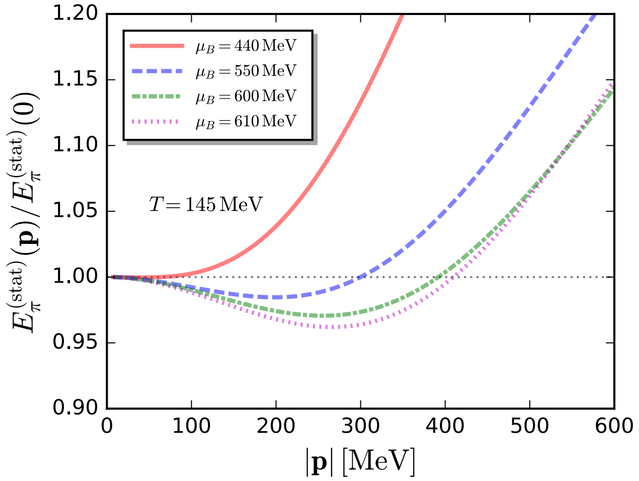
Estimate

Moat regime

Pisarski, Rennecke, PRL 127 (2021) 152302

Regime of quantitative reliability of current best truncation

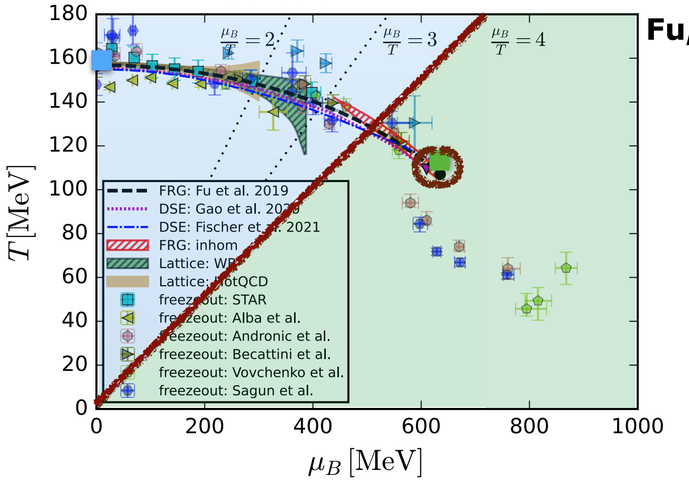
Moat regime is not captured quantitatively



Fu, JMP, Pisarski, Rennecke, Wen, Shi Yin, 2412.15949

Predictions & estimates

Prediction



Fu, JMP, Rennecke, PRD 101 (2020) 054032

Moat regime

Pisarski, Rennecke, PRL 127 (2021) 152302

Regime of quantitative reliability
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current best truncation

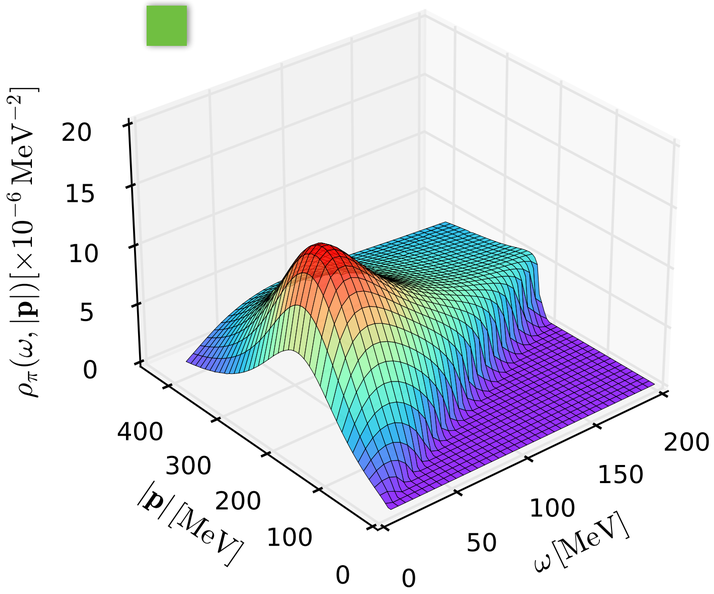
Estimate

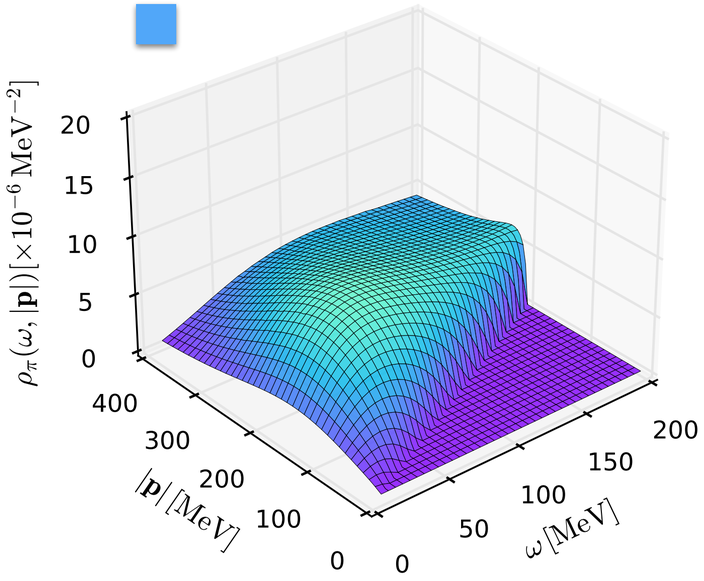
T=114 MeV & μ_B =630 MeV

T=160 MeV & μ_B =0 MeV

Moat regime is not captured quantitatively

Pion spectral functions

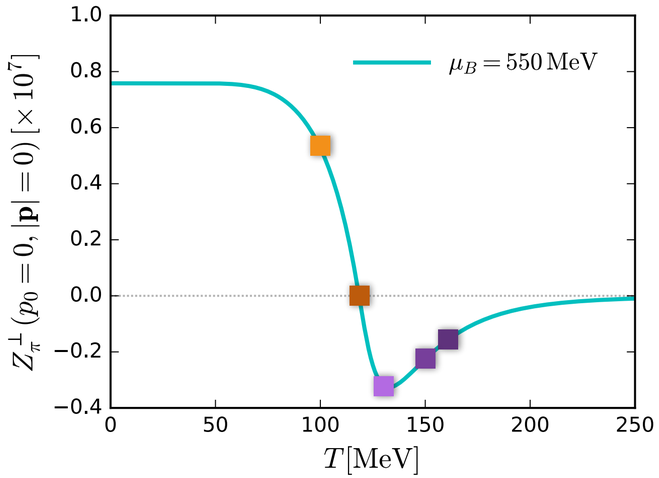


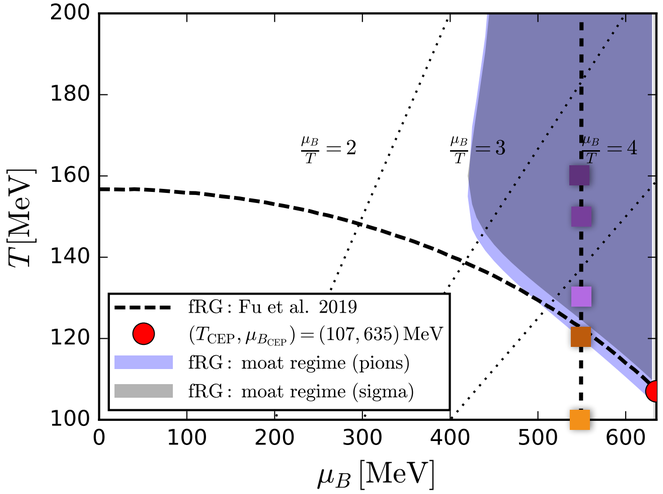


Fu, JMP, Pisarski, Rennecke, Wen, Shi Yin, 2412.15949

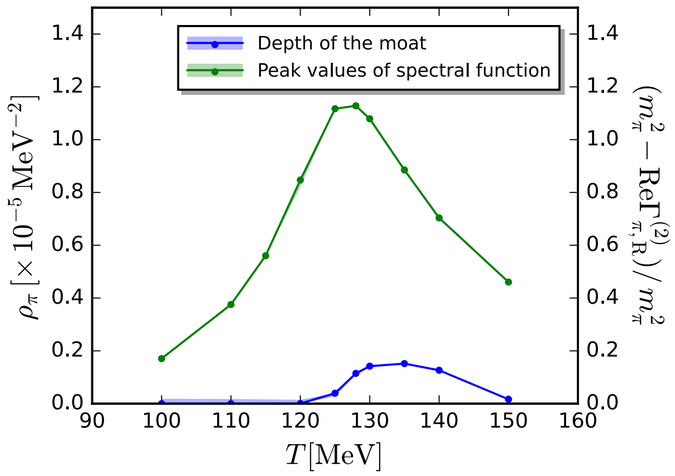
Predictions & estimates

Spatial wave function of the pion at p=0





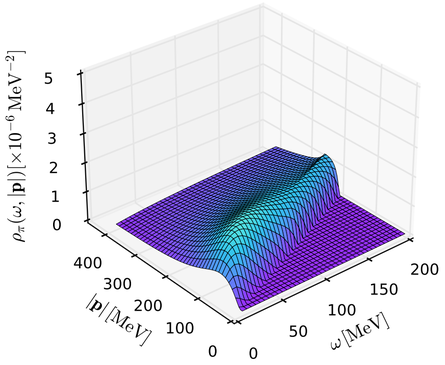
Depth of the moat & spectral peak of the pion



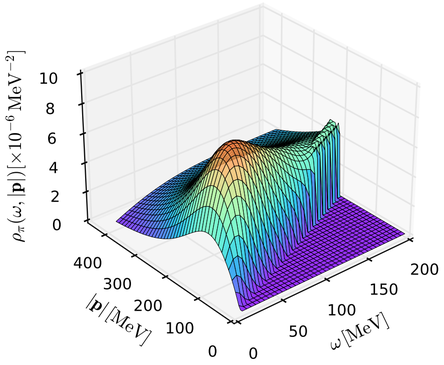
Dissecting the moat & the moaton

μ_B = 650 MeV

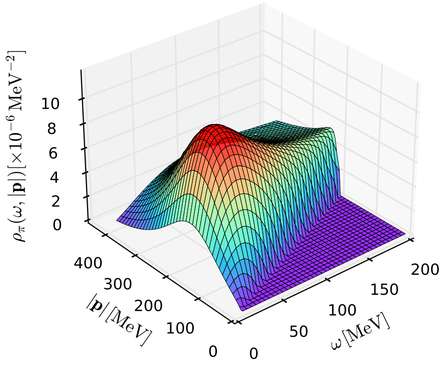
T=100 MeV



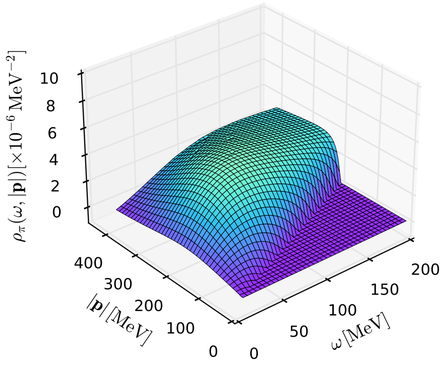
T=120 MeV



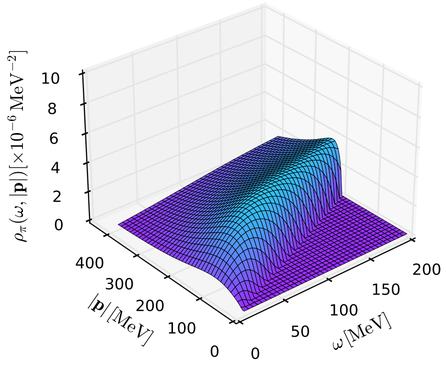
T=130 MeV



T=150 MeV



T=160 MeV

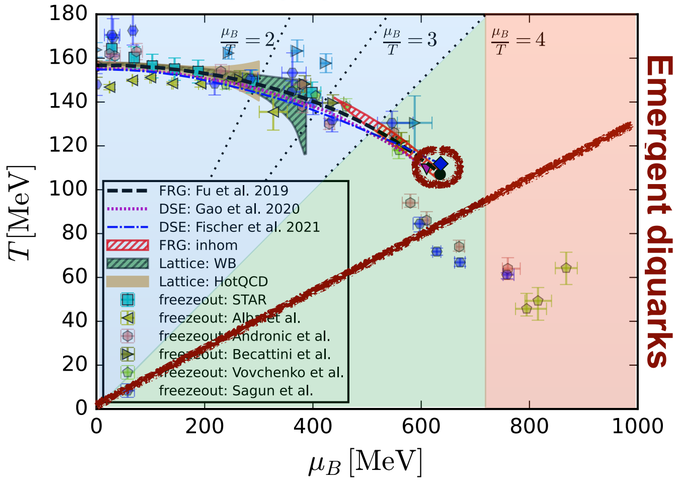


Fu, JMP, Pisarski, Rennecke, Wen, Shi Yin, 2412.15949

Predictions & estimates

Emergent diquarks

Prediction

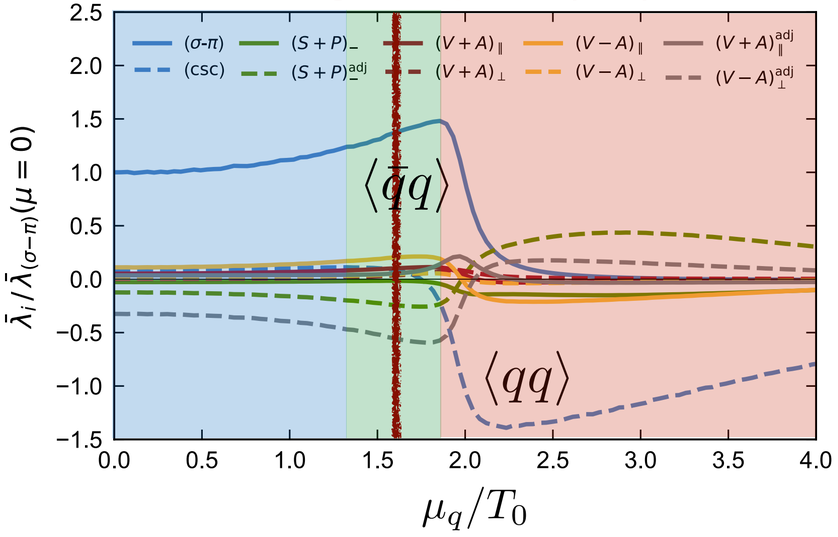


Emergent diquarks

Estimate

Regime of quantitative reliability
of
current best truncation

Emergent diquarks (fRG)

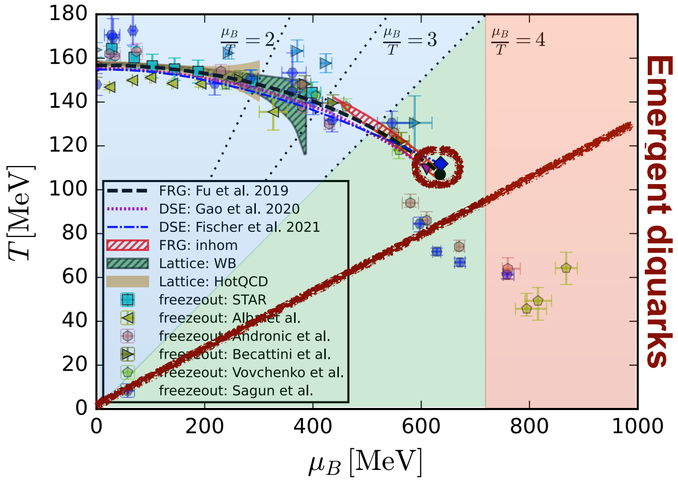


Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004

Predictions & estimates

Emergent diquarks

Prediction



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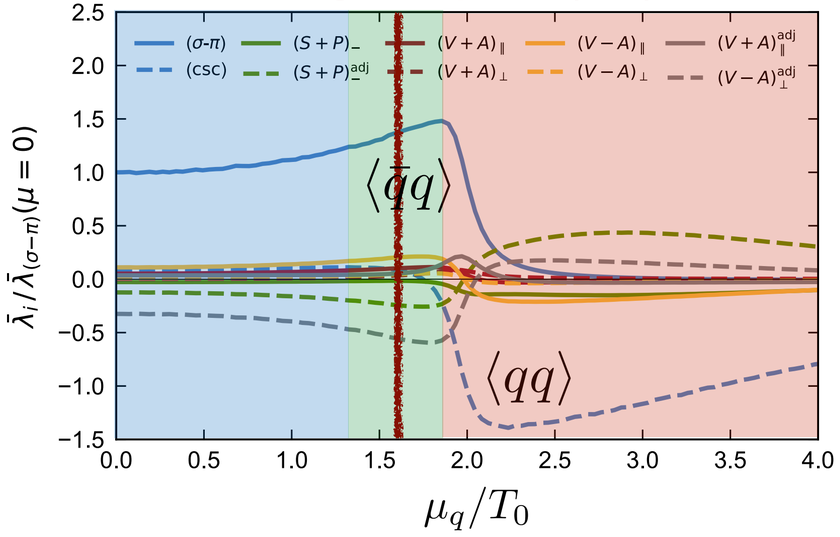


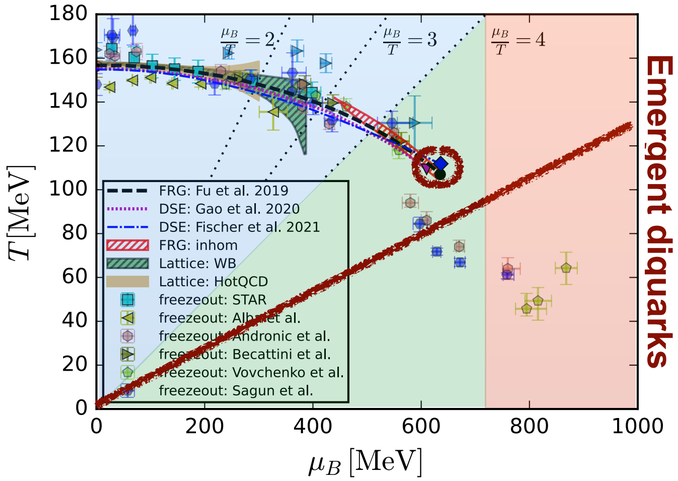
Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004

Emergent diquarks are not captured
by extrapolations

Predictions & estimates

Emergent diquarks

Prediction



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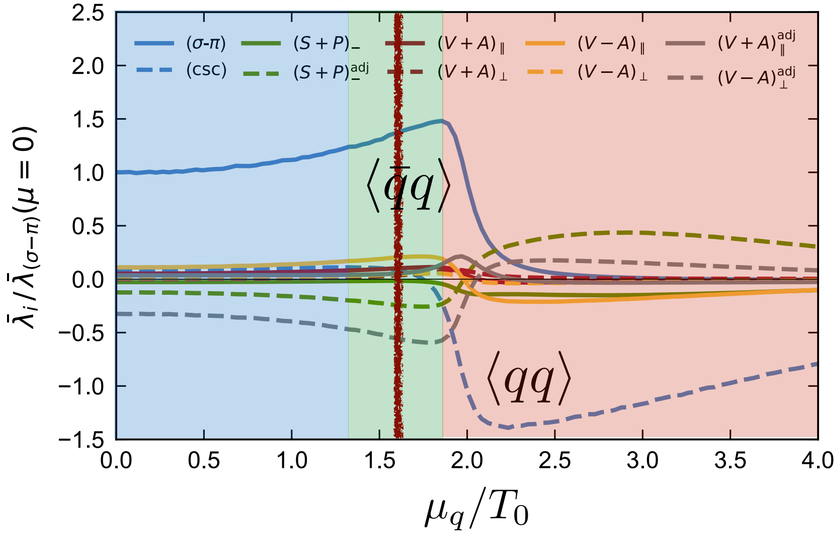
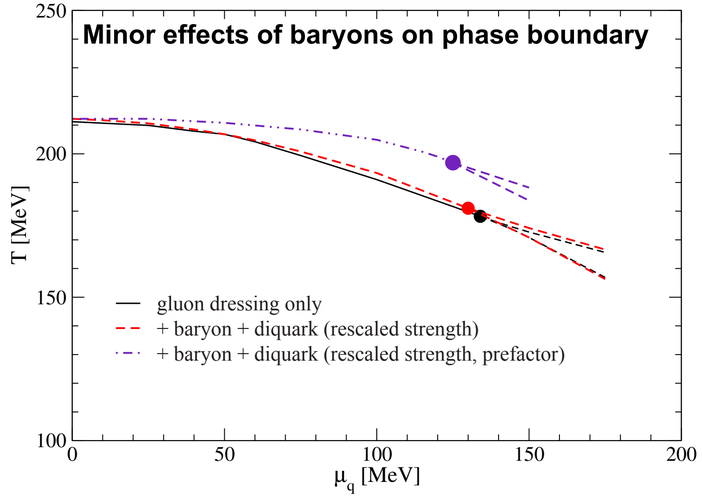


Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004

Emergent baryons (DSE)

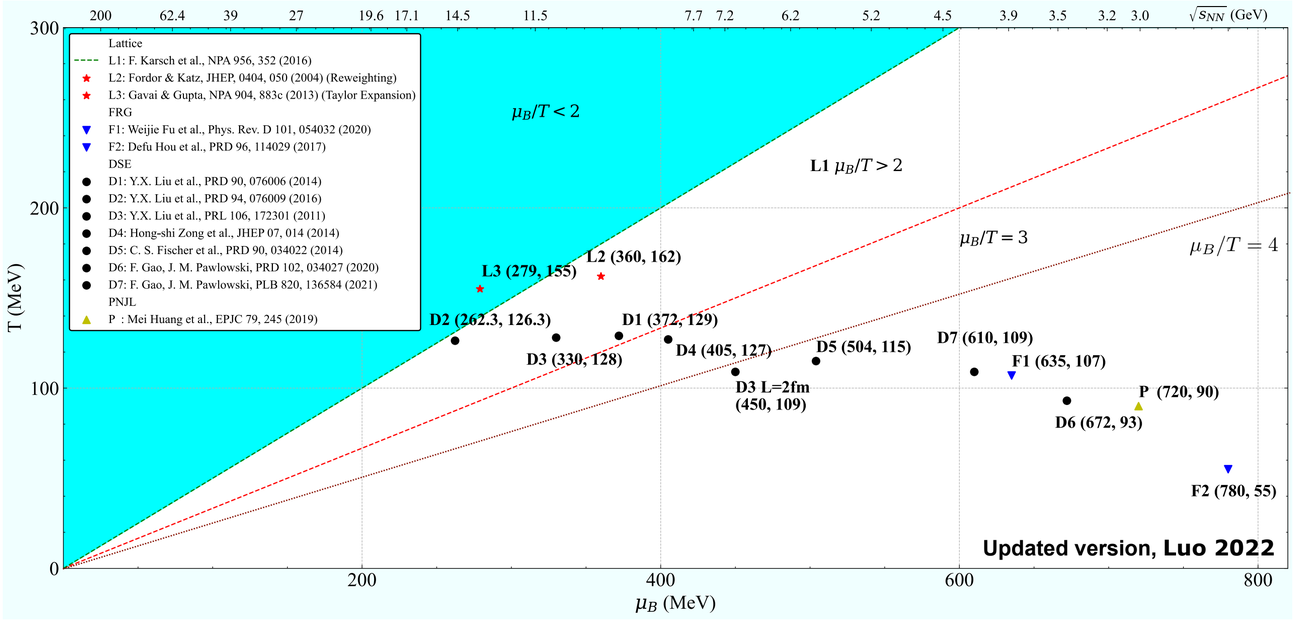


Eichmann, Fischer, Welzbacher, PRD 93 (2016) 034013

Predictions, estimates & extrapolations and how to judge them

Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)



Large uncertainties for the estimation of CP location.

Disclaimer

Most functional computations (LEFT or QCD) have not been set-up for CEP-predictions!

Lack of predictive power for CEP-predictions is no quality measure!

CEP is standing for 'regime with new physics'

Predictions, estimates & extrapolations and how to judge them

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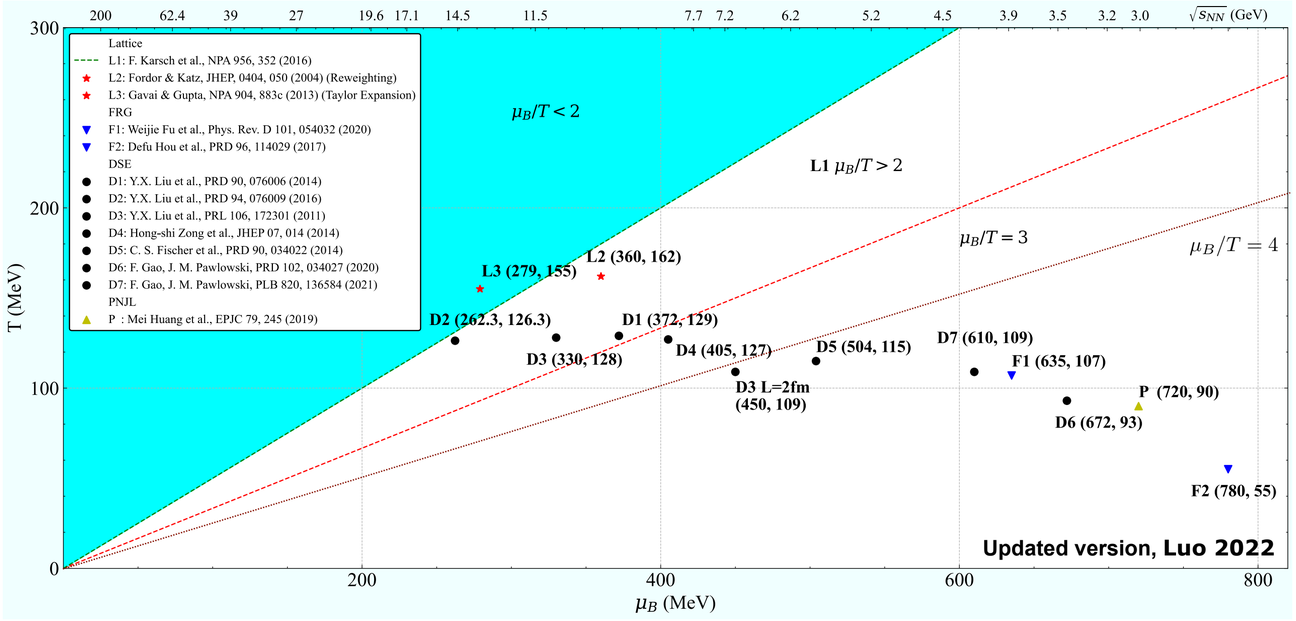

Common folklore since ~2004 ⟷ Large uncertainties for the estimation of CP location.

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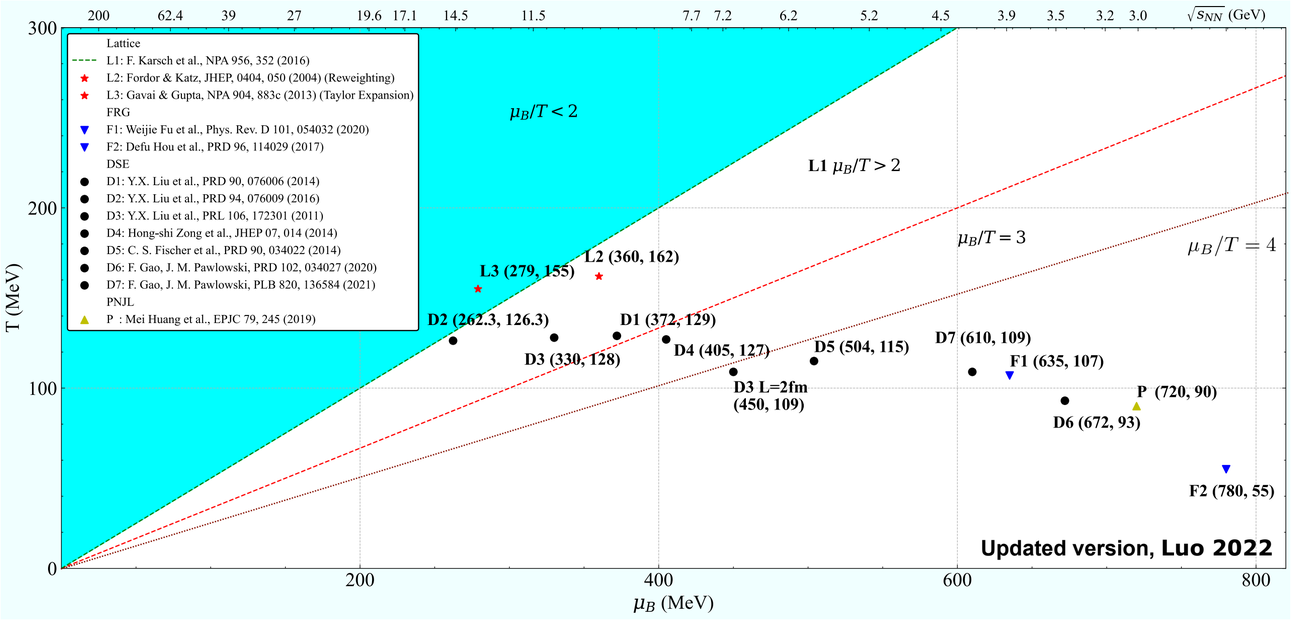

Large uncertainties for the estimation of CP location.

Remove CEP-predictions

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

(i) 'old' CEPs: lattice, Functional QCD approaches, LEFTS (updated computations available)

(ii) LEFTs & Functional Results (qualitative approximations) that miss lattice benchmarks at μ_B =0

(iii) LEFTs with CEPs at large density (missing quark-gluon back reaction)

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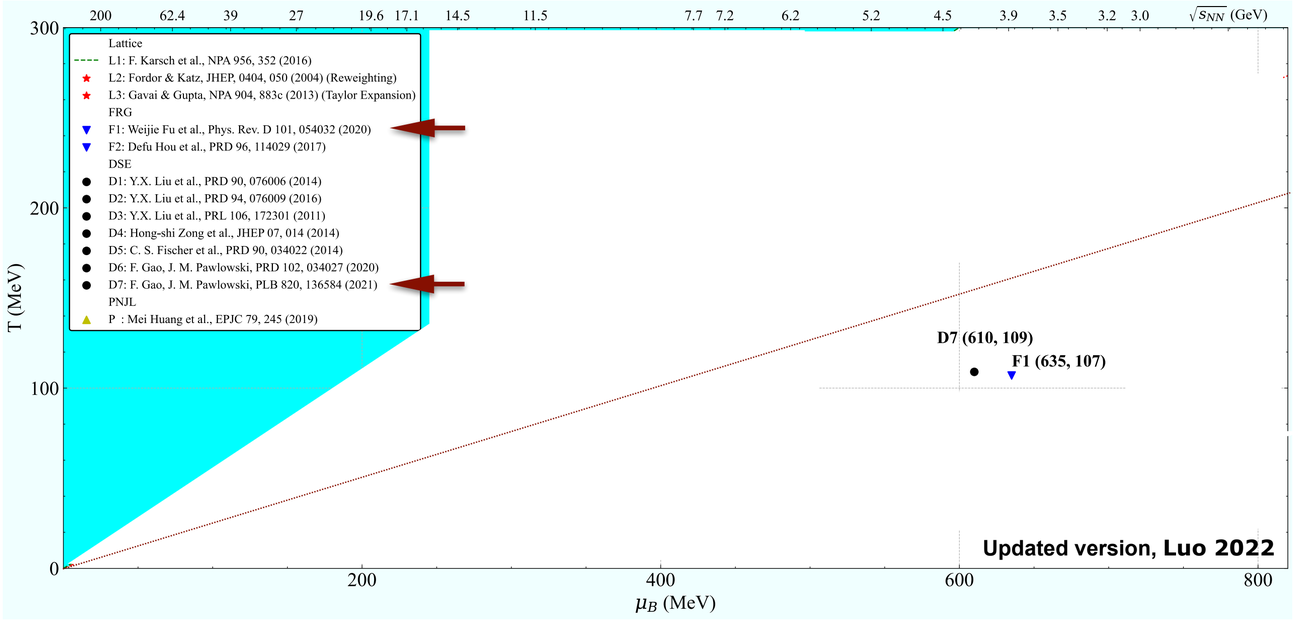

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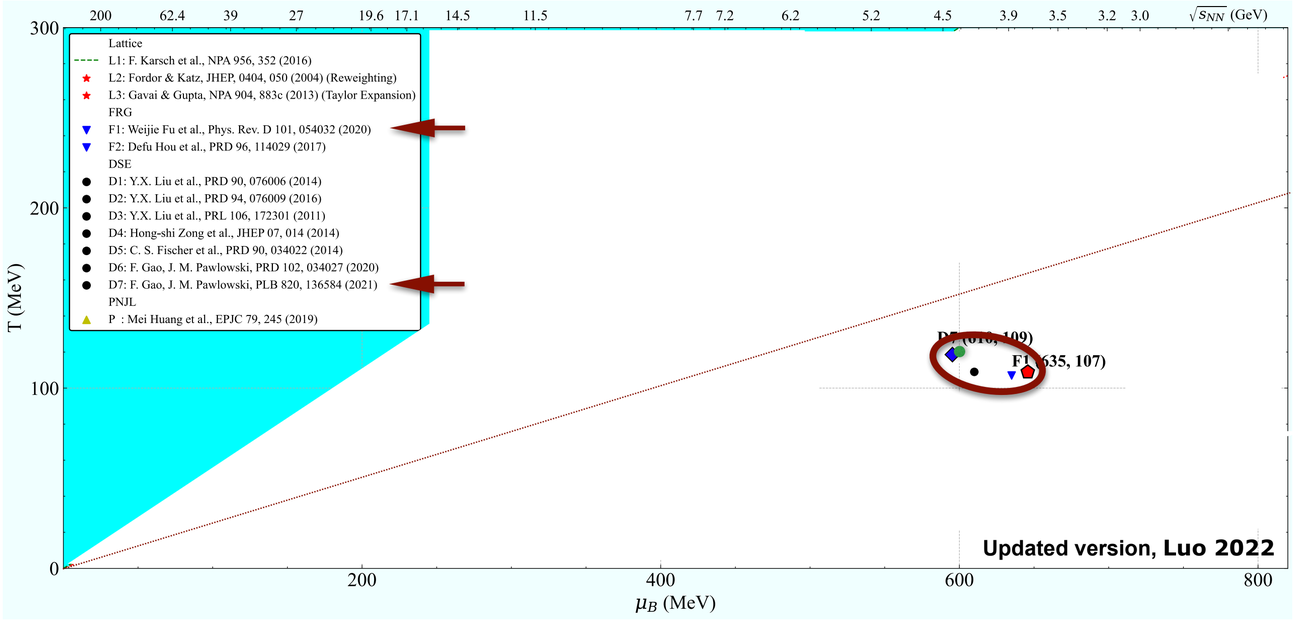
- **Fu, JMP, Rennecke, Wen, Yin, in prep (fRG)**
- **Gunkel, Fischer, PRD 104 (2021) 054022 (DSE)**



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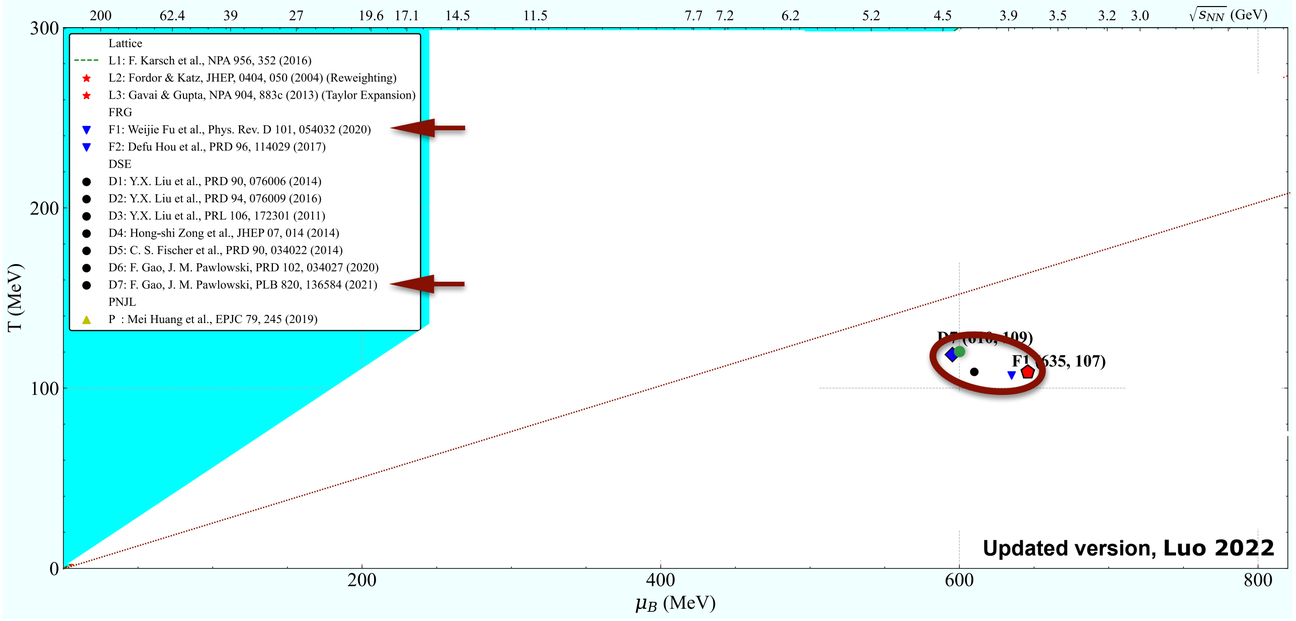

Still small uncertainties for the estimation of CP location **/Onset of new phases**

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Predictions, estimates & extrapolations and how to judge them

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- Gao, Lu, JMP, Schneider, in prep (DSE)
- Fu, JMP, Rennecke, Wen, Yin, in prep (fRG)
- Gunkel, Fischer, PRD 104 (2021) 054022 (DSE)

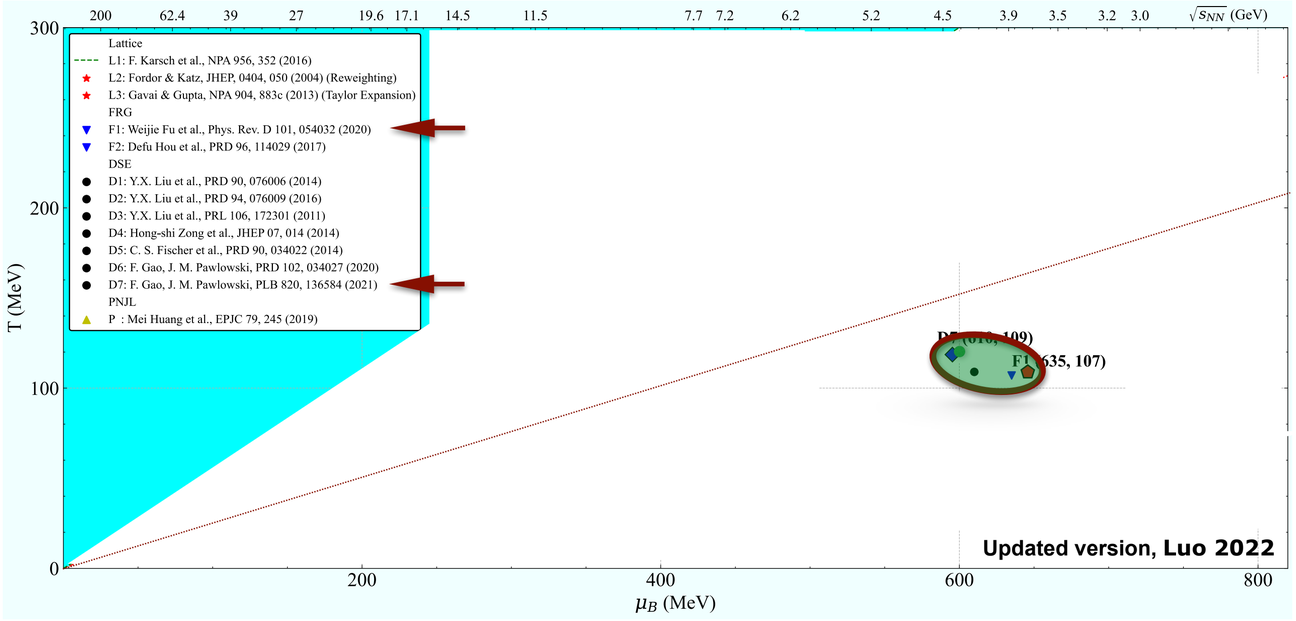
Extrapolations

Lattice extrapolations: Basar, PRC 110 (2024) 015203
Bielefeld-Parma, arXiv:2405.10196

⋮

Holographic models: Hippert, Grefa, Manning, Noronha, Noronha-Hostler, Portillo Vazquez, Ratti, Rougemont, Trujillo, arXiv: 2309.00579

⋮



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Predictions, estimates & extrapolations and how to use them

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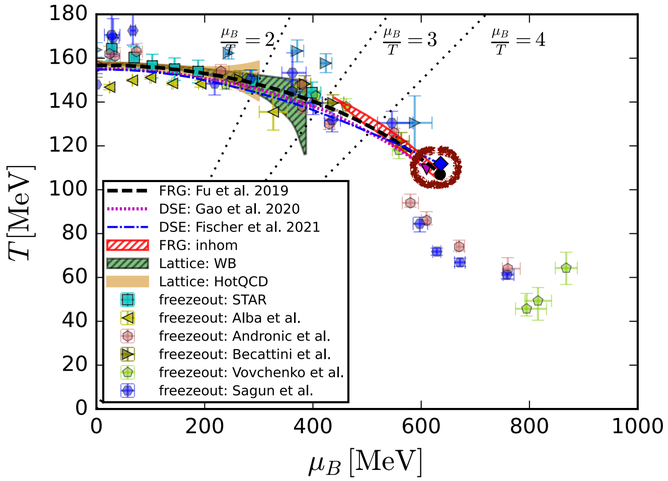
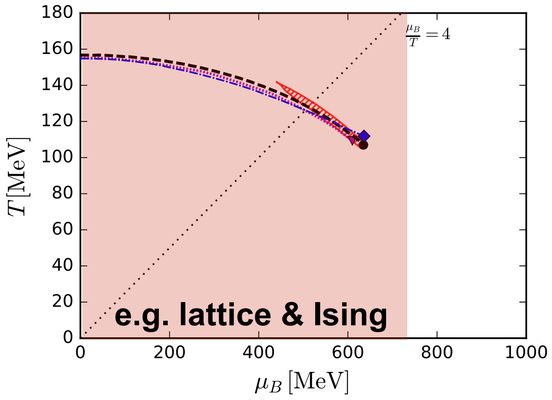


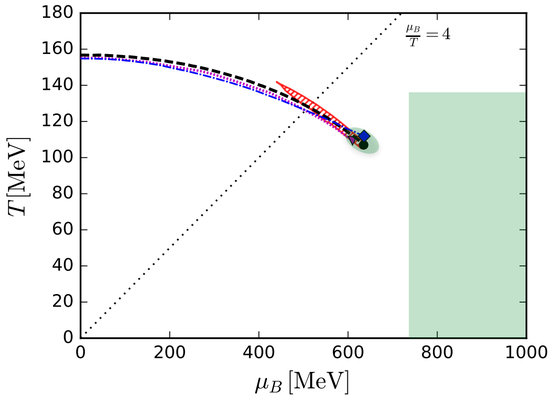
Scenario I

Scaling



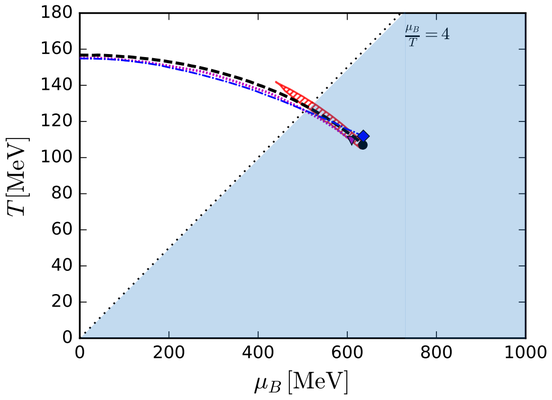
Scenario II

Minimal scaling and new phases



Scenario III

Scaling and/or new phases



Extrapolations for Pheno

✓ ✓ ✗

Predictions, estimates & extrapolations and how to use them

Out by the LEGO ®principle

Fu, JMP, Rennecke, PRD 101 (2020) 054032

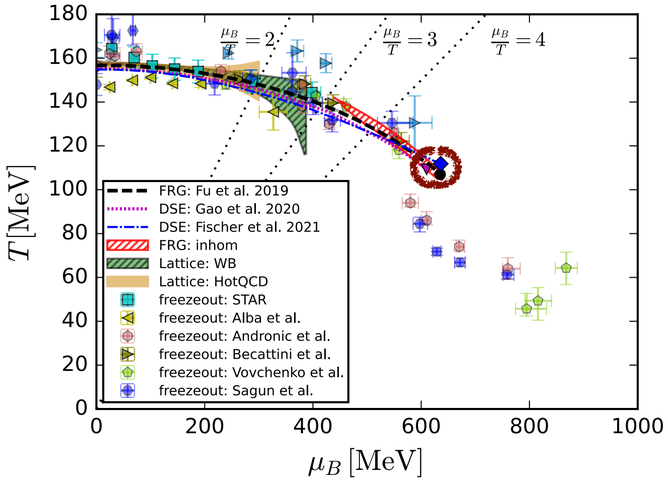
+

Size of scaling regime in LEFTs

Schaefer, Wambach, PRD 75 (2007) 085015

Braun, Klein, Piasecki, EPJC 71 (2011) 1576

⋮



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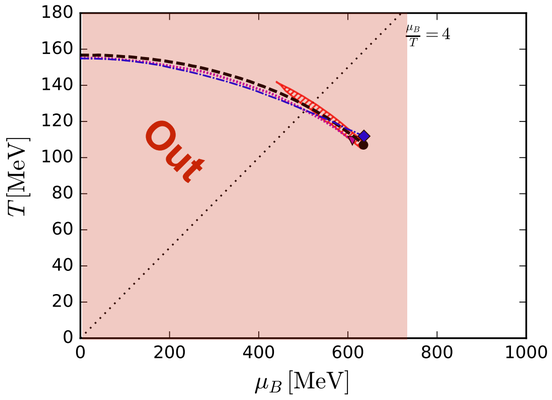
Scaling



Scenario II

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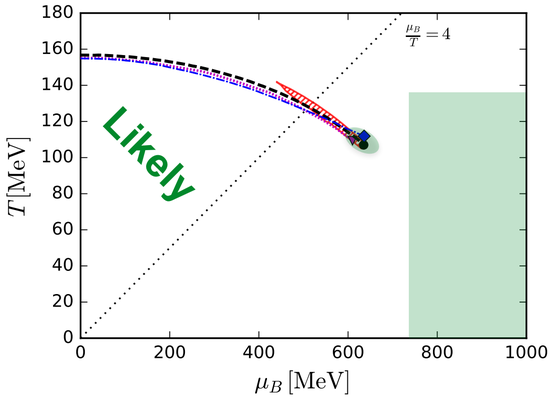

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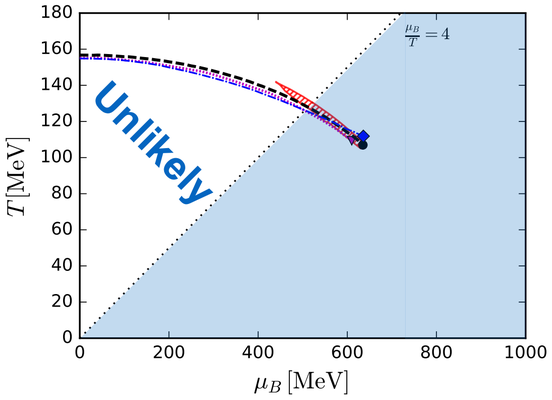


Braun, Fu, JMP, Rennecke, Rosenblüh, Yin, PRD 102 (2020) 056010

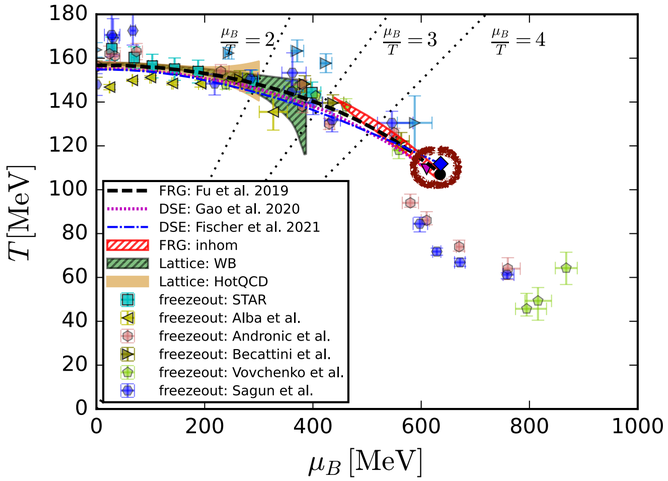
Gao, JMP, PRD 105 (2022) 094020

Soft modes in hot QCD matter: Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, arXiv:2310.19853

+ many results in dynamical low energy effective theories

LEGO® principle

Predictions, estimates & extrapolations and how to use them



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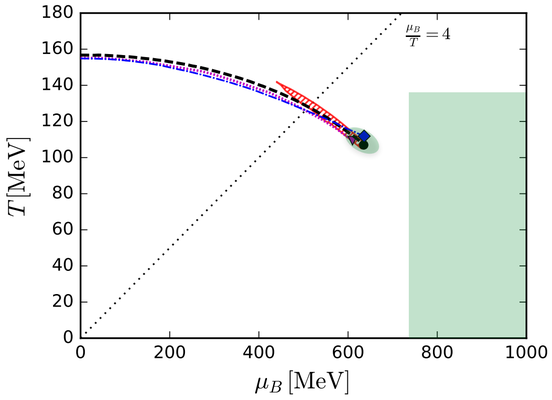
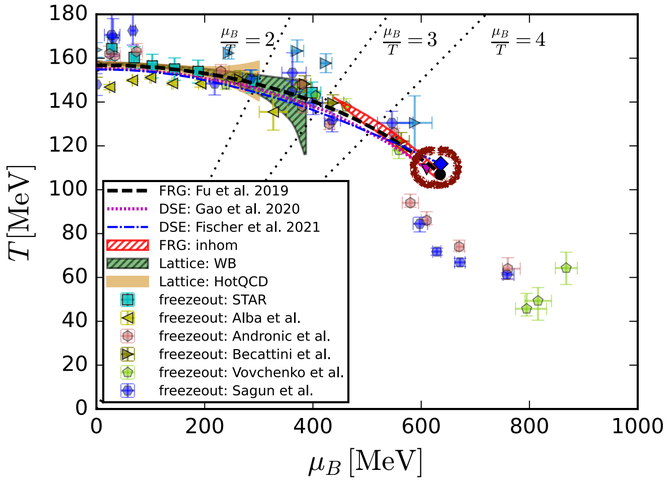


Extrapolations for Pheno

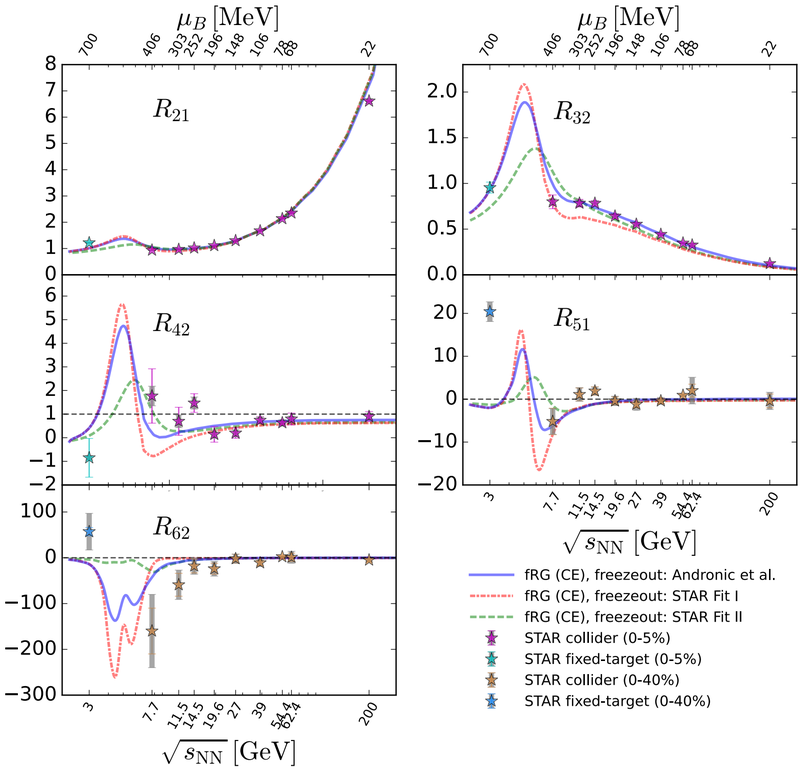
✓

Predictions, estimates & extrapolations and how to use them



Ripples of the critical end point

baryon & proton number fluctuations



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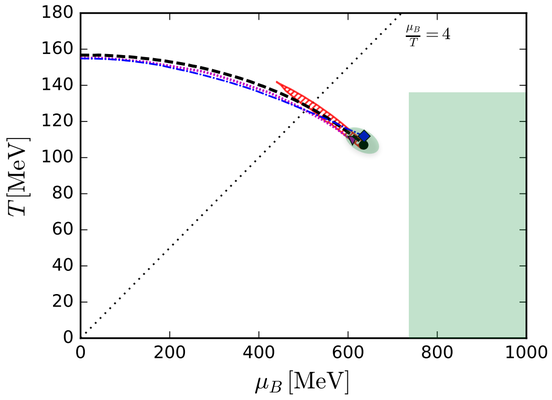


Extrapolations for Pheno

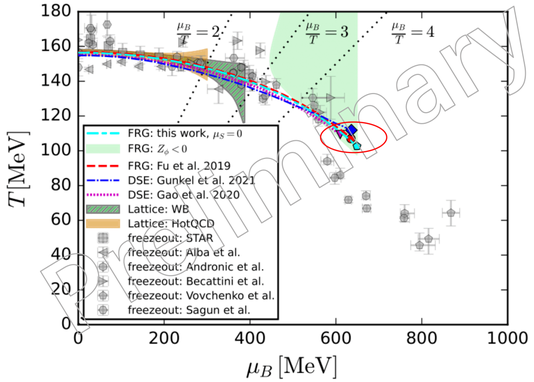
✓

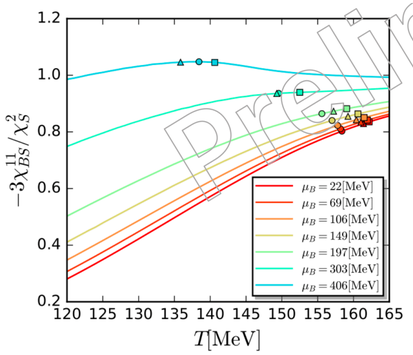
Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 111, L031502

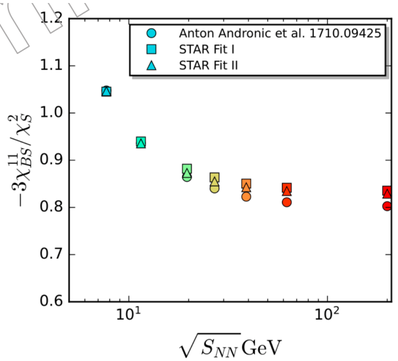
Predictions, estimates & extrapolations and how to use them

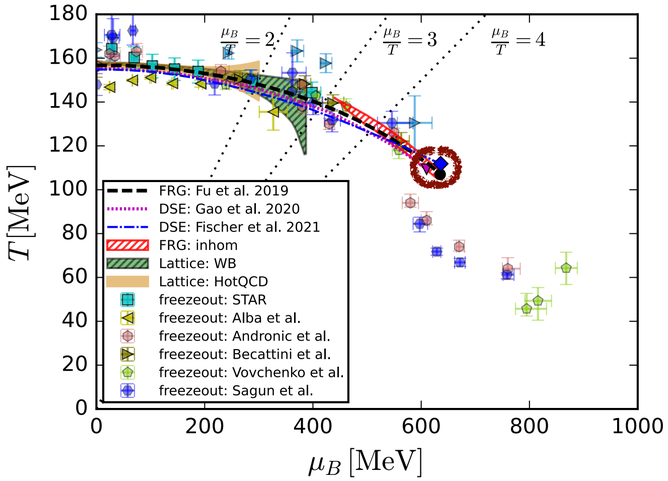
Strangeness neutrality

baryon & proton number fluctuations









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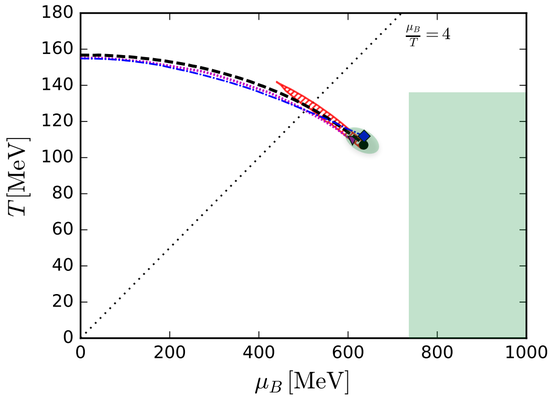

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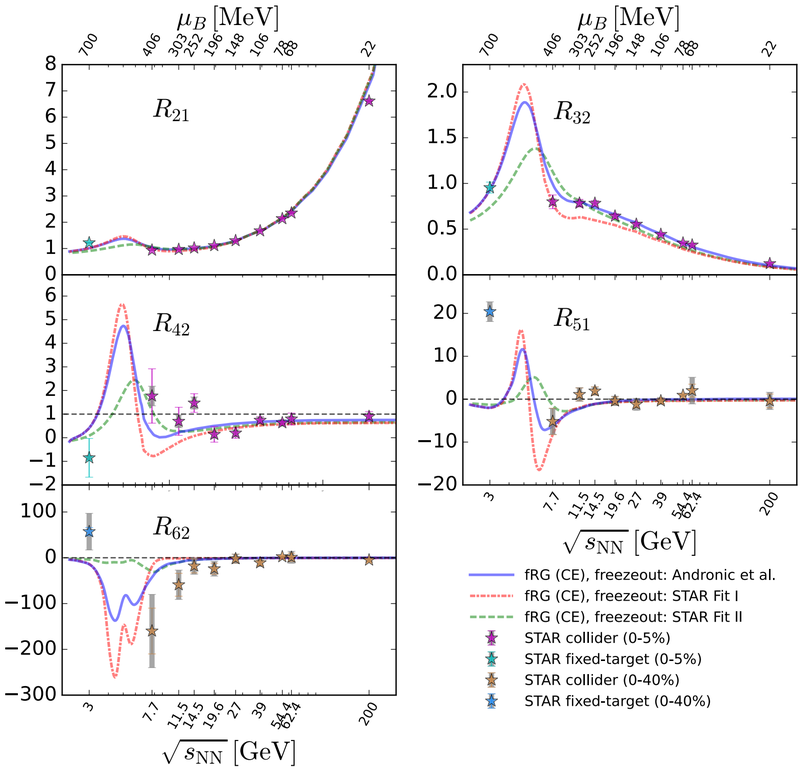


Fu, Huang, JMP, Rennecke, Wen, Yin, in preparation

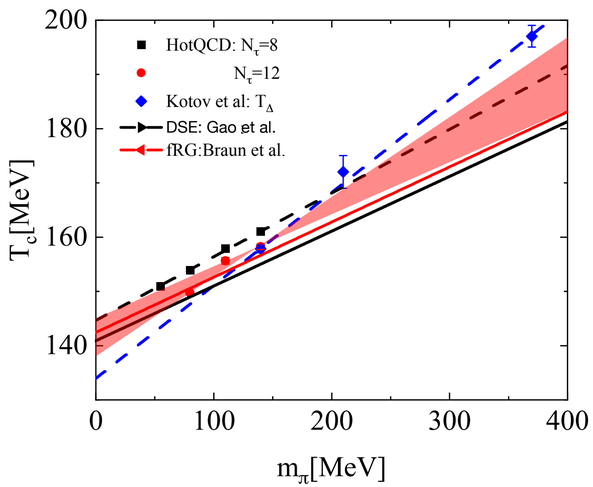
Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 111, L031502

Chiral dynamics & soft modes

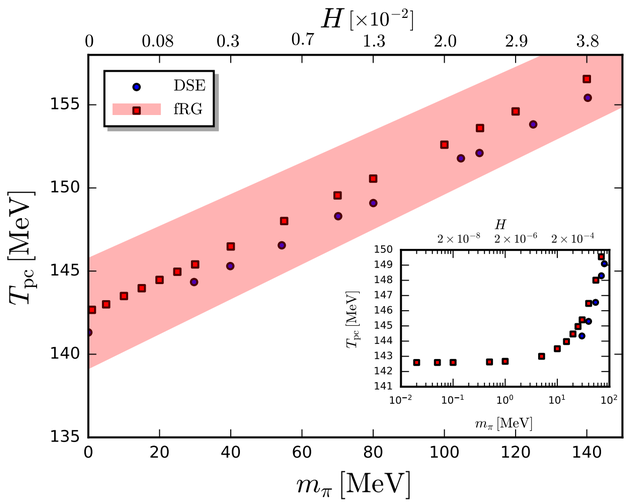
To be (critical) or not (to be)

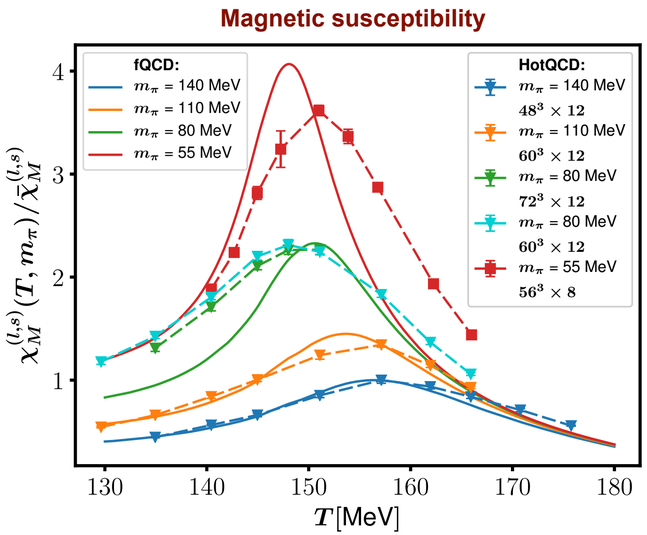
fQCD

Chiral transition temperature



$$H = \frac{m_l}{m_s}$$





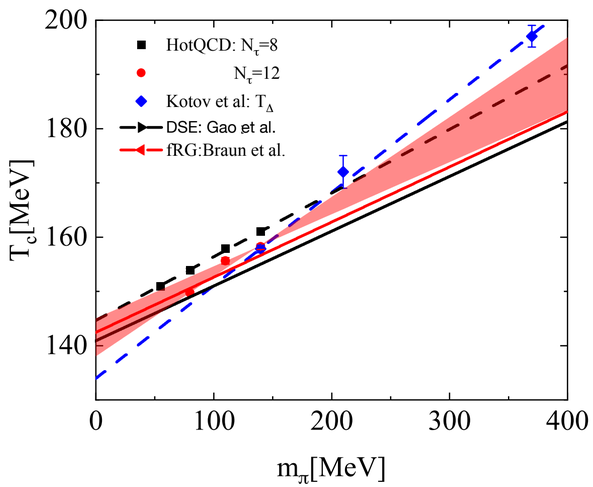
Braun, Fu, JMP, Rennecke, Rosenblüh, Yin, PRD 102 (2020) 056010

Gao, JMP, PRD 105 (2022) 094020

Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, 2310.19853

To be (critical) or not (to be)

fQCD



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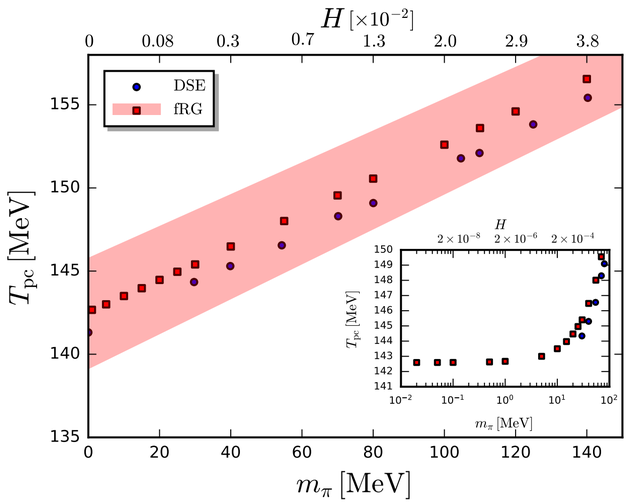
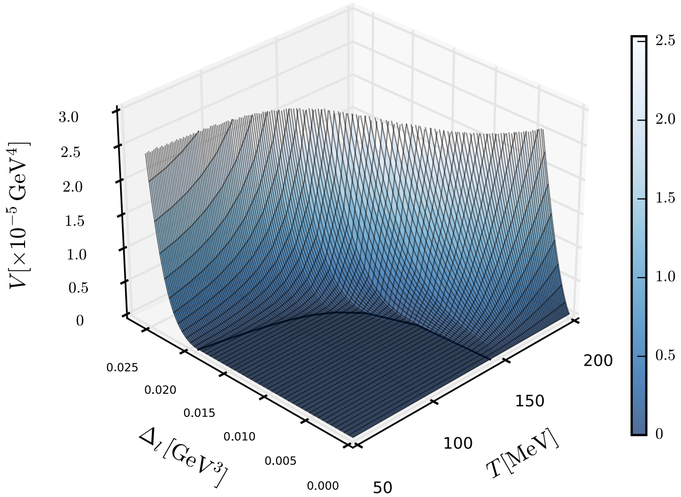


Order parameter potential & scaling

$$V_\chi \approx \Delta_l^n \quad \longleftrightarrow \quad \Delta_l(H) \propto H^{\frac{1}{n-1}}$$



$$V_\chi^{(\mathrm{fRG})} \approx V_\chi^{(\mathrm{DSE})}$$

(Critical) exponent: $\frac{1}{\delta} = \frac{1}{n-1}$

Braun, Fu, JMP, Rennecke, Rosenblüh, Yin, PRD 102 (2020) 056010

Gao, JMP, PRD 105 (2022) 094020

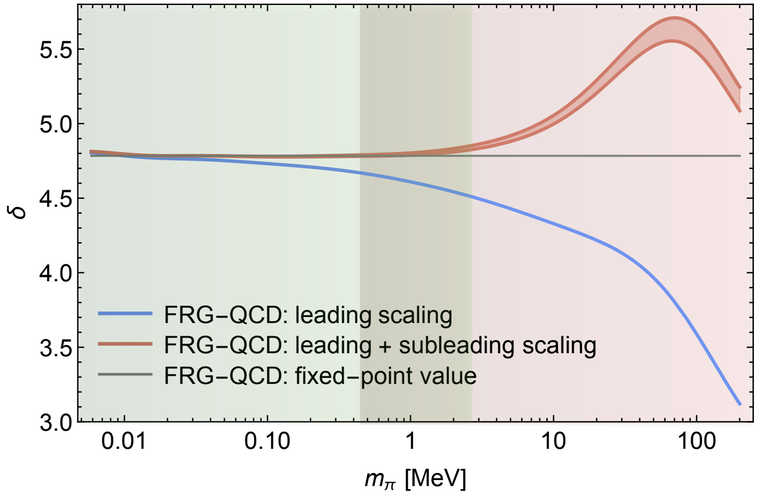
Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, 2310.19853

Chiral dynamics & quasi-massless modes

fQCD

Scaling coefficient as function of the pion mass

Critical O(4) scaling



Trivial $\Delta_l^{1+\delta}$ scaling

Chiral dynamics & quasi-massless modes

Critical scaling

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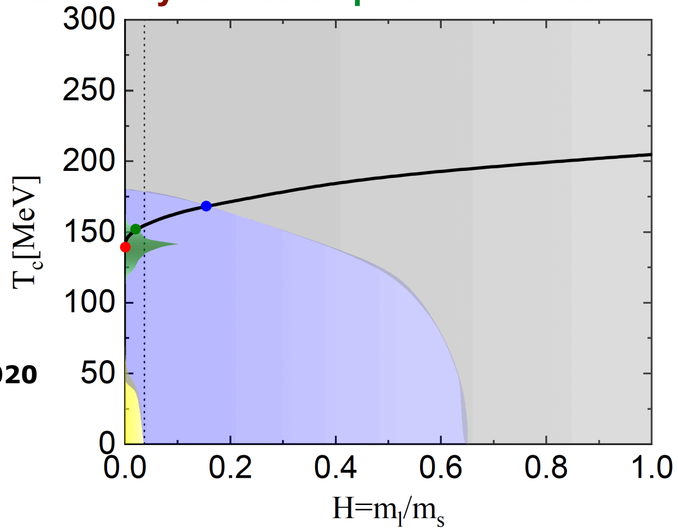

Far away from the critical regime for $m_\pi \gtrsim 1\,\mathrm{MeV}$

$$\Delta_l(T,H) \approx \Delta_{l,\chi}(0)\left(c_0 + c_{\frac{1}{5}} H^{\frac{1}{5}} + c_{\frac{1}{3}} H^{\frac{1}{3}} + c_1 H\right)$$

$$V_\chi(\Delta_l) \propto \qquad \Delta_l^6 \qquad \Delta_l^4 \qquad \Delta_l^2$$

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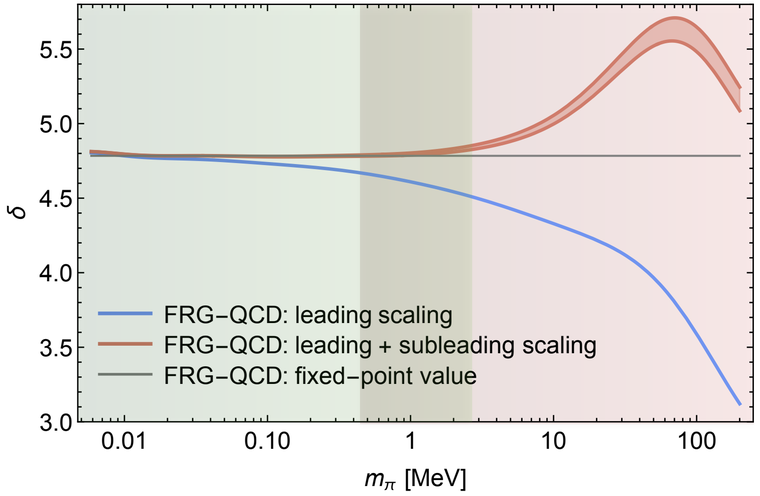
Chiral dynamics & quasi-massless modes

Scaling coefficient as function of the pion mass



Trivial $\Delta_l^{1+\delta}$ scaling

$$\Delta_l(m_\pi) \propto m_\pi^{2/\delta}\left[1 + a_m m_\pi^{2\theta_H} + \cdots\right]$$

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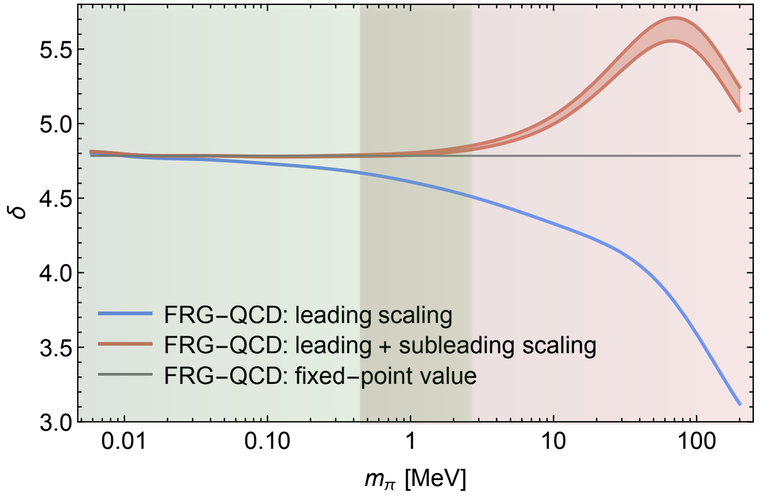
Chiral dynamics & quasi-massless modes

Scaling coefficient as function of the pion mass



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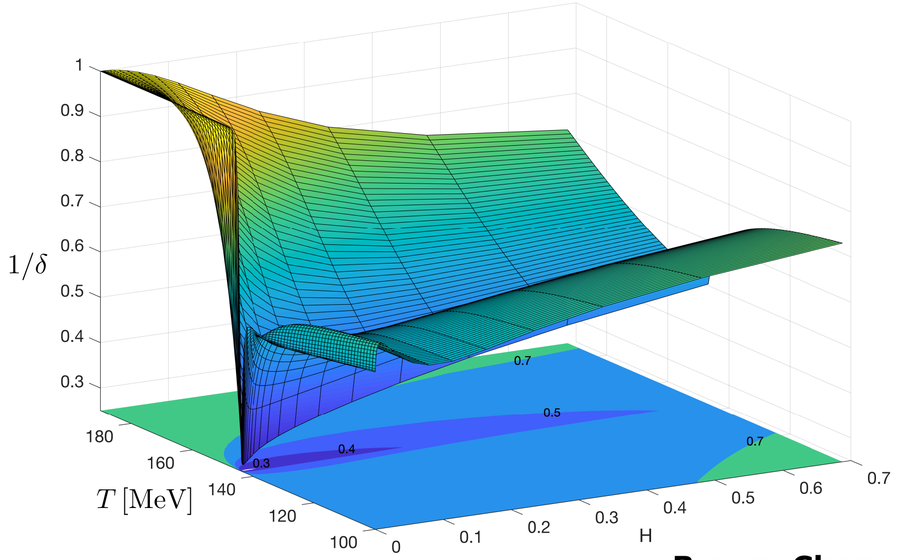
Small chiral scaling regime

↓

Small critical regime around pot. CEP



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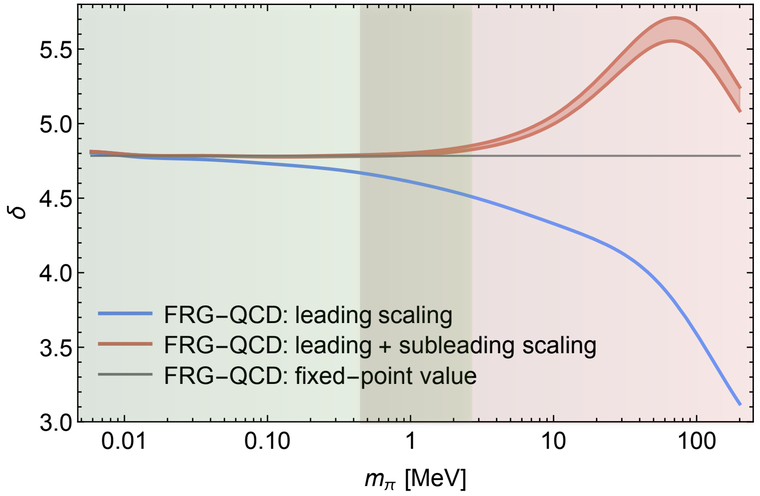
Chiral dynamics & quasi-massless modes

Scaling coefficient as function of the pion mass



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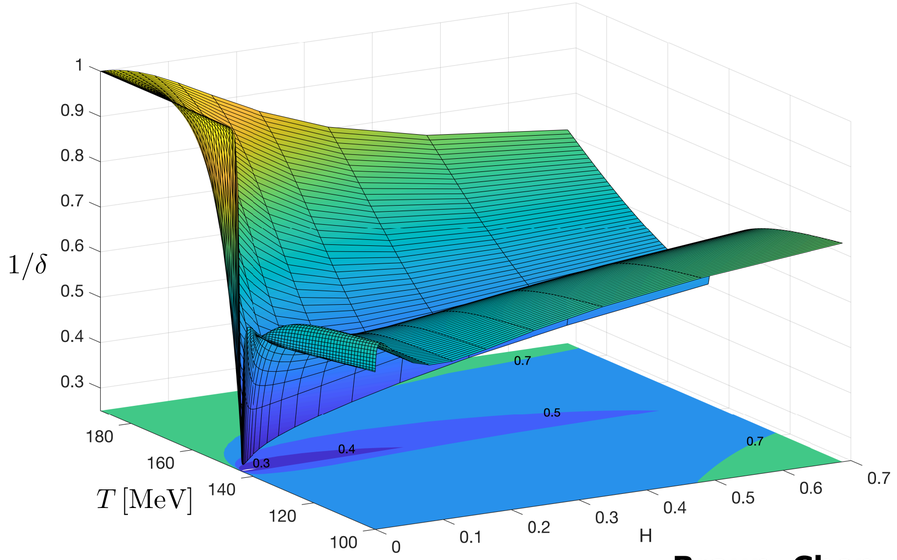

!!Great News!!

Location of CEP/New phase accessible via combination of precision measurements & computations

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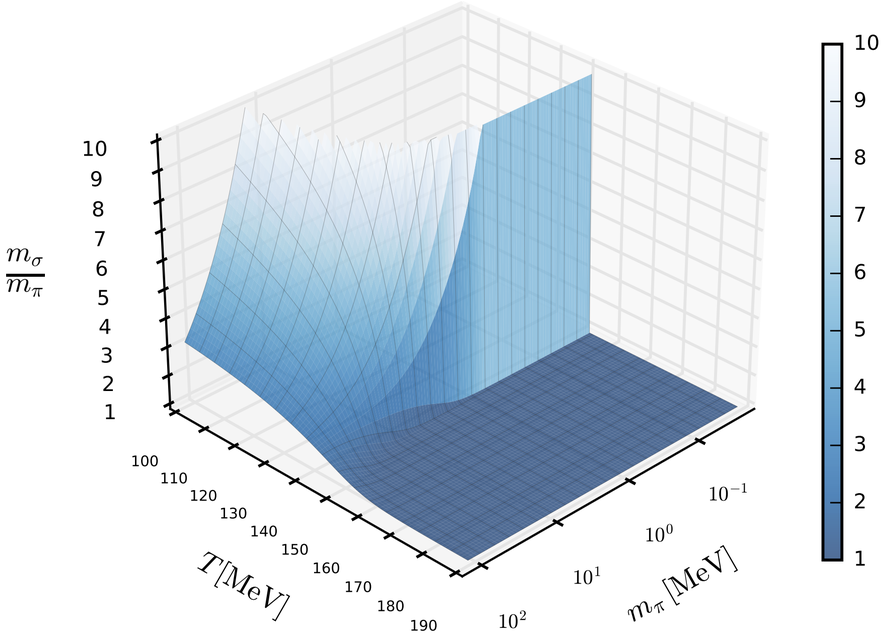
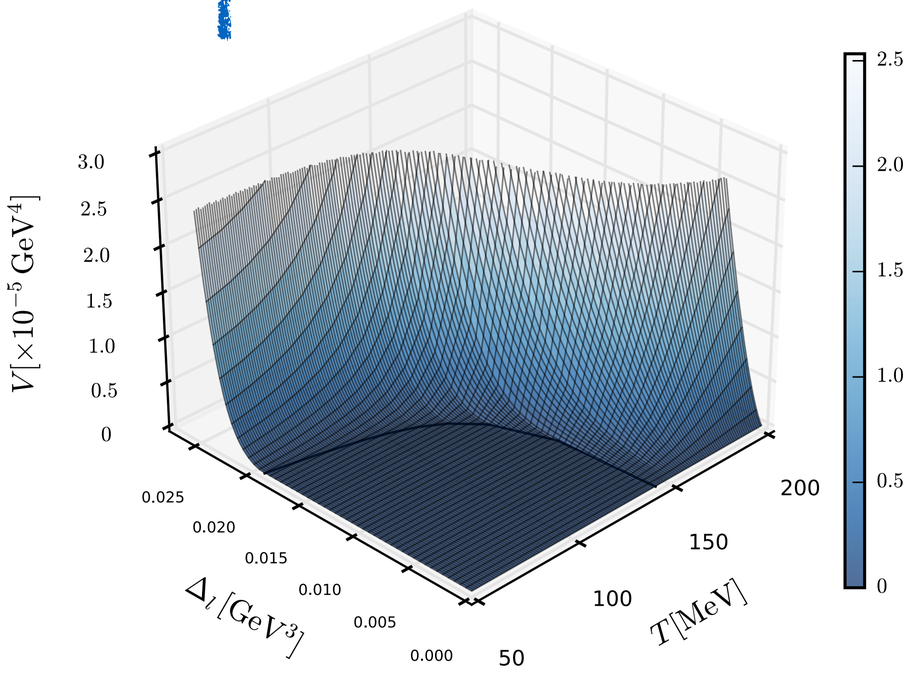
Full order parameter potential

Measure: correlation length



Use for chiral dynamics in heavy ion collisions



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