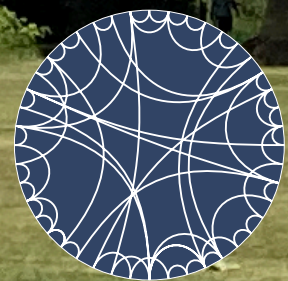


Particle physics and unitarity from asymptotically safe correlation functions

Jan M. Pawłowski

Universität Heidelberg

Nordita August 12th 2024



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Outline

- **Asymptotic safety**
- **Asymptotically safe correlation functions**
- **Applications I: Scattering amplitudes, spectral properties & unitarity**
- **Applications II: Asymptotically safe Standard Model**
- **Summary**

Asymptotic safety

Einstein-Hilbert action

Metric g

Cosmological constant Λ

$$S[g_{\mu\nu}] = \frac{1}{16\pi G_N} \int d^4x \sqrt{-\det g_{\mu\nu}} [R(g_{\mu\nu}) - 2\Lambda]$$

Newton constant G_N

Ricci scalar $R(g)$

Asymptotic safety

Einstein-Hilbert action

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Momentum dimension of couplings

$\dim G_N = -2$

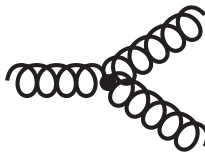
$\dim \Lambda = 2$

perturbatively non-renormalisable

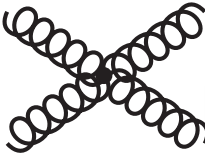
graviton propagator :

 $\propto \frac{1}{p^2}$

3 – grav. vertex :

 $\propto \sqrt{G_N} p^2$

4 – grav. vertex :

 $\propto G_N p^2$

$\vdots$

3

Asymptotic safety

Einstein-Hilbert action

$$S[g_{\mu\nu}] = \frac{1}{16\pi G_N} \int d^4x \sqrt{-\det g_{\mu\nu}} [R(g_{\mu\nu}) - 2\Lambda]$$

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Correlation functions

diffeomorphism invariant

$$\langle R(g(x_1)) \cdots R(g(x_n)) \rangle$$

Ricci scalar correlations

not diffeomorphism invariant

$$\langle g(x_1) \cdots g(x_n) \rangle$$

metric correlations

Asymptotic safety

Weinberg '79: Ultraviolet Divergences in Quantum Theories of Gravitation

Consider an observable $\mathcal{O}(g)$ with fundamental coupling g

Asymptotic safety

Weinberg `79: Ultraviolet Divergences in Quantum Theories of Gravitation

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- Standard perturbation theory

$$\mathcal{O}(g) = O_0 + O_1 g + \frac{1}{2} O_2 g^2 + \dots$$

Asymptotic safety

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Consider an observable $\mathcal{O}(g)$ with fundamental coupling g

- **Standard perturbation theory**

$$\mathcal{O}(g) = O_0 + O_1 g + \frac{1}{2} O_2 g^2 + \dots$$

- **Generalised perturbation theory**

$$\mathcal{O}(g) = O^* + O_1^* (g - g^*) + \frac{1}{2} O_2^* (g - g^*)^2 + \dots$$

e.g. aiming at better convergence

non-perturbative example: analytic perturbation theory in QCD

Asymptotic safety

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- Renormalisation group fixed points

beta functions

dimensionless coupling g $\partial_t g = \beta_g(g, \mu)$

dimensionless mass parameter μ $\partial_t \mu = \beta_\mu(g, \mu)$

Logarithmic momentum (RG) scale: $t = \log \frac{k}{k_0}$

Asymptotic safety

Weinberg `79: Ultraviolet Divergences in Quantum Theories of Gravitation

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- Renormalisation group fixed points

beta functions

Fixed points

dimensionless coupling g

$$\partial_t g = \beta_g(g, \mu)$$

$$\beta_g(g^*, \mu^*) = 0$$

dimensionless mass parameter μ

$$\partial_t \mu = \beta_\mu(g, \mu)$$

$$\beta_\mu(g^*, \mu^*) = 0$$

Asymptotic safety

Consider an observable $\mathcal{O}(g)$ with fundamental coupling g

▪ Ultraviolet running

QCD

$$\beta_g(g) = -\frac{1}{16\pi^2} \frac{22 N_c}{3} g^3$$

Asymptotic freedom

quantum gravity

$$\beta_{g_N} = \left[2 + \eta_N(g_N, \lambda) \right] g_N$$

dimensional running quantum fluctuations

Asymptotic safety

$$g_N = G_n k^2 \qquad \lambda = \frac{\Lambda}{k^2}$$

▪ Renormalisation group fixed points

beta functions

$$\partial_t g = \beta_g(g, \mu)$$

$$\partial_t \mu = \beta_\mu(g, \mu)$$

Fixed points

$$\beta_g(g^*, \mu^*) = 0$$

$$\beta_\mu(g^*, \mu^*) = 0$$

Asymptotic safety

Consider an observable $\mathcal{O}(g)$ with fundamental coupling g

▪ Ultraviolet running

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$$\beta_g(g) = -\frac{1}{16\pi^2} \frac{22 N_c}{3} g^3$$

Asymptotic freedom

Gaußian fixed point

quantum gravity

$$\beta_{g_N} = \left[2 + \eta_N(g_N, \lambda) \right] g_N$$

dimensional running

quantum fluctuations

Asymptotic safety

non-Gaußian fixed point

$$\eta_N = -2$$

$$(g_N^*, \lambda^*) \neq 0$$

▪ Renormalisation group fixed points

beta functions

$$\partial_t g = \beta_g(g, \mu)$$

$$\partial_t \mu = \beta_\mu(g, \mu)$$

Fixed points

$$\beta_g(g^*, \mu^*) = 0$$

$$\beta_\mu(g^*, \mu^*) = 0$$

Computing the effective action

Common approach: the functional renormalisation group

Diagrammatic approach with gauge fixing

Effective action/free energy

$$k\partial_k\Gamma_k[\bar{g}, \phi] = \frac{1}{2} \text{gauge fields} - \text{fermions} + \frac{1}{2} \text{bosons} + \frac{1}{2} \text{gravity}$$

Linear split $g = \bar{g} + h$

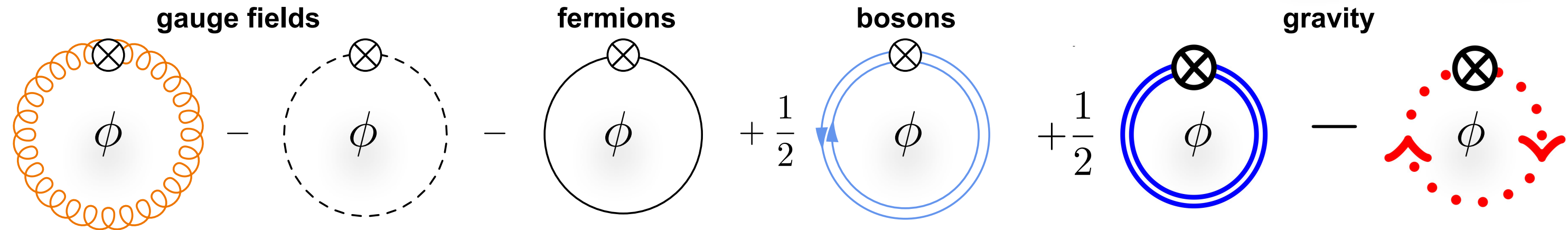
Fluctuation fields $\phi = (\phi_{\text{grav}}, \phi_{\text{mat}})$

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Pure Gravity: $\phi_{\text{grav}} = (h_{\mu\nu}, c_\mu, \bar{c}_\mu)$

Computing the effective action

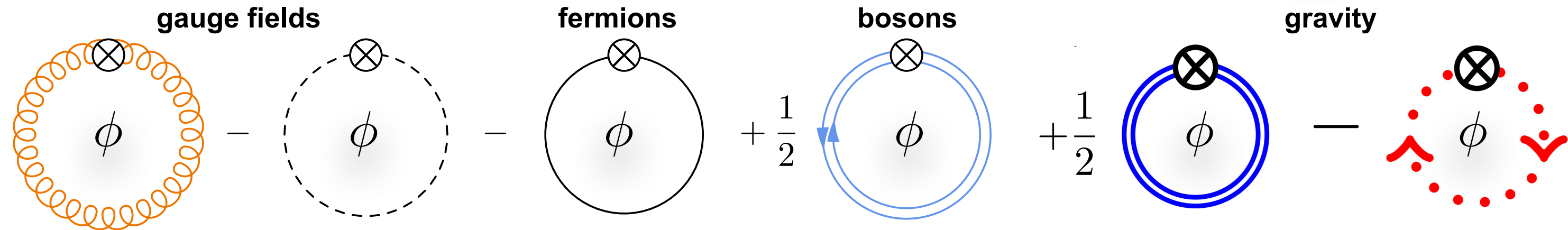
Common approach: the functional renormalisation group

Diagrammatic approach with gauge fixing

Matter: $\phi_{\text{mat}} = (A_\mu, c, \bar{c}, \psi, \bar{\psi}, \varphi, \dots)$

Pure Gravity: $\phi_{\text{grav}} = (h_{\mu\nu}, c_\mu, \bar{c}_\mu)$

Effective action/free energy

$$k\partial_k\Gamma_k[\bar{g}, \phi] = \frac{1}{2} \text{gauge fields} - \text{fermions} - \text{bosons} + \frac{1}{2} \text{bosons} + \frac{1}{2} \text{gravity}$$


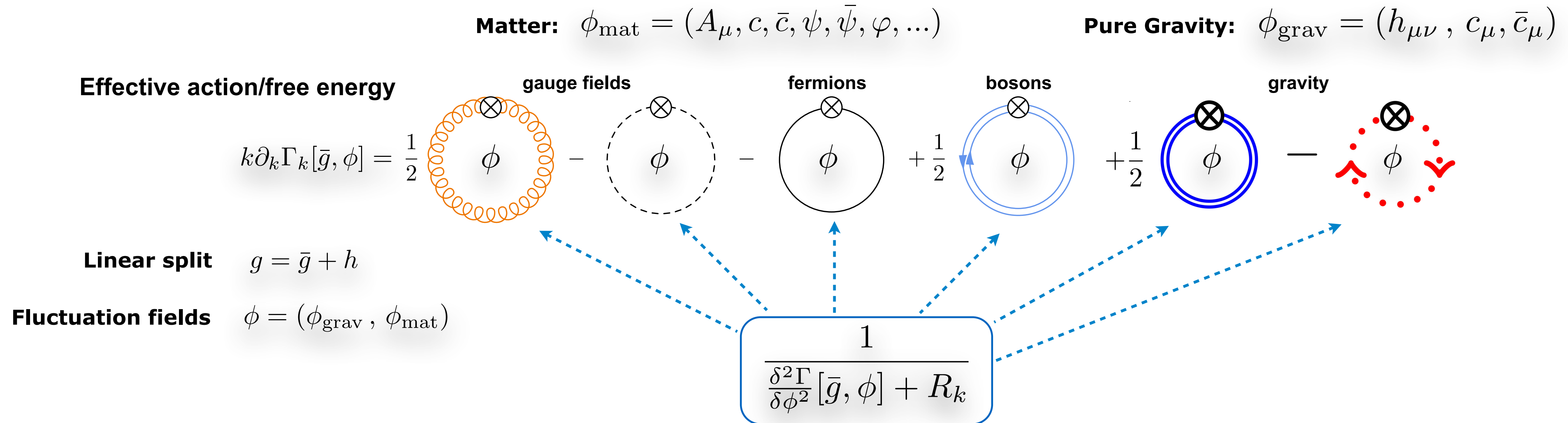
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Computing the effective action

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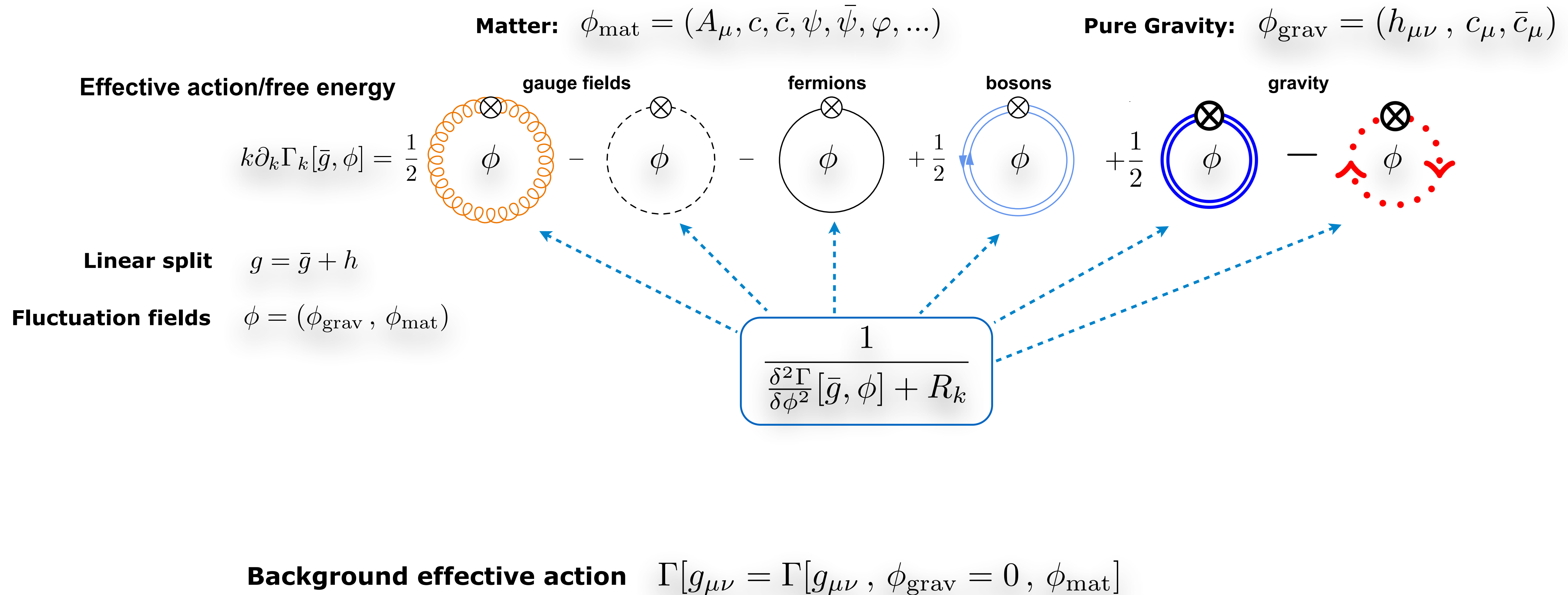
Diagrammatic approach with gauge fixing



Computing the effective action

Common approach: the functional renormalisation group

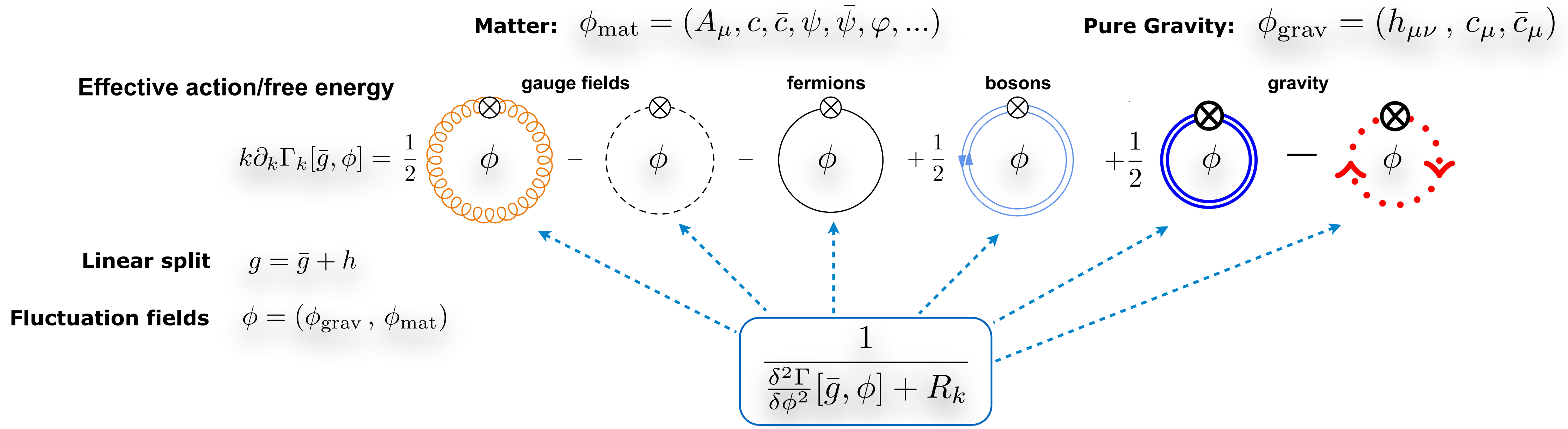
Diagrammatic approach with gauge fixing



Computing the effective action

Common approach: the functional renormalisation group

Diagrammatic approach with gauge fixing



Background effective action $\Gamma[g_{\mu\nu}] = \Gamma[g_{\mu\nu}, \phi_{\text{grav}} = 0, \phi_{\text{mat}}]$

Manifestly background independent effective action

Asymptotically safe observables

How to?

diffeomorphism invariant

$$\langle R(g(x_1)) \cdots R(g(x_n)) \rangle$$

Ricci scalar correlations

not diffeomorphism invariant

$$\langle h(x_1) \cdots h(x_n) \rangle$$

metric correlations

metric

$$g = \bar{g} + h$$

background metric

fluctuation

Asymptotically safe observables

How to?

diffeomorphism invariant

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Ricci scalar correlations

Equations of motion  & derivatives

Manifestly background independent effective action $\Gamma[g_{\mu\nu}]$

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metric correlations

metric

$$g = \bar{g} + h$$

background metric fluctuation

Asymptotically safe observables

How to?

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Ricci scalar correlations

Equations of motion  & derivatives

Manifestly background independent effective action $\Gamma[g_{\mu\nu}]$

 Fluctuation $h_{\mu\nu} = 0$

Gauge-fixed effective action $\Gamma[\bar{g}_{\mu\nu}, h_{\mu\nu}]$

not diffeomorphism invariant

$$\langle h(x_1) \cdots h(x_n) \rangle$$

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background metric fluctuation

Asymptotically safe observables

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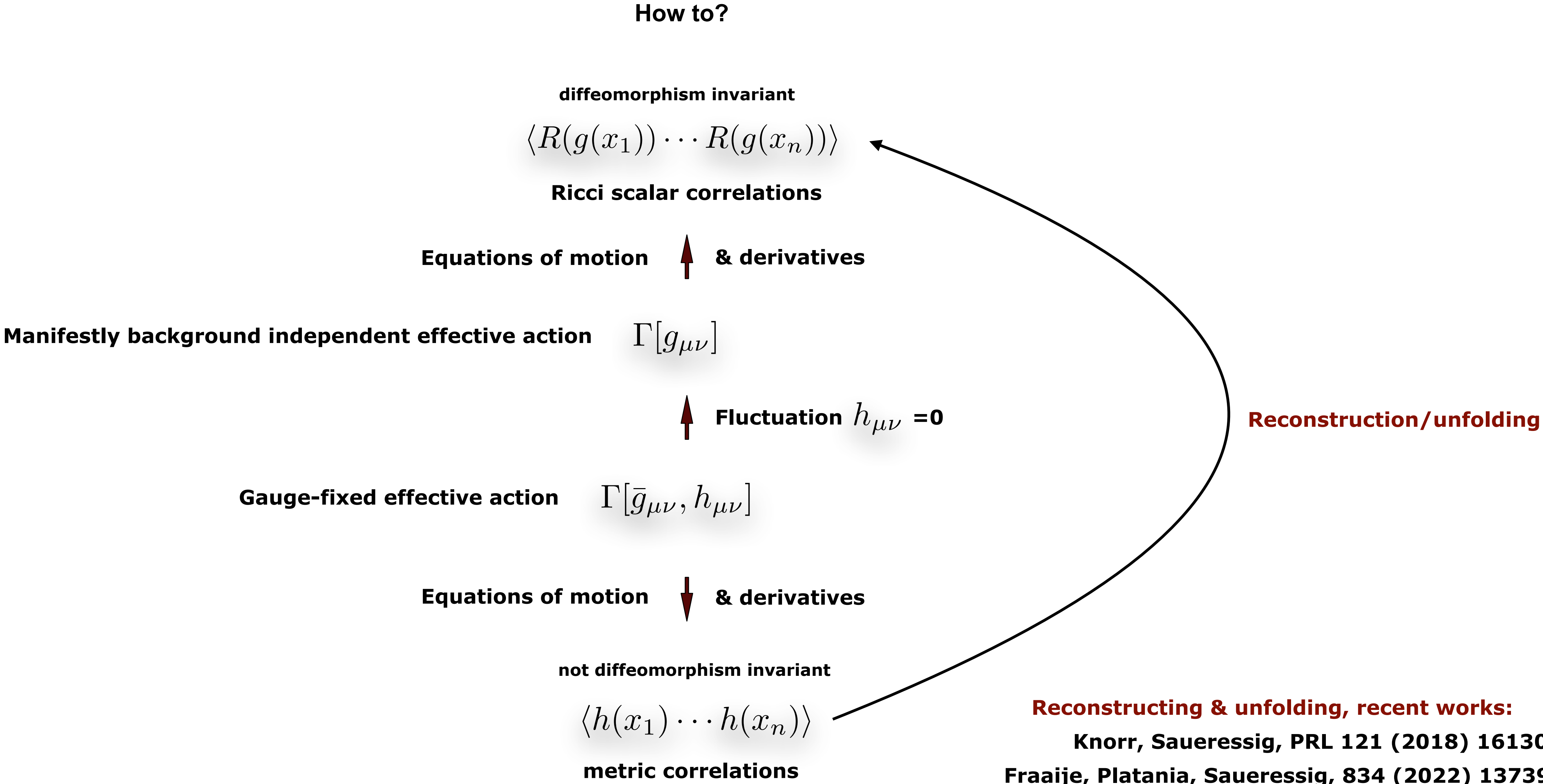
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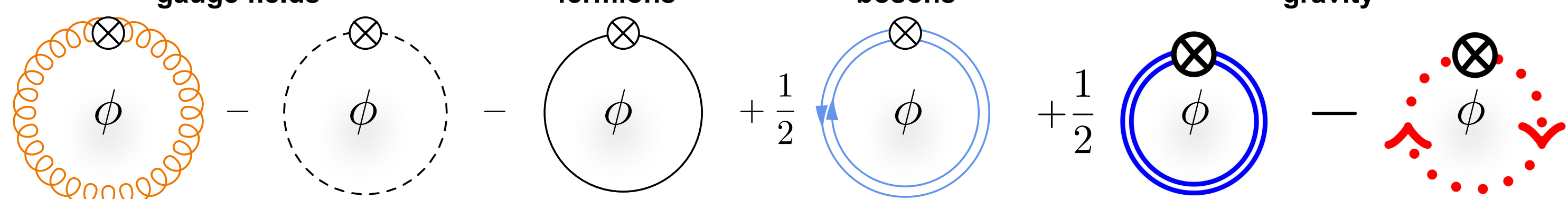


Asymptotically safe correlation functions

JMP, Reichert, Front.in Phys. 8 (2021) 527
2309.10785

Computing the effective action

Effective action/free energy

$$k\partial_k\Gamma_k[\bar{g}, \phi] = \frac{1}{2} \text{gauge fields} - \text{fermions} + \frac{1}{2} \text{bosons} + \frac{1}{2} \text{gravity}$$


The diagram illustrates the components of the effective action/free energy. It consists of six terms separated by minus signs, with the first term having a coefficient of 1/2 and the third and fourth terms having coefficients of 1/2. The terms are:

- gauge fields:** A loop of orange wavy lines with a cross in a circle at the top.
- fermions:** A dashed black circle with a cross in a circle at the top.
- bosons:** A solid black circle with a cross in a circle at the top.
- bosons:** A loop of two blue wavy lines with a cross in a circle at the top and a blue arrow pointing clockwise.
- bosons:** A loop of two blue wavy lines with a cross in a circle at the top.
- gravity:** A loop of red dotted lines with a cross in a circle at the top and red wavy lines at the bottom.

 Each diagram has a ϕ symbol in the center.

Computing the effective action

Effective action/free energy

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Loop: $\text{Tr } f(\nabla_g, R, R_{\mu\nu}, \dots; \partial, \phi, \dots)$

Computing the effective action

Effective action/free energy

$$k\partial_k\Gamma_k[\bar{g}, \phi] = \frac{1}{2} \text{gauge fields} - \text{fermions} - \text{bosons} + \frac{1}{2} \text{bosons} + \frac{1}{2} \text{gravity} -$$

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Fluctuation approach

Background approximation

Computing the effective action

Loop: $\text{Tr } f(\nabla_g, R, R_{\mu\nu}, \dots; \partial, \phi, \dots)$

Fluctuation approach

Compute $\left. \frac{\delta^{n+m} \Gamma[\bar{g}, h]}{\delta h^n \delta \phi^m} \right|_{\phi=0} = f_n(\bar{\nabla}, \bar{R}, \bar{R}_{\mu\nu}, \dots)$

in a given background $\bar{g}$

Background approximation

Computing the effective action

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in a given background $\bar{g}$

$\bar{g}$ **flat:** $f_n(\bar{\nabla}, \bar{R}, \bar{R}_{\mu\nu}, \dots) \rightarrow f_n(p) = f_n(p, 0, 0, \dots)$

Compute ‘standard’ momentum loops’

Background approximation

Computing the effective action

$$\text{Loop: } \text{Tr } f(\nabla_g, R, R_{\mu\nu}, \dots; \partial, \phi, \dots)$$

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Those are my backgrounds (principles),
and if you don't like them...well I have others

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$$\Gamma[\bar{g}, h] = \Gamma_{\text{qu}}[\bar{g}, h] + \Gamma_{\text{gf}}[\bar{g}, h] \approx \Gamma_{\text{qu}}[g, 0] + \Gamma_{\text{gf}}[\bar{g}, h]$$

Computing the effective action

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Compute loops at vanishing fluctuation field $\bar{g} = g$

Computing the effective action

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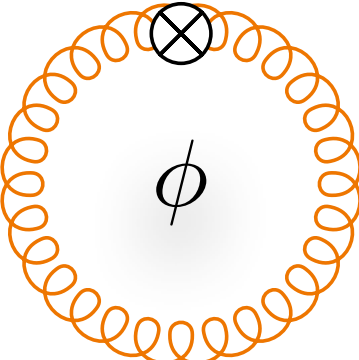
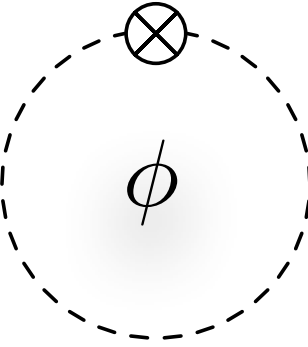
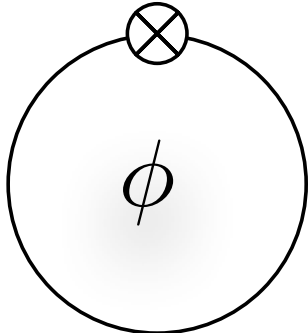
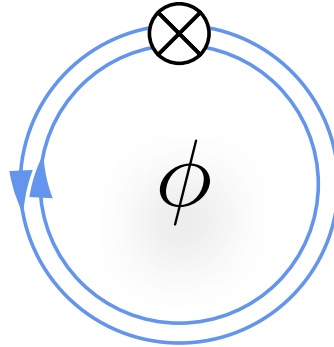
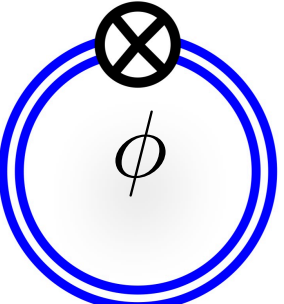
$$\text{Tr } f(\nabla_g, R, R_{\mu\nu}, \dots; \partial, \phi, \dots) \rightarrow \text{Tr } f(\nabla_g, R, R_{\mu\nu}, \dots)$$

Heat kernel techniques with covariantly constant backgrounds

Background (in)dependence in gravity

aka
background and fluctuations fields, modified STIs and their importance

free energy

$k\partial_k\Gamma_k[\bar{g},\phi] = \frac{1}{2}$  $-$  $-$  $+\frac{1}{2}$  $+\frac{1}{2}$  $-$ 

Linear split
 $g = \bar{g} + h$

Effective action

$$\Gamma_k[\bar{g},\bar{h}] = \Gamma_k[\bar{g}] + \Gamma_k^{(0,1)}[\bar{g}] * \bar{h} + \frac{1}{2}\Gamma_k^{(0,2)}[\bar{g}] * \bar{h}^2 + \frac{1}{6}\Gamma_k^{(0,3)}[\bar{g}] * \bar{h}^3 + \dots$$

$\bar{h} = \langle h \rangle$

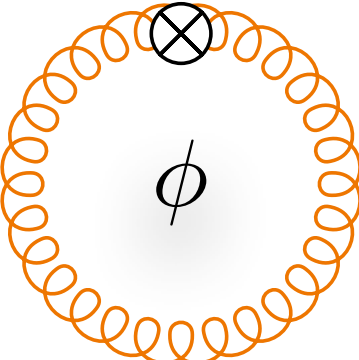
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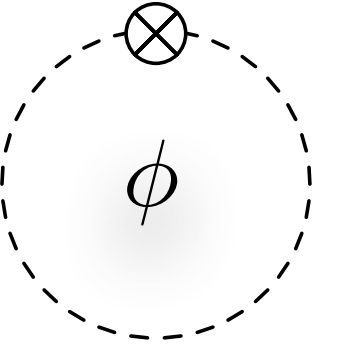
$k\partial_k\Gamma_k[\bar{g},\phi] = \frac{1}{2}$

gauge fields



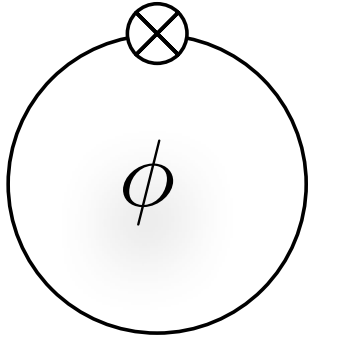
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fermions



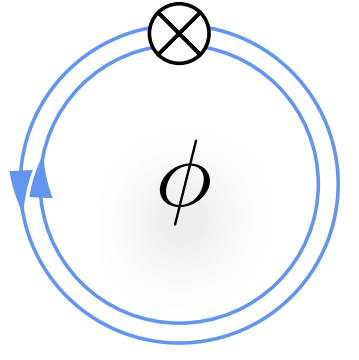
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bosons



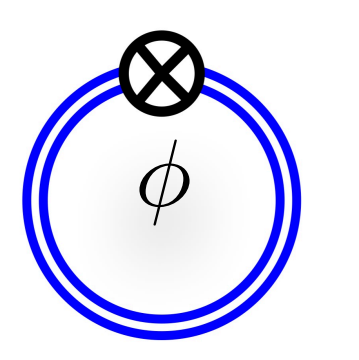
$+\frac{1}{2}$

bosons



$+\frac{1}{2}$

gravity



—

gravity



Linear split
 $g = \bar{g} + h$

Effective action

$$\Gamma_k[\bar{g},\bar{h}] = \Gamma_k[\bar{g}] + \Gamma_k^{(0,1)}[\bar{g}] * \bar{h} + \frac{1}{2}\Gamma_k^{(0,2)}[\bar{g}] * \bar{h}^2 + \frac{1}{6}\Gamma_k^{(0,3)}[\bar{g}] * \bar{h}^3 + \dots$$

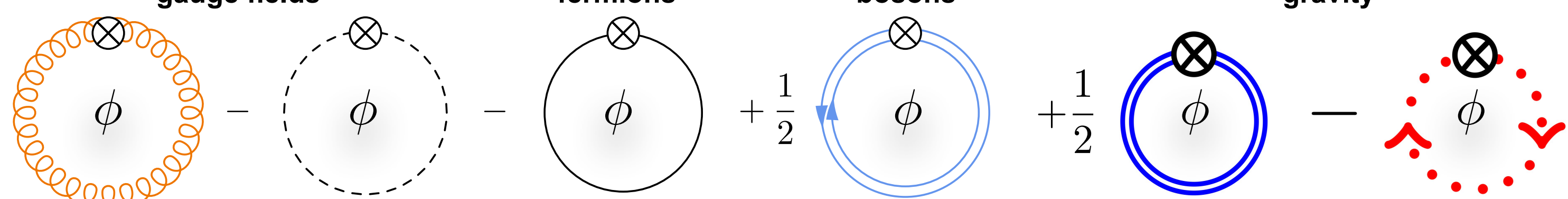
$\bar{h} = \langle h \rangle$

$$\left\{ \Gamma_k[\bar{g}], \Gamma_k^{(0,1)}[\bar{g}], \Gamma_k^{(0,2)}[\bar{g}], \Gamma_k^{(0,3)}[\bar{g}], \dots \right\}$$

Background (in)dependence in gravity

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background and fluctuations fields, modified STIs and their importance

free energy

$$k\partial_k\Gamma_k[\bar{g},\phi] = \frac{1}{2} \text{gauge fields} - \text{fermions} + \frac{1}{2} \text{bosons} + \frac{1}{2} \text{gravity}$$


Linear split
 $g = \bar{g} + h$

Effective action

background & gauge dependent

$$\Gamma_k[\bar{g},\bar{h}] = \Gamma_k[\bar{g}] + \Gamma_k^{(0,1)}[\bar{g}] * \bar{h} + \frac{1}{2}\Gamma_k^{(0,2)}[\bar{g}] * \bar{h}^2 + \frac{1}{6}\Gamma_k^{(0,3)}[\bar{g}] * \bar{h}^3 + \dots$$

$\bar{h} = \langle h \rangle$

background independent

$$\left\{ \Gamma_k[\bar{g}], \Gamma_k^{(0,1)}[\bar{g}], \Gamma_k^{(0,2)}[\bar{g}], \Gamma_k^{(0,3)}[\bar{g}], \dots \right\}$$

Se vogliamo che tutto rimanga come è,
bisogna che tutto cambi.

Il Gattopardo

From vertex dressings/distribution functions to physics

aka
form factors

Effective action

$$\Gamma[\bar{g}, h, c_\mu, \bar{c}_\mu] = \int_x \left[\frac{2\Lambda - R}{16\pi G_N} + R f_R(\Delta) R + C f_C(\Delta) C + \dots \right]_{\text{BRST-inv}} + S_{\text{gf}} + S_{\text{gh}}$$

Background effective action

$$\Gamma[g_{\mu\nu}] = \frac{1}{16\pi} \int_x \sqrt{g} \left\{ \mathcal{R}(\Delta, R) + R f_{R^2}(\Delta) R + R_{\mu\nu} f_{R^2_{\mu\nu}}(\Delta) R^{\mu\nu} + \dots \right\}$$

JMP, Tränkle, 2309.17043

Enforced by IR-UV consistence

$$R f_{R^2}(\Delta, R) R = \mathcal{R}(\Delta, R) + R f_{R^2}(\Delta) R$$

Se vogliamo che tutto rimanga come è,
bisogna che tutto cambi.

Il Gattopardo

From vertex dressings/distribution functions to physics

aka
form factors

Effective action

$$\Gamma[\bar{g}, h, c_\mu, \bar{c}_\mu] = \int_x \left[\frac{2\Lambda - R}{16\pi G_N} + R \boxed{f_R(\Delta)} R + C \boxed{f_C(\Delta)} C + \dots \right]_{\text{BRST-inv}} + S_{\text{gf}} + S_{\text{gh}}$$

gauge dependent

Background effective action

$$\Gamma[g_{\mu\nu}] = \frac{1}{16\pi} \int_x \sqrt{g} \left\{ \boxed{\mathcal{R}(\Delta, R)} + R \boxed{f_{R^2}(\Delta)} R + R_{\mu\nu} \boxed{f_{R^2_{\mu\nu}}(\Delta)} R^{\mu\nu} + \dots \right\}$$

gauge independent

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How much do they differ?

Background effective action

$$\Gamma[g_{\mu\nu}] = \frac{1}{16\pi} \int_x \sqrt{g} \left\{ \mathcal{R}(\Delta, R) + R f_{R^2}(\Delta) R + R_{\mu\nu} f_{R^2_{\mu\nu}}(\Delta) R^{\mu\nu} + \dots \right\}$$

gauge independent

JMP, Tränkle, 2309.17043

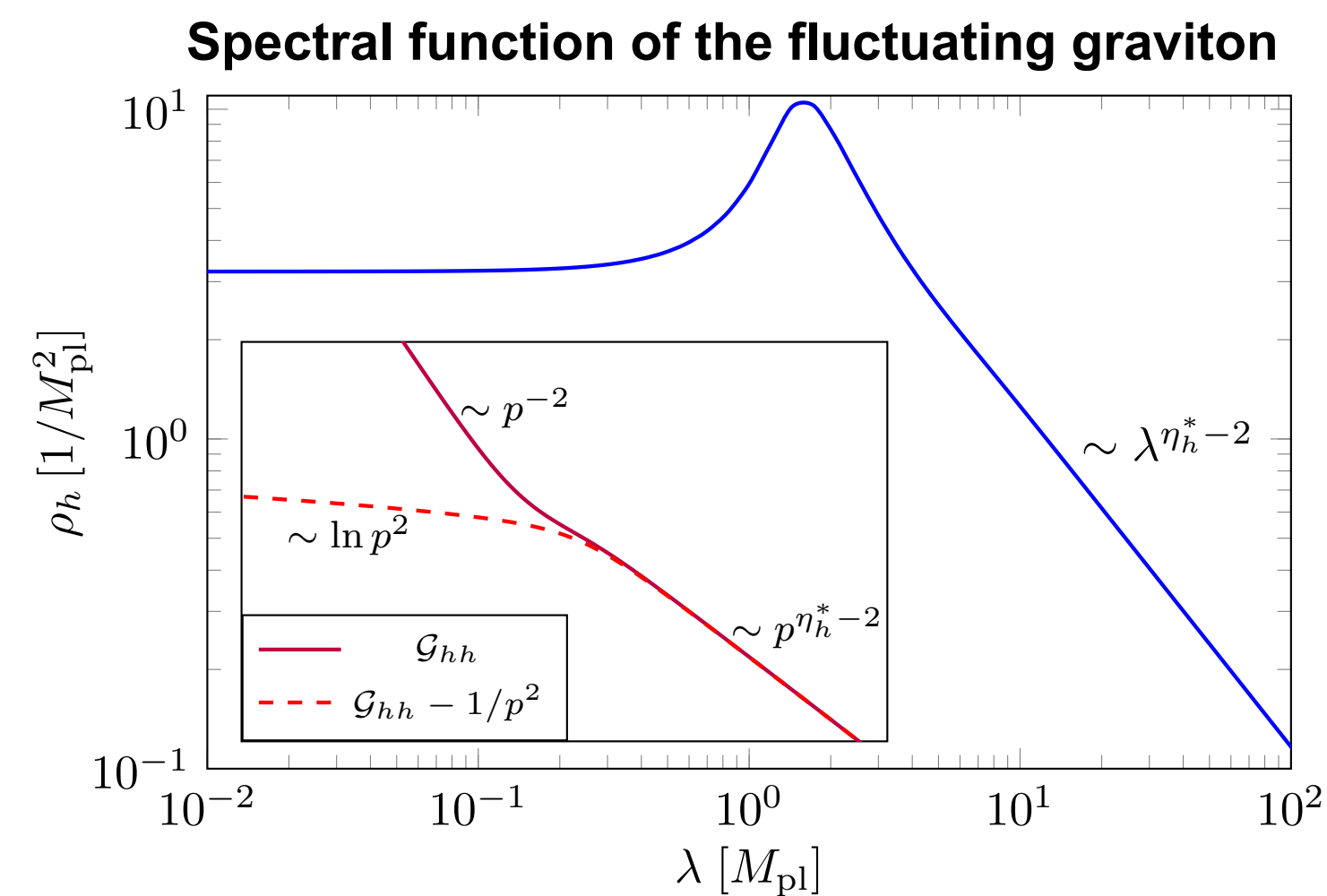
Enforced by IR-UV consistence

$$R f_{R^2}(\Delta, R) R = \mathcal{R}(\Delta, R) + R f_{R^2}(\Delta) R$$

Se vogliamo che tutto rimanga come è,
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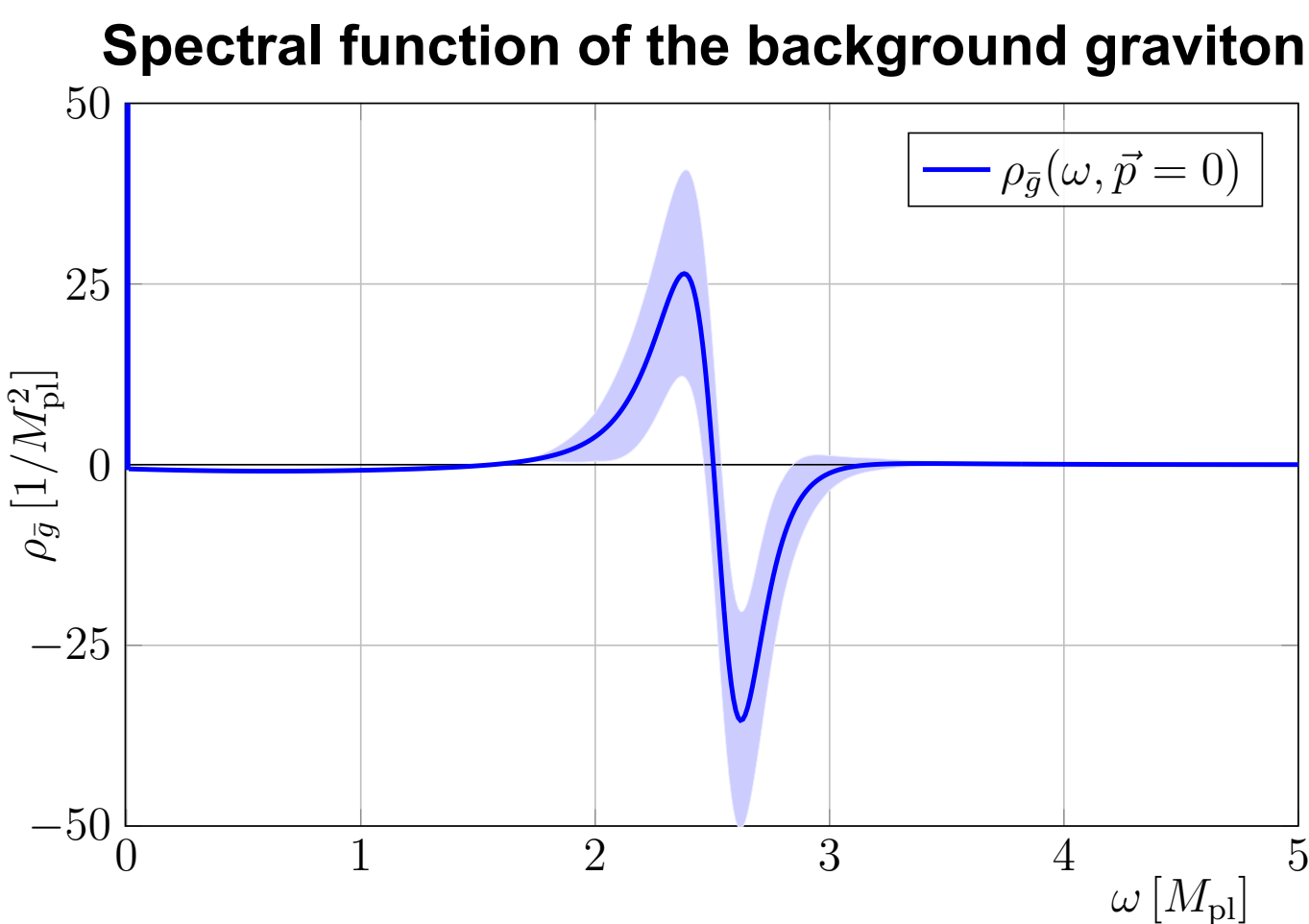
Il Gattopardo

Sneak preview: a lesson from graviton spectral functions in a flat background



Fehre, Litim, JMP, Reichert, PRL 130 (2023) 081501

How much do they differ?

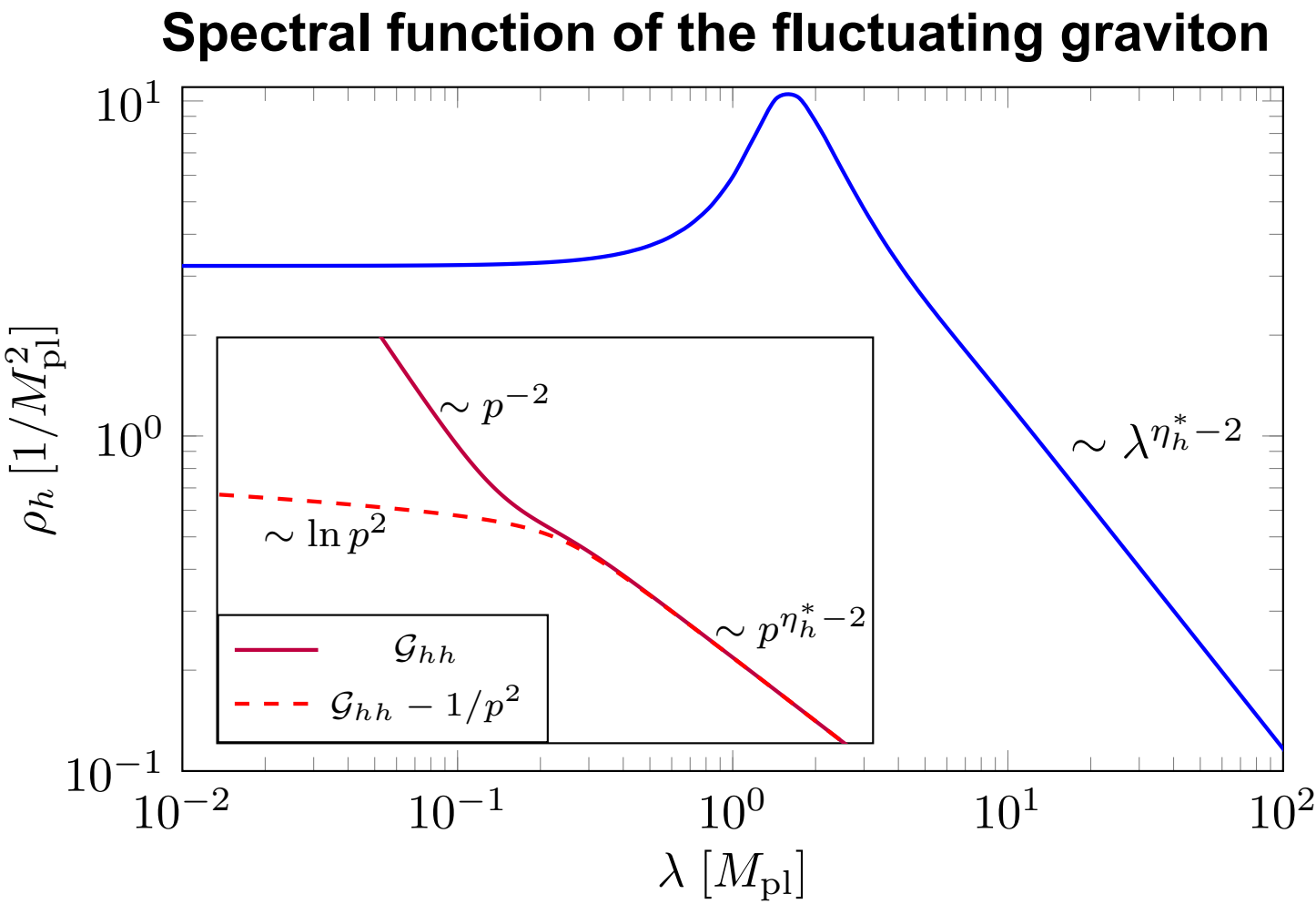


Bonanno, Denz, JMP, Reichert, SciPost Phys. 12 (2022) 1, 001

**Se vogliamo che tutto rimanga come è,
bisogna che tutto cambi.**

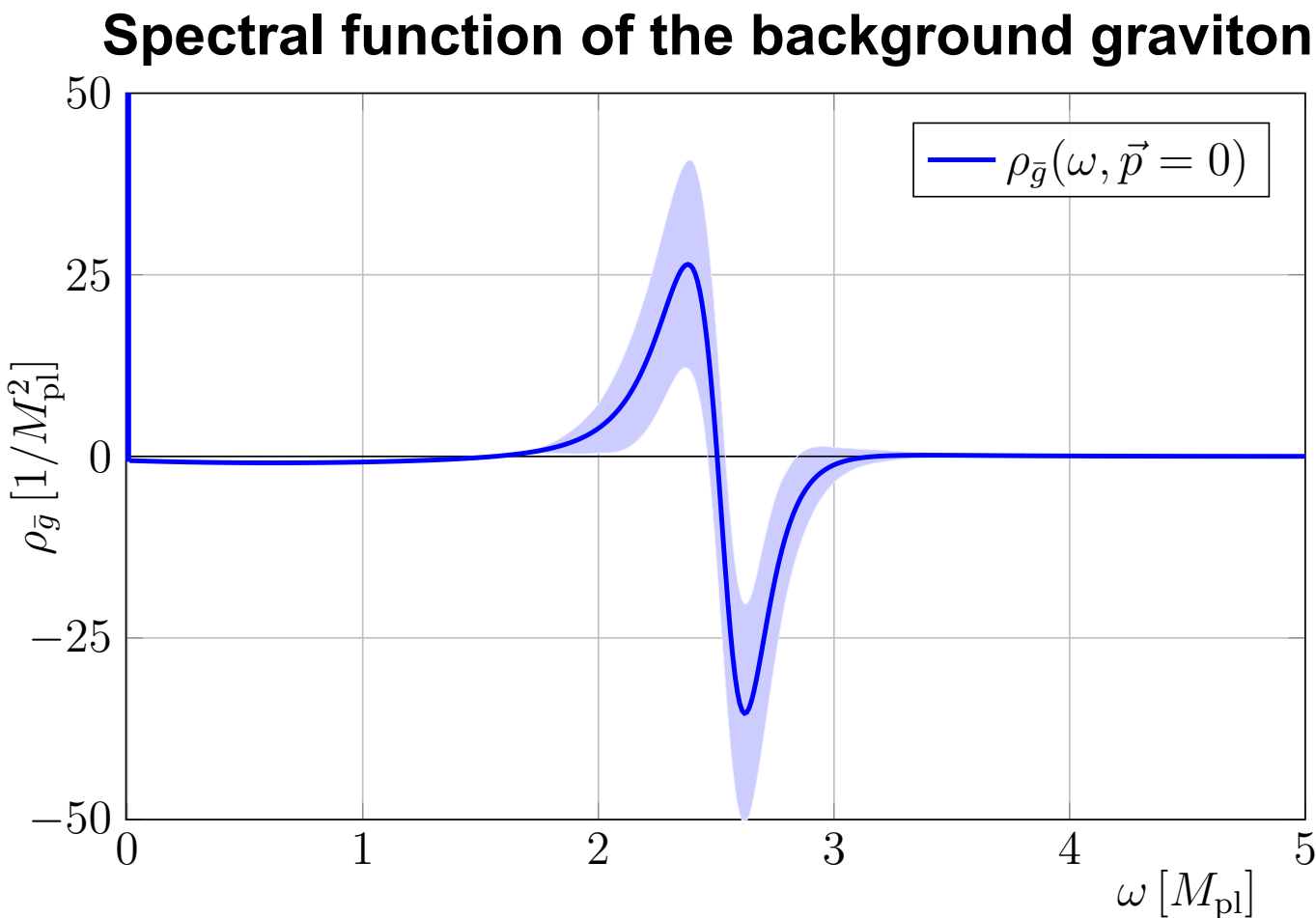
Il Gattopardo

Sneak preview: a lesson from graviton spectral functions in a flat background



Fehre, Litim, JMP, Reichert, PRL 130 (2023) 081501

How much do they differ?



Bonanno, Denz, JMP, Reichert, SciPost Phys. 12 (2022) 1, 001

Spectral properties ‘resemble’ that of an asymptotic state

$$\rho_h(\lambda) \in \mathbb{R}^+ \quad \int_{\mathbb{R}} \frac{d\lambda}{2\pi} \lambda \rho_h(\lambda) = \infty$$

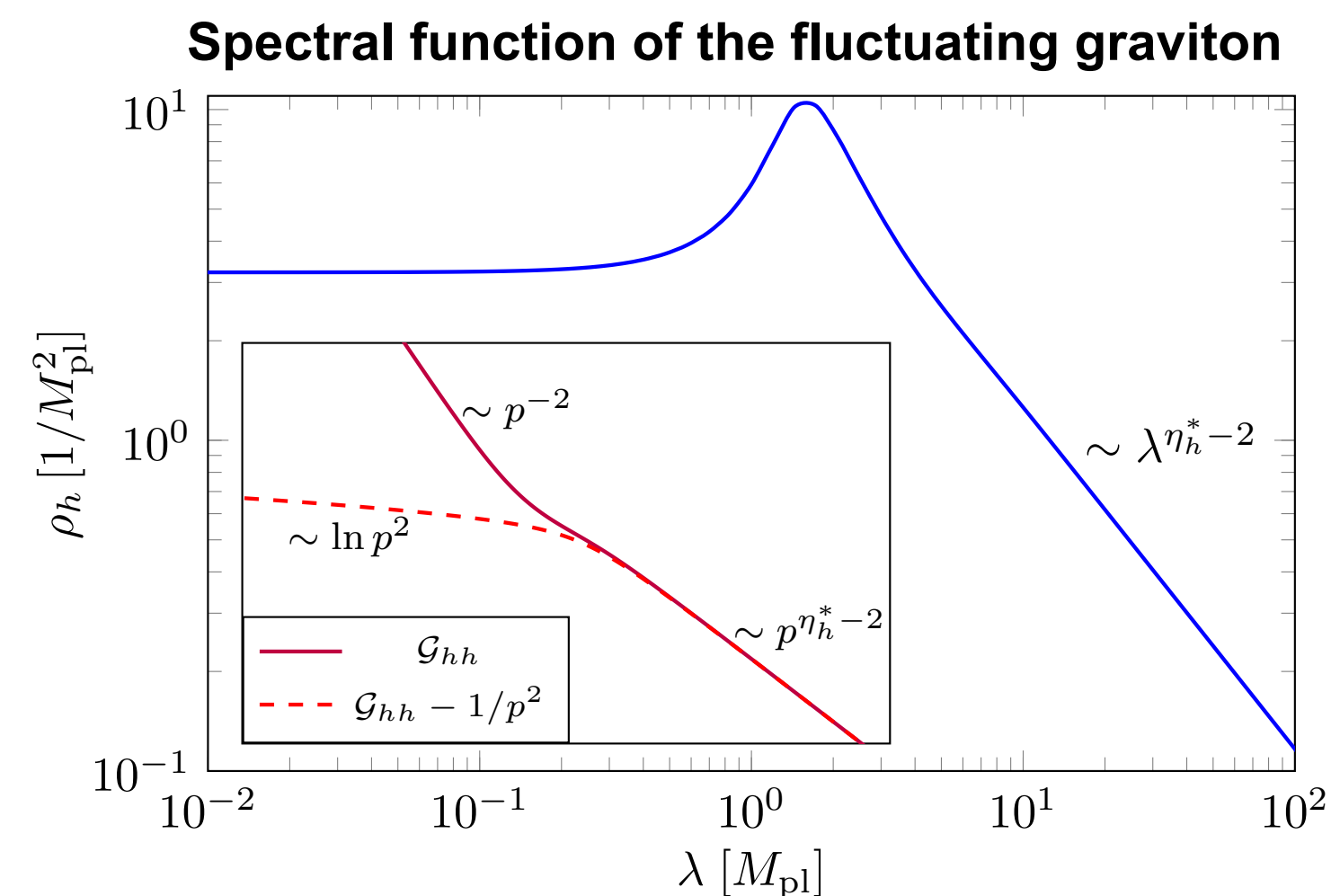
Spectral properties of an unphysical mode

$$\rho_{\bar{g}}(\lambda) \in \mathbb{R} \quad \int_{\mathbb{R}} \frac{d\lambda}{2\pi} \lambda \rho_{\bar{g}}(\lambda) = 0$$

**Se vogliamo che tutto rimanga come è,
bisogna che tutto cambi.**

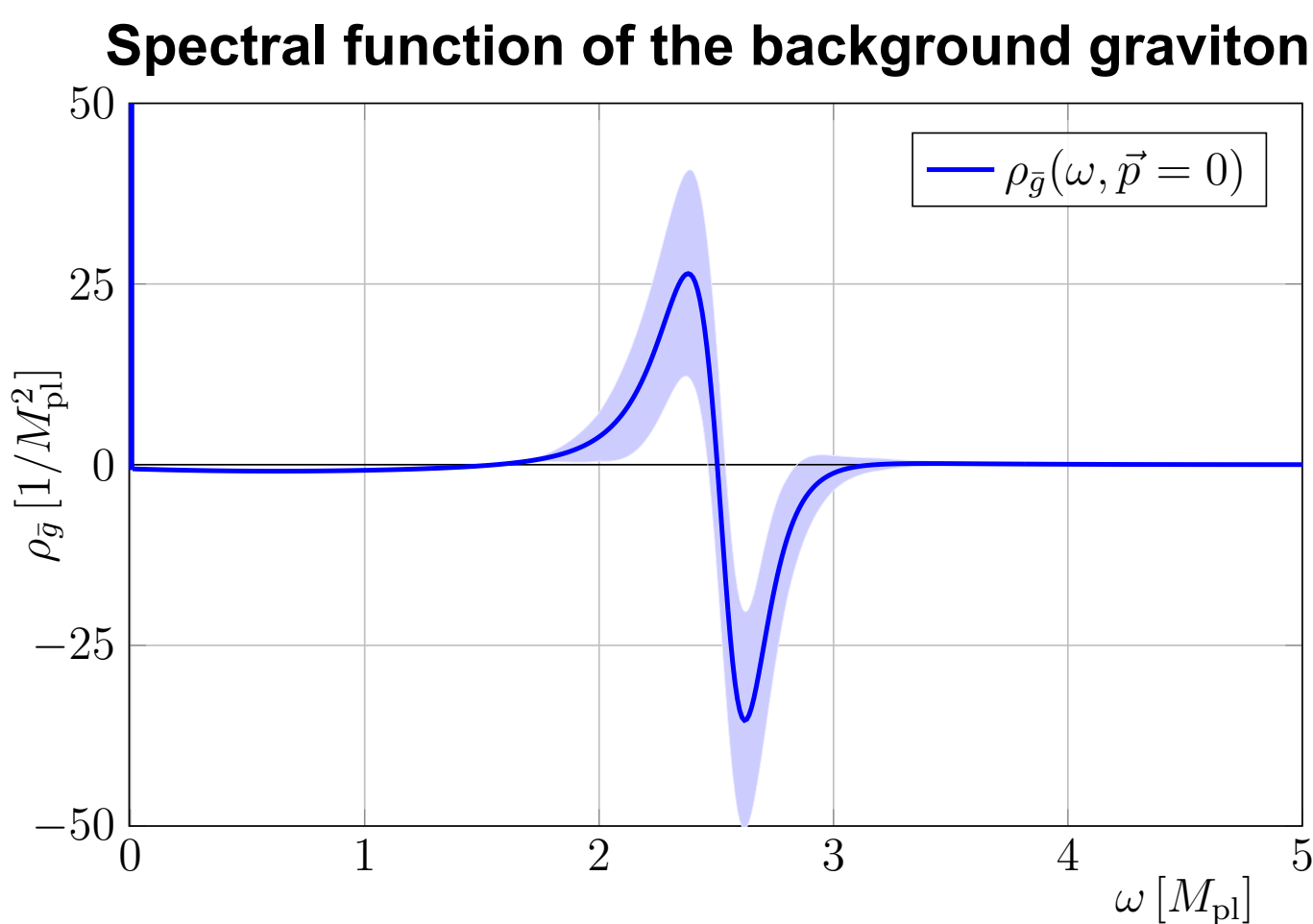
Il Gattopardo

Sneak preview: a lesson from graviton spectral functions in a flat background



Fehre, Litim, JMP, Reichert, PRL 130 (2023) 081501

much !



Bonanno, Denz, JMP, Reichert, SciPost Phys. 12 (2022) 1, 001

Spectral properties ‘resemble’ that of an asymptotic state

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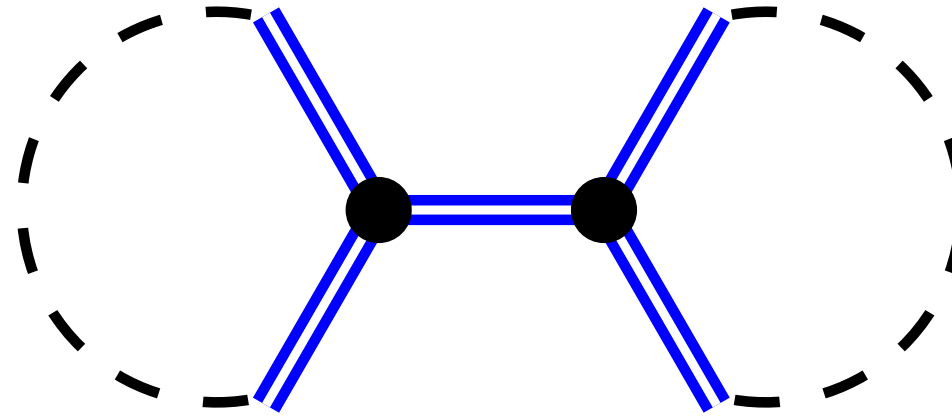
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Il Gattopardo

Scattering amplitudes and cross sections

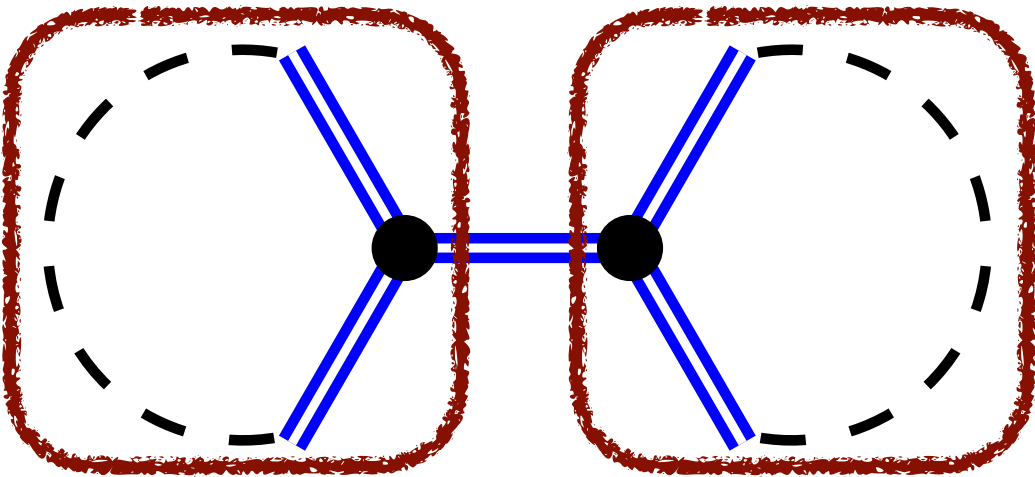
Example: graviton-graviton scattering in a flat background



Bonanno, Denz, JMP, Reichert, *SciPost Phys.* 12 (2022) 1, 001

Scattering amplitudes and cross sections

Example: graviton-graviton scattering in a flat background



Bonanno, Denz, JMP, Reichert, *SciPost Phys.* 12 (2022) 1, 001

RG-invariant vertex

$$\frac{\Gamma_{hhh}^{(3)}(p_1, p_2, p_3)}{Z_h^{\frac{1}{2}}(p_1)Z_h^{\frac{1}{2}}(p_2)Z_h^{\frac{1}{2}}(p_3)}$$

aka

RG-invariant coupling
/form factor

RG-invariant vertex

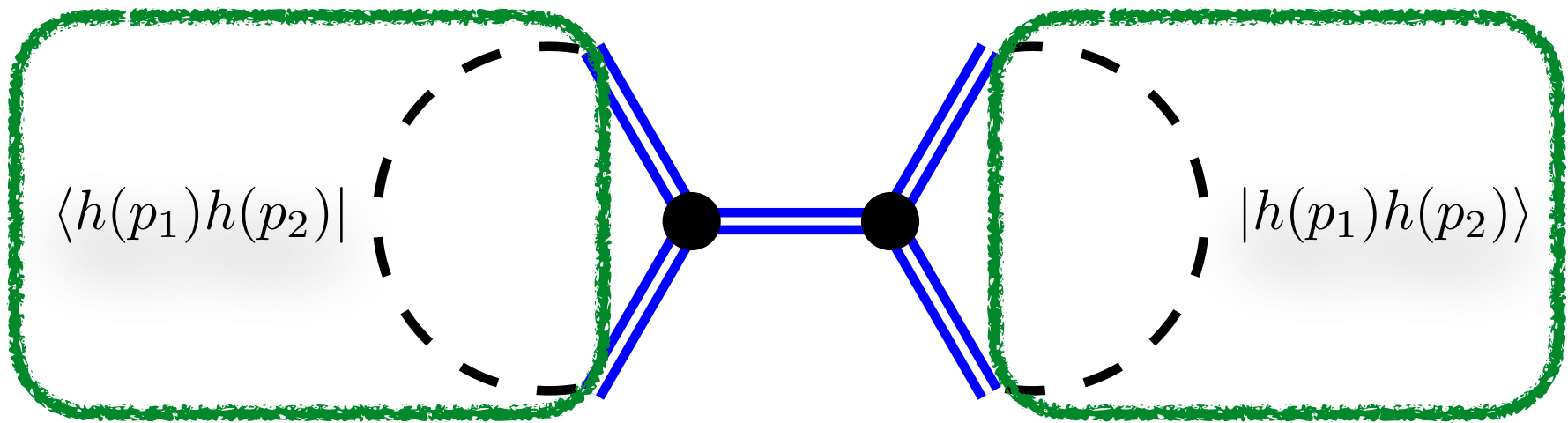
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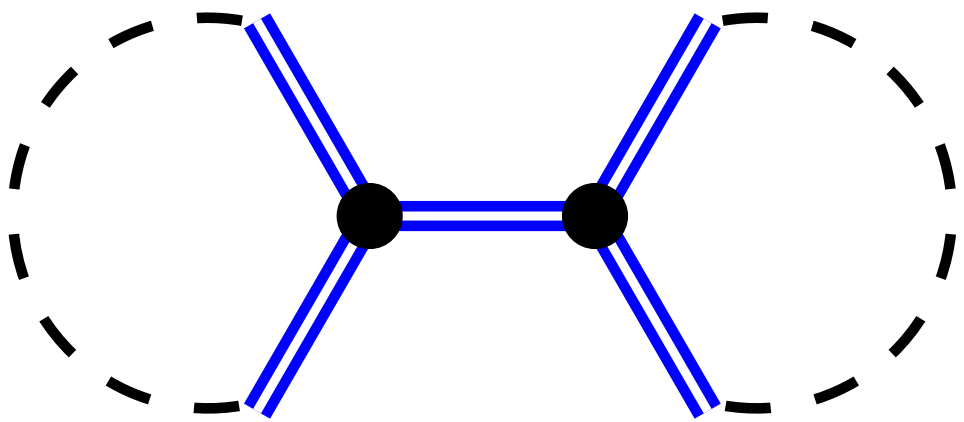
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RG-invariant coupling
/form factor

Bonanno, Denz, JMP, Reichert, *SciPost Phys.* 12 (2022) 1, 001

Fluctuation approach: 2012 ...

Knorr, Reichert,

Form factor approach: 2018 ...

Knorr, Ripken, Saueressig, ...

Scattering amplitudes and cross sections

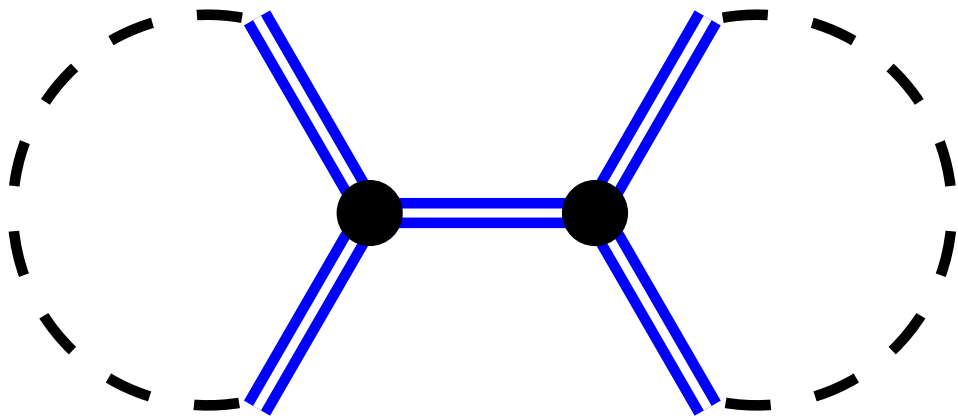
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RG-invariant coupling
/form factor

Bonanno, Denz, JMP, Reichert, *SciPost Phys.* 12 (2022) 1, 001

Suggestive educated guess

$$\bar{\Gamma}_{\bar{g}^n}^{(n)}(p_1,\dots,p_n)\approx \frac{\Gamma_{h^n}^{(n)}(p_1,\dots,p_n)}{Z_h^{\frac{1}{2}}(p_1)\cdots Z_h^{\frac{1}{2}}(p_n)}$$

Scattering amplitudes and cross sections

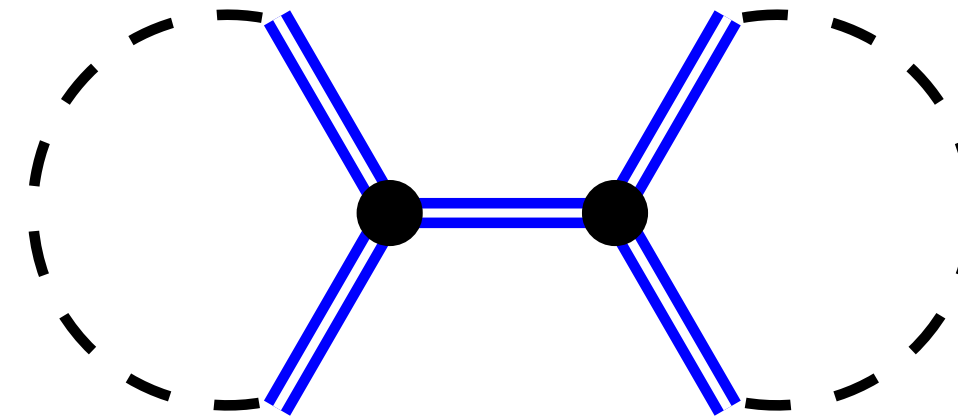
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Bonanno, Denz, JMP, Reichert, *SciPost Phys.* 12 (2022) 1, 001

Suggestive educated guess

$$\bar{\Gamma}_{\bar{g}^n}^{(n)}(p_1, \dots, p_n) \approx \frac{\Gamma_{h^n}^{(n)}(p_1, \dots, p_n)}{Z_h^{\frac{1}{2}}(p_1) \cdots Z_h^{\frac{1}{2}}(p_n)}$$



'Bare' propagator

$$\bar{\Gamma}_{\bar{g}^2}^{(2)}(p_1, p_2) \quad \rho_g(\lambda) \in \mathbb{R}^+$$

$$\int_{\mathbb{R}} \frac{d\lambda}{2\pi} \lambda \bar{\rho}(\lambda) = 1$$

Unfolding the background independent effective action

$$\Gamma[g_{\mu\nu}] = \frac{1}{16\pi} \int_x \sqrt{g} \left\{ \mathcal{R}(\Delta, R) + R f_{R^2}(\Delta) R + R_{\mu\nu} f_{R_{\mu\nu}^2}(\Delta) R^{\mu\nu} + \dots \right\}$$

Revisited

gauge dependent

$$\bar{\Gamma}_{hh}^{(2)}(p)$$

$$\bar{\Gamma}_{h^3}^{(3)}(p)$$

$$\bar{\Gamma}_{h^4}^{(4)}(p)$$



RG-invariant

$$\bar{\Gamma}_{\bar{g}^3}^{(3)}(p)$$

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$$\mathcal{R}(\Delta, R)$$

$$\bar{\Gamma}_{h^3}^{(3)}(p)$$



$$R f_{R^2}(\Delta) R$$

$$\bar{\Gamma}_{h^4}^{(4)}(p)$$

$$\bar{\Gamma}_{\bar{g}^4}^{(4)}(p)$$

$$R_{\mu\nu} f_{R^2_{\mu\nu}}(\Delta) R^{\mu\nu}$$

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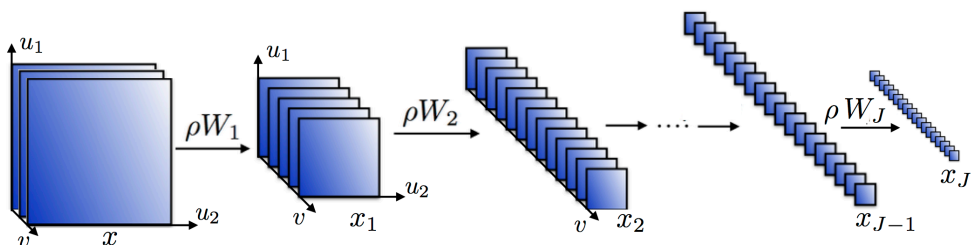
$$\mathcal{R}(\Delta, R)$$

Unfolding

$$R f_{R^2}(\Delta) R$$

Maps

$$R_{\mu\nu} f_{R^2_{\mu\nu}}(\Delta) R^{\mu\nu}$$



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Revisited

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Unfolding

So far educated guess

$$\bar{\Gamma}_{h^3}^{(3)}(p)$$



$$R f_{R^2}(\Delta) R$$

$$\bar{\Gamma}_{\bar{g}^4}^{(4)}(p)$$

Maps

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Revisited

gauge dependent

RG-invariant

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$$\bar{\Gamma}_{\bar{g}^3}^{(3)}(p)$$

$$\mathcal{R}(\Delta, R)$$

Unfolding

So far educated guess

$$\bar{\Gamma}_{h^3}^{(3)}(p)$$



$$R f_{R^2}(\Delta) R$$

Future: Physics-informed RG flows

$$\bar{\Gamma}_{\bar{g}^4}^{(4)}(p)$$

Maps

Ihssen, JMP, in preparation

$$\bar{\Gamma}_{h^4}^{(4)}(p)$$

$$R_{\mu\nu} f_{R^2_{\mu\nu}}(\Delta) R^{\mu\nu}$$

Unfolding the background independent effective action

$$\Gamma[g_{\mu\nu}] = \frac{1}{16\pi} \int_x \sqrt{g} \left\{ \mathcal{R}(\Delta, R) + R f_{R^2}(\Delta) R + R_{\mu\nu} f_{R^2_{\mu\nu}}(\Delta) R^{\mu\nu} + \dots \right\}$$

Revisited

gauge dependent

RG-invariant

gauge independent

$$\bar{\Gamma}_{hh}^{(2)}(p)$$

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Future: Physics-informed RG flows

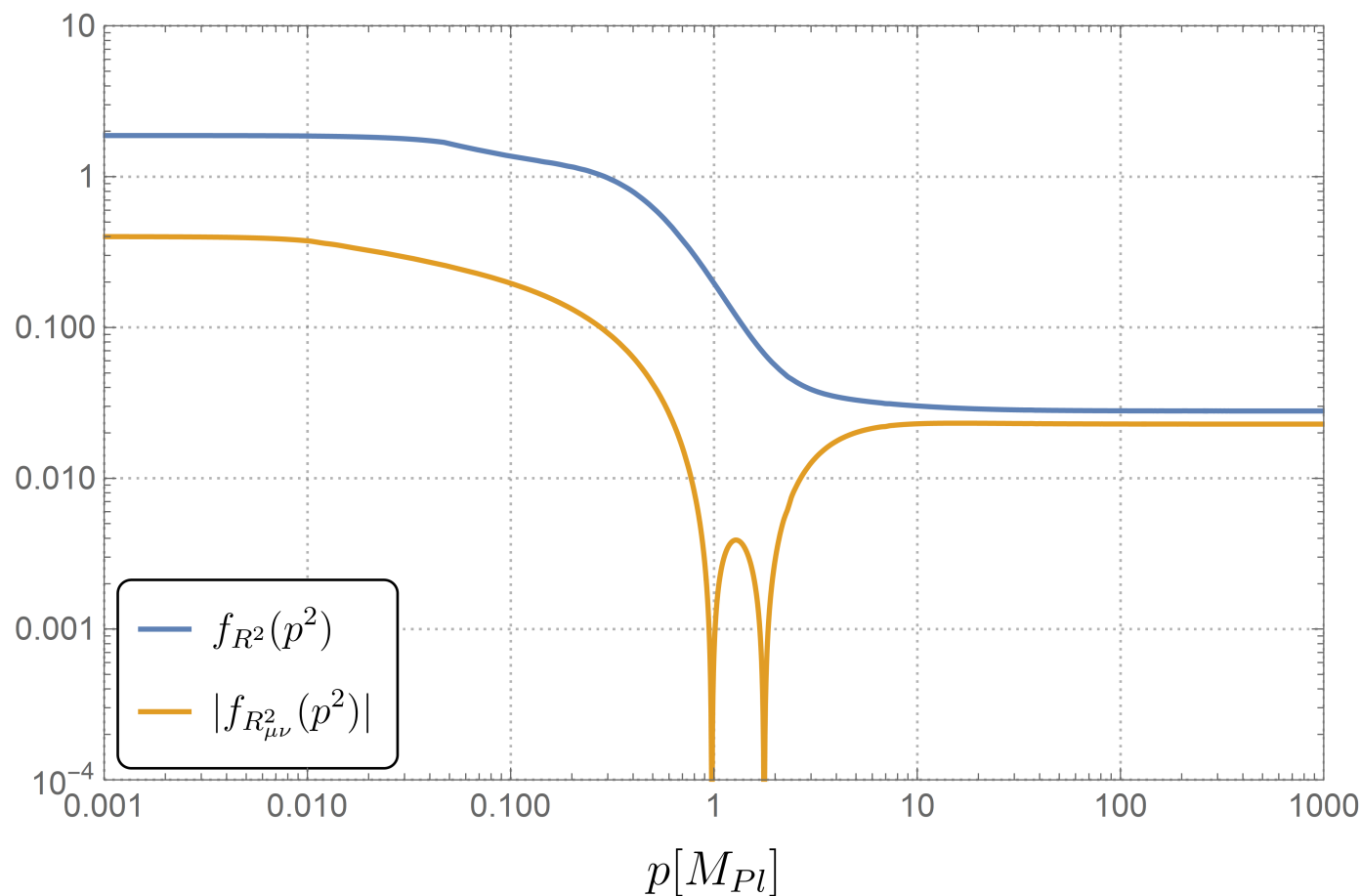
$$\bar{\Gamma}_{\bar{g}^4}^{(4)}(p)$$

Maps

Ihssen, JMP, in preparation

$$\bar{\Gamma}_{h^4}^{(4)}(p)$$

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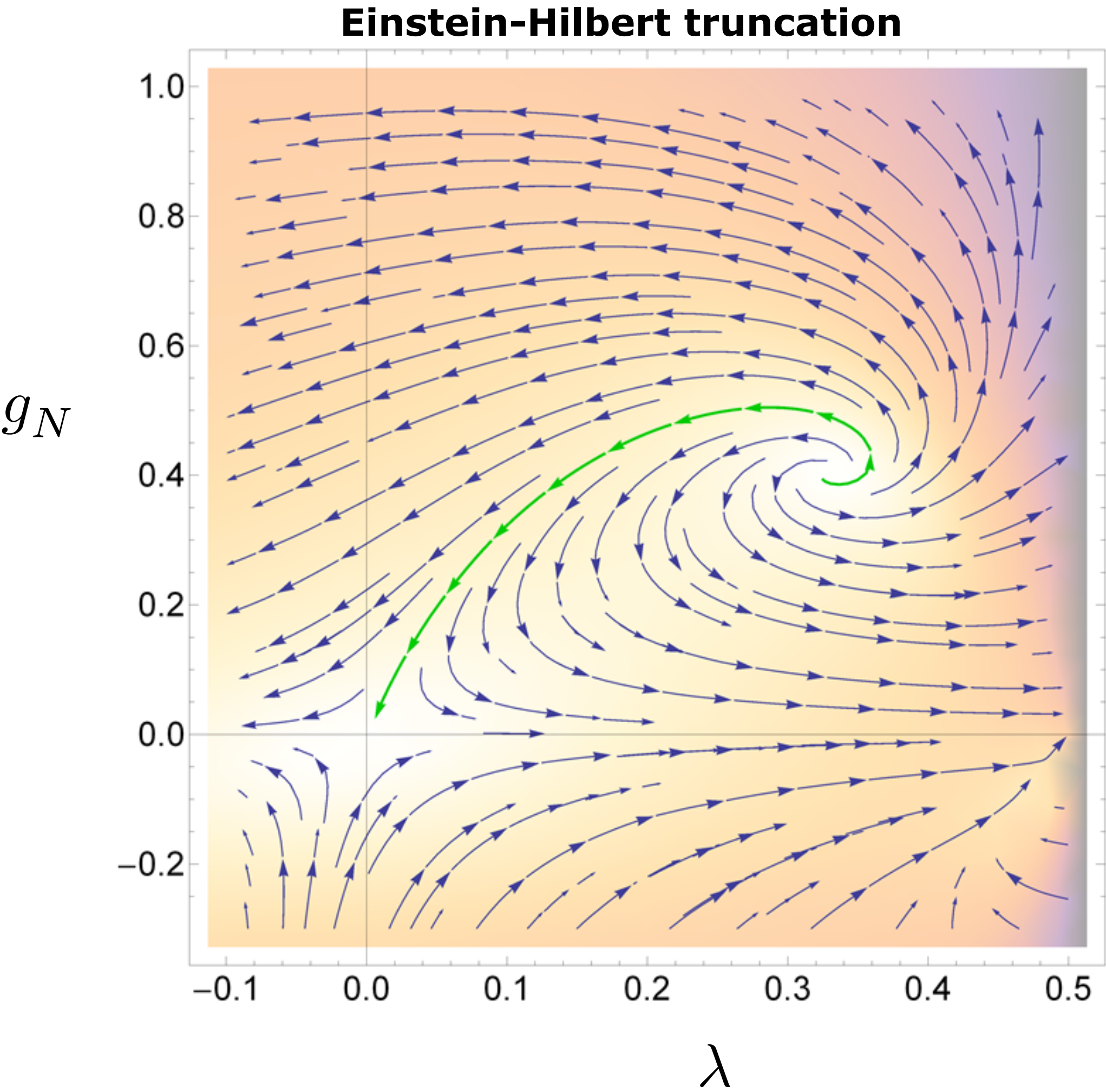
Results for form factors

$$\mathcal{R}(\Delta, R) = R \frac{\gamma_g^{(3)}(\Delta) - \bar{\gamma}_3 \Delta}{\Delta + R} R$$

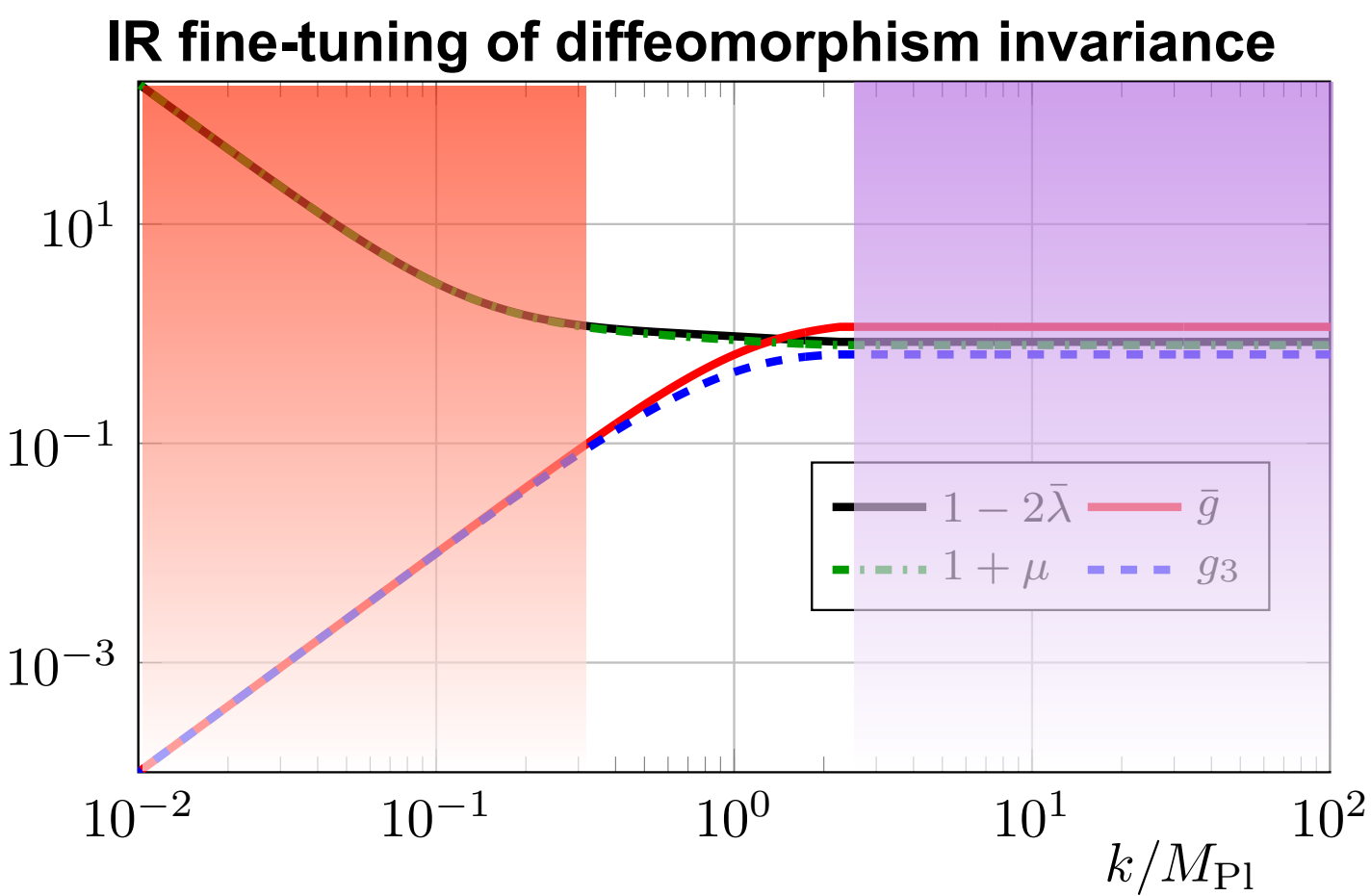
Apparent convergence in asymptotic safety

a community effort

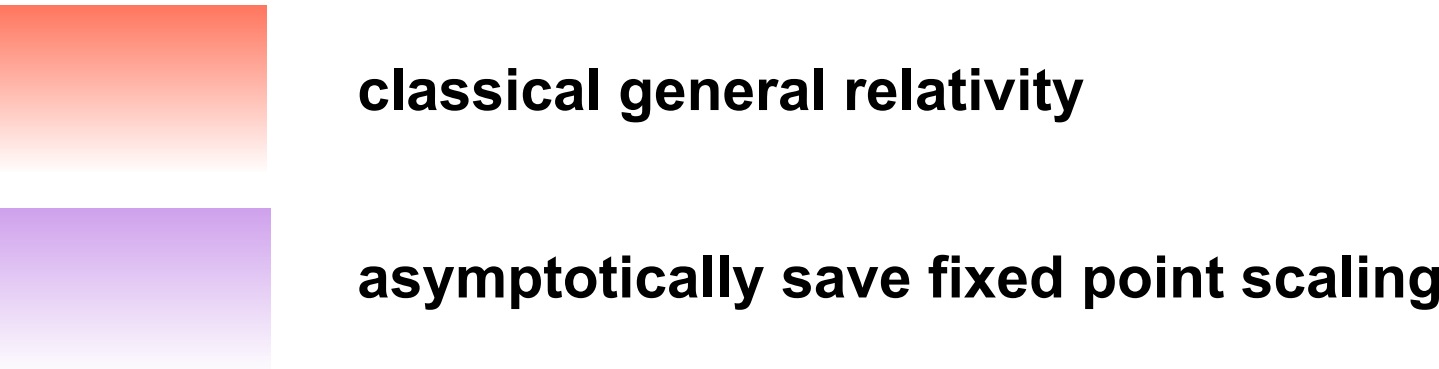
Phase structure of asymptotically safe gravity



Reuter, Saueressig, PRD 65 (2002) 065016



Denz, JMP, Reichert, EPJ C78 (2018) 4, 336



Reuter '96

⋮

Towards apparent convergence in quantum gravity I

expansion in curvature invariants

Quantum Effective Action: $\Gamma[g] = \sum_n c_{n,k} \int d^4x \sqrt{g} \mathcal{O}_n(g)$

$$\mathcal{O}_n$$

$$\Lambda, R$$

Reuter, PRD 57 (1998) 971

$$\Lambda, R, R^2$$

Lauscher, Reuter, PRD 66 (2002) 025026

$$\Lambda, f(R)$$

Codello, Percacci, Rahmede, AP 324 (2009) 414,
 Falls, Litim, Nikolakopoulos, Rahmede,
 arXiv:1301.4191, PRD 93 (2016) 10, 104022

$$\Lambda, R, C^{\mu\nu\kappa\lambda} C^{\kappa\lambda\rho\sigma} C^{\rho\sigma\mu\nu}$$

Gies, Knorr, Lippoldt, Saueressig, PRL 116 (2016) 21, 211302

Towards apparent convergence in quantum gravity I

expansion in curvature invariants

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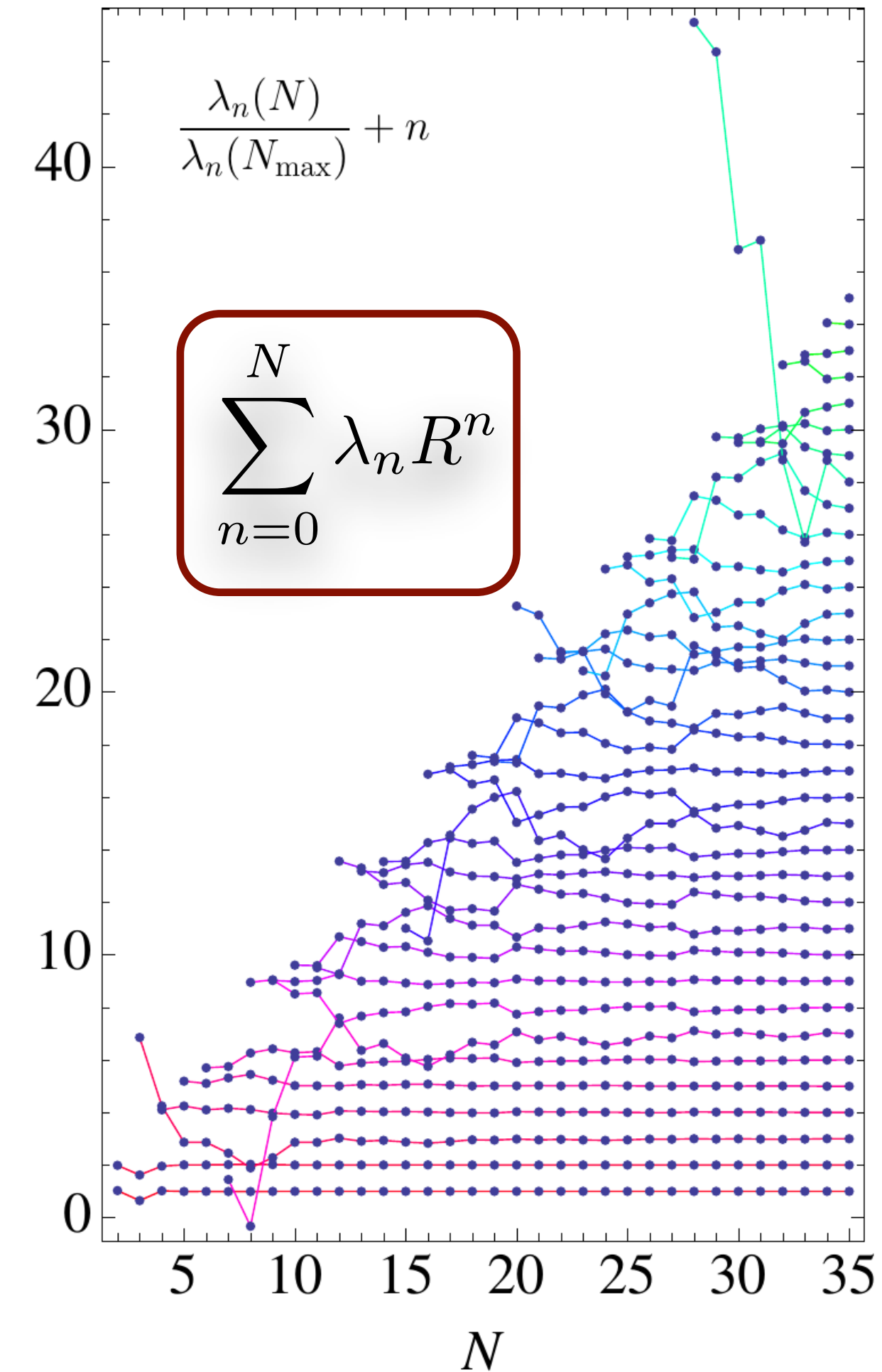
$$\mathcal{O}_n$$

$$\Lambda, R$$

$$\Lambda, R, R^2$$

$$\Lambda, f(R)$$

$$\Lambda, R, C^{\mu\nu\kappa\lambda} C_{\kappa\lambda\rho\sigma} C_{\rho\sigma\mu\nu}$$



Towards apparent convergence in quantum gravity II

vertex expansion

$$\Gamma^{(m,n \geq 2)}$$

$$\partial_t \Gamma_k = \frac{1}{2} \text{ (blue circle with } \otimes \text{) } - \text{ (red dotted circle with } \otimes \text{ and arrows) }$$

$$\partial_t \Gamma_k^{(h)} = -\frac{1}{2} \text{ (blue circle with } \otimes \text{ and one blue line) } + \text{ (red dotted circle with } \otimes \text{ and one red line) }$$

$$\partial_t \Gamma_k^{(2h)} = -\frac{1}{2} \text{ (blue circle with } \otimes \text{ and two blue lines) } + \text{ (blue circle with } \otimes \text{ and two blue lines) } - 2 \text{ (red dotted circle with } \otimes \text{ and two red lines) }$$

$$\partial_t \Gamma_k^{(c\bar{c})} = \text{ (red dotted circle with } \otimes \text{ and two red lines) } + \text{ (blue circle with } \otimes \text{ and two blue lines) }$$

$$\partial_t \Gamma_k^{(3h)} = -\frac{1}{2} \text{ (blue circle with } \otimes \text{ and three blue lines) } + 3 \text{ (blue circle with } \otimes \text{ and three blue lines) } - 3 \text{ (blue circle with } \otimes \text{ and three blue lines) } + 6 \text{ (red dotted circle with } \otimes \text{ and three red lines) }$$

$$\partial_t \Gamma_k^{(4h)} = -\frac{1}{2} \text{ (blue circle with } \otimes \text{ and four blue lines) } + 3 \text{ (blue circle with } \otimes \text{ and four blue lines) } + 4 \text{ (blue circle with } \otimes \text{ and four blue lines) } - 6 \text{ (blue circle with } \otimes \text{ and four blue lines) }$$

$$- 12 \text{ (blue circle with } \otimes \text{ and four blue lines) } + 12 \text{ (blue circle with } \otimes \text{ and four blue lines) } - 24 \text{ (red dotted circle with } \otimes \text{ and four red lines) }$$

Towards apparent convergence in quantum gravity II

vertex expansion

$$\Gamma^{(m,n \geq 2)}$$

Reuter, PRD 57 (1998) 971,

$$\partial_t \Gamma_k = \frac{1}{2} \text{ (blue circle with cross) } - \text{ (red dashed circle with cross) }$$

background approximation: $\Gamma^{(m,n)} \approx \Gamma^{(m+n,0)}$

$$\partial_t \Gamma_k^{(h)} = -\frac{1}{2} \text{ (blue circle with cross and one external line) } + \text{ (red dashed circle with cross and one external line) }$$

$$\partial_t \Gamma_k^{(2h)} = -\frac{1}{2} \text{ (blue circle with cross and two external lines) } + \text{ (blue circle with cross and two external lines) } - 2 \text{ (red dashed circle with cross and two external lines) }$$

$$\partial_t \Gamma_k^{(c\bar{c})} = \text{ (red dashed circle with cross and two external lines) } + \text{ (blue circle with cross and two external lines) }$$

$$\partial_t \Gamma_k^{(3h)} = -\frac{1}{2} \text{ (blue circle with cross and three external lines) } + 3 \text{ (blue circle with cross and three external lines) } - 3 \text{ (blue circle with cross and three external lines) } + 6 \text{ (red dashed circle with cross and three external lines) }$$

$$\begin{aligned} \partial_t \Gamma_k^{(4h)} = & -\frac{1}{2} \text{ (blue circle with cross and four external lines) } + 3 \text{ (blue circle with cross and four external lines) } + 4 \text{ (blue circle with cross and four external lines) } - 6 \text{ (blue circle with cross and four external lines) } \\ & - 12 \text{ (blue circle with cross and four external lines) } + 12 \text{ (blue circle with cross and four external lines) } - 24 \text{ (red dashed circle with cross and four external lines) } \end{aligned}$$

Towards apparent convergence in quantum gravity II

vertex expansion

$$\Gamma^{(m,n \geq 2)}$$

bi-metric approach: Manrique, Reuter, Saueressig, Annals Phys. 326 (2011) 463

$$\begin{aligned} \partial_t \Gamma_k &= \frac{1}{2} \text{[Diagram: Circle with two external lines and a cross]} - \text{[Diagram: Circle with two external lines and a cross, dashed red lines]} \\ \partial_t \Gamma_k^{(h)} &= -\frac{1}{2} \text{[Diagram: Circle with two external lines and a cross]} + \text{[Diagram: Circle with two external lines and a cross, dashed red lines]} \end{aligned}$$

level 1: $\Gamma^{(m,n)} \approx \Gamma^{(m+n-1,1)}$

$$\partial_t \Gamma_k^{(2h)} = -\frac{1}{2} \text{[Diagram: Circle with two external lines and a cross]} + \text{[Diagram: Circle with two external lines and a cross]} - 2 \text{[Diagram: Circle with two external lines and a cross, dashed red lines]}$$

$$\partial_t \Gamma_k^{(c\bar{c})} = \text{[Diagram: Circle with two external lines and a cross, dashed red lines]} + \text{[Diagram: Circle with two external lines and a cross, dashed red lines]}$$

$$\partial_t \Gamma_k^{(3h)} = -\frac{1}{2} \text{[Diagram: Circle with two external lines and a cross]} + 3 \text{[Diagram: Circle with two external lines and a cross]} - 3 \text{[Diagram: Circle with two external lines and a cross]} + 6 \text{[Diagram: Circle with two external lines and a cross, dashed red lines]}$$

$$\begin{aligned} \partial_t \Gamma_k^{(4h)} &= -\frac{1}{2} \text{[Diagram: Circle with two external lines and a cross]} + 3 \text{[Diagram: Circle with two external lines and a cross]} + 4 \text{[Diagram: Circle with two external lines and a cross]} - 6 \text{[Diagram: Circle with two external lines and a cross]} \\ &\quad - 12 \text{[Diagram: Circle with two external lines and a cross]} + 12 \text{[Diagram: Circle with two external lines and a cross]} - 24 \text{[Diagram: Circle with two external lines and a cross, dashed red lines]} \end{aligned}$$

Towards apparent convergence in quantum gravity II

vertex expansion

geometrical approach: Donkin, JMP, arXiv:1203.4207

flat expansion: Christiansen, Litim, JMP, Rodigast, PLB 728 (2014) 114

$$\Gamma^{(m,n\geq 2)}$$

level 2: $\Gamma^{(m,n)} \approx \Gamma^{(m+n-2,2)}$

$$\partial_t \Gamma_k = \frac{1}{2} \text{[blue circle with cross]} - \text{[red dashed circle with cross]}$$
$$\partial_t \Gamma_k^{(h)} = -\frac{1}{2} \text{[blue circle with cross and line]} + \text{[red dashed circle with cross and line]}$$
$$\partial_t \Gamma_k^{(2h)} = -\frac{1}{2} \text{[blue circle with cross and two lines]} + \text{[blue circle with cross and two lines]} - 2 \text{[red dashed circle with cross and two lines]}$$
$$\partial_t \Gamma_k^{(c\bar{c})} = \text{[red dashed circle with cross and dots]} + \text{[blue circle with cross and dots]}$$

$$Z_h(p), \quad Z_c(p), \quad \mu = -2\lambda_2$$

$$\partial_t \Gamma_k^{(3h)} = -\frac{1}{2} \text{[blue circle with cross and three lines]} + 3 \text{[blue circle with cross and three lines]} - 3 \text{[blue circle with cross and three lines]} + 6 \text{[red dashed circle with cross and three lines]}$$
$$\partial_t \Gamma_k^{(4h)} = -\frac{1}{2} \text{[blue circle with cross and four lines]} + 3 \text{[blue circle with cross and four lines]} + 4 \text{[blue circle with cross and four lines]} - 6 \text{[blue circle with cross and four lines]}$$
$$- 12 \text{[blue circle with cross and four lines]} + 12 \text{[blue circle with cross and four lines]} - 24 \text{[red dashed circle with cross and four lines]}$$

Towards apparent convergence in quantum gravity II

vertex expansion

$$\Gamma^{(m,n \geq 2)}$$

Christiansen, Knorr, Maibohm, JMP, Reichert, PRD 92 (2015) 12, 121501

$$\partial_t \Gamma_k = \frac{1}{2} \text{ (loop with cross) } - \text{ (dashed loop with cross) }$$

level 3: $\Gamma^{(m,n)} \approx \Gamma^{(m+n-3,3)}$

$$\partial_t \Gamma_k^{(h)} = -\frac{1}{2} \text{ (loop with cross and external line) } + \text{ (dashed loop with cross and external line) }$$

$$\partial_t \Gamma_k^{(2h)} = -\frac{1}{2} \text{ (loop with cross and two external lines) } + \text{ (loop with cross and two external lines) } - 2 \text{ (dashed loop with cross and two external lines) }$$

$Z_h(p), Z_c(p), \mu = -2\lambda_2$

$$\partial_t \Gamma_k^{(c\bar{c})} = \text{ (dashed loop with cross and external lines) } + \text{ (dashed loop with cross and external lines) }$$

$$\partial_t \Gamma_k^{(3h)} = -\frac{1}{2} \text{ (loop with cross and three external lines) } + 3 \text{ (loop with cross and three external lines) } - 3 \text{ (loop with cross and three external lines) } + 6 \text{ (dashed loop with cross and three external lines) }$$

$g_3(p), \lambda_3$

$$\begin{aligned} \partial_t \Gamma_k^{(4h)} = & -\frac{1}{2} \text{ (loop with cross and four external lines) } + 3 \text{ (loop with cross and four external lines) } + 4 \text{ (loop with cross and four external lines) } - 6 \text{ (loop with cross and four external lines) } \\ & - 12 \text{ (loop with cross and four external lines) } + 12 \text{ (loop with cross and four external lines) } - 24 \text{ (dashed loop with cross and four external lines) } \end{aligned}$$

Towards apparent convergence in quantum gravity II

vertex expansion

$$\Gamma^{(m,n \geq 2)}$$

Denz, JMP, Reichert, EPJ C78 (2018) 4, 336

$$\partial_t \Gamma_k = \frac{1}{2} \text{[diagram: circle with cross]} - \text{[diagram: dashed circle with cross and red arrows]}$$

level 4: $\Gamma^{(m,n)} \approx \Gamma^{(m+n-4,4)}$

$$\partial_t \Gamma_k^{(h)} = -\frac{1}{2} \text{[diagram: circle with cross and one external line]} + \text{[diagram: dashed circle with cross and one external line and red arrows]}$$

$$\partial_t \Gamma_k^{(2h)} = -\frac{1}{2} \text{[diagram: circle with cross and two external lines]} + \text{[diagram: circle with cross and two external lines]} - 2 \text{[diagram: dashed circle with cross and two external lines and red arrows]}$$

$$Z_h(p), \quad Z_c(p), \quad \mu = -2\lambda_2$$

$$\partial_t \Gamma_k^{(c\bar{c})} = \text{[diagram: dashed circle with cross and two external lines and red arrows]} + \text{[diagram: dashed circle with cross and two external lines and red arrows]}$$

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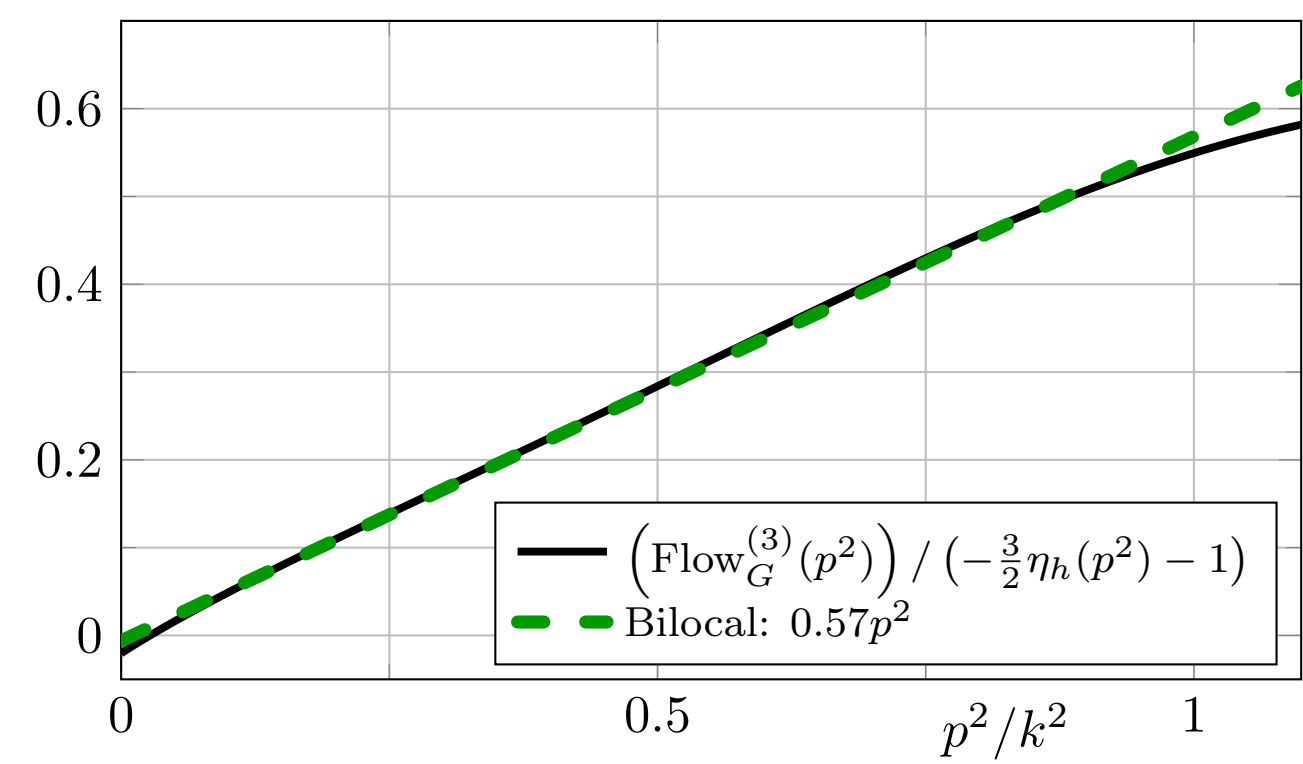
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$$g_4(p), \quad \lambda_4$$

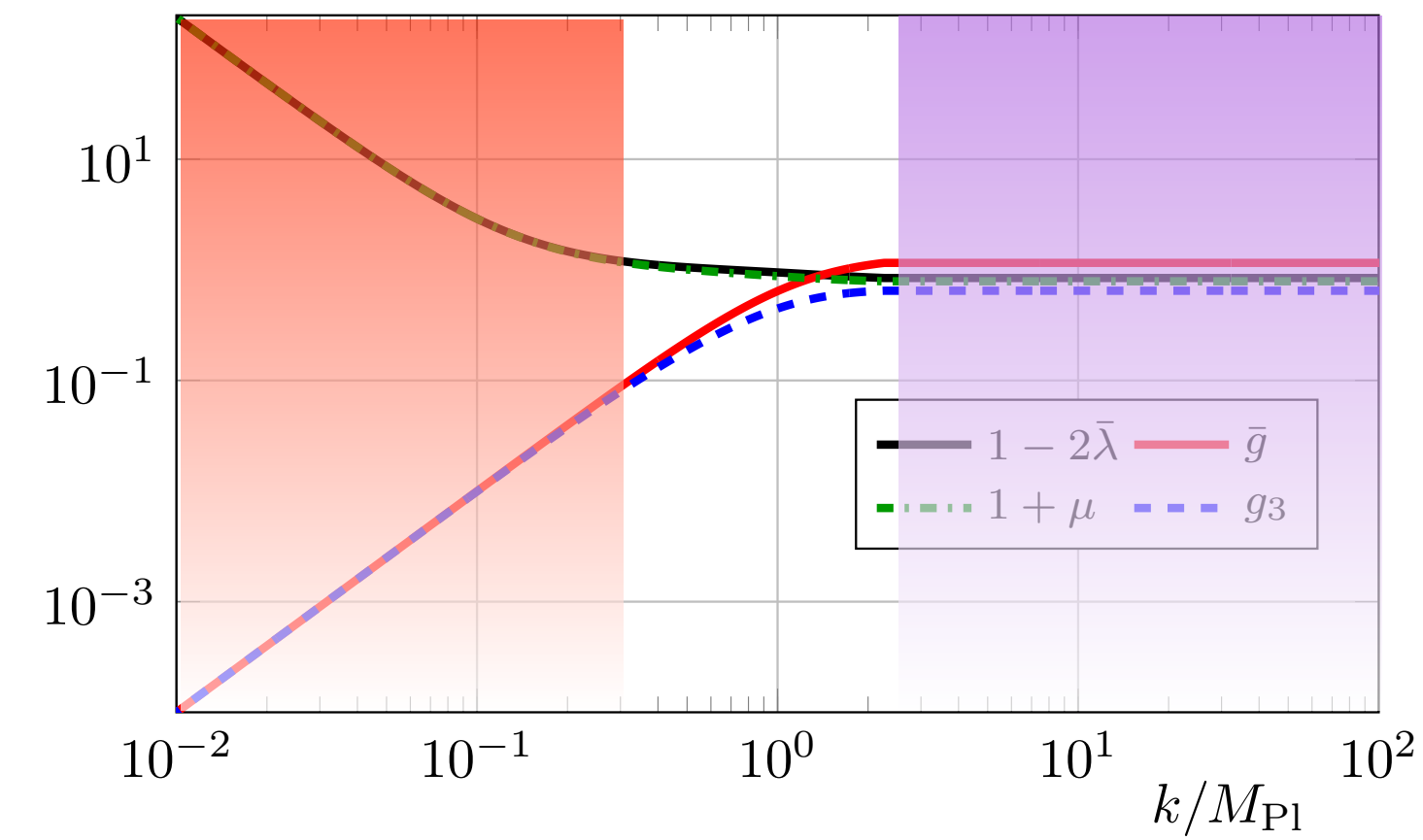
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Momentum-dependent vertices

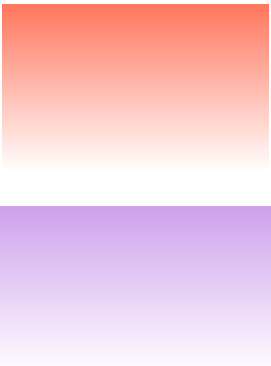
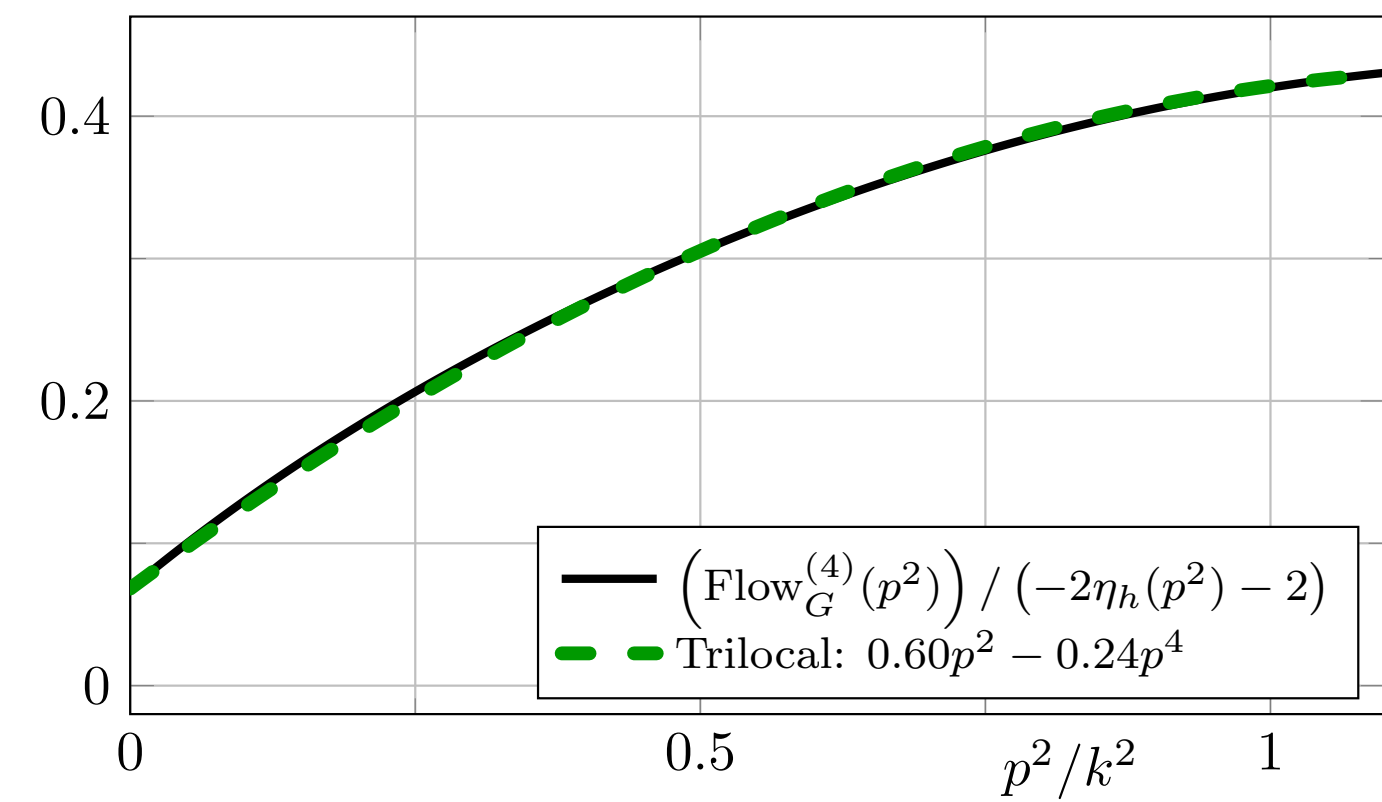
Flow of three-graviton vertex



IR fine-tuning of diffeomorphism invariance



Flow of four-graviton vertex

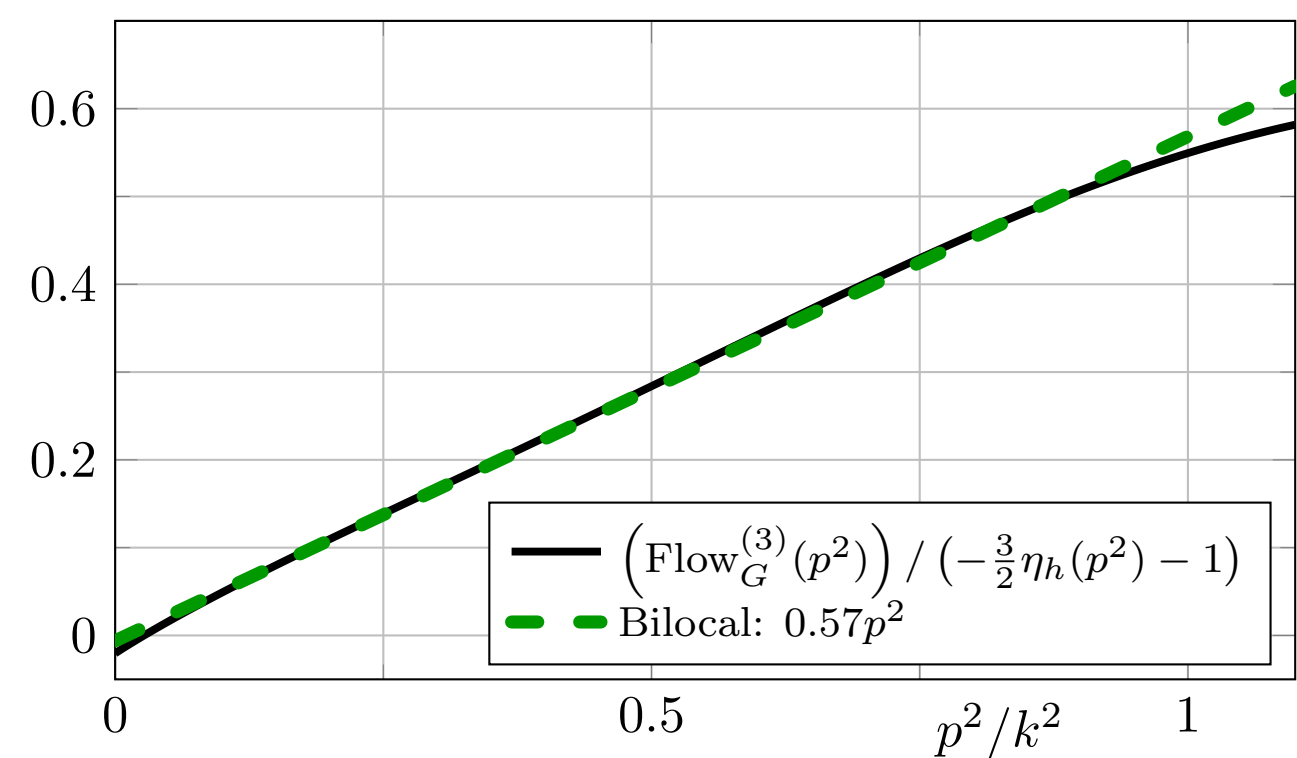


classical general relativity

asymptotically safe fixed point scaling

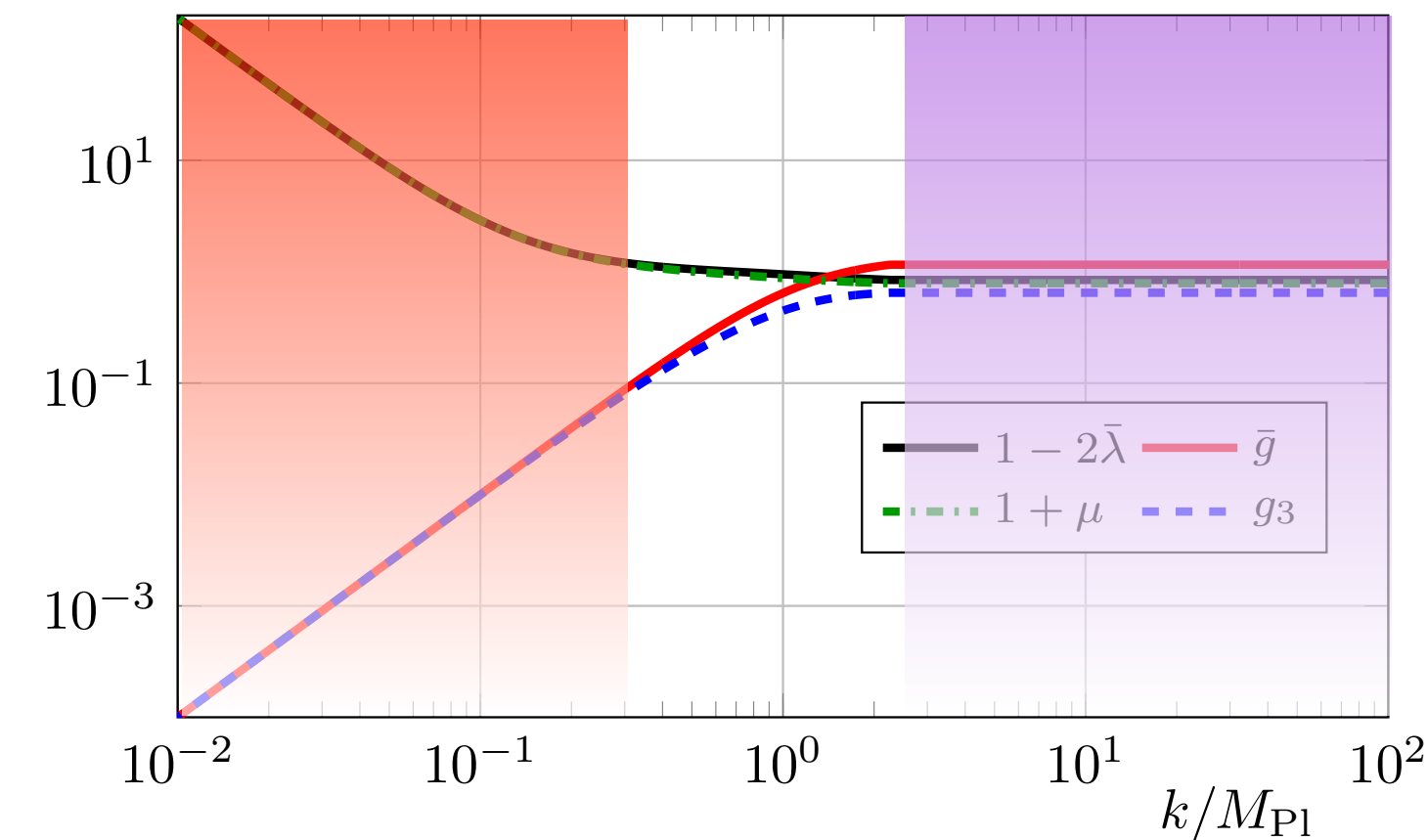
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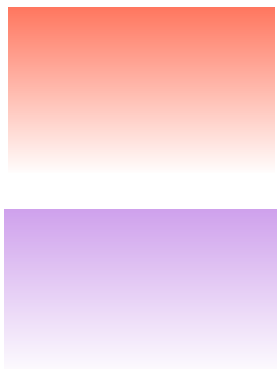
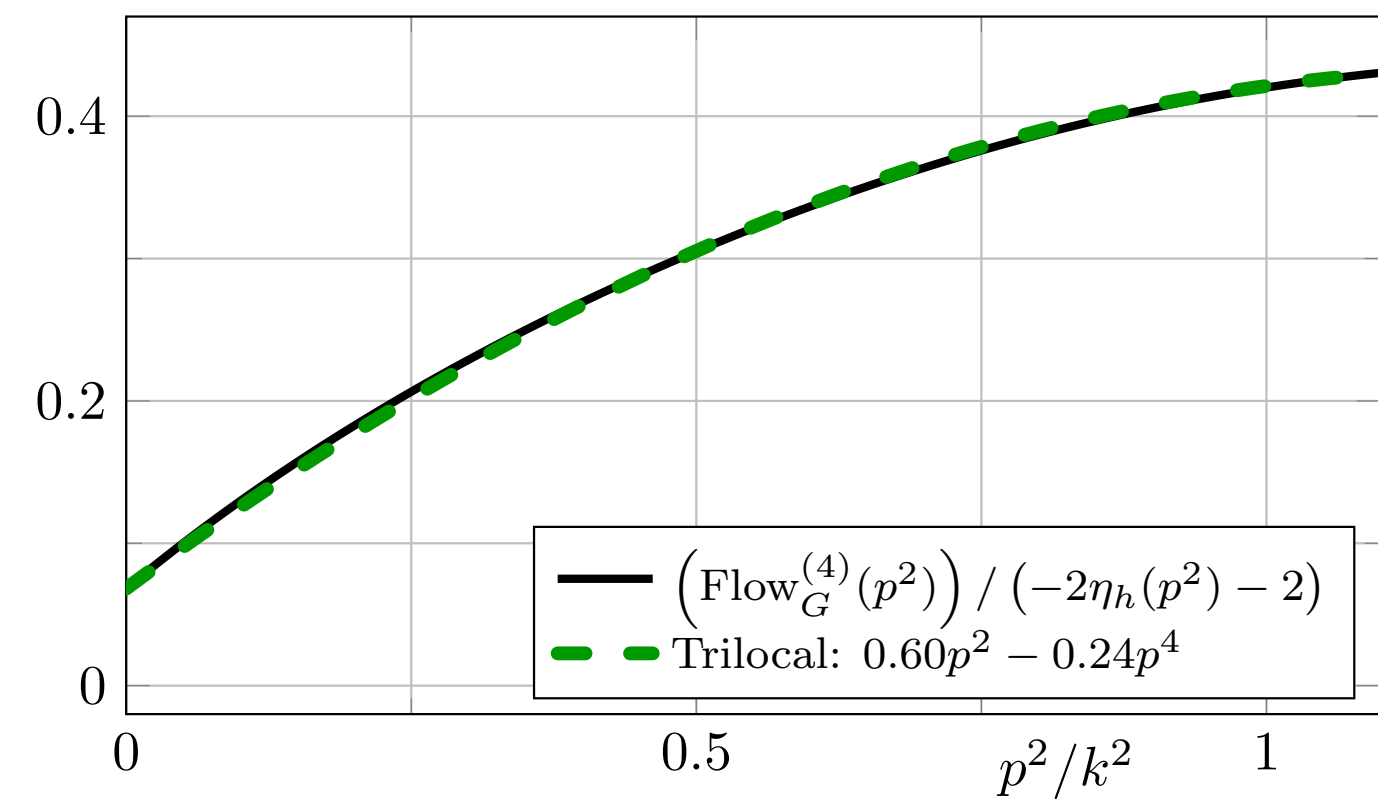


no $\mathbf{R}^2_{\mu\nu}$ - tensorstructure generated!

IR fine-tuning of diffeomorphism invariance



Flow of four-graviton vertex

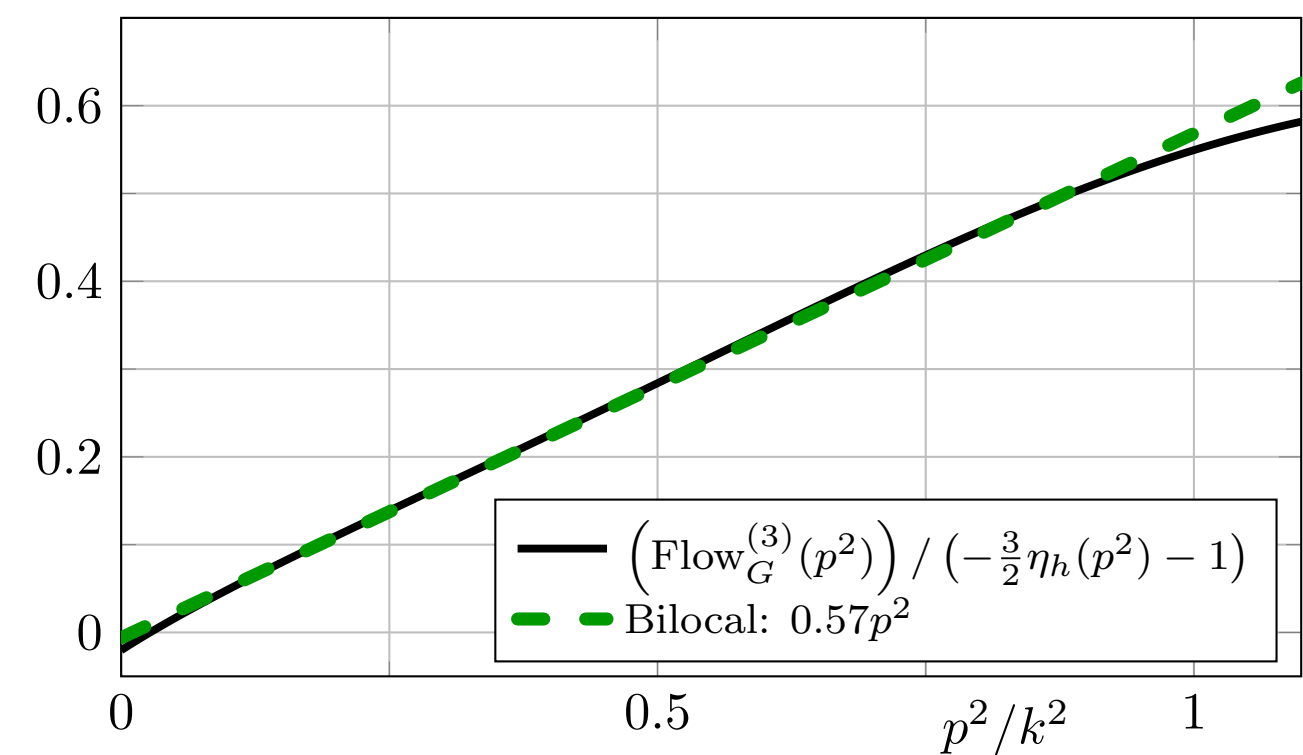


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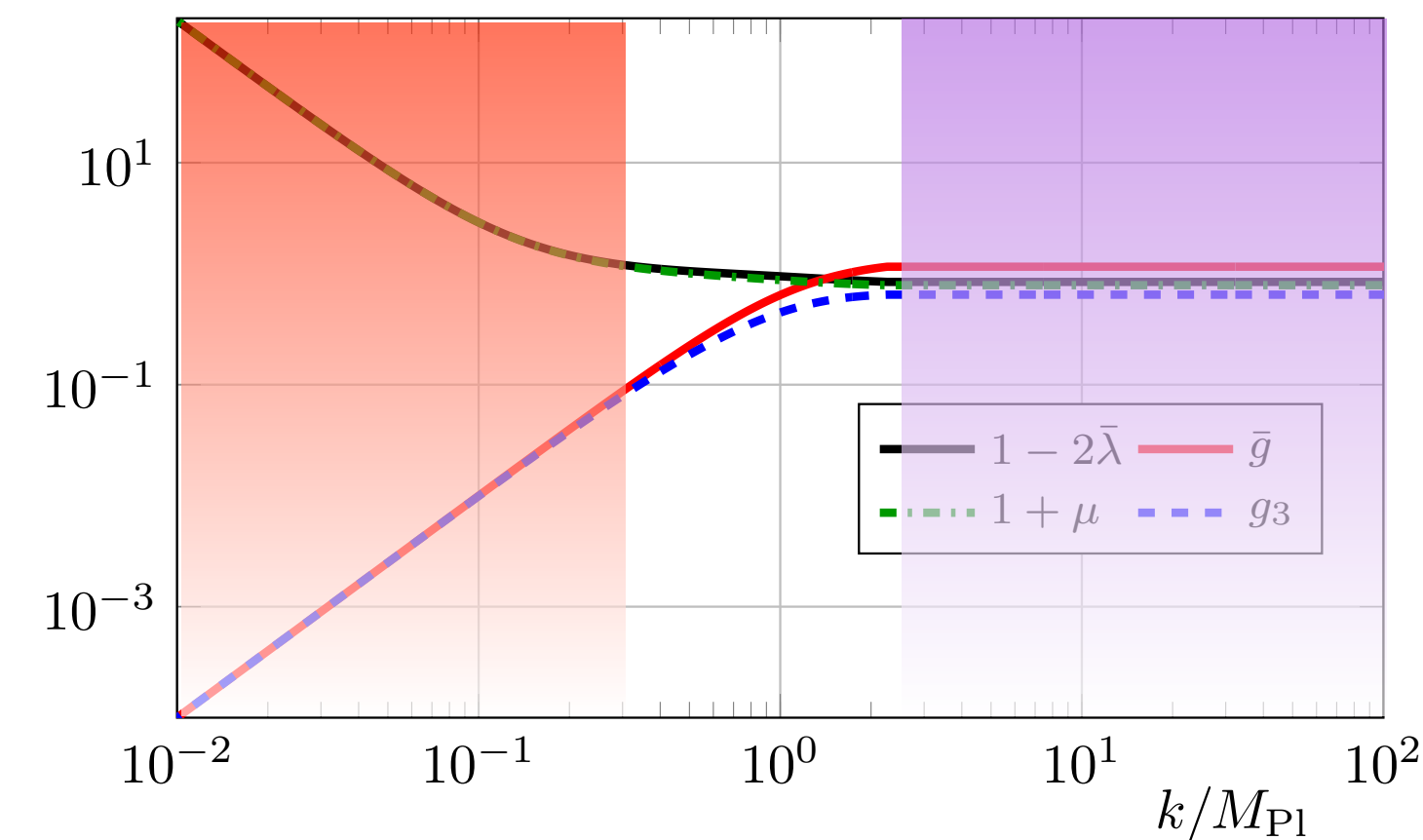
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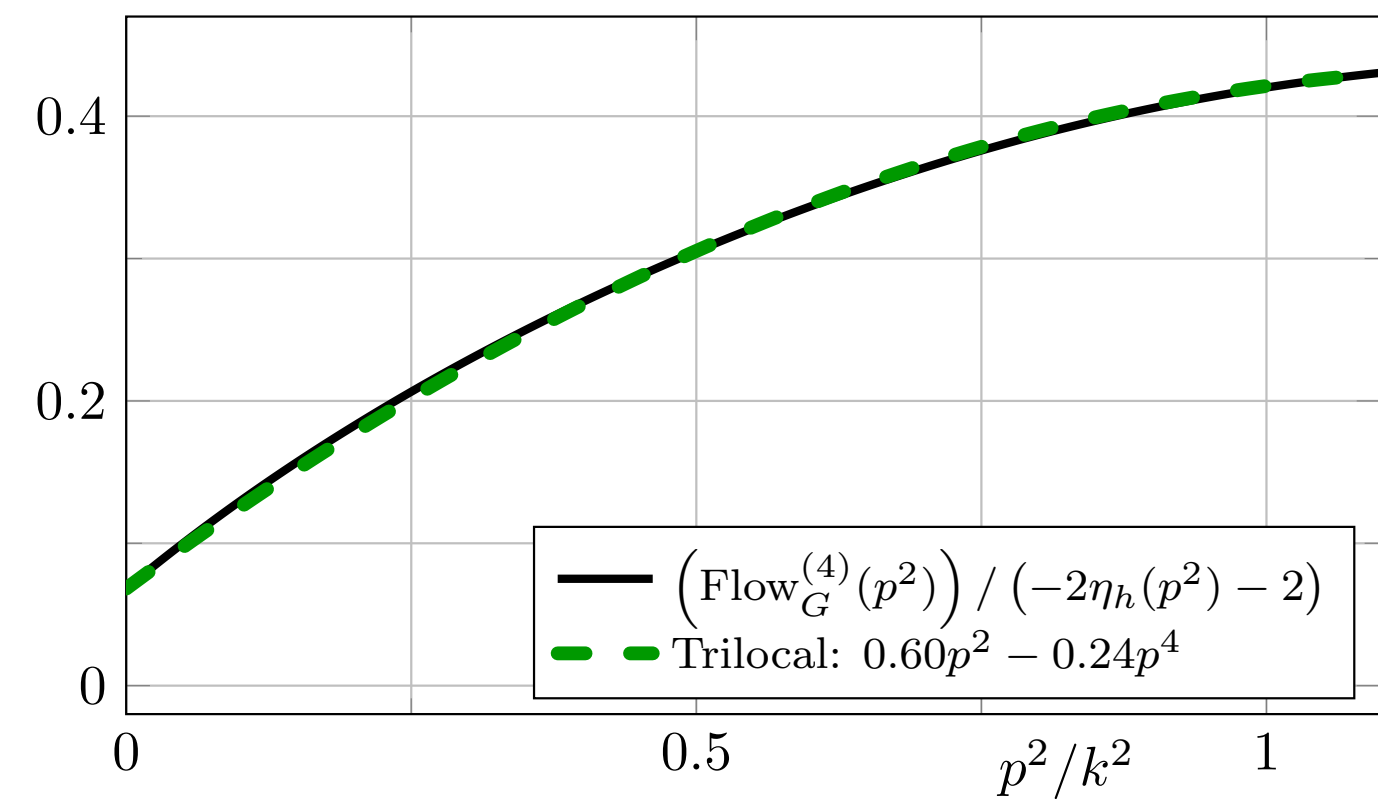


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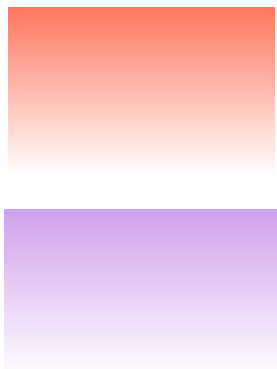


Flow of four-graviton vertex



$\mathbf{R}^2$ - tensorstructure generated

$\mathbf{R}$ - tensorstructure sustained

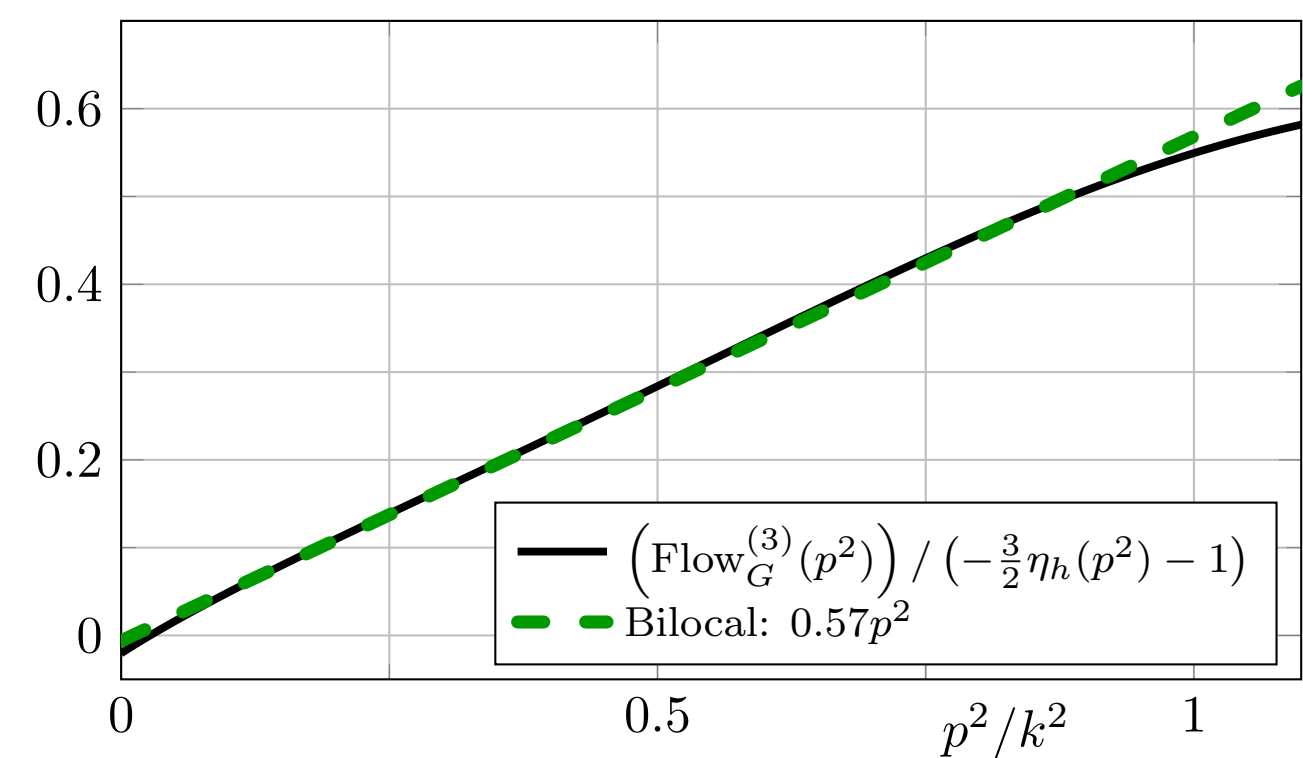


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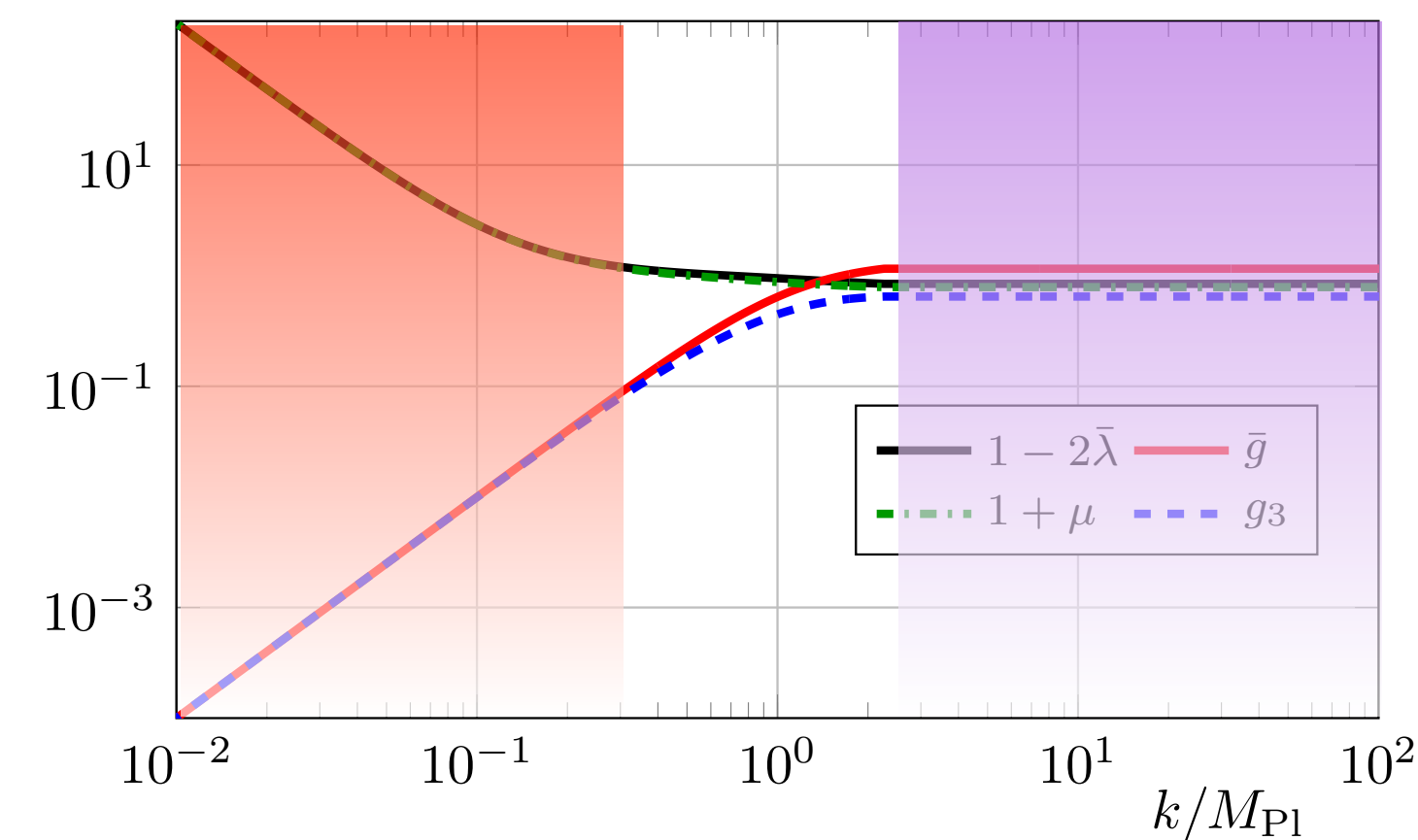
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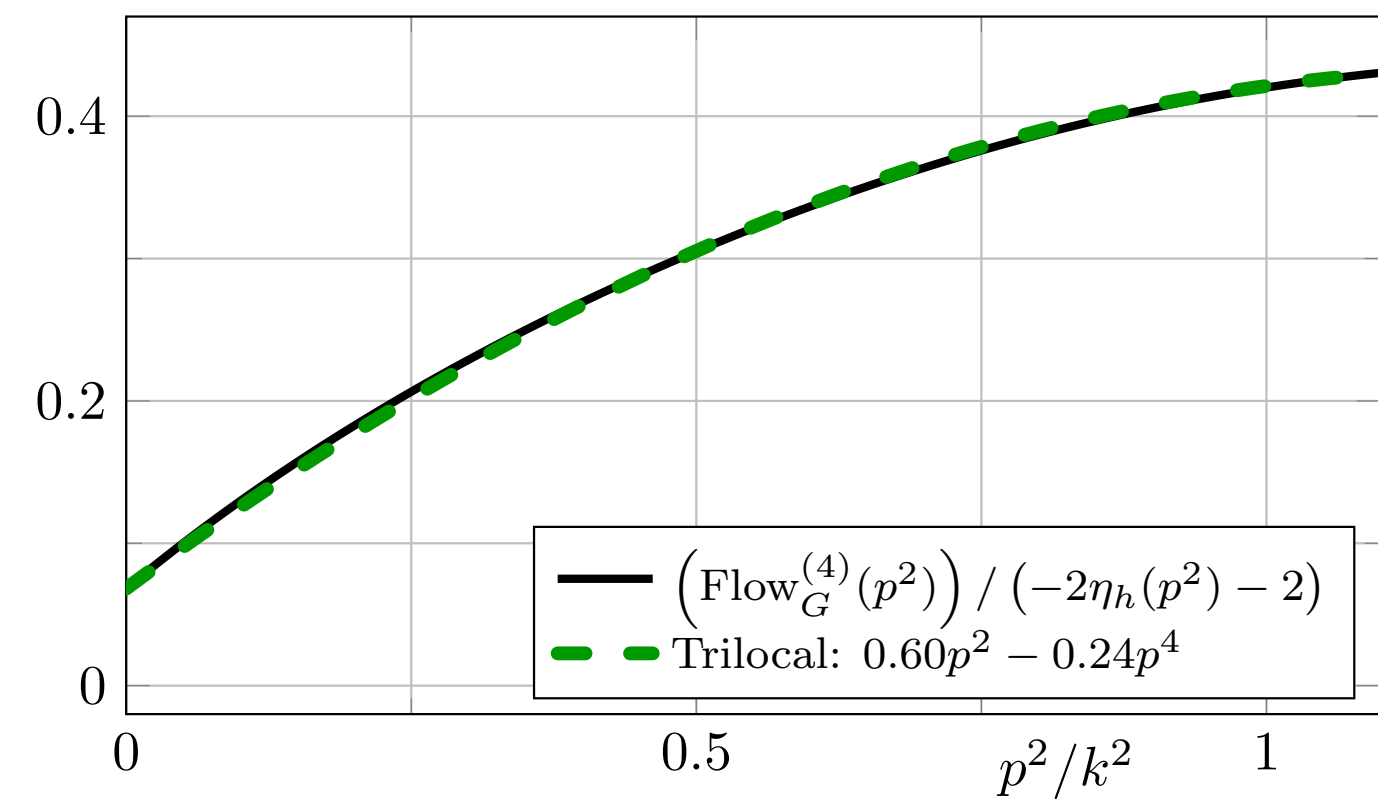


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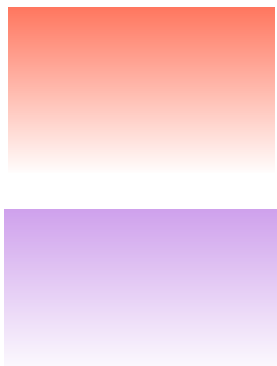


Flow of four-graviton vertex



$\mathbf{R}^2$ - tensorstructure generated

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classical general relativity

asymptotically safe fixed point scaling

Full momentum dependence
of
two-, three- and four-point function
at $k=0$

➡ Spectral properties

Towards apparent convergence in quantum gravity

Why does/could it work?

Typically diagrams with higher order vertices are strongly suppressed

(a) couplings stay finite

(b) combinatorial suppression of diagrams with higher vertices

(c) phase space (angular) suppression of diagrams with higher vertices

turns out to be very efficient!

Towards apparent convergence in quantum gravity

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Resonant interaction channels and their interactions circumvent (b) and make (a) irrelevant

(a) couplings diverge

(b) hadrons, diquarks, glueballs, ... in QCD



Emergent composites, BSE

Gies, Wetterich, PRD 65 (2002) 0650016

JMP, AP 322 (2007) 2831

Flörchinger, Wetterich, PLB 680 (2009) 371

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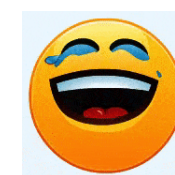
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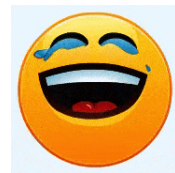
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JMP, AP 322 (2007) 2831

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QG as perturbative as possible & apparently converging

... slight oversimplification for the sake of this talk ...

JMP, Reichert, Front.in Phys. 8 (2021) 527

2309.10785

The physics of thresholds

Bonanno et al., Critical reflections on asymptotically safe gravity, Front.in Phys. 8 (2020) 269

QCD & SM thresholds in the RG since (many) decades

QCD with the fRG since a decade

The physics of thresholds

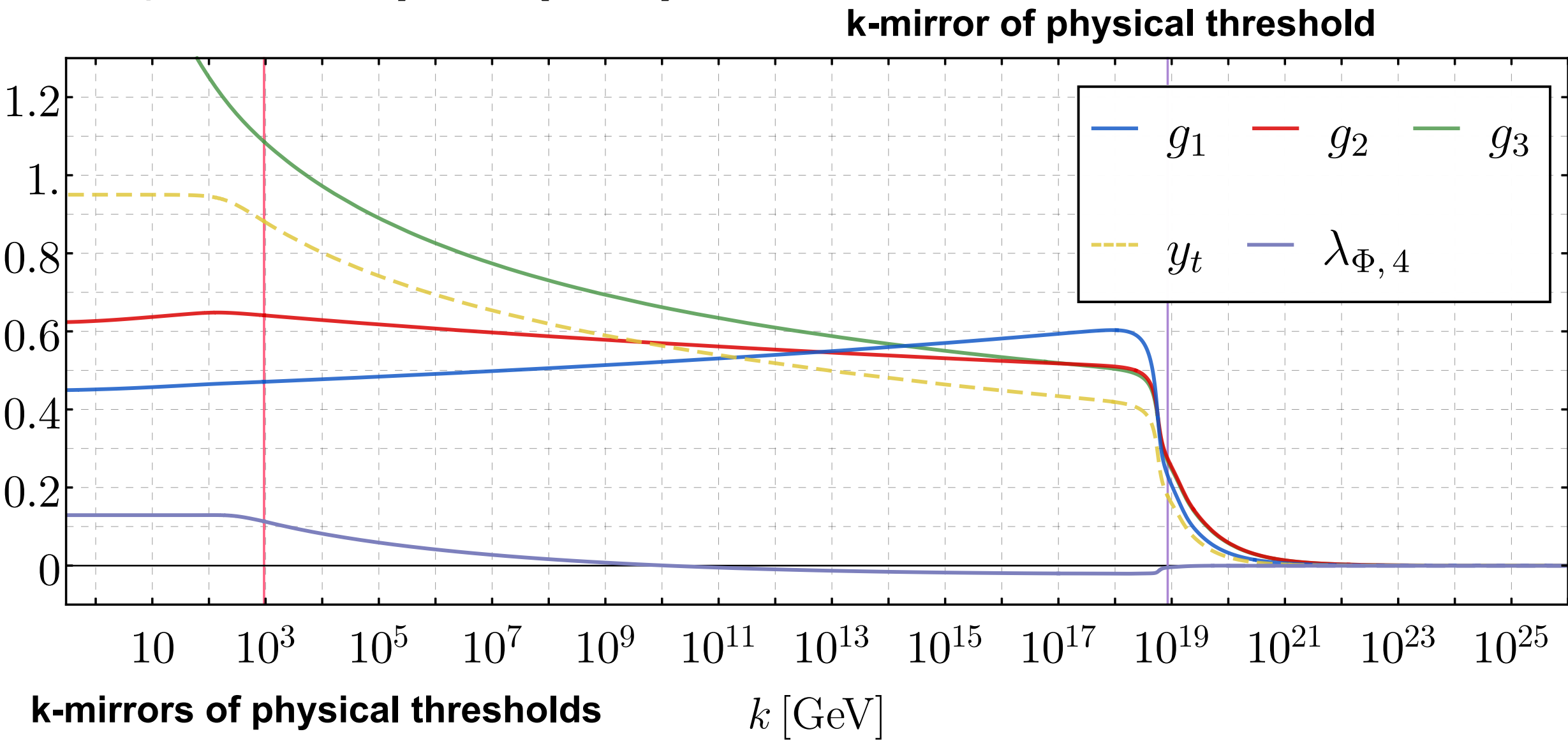
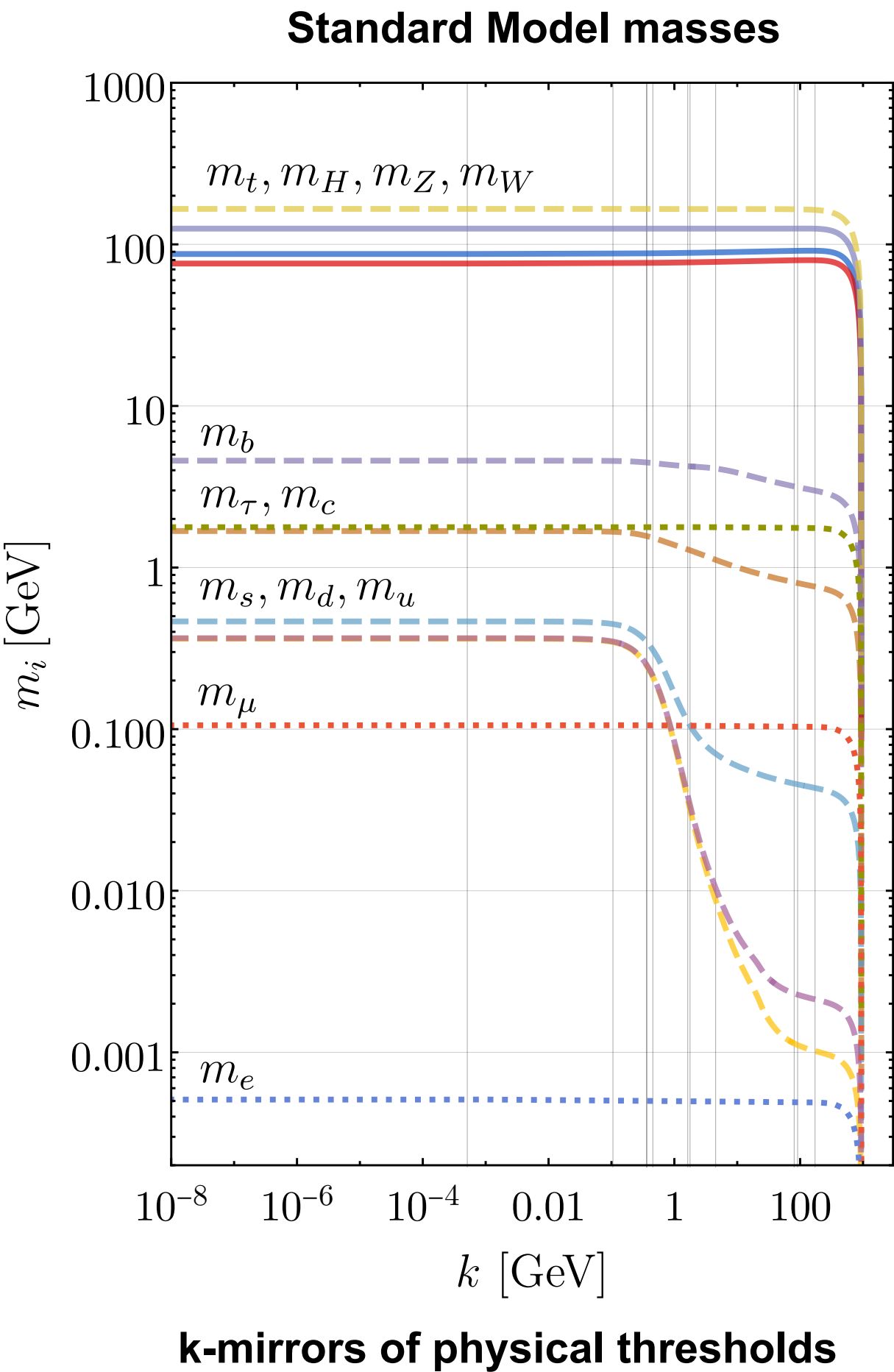
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Pastor-Gutiérrez, JMP, Reichert, SciPost Phys. 15 (2023) 105



The physics of thresholds

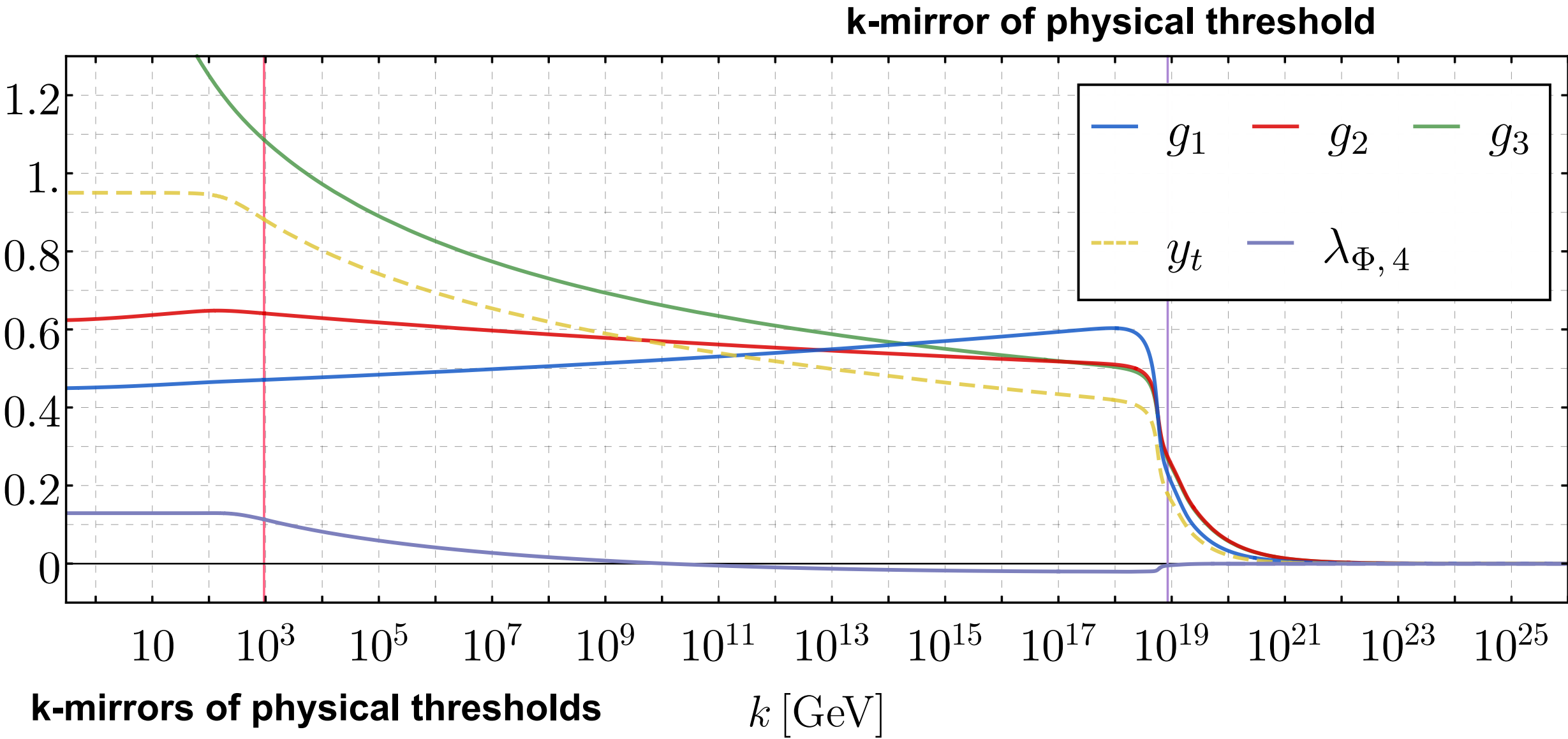
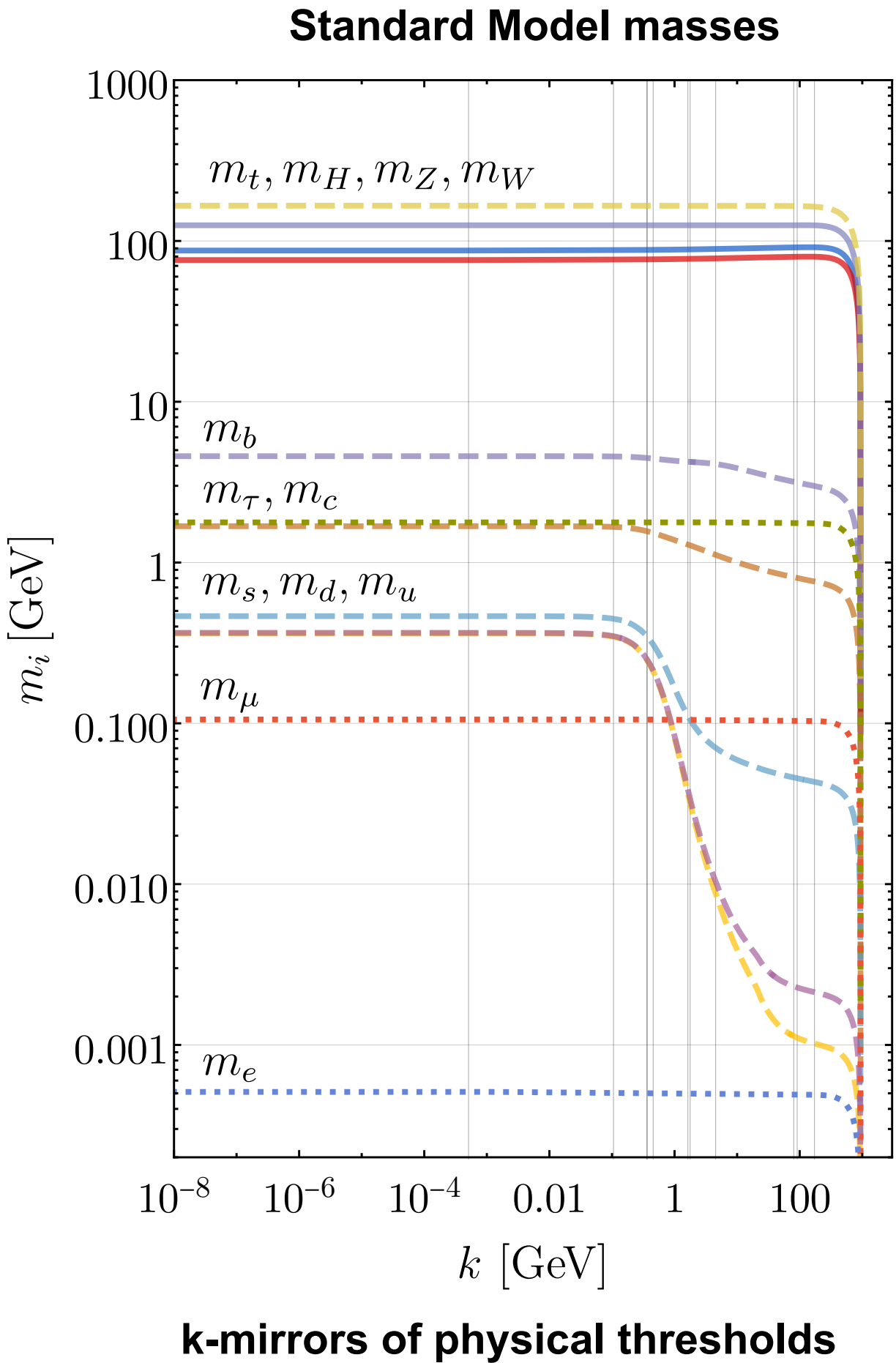
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k-mirrors of physical thresholds: feature, not bug!

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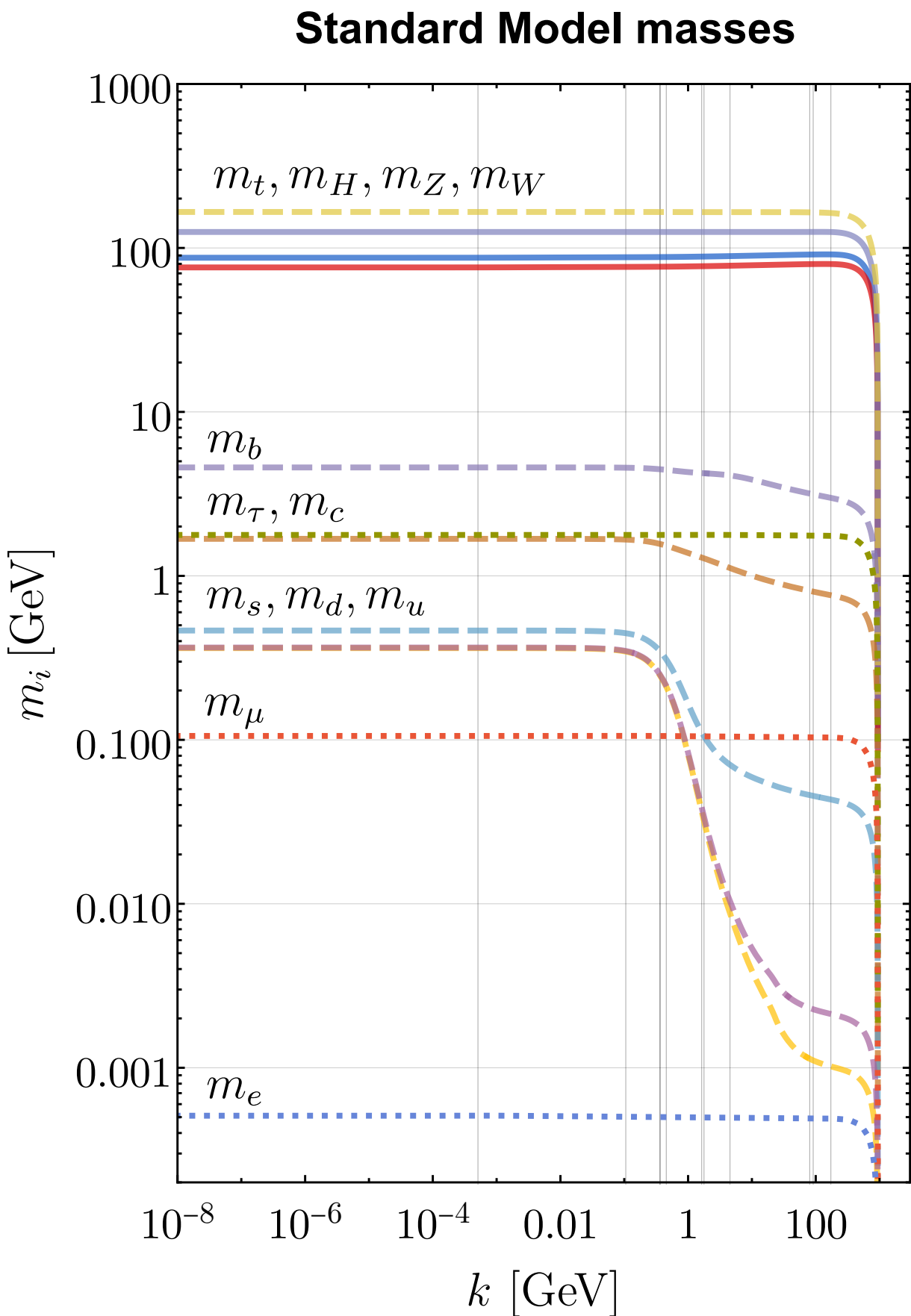
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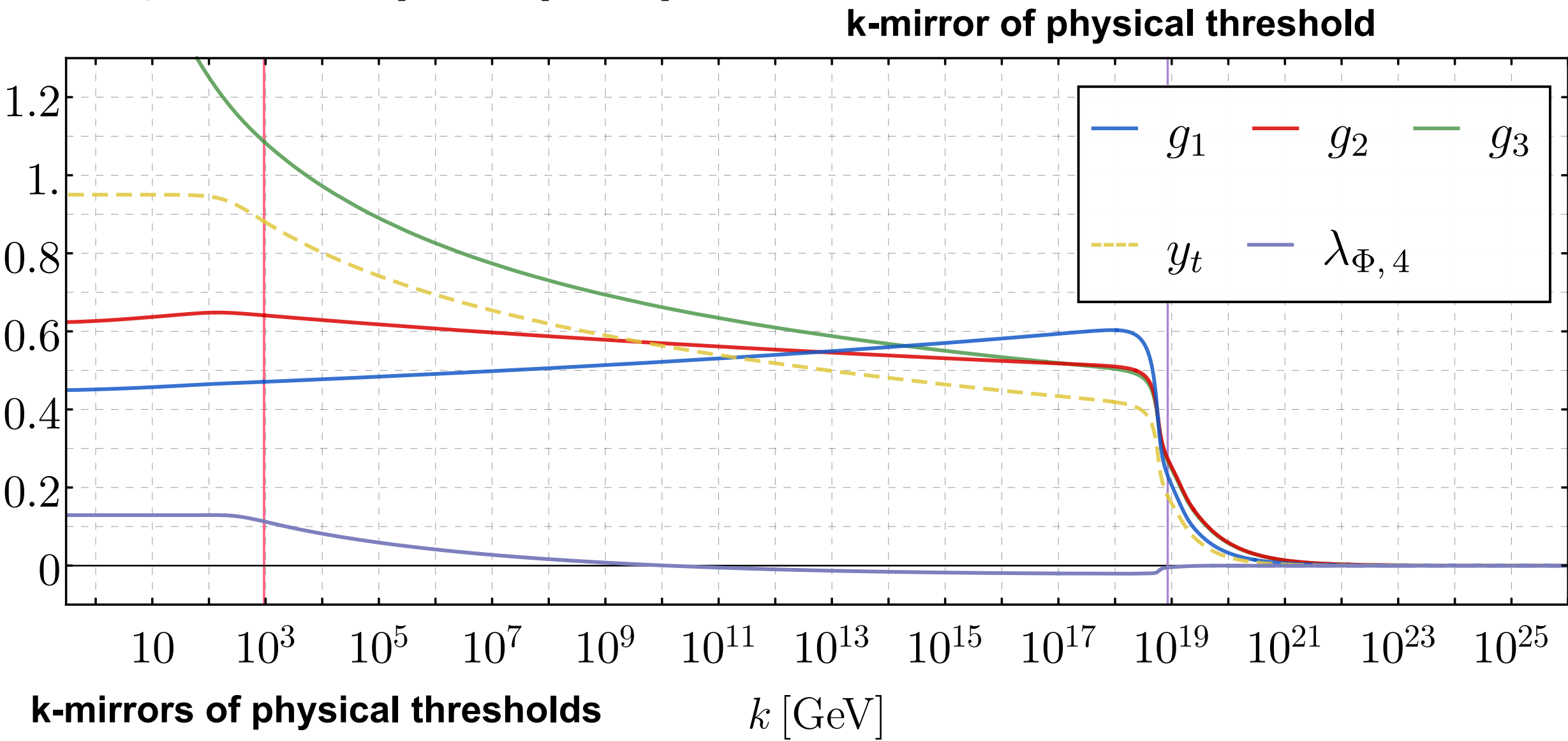
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k-mirrors of physical thresholds



k-mirrors of physical thresholds: feature, not bug!

Full physics: momentum-dependent correlation functions & S-matrix elements at k=0

Scattering amplitudes, spectral properties & unitarity

Donoghue, **Knorr**, Litim, **Platania**, JMP, Percacci, **Reichert**, Saueressig, Wetterich, ...

Spectral properties

$$\rho \simeq i \langle [\phi(x), \phi(y)] \rangle$$

Spectral function

$$\rho(\omega, \vec{p}) \simeq 2 \operatorname{Im} G_{\text{Eucl}}(-i(\omega + i\varepsilon), \vec{p})$$

$$F \simeq \langle \{\phi(x), \phi(y)\} \rangle$$

Statistical function

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Källen-Lehmann spectral representation

$$G(p) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \frac{\lambda \rho_k(\lambda, \vec{p})}{\lambda^2 + p_0^2}$$

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Normalisation for physical fields

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} d\lambda \lambda \rho_{\phi}(\lambda) = 1$$



Canonical computation relation

$$[\phi(t, \vec{x}), \dot{\phi}(t, \vec{y})] = \delta(\vec{x} - \vec{y})$$

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with spectral representation

Bjorken-Johnson-Low limit

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‘The wave function Z(p) of physical fields has a trivial UV-limit’

$$G_{\phi}(p) = \frac{1}{Z_{\phi}(p)} \frac{1}{p^2} \xrightarrow{p^2 \rightarrow \infty} \frac{1}{p^2}$$

Beware
inherently gauge-dependent

Properties of the spectral function

$$G_{\phi}^{(\text{IR})}(p) = \frac{Z_{\phi}^{(\text{IR})}}{p^{2(1-\frac{\eta_{\phi}}{2})}}$$

Analytic consequences & constraints

UV-asymptotics

$$\text{sign}[\rho_{\phi}(\lambda \rightarrow 0)] = -\text{sign}[\partial_p^2 G_{\phi}(p^2 \rightarrow 0)] \qquad \lambda \geq 0$$

Cyrol, JMP, Rothkopf, Wink SciPost Phys. 5 (2018) 6, 065
Bonanno, Denz, JMP, Reichert, SciPost Phys. 12 (2022) 1, 001
Horak, JMP, Wink, 2202.09333, 2210.07597 (SciPost Phys.)

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Oehme-Zimmermann superconvergence

Quarks, gluons, **background graviton**

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No field in the SM!
Example: scalar field in d<4

Cyrol, JMP, Rothkopf, Wink SciPost Phys. 5 (2018) 6, 065
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No field in the SM!
Example: scalar field in d<4

(3)

$\gamma_{\phi} < 0$

$\eta_{\phi} > 0$



$G_{\phi}^{(\text{UV})}(p) = \frac{Z_{\phi}^{(\text{UV})}}{p^{2(1-\frac{\eta_{\phi}}{2})}}$

$$\lim_{\Lambda \rightarrow \infty} \left| \frac{1}{2\pi} \int_{-\Lambda}^{\Lambda} d\lambda \, \lambda \rho_{\phi}(\lambda) \right| \rightarrow \infty$$

fluctuating graviton

Cyrol, JMP, Rothkopf, Wink SciPost Phys. 5 (2018) 6, 065
 Bonanno, Denz, JMP, Reichert, SciPost Phys. 12 (2022) 1, 001
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$$\lambda \geq 0$$

(1)

$$\eta_{\phi} > 2$$



$$\rho_{\phi}^{(\text{IR})}(\lambda) = -\frac{Z_{\phi}^{(\text{IR})}}{\lambda^{2(1-\frac{\eta_{\phi}}{2})}}$$

Scaling gluon prop

Beware
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Scaling gluon prop

(2) $0 < \eta_{\phi} < 2$ 

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Scaling ghost dressing
Scalar field in 3d at Wilson-Fisher FP

Cyrol, JMP, Rothkopf, Wink SciPost Phys. 5 (2018) 6, 065
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Properties of the spectral function

$$G_{\phi}^{(\text{IR})}(p) = \frac{Z_{\phi}^{(\text{IR})}}{p^{2(1-\frac{\eta_{\phi}}{2})}}$$

Analytic consequences & constraints

IR-asymptotics

$$\text{sign}[\rho_{\phi}(\lambda \rightarrow 0)] = -\text{sign}[\partial_p^2 G_{\phi}(p^2 \rightarrow 0)]$$

$$\lambda \geq 0$$

(1) $\eta_{\phi} > 2$ 

$$\rho_{\phi}^{(\text{IR})}(\lambda) = -\frac{Z_{\phi}^{(\text{IR})}}{\lambda^{2(1-\frac{\eta_{\phi}}{2})}}$$

Scaling gluon prop

(2) $0 < \eta_{\phi} < 2$ 

$$\rho_{\phi}^{(\text{IR})}(\lambda) = \frac{Z_{\phi}^{(\text{IR})}}{\lambda^{2(1-\frac{\eta_{\phi}}{2})}}$$

Scaling ghost dressing
Scalar field in 3d at Wilson-Fisher FP

(3) $\eta_{\phi} = 0, 2$ 

$$\rho_{\phi}^{(\text{IR})}[c_{\log} \log p^2, m_{\text{gap}}^2]$$

‘Physical’ fields,
decoupling gluon prop & ghost dressing
Fluctuating & background gravitons

Cyrol, JMP, Rothkopf, Wink SciPost Phys. 5 (2018) 6, 065
Bonanno, Denz, JMP, Reichert, SciPost Phys. 12 (2022) 1, 001
Horak, JMP, Wink, 2202.09333, 2210.07597 (SciPost Phys.)

Unitarity & the graviton spectral function

Reconstruction

Flows for two- and three point functions

$$\partial_t \left(\text{fluctuation propagator} \right)^{-1} = -\frac{1}{2} \left(\text{fluctuation propagator with regulator insertion} \right) + \left(\text{fluctuation propagator with regulator insertion} \right) - 2 \left(\text{ghost propagator with regulator insertion} \right)$$

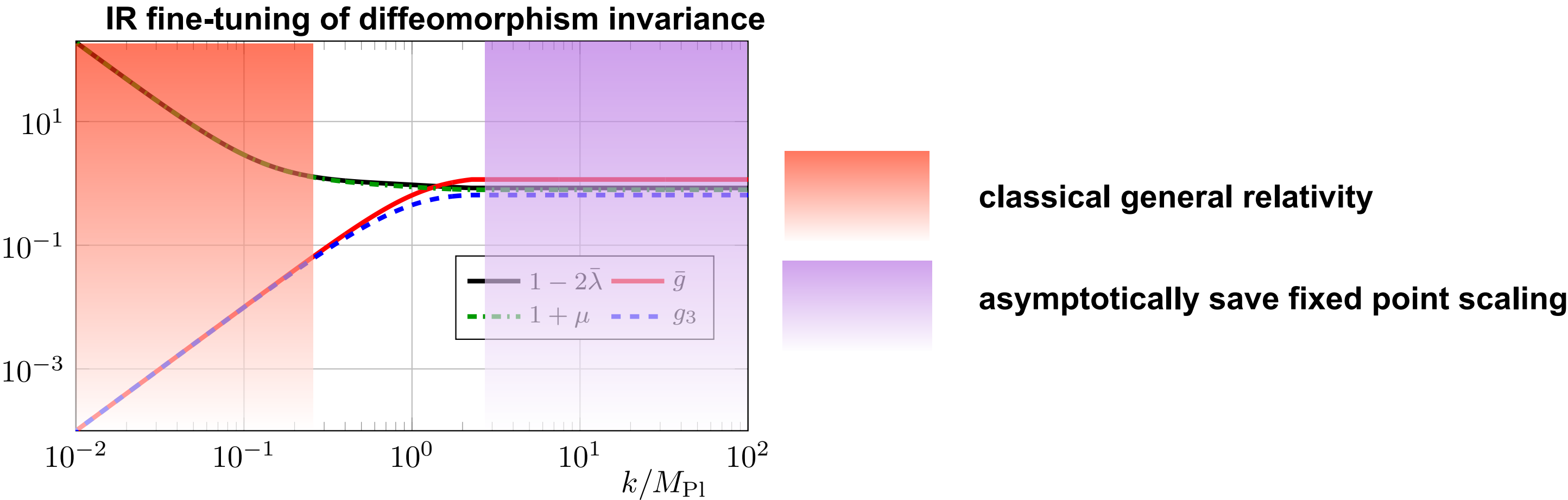
$$\partial_t \left(\text{background propagator} \right)^{-1} = -\frac{1}{2} \left(\text{fluctuation propagator with regulator insertion} \right) + \left(\text{background propagator with regulator insertion} \right) + \left(\text{ghost propagator with regulator insertion} \right) - 2 \left(\text{background propagator with regulator insertion} \right)$$

$$\partial_t \left(\text{three-point vertex} \right) = -\frac{1}{2} \left(\text{fluctuation propagator with regulator insertion} \right) + 3 \left(\text{fluctuation propagator with regulator insertion} \right) - 3 \left(\text{ghost propagator with regulator insertion} \right) + 6 \left(\text{ghost propagator with regulator insertion} \right)$$

• vertex ⊗ regulator insertion

==== fluctuation propagator == background propagator

—→ ghost propagator

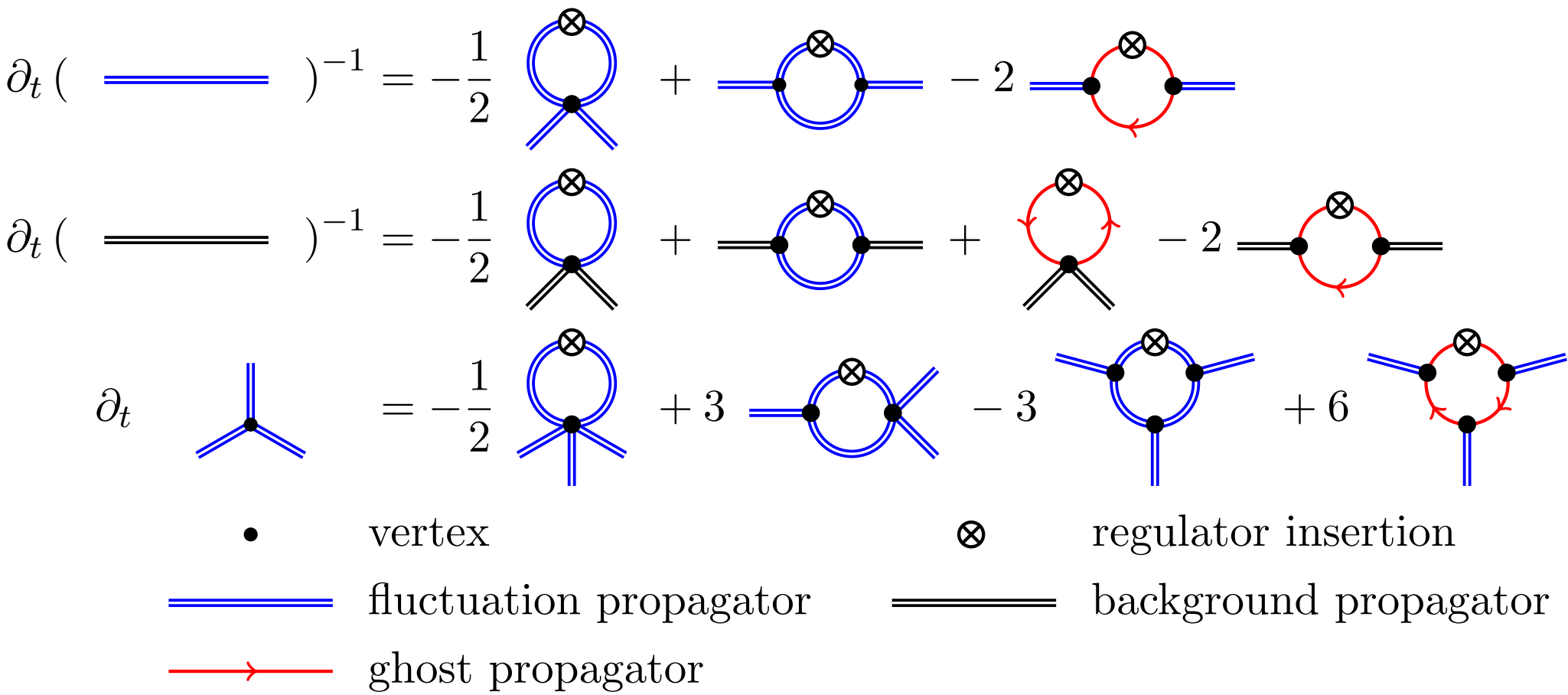


Denz, JMP, Reichert, EPJ C78 (2018) 4, 336

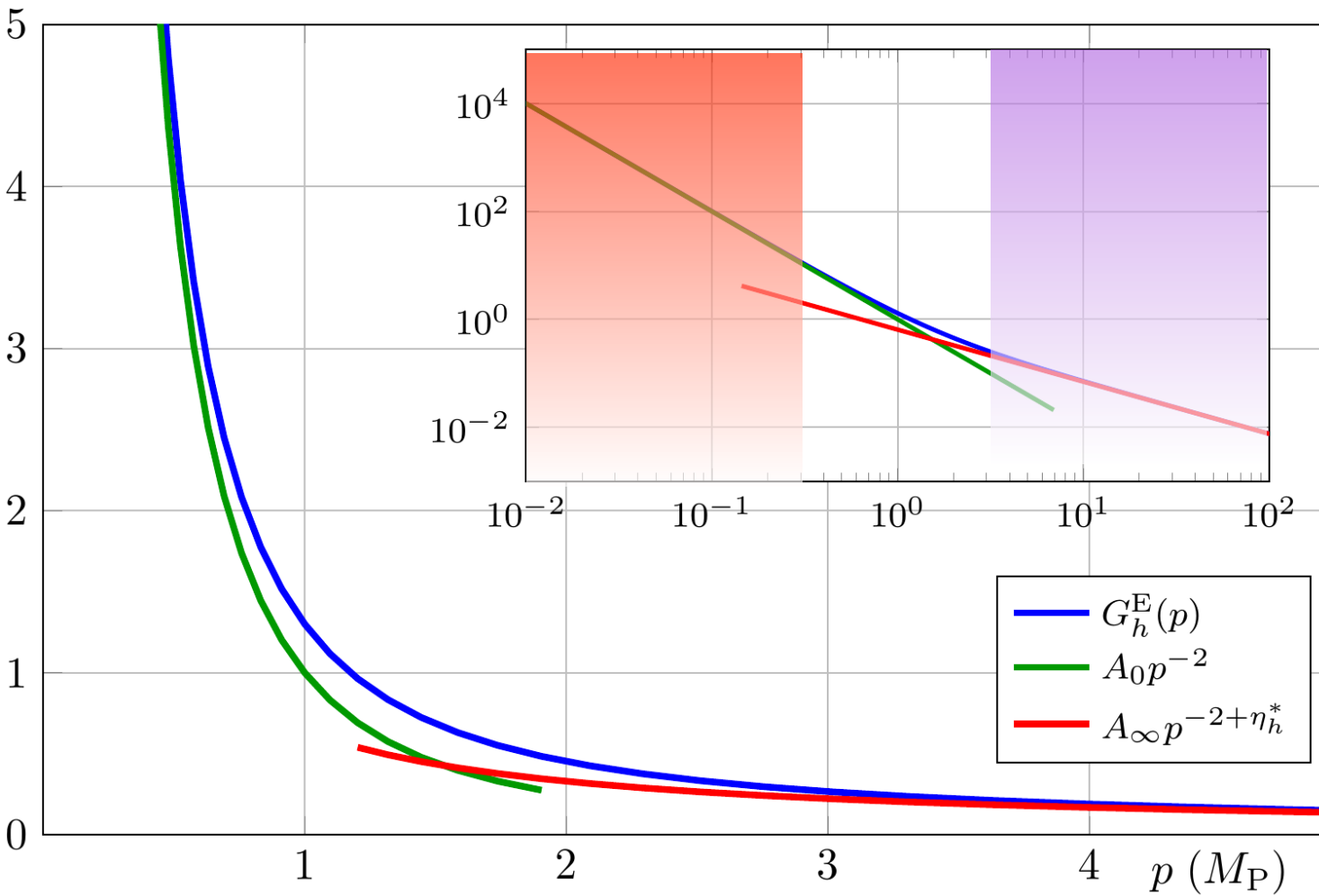
Unitarity & the graviton spectral function

Reconstruction

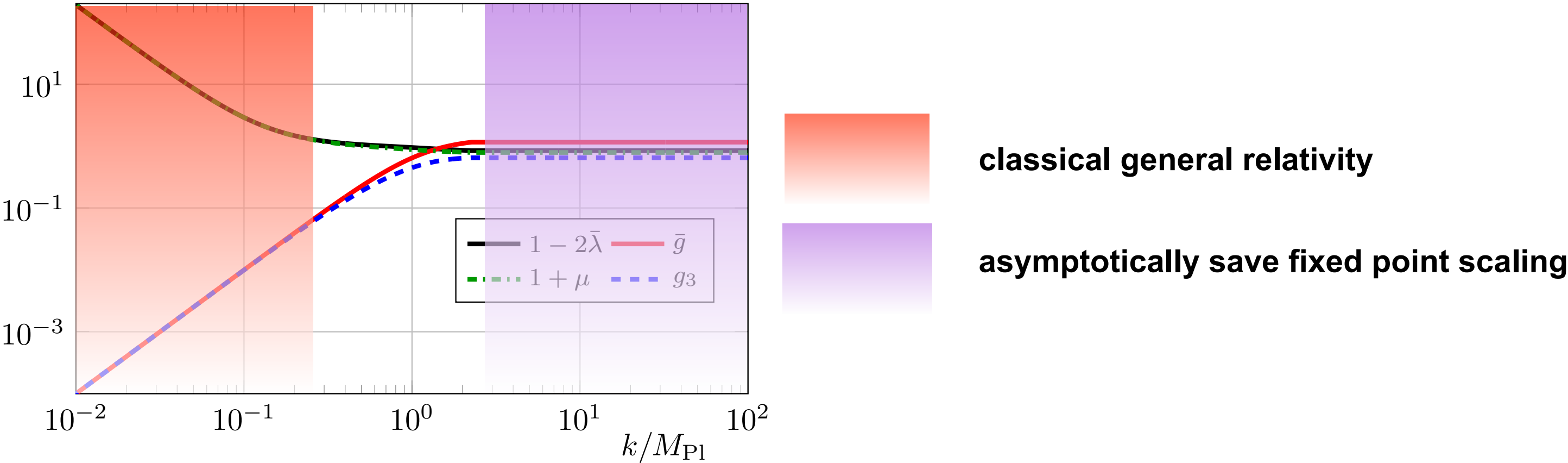
Flows for two- and three point functions



Fluctuation propagator at vanishing cutoff scale



IR fine-tuning of diffeomorphism invariance



General basis

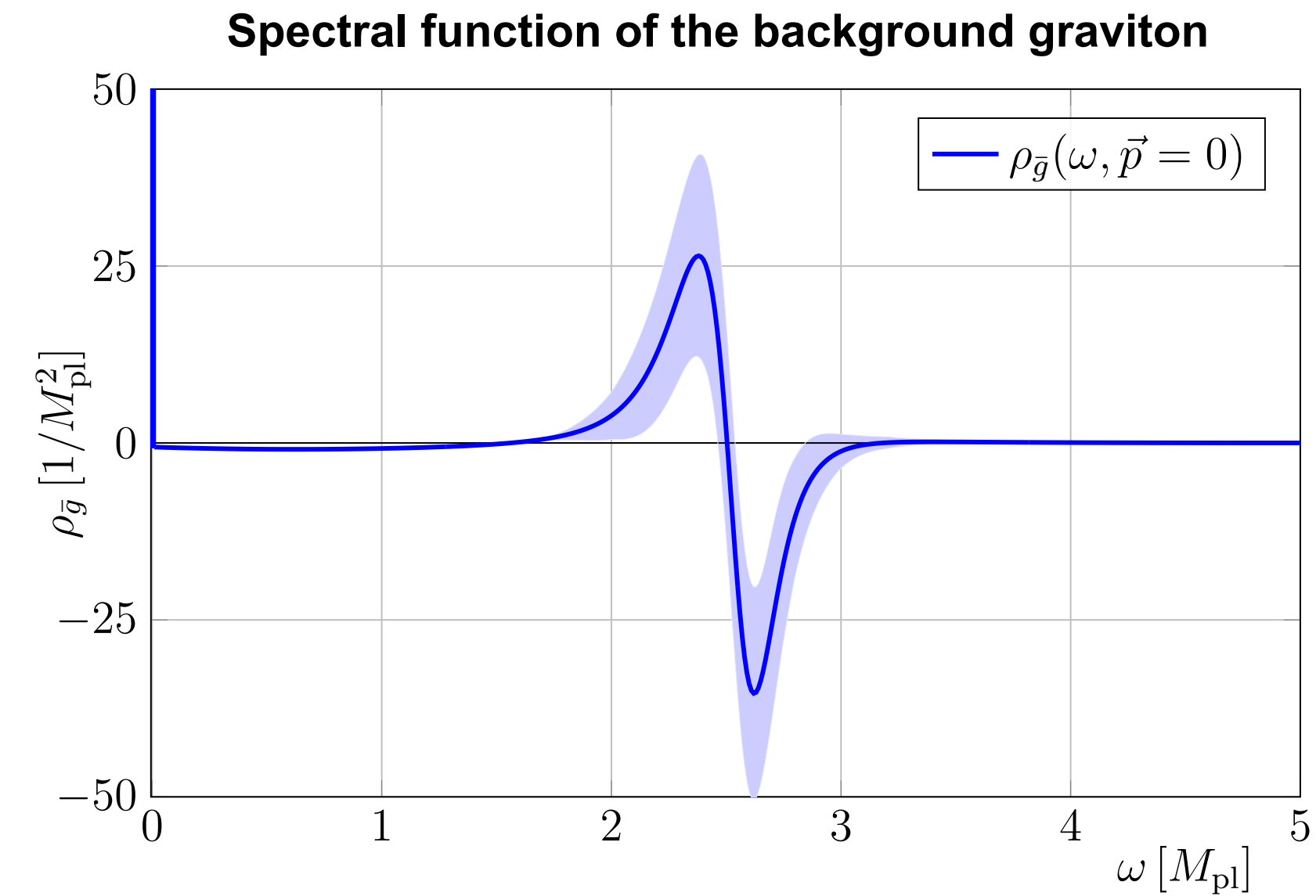
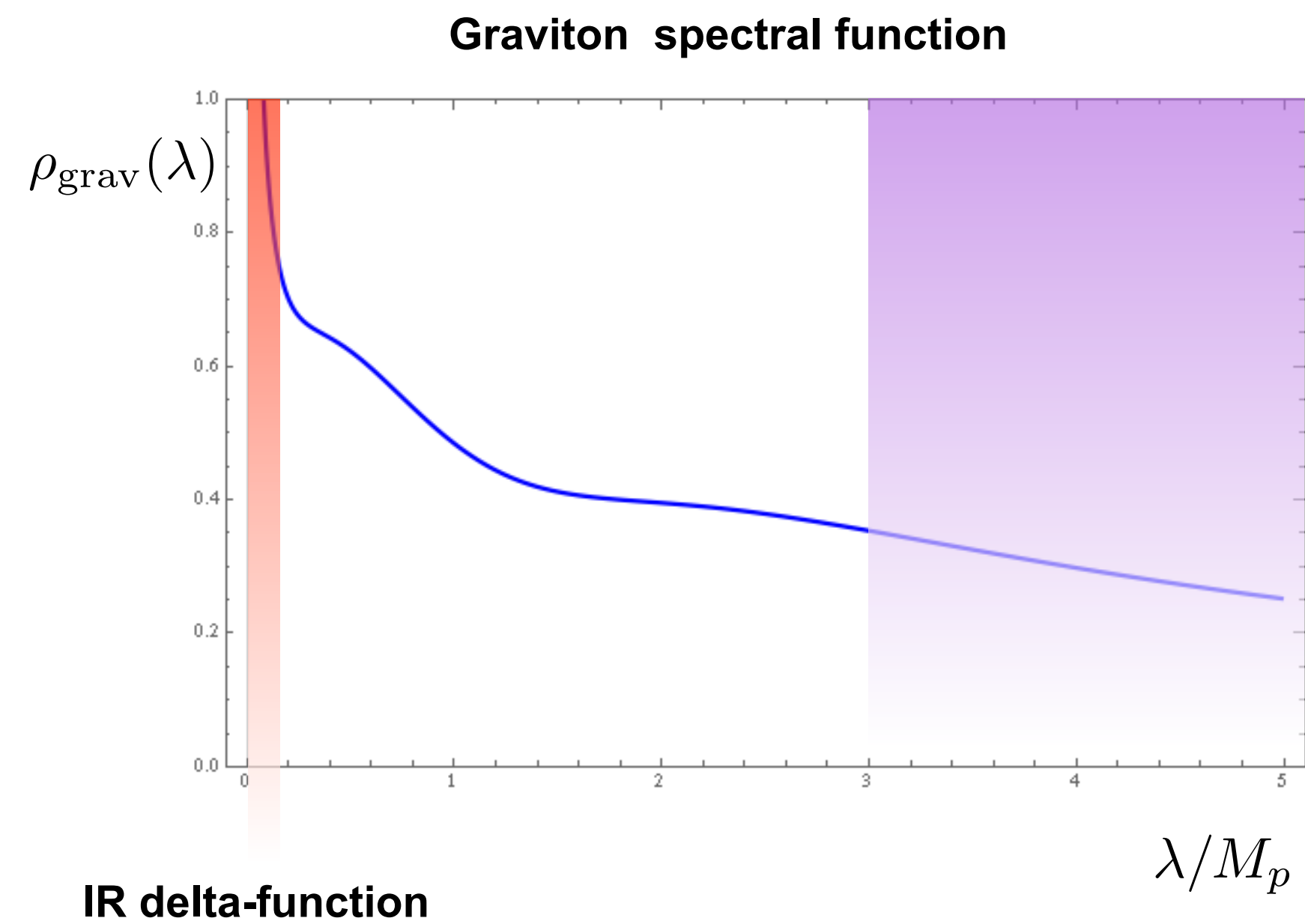
$$\hat{G}^{BW}(p_0) = \sum_{k=1}^{N_{ps}} \prod_{j=1}^{N_{pp}^{(k)}} \left(\frac{\hat{\mathcal{N}}_k}{(\hat{p}_0 + \hat{\Gamma}_{k,j})^2 + \hat{M}_{k,j}^2} \right)^{\delta_{k,j}}$$

$$G^{BW}(p_0) = \mathcal{K} (p_0^2)^{-1-2\alpha} \left(\sum_{j=1}^{N_{poly}} \hat{a}_j (\hat{p}_0^2)^{\frac{j}{2}} \right) \hat{G}^{BW}(p_0)$$

Denz, JMP, Reichert, EPJ C78 (2018) 4, 336

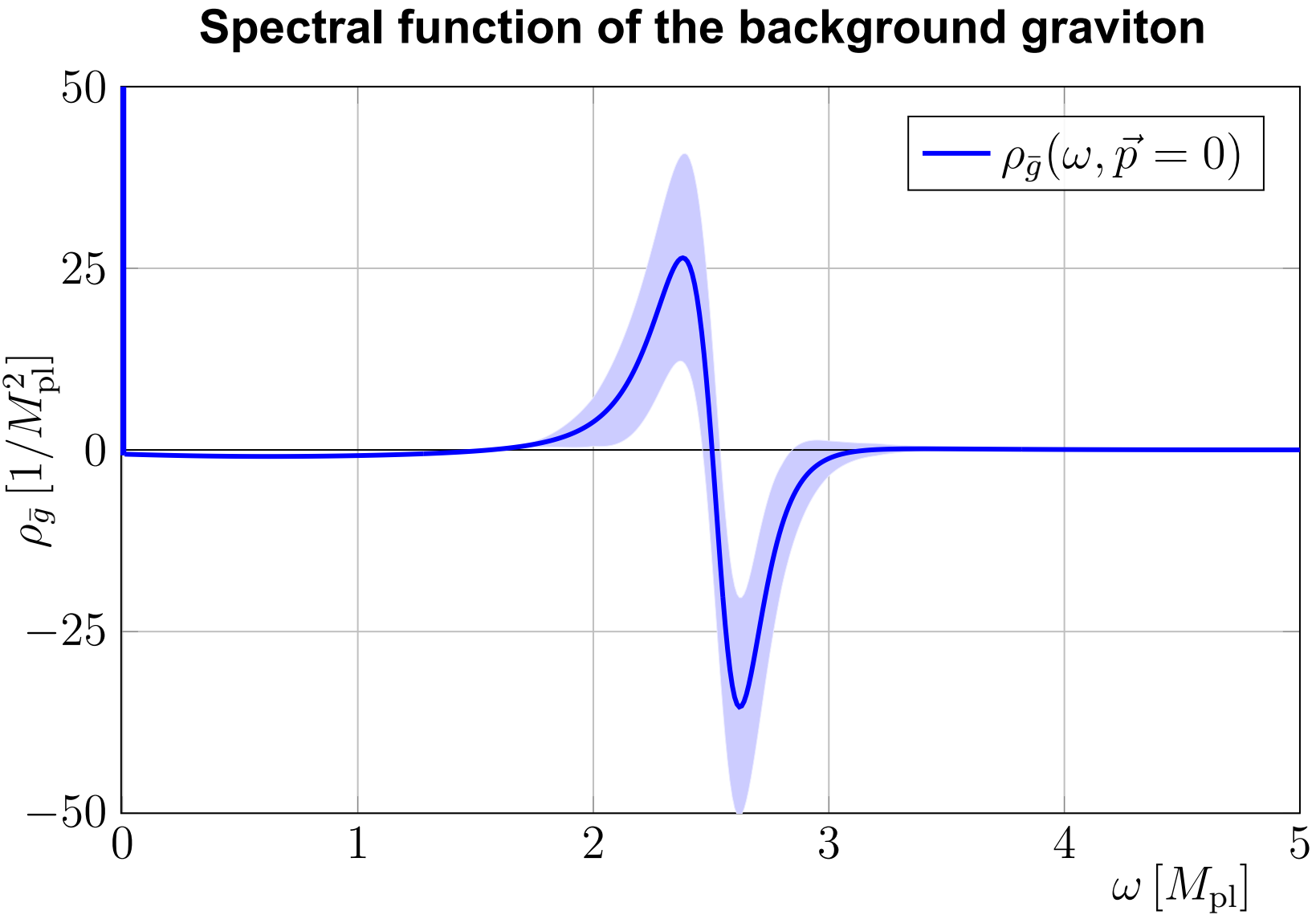
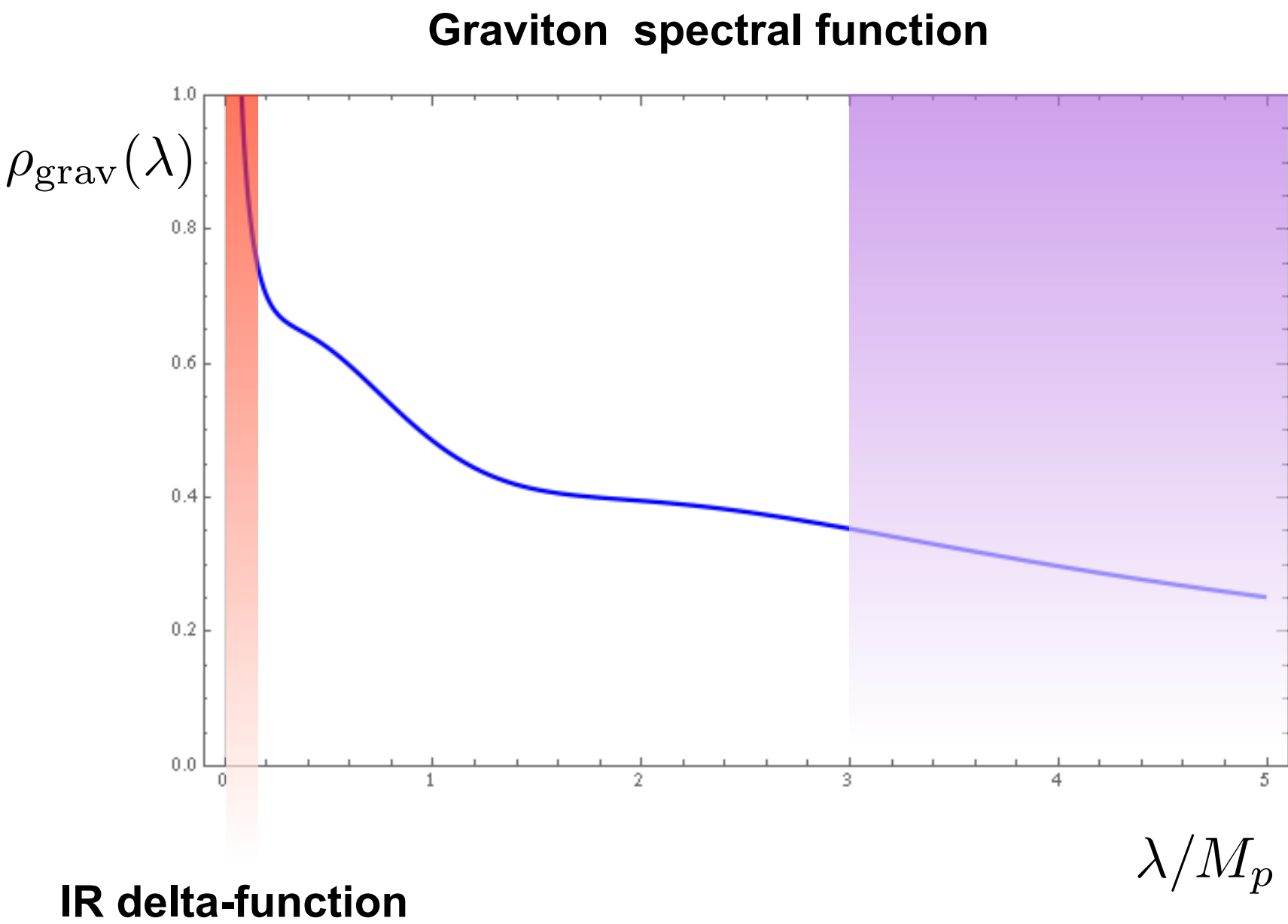
Unitarity & the graviton spectral function

Reconstruction



Unitarity & the graviton spectral function

Reconstruction



Spectral properties ‘resemble’ that of an asymptotic state

$$\rho_h(\lambda) \in \mathbb{R}^+ \qquad \int_{\mathbb{R}} \frac{d\lambda}{2\pi} \lambda \rho_h(\lambda) = \infty$$

Spectral properties of an unphysical mode

$$\rho_{\bar{g}}(\lambda) \in \mathbb{R} \qquad \int_{\mathbb{R}} \frac{d\lambda}{2\pi} \lambda \rho_{\bar{g}}(\lambda) = 0$$

Spectral flows

Fehre, Litim, JMP, Reichert, PRL 130 (2023) 8, 081501

Braun, Chen, Fu, Geißel, Horak, Huang, Ihssen, JMP, Reichert, Rennecke,

Tan, Töpfel, Wessely, Wink, SciPost Phys.Core 6 (2023) 061

Horak, Wessely, JMP, Wink, arXiv:2303.16719

Spectral flows

Fehre, Litim, JMP, Reichert, PRL 130 (2023) 8, 081501

fQCD, Reichert, SciPost Phys.Core 6 (2023) 061

Horak, Wessely, JMP, Wink, arXiv:2303.16719

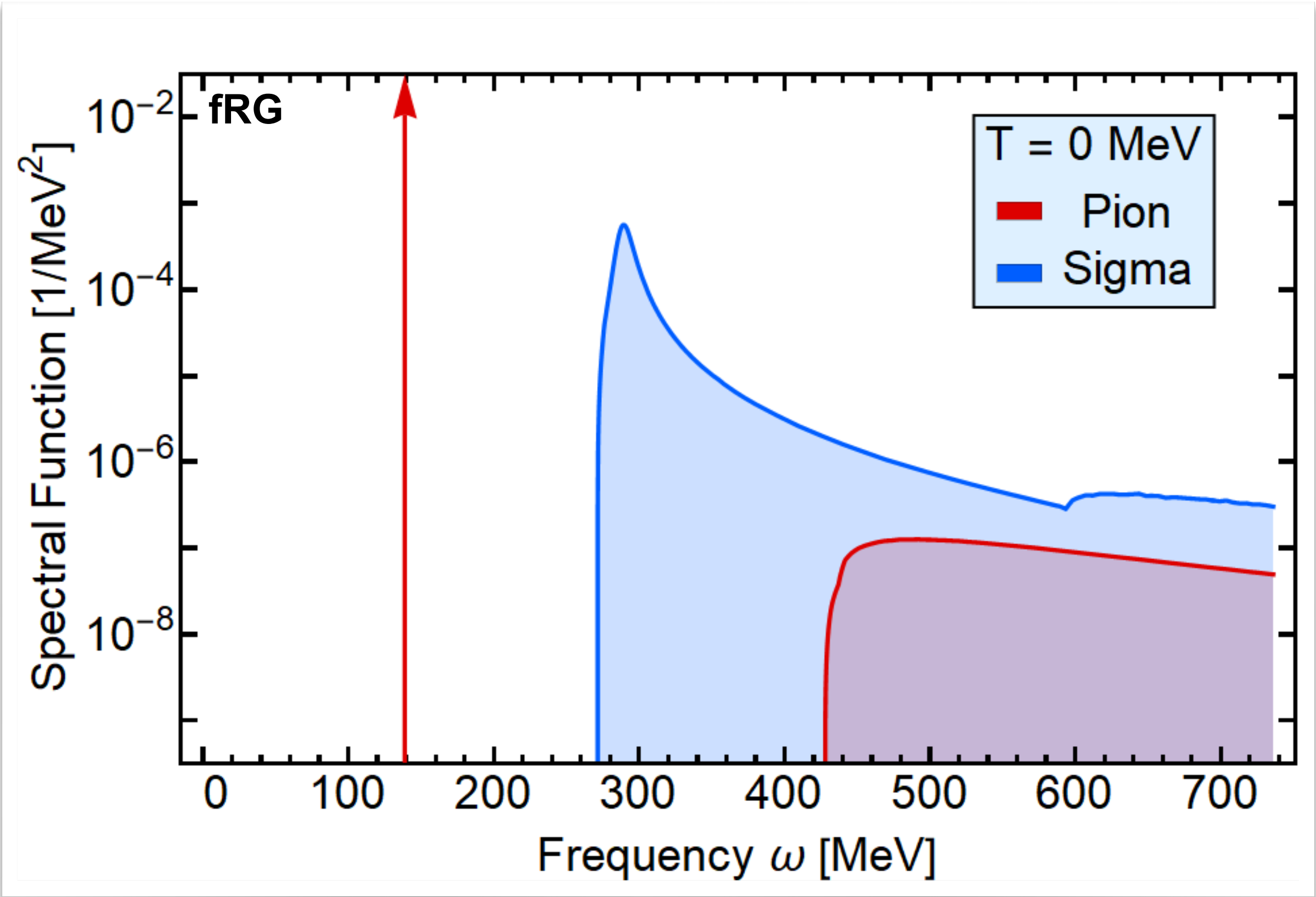
Realtime functional approaches at work in standard QFT



'... and now for something
Completely different...'

Finite T Sigma & Pion spectral functions

Linear sigma model/ scalar O(4)-model



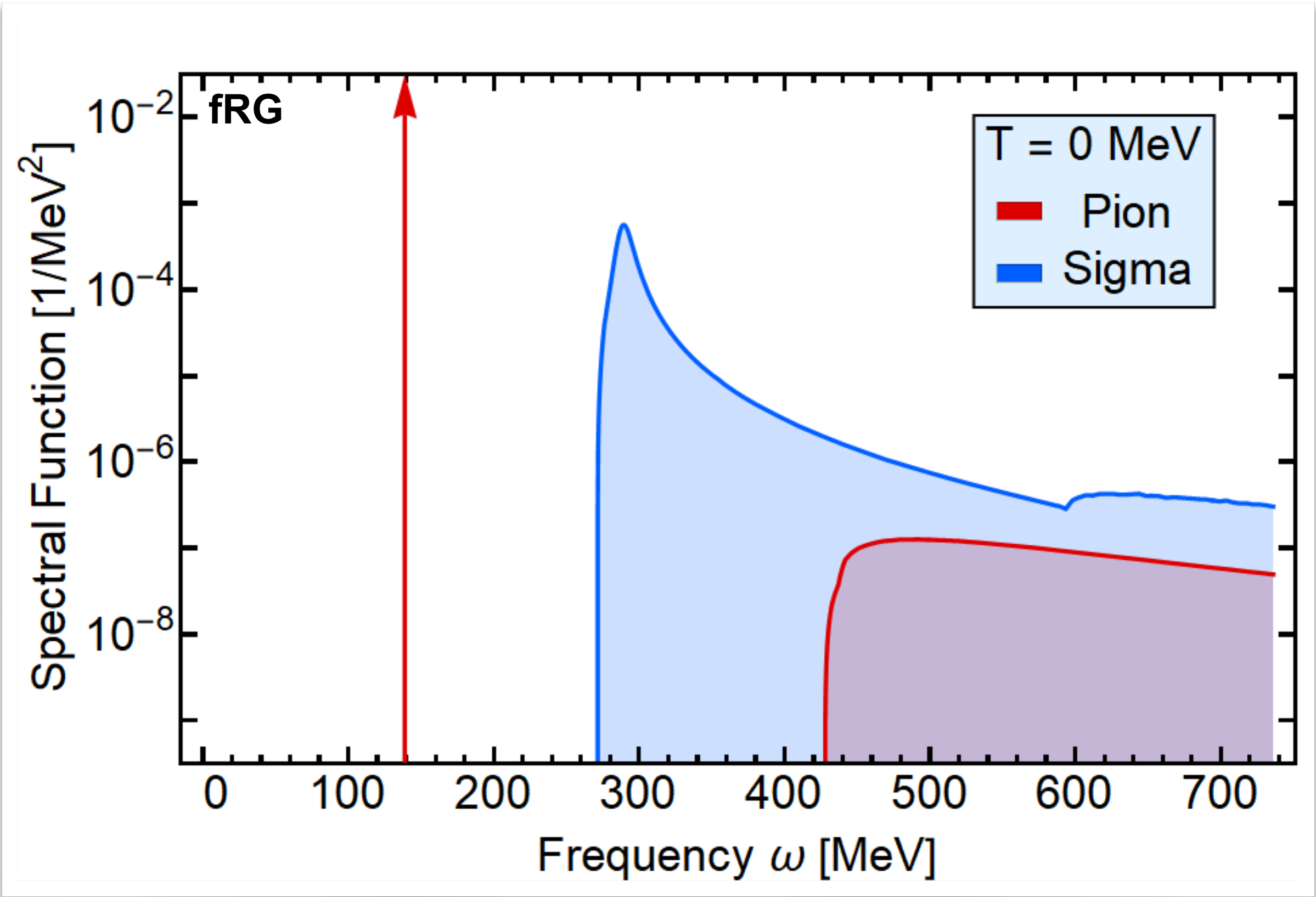
Realtime functional approaches at work in standard QFT



'... and now for something
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Finite T Sigma & Pion spectral functions

Linear sigma model/ scalar O(4)-model



Real time flows

Spectral representation

$$G_k(p) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \frac{\lambda \rho_k(\lambda, \vec{p})}{\lambda^2 + p_0^2}$$

$$\rho \simeq i \langle [\phi(x), \phi(y)] \rangle$$

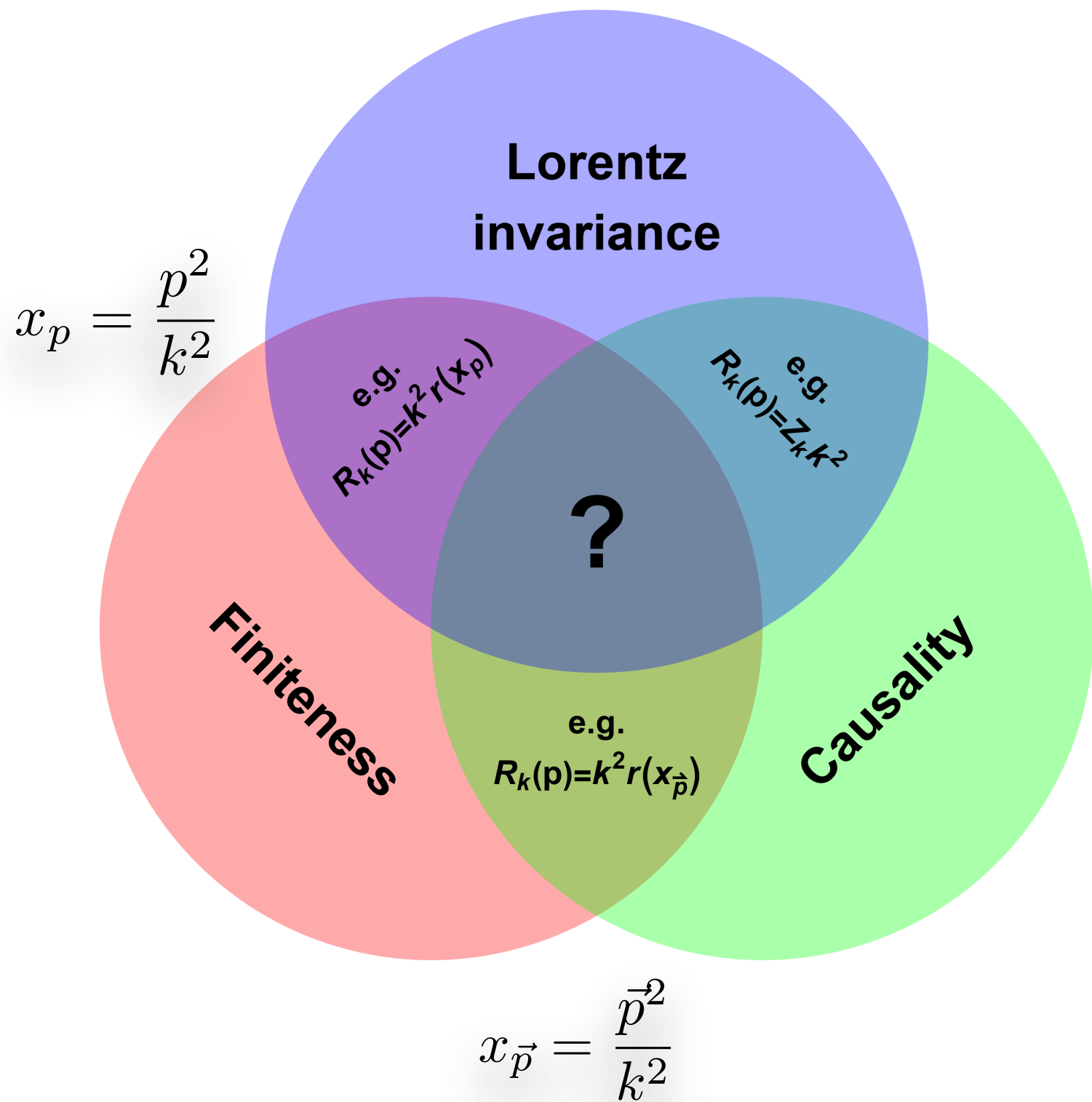
$$F \simeq \langle \{ \phi(x), \phi(y) \} \rangle$$

Real time flows

Spectral representation

$$G_k(p) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \frac{\lambda \rho_k(\lambda, \vec{p})}{\lambda^2 + p_0^2}$$

Symmetry preserving regulators

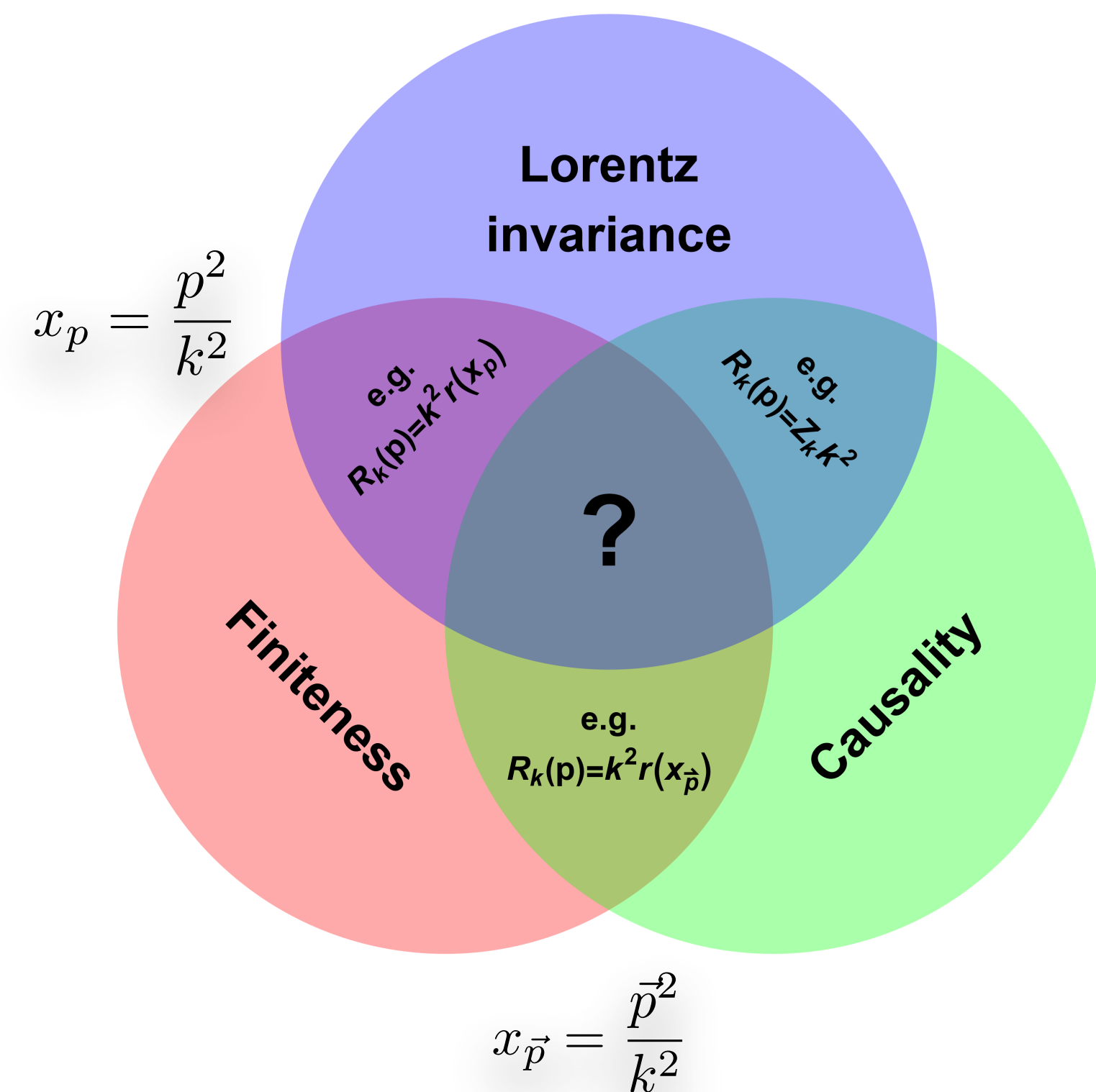


Real time flows

Spectral representation

$$G_k(p) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \frac{\lambda \rho_k(\lambda, \vec{p})}{\lambda^2 + p_0^2}$$

Symmetry preserving regulators



Lorentz invariance & Causality

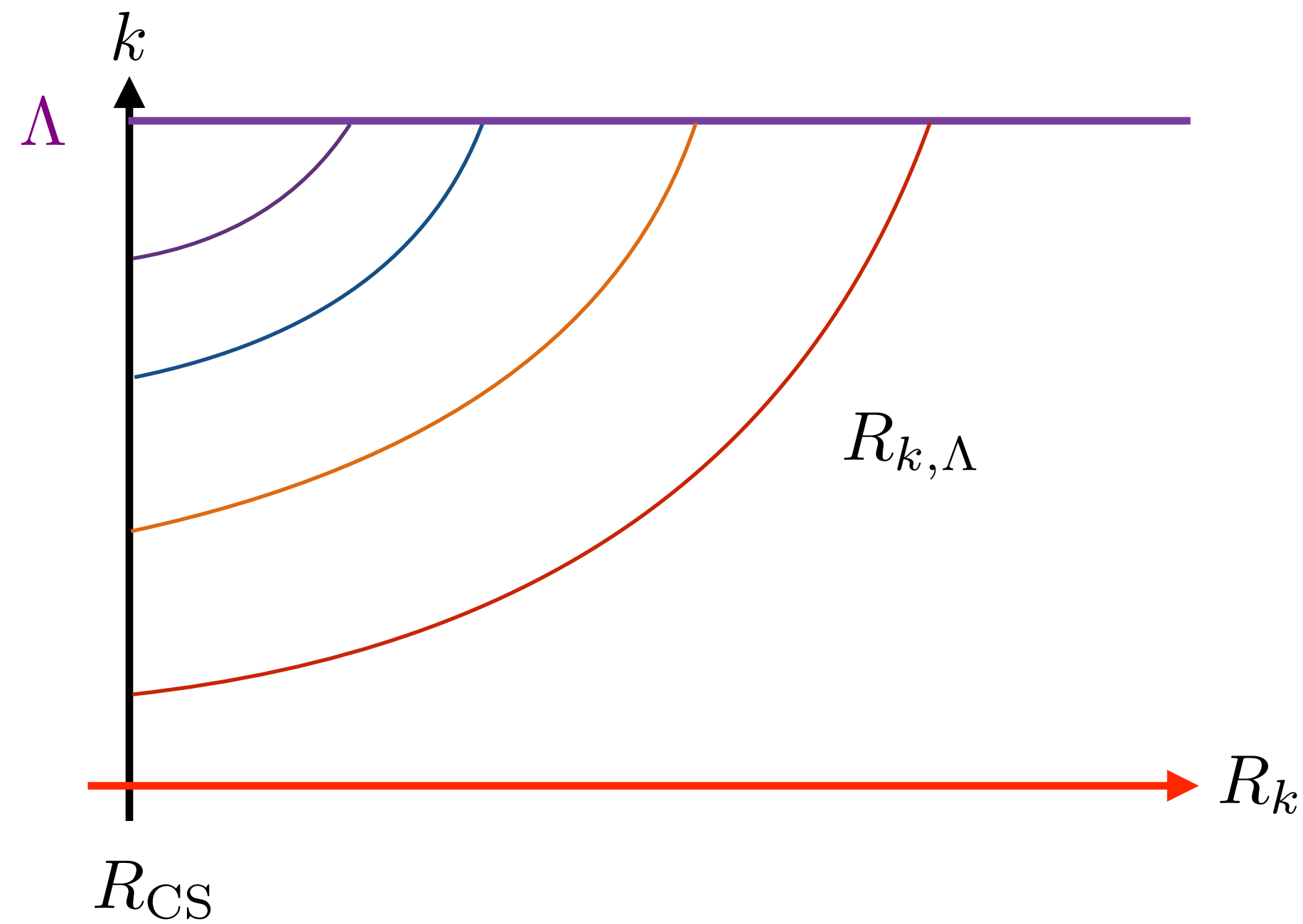
Callan-Symanzik regulator

$$R_{\text{CS}} = Z_{\phi} k^2$$

Finiteness lost

Renormalised renormalisation group flows

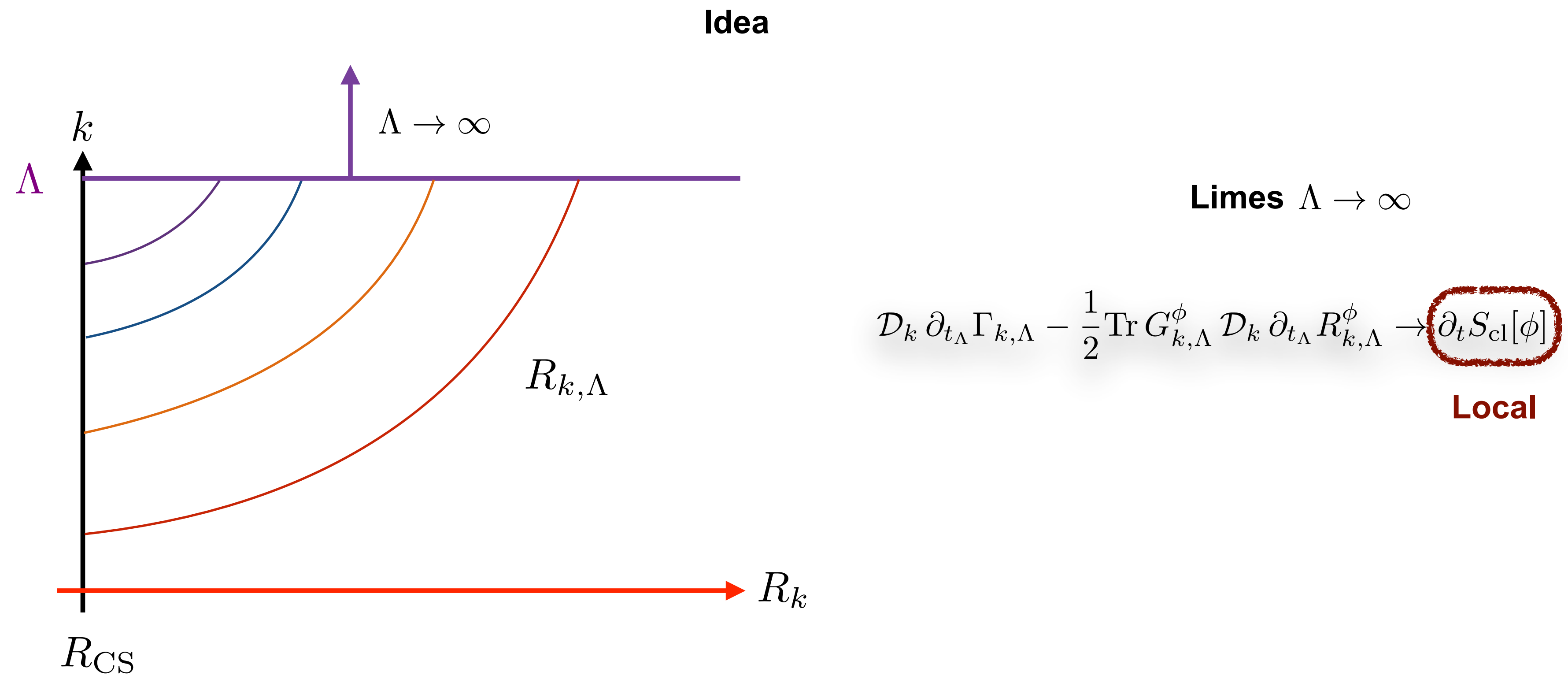
Idea



Composite UV-IR fRG flow

$$(\partial_t|_{\Lambda} + \mathcal{D}_k \partial_{t_{\Lambda}}) \Gamma_{k,\Lambda} = \frac{1}{2} \text{Tr} G_{k,\Lambda}^{\Phi} (\partial_t|_{\Lambda} R_{k,\Lambda}^{\Phi} + \mathcal{D}_k \partial_{t_{\Lambda}} R_{k,\Lambda}^{\Phi})$$

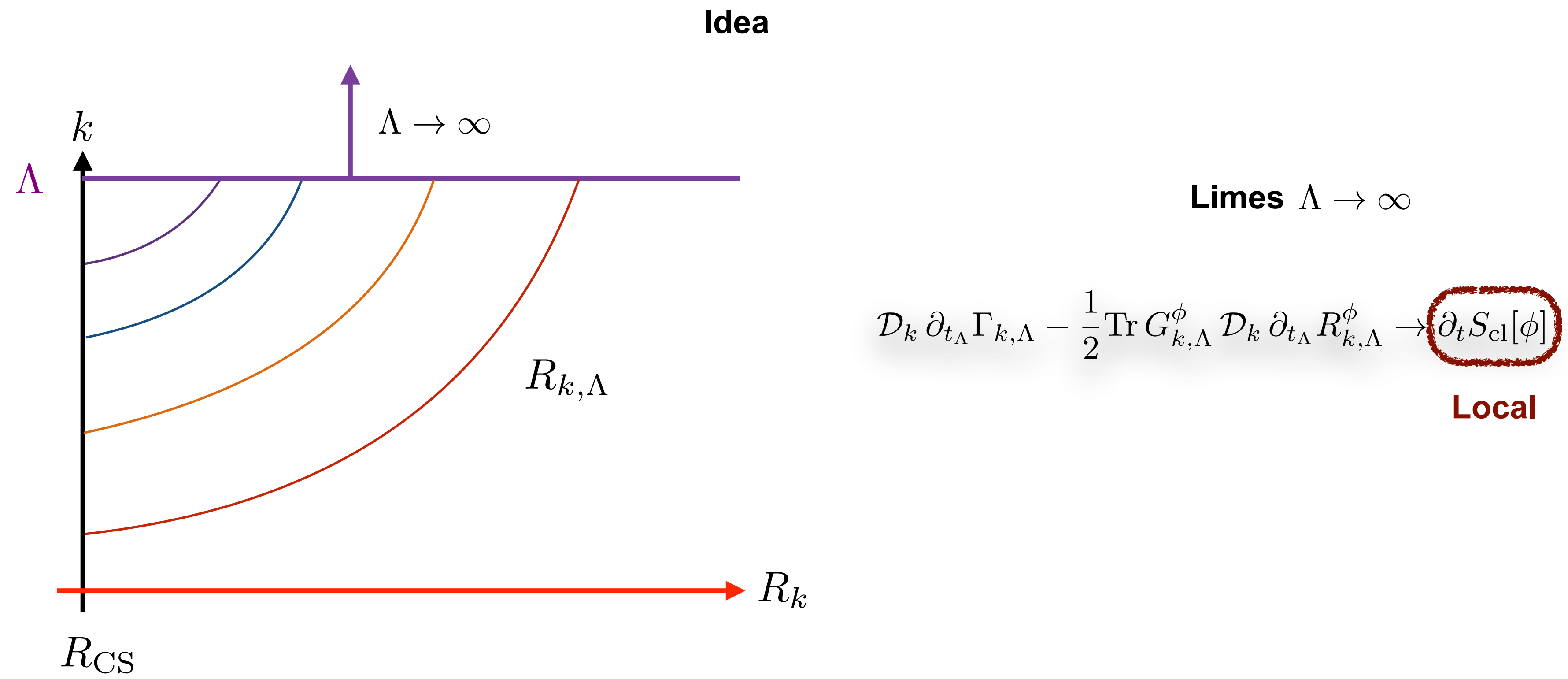
Renormalised renormalisation group flows



Composite UV-IR fRG flow

$$\left(\partial_t\big|_\Lambda + \mathcal{D}_k \partial_{t_\Lambda}\right) \Gamma_{k,\Lambda} = \frac{1}{2} \text{Tr} G_{k,\Lambda}^\Phi \left(\partial_t\big|_\Lambda R_{k,\Lambda}^\Phi + \mathcal{D}_k \partial_{t_\Lambda} R_{k,\Lambda}^\Phi\right)$$

Renormalised renormalisation group flows



fRG flow with flowing renormalisation

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{Tr} G_\phi[\phi] \partial_t R^\phi - \partial_t S_{\text{ct}}[\phi]$$

Renormalised renormalisation group flows

Spectral CS-fRG flow with flowing renormalisation

$$\partial_t \Gamma_k[\phi] = \text{Tr } G_\phi[\phi] k^2 - \partial_t S_{\text{ct}}[\phi]$$

$$G_k[\phi](p) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \frac{\lambda \rho_k[\phi](\lambda, \vec{p})}{\lambda^2 + p_0^2}$$

Renormalised renormalisation group flows

Spectral CS-fRG flow with flowing renormalisation

$$\partial_t \Gamma_k[\phi] = \text{Tr} G_\phi[\phi] k^2 - \partial_t S_{\text{ct}}[\phi] \qquad G_k[\phi](p) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \frac{\lambda \rho_k[\phi](\lambda, \vec{p})}{\lambda^2 + p_0^2}$$

Flowing on-shell renormalisation

$$\lim_{\Lambda \rightarrow \infty} \Gamma_{k,\Lambda}^{(2)}[\bar{\phi}](p) \Big|_{p_0^2 = -\mu^2(k)} = -k^2$$

Controlled UV-limit

Renormalised renormalisation group flows

Spectral CS-fRG flow with flowing renormalisation

$$\partial_t \Gamma_k[\phi] = \text{Tr} G_\phi[\phi] k^2 - \partial_t S_{\text{ct}}[\phi] \qquad G_k[\phi](p) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \frac{\lambda \rho_k[\phi](\lambda, \vec{p})}{\lambda^2 + p_0^2}$$

Flowing on-shell renormalisation

$$\lim_{\Lambda \rightarrow \infty} \Gamma_{k,\Lambda}^{(2)}[\bar{\phi}](p) \Big|_{p_0^2 = -\mu^2(k)} = -k^2 \qquad \longleftarrow \qquad \lim_{\Lambda \rightarrow \infty} \partial_t \left[\Gamma_{k,\Lambda}^{(2)}[\bar{\phi}](p) \Big|_{p_0^2 = -\mu^2} \right] = -2 k^2$$

Controlled UV-limit

Controlled UV-limit

Renormalised renormalisation group flows

Spectral CS-fRG flow with flowing renormalisation

$$\partial_t \Gamma_k[\phi] = \text{Tr} \, G_\phi[\phi] \, k^2 - \partial_t S_{\text{ct}}[\phi] \qquad G_k[\phi](p) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \frac{\lambda \, \rho_k[\phi](\lambda, \vec{p})}{\lambda^2 + p_0^2}$$

Flowing on-shell renormalisation

$$\lim_{\Lambda \rightarrow \infty} \Gamma_{k,\Lambda}^{(2)}[\bar{\phi}](p) \Big|_{p_0^2 = -\mu^2(k)} = -k^2 \qquad \longleftarrow \qquad \lim_{\Lambda \rightarrow \infty} \partial_t \left[\Gamma_{k,\Lambda}^{(2)}[\bar{\phi}](p) \Big|_{p_0^2 = -\mu^2} \right] = -2 \, k^2$$

Controlled UV-limit

Controlled UV-limit

Similarly for wave function and coupling renormalisation

Spectral functional approaches at work

DSE

$$\text{fermion line with blue dot} = \text{bare fermion line} + \frac{1}{2} \text{gluon loop} - \frac{1}{2} \text{fermion loop} - \frac{1}{6} \text{two gluon loops} + \frac{1}{2} \text{gluon loop and fermion loop}$$

fRG

$$\partial_t \Gamma_k^{(2)}(p) = \text{fermion loop with gluon loop} - \frac{1}{2} \text{fermion loop with gluon loop} - \partial_t S_{ct,k}^{(2)}$$

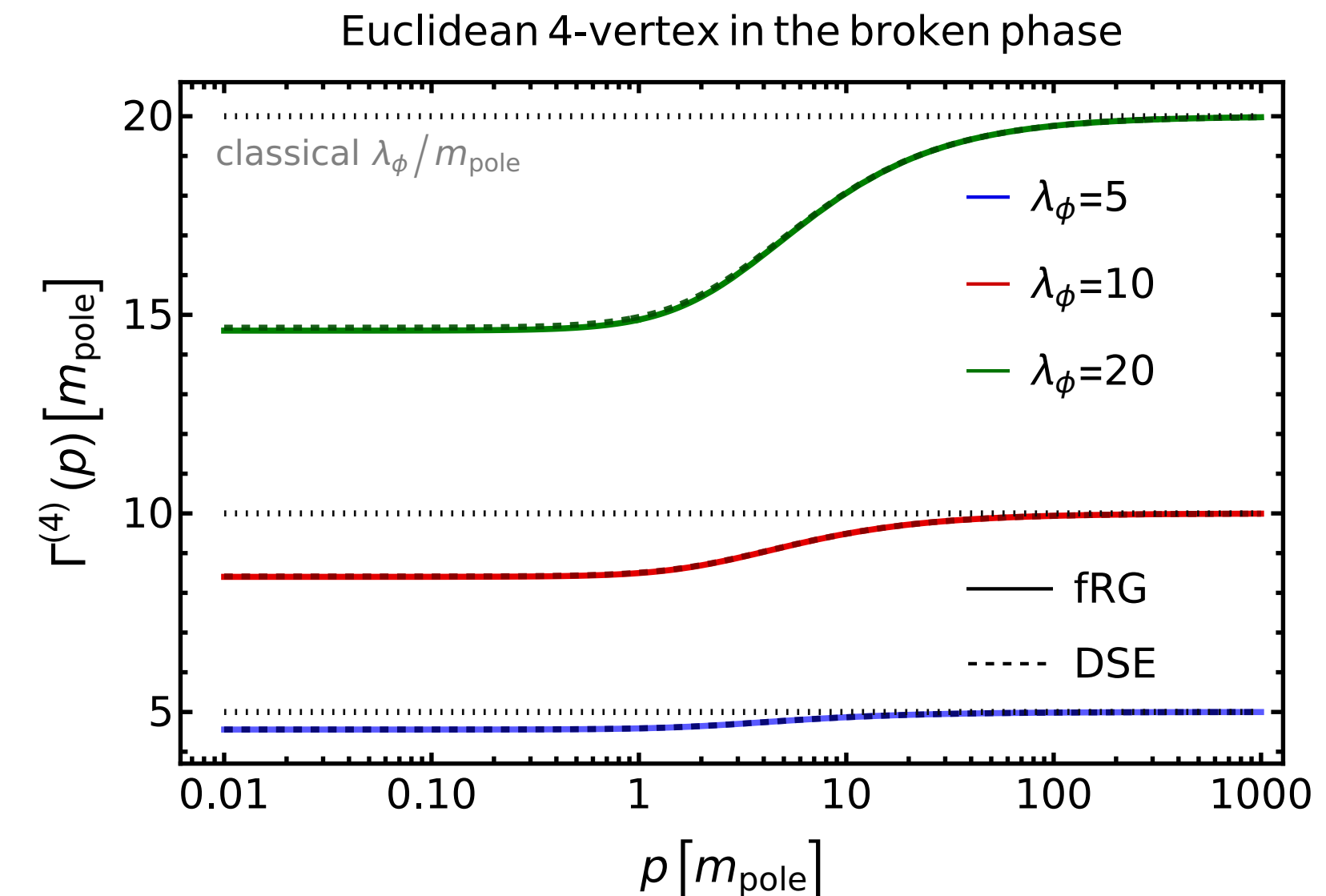
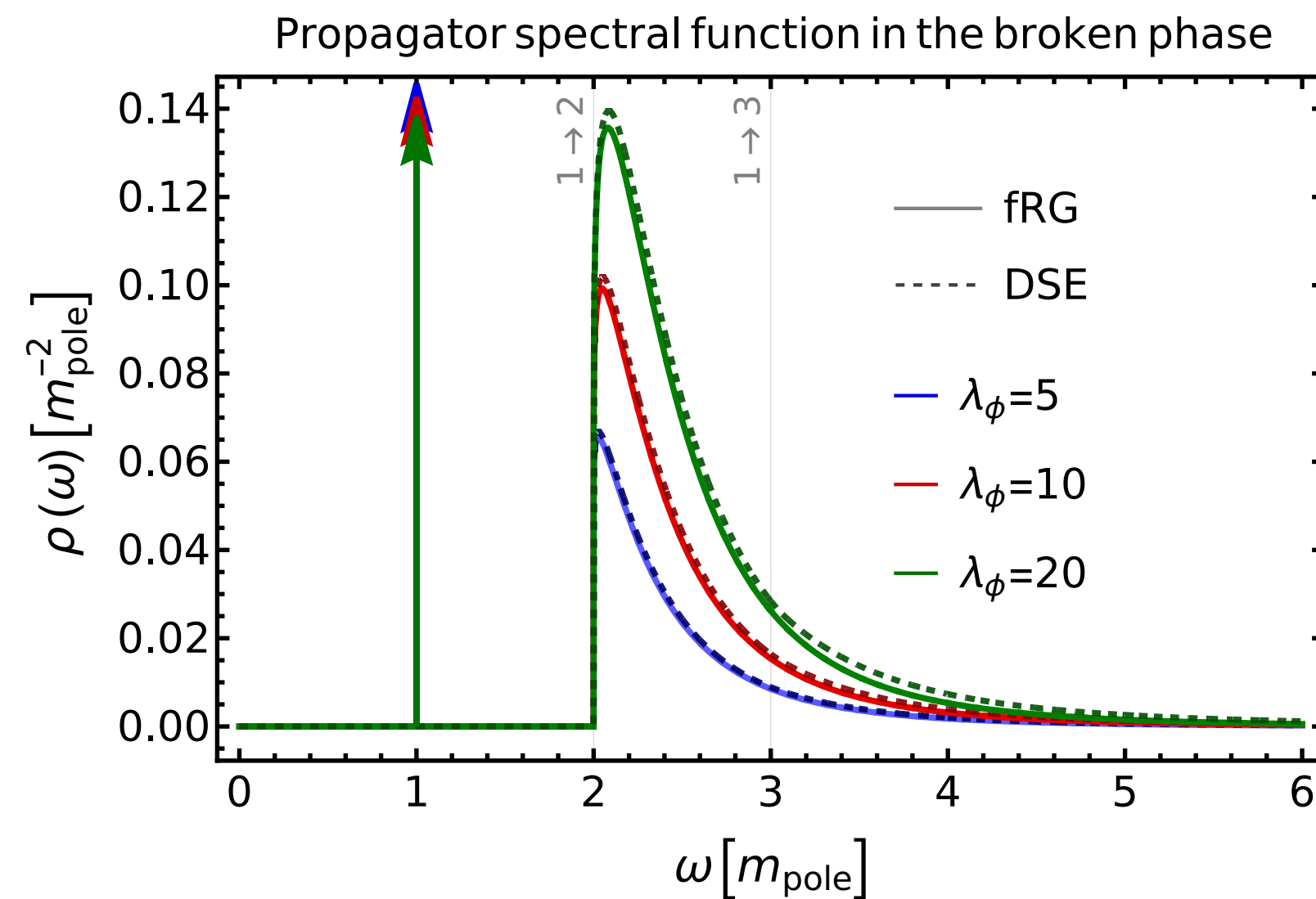
Spectral functional approaches at work

DSE

$$\text{fermion line with blue dot} = \text{bare fermion line} + \frac{1}{2} \text{self-energy loop} - \frac{1}{2} \text{bubble diagram} - \frac{1}{6} \text{triangle diagram} + \frac{1}{2} \text{complex diagram}$$

fRG

$$\partial_t \Gamma_k^{(2)}(p) = \text{loop diagram with cross and } q - \frac{1}{2} \text{self-energy loop with cross and } q - \partial_t S_{\text{ct},k}^{(2)}$$



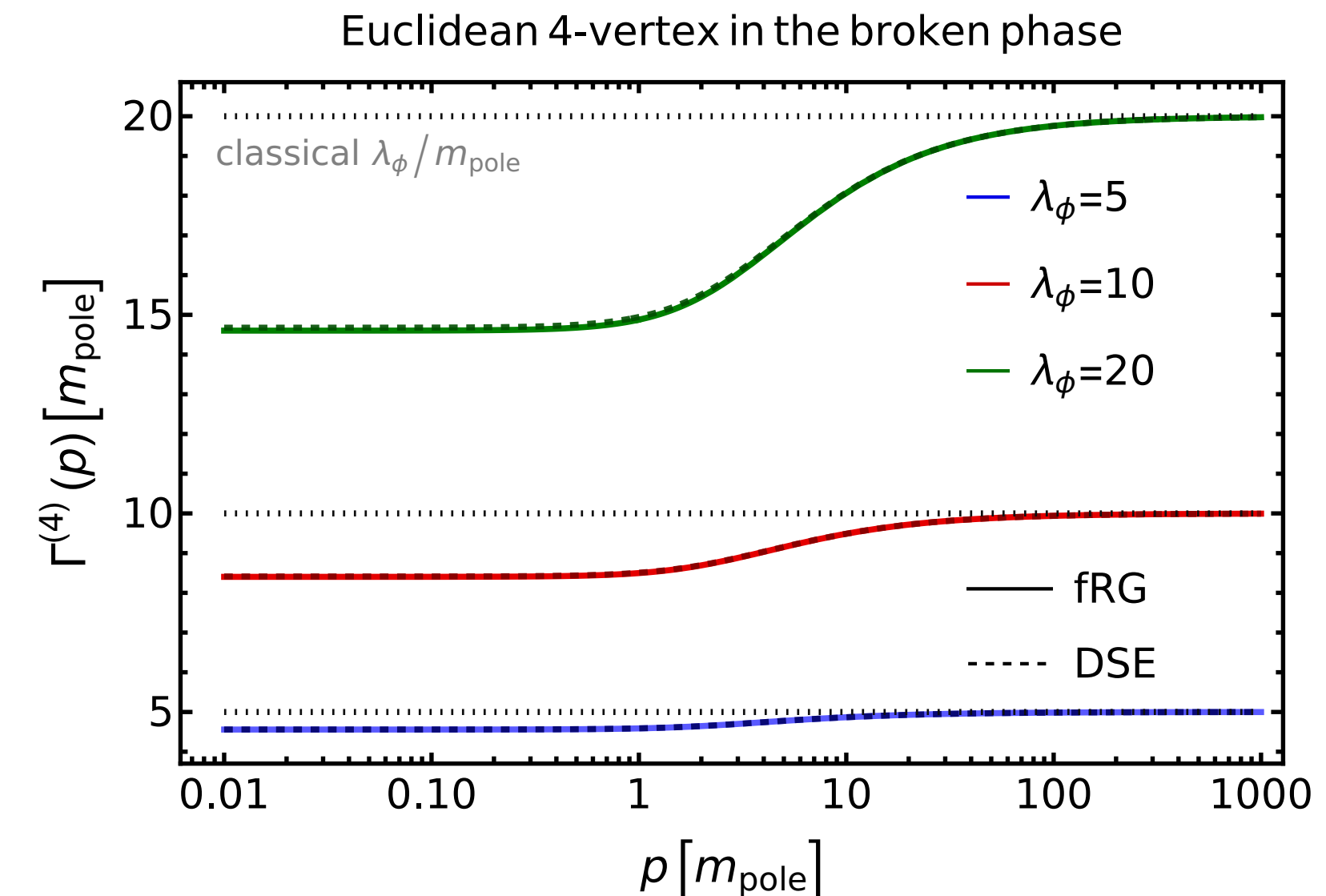
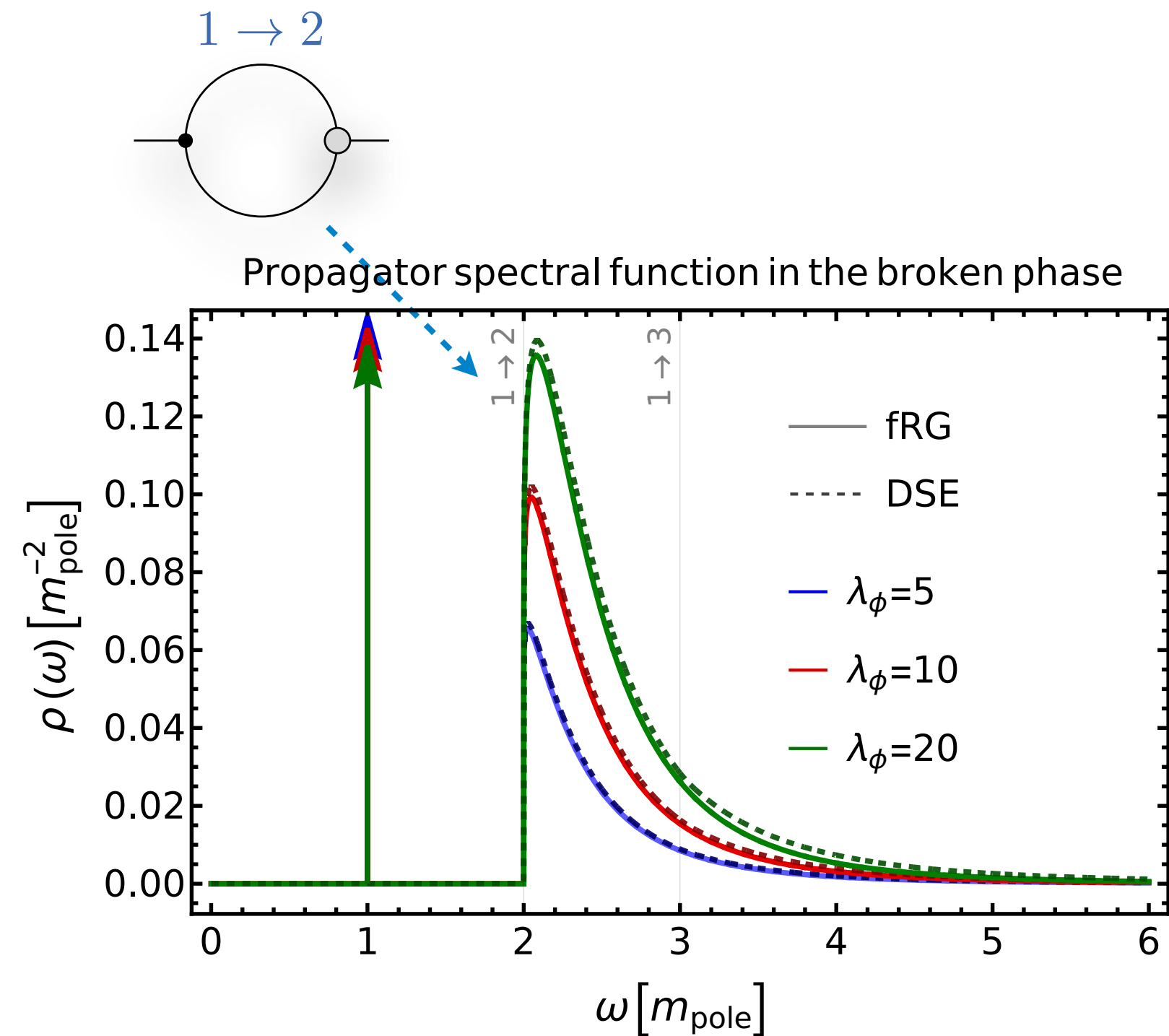
Spectral functional approaches at work

DSE

$$\text{fermion line with blue dot} = \text{bare fermion line} + \frac{1}{2} \text{self-energy loop} - \frac{1}{2} \text{bubble diagram} - \frac{1}{6} \text{triangle diagram} + \frac{1}{2} \text{four-loop diagram}$$

fRG

$$\partial_t \Gamma_k^{(2)}(p) = \text{loop diagram}(p, q) - \frac{1}{2} \text{self-energy loop}(p) - \partial_t S_{\text{ct},k}^{(2)}$$



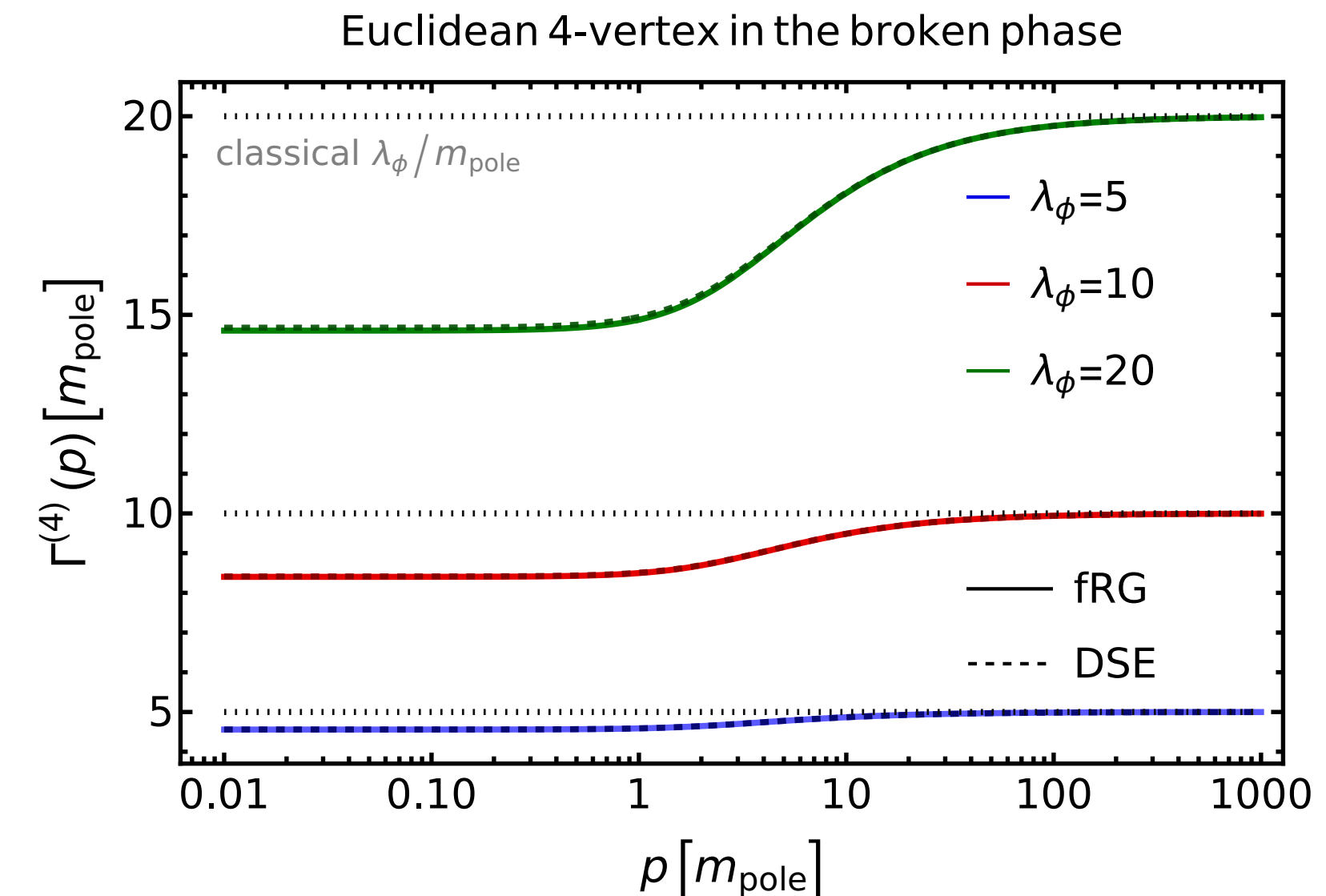
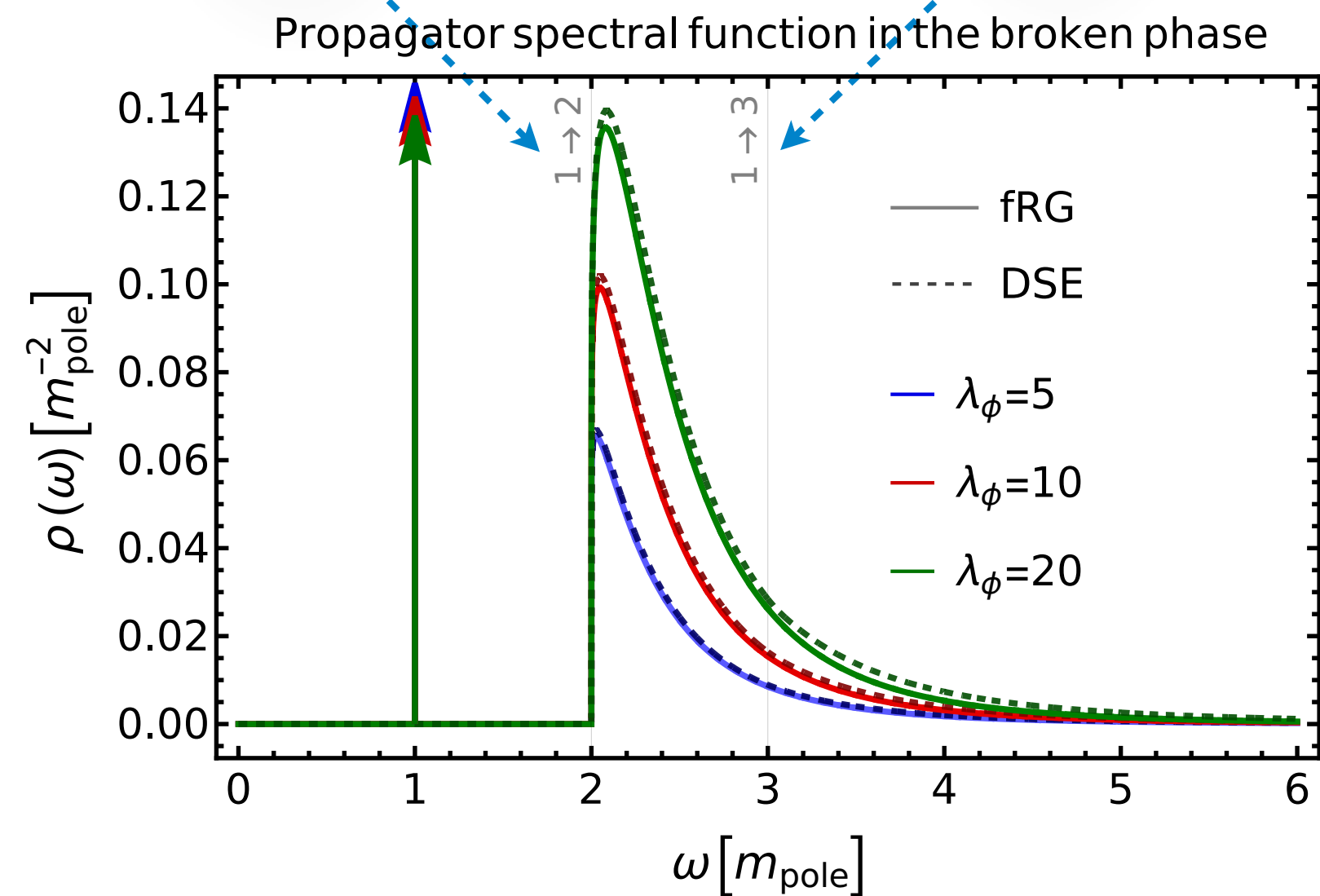
Spectral functional approaches at work

DSE

$$\text{---} \bullet \text{---} = \text{---} \bullet \text{---} + \frac{1}{2} \text{---} \bullet \text{---} \text{---} \text{---} - \frac{1}{2} \text{---} \bullet \text{---} \text{---} \text{---} \text{---} - \frac{1}{6} \text{---} \bullet \text{---} \text{---} \text{---} \text{---} \text{---} + \frac{1}{2} \text{---} \bullet \text{---} \text{---} \text{---} \text{---} \text{---}$$

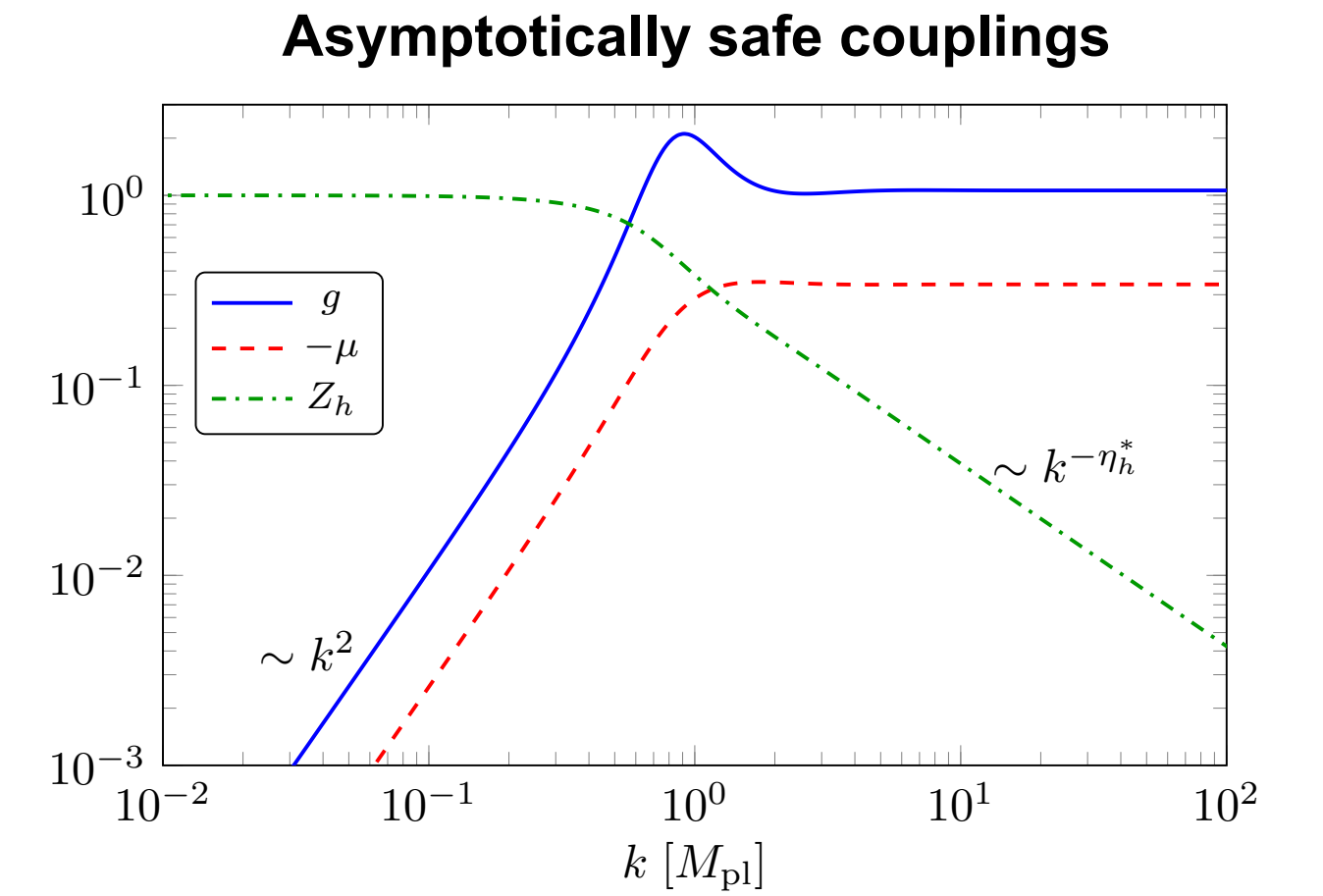
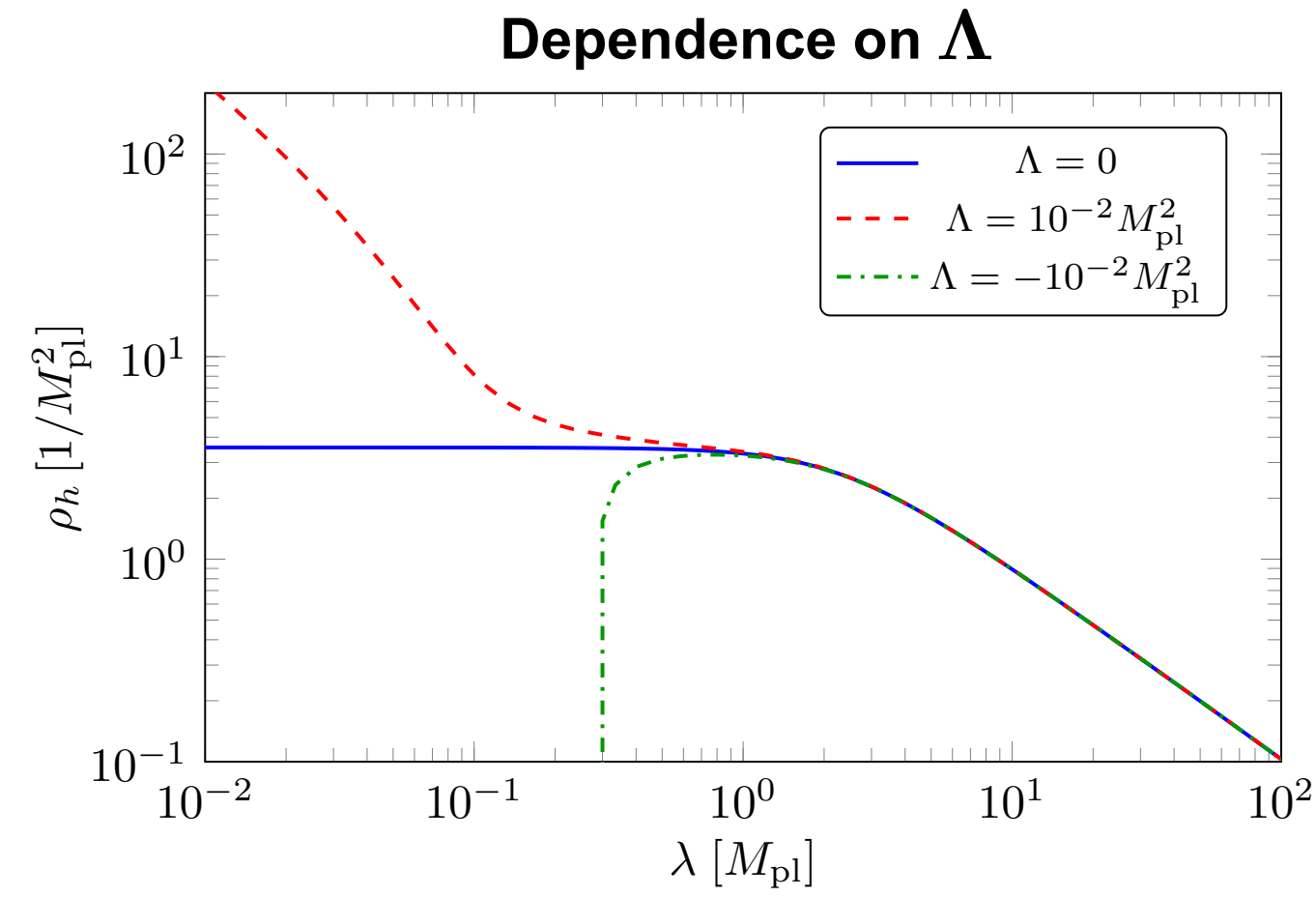
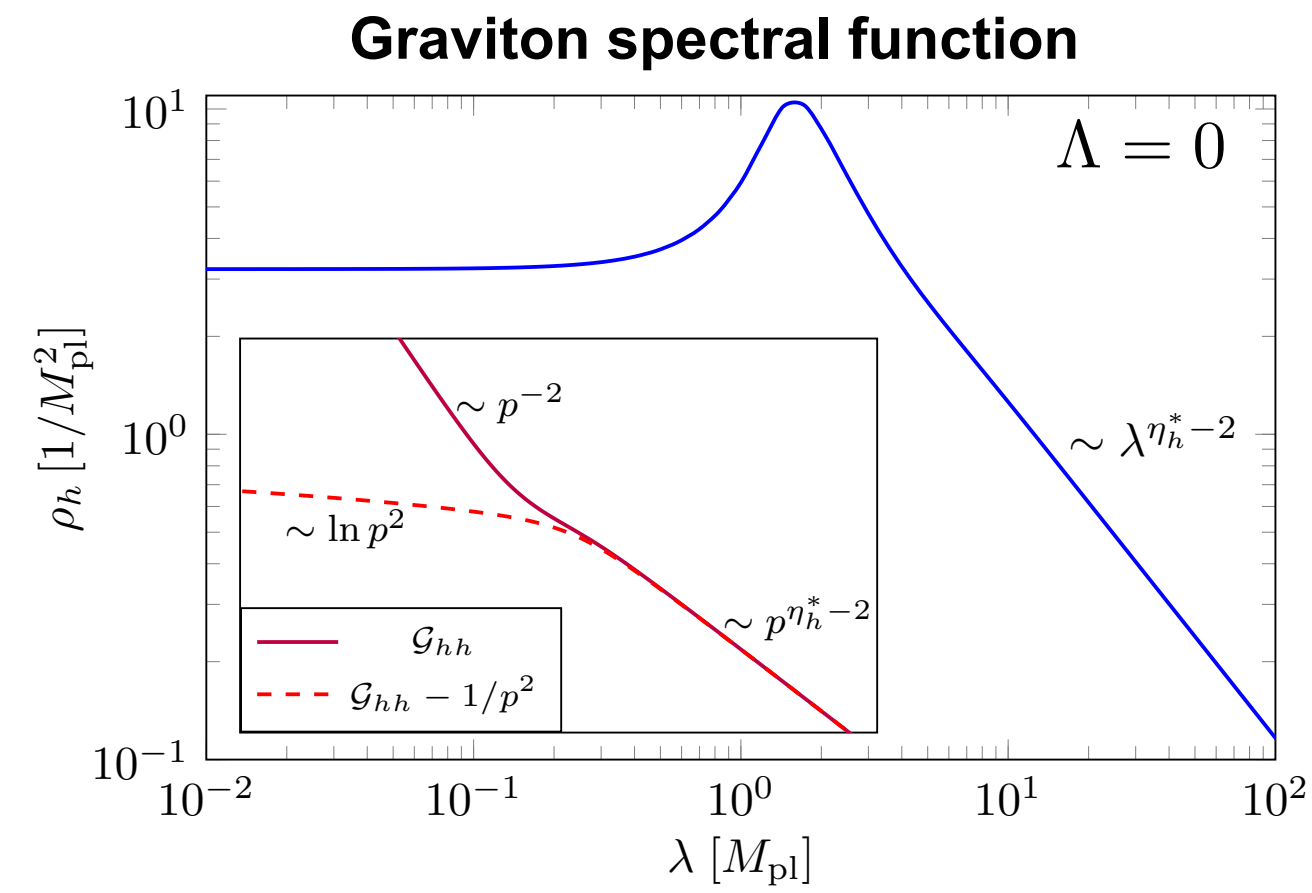
fRG

$$\partial_t \Gamma_k^{(2)}(p) = \text{---} \bullet \text{---} \text{---} \text{---} - \frac{1}{2} \text{---} \bullet \text{---} \text{---} \text{---} - \partial_t S_{\text{ct},k}^{(2)}$$



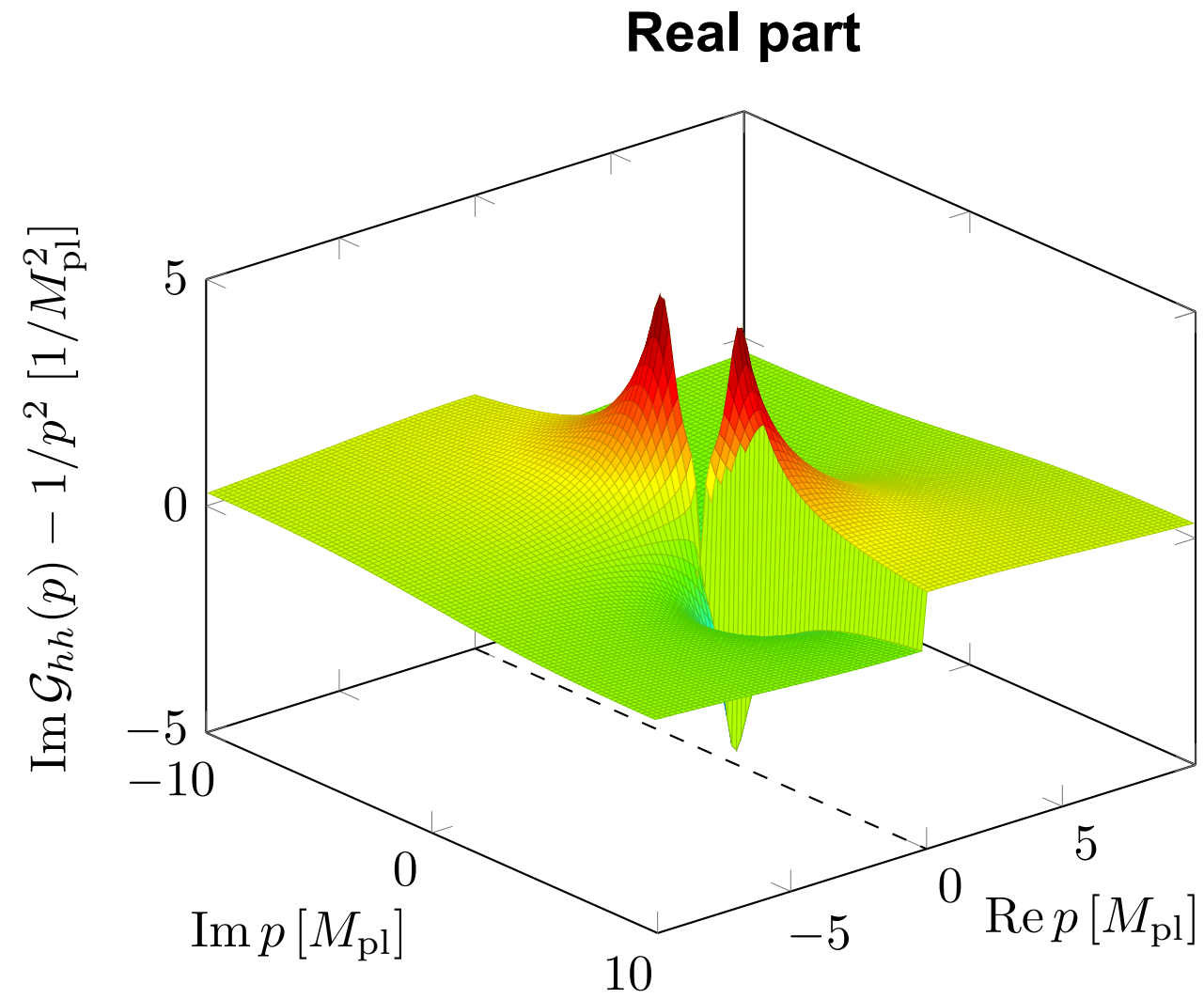
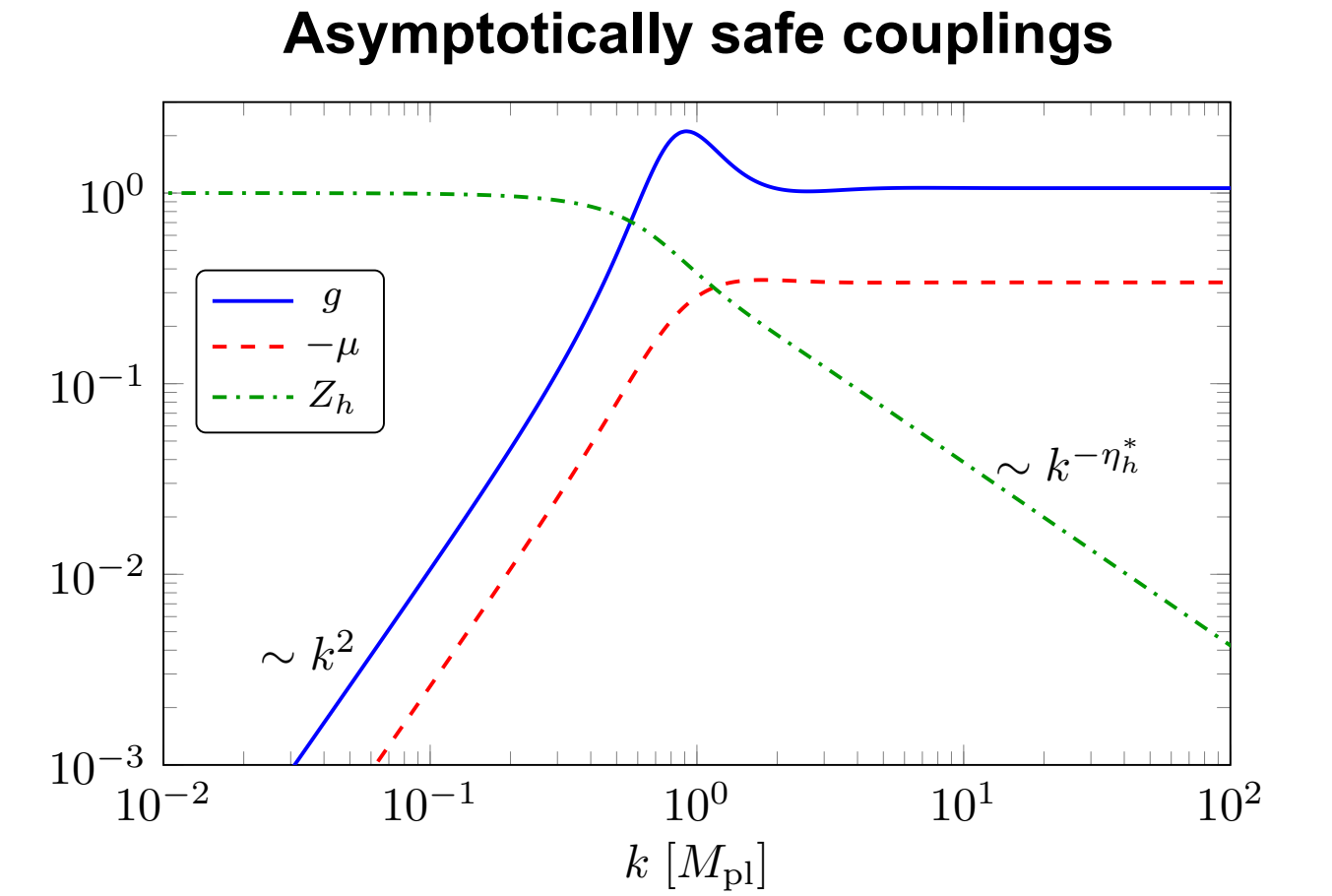
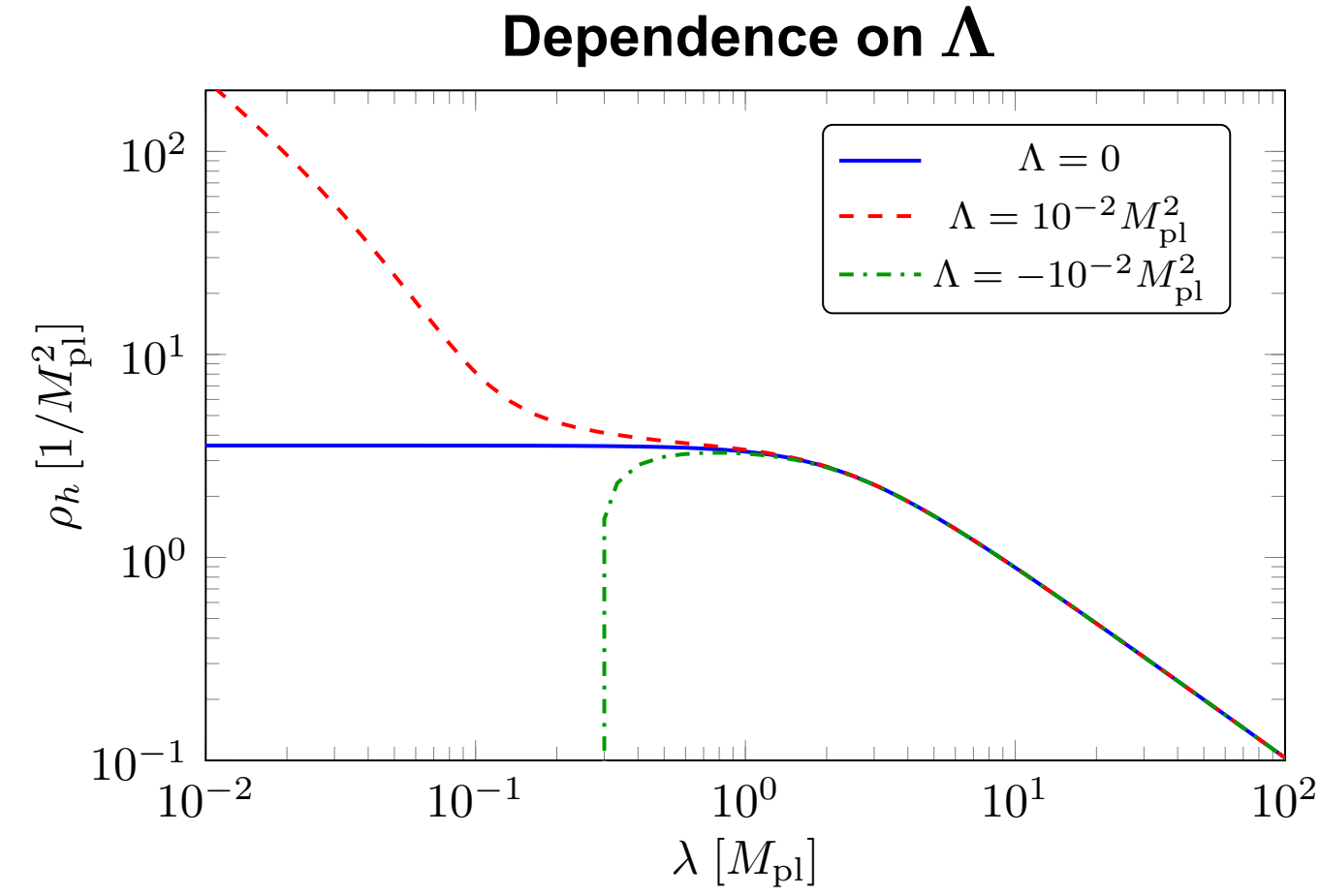
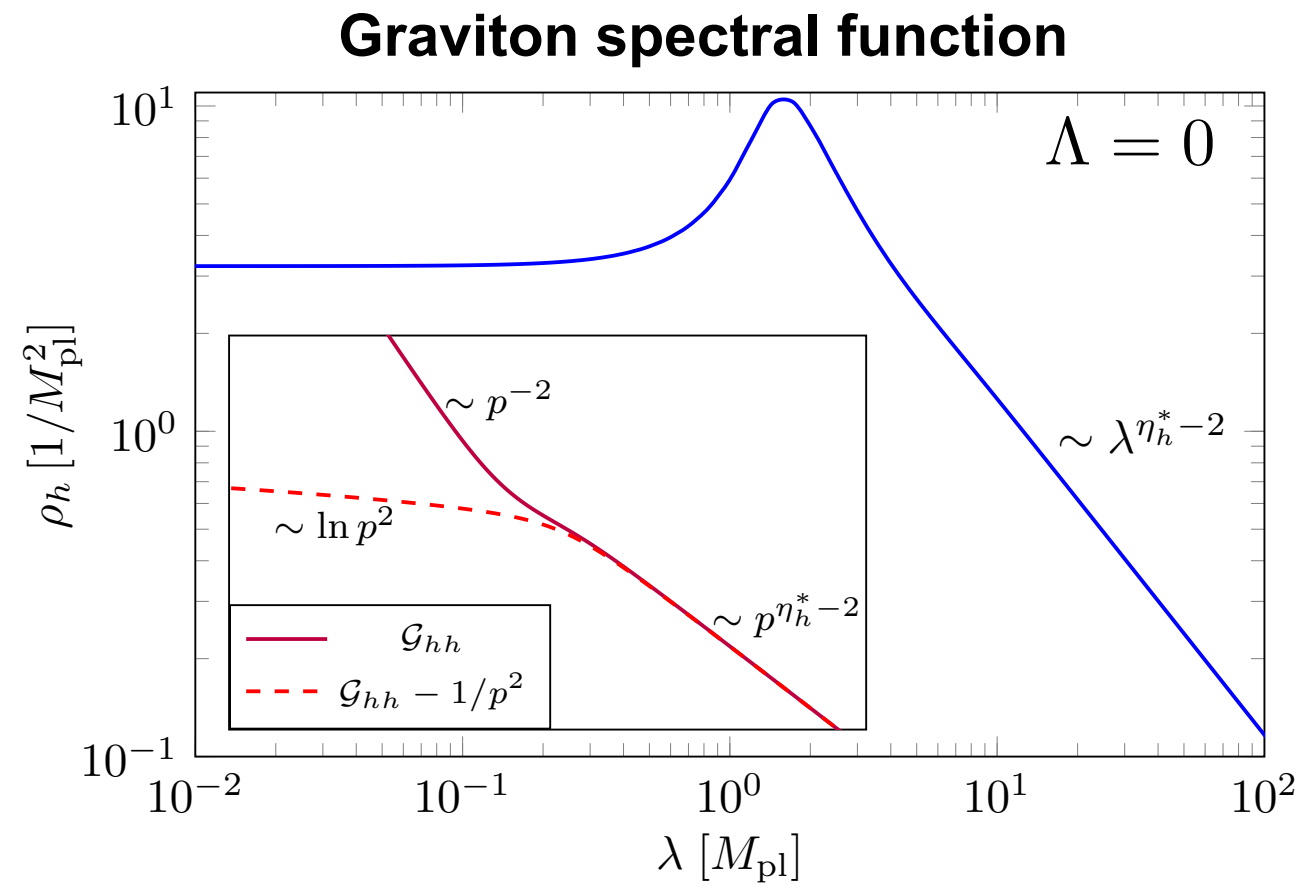
Renormalised renormalisation group flows

Spectral properties of Lorentzian asymptotically safe gravity

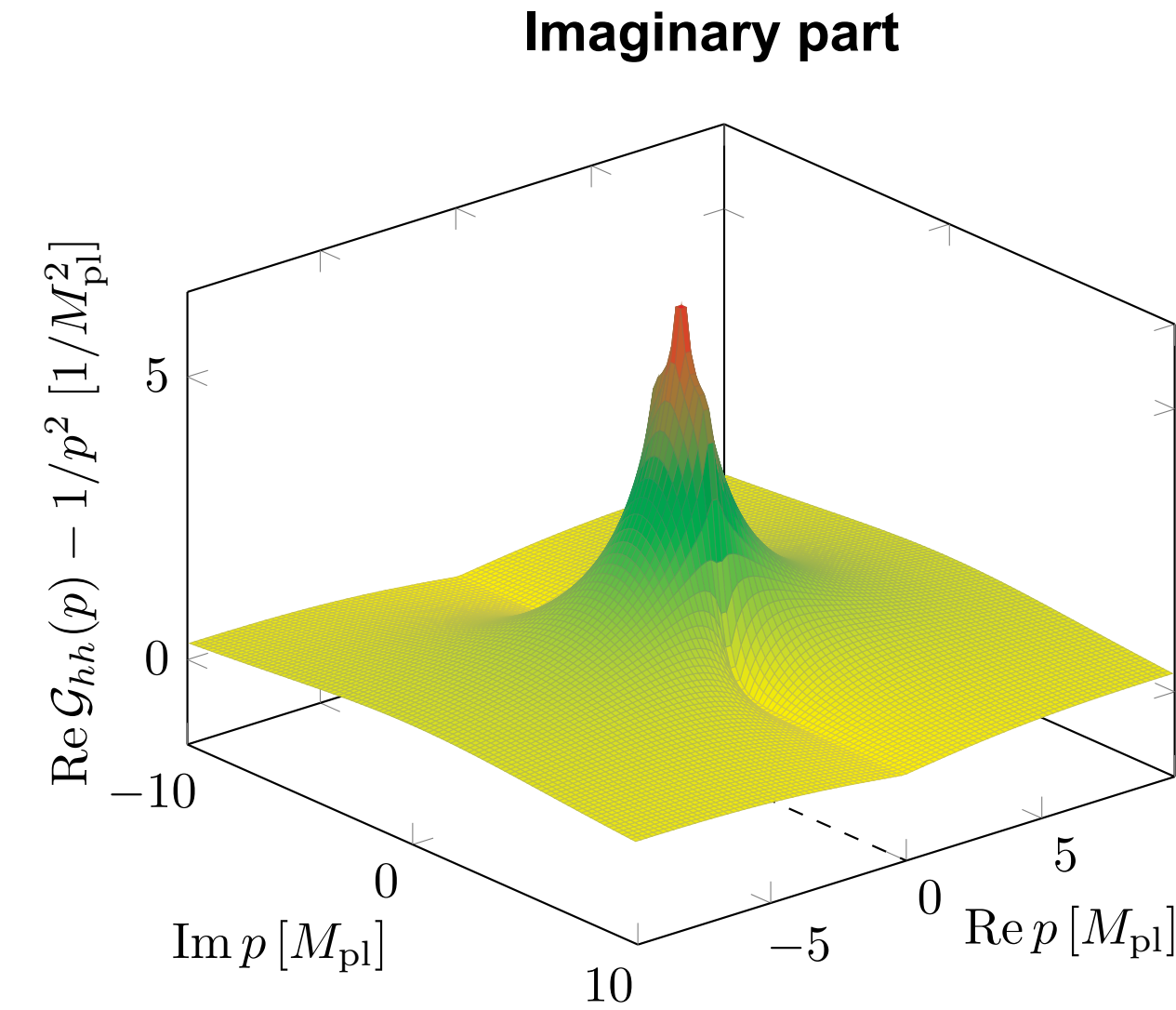


Renormalised renormalisation group flows

Spectral properties of Lorentzian asymptotically safe gravity

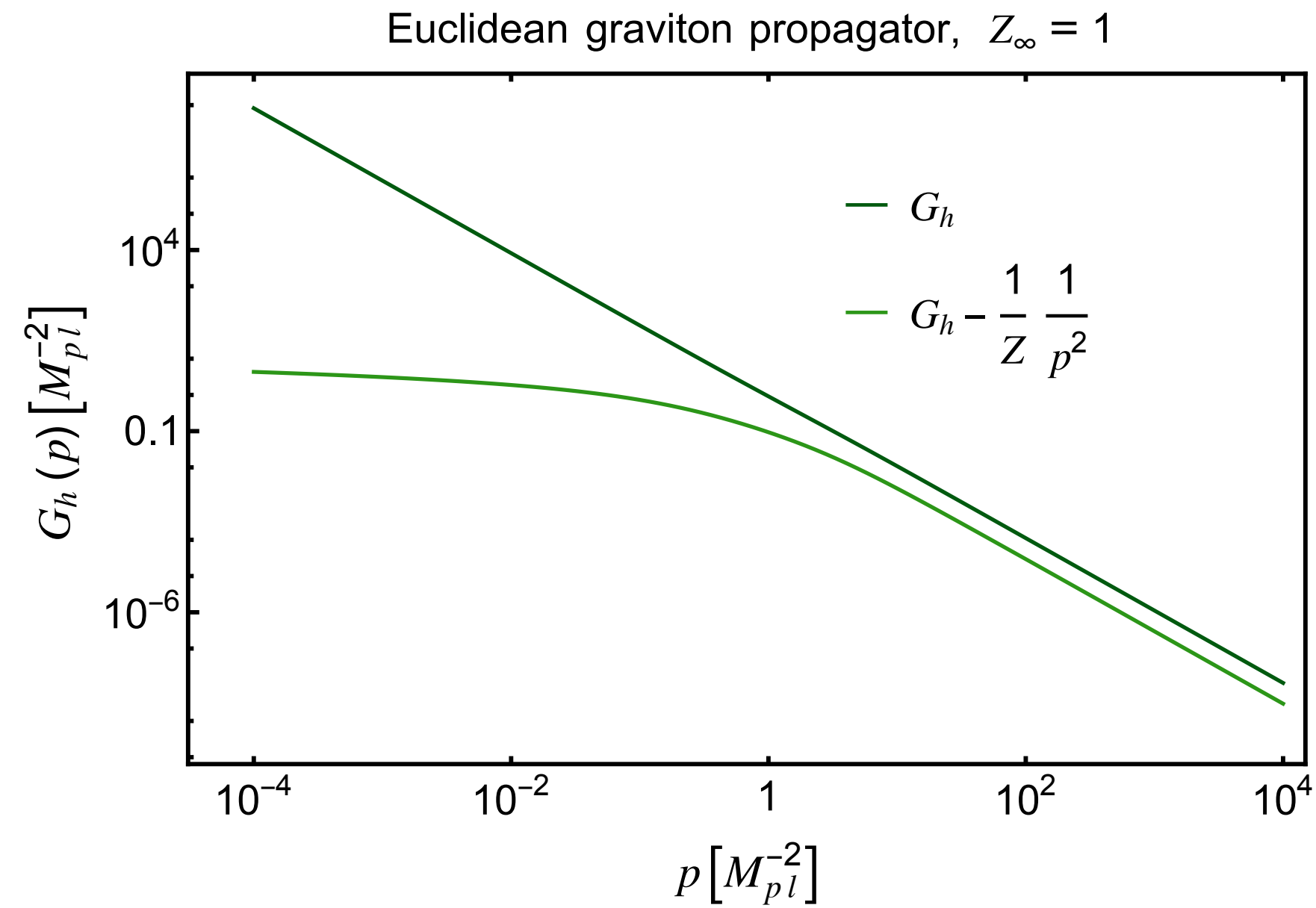


$$\langle h_{TT}(p) h_{TT}(-p) \rangle$$

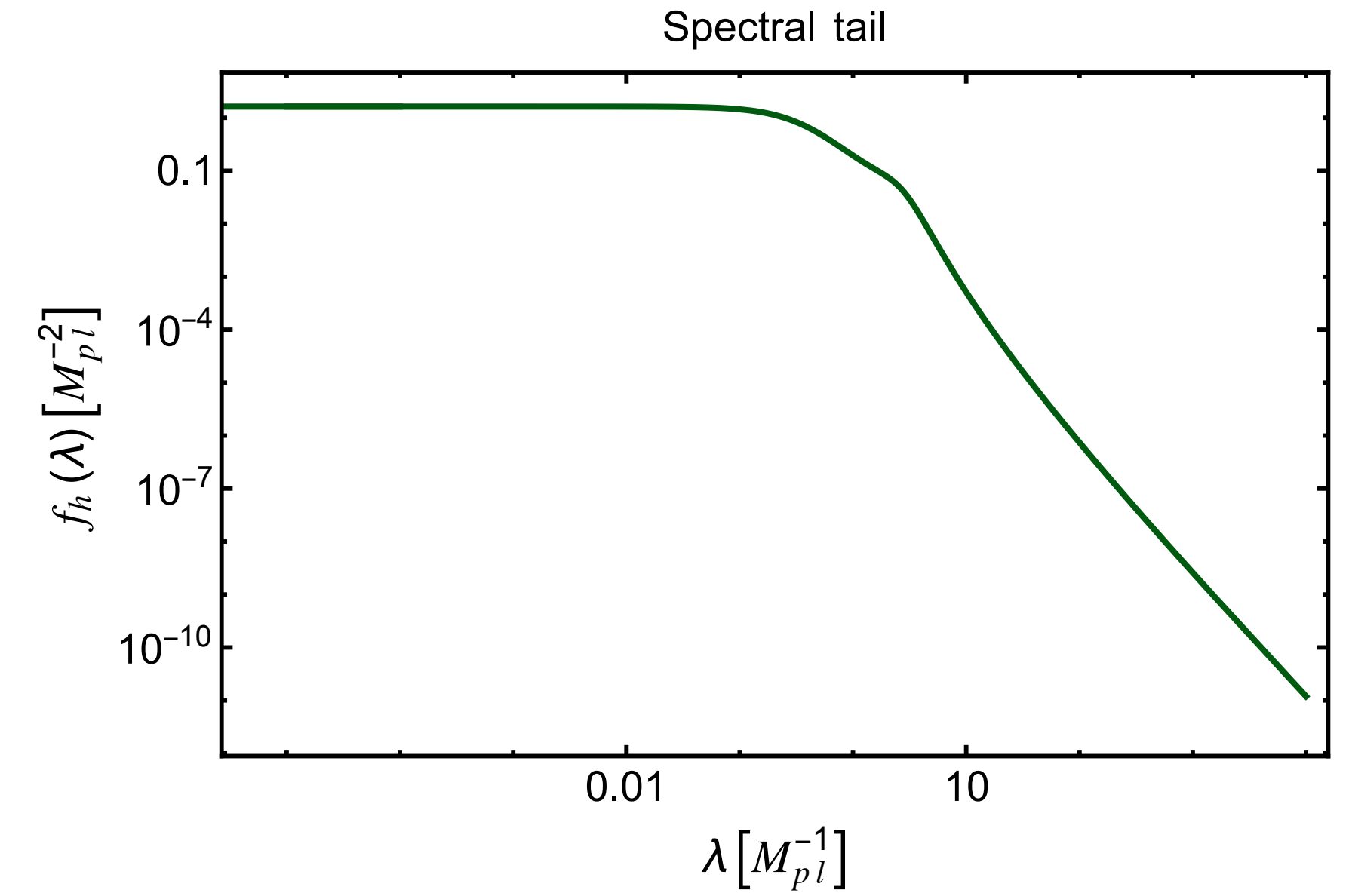


Fully self-consistent graviton spectral function with ‘on-shell’ renormalisation

Space-like fluctuation propagator

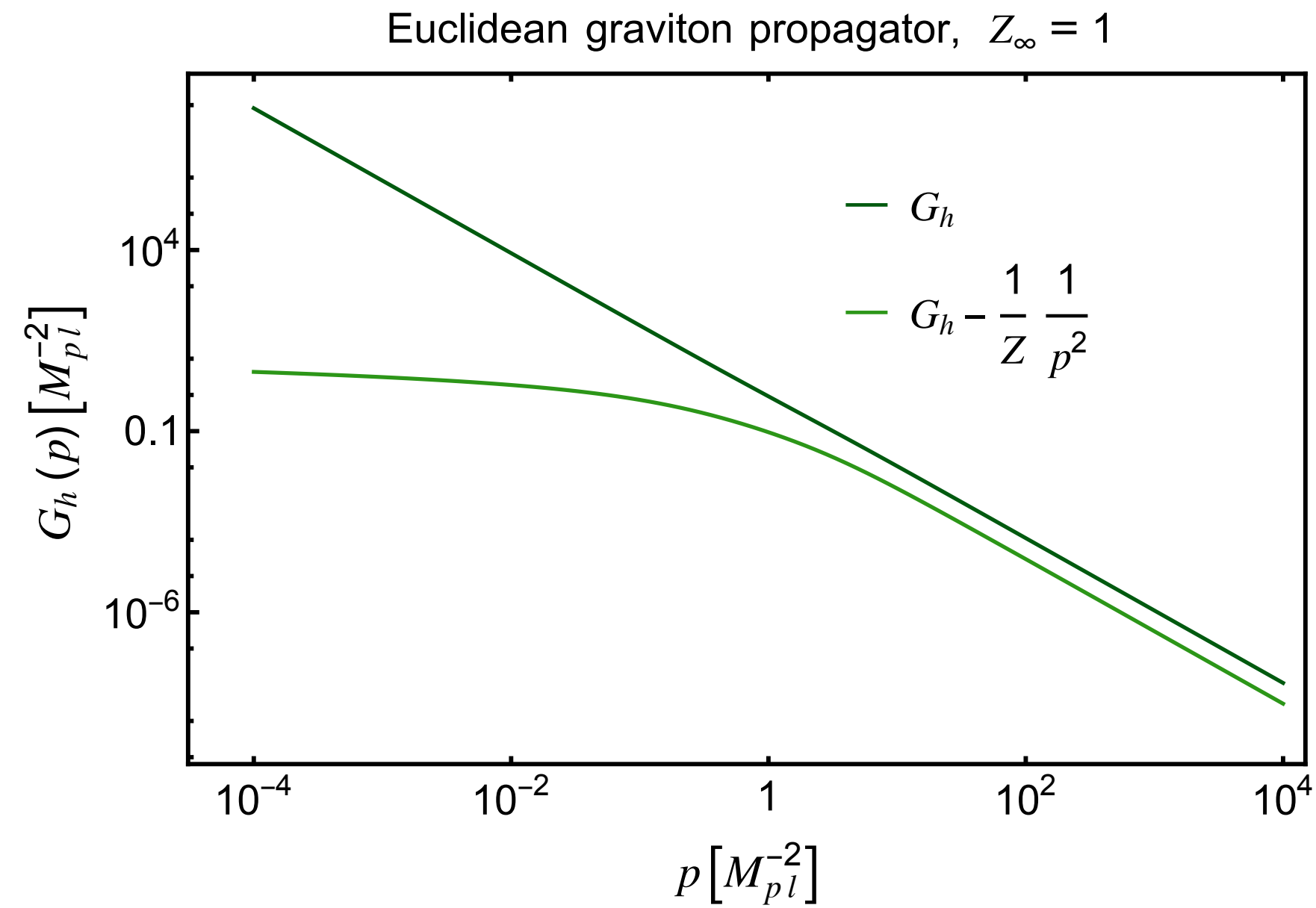


Spectral function of the fluctuation graviton

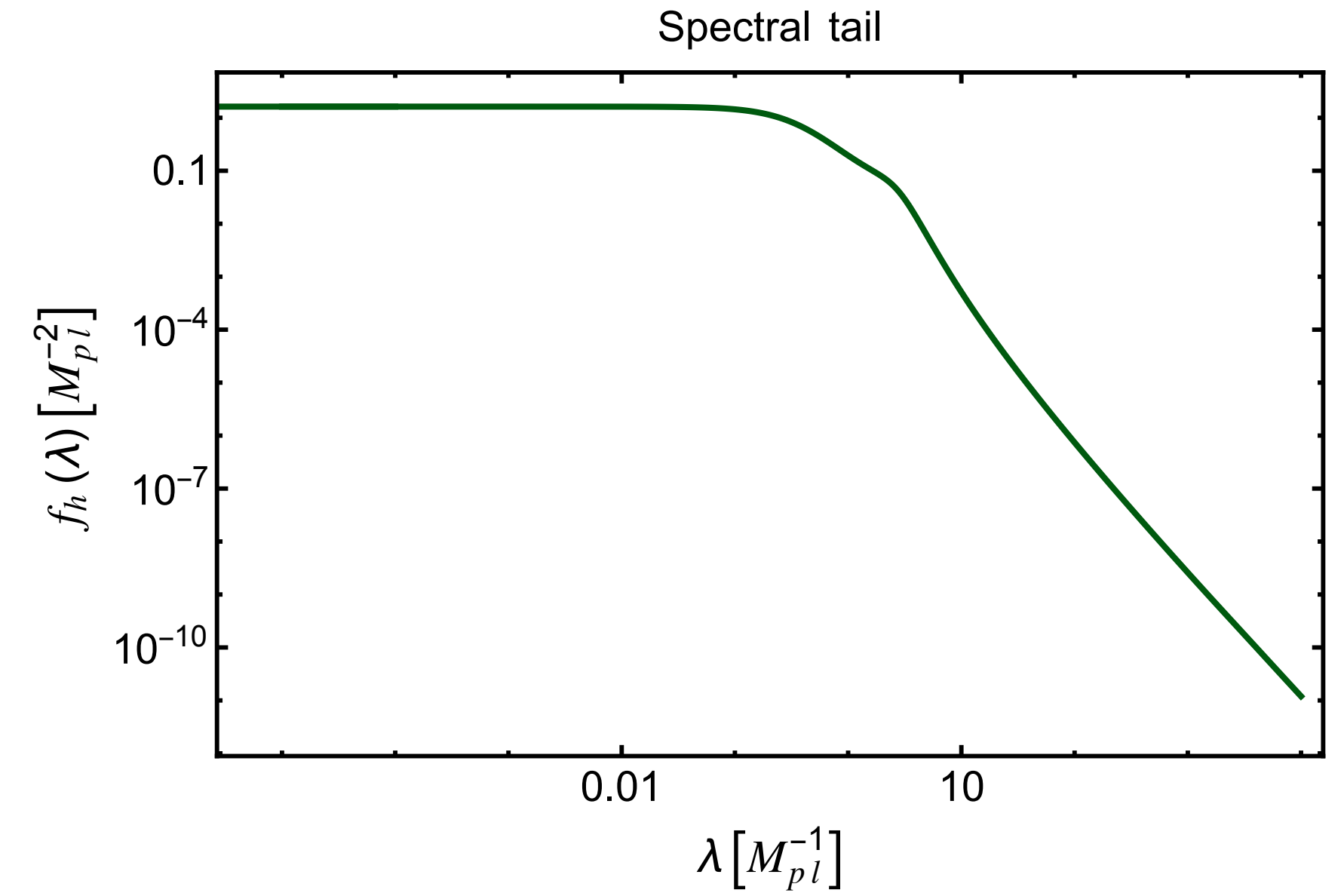


Fully self-consistent graviton spectral function with ‘on-shell’ renormalisation

Space-like fluctuation propagator



Spectral function of the fluctuation graviton

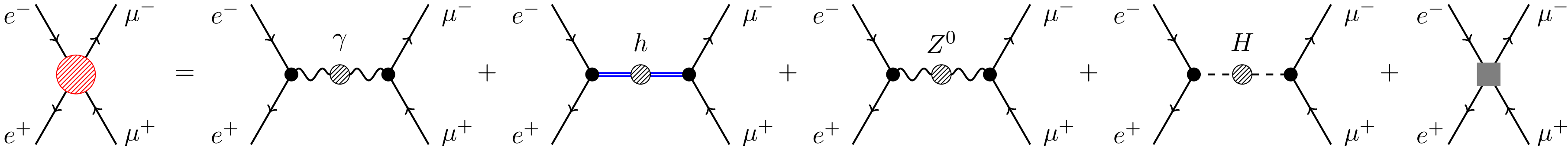
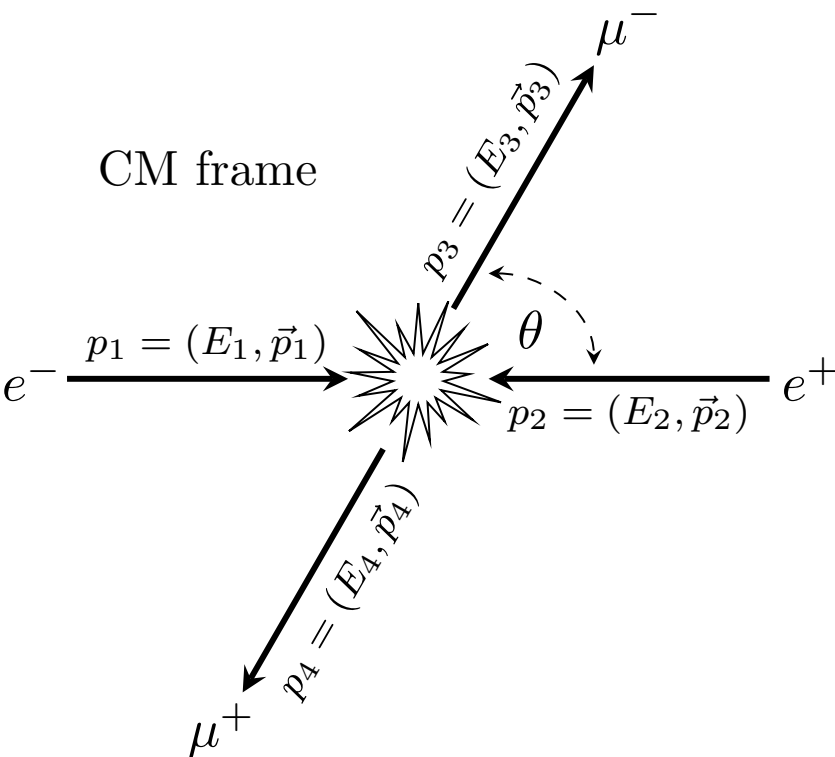


Spectral sum rule

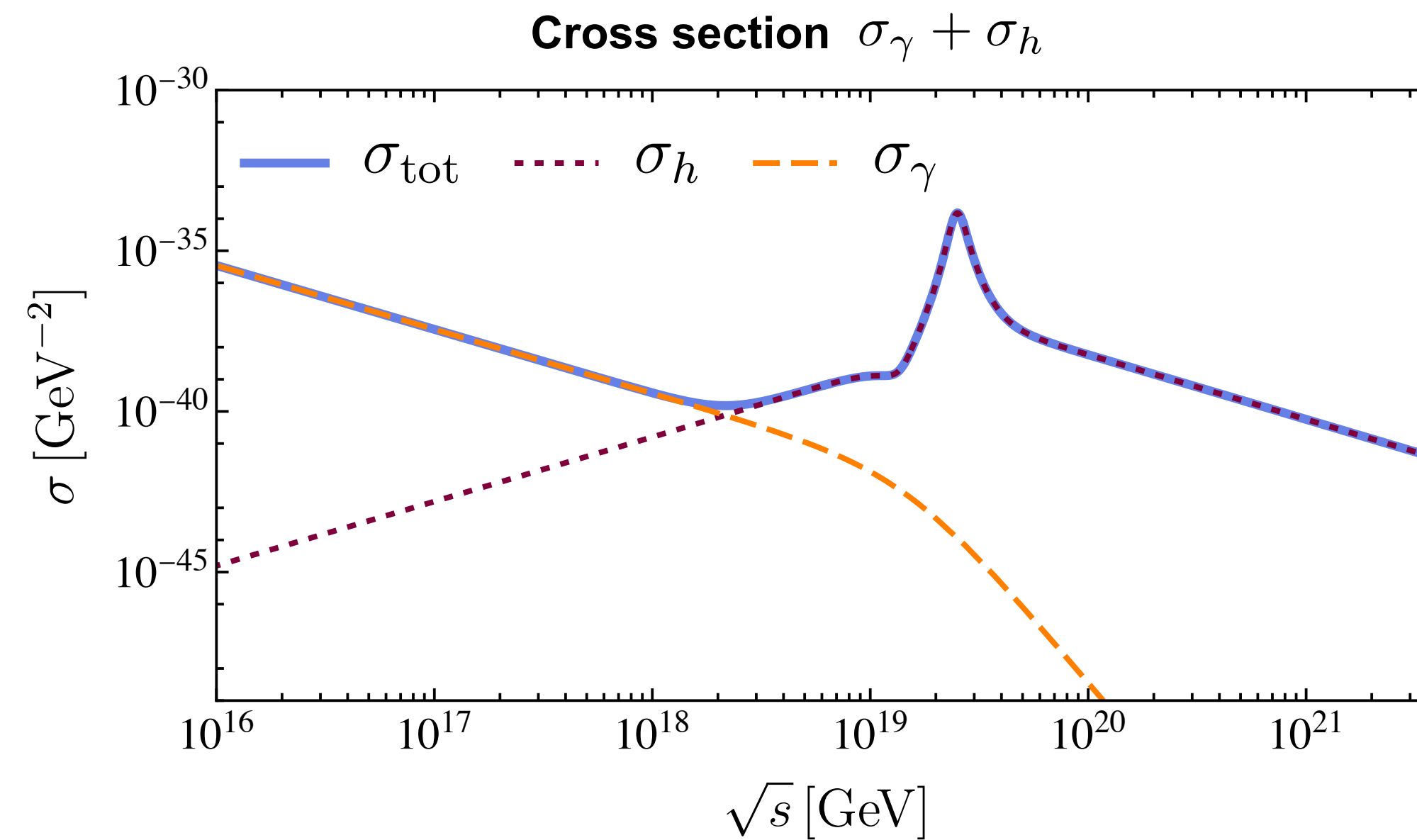
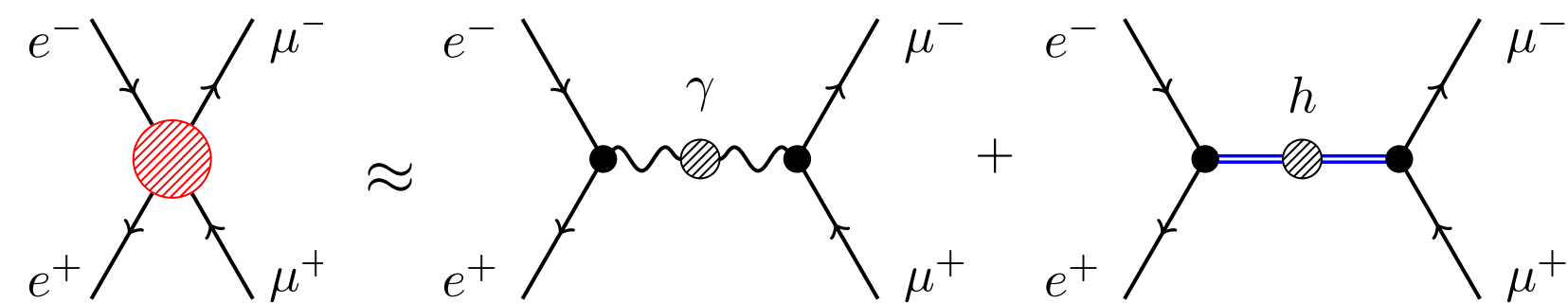
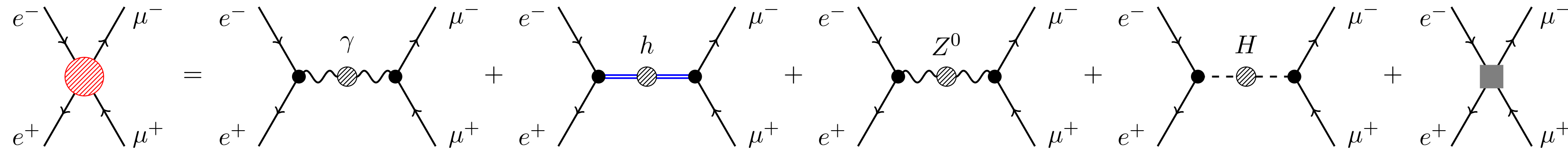
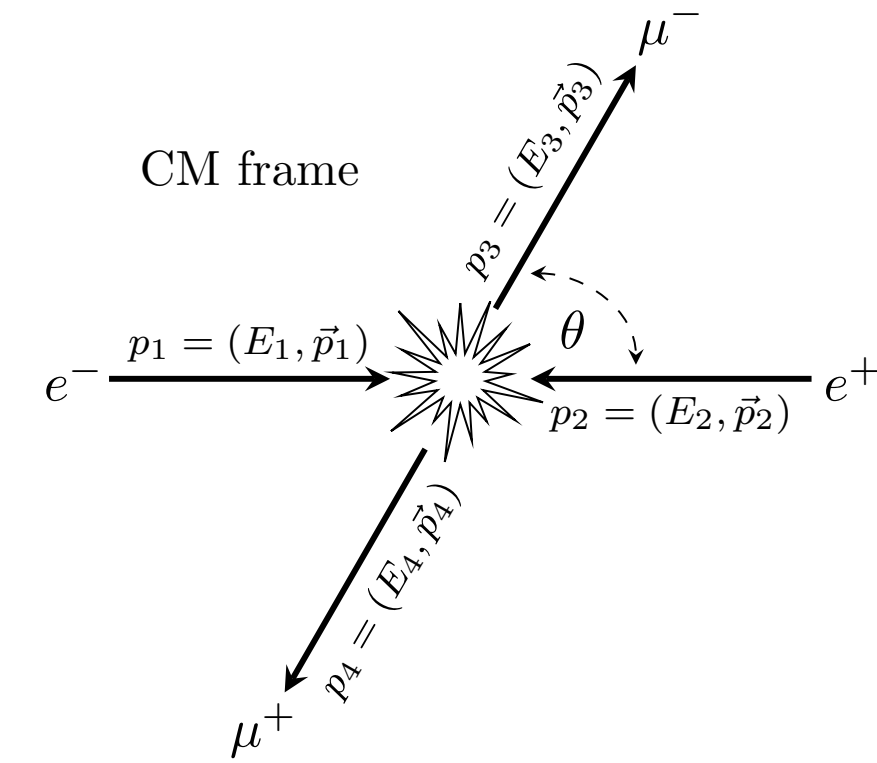
$$\int_{\mathbb{R}} \frac{d\lambda}{2\pi} \lambda \rho_h(\lambda) = 1$$

Scattering amplitudes

Scattering amplitude $e^+e^- \rightarrow \mu^+\mu^-$

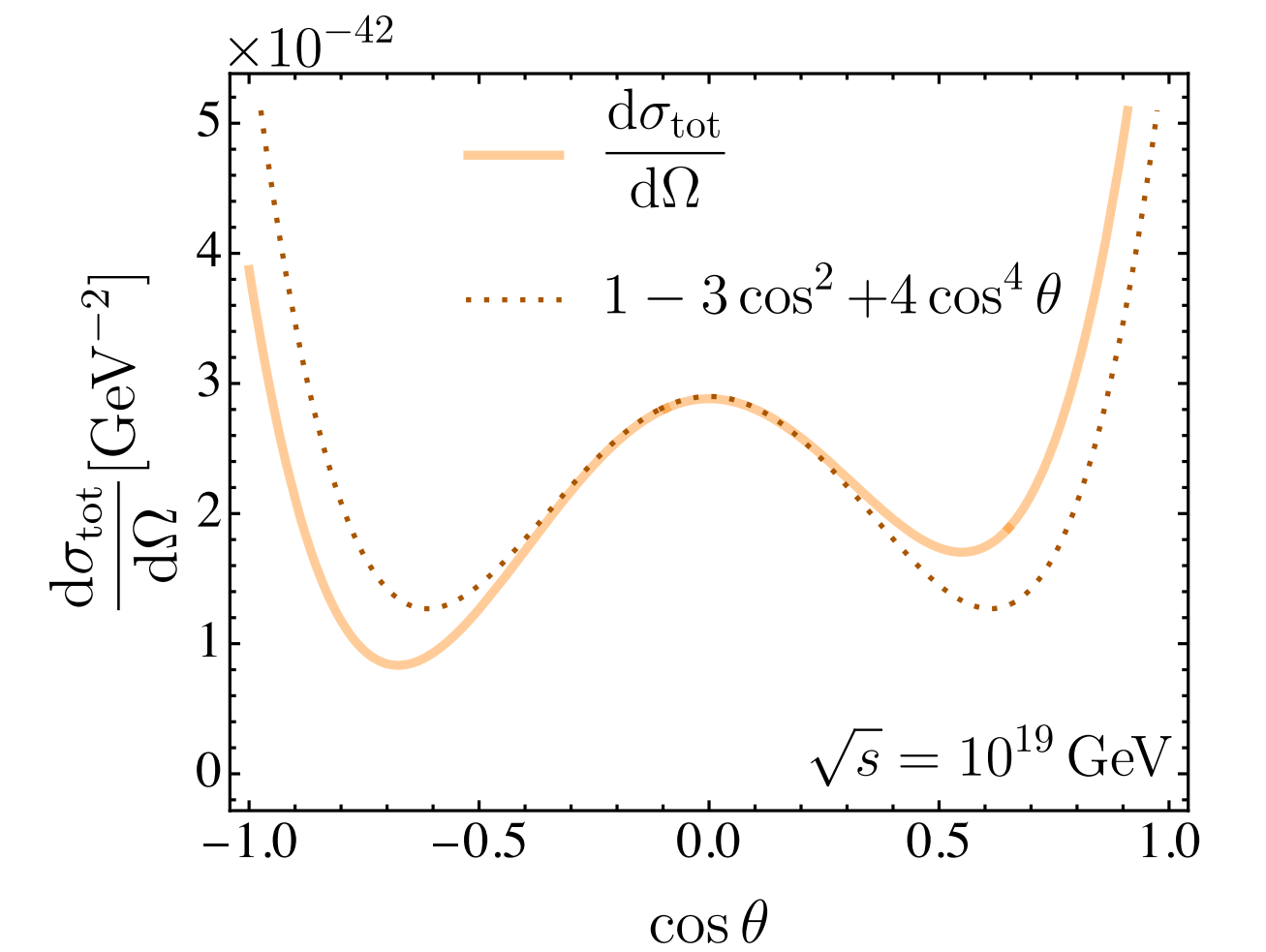
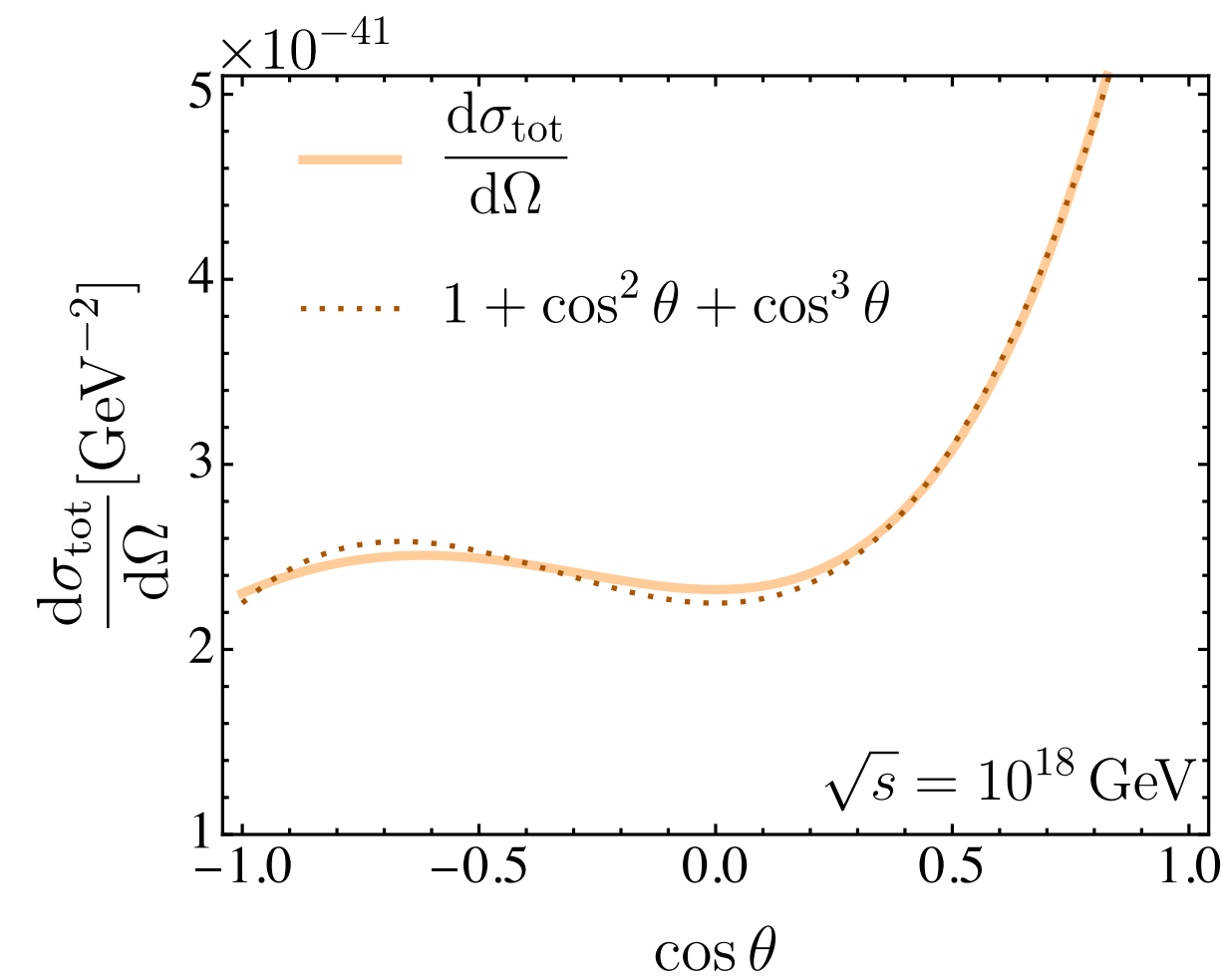
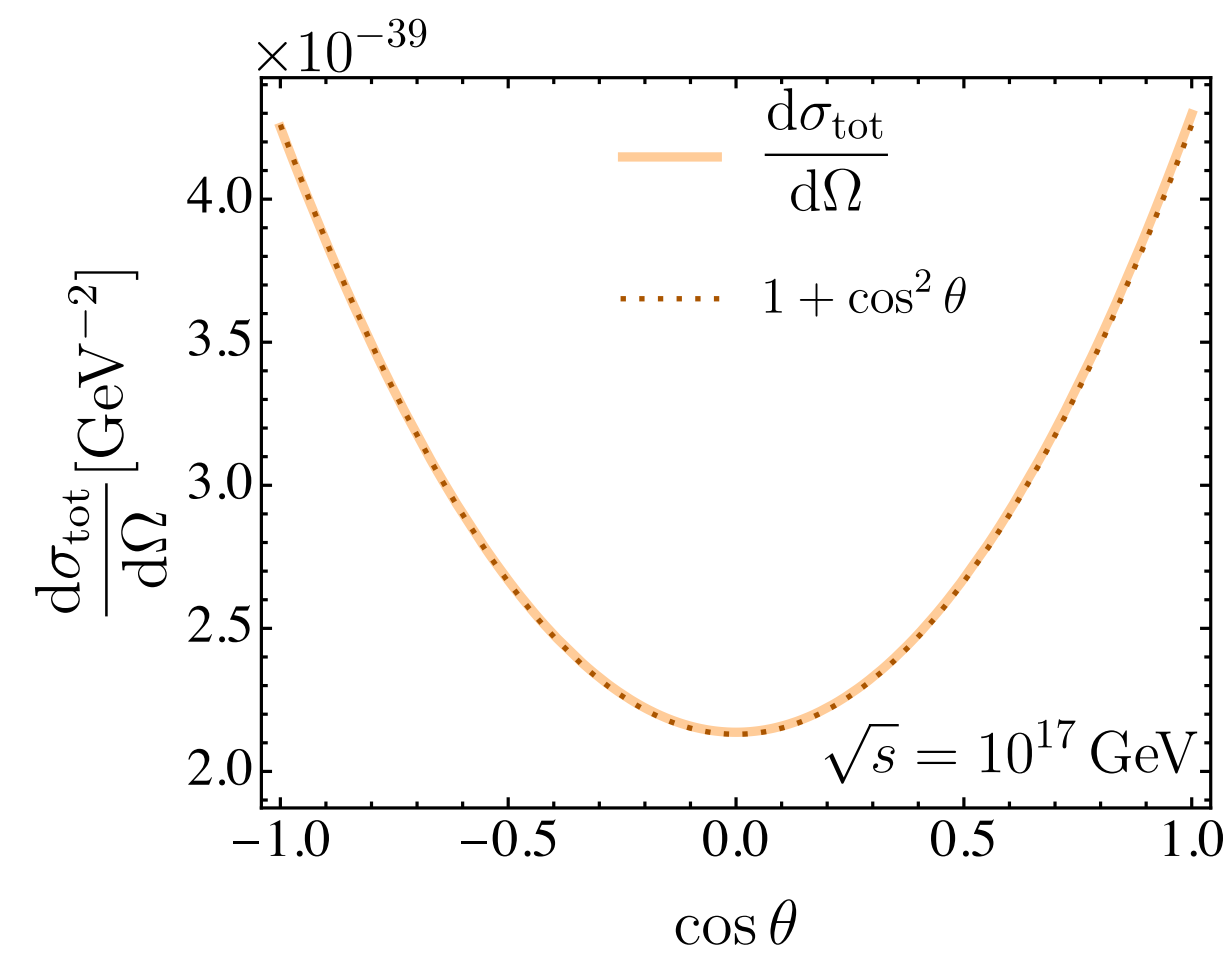
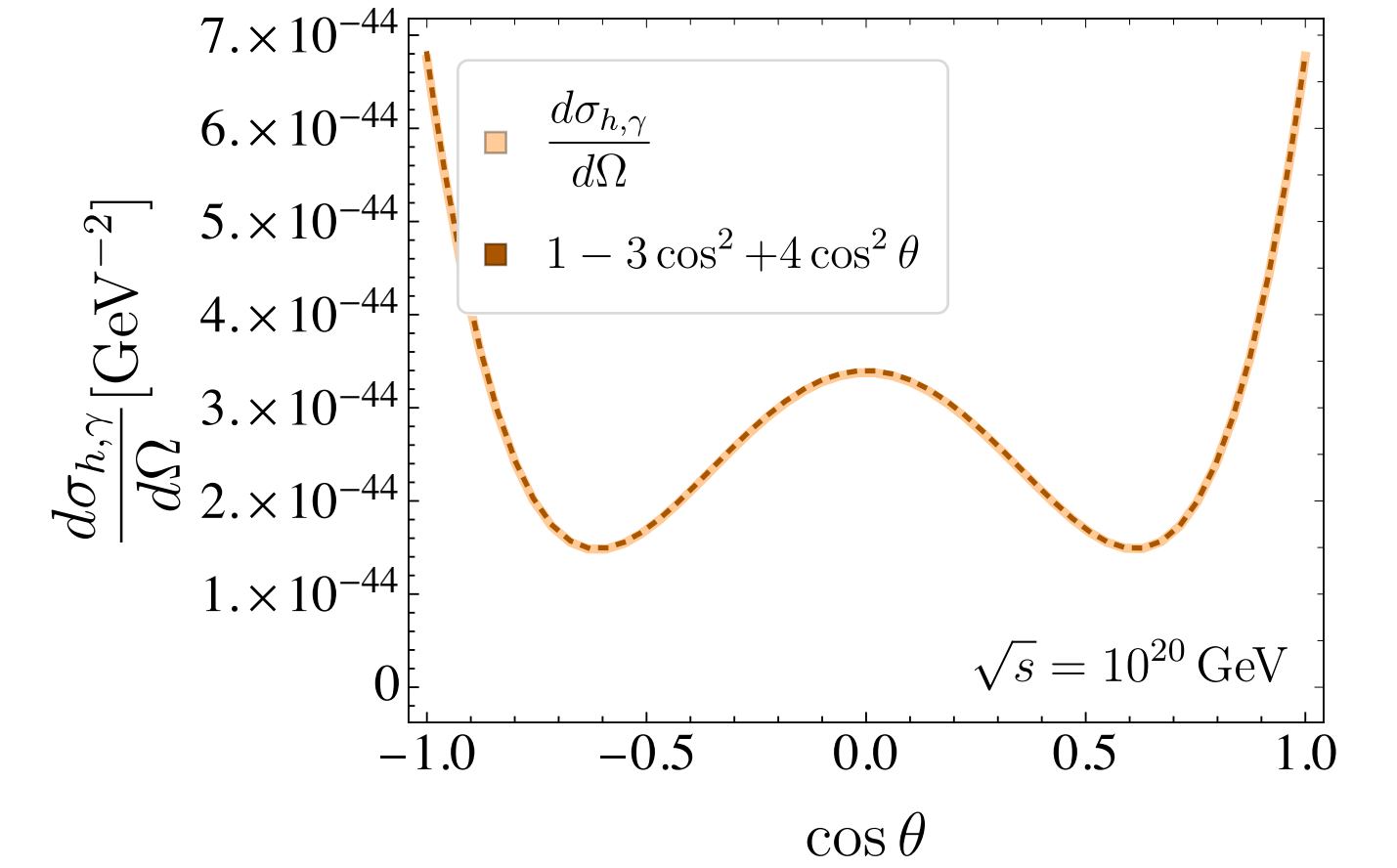
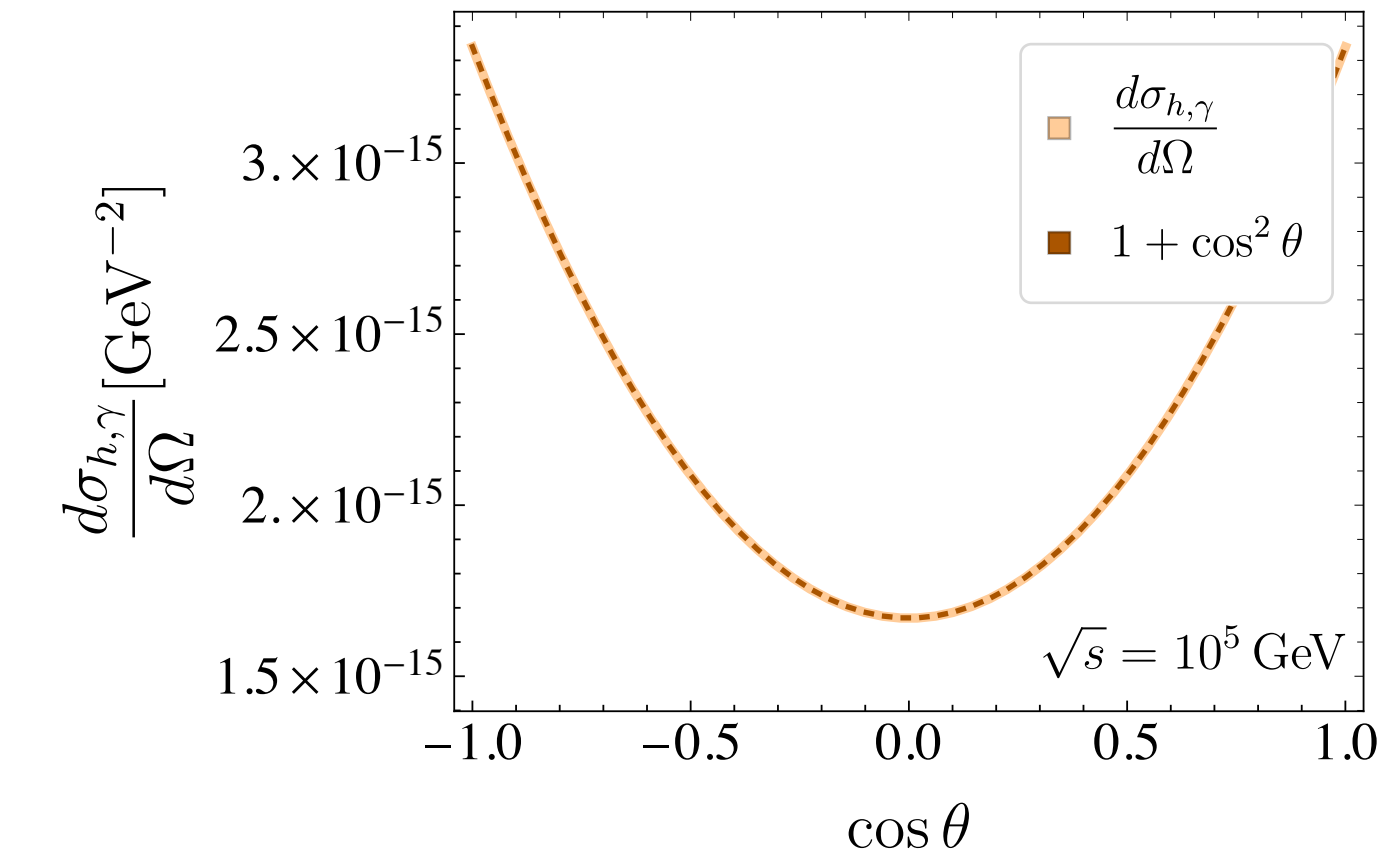


Scattering amplitude $e^+e^- \rightarrow \mu^+\mu^-$



Scattering amplitude $e^+e^- \rightarrow \mu^+\mu^-$

Angular dependence of the differential cross section



Asymptotically safe Standard Model

Pastor-Gutiérrez, JMP, Reichert, SciPost Phys. 15 (2023) 105

Dona, Eichhorn, Percacci, PRD 89 (2014) 084035

Meibohm, JMP, Reichert, EPJC 76 (2016) 285

Christiansen, Litim, JMP, Reichert PRD 97 (2018) 4, 046007



Shaposhnikov, Wetterich, PLB 683 (2010) 196

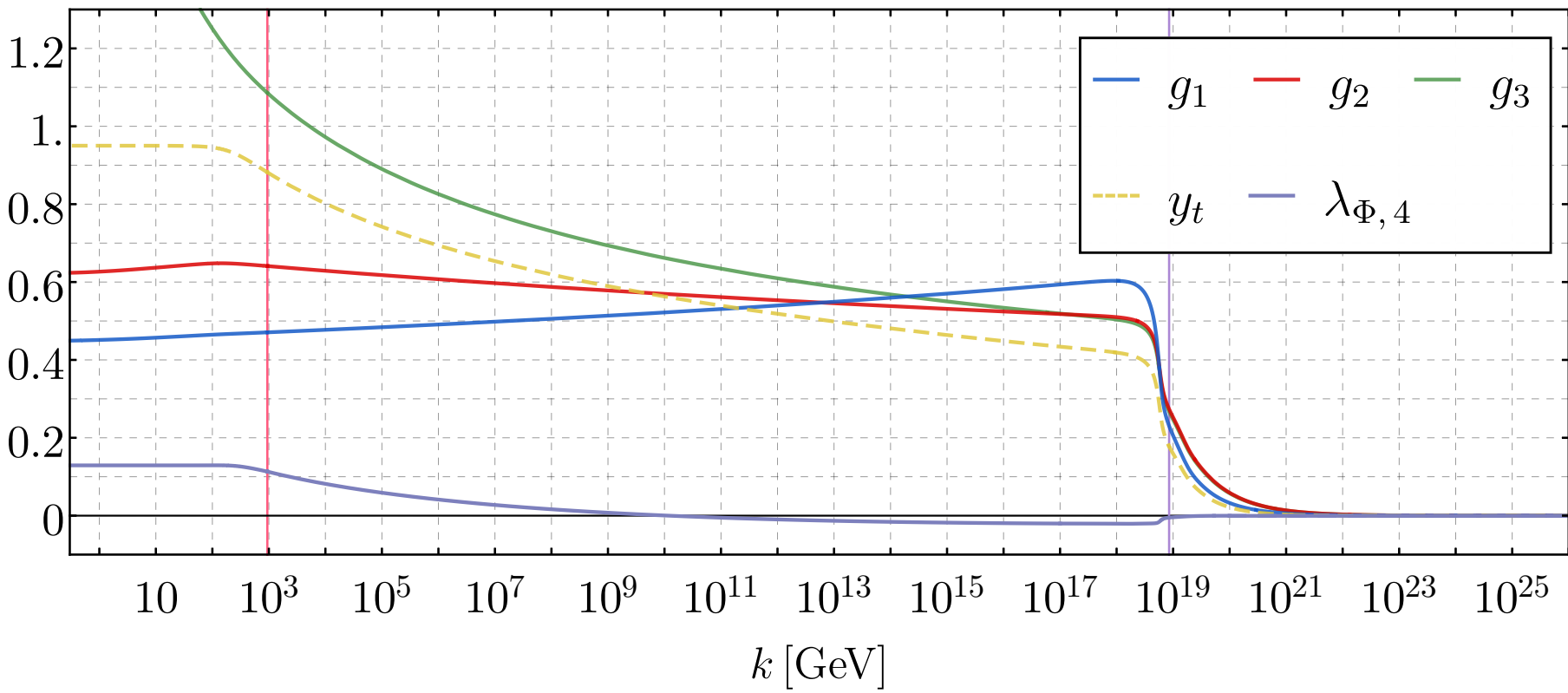
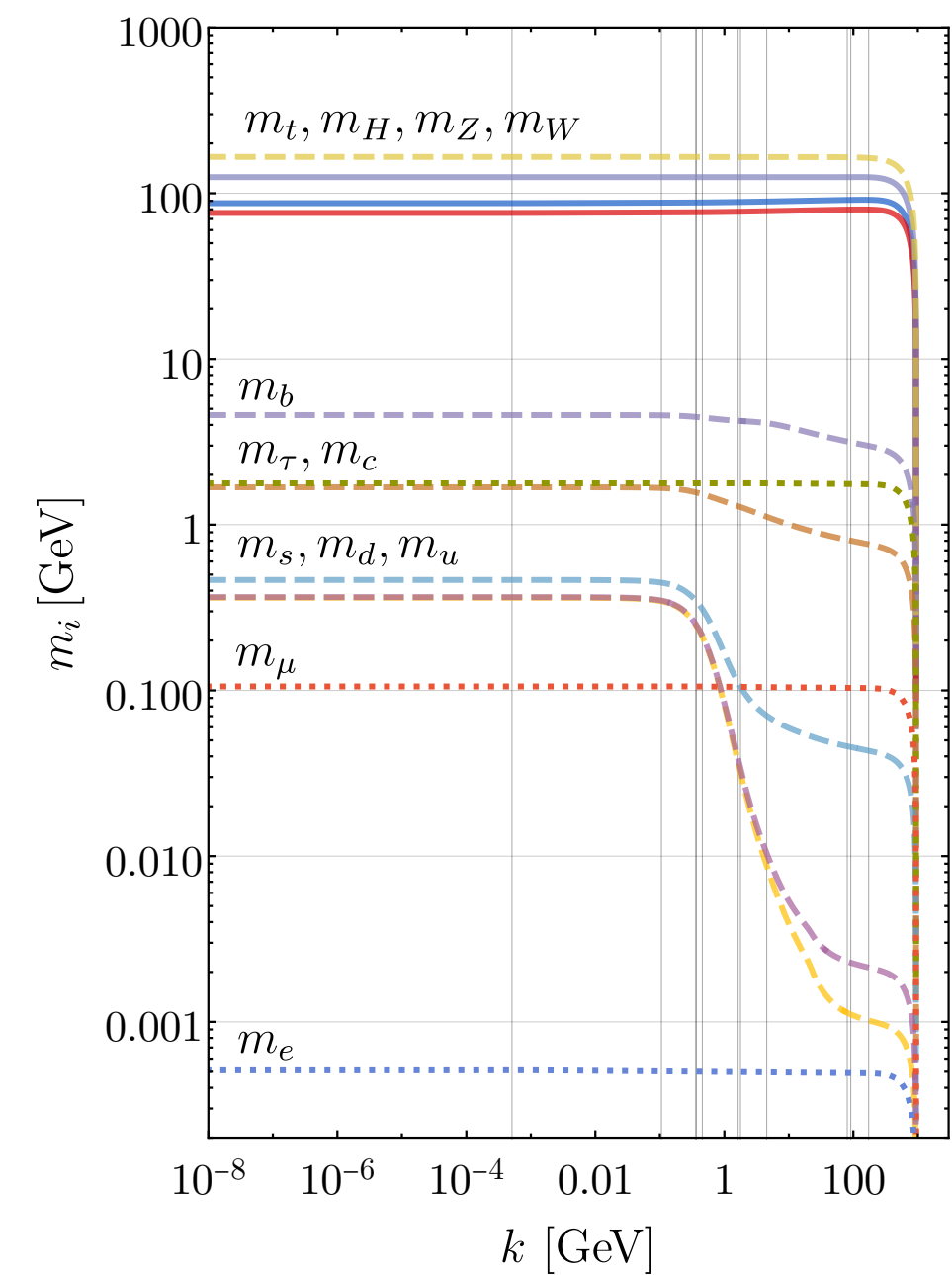
Eichhorn, Versteegen, JHEP 1801 (2018) 030

Eichhorn, Held, PRL 121 (2018) 151302



Latest ‘status report’: Eichhorn, Schiffer, 2212.07456

Asymptotically safe Standard Model

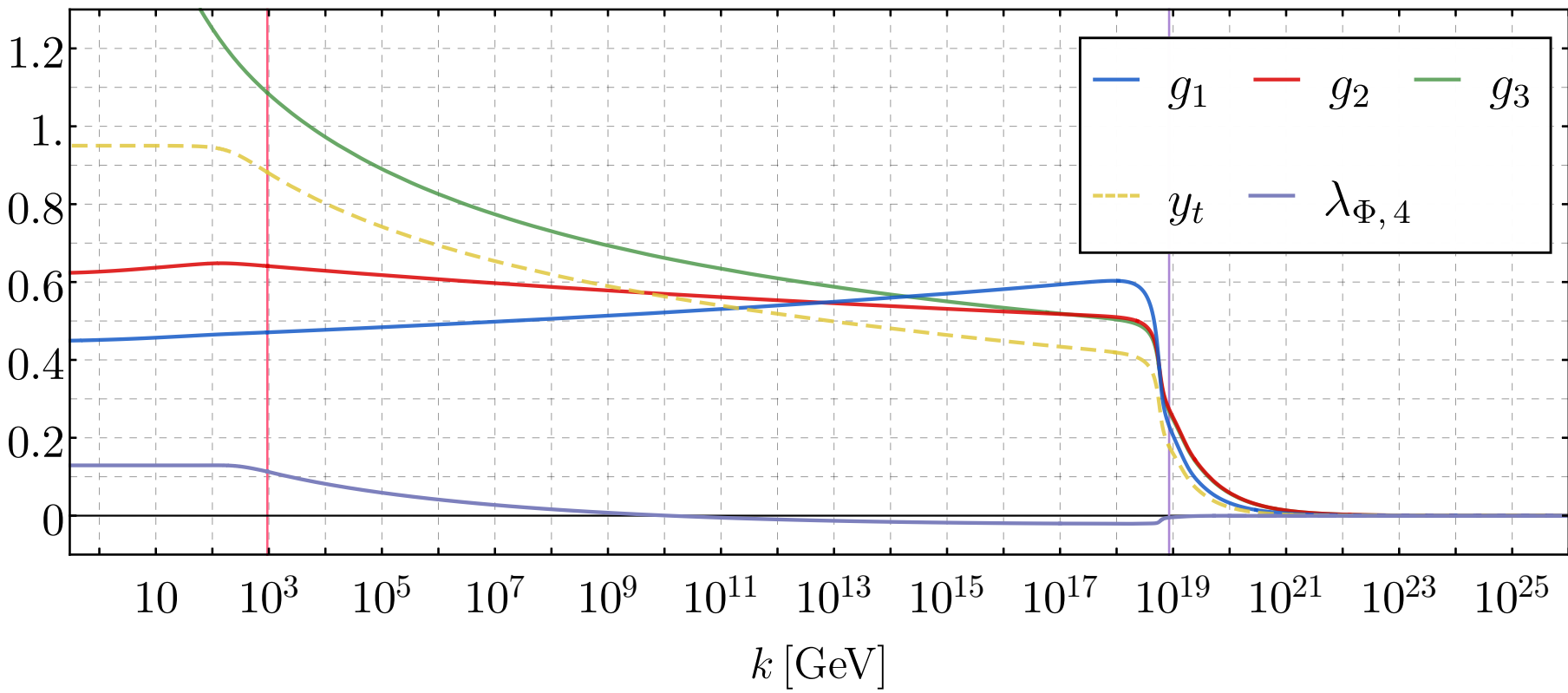
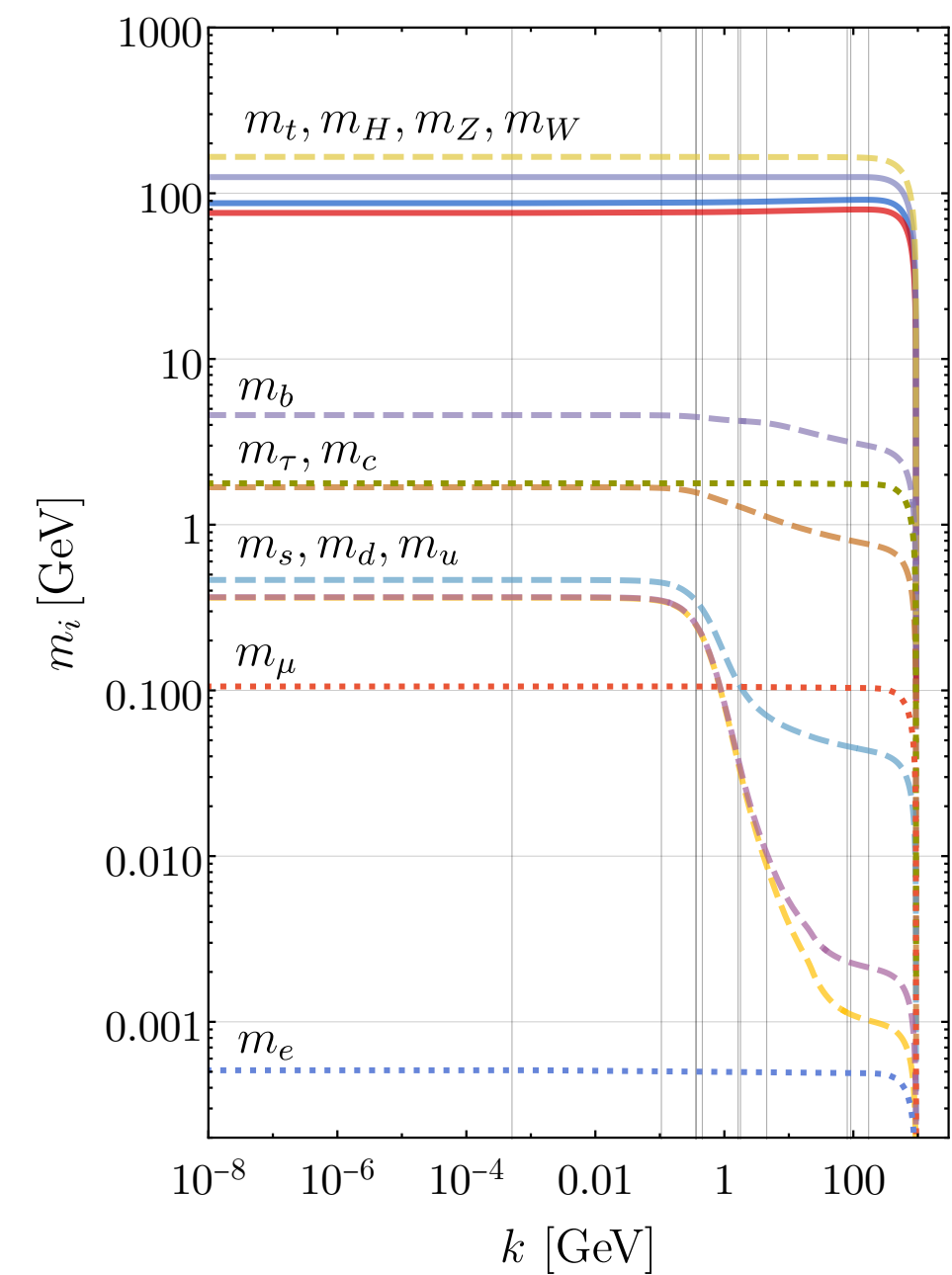


top pole mass (getting real)

$$M_{t,\text{pole}}^{(\text{exp})} = 172.5 \pm 0.7 \text{ GeV}$$

Experimental value (PDG)

Asymptotically safe Standard Model

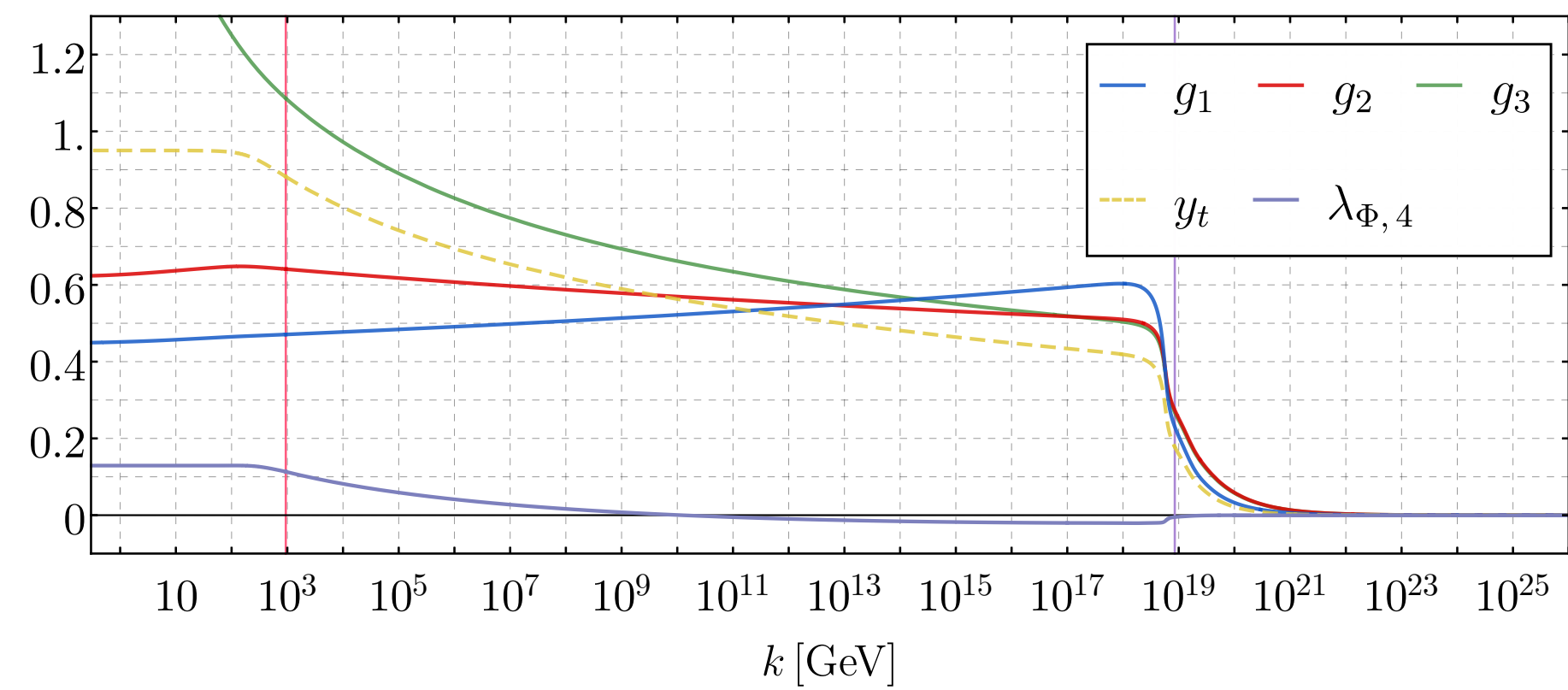
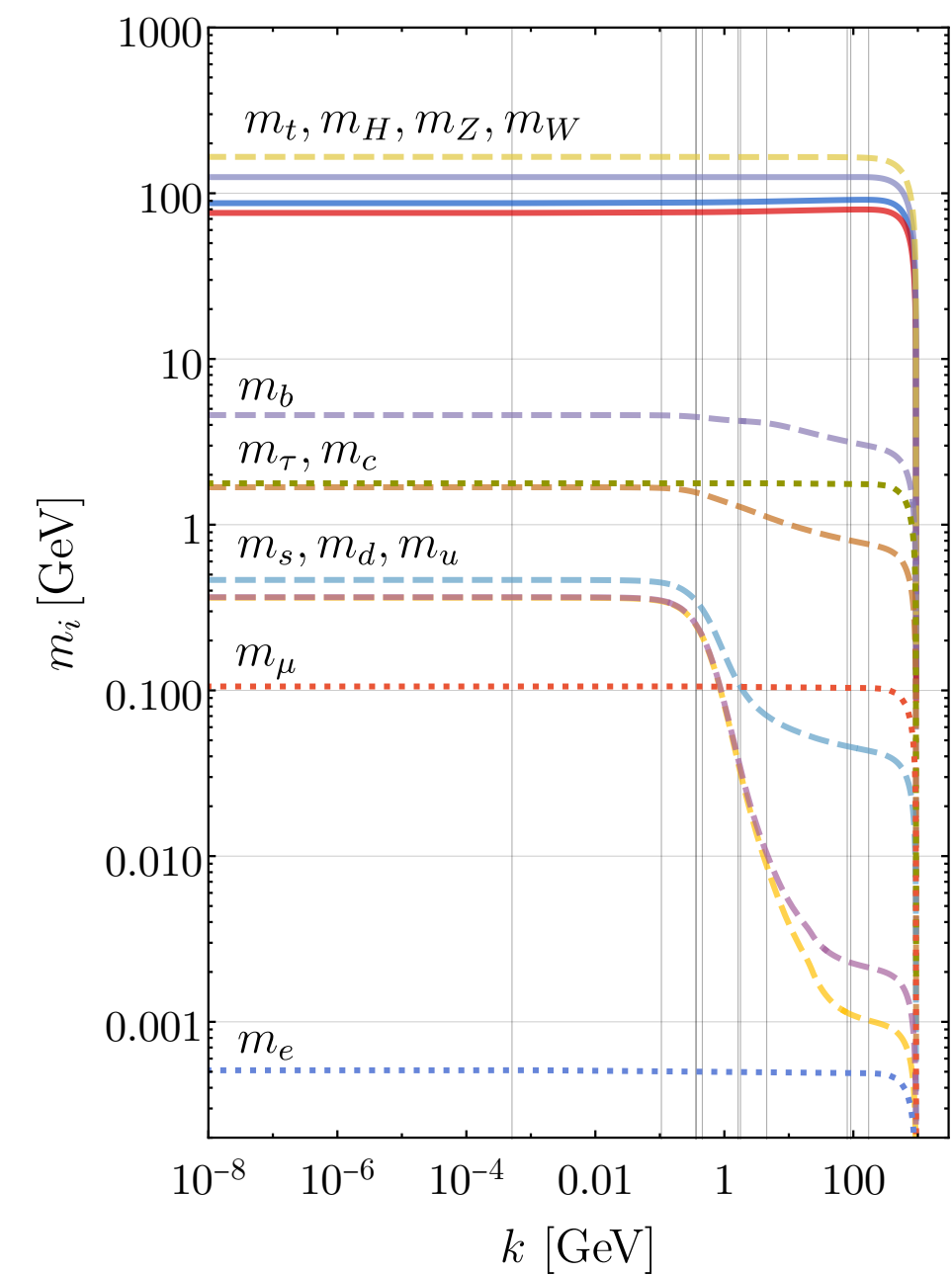


top pole mass (getting real)

$$M_{t,\text{pole}}^{(\text{exp})} = 172.5 \pm 0.7 \text{ GeV} \quad \leftarrow \quad m_t = 165.4_{-0.2}^{+0.9} \text{ GeV}$$

Experimental value (PDG) Euclidean curvature mass

Asymptotically safe Standard Model



top pole mass (getting real)

$$M_{t,\text{pole}}^{(\text{exp})} = 172.5 \pm 0.7 \text{ GeV}$$

Experimental value (PDG)

$$m_t = 165.4^{+0.9}_{-0.2} \text{ GeV}$$

Euclidean curvature mass

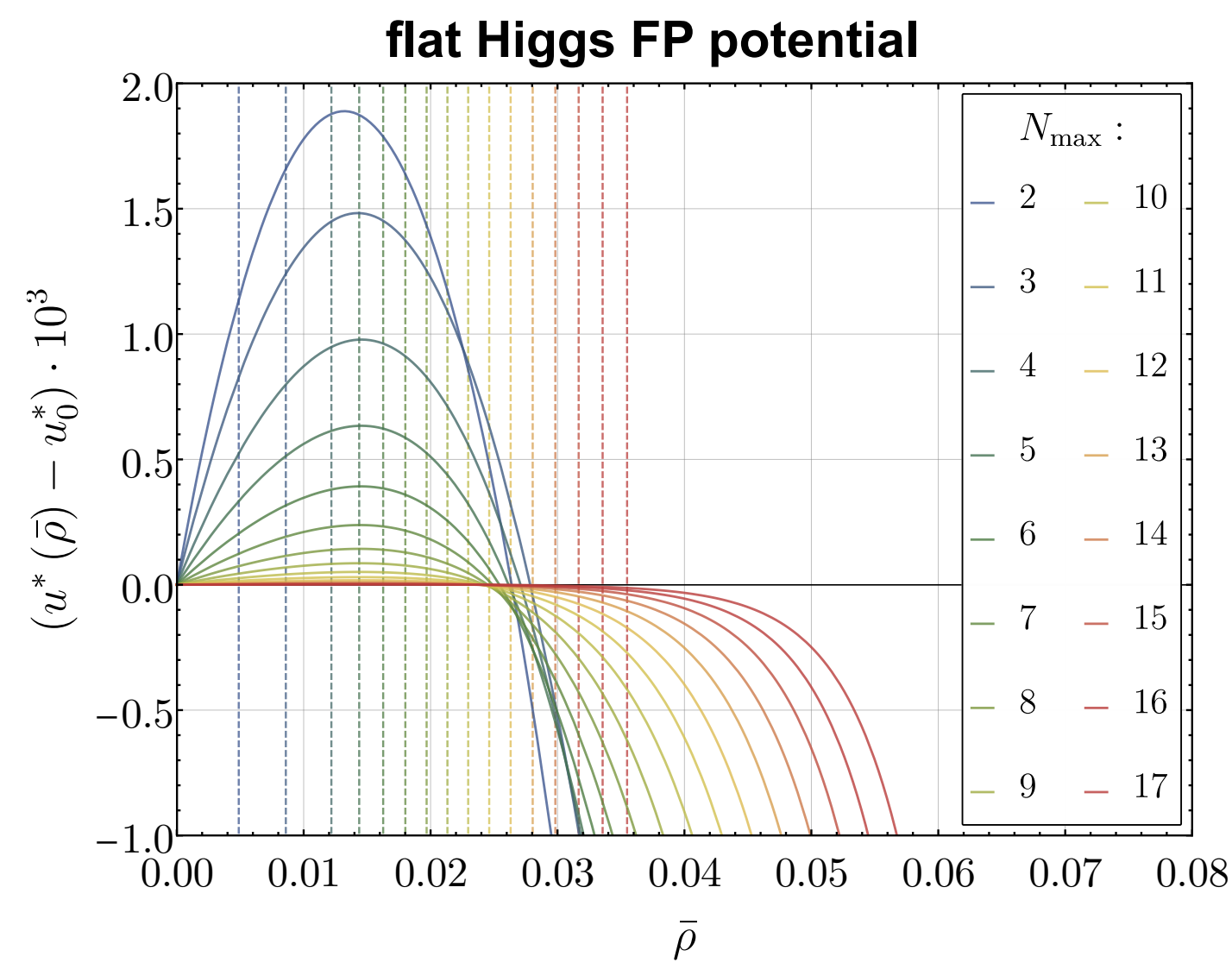
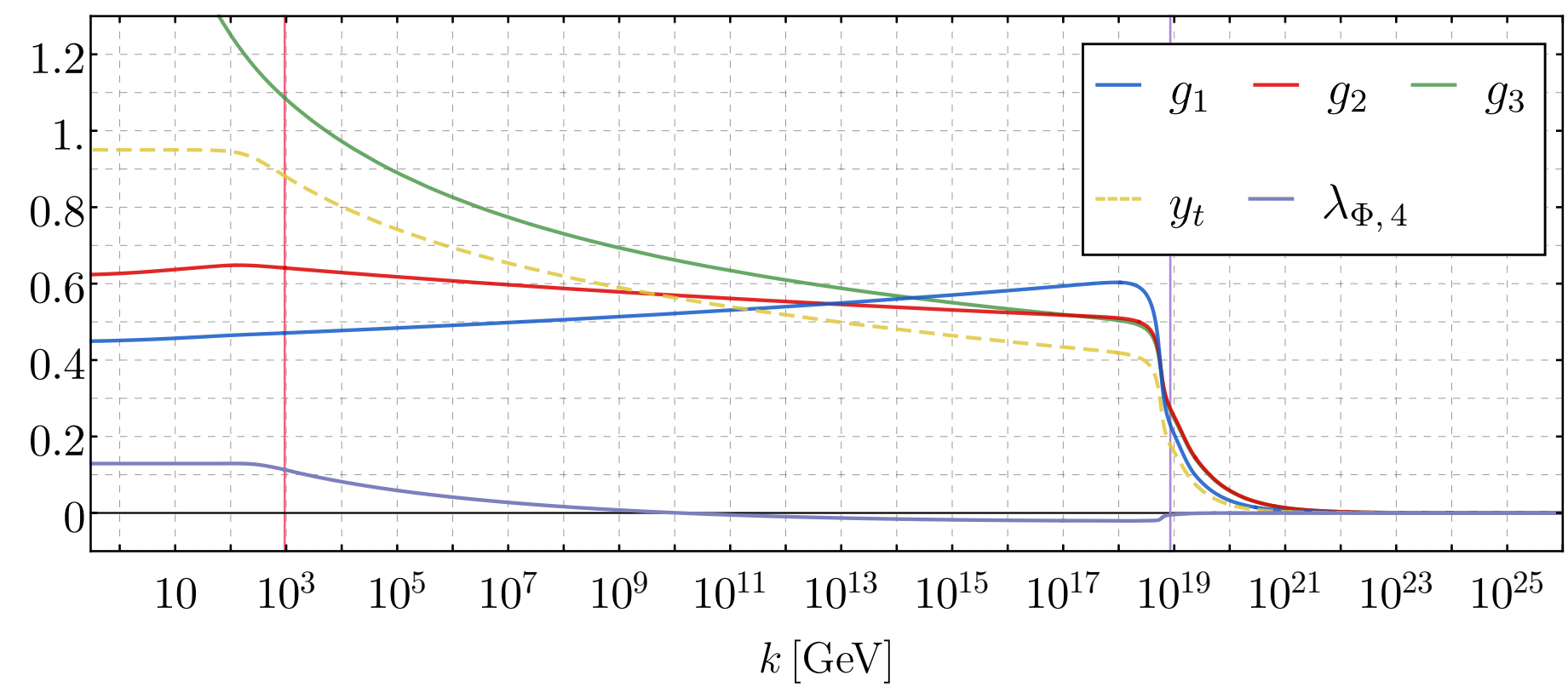
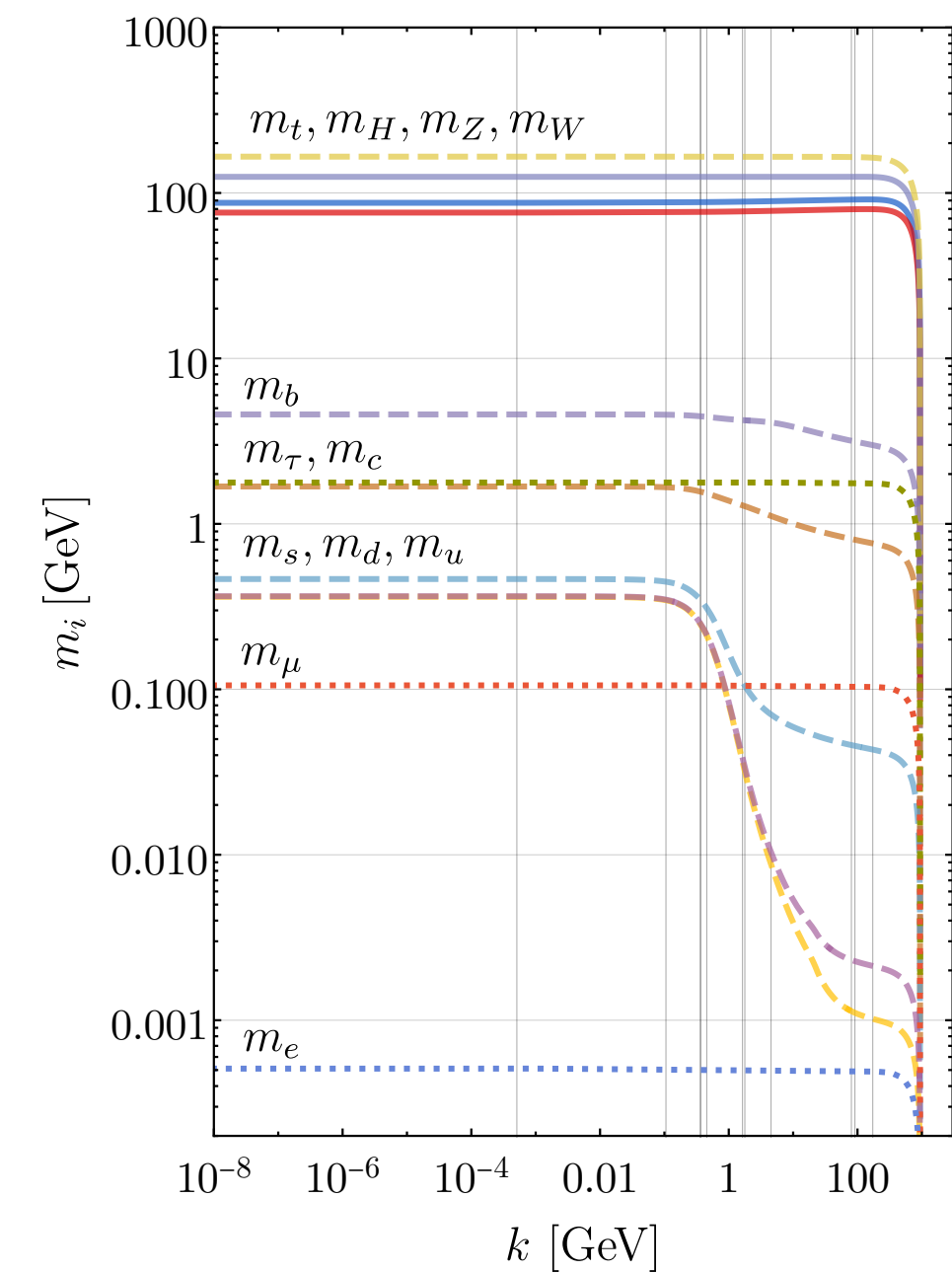
Prediction of decay width

$$\Gamma_{t,\text{pole}}^{(\text{theo})} = 1.72^{+0.09}_{-0.41} \text{ GeV}$$

$$\Gamma_{t,\text{pole}}^{(\text{exp})} = 1.42^{+0.19}_{-0.15} \text{ GeV}$$

Experimental value (PDG)

Asymptotically safe Standard Model



Two relevant directions

top pole mass (getting real)

$$M_{t,\text{pole}}^{(\text{exp})} = 172.5 \pm 0.7 \text{ GeV} \quad \leftarrow \quad m_t = 165.4_{-0.2}^{+0.9} \text{ GeV}$$

Experimental value (PDG)

Euclidean curvature mass

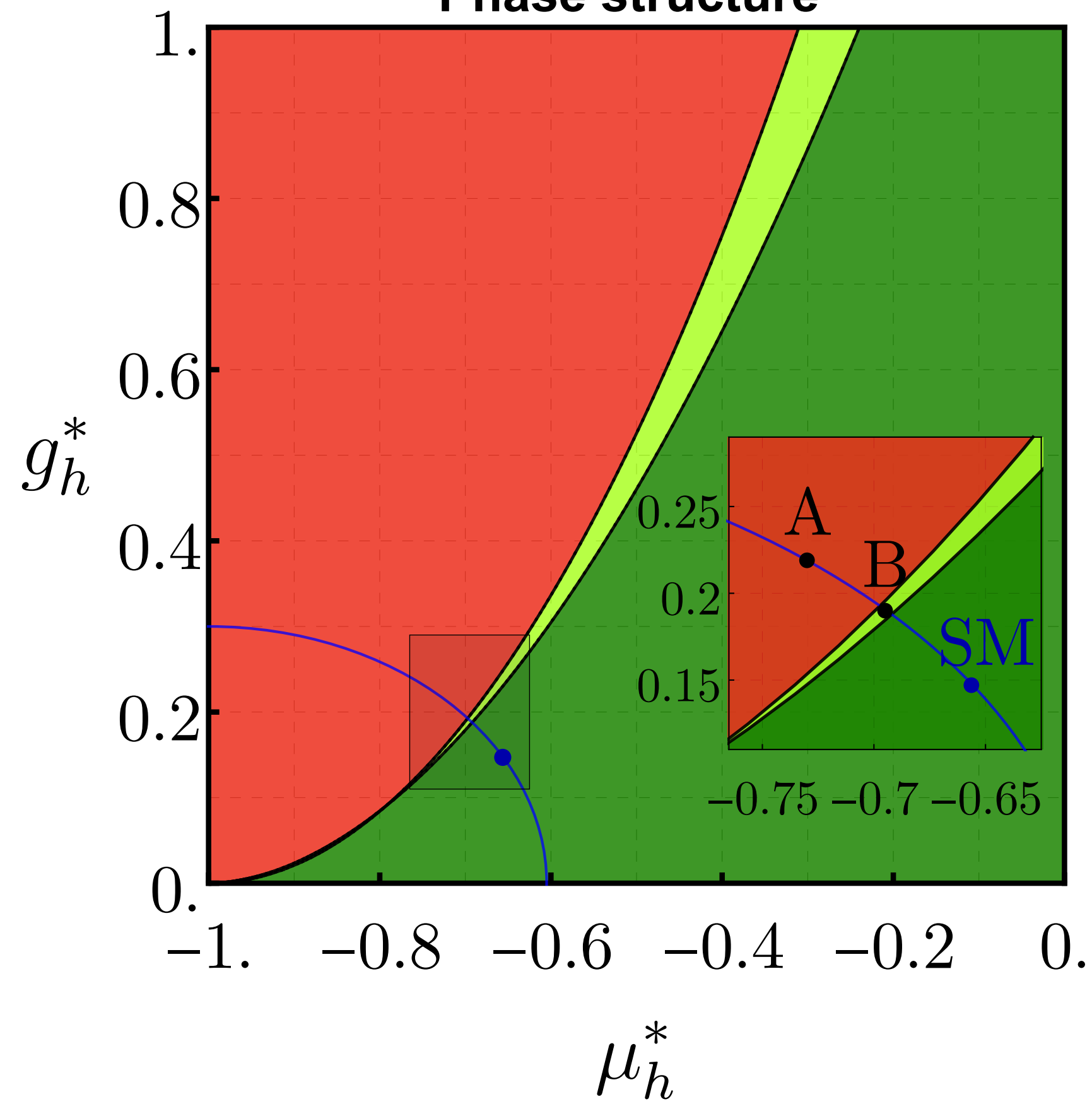
Prediction of decay width

$$\Gamma_{t,\text{pole}}^{(\text{theo})} = 1.72_{-0.41}^{+0.09} \text{ GeV} \quad \Gamma_{t,\text{pole}}^{(\text{exp})} = 1.42_{-0.15}^{+0.19} \text{ GeV}$$

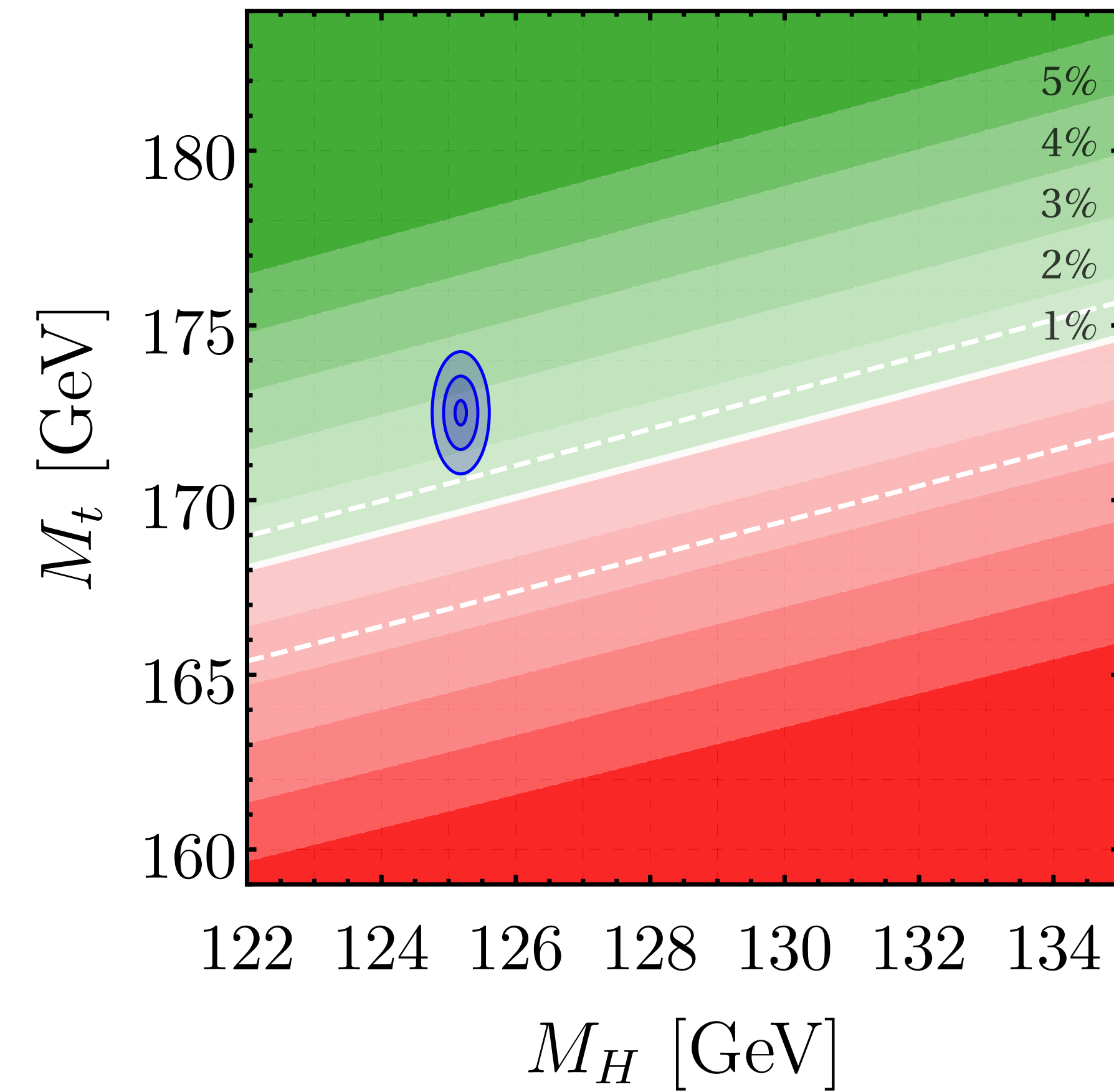
Experimental value (PDG)

Asymptotically safe Standard Model

Phase structure



Where are we?



Summary

Summary

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \left(\text{orange wavy circle} - \text{dashed circle} - \text{solid black circle} + \frac{1}{2} \text{blue double circle with arrow} + \frac{1}{2} \text{blue double circle with cross} - \text{red dotted circle with arrows} \right)$$

Getting real I: Apparent convergence

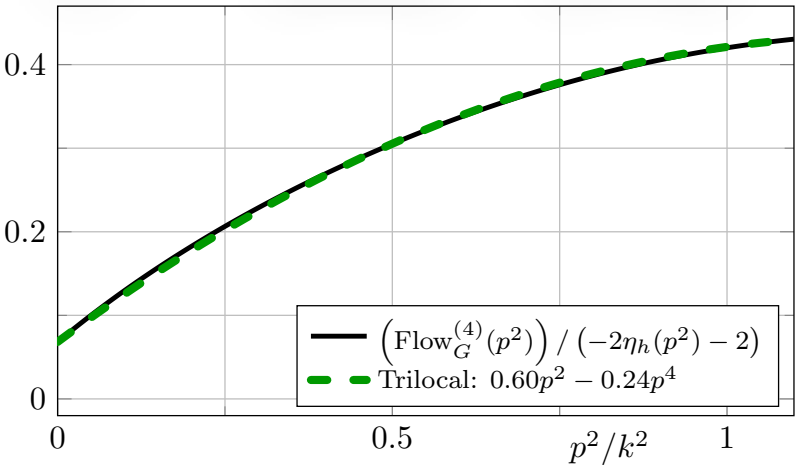
Getting real II: Scattering & spectral properties

Summary

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \text{ (orange wavy loop) } - \text{ (dashed black loop) } - \text{ (solid black loop) } + \frac{1}{2} \text{ (blue loop with arrow) } + \frac{1}{2} \text{ (double blue loop) } - \text{ (red dotted loop) }$$

Getting real I: Apparent convergence

$$\begin{aligned} \partial_t \Gamma_k &= \frac{1}{2} \text{ (blue loop) } - \text{ (red dotted loop) } \\ \partial_t \Gamma_k^{(h)} &= -\frac{1}{2} \text{ (blue loop with arrow) } + \text{ (red dotted loop with arrow) } \\ \partial_t \Gamma_k^{(2h)} &= -\frac{1}{2} \text{ (blue loop with arrow) } + \text{ (blue loop with arrow) } - 2 \text{ (red dotted loop) } \\ \partial_t \Gamma_k^{(c\bar{c})} &= \text{ (blue loop with arrow) } + \text{ (red dotted loop with arrow) } \\ \partial_t \Gamma_k^{(3h)} &= -\frac{1}{2} \text{ (blue loop with arrow) } + 3 \text{ (blue loop with arrow) } - 3 \text{ (blue loop with arrow) } + 6 \text{ (red dotted loop) } \\ \partial_t \Gamma_k^{(4h)} &= -\frac{1}{2} \text{ (blue loop with arrow) } + 3 \text{ (blue loop with arrow) } + 4 \text{ (blue loop with arrow) } - 6 \text{ (blue loop with arrow) } \\ &\quad - 12 \text{ (blue loop with arrow) } + 12 \text{ (blue loop with arrow) } - 24 \text{ (red dotted loop) } \end{aligned}$$



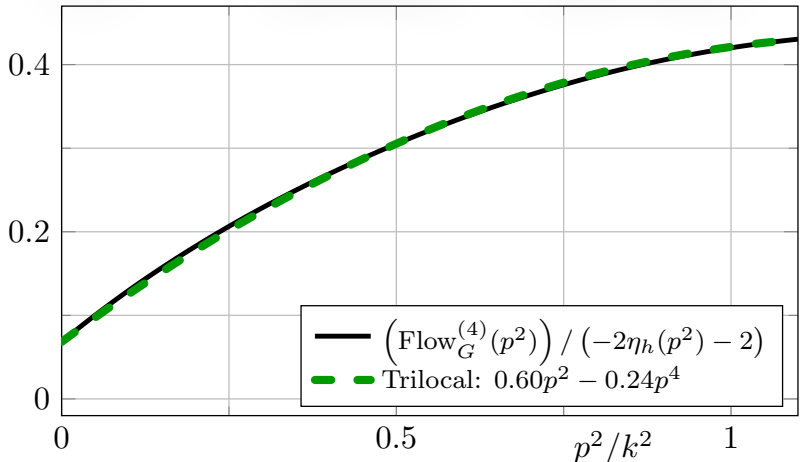
Getting real II: Scattering & spectral properties

Summary

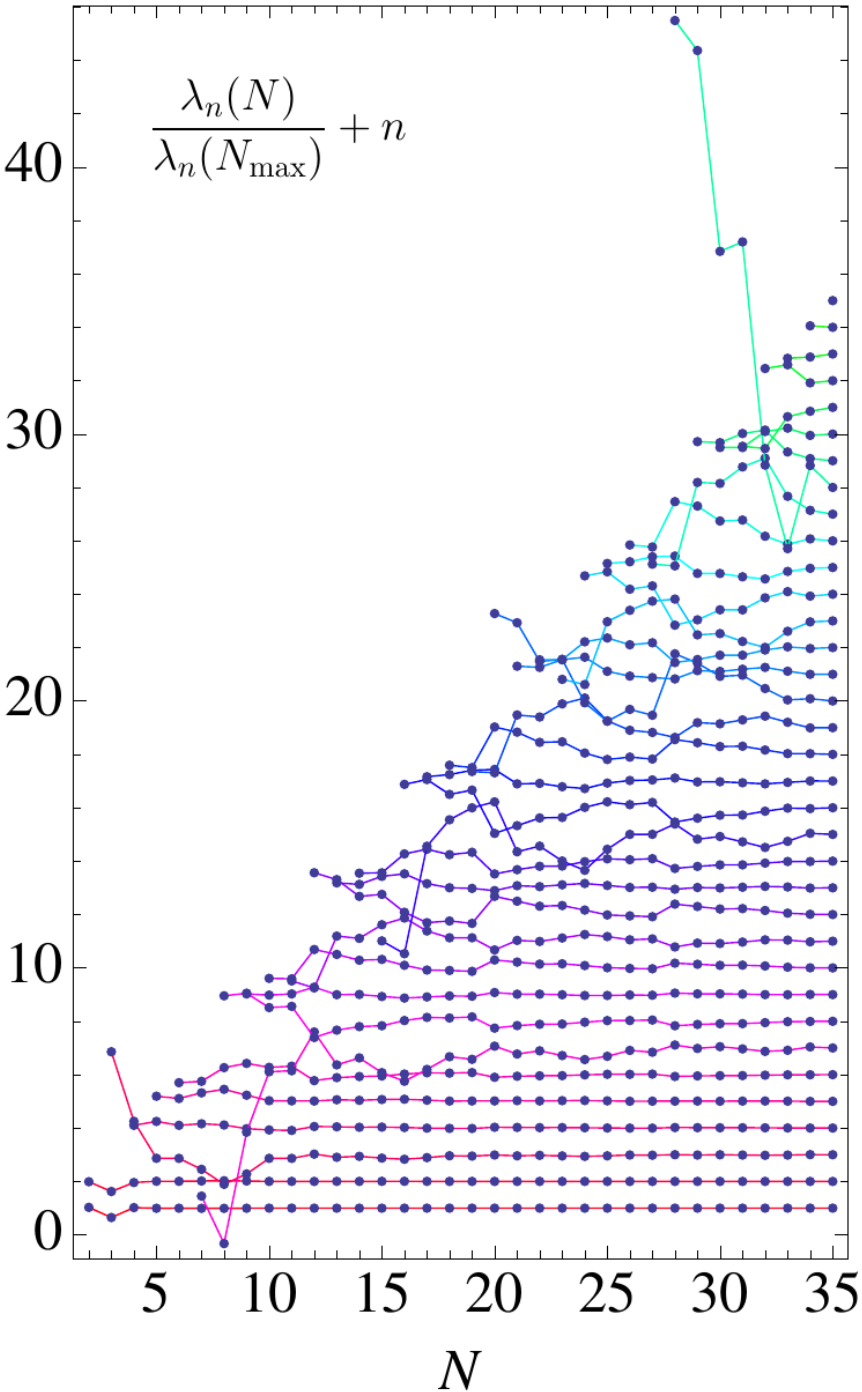
$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \left(\text{orange loop} - \text{dashed loop} - \text{solid black loop} + \frac{1}{2} \text{blue loop with arrow} + \frac{1}{2} \text{double blue loop} - \text{red dotted loop} \right)$$

Getting real I: Apparent convergence

$$\begin{aligned} \partial_t \Gamma_k &= \frac{1}{2} \left(\text{blue loop} - \text{red dotted loop} \right) \\ \partial_t \Gamma_k^{(h)} &= -\frac{1}{2} \left(\text{blue loop} + \text{red dotted loop} \right) \\ \partial_t \Gamma_k^{(2h)} &= -\frac{1}{2} \left(\text{blue loop} + \text{blue loop} - 2 \text{ red dotted loop} \right) \\ \partial_t \Gamma_k^{(c\bar{c})} &= \text{red dotted loop} + \text{red dotted loop} \\ \partial_t \Gamma_k^{(3h)} &= -\frac{1}{2} \left(\text{blue loop} + 3 \text{ blue loop} - 3 \text{ blue loop} + 6 \text{ red dotted loop} \right) \\ \partial_t \Gamma_k^{(4h)} &= -\frac{1}{2} \left(\text{blue loop} + 3 \text{ blue loop} + 4 \text{ blue loop} - 6 \text{ blue loop} \right. \\ &\quad \left. - 12 \text{ blue loop} + 12 \text{ blue loop} - 24 \text{ red dotted loop} \right) \end{aligned}$$



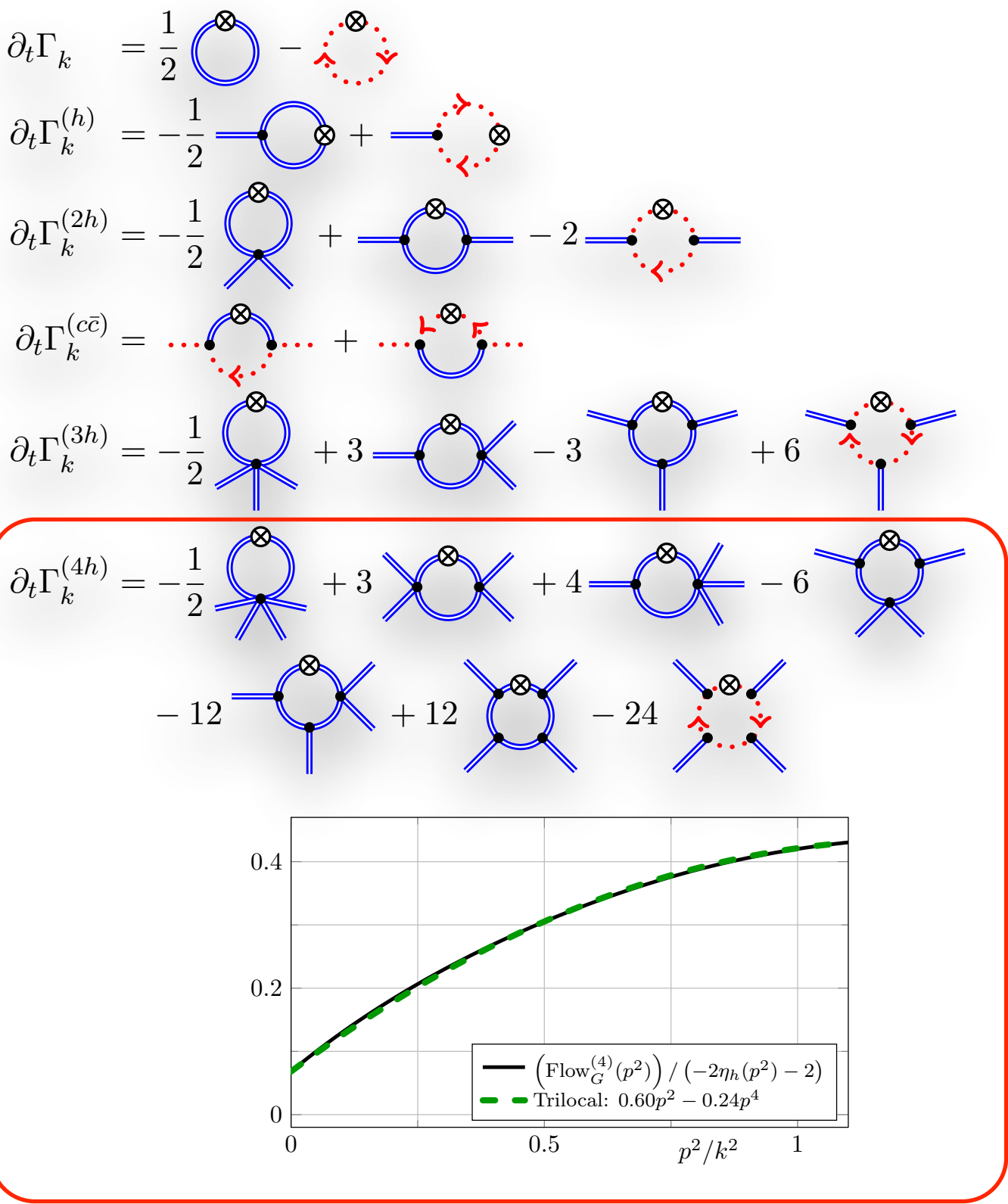
Getting real II: Scattering & spectral properties



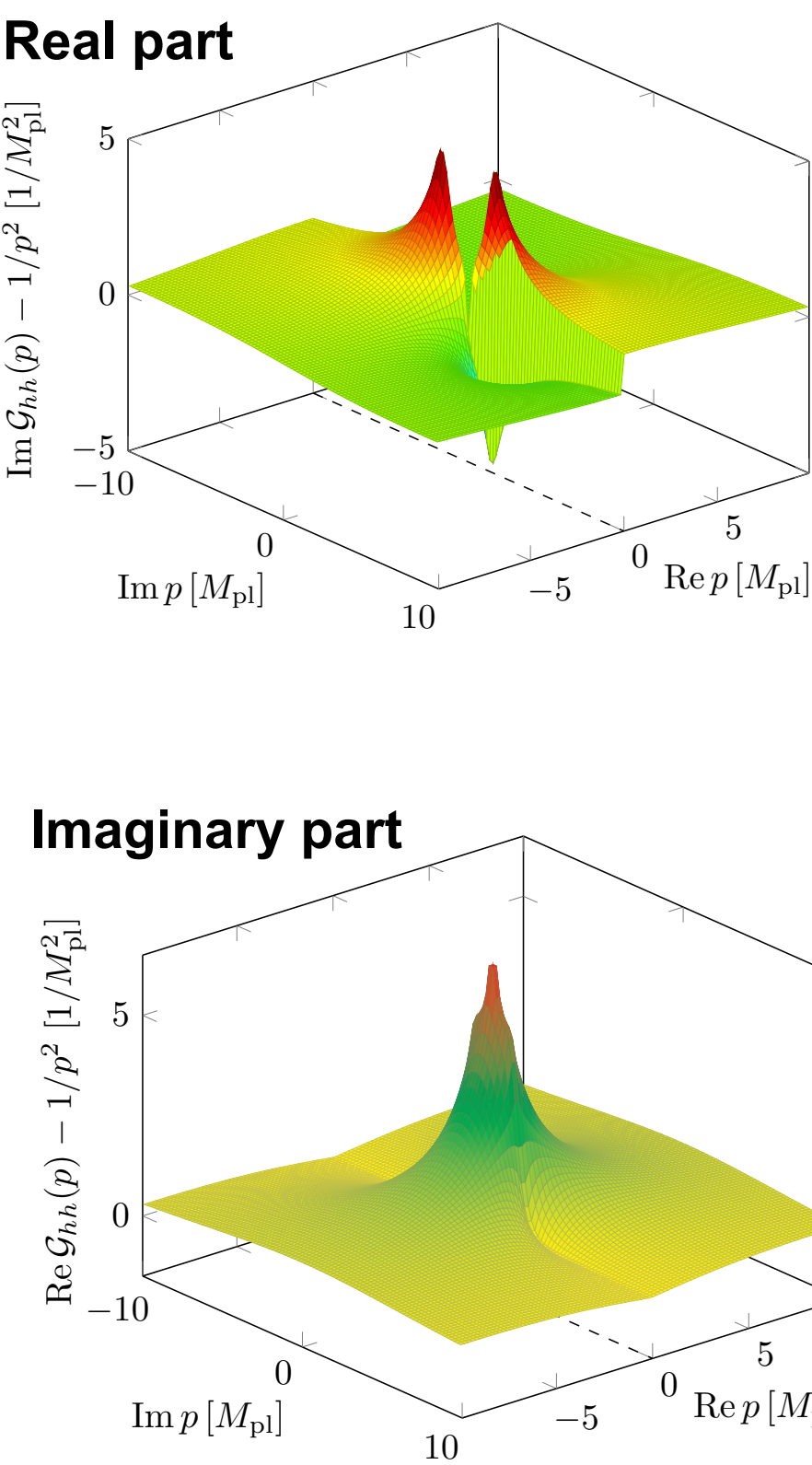
Summary

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \text{ (orange wavy loop) } - \text{ (dashed black loop) } - \text{ (solid black loop) } + \frac{1}{2} \text{ (blue double loop with arrow) } + \frac{1}{2} \text{ (blue double loop) } - \text{ (red dotted loop) }$$

Getting real I: Apparent convergence



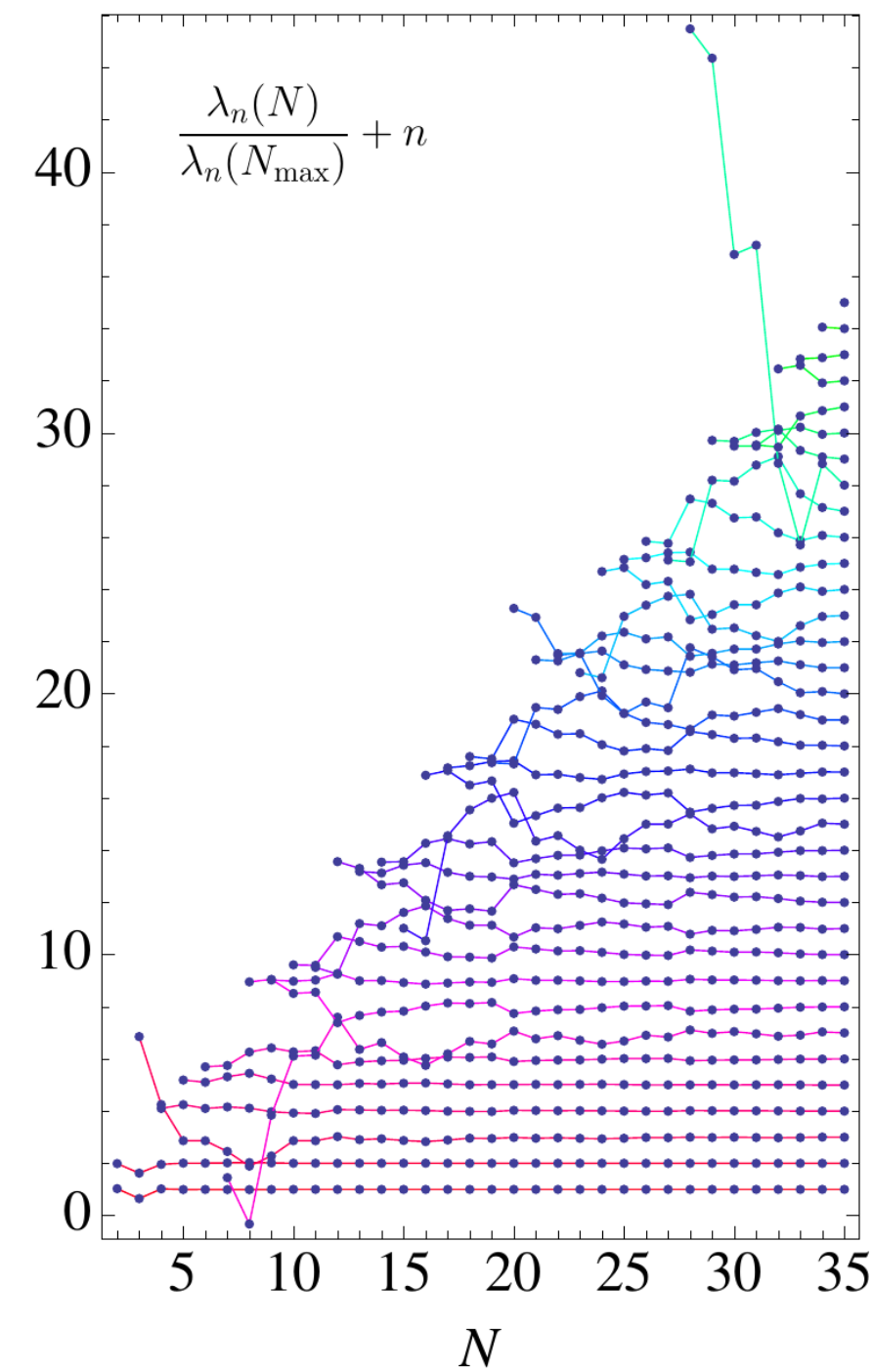
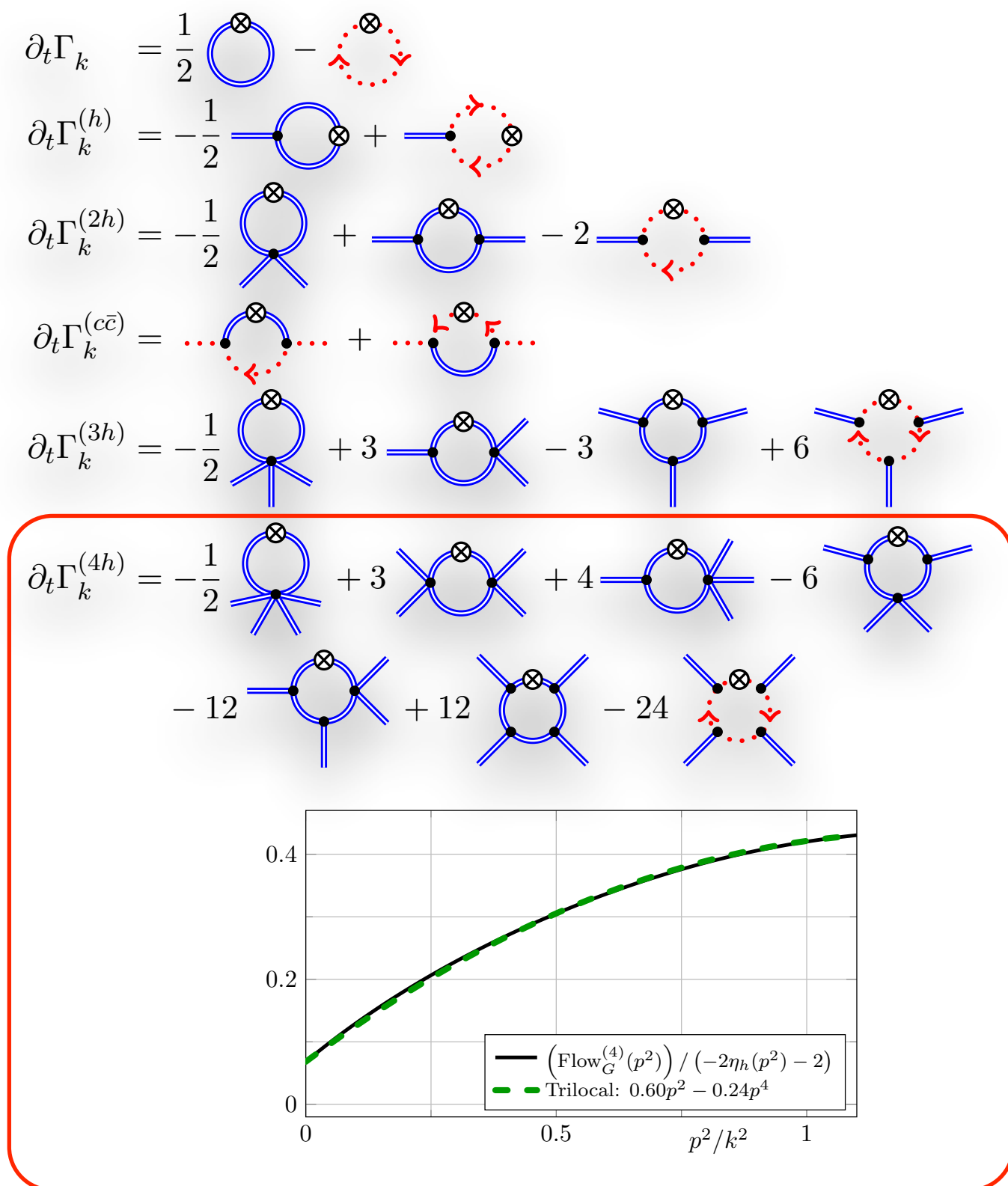
Getting real II: Scattering & spectral properties



Summary

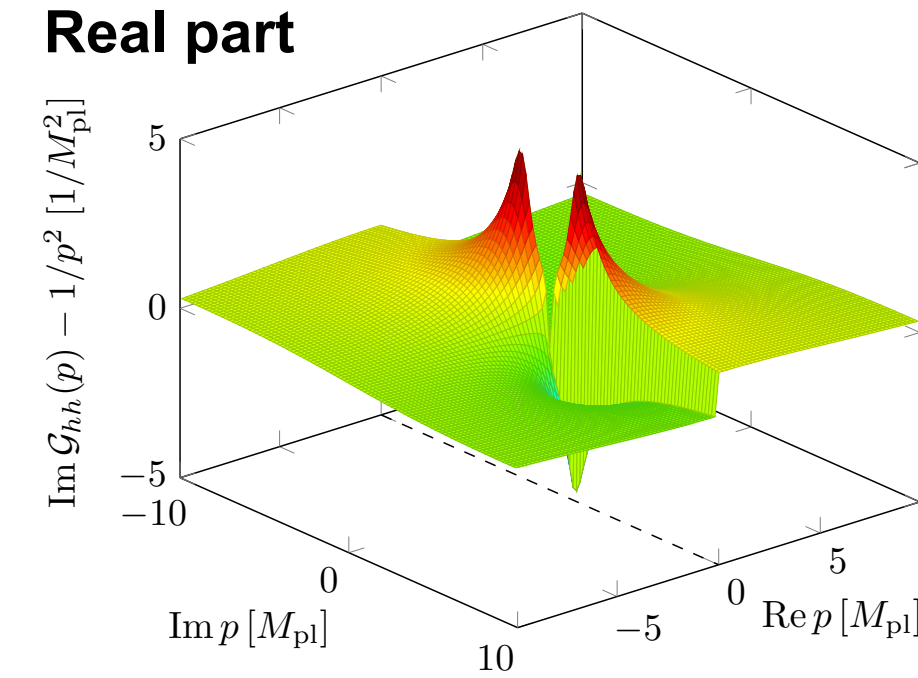
$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \left(\text{orange loop} - \text{dashed loop} - \text{solid loop} + \frac{1}{2} \text{blue loop with arrow} + \frac{1}{2} \text{blue loop with cross} - \text{red loop with cross} \right)$$

Getting real I: Apparent convergence

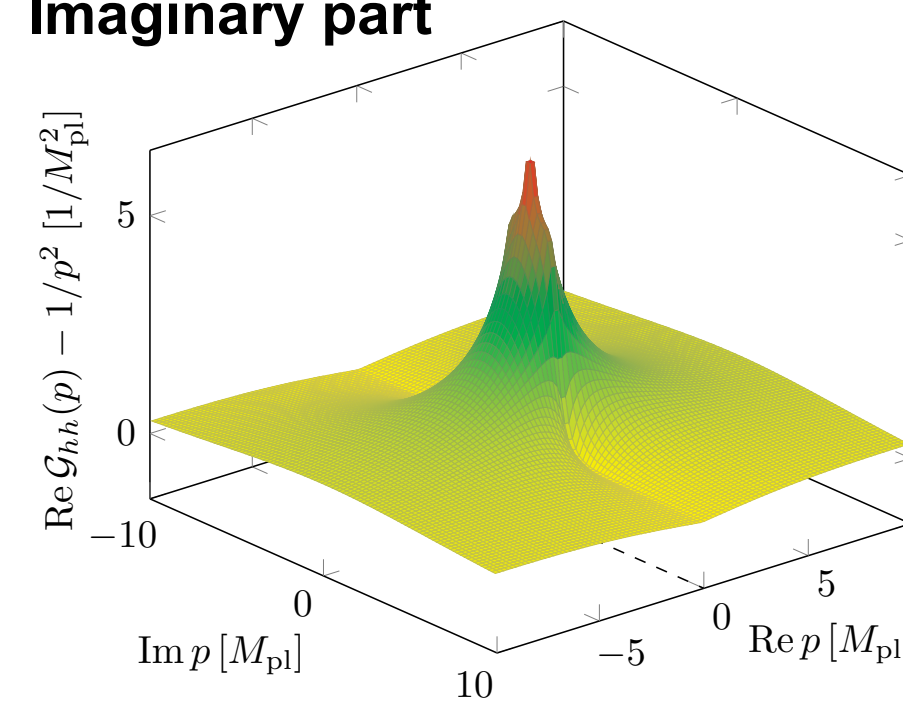


Getting real II: Scattering & spectral properties

Real part

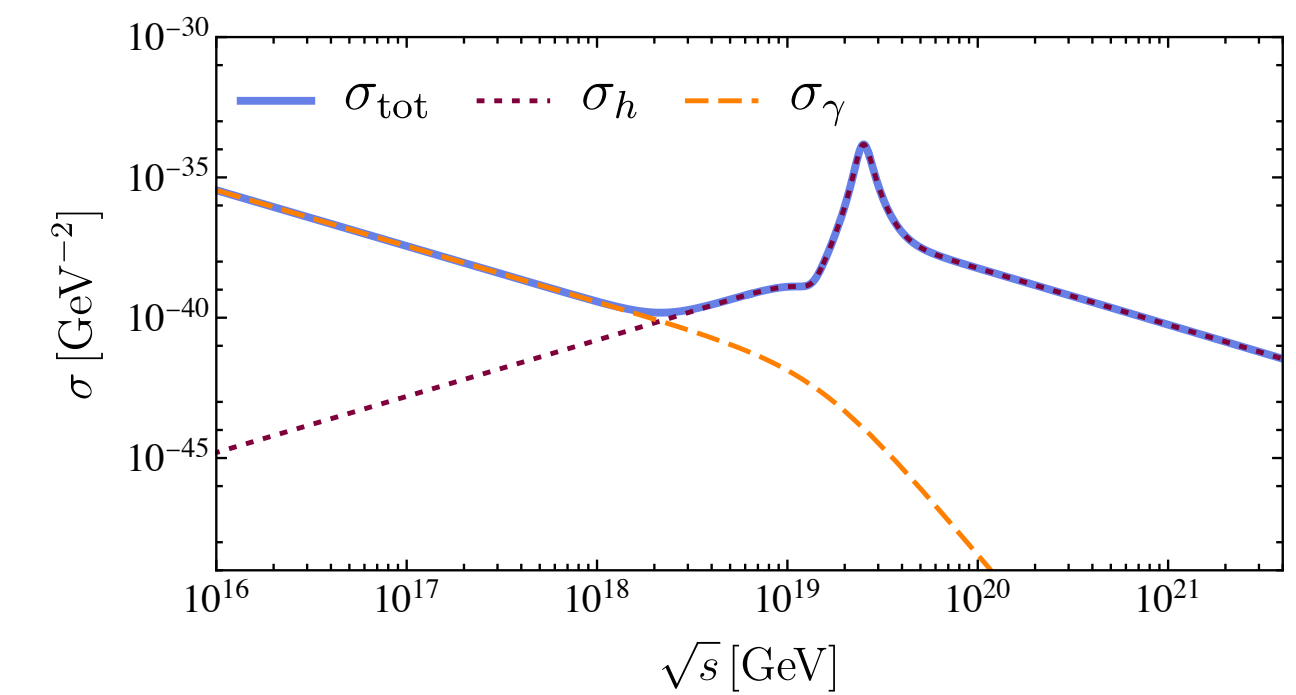


Imaginary part



Scattering process diagram:

$$e^- \mu^- \rightarrow e^+ \mu^+ \approx \text{photon exchange} + \text{Higgs exchange}$$



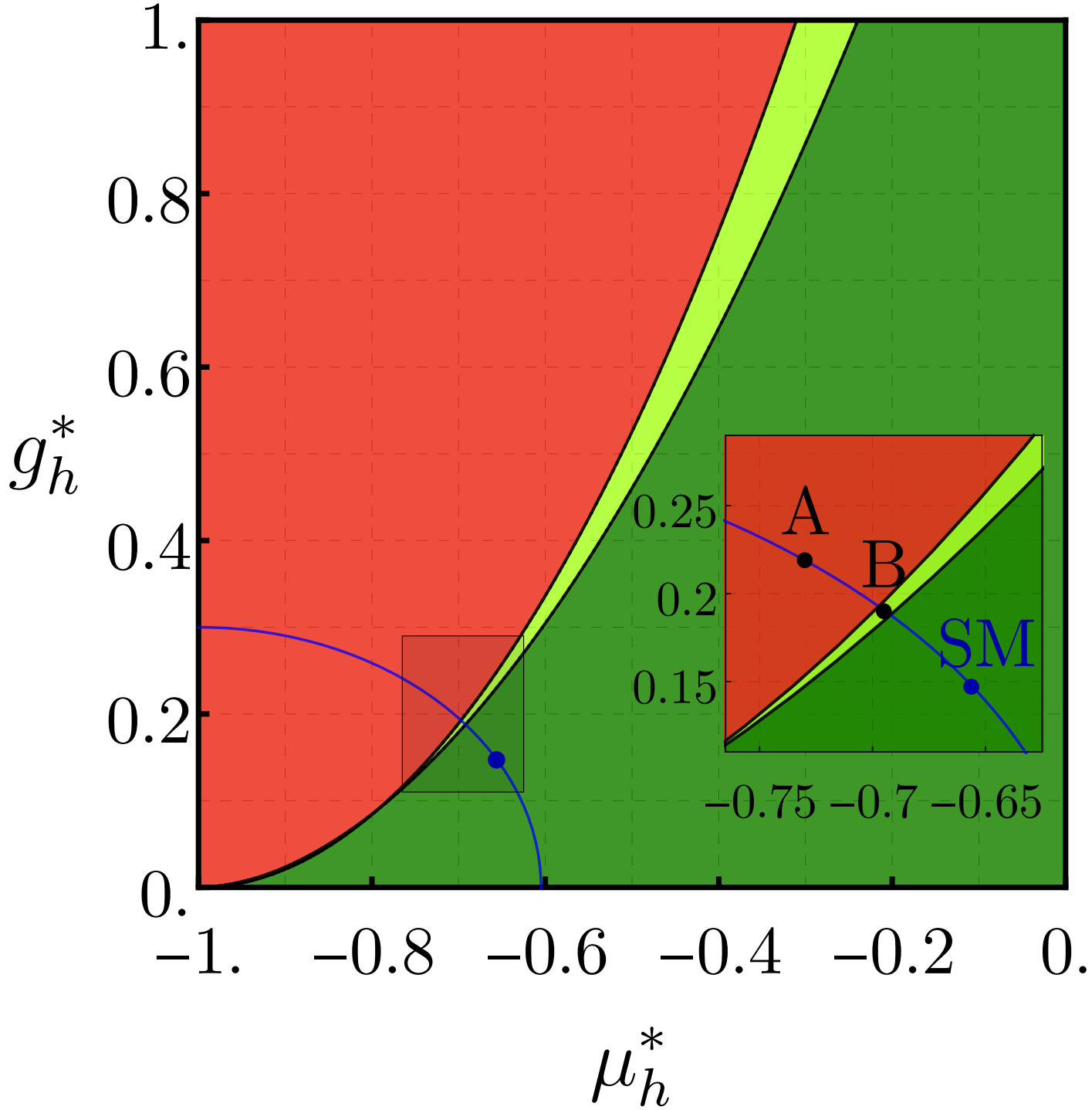
Summary

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \text{ (orange loop) } - \text{ (dashed loop) } - \text{ (solid loop) } + \frac{1}{2} \text{ (blue double loop with arrow) } + \frac{1}{2} \text{ (blue double loop with cross) } - \text{ (red dotted loop with cross) }$$

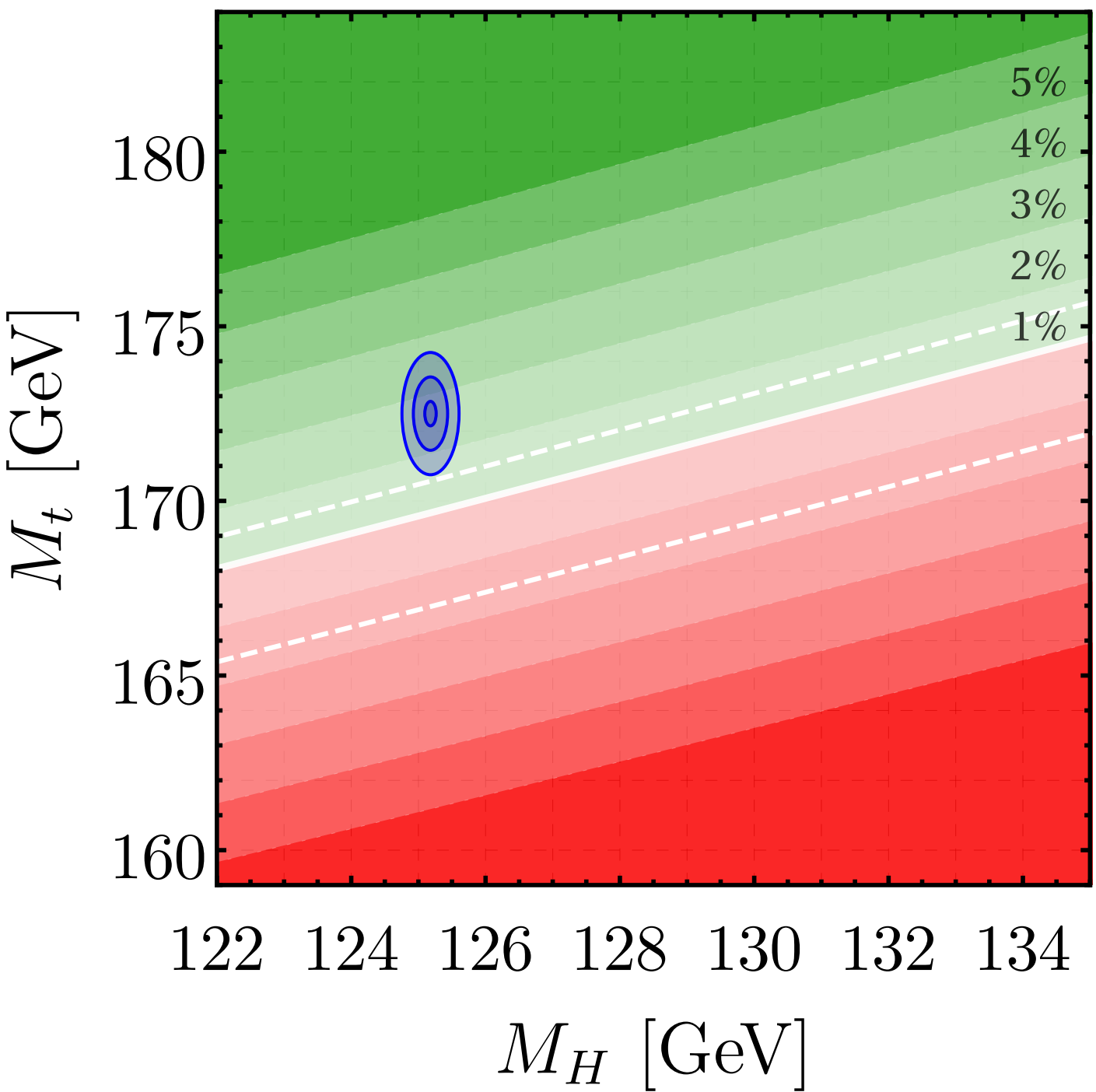
Asymptotically safe SM

Quantum black holes

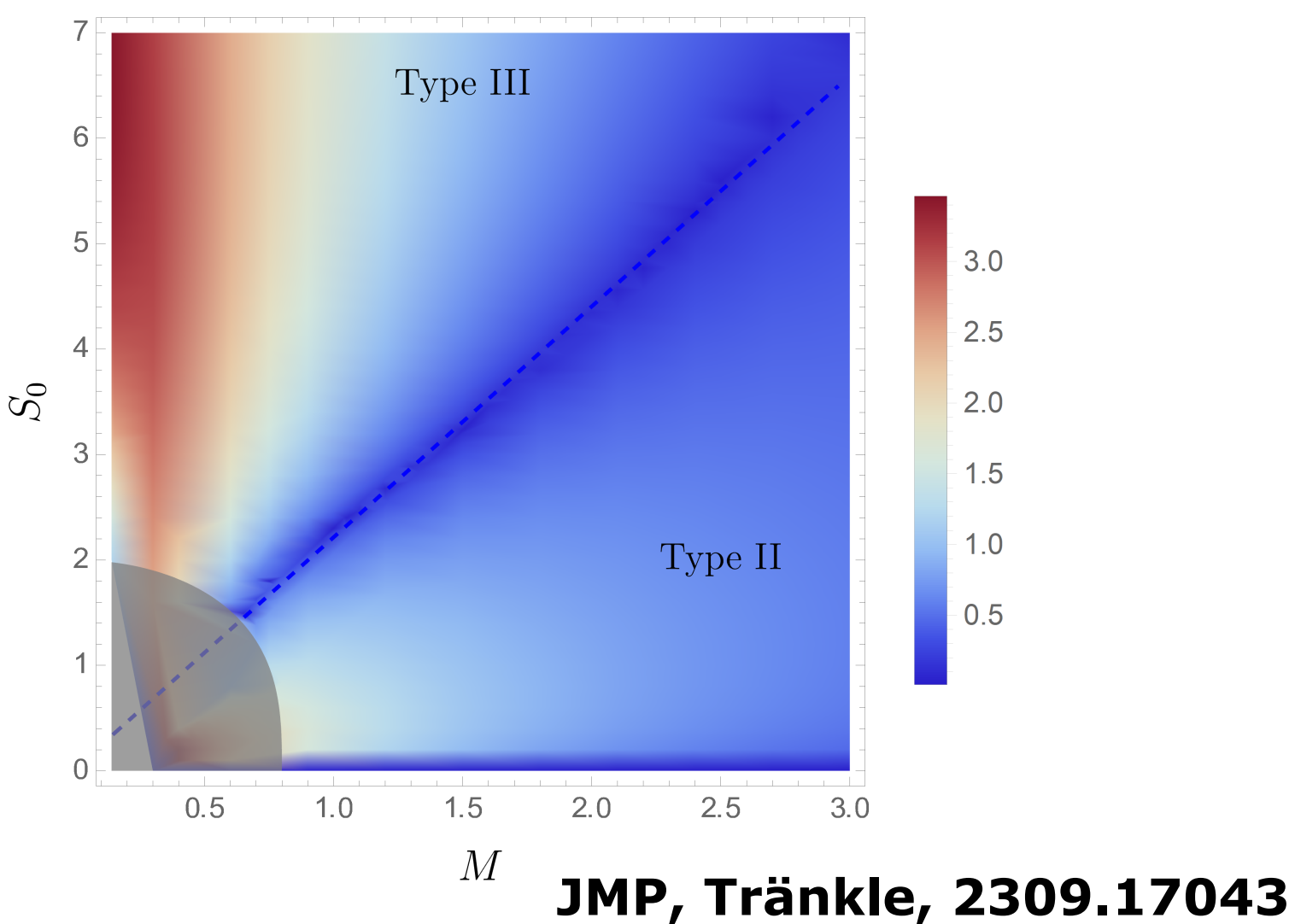
Phase structure



Where are we?



Phase structure



JMP, Tränkle, 2309.17043