

Theoretische Physik III (Lehramt)

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Übungsblatt 3

Summe der Punkte: 20 + 6 Extrapunkte

Abgabe am Donnerstag, dem 05.05.2011 zu Beginn der Vorlesung.**Aufgabe 3.1** Eine elektrische Ladungsverteilung erzeugt ein elektrisches Feld

$$\vec{E}(\vec{r}) = c(1 - e^{-\alpha r}) \frac{\vec{r}}{r^3}$$

wobei c und α Konstanten sind ($\alpha > 0$). Bestimmen Sie die innerhalb des Radius $r = 1/\alpha$ befindliche Gesamtladung. **(6 Punkte)**

Aufgabe 3.2 *Mittelwerteigenschaft harmonischer Funktionen* **(8 Punkte)**

- a) Beweisen Sie, dass im ladungsfreien Raum der Wert des elektrostatischen Potentials $\phi(\vec{x})$ an einem Aufpunkt \vec{x} gleich dem Mittelwert des Potentials über jede Kugeloberfläche mit Mittelpunkt \vec{x} ist. **(4 Punkte)**

(*Tipp: Verwenden Sie den Satz von Green mit $\psi_1 = \phi$ und $\psi_2 = \frac{1}{|\vec{x}-\vec{x}'|}$.)*)

- b) Zeigen Sie damit, dass eine beliebige Anordnung von endlich vielen Punktladungen sich nicht in einem **stabilen** Gleichgewicht befinden kann (**Earnshaw Theorem**). **(4 Punkte)**
 (*Tipp: Untersuchen Sie, ob sich eine beliebig herausgegriffene Punktladung in einem Minimum des Potentials aller anderen Punktladungen befinden kann.*)

Aufgabe 3.3: *Homogen geladene Vollkugel, klassischer Elektronenradius* **(6 Punkte)**

- a) Betrachten Sie eine homogen geladene Vollkugel mit der Gesamtladung Q . Berechnen Sie das Potential ϕ und die elektrische Feldstärke \vec{E} im Inneren und Äußeren der Vollkugel und skizzieren Sie $\phi(r)$ und $E_r(r) = \vec{E}(\vec{x}) \cdot \frac{\vec{x}}{r}$ als Funktion von $r = |\vec{x}|$. **(2 Punkte)**
- b) Berechnen Sie die elektrostatische Feldenergie der homogen geladenen Vollkugel. **(2 Punkte)**
- c) Nehmen Sie an, das Elektron wäre eine homogen geladene Vollkugel. Was ergibt sich in diesem Fall für den Radius r_e des Elektrons, wenn man die gesamte Ruheenergie als elektrostatisch Energie deutet? **(2 Punkte)**

Freiwillige Zusatzaufgabe (6 Extrapunkte):**Aufgabe 3.4:** *Punktladung vor einer unendlich ausgedehnten Metallplatte* **(6 Extrapunkte)**

Eine Ladung q befinde sich im Abstand $a/2$ vor einer elektrisch leitfähigen, geerdeten Platte (von unendlicher Ausdehnung). Berechnen Sie das elektrostatische Potential dieser Anordnung. Legen Sie dazu das Koordinatensystem so, daß die Platte in der durch $x = 0$ festgelegten Ebene und die Ladung auf der x -Achse liegt.

Tipp: Auf der Leiteroberfläche verschwindet die Tangentialkomponente des elektrischen Feldes.

Samuel Earnshaw (1805-1888), Cleric and Mathematical Physicist Originator of *Earnshaw's Theorem*

Samuel Earnshaw was born February 1, 1805, to Joseph Earnshaw, a file cutter of Sheffield, and his wife Martha. When Samuel was about 13, his father became master of a Sheffield school. Having studied at the Carver St. National School and under the Rev. William Ellis at nearby Doncaster, he was admitted as Pensioner to St. John's College, Cambridge, at the age of 22 on 5 July, 1827, with Mr. Tatham as his tutor.

As a Pensioner Earnshaw would have been responsible for meeting his own expenses. Presumably this was not possible, even with some help from several Sheffield gentlemen, so he was not, in fact, recorded as completing a term of residence until Michaelmas (Fall) term of 1828, after he had been readmitted as Sizar on 29 January, 1828. A Sizar seved the Fellows at High Table and was entitled to dine on their left-overs. Earnshaw would spend the next 19 years in Cambridge.

During his undergraduate years he progressed steadily from Sizar to Exhibitioner to Scholar. From Midsummer 1829 through Midsummer 1830 he held three Exhibitioner appointments (Hewytt's, Allott's, Baker), and in November 1830 was admitted Hebblethwaite Scholar. Having coached with John Birkett he finished as the top mathematician, becoming Senior Wrangler and 1st Smith's Prizeman in 1831.

He continued studying for the M.A. (1834), and under normal circumstances would have been a



A treatise on statics containing the theory of the equilibrium of forces; and numerous examples illustrative of the general principles of the science
3rd ed. enlarged 270 pp. (1845)

Although the monographs on dynamics and statics went through three and four editions, respectively, the circulation of his works outside Cambridge may have been limited. The on-line Oxford Library catalogue lists only one Earnshaw holding (Dynamics 1844). Leeds and Edinburgh each list 3; Dublin, 1. The Cambridge Libraries list 11 (mostly in St. John's College Library), and the British Library lists 16. Harvard lists four (Dynamics 1844, Statics 1858, Partial Differential Equations 1871, and Germs 1881). Yale, Library of Congress, and the other ten members of the British Consortium of University Research Libraries, including the University of Sheffield, list none. Of course some of these on-line catalogues are incomplete. For example the Sheffield Library does hold four Earnshaw titles (Solitary Waves reprint, 1846; Tradition of the Elders, 1860; Etherspheres, 1879; Germs, 1881).

In 1847, at the age of 42, Earnshaw apparently suffered a breakdown in health (and perhaps of local prospects) and returned to Sheffield, where he was appointed chaplain on Queen Mary's foundation, serving essentially as assistant minister of the Sheffield parish church. He held this post for the second half of his life.

From Sheffield he continued to publish occasional works on mathematics and physics including a 240-page 4th revised edition of his *Treatise on Statics* (1858) and the following:

- On the mathematical theory of **sound** 133 pp. (1860)

Description and use of the Key of **Musical Keys**, a new apparatus for solving ... questions about musical scales, intervals and chords, etc. (1869)

Partial differential equations: An essay towards an entirely new method of integrating them 158 pp. (1871)

Etherspheres : a vera causa in natural philosophy; with special application to the theory of heat 46 pp. (1879)

The **doctrine of germs**: or the integration of certain partial differential equations which occur in mathematical physics. 113 pp. (1881)

Not surprisingly he also published a number of sermons and lectures including the following:

- **Parental Responsibility**: a sermon [on Deut. vi. 6, 7 "And these words, which I command thee this day, shall be in thine heart: And thou shalt teach them diligently unto thy children, and shalt talk of them when thou sittest in thine house, and when thou walkest by the way, and when liest down, and when thou risest up."] (1848)

The Tradition of the Elders. A sermon [on Mark vii. 2, 3 "And when they saw some of his disciples eat bread with defiled, that is to say, with unwashed, hands, they found fault. For the Pharisees, and all the Jews, except they wash their hands oft, eat not, holding the tradition of the elders.']. ... Fourth edition. (1860)

The Love of the World. A sermon [on 1 John ii. 17 "And the world passeth away, and the lust thereof: but he that doeth the will of God abideth for ever."] preached as a sequel to "the Tradition of the Elders." (1860)

No more Death. A sermon [on Luke XX. 36 "Neither can they die any more; for they are equal unto the angels; and are the children of God, being the children of the resurrection."] (1861)

The Church and the Artizan. A sermon [on Matth. v. 16 "Let your light so shine before men, that they may see your good works, and glorify your Father which is in heaven."] (1861)

candidate for fellowship at St. John's, but had become ineligible on 6 January, 1832, when he married his hometown sweetheart, Ann Wall of Kirk Sandall near Doncaster (second daughter of the late John Wall). They resided at Gonville Place where their son Samuel Walter was born in 1833.

(Samuel Walter would become, like his father, a graduate of St. John's, B.A. 1857, M.A. 1868, and a cleric. He is reported to have been a "great lover of chess, visiting London at frequent intervals to compete with the leading players.")

From 1831 to 1847 Earnshaw supported his family comfortably in Cambridge as "a very successful coach", presumably of mathematics and physics, and a rising cleric. Like most wranglers in this period [according to Andrew Warwick] he apparently aspired to be more a parson than a mathematician. He was ordained deacon 21 September 1834 at Buckden (Lincoln), and priest 20 September 1846 in the parish church of St. Michael, Cambridge (Ely). In 1837 he acted as Proctor. In June 1838 he was appointed to the sinecure Barnaby Lecturship in Philosophy at the Congregation. For a time he acted as curate to the Rev. Charles Simeon.

During this period he published a number of papers and books on mathematics and mathematical physics including:

in **Transactions of the Cambridge Philosophical Society**, Vols.6-8, **1837-1846**

On the fluid motion so far as it is expressed by the **equation of continuity**

On the **diffraction of an object-glass**...

On the nature of the **molecular forces** which regulate the constitution of the luminiferous ether
(This is the paper that presented the proof of "[Earnshaw's Theorem](#)". It was read to the Society in March, 1839, but not printed until 1842)

On the values of the **sine and cosine of an infinite angle**

The mathematical theory of the **two great solitary waves** of the first order

On the notation of the **Differential Calculus**. (1832) [British Library only, octavo]

Dynamics, or an elementary treatise on motion: with a great variety of examples...to which is added, a short treatise on attractions. 193pp. (1832)

- **2nd ed. enlarged** 359pp. (1839)

3rd ed. revised 396 pp. (1844)

The theory of **statics**: with numerous practical applications 299 pp. (1834)

- A treatise on statics: containing a new proof of the principle of virtual velocities, and numerous examples illustrative of the general principles of the science
2nd ed. revised 230 pp. (1842)

Is there a Sacrificing Priesthood in the New Testament? A lecture ... at the Cutler's Hall, Sheffield, February 18, 1868.

The attitude of the working class towards religion: a lecture. (1870)

Upon the state of education among the working classes of the parish of Sheffield. (undated)

His "Tradition of the Elders" sermon not only deserved four editions and a sequel, it also prompted an anonymous Elder to respond with

- The Blind leading the Blind: a dialogue, containing strictures on the Rev. S. Earnshaw's Sermon, entitled "The Tradition of the Elders," by an Elder, who holds no "traditions" but those of Christ and His Apostles, etc. (1860)

In Sheffield he introduced the University Local Examinations. The Cambridge University Extension Movement was a scheme whereby the University would, in Earnshaw's words, "place its resources at the disposal of the towns of the country, bringing the advantages of University education to the very doors of the people." He subsequently could report that to his certain knowledge "the Cambridge lectures produced a most marvellous effect in this town upon the upper middle class. The conversation now heard and the books now read were of a very superior kind." This success led Mark Firth, a Sheffield steel manufacturer, to establish Firth College in 1879. Earnshaw was elected Vice President under President Firth, whom he succeeded the following year. The present University of Sheffield developed from amalgamation of Firth College with the Sheffield School of Medicine and the Sheffield Technical School in 1897.

In 1887 an Earnshaw Scholarship was founded at Firth College in his honor. In the same year his son died.

Earnshaw himself died 6 December, 1888, at the age of 83. His obituary notice in the *Sheffield Daily Telegraph* suggests that he looked back on his life with contentment:

As a preacher, a scholar, a mathematician, a controversialist, an educationist, a philanthropist, and a simple-hearted Christian gentleman, he was alike conspicuous. Notwithstanding a somewhat stern and forbidding exterior, his was one of the gentlest hearts that ever beat... Pride and ambition were absolutely wanting. "I never had the slightest spark of ambition," he once remarked. "All my good fortune came to me. I never sought it except by hard work."

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References and Acknowledgements

The sources for most of the above material are J.A.Venn *Alumni Cantabrigienses: a biographical list of all known students, graduates, and holders of office at the University of Cambridge, from the earliest times to 1900*, Part II from 1752 to 1900, Vol II (Cambridge: Cambridge University Press, 1944) p.374, and three pages from the "*Biographical Collection*" of the Library of St. John's College, Cambridge. I am indebted to Jonathan A. Harrison, Special Collections Librarian at St. John's, for providing this information on very short notice, to Dr.