Part 2

In Lecture 1 we covered supersymmetry (the symmetry); supermultiplets; superpotential and supersymmetric Lagrangians; R-parity; the MSSM particle content.

Now let's look at supersymmetry breaking/mediation schemes; electroweak symmetry breaking in the MSSM; implications for models and LHC searches.

Tomorrow we'll look at flavor, specifically the supersymmetric flavor problem; schemes and models that reduce the parameter space; production of superpartners and decays; signals at LHC.

Supersymmetric Breaking

Origins of Supersymmetry Breaking

To gain deeper understanding, let us consider how SUSY could be spontaneously broken. This means that the Lagrangian is invariant under SUSY transformations, but the ground state is not:

$$Q_\alpha|0\rangle \neq 0, \qquad\qquad Q^\dagger_{\dot\alpha}|0\rangle \neq 0.$$

The SUSY algebra tells us that the Hamiltonian is related to the SUSY charges by:

$$H = P^0 = \tfrac{1}{4}(Q_1 Q_1^\dagger + Q_1^\dagger Q_1 + Q_2 Q_2^\dagger + Q_2^\dagger Q_2).$$

Therefore, if SUSY is unbroken in the ground state, then $H|0\rangle = 0$, so the ground state energy is 0. Conversely, if SUSY is spontaneously broken, then the ground state must have positive energy, since

$$\langle 0|H|0\rangle = \tfrac{1}{4}\left(\|Q_1^\dagger|0\rangle\|^2 + \|Q_1|0\rangle\|^2 + \|Q_2^\dagger|0\rangle\|^2 + \|Q_2|0\rangle\|^2\right) > 0$$

To achieve spontaneous SUSY breaking, we need a theory in which the prospective ground state $|0\rangle$ has positive energy.

In SUSY, the potential energy can be written, using the equations of motion, as:

$$V = \sum_i |F_i|^2 + \frac{1}{2} \sum_a D^a D^a,$$

a sum of squares of auxiliary fields. So, for spontaneous SUSY breaking, one must arrange a stable (or quasi-stable) ground state with either $\langle F_i \rangle \neq 0$ or $\langle D^a \rangle \neq 0$, for at least one i or a.

Models of SUSY breaking where

- $\langle F_i \rangle \neq 0$ are called "O'Raifeartaigh models" or "F-term breaking models"
- $\langle D^a \rangle \neq 0$ are called "Fayet-Iliopoulis models" or "D-term breaking models"

F-term breaking is used in (almost) all known realistic models.

This can only happen if the chiral supermultiplet is a singlet.

(otherwise a gauge symmetry would be simultaneously broken)

Spontaneous Breaking of SUSY requires us to extend the MSSM

There is no gauge-singlet chiral supermultiplet in the MSSM that could get a non-zero F-term VEV.

Even if there were such an $\langle F \rangle$, there is another general obstacle. Gaugino masses cannot arise in a renormalizable SUSY theory at tree-level. This is because SUSY does not contain any (gaugino)-(gaugino)-(scalar) coupling that could turn into a gaugino mass term when the scalar gets a VEV.

This leads to the following general schematic picture of SUSY breaking...

The MSSM soft SUSY-breaking terms arise indirectly or radiatively, not from tree-level renormalizable couplings directly to the SUSY-breaking sector.

Spontaneous SUSY breaking occurs in a “hidden sector” of particles with no (or tiny) direct couplings to the “visible sector” chiral supermultiplets of the MSSM. However, the two sectors do share some mediating interactions that transmit SUSY-breaking effects indirectly.

By dimensional analysis,

$$m_{\rm soft} \sim \frac{\langle F \rangle}{M}$$

where M is a mass scale associated with the physics that mediates between the two sectors.

Planck-scale Mediated SUSY Breaking ("gravity mediation")

The idea: SUSY breaking is transmitted from a hidden sector to the MSSM by the new interactions, including gravity, that enter near the Planck mass scale M_P.

If SUSY is broken in the hidden sector by some VEV $\langle F \rangle$, then the MSSM soft terms should be of order:

$$m_{\rm soft} \sim \frac{\langle F \rangle}{M_P}$$

This follows from dimensional analysis, since $m_{\rm soft}$ must vanish in the limit that SUSY breaking is turned off $(\langle F \rangle \rightarrow 0)$ and in the limit that gravity becomes irrelevant $(M_P \rightarrow \infty)$.

Since we know $m_{\rm soft} \sim$ few hundred GeV, and $M_P \sim 2.4 \times 10^{18}$ GeV:

$$\sqrt{\langle F \rangle} \sim 10^{11} \text{ or } 10^{12} \text{ GeV}$$

Planck-scale Mediated SUSY Breaking (continued)

Write down an effective field theory non-renormalizable Lagrangian that couples F to the MSSM scalar fields ϕ_i and gauginos λ^a:

$$\mathcal{L}_{\text{PMSB}} = -\left(\frac{f^a}{2M_P} F\lambda^a\lambda^a + \text{c.c.}\right) - \frac{k_i^j}{M_P^2} FF^*\phi_i\phi^{*j}$$
$$-\left(\frac{\alpha^{ijk}}{6M_P} F\phi_i\phi_j\phi_k + \frac{\beta^{ij}}{2M_P} F\phi_i\phi_j + \text{c.c.}\right)$$

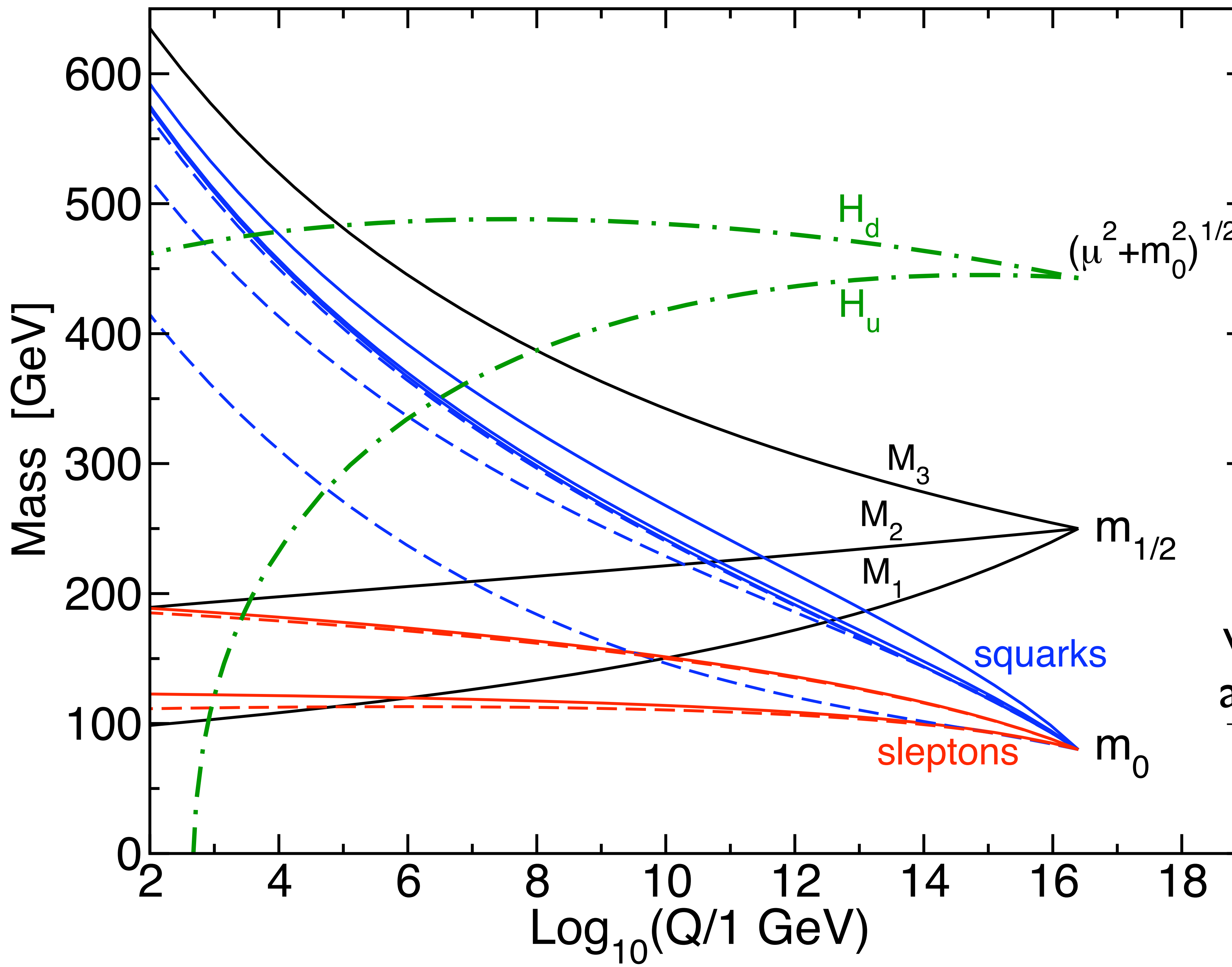
This is (part of) a fully supersymmetric Lagrangian that arises in supergravity. When we replace F by its VEV $\langle F\rangle$, we get exactly the MSSM soft SUSY-breaking Lagrangian, with:

- Gaugino masses: $M_a = f^a\langle F\rangle/M_P$
- Scalar squared massed: $(m^2)_i^j = k_i^j|\langle F\rangle|^2/M_P^2$ and $b^{ij} = \beta^{ij}\langle F\rangle/M_P$
- Scalar^3 couplings $a^{ijk} = \alpha^{ijk}\langle F\rangle/M_P$

Unfortunately, it is **not** obvious why these are flavor-blind!

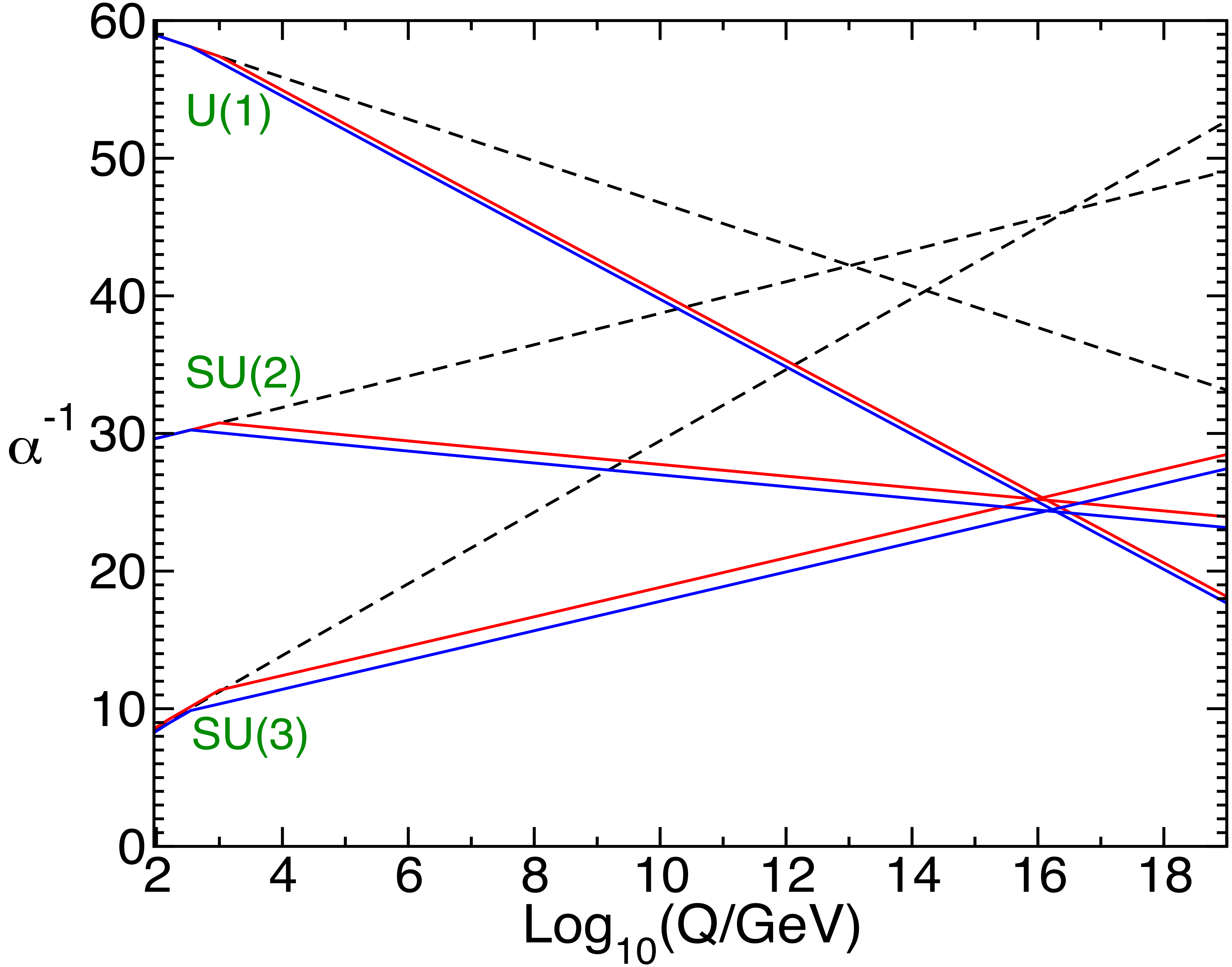
These SUSY breaking masses are generated at the messenger scale (in this case M_P). We then must use the renormalization group to evolve these parameters from the Planck scale to the weak scale...

Renormalization Group Evolution Sparticle Masses

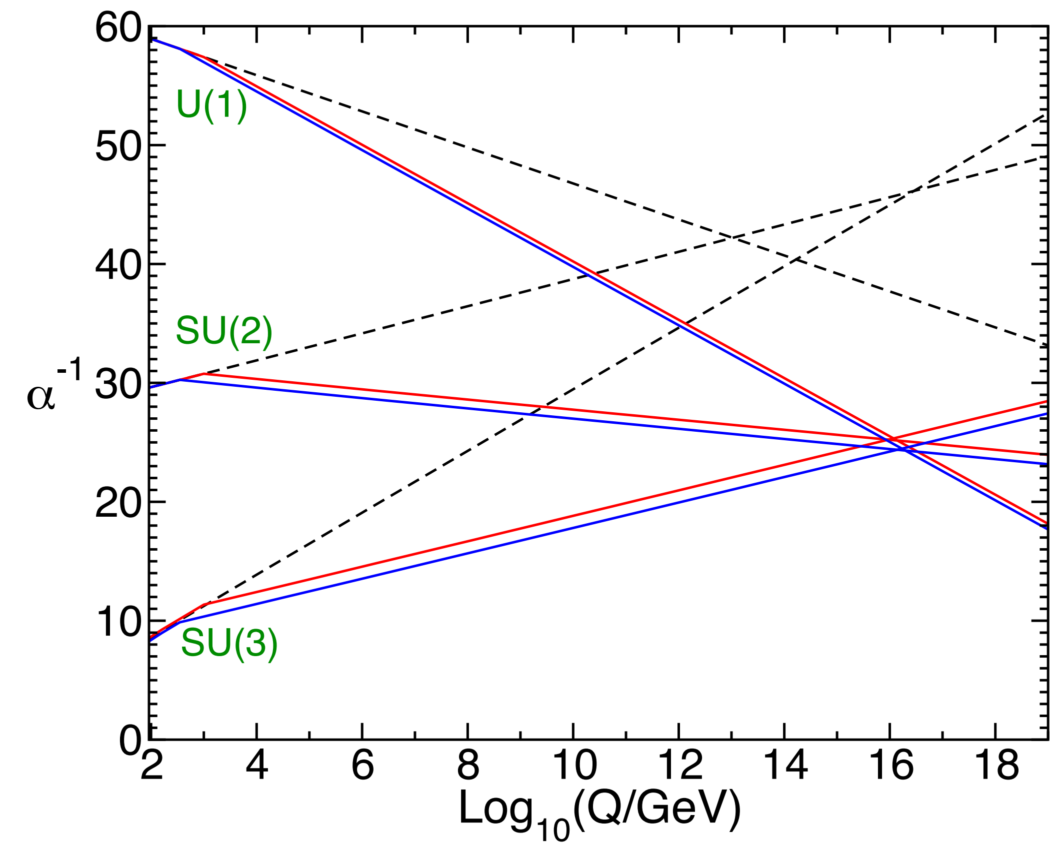


(If the masses were "unified" at a high scale)

Renormalization Group Evolution Gauge Couplings



Unification: Clue or Clueless?



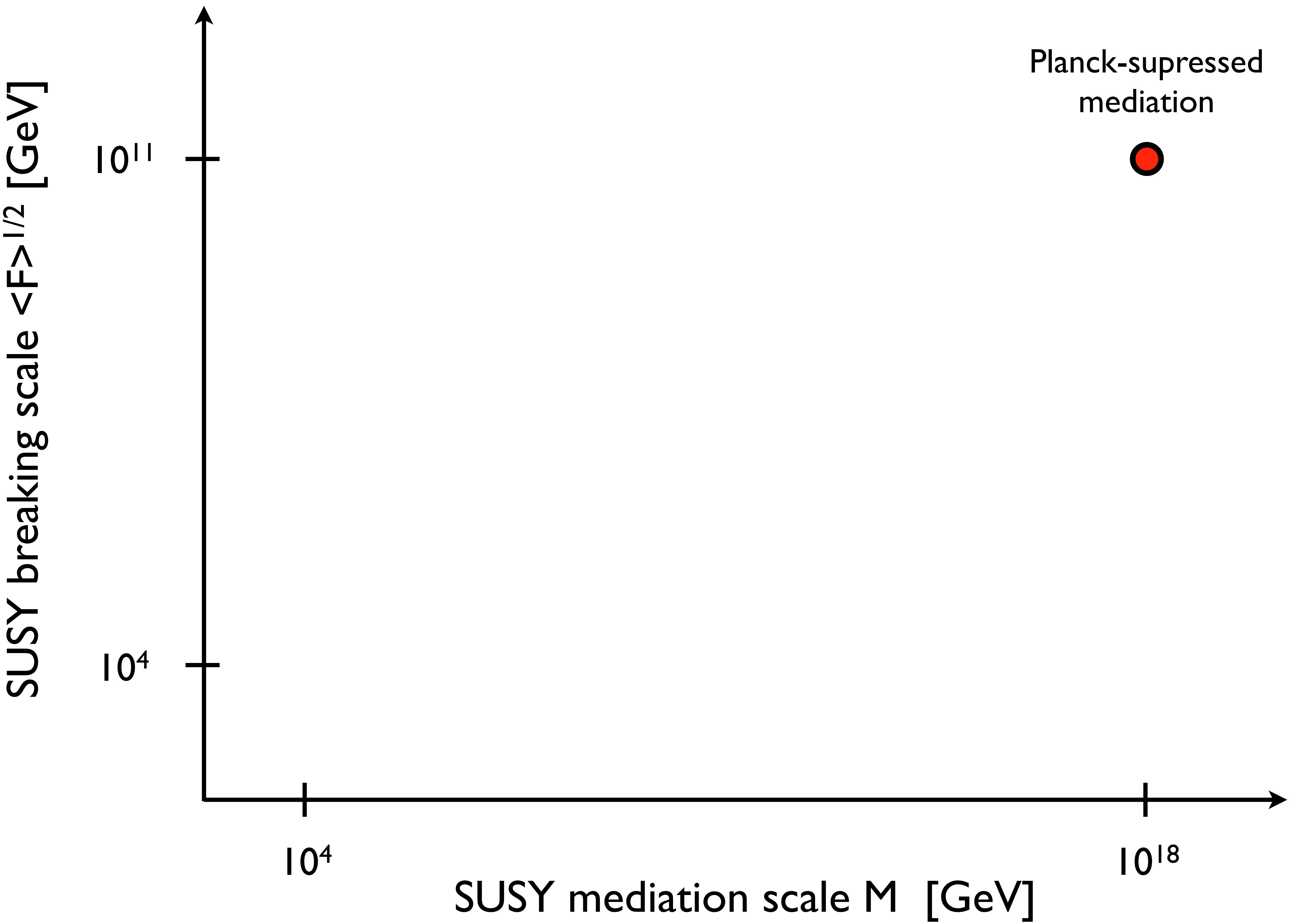
The apparent unification of the gauge couplings has remained a tantalizing hint about physics at high scales.

Unification should simplify the high scale physics somewhat [gaugino masses universal in simple GUTs like SU(5)]

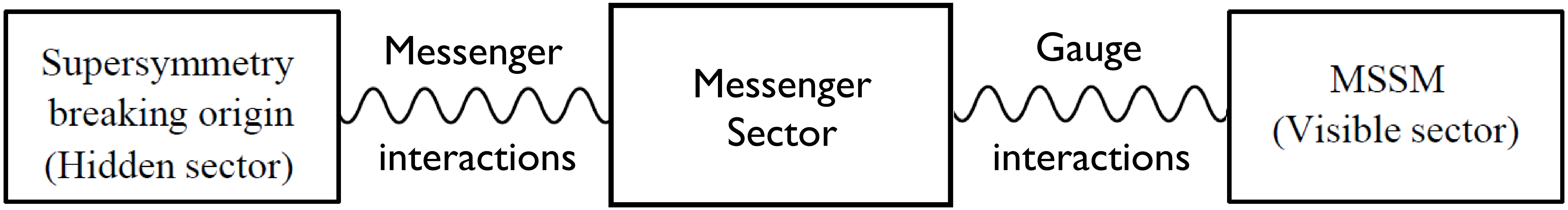
In my view, while this is intriguing, unification remains very difficult to realize in terms of an actual "workable" GUT model. Various problems inevitably arise, including doublet/triplet splitting, rapid Higgs-triplet-mediated proton decay, etc.

Is this the clue to move forward? Despite what some people might have told you, we just don't know.

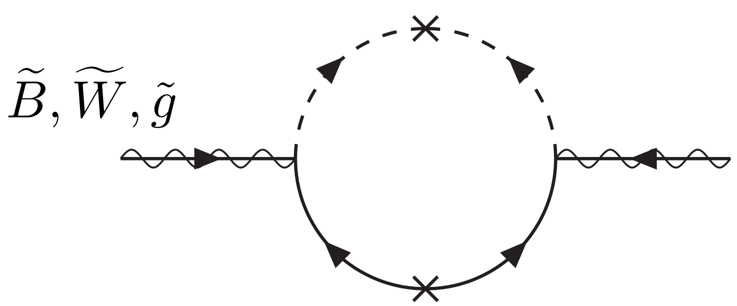
SUSY Mediation Map



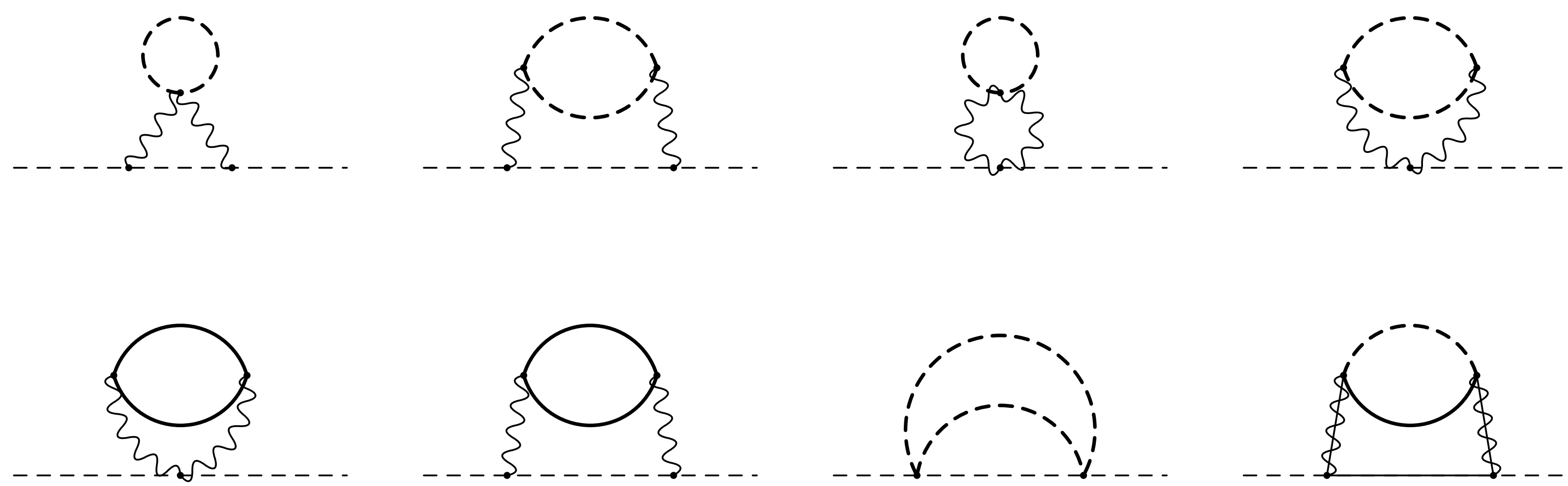
Low Scale ("Gauge") Mediation



Gaugino masses



Scalar masses



$$M_a = n_a \frac{\alpha_a}{4\pi} \frac{F}{M_{\rm mess}}$$

$$m_i^2 = \sum_{a=1,2,3} C_a(i) \left(\frac{\alpha_a}{4\pi}\right)^2 \left(\frac{F}{M_{\rm mess}}\right)^2$$

Gauge Mediation

Features:

- $M_{gaugino}$ is one-loop suppressed relative to $\frac{F}{M_{\rm mess}}$
- m^2_{scalar} is two-loop suppressed relative to $\left(\frac{F}{M_{\rm mess}}\right)^2$

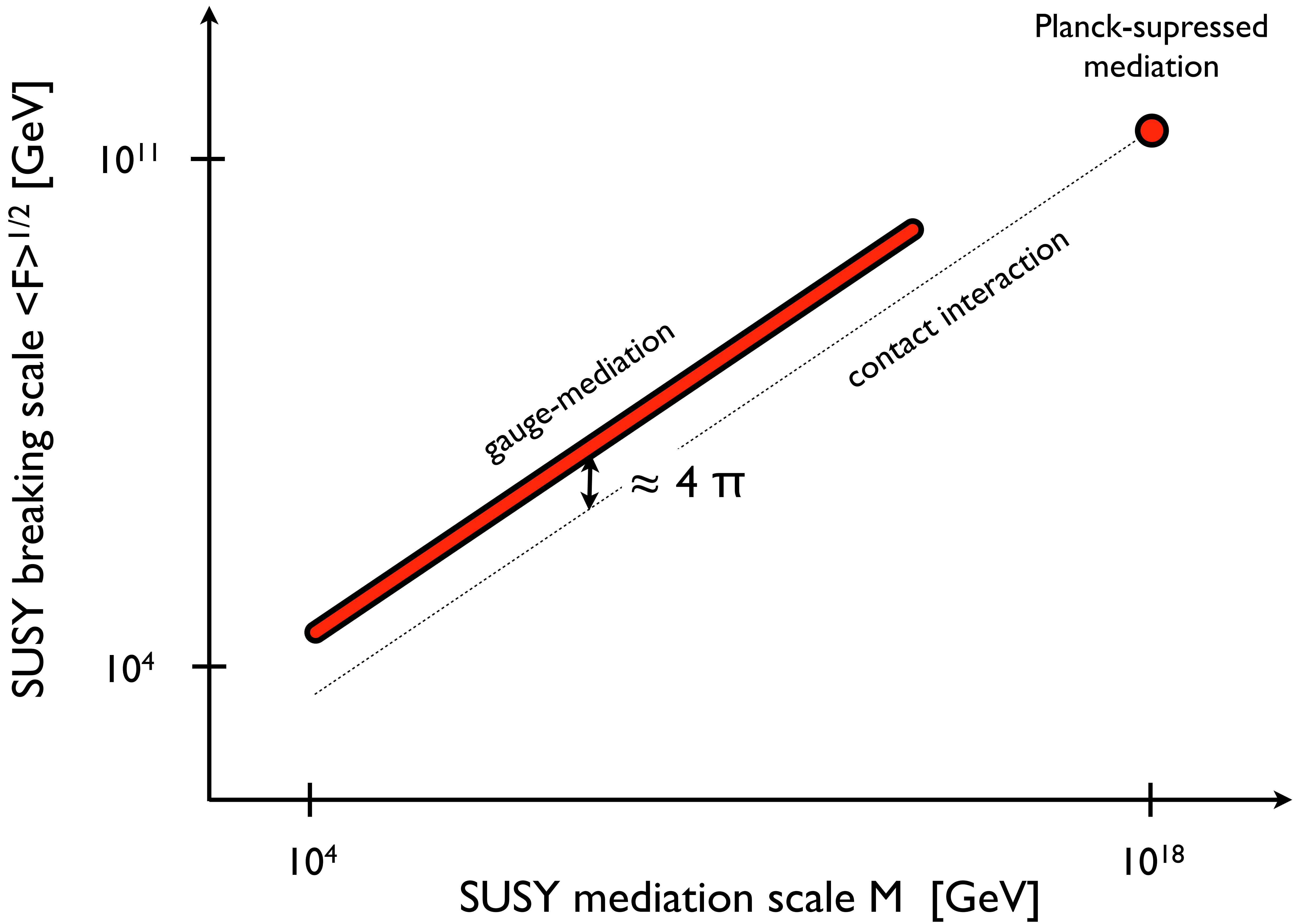
  --> hence, $M_{gaugino} \approx M_{scalar}$

- This means $F/M_{\rm mess} \approx$ 10-20 TeV to get $M_{gaugino} \approx M_{scalar} \approx$ TeV scale

- Planck scale mediation present, but subdominant
  (so long as mediation scale << Planck scale)

- Mediation through gauge interactions implies supersymmetry breaking parameters are flavor-blind!

SUSY Mediation Map



Gauge Mediation: LSP is the Gravitino

Gravitino gets a mass purely from Planck-scale mediated operator:

$$m_{3/2} \sim \frac{F_S}{M_{\rm Pl}} \ll M_{\rm gaugino,\ squark...}$$

Gravitino mass could be ≈ 1 eV to 1 GeV, depending on the mediation scale (while preserving flavor-blindness).

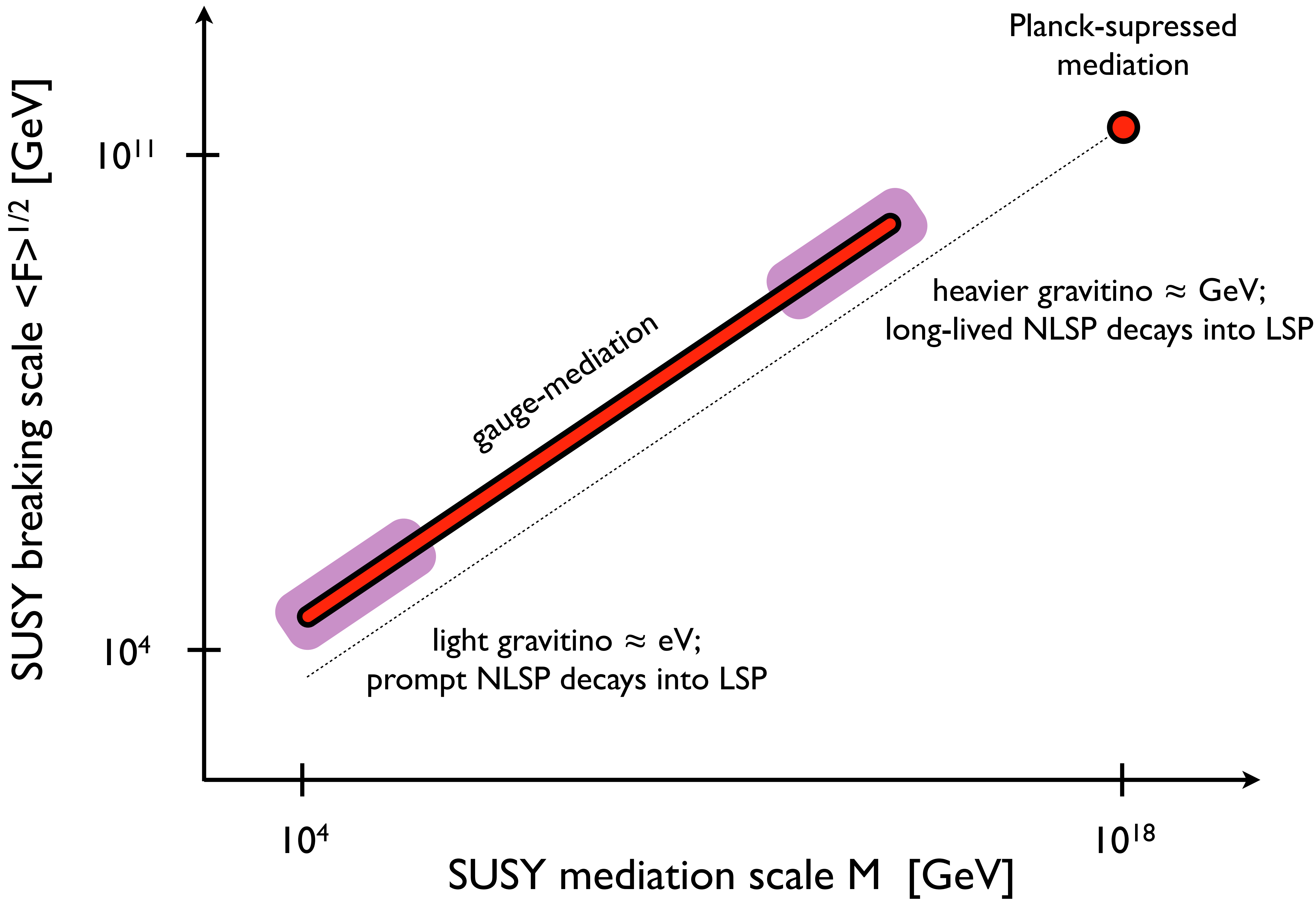
Phenomenology qualitatively affected:

- MSSM superpartners decay to lightest state, the "next-to-lightest sparticle" (NLSP)
- The NLSP decays to the gravitino (LSP), possibly with a lifetime that is long on collider time scales:

$$\Gamma(\widetilde{N}_1 \to \gamma \widetilde{G}) \;=\; 2 \times 10^{-3}\, \kappa_{1\gamma} \left(\frac{m_{\widetilde{N}_1}}{100\ \mathrm{GeV}}\right)^5 \left(\frac{\sqrt{\langle F \rangle}}{100\ \mathrm{TeV}}\right)^{-4} \mathrm{eV}$$

$$d = 9.9 \times 10^{-3}\, \frac{1}{\kappa_{1\gamma}} \,(E^2/m^2_{\widetilde{N}_1} - 1)^{1/2} \left(\frac{m_{\widetilde{N}_1}}{100\ \mathrm{GeV}}\right)^{-5} \left(\frac{\sqrt{\langle F \rangle}}{100\ \mathrm{TeV}}\right)^{4} \mathrm{cm}$$

SUSY Mediation Map



Gauge Mediation: Challenges

- New messenger sector adds structure, some potential problems (messenger/matter mixing)

- Light gravitino may or may not be viable dark matter, depending on mass and abundance source (generally don't get the right thermal abundance, since gravitino has interactions suppressed by $1/\sqrt{F}$)

- Generating μ, $B\mu$ problematic (Higgs sector)

- Difficult to get large A-terms; hence difficult to get 125 GeV Higgs mass without several-TeV superpartners (e.g., Craig, Knapen, Shih, Zhao)

Other Approaches

Gaugino Mediation

- Gaugino masses >> scalar masses, trilinear scalar couplings at some large scale. Renormalization group evolution will regenerate flavor-blind scalar masses.
  - main challenge is that the stops need to be heavy enough to get 125 GeV Higgs; this requires excessively large gauginos...

Anomaly Mediation

- Superconformal anomaly leads to ≈ $b_a \frac{\alpha_a}{4\pi} \frac{F}{M_{Pl}}$ and two-loop flavor-blind scalar masses

  But, sleptons get negative (mass)^2 (solutions, complicated); continuing debate as to nature of (and even existence of) anomaly mediated contributions [Dine, Seiberg; Thaler et al; de Alvies; ...]

R-symmetry

- Extend MSSM so that gauginos have Dirac masses that are ≈ 4π heavier than squarks/sleptons. Flavor interesting and nontrivial -- more on this if I have time...

EWSB and Higgs physics in the MSSM

EWSB
SM versus SUSY

$$V = m_H^2|H|^2 + \lambda|H|^4$$

$$v = \langle H \rangle = \sqrt{\frac{-m_H^2}{2\lambda}} = 174 \text{ GeV}$$

$$\begin{aligned} V &= (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 \\ &\quad + \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2 \\ &\quad - (b\, H_u^0 H_d^0 + \text{c.c.}) \end{aligned}$$

$$v_u^2 + v_d^2 = v^2 = 2m_Z^2/(g^2 + g'^2) \approx (174 \text{ GeV})^2$$

$$v_u = \langle H_u^0 \rangle \qquad \tan\beta \equiv \frac{v_u}{v_d}$$

$$v_d = \langle H_d^0 \rangle$$

$$\sin(2\beta) = \frac{2b}{m_{H_u}^2 + m_{H_d}^2 + 2|\mu|^2}$$

$$m_Z^2 = -2(m_{H_u}^2 + |\mu|^2) + \frac{2}{\tan^2\beta}(m_{H_d}^2 - m_{H_u}^2) + \mathcal{O}(1/\tan^4\beta)$$

EWSB
SM versus SUSY

Yukawa couplings

$$y_t = \frac{m_t}{v}, \quad y_b = \frac{m_b}{v}, \quad y_\tau = \frac{m_\tau}{v}$$

$$m_h = \sqrt{\lambda} v$$

Yukawa couplings

$$y_t = \frac{m_t}{v \sin\beta}, \qquad y_b = \frac{m_b}{v \cos\beta}, \qquad y_\tau = \frac{m_\tau}{v \cos\beta},$$

To keep Yukawas less than about one,

$$1.5 \lesssim \tan\beta \lesssim 55$$

In the SUSY two-Higgs doublet model...

Define mass-eigenstate Higgs bosons: h^0, H^0, A^0, G^0, H^+, G^+ by:

$$\begin{pmatrix} H_u^0 \\ H_d^0 \end{pmatrix} = \begin{pmatrix} v_u \\ v_d \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h^0 \\ H^0 \end{pmatrix} + \frac{i}{\sqrt{2}} \begin{pmatrix} \sin\beta & \cos\beta \\ -\cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}$$

$$\begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix} = \begin{pmatrix} \sin\beta & \cos\beta \\ -\cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix}$$

Now, expand the potential to second order in these fields to obtain the masses:

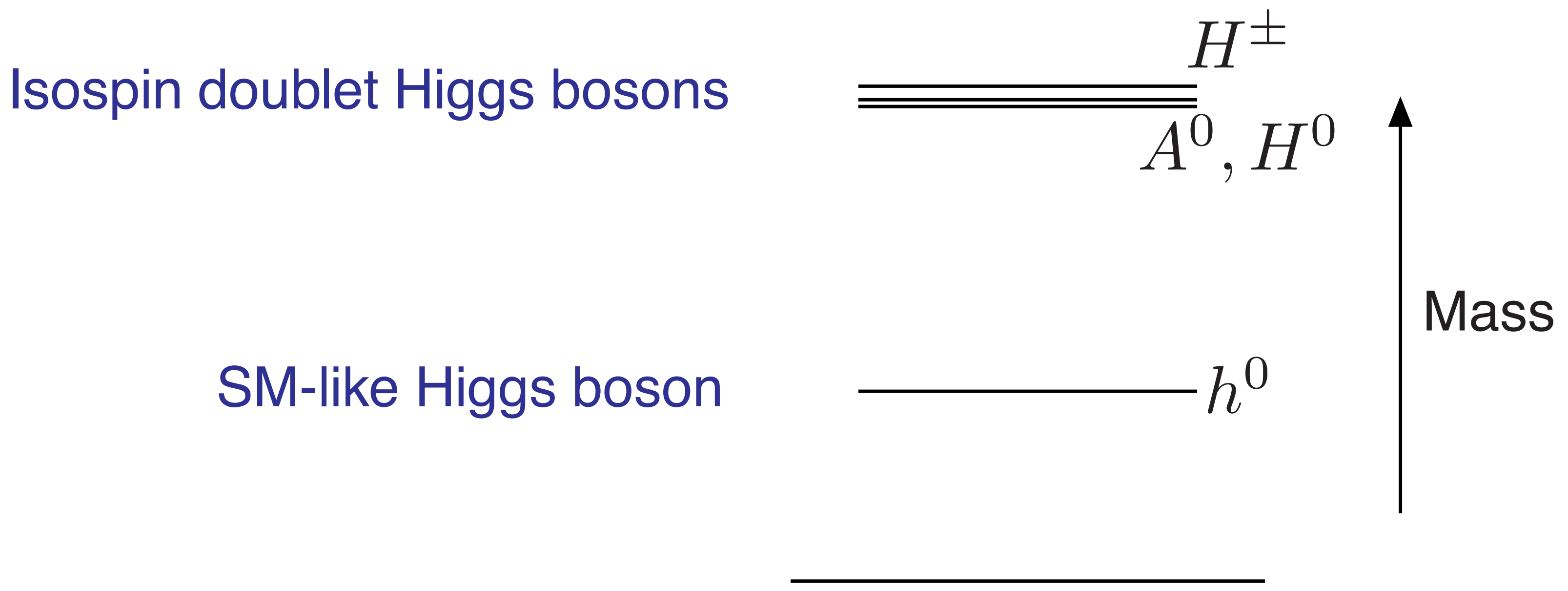
$$\begin{aligned} m_{A^0}^2 &= 2b/\sin 2\beta \\ m_{h^0,H^0}^2 &= \tfrac{1}{2}\left(m_{A^0}^2 + m_Z^2 \mp \sqrt{(m_{A^0}^2 + m_Z^2)^2 - 4m_Z^2 m_{A^0}^2 \cos^2 2\beta}\right), \\ m_{H^\pm}^2 &= m_{A^0}^2 + m_W^2 \end{aligned}$$

The Goldstone bosons have $m_{G^0} = m_{G^\pm} = 0$; they are absorbed by the Z, $W^\pm$ bosons to give them masses, just as in the Standard Model.

The decoupling limit for the Higgs bosons

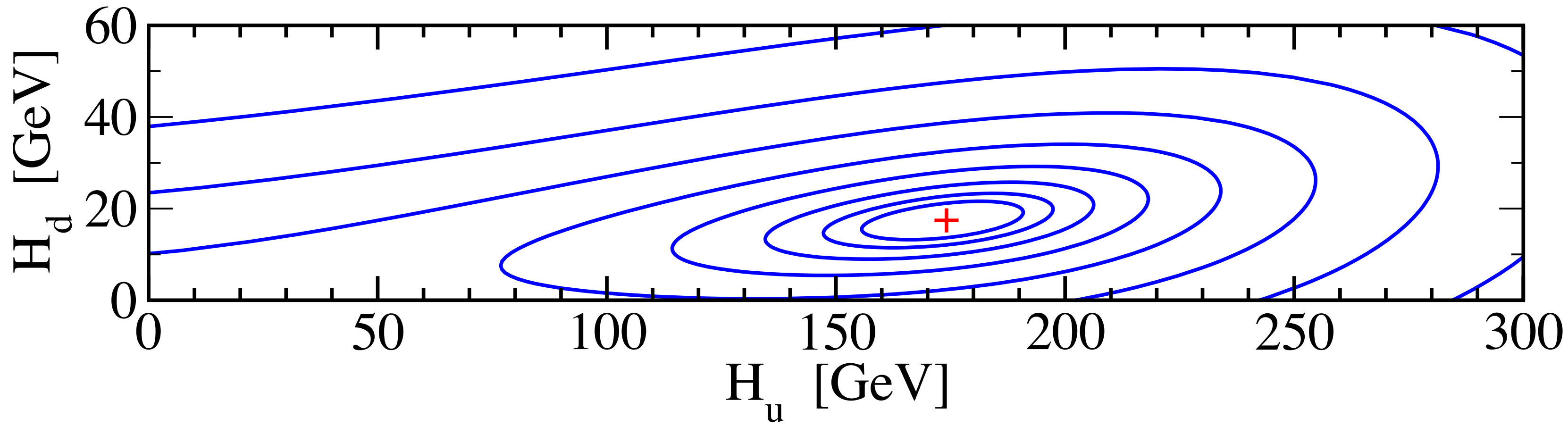
If $m_{A^0} \gg m_Z$, then:

- h^0 has the same couplings as would a Standard Model Higgs boson of the same mass
- $\alpha \approx \beta - \pi/2$
- $A^0, H^0, H^\pm$ form an isospin doublet, and are much heavier than h^0
- h^0 mass is maximized



Many models of SUSY breaking approximate this decoupling limit.

Typical contour map of the Higgs potential in SUSY:



The Standard Model-like Higgs boson h^0 corresponds to oscillations along the shallow direction with $(H_u^0 - v_u, H_d^0 - v_d) \propto (\cos\alpha, -\sin\alpha)$. At tree-level,

$$m_{h^0} < m_Z.$$

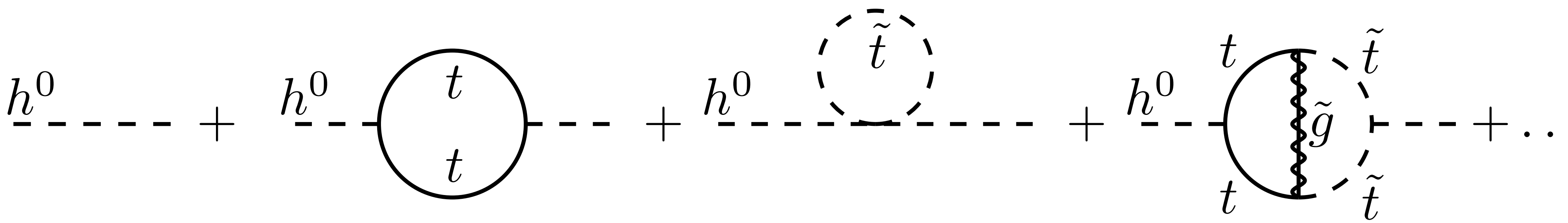
This has been ruled out by LEP2. However, taking into account loop effects, m_{h^0} is considerably larger. Assuming that all superpartners are lighter than 1000 GeV, and that perturbation theory is valid to very high energies, one finds:

$$m_{h^0} \lesssim 130 \text{ GeV}$$

in the MSSM. By adding more supermultiplets, or not requiring that the theory stays perturbative, one can get up to 200 GeV.

Radiative corrections to the Higgs mass in SUSY:

$$m_{h^0}^2 = m_Z^2 \cos^2(2\beta) + \frac{3}{4\pi^2} y_t^2 m_t^2 \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + \ldots$$



At tree-level: m_Z^2 pure electroweak

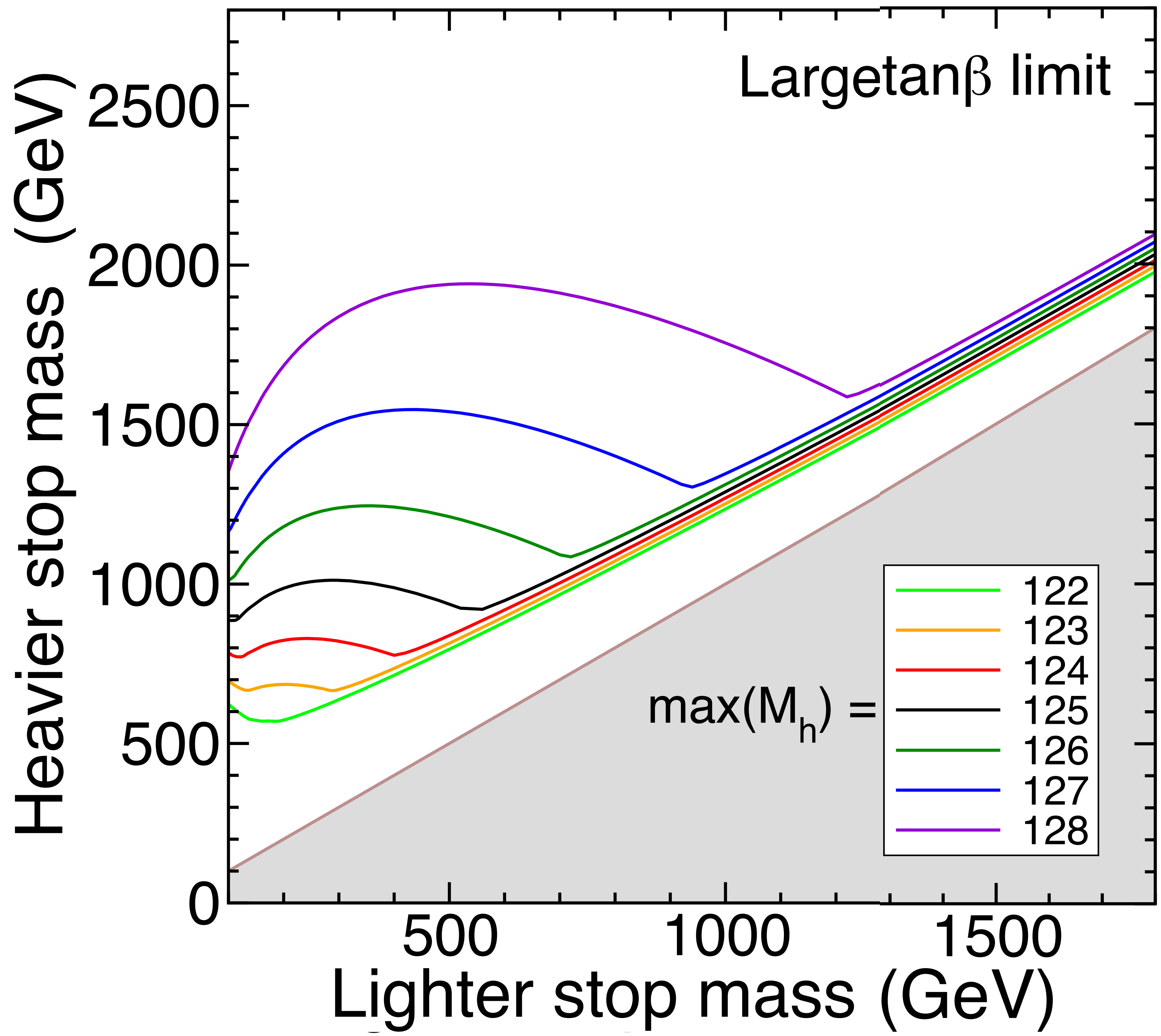
At one-loop: $y_t^2 m_t^2$ top Yukawa comes in

At two-loop: $\alpha_S y_t^2 m_t^2$ SUSYQCD comes in

At three-loop: $\alpha_S^2 y_t^2 m_t^2$

Even the three-loop corrections can add 1 GeV or so to m_{h^0}.

For each $\tilde{t}_1$, $\tilde{t}_2$ masses, scan over other parameters, find maximum M_h:



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Clearly this imposes significant constraints on the MSSM parameter space to obtain a Higgs mass consistent with ≈ 125 GeV.