1 The Standard Model and Beyond

Standard Model: gauge theory with group structure $SU(3) \times SU(2) \times U(1)$
gauge boson masses through spontaneous symmetry breaking
(Higgs mechanism; vev $v = 246$ GeV; m_H unknown)

1.1 Experiments:

LEP & Tevatron

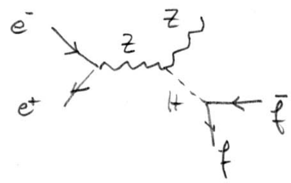
* gauge bosons found, $m_W \sim 80$ GeV
  $m_Z \sim 91$ GeV

  no anomalous W, Z decays

* 6 quarks found, $m_t \sim 174$ GeV
  typical decay $t \to bW^+$ observed

* leptons, including τ as expected

* electroweak fit: $m_H \sim 110$ GeV best value
  $m_H \lesssim 250$ GeV 1σ bound from fit
  $m_H > 114$ GeV direct search for SM-like Higgs



mostly $H \to b\bar{b}$
$Z \to l\bar{l}, \nu\bar{\nu}$

problem? fit is not good, b-observables might be inconsistent

$\rightsquigarrow$ not conclusive

$(g-2)_\mu$

* determine anomalous magnetic moment of μ

$$a_\mu^{exp} = \frac{1}{2}(g-2)_\mu^{exp} = (11659208 \pm 6) \cdot 10^{-10}$$

direct measurement, likely terminated

SM prediction from 2 groups

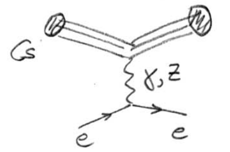
$$a_\mu^{exp} - a_\mu^{SM} = (31.7 \pm 9.5) \cdot 10^{-10} \quad : 3.3\sigma$$

$$a_\mu^{exp} - a_\mu^{SM} = (20.2 \pm 9.0) \cdot 10^{-10} \quad : 2.1\sigma$$

$\boxed{\text{General agreement: } 5\sigma \equiv \text{discovery}}$

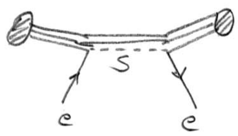
$\rightsquigarrow$ not conclusive

Atomic parity violation



parity violation through Z

look for contributions e.g. of the kind



terminated at $\sim 2\sigma$ discrepancy between SM and measurement

$\rightsquigarrow$ not conclusive

Cosmology

WMAP $\rightsquigarrow$ there is dark matter in the universe:

$$\boxed{\Omega_{DM} h^2 = 0.094 \cdots 0.129}$$

where $\Omega = 1$ is critical density for flat universe
and $h = \frac{H_0}{100}$ km s^{-1} Mpc^{-1} ~ 0.7 with $H_0 \sim 73$ km s^{-1} Mpc^{-1}

baryon density $\Omega_b h^2 = 0.024 \pm 0.001$
matter density $\Omega_M h^2 = 0.14 \pm 0.02$

(errors are mixture of chemistry & miracles)

$\rightsquigarrow$ conclusive

[Bertone, Hooper, Silk hep-ph/0404175]

sign for physics beyond the SM, dark matter could (for example) be a weakly interacting particle with a mass of $\mathcal{O}(100\,\text{GeV})$, yet undetected by direct and indirect searches

Flavor physics

* proton decay not observed
* flavor-changing neutral currents not observed
* anomalous B decays not observed

⋮

$\rightsquigarrow$ unfortunately / fortunately also conclusive

1.2 Theory :

start from assuming that the SM as a renormalizable theory (i.e. no inverse powers of mass in the Lagrangian) does not have a built-in energy scale where it breaks down.

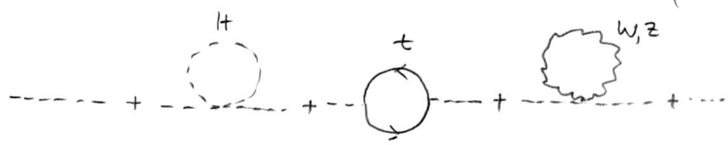
exception : gravity , ⇝ energy scales above 10^{19} GeV should be described by some combination of SM & quantum gravity

all current observables probe scales $E \lesssim 100\ \mathrm{TeV} = 10^5\ \mathrm{GeV}$

⇝ no need to care about

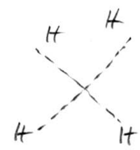
SM beyond tree-level

Higgs mass gets corrected by loops in 2-point function



[Schmaltz hep-ph/0210415]

and we can calculate e.g. the H loop :



$$-\frac{3}{4} i g^2 \frac{m_H^2}{m_W^2}$$

amplitude :

$$\mathcal{A} = \int^{\Lambda} \frac{d^4q}{(2\pi)^4} \left(-\frac{3}{4} i \ldots\ldots\right) \frac{1}{q^2 - m_H^2} \overset{!\ \text{for simplicity}}{=} \int \frac{d^4q}{(2\pi)^4} \left(\frac{1}{q^2 - m_H^2} - \frac{1}{q^2 - \Lambda^2} \right) \leftarrow \begin{cases} \frac{1}{q^2 - m^2} & \text{for } q^2 \ll \Lambda^2 \\ \frac{1}{q^2} - \frac{1}{q^2} = 0 & \text{for } q^2 \gg \Lambda \end{cases}$$

Pauli-Villars regularization

$$= (m^2 - \Lambda^2) \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2 - m_H^2)(q^2 - \Lambda^2)}$$

$$= (m^2 - \Lambda^2) \int \frac{d^4q}{(2\pi)^4} \int_0^1 dx \int_0^1 dy \frac{2\,\delta(1-x-y)}{[(q^2 - m_H^2)x + (q^2 - \Lambda^2)y]^2}$$ Feynman parametrization

$$= 2(m^2 - \Lambda^2) \int_0^1 dx \int_0^1 dy\ \delta(1-x-y) \int \frac{d^4q}{(2\pi)^4} \frac{1}{[q^2 - x m_H^2 - y \Lambda^2]^2}$$

$$= 2(m^2 - \Lambda^2) \int_0^1 dx \int_0^1 dy\ \delta(1-x-y) \frac{i}{16\pi^2} = -\frac{2i\Lambda^2}{16\pi^2} \left(-\frac{3}{4} i g^2 \frac{m_H^2}{m_W^2}\right) = -\frac{3}{32\pi^2} g^2 \frac{m_H^2}{m_W^2} \Lambda^2$$

using $\displaystyle \int \frac{d^4q}{(2\pi)^4} \frac{(q^2)^R}{(q^2 - C)^M} = \frac{i(-1)^{R-M}}{16\pi^2} C^{R-M+2} \frac{\Gamma(R+2)\Gamma(M-R-2)}{\Gamma(M)}$ for $R=0$, $M=2$

how does this affect the mass:

$$= \frac{1}{q^2-m_H^2} + \frac{1}{q^2-m_H^2}\mathcal{A}\frac{1}{q^2-m_H^2} + \frac{1}{q^2-m_H^2}\mathcal{A}\frac{1}{q^2-m_H^2}\mathcal{A}\frac{1}{q^2-m_H^2} + \cdots$$

$$= \frac{1}{q^2-m_H^2}\sum_{n=0}^{\infty}\left(\mathcal{A}\frac{1}{q^2-m_H^2}\right)^n \qquad \text{geometric series}$$

$$= \frac{1}{q^2-m_H^2}\,\frac{1}{1-\mathcal{A}\frac{1}{q^2-m_H^2}} = \frac{1}{q^2-m_H^2-\mathcal{A}}$$

$$\Rightarrow \quad m_H^2 = m_H^{(LO)^2} + \mathcal{A}$$

calculate everything proportional to Λ^2

$$\boxed{m_H^2 = m_H^{(LO)2} + \frac{3g^2}{32\pi^2}\frac{\Lambda^2}{m_W^2}\left[m_H^2 + 2m_W^2 + m_Z^2 - \frac{4n_f}{3}m_t^2\right]}$$

$[\ldots]=0$ called Veltman's condition, of course only a 1-loop solution.

usually we use dimensional regularization $d^4q \mapsto d^N q$

$$\int \frac{d^N q}{(2\pi)^N} \frac{(q^2)^R}{(q^2-m^2)^M} = \frac{i(-1)^{R-M}}{(16\pi^2)^{N/4}} (m^2)^{R-M+\frac{N}{2}} \frac{\Gamma(R+\frac{N}{2})\,\Gamma(M-R-\frac{N}{2})}{\Gamma(\frac{N}{2})\,\Gamma(M)}$$ [Field's book]

$$\Rightarrow \int \frac{d^N q}{(2\pi)^N} \frac{1}{q^2-m_H^2} \overset{\substack{R=0\\ M=1}}{=} \frac{-i}{(16\pi^2)^{N/4}} (m_H^2)^{-1+\frac{N}{2}} \frac{\Gamma(\frac{N}{2})\,\Gamma(1-\frac{N}{2})}{\Gamma(\frac{N}{2})\,\Gamma(1)}$$

$N = 4-2\epsilon$

$\Downarrow$

$\frac{N}{4} = 1-\frac{\epsilon}{2}$; $\frac{N}{2} = 2-\epsilon$;

$-1+\frac{N}{2} = 1-\epsilon$; $1-\frac{N}{2} = \epsilon-1$

$\Downarrow$

$\Gamma(1-\frac{N}{2}) = \Gamma(\epsilon-1) = \frac{\Gamma(\epsilon)}{\epsilon-1}$

because $x\Gamma(x) = \Gamma(x+1)$

$$= \frac{-i}{(16\pi^2)^{1-\epsilon/2}}\, m_H^{2-2\epsilon}\, \frac{\Gamma(\epsilon)}{\epsilon-1}$$

$$= \frac{-i}{(16\pi^2)^{1-\epsilon/2}}\, m_H^{2-2\epsilon}\, \frac{e^{-\gamma_E \epsilon}}{\epsilon-1} \left(\frac{1}{\epsilon} + \frac{S_2}{2}\epsilon + \cdots\right)$$

divergent for $N\to 4$
we know: UV divergence

$$\overset{\epsilon\to 0}{\simeq} \frac{+i}{16\pi^2}\, m_H^2 \left(\frac{1}{\epsilon} + \mathcal{O}(\epsilon^0)\right)$$

trick: $x^\epsilon = e^{\log x^\epsilon} = e^{\epsilon \log x}$
$= 1+\epsilon \log x + \frac{\epsilon^2}{2}\log^2 x + \cdots$

$\Rightarrow$ loop contribution $\boxed{\delta m_H^{2\,(H\,loop)} \sim \frac{m_H^2}{\epsilon} + \mathcal{O}(\epsilon^0)}$

philosophical discussion: If dimensional regularization is a physical scheme, there is no quadratic divergence.
But how can $(4-2\epsilon)$ dimensions make physical sense

all SM loops $\quad m_H^2 = m_H^{(LO)\,2} + x$

with $x = \begin{cases} -\frac{3}{8\pi^2}\lambda_t^2 \Lambda^2 \sim -(2\text{ TeV})^2 & t \\ +\frac{1}{16\pi^2} g^2\Lambda^2 \sim (700\text{ GeV})^2 & W \\ +\frac{1}{16\pi^2}\lambda^2\Lambda^2 \sim (500\text{ GeV})^2 & H \end{cases}$ ($\Lambda = 10$ TeV)

in terms of respective couplings

tuning to $\frac{1}{100}$

$$m_H^2 = \begin{cases} m_H^{(LO)2} + (-100+10+5)(200\text{ GeV})^2 & \text{for } \Lambda = 10\text{ TeV} \\ m_H^{(LO)} + (-10000+1000+500)(200\text{ GeV})^2 & \text{for } \Lambda = 100\text{ TeV} \\ \vdots & \vdots \end{cases}$$

$\Downarrow$

* The only experimental reason to believe in BSM physics is dark matter (or the experience that until now every increase in energy has brought us new physics). Any new 100-GeV WIMP can do that ...

* The theoretical reason to believe in BSM physics is the lack of stability for fundamental scalar masses in perturbative field theory.

$\Downarrow$

New physics at the TeV scale.

* supersymmetry	cancel Λ^2 terms
* little Higgs ("bosonic supersymmetry")	cancel Λ^2 terms
* composite-Higgs models: technicolor, topcolor, ...	cut off integral
* extra space dimensions	$\Lambda_{Planck} \to \text{TeV}$