2.5 Gauge bosons and SUSY

Until now we managed to write a supersymmetric partner of a scalar electron; now try the photon part of scalar QED with a photino fermion

(on-shell photon: 2 d.o.f. ; Weyl fermion: 2 d.o.f.)

$\Rightarrow$ try $$\boxed{\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \lambda^\dagger \bar{\sigma}^\mu \partial_\mu \lambda}$$

usual Weyl fermion as before

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ — photon field, real

for abelian gauge theory $U(1)$

> by the way – from the F field argument in the Wess-Zumino model we know that there will be another auxiliary field absorbing the off-shell γ d.o.f., but one step after another...

$$\boxed{\delta_\xi A^\mu = \xi^\dagger \bar{\sigma}^\mu \lambda + \lambda^\dagger \bar{\sigma}^\mu \xi}$$

$[\delta_\xi A^\mu] = M$; $[\xi] = M^{-1/2}$; $[\lambda] = M^{3/2}$

$\delta_\xi A^\mu \sim \xi \cdot \lambda$ fine, but index missing on r.h.s

$\leadsto \delta_\xi A^\mu \sim \xi^\dagger \bar{\sigma}^\mu \lambda$ because $\partial^\mu \lambda$ would have wrong dimension; no $A^\mu \in \mathbb{R}$

$\leadsto \delta_\xi A^\mu \sim \xi^\dagger \bar{\sigma}^\mu \lambda + \text{h.c.}$
$= \xi^\dagger \bar{\sigma}^\mu \lambda + \lambda^\dagger \bar{\sigma}^\mu \xi$

$$\boxed{\delta_\xi \lambda = C \cdot \frac{i}{2} \sigma^\mu \bar{\sigma}^\nu \xi F_{\mu\nu}}$$

$[F_{\mu\nu}] = M^2$

$\delta_\xi \lambda \sim \xi F_{\mu\nu}$ again indices missing, avoid $\sigma^\mu \sigma^\nu$ because of $SU(2)$ algebra

$\leadsto \delta_\xi \lambda \sim \sigma^\mu \bar{\sigma}^\nu \xi F_{\mu\nu}$

$\Rightarrow \delta_\xi \lambda^\dagger = -C^* \frac{i}{2} \xi^\dagger \bar{\sigma}^\nu \sigma^\mu F_{\mu\nu}$ as naively expected as usually on foot:

$$F_{\mu\nu} = -F_{\nu\mu}$$

$$\delta_\xi\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right) = -\frac{1}{2}F_{\mu\nu}\left(\partial^\mu\delta_\xi A^\nu - \partial^\nu\delta_\xi A^\mu\right) = -\frac{1}{2}F_{\mu\nu}\partial^\mu\delta_\xi A^\nu + \frac{1}{2}F_{\nu\mu}\partial^\nu\delta_\xi A^\mu$$

$$= -F_{\mu\nu}\partial^\mu\delta_\xi A^\nu = -F_{\mu\nu}\partial^\mu\left(\xi^\dagger\bar\sigma^\nu\lambda + \lambda^\dagger\bar\sigma^\nu\xi\right)$$

$$\delta_\xi\left(i\lambda^\dagger\bar\sigma^\mu\partial_\mu\lambda\right) = i(\delta_\xi\lambda^\dagger)\bar\sigma^\mu\partial_\mu\lambda + i\lambda^\dagger\bar\sigma^\mu\partial_\mu(\delta_\xi\lambda)$$

$$= +\frac{C^*}{2}\xi^\dagger\bar\sigma^\nu\sigma^\mu F_{\mu\nu}\bar\sigma^\rho\partial_\rho\lambda - \frac{C}{2}\lambda^\dagger\bar\sigma^\rho\partial_\rho\left(\sigma^\mu\bar\sigma^\nu\xi F_{\mu\nu}\right)$$

$$= \frac{C^*}{2}\xi^\dagger\underbrace{\bar\sigma^\nu\sigma^\mu\bar\sigma^\rho}\,F_{\mu\nu}\partial_\rho\lambda - \frac{C}{2}\lambda^\dagger\bar\sigma^\rho\sigma^\mu\bar\sigma^\nu\xi\partial_\rho F_{\mu\nu}$$

only $\xi^\dagger$

$$= \frac{C^*}{2}\xi^\dagger\left[\cancel{g^{\nu\mu}\bar\sigma^\rho} - g^{\nu\rho}\bar\sigma^\mu + g^{\mu\rho}\bar\sigma^\nu - i\epsilon^{\mu\nu\rho\sigma}\bar\sigma_\sigma\right]\underline{F_{\mu\nu}\partial_\rho\lambda}$$

because $F_{\mu\nu}$ antisymmetric

rewrite as $-\lambda\partial_\rho F_{\mu\nu}$; $\epsilon^{\mu\nu\rho\sigma}\left(\partial_\rho\partial_\mu A_\nu - \partial_\rho\partial_\nu A_\mu\right) = 0$

$$= \frac{C^*}{2}\xi^\dagger\left[-g^{\nu\rho}\bar\sigma^\mu + g^{\mu\rho}\bar\sigma^\nu\right]F_{\mu\nu}\partial_\rho\lambda$$

$$= \frac{C^*}{2}F_{\mu\nu}\xi^\dagger\left[-\bar\sigma^\mu\partial^\nu\lambda + \bar\sigma^\nu\partial^\mu\lambda\right]$$

$$= \frac{C^*}{2}F_{\mu\nu}\xi^\dagger\left[+\bar\sigma^\nu\partial^\mu\lambda + \bar\sigma^\nu\partial^\mu\lambda\right]$$ using again $F_{\mu\nu} = -F_{\nu\mu}$

$$= \frac{C^*}{2}F_{\mu\nu}\xi^\dagger\bar\sigma^\nu\partial^\mu\lambda$$

cancels the contribution to $F_{\mu\nu}F^{\mu\nu}$ term, term proportional to ξ dto. if $C=1$

$$\Rightarrow \boxed{\delta_\xi\lambda = \frac{i}{2}\sigma^\mu\bar\sigma^\nu\xi F_{\mu\nu}}$$

and photon-photino Lagrangian is supersymmetric

Same as before: calculate $[\delta_\eta, \delta_\xi]\left\{\begin{matrix} A^\mu \\ \lambda \end{matrix}\right\}$ ⇝ algebra does not close for λ

⇝ try auxiliary scalar again

⇝ 1 d.o.f. left in photon, mapped onto real scalar D

$$\boxed{\delta_\xi D = -i\left(\xi^\dagger \bar{\sigma}^\mu \partial_\mu \lambda - (\partial_\mu \lambda)^\dagger \bar{\sigma}^\mu \xi\right)}$$

$[D] = M^2$

⇝ try F-scalar result, $[F] = M^2$

⇝ $\delta_\xi D \sim -i\, \xi^\dagger \bar{\sigma}^\mu \partial_\mu \chi$,

but D is real, only one d.o.f., F was two...

⇝ add h.c.

$$\boxed{\delta_\xi \lambda = \frac{i}{2} \bar{\sigma}^\mu \bar{\sigma}^\nu \xi F_{\mu\nu} + \xi D}$$

just like $\delta_\xi \chi$ before...

Lagrangian

$$\delta_\xi \left(\tfrac{1}{2} D^2\right) = D\, \delta_\xi D = -iD\, \xi^\dagger \bar{\sigma}^\mu \partial_\mu \lambda + iD\,(\partial_\mu \lambda)^\dagger \bar{\sigma}^\mu \xi$$

$$\delta_\xi (\lambda^\dagger i \bar{\sigma}^\mu \partial_\mu \lambda)\Big|_D = (\delta_\xi \lambda^\dagger)\, i \bar{\sigma}^\mu \partial_\mu \lambda + \lambda^\dagger i \bar{\sigma}^\mu \partial_\mu (\delta_\xi \lambda)\Big|_D$$

$$= iD\, \xi^\dagger \bar{\sigma}^\mu \partial_\mu \lambda + i\lambda^\dagger \bar{\sigma}^\mu \partial_\mu (\xi D)$$

$$= iD\, \xi^\dagger \bar{\sigma}^\mu \partial_\mu \lambda + i \lambda^\dagger \bar{\sigma}^\mu \xi\, \partial_\mu D$$

add to total derivative

$$\Rightarrow \quad \boxed{\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\lambda^\dagger \bar{\sigma}^\mu \partial_\mu \lambda + \frac{1}{2} D^2}$$

is supersymmetric and $\{\lambda, A_\mu, D\}$ form a multiplet with closed algebra

2.6 Interactions and scalar QED

We have Lagrangians for the scalar electron and its SUSY multiplet and for the photon and its SUSY multiplet, let's build scalar QED!

How does the $ee\gamma$ interaction appear in non-SUSY QED?

replace all derivatives with covariant (abelian) derivative

$$\partial^\mu \mapsto D^\mu := \partial^\mu + iqA^\mu$$, so the Lagrangian is gauge invariant

SUSY : ∂^μ in $\mathcal{L}$: replace by D^μ to keep $\mathcal{L}$ gauge invariant

∂^μ in δ_ξ : replace by D^μ to keep SUSY from breaking gauge invariance

$$\Rightarrow \boxed{\mathcal{L} = (D_\mu\phi)^\dagger(D^\mu\phi) + i\chi^\dagger\bar{\sigma}^\mu D_\mu\chi + F^\dagger F^\dagger - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\lambda^\dagger\bar{\sigma}^\mu D_\mu\lambda + \frac{D^2}{2}}$$

note the annoying double use of D, D^μ and $F, F^{\mu\nu}$ $\rightsquigarrow$ be careful!

$$\delta_\xi\phi = \xi\cdot\chi$$
$$\delta_\xi\chi = \sigma^\mu\sigma_2\,\xi^* D_\mu\phi + \xi F$$
$$\delta_\xi F = -i\,\xi^\dagger\bar{\sigma}^\mu D_\mu\chi$$

as before

$\Downarrow$

first 3 terms in $\mathcal{L}$ SUSY invariant under ξ

$$\delta_\xi A^\mu = \alpha\,(\xi^\dagger\bar{\sigma}^\mu\lambda + h.c.)$$
$$\delta_\xi\lambda = \alpha\frac{i}{2}\sigma^\mu\bar{\sigma}^\nu\xi\,F_{\mu\nu} + \alpha\,\xi D$$
$$\delta_\xi D = -\alpha(i\,\xi^\dagger\bar{\sigma}^\mu D_\mu\lambda + h.c.)$$

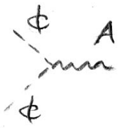
as before but with $\xi \mapsto \alpha\xi$

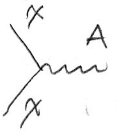
$\Downarrow$

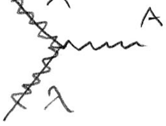
last 3 terms in $\mathcal{L}$ SUSY invariant under $\alpha\xi$

$\longleftrightarrow$

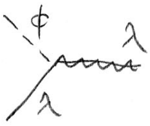
more symmetries than we need....

this Lagrangian includes the vertex 

and 

and 

(in principle)

but not 

, which should be there!

⇝ what can we add to $\mathcal{L}$ while keeping it renormalizable and SUSY invariant?

$$\mathcal{L} \mapsto \mathcal{L} + \underbrace{}_{}A q \left[(\underset{\xi}{\phi^{\dagger}} \underset{\xi}{\chi}) \cdot \underset{\alpha\xi}{\lambda} + h.c. \right] + B q\, \underset{\xi}{\phi^{\dagger}} \phi\, \underset{\alpha\xi}{D} \qquad [A] = M^{0} = [B]$$

n.b.: interactions term will mix the two sub-SUSY-transformations ξ and $\alpha\xi$ and get rid of the unnecessary SUSY invariance of the two halfs of $\mathcal{L}$ by telling us what α is (together with A and B)

⇝ compute $\delta_{\xi} \left\{ A q \left[(\phi^{\dagger}\chi) \cdot \lambda + h.c. \right] + B q\, \phi^{\dagger} \phi D \right\}$

e.g.: $A\, \delta_{\xi} \lambda \to A \xi D$ mimicks B-type contribution

$\left.\begin{array}{l} B\, \delta_{\xi} D \to \alpha B \cdot \xi^{\dagger} \bar{\sigma}^{\mu} D_{\mu} \lambda \\ B\, \delta_{\xi} \phi \to B \xi \chi \end{array}\right\}$ mimicks A-type contribution

$\Rightarrow$ $A\alpha = -B$ from all B-type contributions (involving D) in additional $\mathcal{L}$

$A = -2\alpha$ from A-type contributions plus $\chi\chi A^{\mu}$ term in original $\mathcal{L}$

⋮

$B = -1$ once we include $\phi\phi A^{\mu}$ term in original $\mathcal{L}$

(we know by now how to compute $\delta_{\xi}\mathcal{L}$ and make sure it only produces total derivatives)

$\Rightarrow$ $A = \pm 2\sqrt{\tfrac{1}{2}}$ ⇝ $A = -\sqrt{2}$

$B = -1$

$\alpha^{2} = \frac{1}{2}$ ⇝ $\alpha = -\frac{1}{\sqrt{2}}$

$$\Rightarrow \quad \boxed{\begin{aligned} \mathcal{L}_{S\text{-}SQED} &= (D_\mu \phi)^\dagger (D^\mu \phi) + i\chi^\dagger \bar{\sigma}^\mu D_\mu \chi + F^\dagger F \\ &\quad - \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \lambda^\dagger \bar{\sigma}^\mu D_\mu \lambda + \frac{D^2}{2} \\ &\quad - \sqrt{2} q \left[(\phi^\dagger \chi) \cdot \lambda + h.c. \right] - q \phi^\dagger \phi D \end{aligned}}$$

ϕ: scalar $e^\pm$
χ: $e^\pm$-ino
A^μ: photon
λ: photino

where charge / coupling q is given by $D^\mu = \partial^\mu + iqA^\mu$ (charge of ϕ, χ, λ)

and SUSY transforms multiplets $\{\phi, \chi, F\}$ and $\{A^\mu, \lambda, D\}$

and F is a complex auxiliary field, D is a real auxiliary field.

equations of motion:

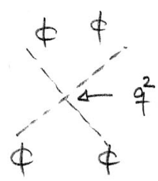
$$\frac{\partial \mathcal{L}}{\partial F} \overset{!}{=} \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu F^\dagger)} \right) = 0 \quad \Leftrightarrow \quad F \text{ free and not propagating, } F = 0$$

$$\frac{\partial \mathcal{L}}{\partial D} = D - q\phi^\dagger \phi \overset{!}{=} \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu D^\dagger)} \right) = 0 \quad \Leftrightarrow \quad D = q\phi^\dagger \phi$$

not propagation, but not zero.

$\Rightarrow$ we can replace one auxiliary field in $\mathcal{L}_{S\text{-}SQED}$

$$\frac{D^2}{2} - q\phi^\dagger \phi D = \frac{q^2}{2} (\phi^\dagger \phi)(\phi^\dagger \phi) - q^2 (\phi^\dagger \phi)(\phi^\dagger \phi) = -\frac{1}{2} q^2 (\phi^\dagger \phi)^2$$



these 4-scalar couplings in SUSY Lagrangians, governed by the gauge coupling q, are called D-terms.

2.7 Scalar potential

$\mathcal{L}_{S\text{-}SQED}$ still leaves us with $F^\dagger F$, which we still have not made sense of....

What are the Lagrangian terms still missing & renormalizable & Lorentz-invariant

$$\boxed{\mathcal{L}_W = \underbrace{W_i(\phi,\phi^\dagger)F}_{\text{generally allowed with } [W]=M^2} - \frac{1}{2}\underbrace{W_{ij}(\phi,\phi^\dagger)\chi_i\chi_j}_{\text{quasi-mass term for Weyl spinor}} + \text{h.c}}$$

$\{\phi, \chi, F\}$ multiplet only

$\delta_\xi \mathcal{L}_W = 0$:

(1) $\frac{\partial W_{ij}}{\partial \phi_k}$ symm. in (i,j,k)

(2) $\frac{\partial W_{ij}}{\partial \phi_k^\dagger} = 0$

$$\boxed{W_{ij} = M_{ij} + y_{ijk}\phi_k}$$

or $W_{ij} = \frac{\partial^2 W}{\partial \phi_i \partial \phi_j}$; $W = \frac{M_{ij}}{2}\phi_i\phi_j + \frac{y_{ijk}}{6}\phi_i\phi_j\phi_k$

(3)
$$0 \overset{?}{=} -iW_{ij}\,\xi^\dagger\bar\sigma^\mu\chi_i\partial_\mu\phi_j - iW_i\,\xi^\dagger\bar\sigma^\mu\partial_\mu\chi_i$$
$$\Leftrightarrow \quad 0 \overset{?}{=} -i\,\xi^\dagger\bar\sigma^\mu\chi_i\partial_\mu\left(\frac{\partial W}{\partial \phi_i}\right) - iW_i\,\xi^\dagger\bar\sigma^\mu\partial_\mu\chi_i$$

not zero, but total derivative, provided

$$\boxed{W_i = \frac{\partial W}{\partial \phi_i} = M_{ij}\phi_j + \frac{1}{2}y_{ijk}\phi_j\phi_k}$$

now, try equation of motion for F_i, assuming Lagrangian

$$\mathcal{L} \sim F_i^\dagger F_i + W_i F_i + W_i^\dagger F_i^\dagger$$

$$\frac{\partial \mathcal{L}}{\partial F} = F_i^\dagger + W_i \overset{!}{=} \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu F^\dagger)} \right) = 0 \quad \Leftrightarrow \quad F_i^\dagger = -W_i$$

$$F_i = -W_i^\dagger$$

$\Rightarrow$ Wess-Zumino model Lagrangian part becomes

$$\boxed{\mathcal{L}_W \sim -|W_i|^2 - \frac{1}{2}\left[W_{ij}\, \chi_i \cdot \chi_j + h.c. \right]}$$

ϕ mass term M_{ij}^2
$\phi\phi\phi$ coupling $M_{ij} Y_{ijk}$
$\phi\phi\phi\phi$ coupling $Y_{ijk} \cdot Y_{ijk}$

χ mass term M_{ij}
$\phi\chi\chi$ coupling Y_{ijk}

$\Rightarrow$ ϕ and χ masses match
ϕ^3 and $\phi\chi\chi$ coupling matches
ϕ^4 couplings now Yukawa-type in addition to D-term