3. Superfields

Unfortunately, at some point the algebra of SUSY generators has to drop on our heads.... Remember the (chiral) supermultiplet $\{\phi, \chi, F\}$ which closed under

$$[\delta_\eta, \delta_\xi] = i\eta^\dagger \bar{\sigma}^\mu \xi^* \partial_\mu \quad + h.c.$$

Now write SUSY transformation in terms of generators of the supersymmetry

$$\phi' = \phi + \delta_\xi \phi \qquad \xi\cdot Q \equiv -i\xi^T \sigma_2 Q \text{ as usual}$$

$$\parallel$$

$$(1 + i\xi\cdot Q)\phi(1 - i\xi\cdot Q) = \phi + i\,\xi\cdot Q\,\phi - i\phi\,\xi\cdot Q = \phi + i[\xi\cdot Q, \phi]$$

infinitesimal transformation in Lorentz-invariant form; ξ: SUSY-shift
Q: SUSY generator

$\Rightarrow$ try to write something like

$$\boxed{\delta_\xi \phi = -i\,\xi^T \sigma_2 \chi \stackrel{?}{=} i[\xi\cdot Q, \phi]}$$

Remember SUSY transformations involved ξ and $\xi^\dagger$; define analogue to $\xi\cdot Q$ but with ξ^* as $\bar{\xi}\cdot\bar{Q} := \xi^\dagger i\sigma_2 Q^*$ and generalize the commutator above to

$$\boxed{\delta_\xi \phi = i[\xi\cdot Q + \bar{\xi}\cdot\bar{Q}, \phi]}$$

And now let the algebra of Q and $Q^\dagger$ fall on our heads:

$$\boxed{\begin{aligned} \{Q_a, Q_b\} &= 0 \\ \{Q_a^\dagger, Q_b^\dagger\} &= 0 \\ \{Q_a, Q_b^\dagger\} &= (\sigma^\mu)_{ab} P_\mu \end{aligned}}$$

fermions anticommute

as above, $i\partial_\mu \leftrightarrow P_\mu$
$\{\,\}$ because of Grassmann ξ, η

$a, b = 1, 2$ (components)

Beyond infinitesimal transformations and interpret ξ as a coordinate θ, to form a superspace (x, θ, θ^*) with transformation

$$\boxed{\Phi(x,\theta,\theta^*) = \mathcal{U}(x,\theta,\theta^*)\,\Phi(0,0,0)\,\mathcal{U}^{-1}(x,\theta,\theta^*)}$$

with the generalized finite unitary transformation

$$\mathcal{U}(x,\theta,\theta^*) = e^{ix\cdot P}\, e^{i\theta\cdot Q}\, e^{i\bar\theta\cdot\bar Q}$$

with the proper definitions of $x\cdot P$, $\theta\cdot Q$, $\bar\theta\cdot\bar Q$

and the superfield $\Phi(x,\theta,\theta^*)$ which we still have to define.

Two successive superspace transformations:

$$0 \mapsto x^\mu \overset{?}{\mapsto} x^\mu + a^\mu$$
$$0 \mapsto \theta \mapsto \theta + \xi$$
$$0 \mapsto \theta^* \mapsto \theta^* + \xi^*$$

defined by $\mathcal{U}(a,\xi,\xi^*)\,\mathcal{U}(x,\theta,\theta^*)\,\Phi(0)\,\mathcal{U}^{-1}(x,\theta,\theta^*)\,\mathcal{U}^{-1}(a,\xi,\xi^*)$

can be calculated using Baker-Campbell-Hausdorf formula (see Aitchison p. 68ff)

$$e^A e^B = e^{A+B+\frac{1}{2}[A,B]+\frac{1}{6}[[A,B],B]+\cdots}$$

and gives additional term due to SUSY algebra: $0 \to x^\mu \mapsto x^\mu + a^\mu - i\theta\cdot\sigma^\mu\xi^*$

$$\boxed{\mathcal{U}(a,\dots)\,\mathcal{U}(x,\dots)\,\Phi(0)\,\mathcal{U}^{-1}(x,\dots)\,\mathcal{U}^{-1}(a,\dots) = \Phi(x+a-i\theta\sigma\xi^*,\ \theta+\xi,\ \theta^*+\xi^*)}$$

Question: Can we make use of these SUSY generators and the superfields Φ living in superspace $(x^\mu, \theta, \theta^*)$?

Remember that θ, θ^* are Grassmann variables, i.e. $(\theta_1)^2 = 0 = (\theta_2)^2$

$\Rightarrow$ write superfield as finite Taylor series

$$\Phi(x,\theta,\theta^*) = \Phi_{00}(x) + \theta\cdot\Phi_{10}(x) + \frac{1}{2}\theta\cdot\theta\,\Phi_{20}(x) + \theta^*\Phi_{01}(x) + \frac{1}{2}\theta^*\theta^*\Phi_{02}(x) + \ldots\ldots$$

and SUSY shift as (only shift in super-coordinates by ξ or ξ^*)

$$\delta\Phi = -i\theta\sigma^\mu\xi^*\partial_\mu\Phi + \xi^a\frac{\partial\Phi}{\partial\theta^a} + \xi^*_a\frac{\partial\Phi}{\partial\theta^*_a}$$

<u>Example</u>: simple superfield $\boxed{\Phi(x,\theta) = \Phi_0(x) + \theta\cdot\Phi_1(x) + \frac{1}{2}\theta\theta\,\Phi_2(x)}$

with shift $\boxed{\delta\Phi = -i\theta\sigma^\mu\xi^*\partial_\mu\Phi + \xi^a\frac{\partial\Phi}{\partial\theta_a}}$

systematically produces ($\delta\Phi$ now expanded in θ)

$$\begin{aligned}\delta\Phi &= (\delta\Phi)_0 + \theta\cdot(\delta\Phi)_1 + \frac{1}{2}\theta\cdot\theta\,(\delta\Phi)_2 \\ &= -i\theta\sigma^\mu\xi^*\left(\partial_\mu\Phi_0 + \theta\cdot\partial_\mu\Phi_1 + \frac{1}{2}\theta\theta\,\partial_\mu\Phi_2\right) + \xi^a\left(\Phi_{1a} + \theta_a\Phi_2\right) \\ &= \xi\cdot\Phi_1 - i\theta\sigma^\mu\xi^*\partial_\mu\Phi_0 + \xi\cdot\theta\,\Phi_2 - i\theta\sigma^\mu\xi^*(\theta\,\partial_\mu\Phi_1)\end{aligned}$$

compare with chiral multiplet

$$\begin{aligned}\delta_\xi\phi &= \xi\cdot\chi \\ \delta_\xi\chi &= \sigma^\mu\sigma_2\xi^*\partial_\mu\phi + \xi F \\ \delta_\xi F &= -i\xi^\dagger\bar\sigma^\mu\partial_\mu\chi\end{aligned}$$

$\ulcorner \sigma_2\sigma^\mu\sigma_2 = \bar\sigma^{\mu T} \lrcorner$

$\Rightarrow$ SUSY transformations correct if we identify $\Phi_0 = \phi$; $\Phi_1 = \chi$; $\Phi_2 = F$

$\Rightarrow$ <u>chiral superfield</u>

$$\boxed{\Phi(x,\theta) = \phi(x) + \theta\cdot\chi(x) + \frac{1}{2}\theta\cdot\theta\,F(x)}$$

4 The MSSM

Remember the particle content of our scalar SUSY-QED after elimination all auxiliary fields:

$$\phi\,,\ \chi\,,\ A^\mu\,,\ \lambda$$

ϕ: scalar $e_L^\pm$, 2 d.o.f. (on-shell)
χ: Weyl $\tilde{e}$ 2 d.o.f
A^μ: photon 2 d.o.f
λ: Weyl $\tilde{\gamma}$ 2 d.o.f

add another, now right handed scalar electron
right handed $\Leftrightarrow$ partner of ψ field in $\Psi = \begin{pmatrix} \psi \\ \chi \end{pmatrix}$

$\rightsquigarrow$ particle content $\quad \underbrace{e_L^\pm, e_R^\pm}_{\text{two scalars}}, \underbrace{\tilde{e}_\chi, \tilde{e}_\psi}_{\text{Dirac fermion}}, A^\mu, \lambda$

$\rightsquigarrow$ relabel $e_L^\pm, e_R^\pm \mapsto \tilde{e}_L^\pm, \tilde{e}_R^\pm$ scalar electrons (SUSY partners)
$\qquad \tilde{e}_\chi, \tilde{e}_\psi \mapsto \Psi_e$ Dirac electron (SM field)

$\rightsquigarrow$ What is λ? Build 4-spinor out of one Weyl spinor:

$$\Psi_M := \begin{pmatrix} \lambda \\ -i\sigma_2 \lambda^* \end{pmatrix}$$

has correct Lorentz transformations for λ and $(-i\sigma_2 \lambda^*)$

charge conjugation

$$\Psi_M^C = -i\gamma^2 \Psi_M^* = \begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix} \begin{pmatrix} \lambda^* \\ -i\sigma_2 \lambda \end{pmatrix} = \begin{pmatrix} +\sigma_2^2 \lambda \\ -i\sigma_2 \lambda^* \end{pmatrix} = \begin{pmatrix} \lambda \\ -i\sigma_2 \lambda^* \end{pmatrix} \equiv \Psi_M$$

photino is Majorana 4-spinor and its own anti-particle!

Side remark: charges of Majorana spinors have to be real, so Lagrangian is hermitian
e.g. adjoint representation of SU(N)

Chiral multiplets in MSSM:

representation

superfield	scalar	fermion	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
Q	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	$\underline{3}$	$\underline{2}$	$1/3$ ← quantum number
$\bar{u}$	$\tilde{u}_R$	u_R^c	$\underline{\bar{3}}$	$\underline{1}$	$-4/3$
$\bar{d}$	$\tilde{d}_R$	d_R^c	$\underline{\bar{3}}$	$\underline{1}$	$2/3$
H_u	(H_u^+, H_u^0)	$(\tilde{H}_u^+, H_u^0)$	$\underline{1}$	$\underline{2}$	1
H_d	(H_d^0, H_d^-)	$(\tilde{H}_d^0, \tilde{H}_d^-)$	$\underline{1}$	$\underline{2}$	-1

n.b. remember that Yukawa coupling now come from $W_{ij}(\phi, \phi^+)$ and supersymmetry required $\frac{\partial W_{ij}}{\partial \phi_k^+} = 0$

(only one reason why we need 2 doublets to give mass to b and t, instead of H and H^+ in the SM)

gauge multiplets in MSSM:

fermion	boson	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$\tilde{g}$	g	$\underline{8}$	$\underline{1}$	0
$\tilde{W}^\pm, \tilde{W}^0$	$\tilde{W}^\pm\ W^0$	$\underline{1}$	$\underline{3}$	0
$\tilde{B}$	B	$\underline{1}$	$\underline{1}$	0

with neutralino / chargino mass matrices

$\tilde{H}\tilde{B}Z$ Yukawa couplings.

$$\begin{pmatrix} m_{\tilde{B}} & 0 & -c_\beta s_W m_Z & s_\beta s_W m_Z \\ 0 & m_{\tilde{W}} & +c_\beta c_W m_Z & -s_\beta c_W m_Z \\ & & 0 & -\mu \\ \text{symmetric matrix} & & -\mu & 0 \end{pmatrix} \qquad \begin{pmatrix} m_{\tilde{W}^\pm} & \sqrt{2}\, s_\beta m_W \\ \sqrt{2}\, c_\beta m_W & \mu \end{pmatrix}$$

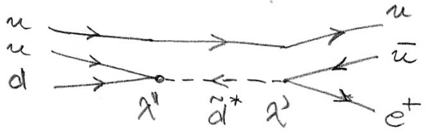
$s_\beta \equiv \sin\beta$
$c_\beta \equiv \cos\beta$

R parity

gauge invariant, renormalizable and perfectly allowed terms in the superpotential

$$W = \lambda_{ijk} \, L_i L_j \bar{e}_k + \lambda'_{ijk} \, L_i Q_j \bar{d}_k + \lambda''_{ijk} \, \bar{u}_i \bar{d}_j \bar{d}_k$$

for example mediate proton decay through renormalizable interactions



but proton life time experimentally excluded below $\sim 10^{16}$ GeV

↳ well over 10 orders of magnitude suppressed

↳ decay operator should not be renormalizable, $\sim \frac{1}{M_{GUT}}$ or $\frac{1}{M_{Planck}}$ instead

$\Rightarrow$ postulate R parity conservation

$$R = \begin{cases} +1 & \text{SM particles and 2HDM} \\ -1 & \text{SUSY partners} \end{cases}$$

or $\boxed{R = (-1)^{3B+L+2s}}$

SUSY breaking

Remember, SUSY predicted $m_{\tilde{e}} = m_e = 511\,\text{keV}$, which again is experimentally excluded

↳ Go back to original motivation and define all explicitly SUSY-breaking Lagrangian terms which do not produce a quadratic divergence of m_H: soft SUSY breaking

e.g.: scalar masses, fermion masses,
trilinear scalar terms

↳ Unfortunately there is no time to discuss SUSY breaking scenarios. Great reference is S. Martin hep-ph/9709356

* gravity mediation (mSUGRA)
* gauge mediation (GMSB)
* anomaly mediation (AMSB)
* gaugino mediation ($\tilde{g}$MSB)

⋮