4 MSSM Higgs Sector

Remember that supersymmetry required $\delta_\xi \mathcal{L}_W = 0 \Leftrightarrow \begin{cases} \frac{\partial W_{ij}}{\partial \phi_k} \text{ symmetric in } (i,j,k) \\ \frac{\partial W_{ij}}{\partial \phi_k^\dagger} = 0 \\ \vdots \end{cases}$

which (1) allowed us to write a scalar potential

$$W_i = \frac{\partial W}{\partial \phi_i} = M_{ij}\phi_j + \frac{1}{2} y_{ijk}\phi_j\phi_k \quad , \quad \mathcal{L}_W \sim -|W_i|^2$$

(2) forces us to introduce two Higgs doublets to write mass terms for up-type and down-type fermions.

Also, remember the different sources of scalar terms in the Lagrangian:

$W \sim \mu \cdot H_u \cdot H_d$ or more general for $H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$ $\quad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$

$$\mathcal{L}_W \sim |\mu|^2 \left(|H_u^+|^2 + |H_d^-|^2 + |H_u^0|^2 + |H_d^0|^2 \right)$$

from superpotential (supersymmetric!)

soft SUSY breaking

$$\mathcal{L}_{SSB} \sim -m_{H_u}^2 \left(|H_u^+|^2 + |H_u^0|^2 \right) - m_{H_d}^2 \left(|H_d^-|^2 + |H_d^0|^2 \right) + b \left[\left(H_u^+ H_d^- - H_u^0 H_d^0 \right) + h.c. \right]$$

D terms (U(1) and SU(2) gauge couplings)

$$\mathcal{L}_D \sim \frac{g^2}{8} \left\{ \left[\left(|H_u^+|^2 + |H_u^0|^2 \right) - \left(|H_d^-|^2 + |H_d^0|^2 \right) \right]^2 + 4 \left| H_u^+ H_d^{0\dagger} + H_u^0 H_d^{-\dagger} \right|^2 \right\}$$
$$+ \frac{g'^2}{8} \left[\left(|H_u^+|^2 + |H_u^0|^2 \right) - \left(|H_d^-|^2 + |H_d^0|^2 \right) \right]^2$$

remember $D = q\phi^\dagger\phi$ or non-abelian $D^\alpha = g \sum_i \phi_i^\dagger T^\alpha \phi_i$

charge, e.g. U(1) hypercharge ± 1

non-abelian generators i.e. Pauli matrices collect all terms and write a Higgs potential for 2HDM:

$$V = (|\mu|^2 + m_{H_u}^2)(|H_u^+|^2 + |H_u^0|^2) + (|\mu|^2 + m_{H_d}^2)(|H_d^0|^2 + |H_d^-|^2)$$

sign not fixed!

$$+ b\left[(H_u^+ H_d^- - H_u^0 H_d^0) + h.c.\right]$$

$$+ \frac{g^2+g'^2}{8}(|H_u^+|^2 + |H_u^0|^2 - |H_d^-|^2 - |H_d^0|^2)^2 + \frac{g^2}{2}|H_u^+ H_d^{0\dagger} + H_u^0 H_d^{-\dagger}|^2$$

just collected from before.

all ϕ^4 terms with prefactor >0

remember that we can rotate H_u and H_d simultaneously

$\Rightarrow$ choose $H_u^+ = 0$ at minimum of V, i.e. at $\frac{\partial V}{\partial H_u^+} = 0$

$$\Leftrightarrow \quad H_d^- = 0 \quad \underline{\text{or}} \quad b + \frac{g^2}{2} H_d^{0\dagger} H_u^{0\dagger} = 0$$

$$\Downarrow$$

$$b\left[(H_u^+ H_d^- - H_u^0 H_d^0) + h.c.\right] = b(-H_u^0 H_d^0 + h.c.) = -2b\, H_u^0 H_d^0$$

$$= +g^2 H_d^{0\dagger} H_u^{0\dagger} H_u^0 H_d^0 = g^2 |H_u^0|^2 |H_d^0|^2 > 0$$

which does not help finding a minimum

$\Rightarrow$ at minimum $H_u^+ = 0$ and $H_d^- = 0$

take $b \in \mathbb{R}$, absorb phase into $H_d^0 H_u^0$

$\in \mathbb{R}$ at minimum

remember: D-terms

$$V = (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 - b(H_u^0 H_d^0 + h.c.) + \frac{g^2+g'^2}{8}(|H_u^0|^2 - |H_d^0|^2)^2$$

$$\Rightarrow \boxed{V = (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 - 2b\,|H_u^0||H_d^0| + \frac{g^2+g'^2}{8}(|H_u^0|^2 - |H_d^0|^2)^2}$$

special direction: $|H_u^0| = |H_d^0| \equiv |H^0|$

$$V = \underbrace{(2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2 - 2b)}_{\overset{!}{>}0}|H^0|^2$$

$\overset{!}{>} 0$, otherwise not bounded from below

$\Leftrightarrow$ EWSB requires $2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2 > 2b$

Find stationary minimum

$$0 \overset{!}{=} \frac{\partial V}{\partial |H_u^0|}\Bigg|_{|H_i^0|=v_i} = 2\left(|\mu|^2 + m_{H_u}^2\right)|H_u^0| - 2b\,|H_d^0| + \frac{g^2+g'^2}{4}\left(|H_u^0|^2 - |H_d^0|^2\right)\cdot 2|H_u^0| \Bigg|_{|H_i^0|=v_i}$$

$$0 \overset{!}{=} \frac{\partial V}{\partial |H_d^0|}\Bigg|_{|H_i^0|=v_i} = 2\left(|\mu|^2 + m_{H_d}^2\right)|H_d^0| - 2b\,|H_u^0| - \frac{g^2+g'^2}{4}\left(|H_u^0|^2 - |H_d^0|^2\right) 2|H_d^0| \Bigg|_{|H_i|=v_i}$$

$$\Leftrightarrow \quad \begin{cases} \left(|\mu|^2 + m_{H_u}^2\right) v_u = b\, v_d + \frac{g^2+g'^2}{4}\left(v_d^2 - v_u^2\right) v_u \\ \left(|\mu|^2 + m_{H_d}^2\right) v_d = b\, v_u - \frac{g^2+g'^2}{4}\left(v_d^2 - v_u^2\right) v_d \end{cases}$$

check gauge boson masses:

$$m_Z^2 = \frac{g^2+g'^2}{2}\left(v_u^2 + v_d^2\right)$$

$$m_W^2 = \frac{g^2}{2}\left(v_u^2 + v_d^2\right)$$

and define: $\boxed{\tan\beta = \frac{v_u}{v_d}}$

$\Leftrightarrow \quad v_u = v \sin\beta$, $v_d = v\cos\beta$

$$\Rightarrow \quad \begin{cases} |\mu|^2 + m_{H_u}^2 = b\cot\beta + \frac{m_Z^2}{2}\cos 2\beta \\ |\mu|^2 + m_{H_d}^2 = b\tan\beta - \frac{m_Z^2}{2}\cos 2\beta \end{cases}$$

fixes e.g. b, but we will for now keep it to shorten formulas

Now, count degrees of freedom in Higgs doublets:

$$\begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} = \begin{pmatrix} \underbrace{\mathrm{Re}\, H_u^+}_{H^+} + i\, \underbrace{\mathrm{Im}\, H_u^+}_{\text{long } W^+} \\ v_u + \underbrace{\mathrm{Re}\, H_u^0}_{\text{scalars } h^0, H^0} + i\, \underbrace{\mathrm{Im}\, H_u^0}_{\text{long. } W_3^0 \,\&\, A^0} \end{pmatrix} \qquad \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} = \begin{pmatrix} v_d + \underbrace{\mathrm{Re}\, H_d^0}_{\text{scalars } h^0, H^0} + i\, \underbrace{\mathrm{Im}\, H_d^0}_{\text{long } W_3^0 \,\&\, A^0} \\ \underbrace{\mathrm{Re}\, H_d^-}_{H^-} + i\, \underbrace{\mathrm{Im}\, H_d^-}_{\text{long } W^-} \end{pmatrix}$$

and remember how masses are given by the potential:

$$\boxed{2 m_i^2 = \left.\frac{\partial^2 V}{\partial |H_i|^2}\right|_{|H_j^0| = v_j}}$$

get factors right between kinetic terms and potential

First, compute pseudoscalar mass m_{A^0}

$$V \sim (|\mu|^2 + m_{H_u}^2)(\mathrm{Im}\, H_u^0)^2 + (|\mu|^2 + m_{H_d}^2)(\mathrm{Im}\, H_d^0)^2 + 2b\, (\mathrm{Im}\, H_u^0)(\mathrm{Im}\, H_d^0)$$
$$+ \frac{g^2+g'^2}{8}\left[(\mathrm{Re}\, H_u^0)^2 + (\mathrm{Im}\, H_u^0)^2 - (\mathrm{Re}\, H_d^0)^2 - (\mathrm{Im}\, H_d^0)^2\right]^2$$

$$\Rightarrow \frac{\partial V}{\partial(\mathrm{Im}\, H_u^0)} \sim 2\,(|\mu|^2 + m_{H_u}^2)\, \mathrm{Im}\, H_u^0 + 2b\, \mathrm{Im}\, H_d^0 + \frac{g^2+g'^2}{8}[\dots]\, 2 \cdot 2 \cdot \mathrm{Im}\, H_u^0$$

$$\Rightarrow \frac{\partial^2 V}{\partial(\mathrm{Im}\, H_u^0)^2} \sim 2\,(|\mu|^2 + m_{H_u}^2) + \frac{g^2+g'^2}{2}[\dots] + \frac{g^2+g'^2}{2}\, \mathrm{Im}\, H_u^0 \cdot 2\, \mathrm{Im}\, H_u^0$$

$$\Rightarrow \left.\frac{\partial^2 V}{\partial(\mathrm{Im}\, H_u^0)^2}\right|_{\text{vev}} \sim 2\,(|\mu|^2 + m_{H_u}^2) + \frac{g^2+g'^2}{2}(v_u^2 - v_d^2) \overset{\text{minimum condition}}{=} 2b \cot\beta$$

$\Rightarrow$ CP-odd mass matrix for $\mathrm{Im}\, H_u^0$, $\mathrm{Im}\, H_d^0$

$$\mathcal{M}_A^2 = \begin{pmatrix} b\cot\beta & b \\ b & b\tan\beta \end{pmatrix} \qquad \text{with eigenvalues} \quad m_\pm^2 = \begin{cases} 0 & \text{Goldstone in } Z^0 \\ \dfrac{2b}{\sin 2\beta} & \text{pseudoscalar } A^0 \end{cases}$$

$$\Rightarrow \boxed{m_A = \sqrt{\frac{2b}{\sin 2\beta}}}$$

Next, do the same for two scalar Higgses

$$V \sim (|\mu|^2 + m_{H_u}^2)(\mathrm{Re}\,H_u^0)^2 - 2b\,(\mathrm{Re}\,H_u^0)(\mathrm{Re}\,H_d^0) + \frac{g^2+g'^2}{8}[\ldots]^2$$

$$\Rightarrow \frac{\partial V}{\partial(\mathrm{Re}\,H_u^0)} = 2(|\mu|^2+m_{H_u}^2)\,\mathrm{Re}\,H_u^0 - 2b\,\mathrm{Re}\,H_d^0 + \frac{g^2+g'^2}{8}[\ldots]\,2\cdot 2\,\mathrm{Re}\,H_u^0$$

$$\Rightarrow \frac{\partial^2 V}{\partial(\mathrm{Re}\,H_u^0)^2} = 2(|\mu|^2+m_{H_u}^2) + \frac{g^2+g'^2}{2}[\ldots] + \frac{g^2+g'^2}{2}\mathrm{Re}\,H_u^0 \cdot 2\,\mathrm{Re}\,H_u^0$$

$$= 2(|\mu|^2+m_{H_u}^2) + \frac{g^2+g'^2}{2}[\ldots] + (g^2+g'^2)(\mathrm{Re}\,H_u^0)^2$$

$$\Rightarrow \left.\frac{\partial^2 V}{\partial(\mathrm{Re}\,H_u^0)^2}\right|_{vev} = 2(|\mu|^2+m_{H_u}^2) + \frac{g^2+g'^2}{2}(v_u^2 - v_d^2 + 2v_u^2) = 2(|\mu|^2+m_{H_u}^2) + \frac{g^2+g'^2}{2}(3v_u^2 - v_d^2)$$

$$= 2b\cot\beta + \frac{g^2+g'^2}{2}(v_d^2 - v_u^2) + \frac{g^2+g'^2}{2}(3v_u^2 - v_d^2) = 2b\cot\beta + \frac{g^2+g'^2}{2}(2v_u^2)$$

$$= 2b\cot\beta + \frac{g^2+g'^2}{2}\,2v^2\sin^2\beta = 2\left[b\cot\beta + \frac{m_Z^2}{2}\,2\sin^2\beta\right]$$

$\Rightarrow$ CP-even mass matrix for $\mathrm{Re}\,H_u^0$, $\mathrm{Re}\,H_d^0$

$$\mathcal{M}_{h,H}^2 = \begin{pmatrix} b\cot\beta + m_Z^2\sin^2\beta & -b - \frac{1}{2}m_Z^2\sin 2\beta \\ -b - \frac{1}{2}m_Z^2\sin 2\beta & b\tan\beta + m_Z^2\cos^2\beta \end{pmatrix}$$

with eigenvalues

$$m_{h,H^0}^2 = \frac{1}{2}\left[m_A^2 + m_Z^2 \mp \left((m_A^2+m_Z^2)^2 - 4m_A^2 m_Z^2\cos^2 2\beta\right)^{1/2}\right]$$

in limit $m_A \gg m_Z$:

$$m_{h^0}^2 = \frac{1}{2}\left[m_A^2 \mp \left(m_A^4 - 4m_A^2 m_Z^2\cos^2 2\beta\right)^{1/2}\right] \simeq \frac{1}{2}\left[m_A^2 \mp m_A^2\left(1 - \frac{4m_Z^2}{m_A^2}\cos^2 2\beta\right)^{1/2}\right]$$

$$= \frac{1}{2}\left[m_A^2 \mp m_A^2\left(1 - \frac{2m_Z^2}{m_A^2}\cos^2 2\beta\right)\right] = \begin{cases} m_Z^2\cos^2 2\beta & \text{bound from above} \\ m_A^2 & \text{high-mass scale} \end{cases}$$

Summary: 2HDM in the MSSM

8 d.o.f in doublets, 3 needed for long. W, Z

$\Rightarrow$ 5 physical d.o.f $h^0, H^0, A^0, H^\pm$

$\Rightarrow$ mass spectrum