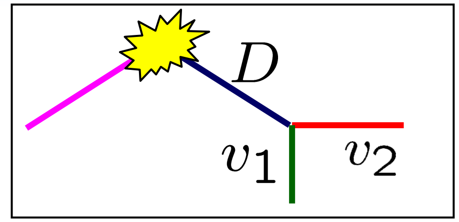
Mass Measurement in Events with Missing Transverse Energy

Dan Tovey,
University of Sheffield

Plan

- Summary of mass measurement techniques
- Point out assumptions, advantages, disadvantages
- Start with conventional techniques developed for SM
- Move onto evolved techniques for multiple invisible FS particles (DM)
- Apologies if I've missed / omitted your favourite technique!

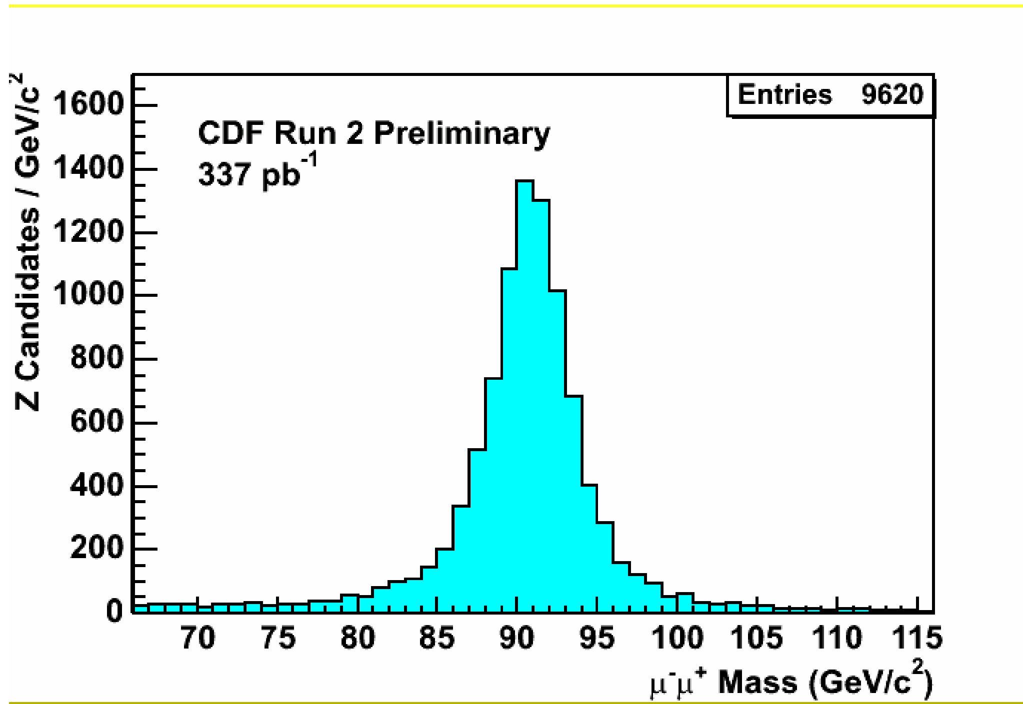
SM Mass Measurements



- **Easiest to focus where possible on channels without neutrinos**
  - **Trivial: obtain mass from invariant mass of decay products**
  - **E.g. Z→ll, t→bqq'**
- **If v1 and v2 massless then:**

$$M^2(v_1, v_2) = 2p(v_1)p(v_2)(1 - \cos\theta_{12})$$

- **Mass peak ☺☺☺**
- **No use of E_T^{miss} ☺**
- **Fully boost invariant ☺**



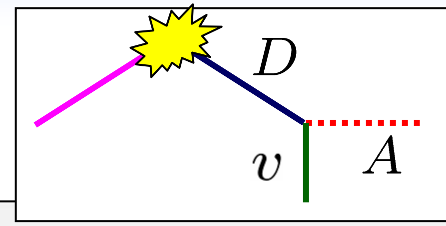
$$\begin{aligned} m^2(v_1, v_2) &= [E(v_1) + E(v_2)]^2 - [\mathbf{p}(v_1) + \mathbf{p}(v_2)]^2 \\ &= m^2(v_1) + m^2(v_2) + 2[E(v_1)E(v_2) - \mathbf{p}(v_1) \cdot \mathbf{p}(v_2)] \end{aligned}$$

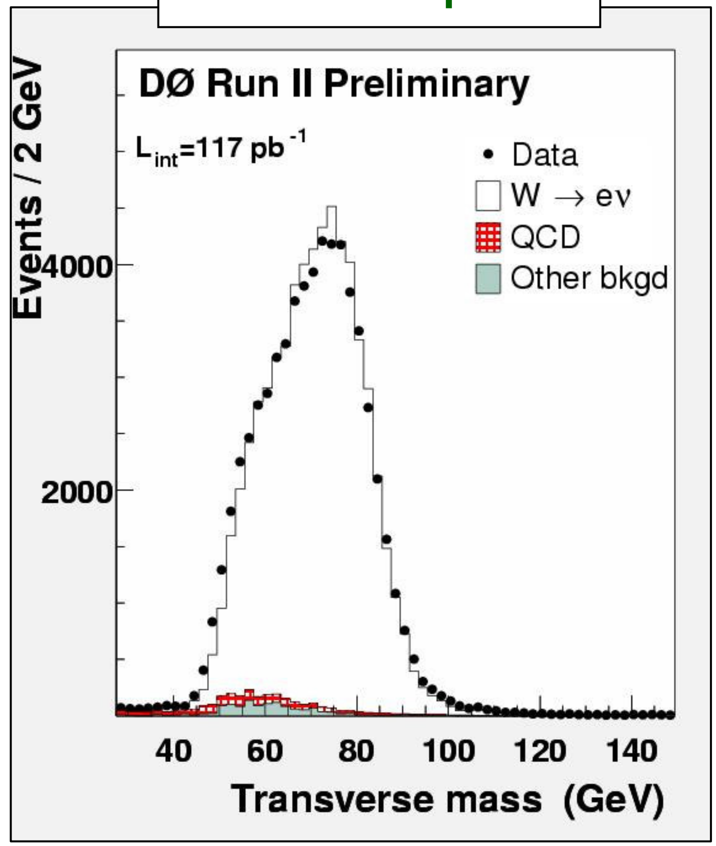
Single 1-Step SI Chain

- Single-step semi-invisible (SI) chain: use transverse mass ($p_T(A) = E_T^{miss}$)
  - e.g. $W \rightarrow l\nu$
- If v and A massless then:

$$M_T^2(v,A) = 2p_T(v)p_T(A)(1-\cos\phi_{12})$$

- Requires use of E_T^{miss} ☹
- Unconstrained $p_z(v)$ destroys mass peak ☹☹☹
- m_T has kinematic endpoint at M_D ☺
- (Trivially) invariant under longitudinal boosts ☺
- Invariant under transverse boosts ☺

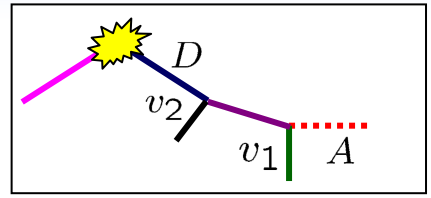


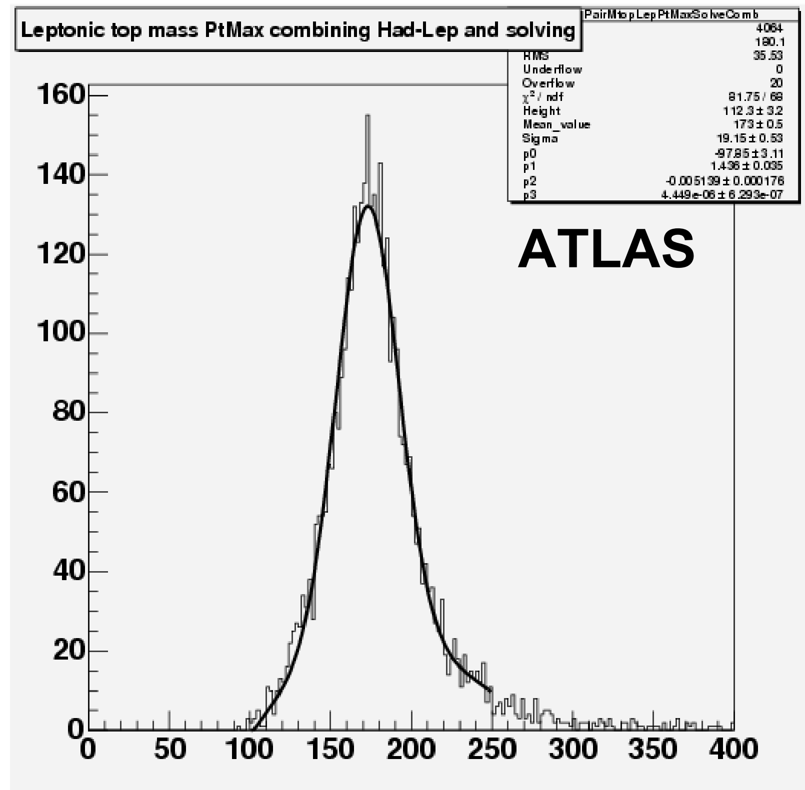


$$\begin{aligned} m_T^2(v,A) &= [E_T(v)+E_T(A)]^2 - [\mathbf{p_T}(v)+\mathbf{p_T}(A)]^2 \\ &= m^2(v)+m^2(A)+2[E_T(v)E_T(A)-\mathbf{p_T}(v)\cdot\mathbf{p_T}(A)] \end{aligned}$$

Single Multi-Step SI Chain

- Multi-step SI chain with mass constraint: solve constraint equation to reconstruct $p_z(\nu_1)$ (up to quadratic ambiguity)
  - e.g. $t \to Wb \to bl\nu$
- 'Hybrid' method (see later)
- Recovers mass peak ☺☺☺
- Fully boost invariant ☺
- Requires use of E_T^{miss} ☹

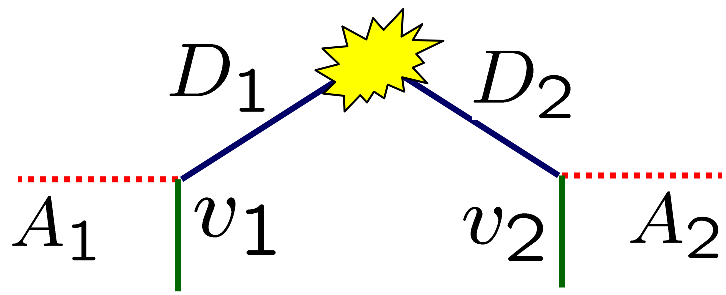




$$p_z(\nu) = \frac{[\frac{1}{2}(m_W^2 - m_T^2) + |p_T(l)||E_T^{miss}|]p_z(l) \pm E(l)\sqrt{\frac{1}{4}(m_W^2 - m_T^2) + |p_T(l)||E_T^{miss}|}}{p_T^2(l)}$$

Double SI Chains

- The rest of this talk ….
- Assume masses of invisibles equal
- Ubiquitous in
  - R-Parity conserving SUSY
  - Most models with prompt DM
  - SM: tt→blνblν
  - SM: WW→lνlν



- Extra degrees of freedom lead to under-constrained systems, even for long chains ➔ kinematic endpoint techniques
- In general: More DoF ←➔ less prominent endpoint
- Much work over past 12 years (SUSY)
- "KE" to SM (m_{top}, WW) measurements ….?

Inclusive Measurements

- Historically first techniques to be developed
- Widely applicable ☺
- Large signal acceptance ➔ small lumi ☺
- Signal often lacks clear features ☹
- Model-independent interpretation difficult ☹

Effective Mass M_{eff} / H_T

- Most widely used: simple ☺
- Measures ~$2(M_D^2 - M_A^2)/M_D$ (see later) via total energy of products
- Approximation 1: neutrals co-linear
- Approximation 2: heavy states produced at threshold / rest
- Frequently use p_T sum of 4 hardest jets:

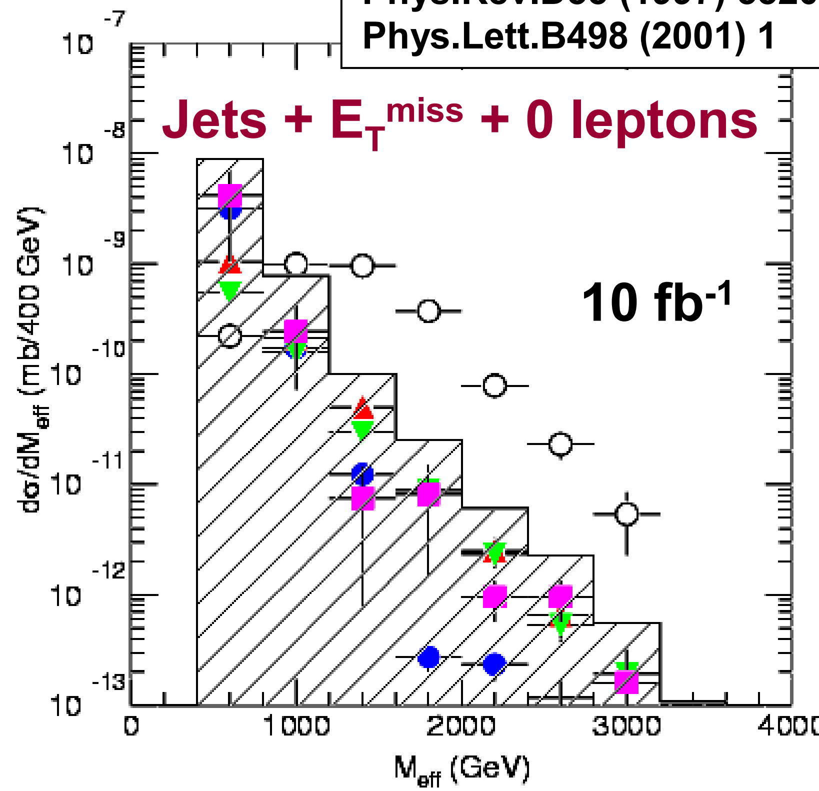
$$M_{\text{eff}} = \sum_{i=1}^{4} p_T(j_i) + E_T^{\text{miss}}$$

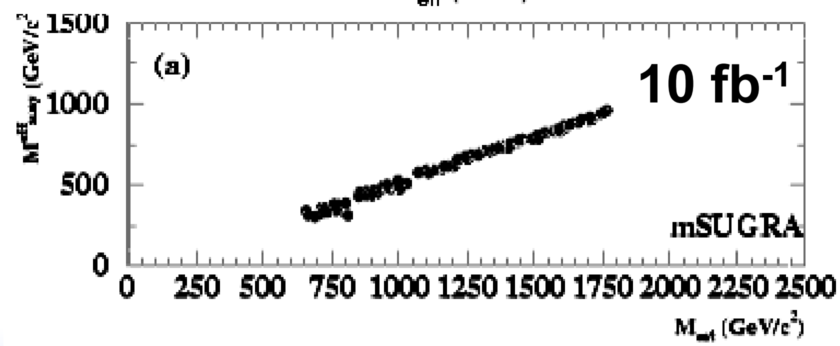
- Sum over all visible states gives better measure

$$M_{\text{eff}} = \sum_{i=1}^{n} p_T(j_i) + E_T^{\text{miss}}$$

- Non-zero boost smears endpoint ☹
- Gaussian fit gives mass scale precision ~10% for light mSUGRA, 10 fb^{-1}

Phys.Rev.D55 (1997) 5520;
Phys.Lett.B498 (2001) 1





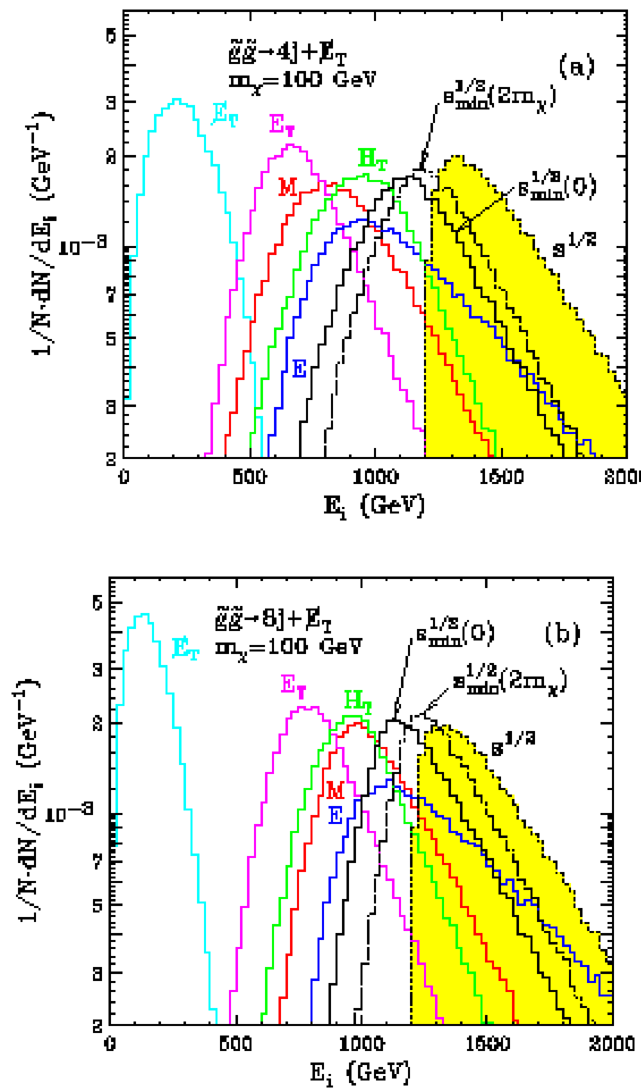
Other Techniques

arXiv:0812.1042 [hep-ph]

- Attempt to reconstruct CoM energy of hard process: $s_{min}^{1/2}$:

$$\hat{s}^{1/2}_{\text{min}} = \sqrt{E^2 - p_z^2} + \sqrt{(E_T^{\text{miss}})^2 + M_{\text{inv}}^2}$$

- Defn: minimum $s^{1/2}$ consistent with the observed particles
- Expect minimum endpoint (threshold) in $s^{1/2}$
- Endpoint lost in $s_{min}^{1/2}$ as in principle $E=p_z$ (if visible decay products co-linear along beam line ☹
- BUT retains a peak correlated with mass scale ☺
- Requires knowledge of mass of invisible state OR assume massless ☹



Exclusive Techniques

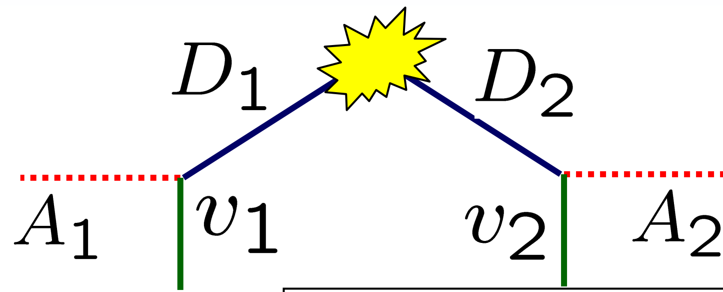
- Subject of most work over past decade
- Require knowledge of decay chain ☹
- Smaller signal acceptance ➔ large lumi ☹
- Signal often possesses clear feature e.g. endpoint ☺
- Once decay chain assumed interpretation *can* be easier ☺
- Two classes of technique:
  - Transverse momentum-based (short chains, not invariant)
  - Invariant mass-based (long chains, invariant)

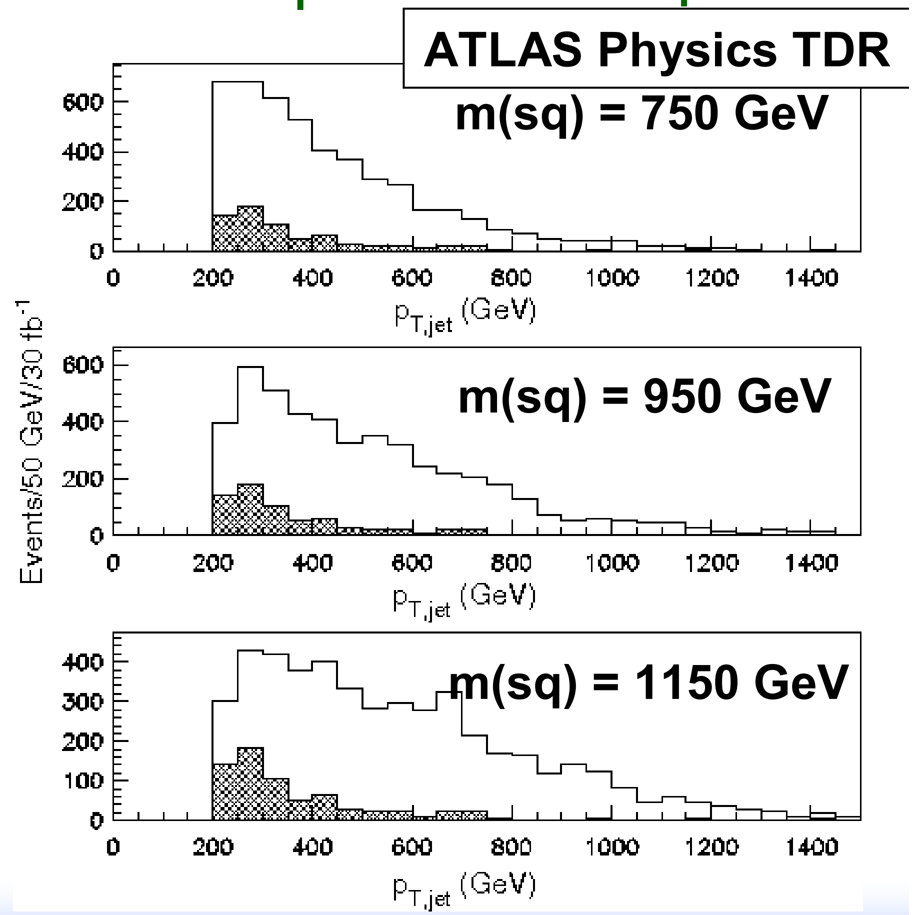
Double Symmetric 1-Step SI

- Transverse momentum of visible decay products given by:

$$p_0 = \frac{m_D^2 - m_A^2}{2m_D}$$

- Simplest approach ➔ plot $p_T(v)$ of decay product
  - Should display endpoint at p_0
- Assumption: heavy states produced at rest
- Non-zero boost of D smears endpoint ☹

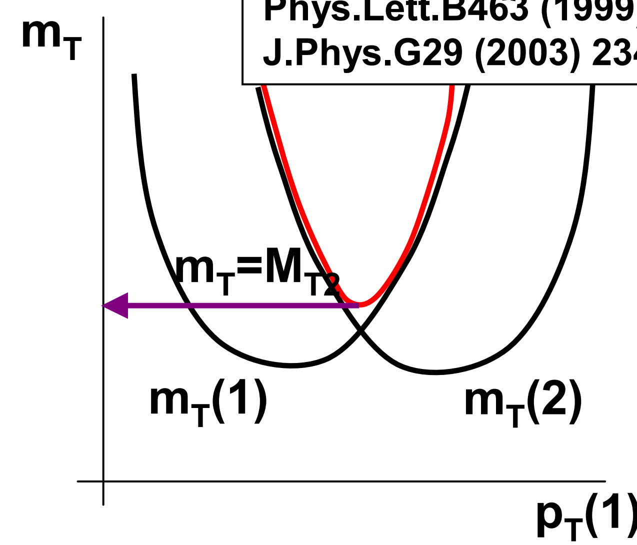


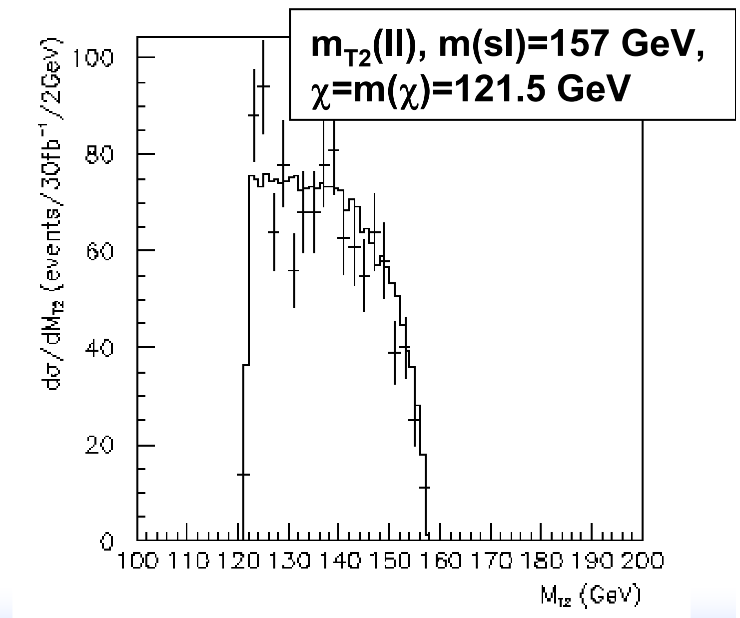


Stransverse Mass M_{T2}

- **Start from m_T of each leg**
- **$m_T(A_1,\nu_1) < m_D$**
  - **But don't know $p_T(A_1)$**
- **$m_T(A_2,\nu_2) < m_D$**
  - **Don't know $p_T(A_2)$, <u>unless</u> know $p_T(A_1)$, via E_T^{miss}.**
- **Hence vary assumed p_x, p_y of A_1 until minimise maximum m_T of D_1 and D_2 decay products.**
  - **Resulting m_T must be less than mass of D ➔ M_{T2} ☺ ☺ ☺**
- **A function not a variable: value depends on assumed mass χ ☹**
- **Analytical formula only exists for zero transverse boosts ☹**
- **Invariant under transervse CoM boosts if $\chi=m_A$ ☺ but not otherwise ☹**

Phys.Lett.B463 (1999) 99;
J.Phys.G29 (2003) 2343

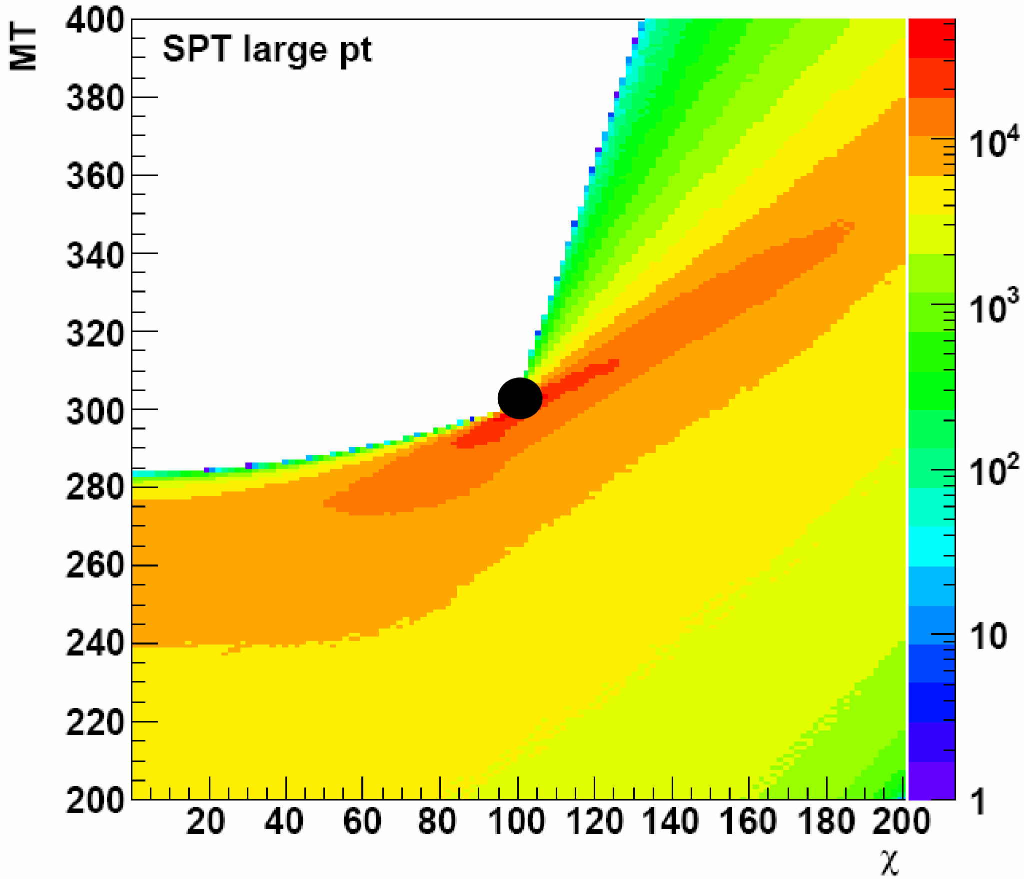




Stransverse Mass M_{T2}

Phys.Rev.Lett.100 (2008) 171801;
JHEP 0802 (2008) 014

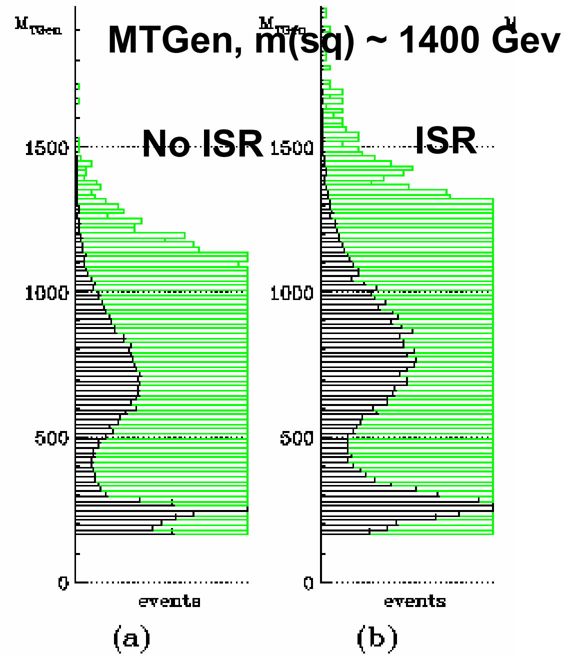
- Bonus: measurement of individual masses ☺
- m_T not invariant under transverse boosts (except when $\chi=m_A$) so m_T curves change
- Each event contributes locus as function of trial mass χ
- Envelope of loci experiences kink at $\chi=m_A$
- Nice idea in theory – difficult to observe in practice (ISR, smearing etc.)?

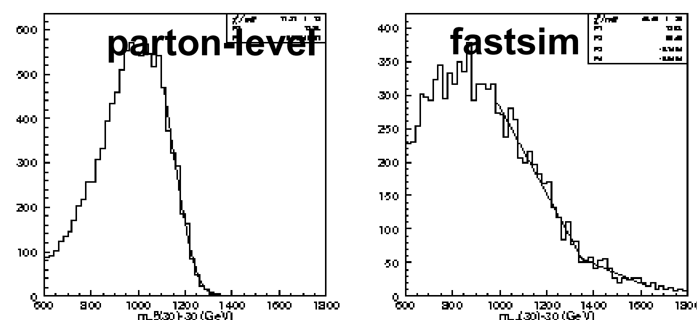


Inclusive M_{T2}

JHEP 0712 (2007) 102;
JHEP 0806 (2008) 035

- Inclusive application of M_{T2}
- Option 1: minimise M_{T2} over all possible partitions of visible decay products between chains ('MTGEN')
- Option 2: allocate visible decay products to chains with hemisphere algorithm
- Clearer endpoint than M_{eff} ☺
- Requires knowledge of mass of invisible state OR assume massless ☹
- Endpoint smeared by presence of ISR jets ☹
  - Endpoint not boost invariant (unless $\chi=m_A$)
  - Risk including ISR jets in decay products





Hemisphere mT2, m(gl)~1500 GeV, m(χ)=487GeV, χ=900Gev

Contransverse Mass M_{CT}

JHEP 0804 (2008) 034

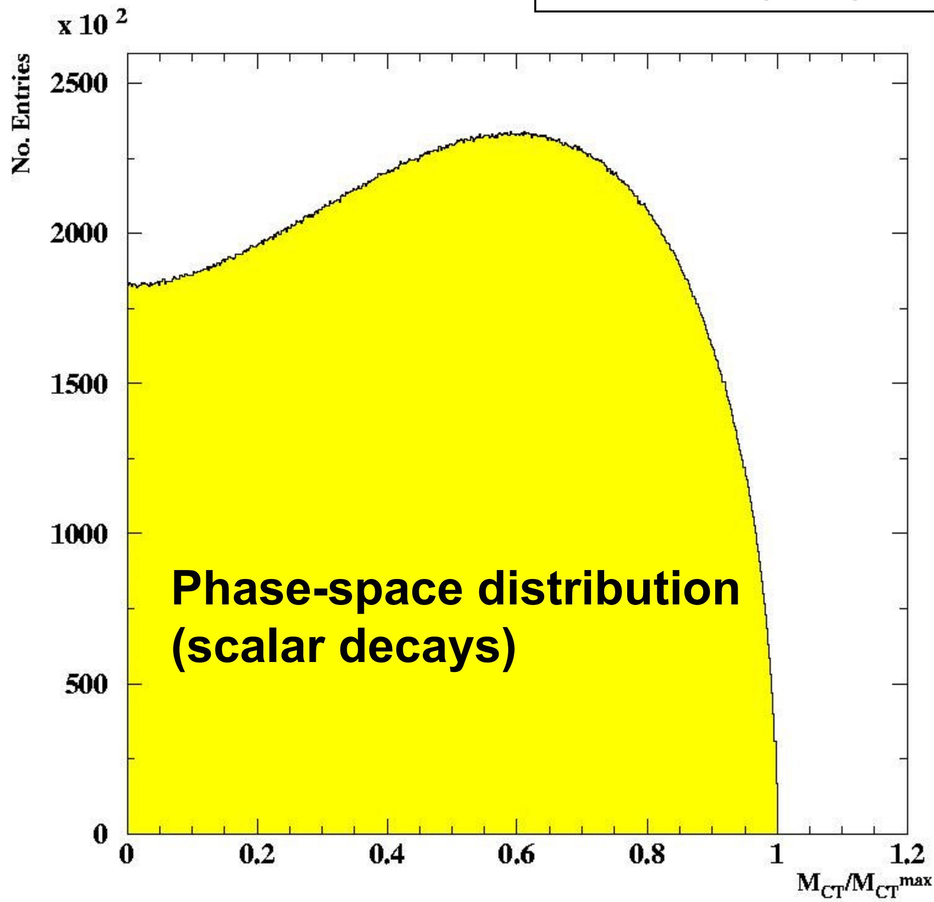
- **Aim: cancel smearing of transverse momentum endpoint by transverse boost of D**
- **If v_1 and v_2 massless then**

$$M^2_{CT}(v_1, v_2) = 2p_T(v_1)p_T(v_2)(1 + \cos\phi_{12})$$

- **End-point expected at $2p_0$:**

$$M^{max}_{CT} = \frac{m^2_D - m^2_A}{m_D}$$

- **Not invariant under transverse CoM boosts ☹**
- **Simple analytical formula ☺**
- **One value, valid for all m_A ☺**

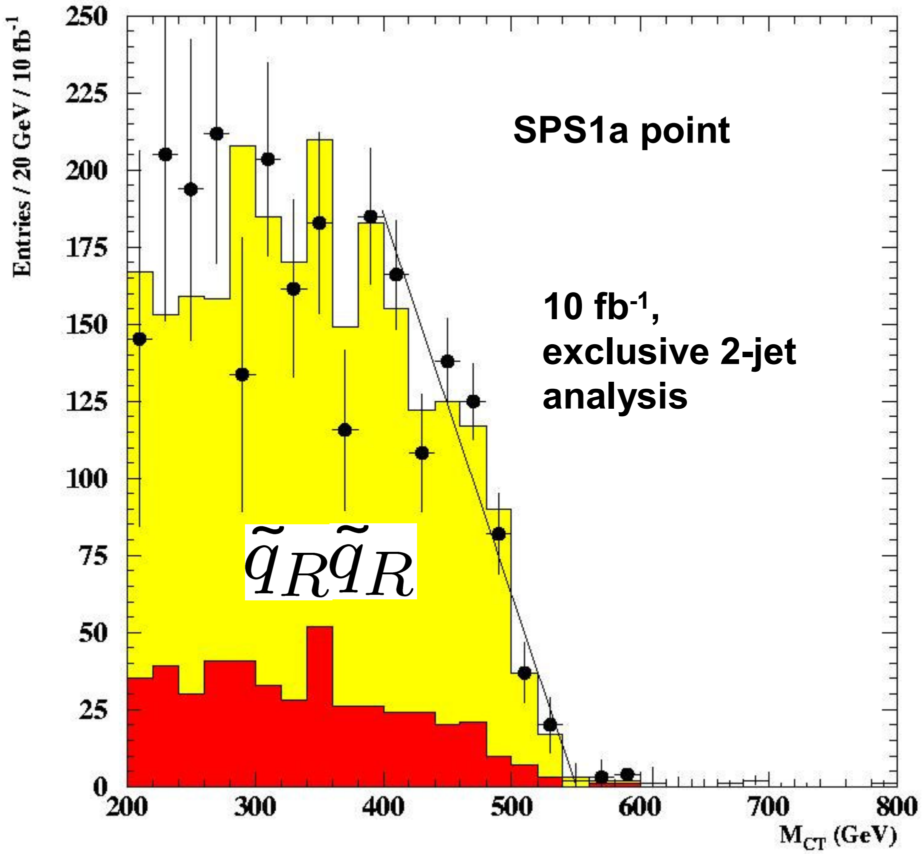


$$\begin{aligned} M^2_{CT}(v_1, v_2) &\equiv [E_T(v_1) + E_T(v_2)]^2 - [\mathbf{p_T}(v_1) - \mathbf{p_T}(v_2)]^2 \\ &= m^2(v_1) + m^2(v_2) + 2[E_T(v_1)E_T(v_2) + \mathbf{p_T}(v_1) \cdot \mathbf{p_T}(v_2)] \end{aligned}$$

Contransverse Mass M_{CT}

JHEP 0804 (2008) 034

- **Events at end-point maximise D-A mass difference.**
  - 2 jets: dominated by D=right-squark, A=LSP
  - 3/4/n jets: larger contribution from D= left-squark
- **Contributions with smaller mass difference smear end-point**
- **Gluino decays in principle give greater mass difference / larger M_{CT}**



QCD Background Rejection

JHEP 0804 (2008) 034

- New basis for E_T^{miss} provided by M_{CT} for dijet events

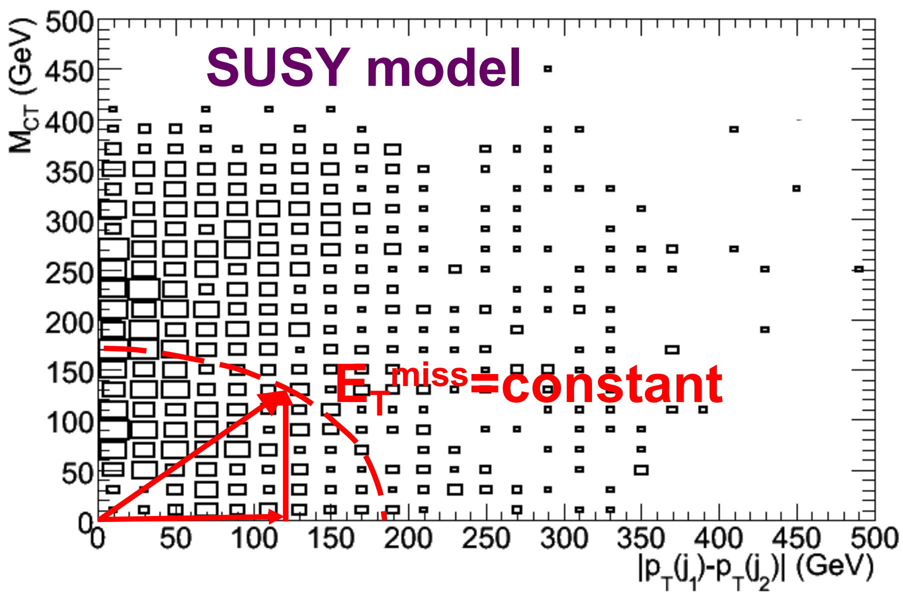
$$\mathrm{E_T}^{miss} = \{p_T(v_1) - p_T(v_2), M_{CT}\}$$

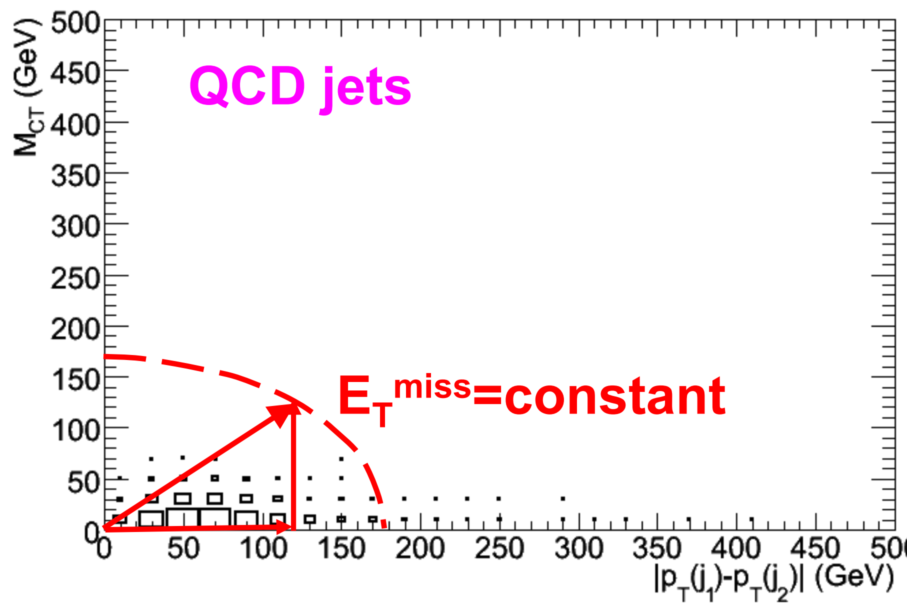
Asymmetry Topology

- M_{CT} measures contribution to E_T^{miss} from event topology:

$$E_T^{miss} = \sqrt{(p_T(v_1) - p_T(v_2))^2 + M_{CT}^2}$$

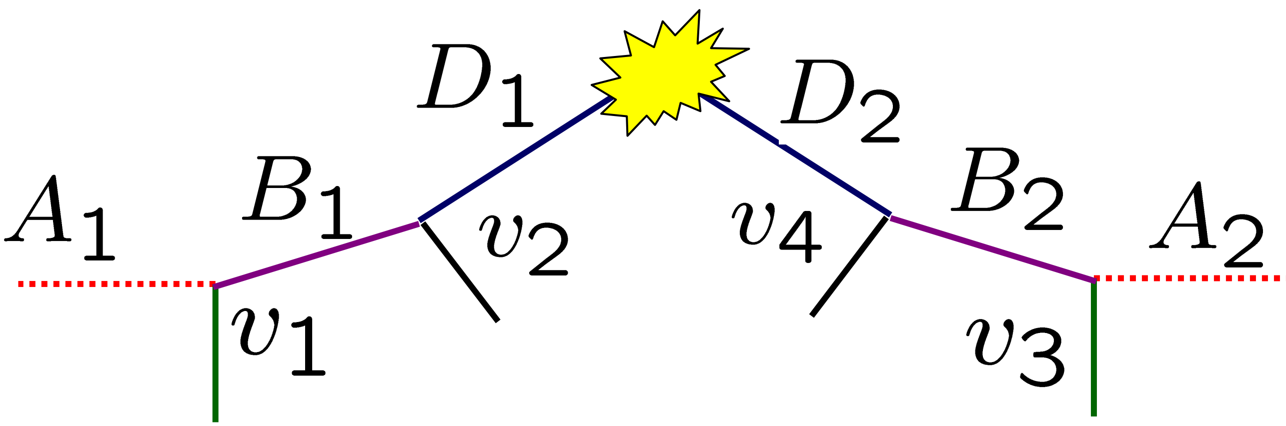
- Jet mis-measurement mainly modifies asymmetry but topology less so: powerful discrimination tool … ☺





Double Symmetric 2-Step SI

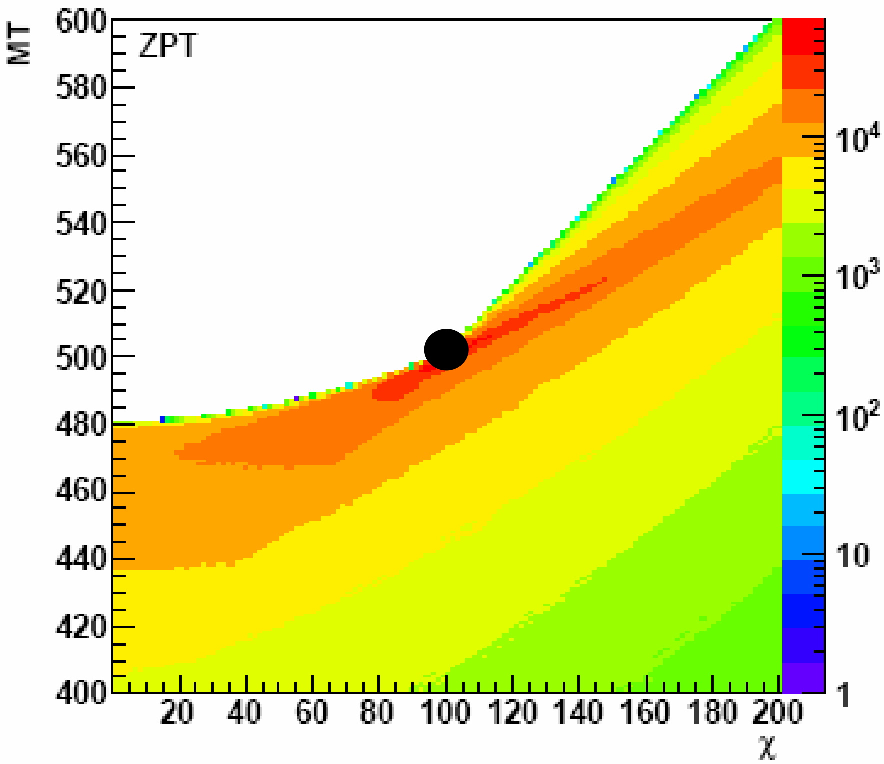
- Extra mass constraints ➔ more possibilities to measure individual masses ☺

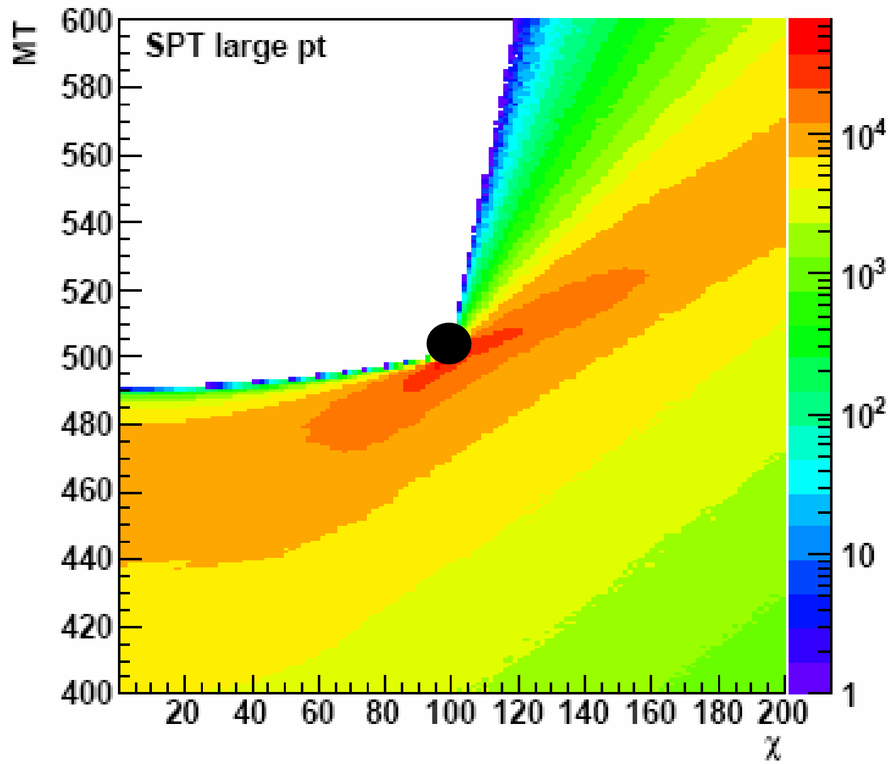


M_{T2} Mass Measurement

Phys.Rev.Lett.100 (2008) 171801;
JHEP 0802 (2008) 014

- **Kink structure enhanced when m_ν non-zero ➔ kink even when transverse CoM boost zero ☺**
- **Gradient of $m_{T2}(\chi)$ curve depends on m_ν**





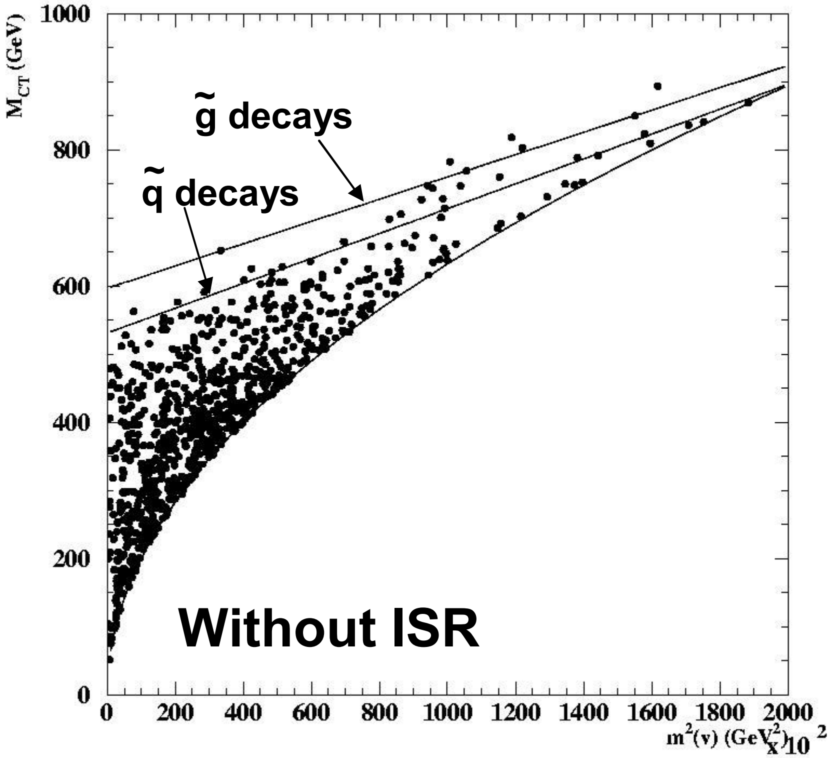
M_{CT} Mass Measurement

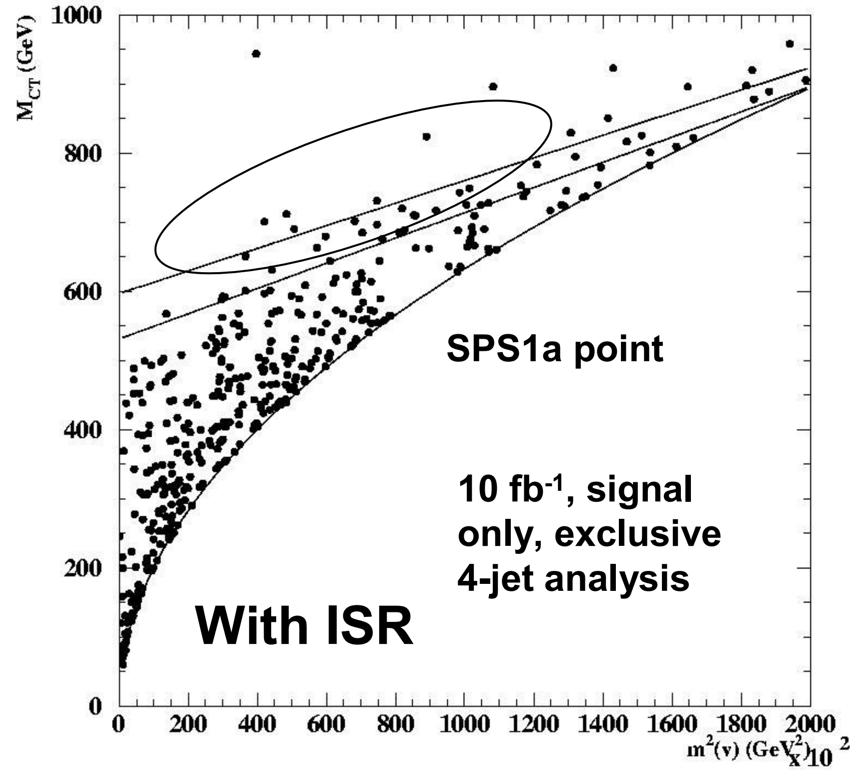
JHEP 0804 (2008) 034

- Use gradient of M_{CT} endpoint vs. m_v : M_{CT}^{max} depends on squared mass of visible products. E.g. when masses identical ☺ :

$$M_{CT}^{max} = \frac{1}{m_D} m^2(v) + \frac{m_D^2 - m_A^2}{m_D}$$

- Similar issues to M_{T2} with ISR in inclusive approach ☹ (exclusive OK)

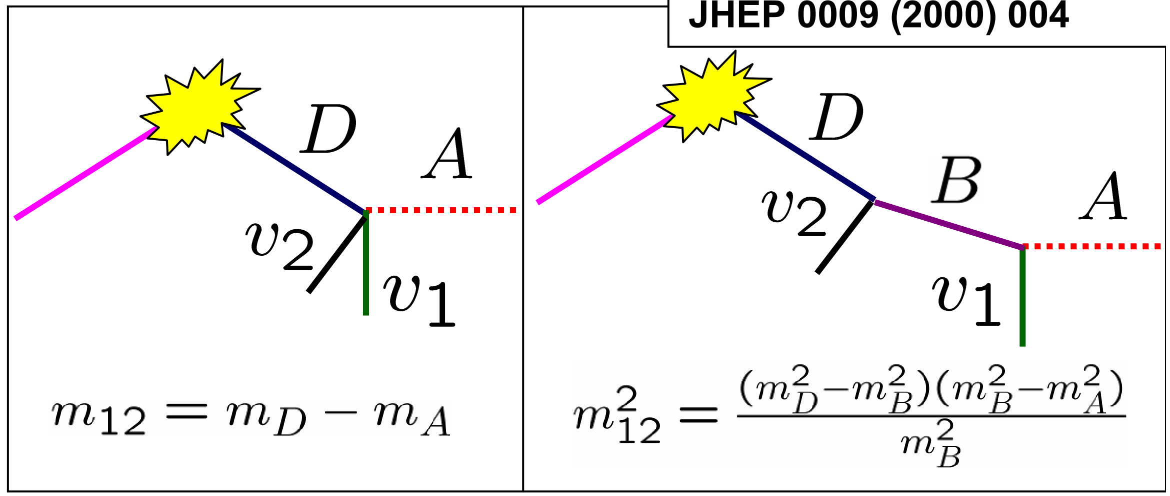


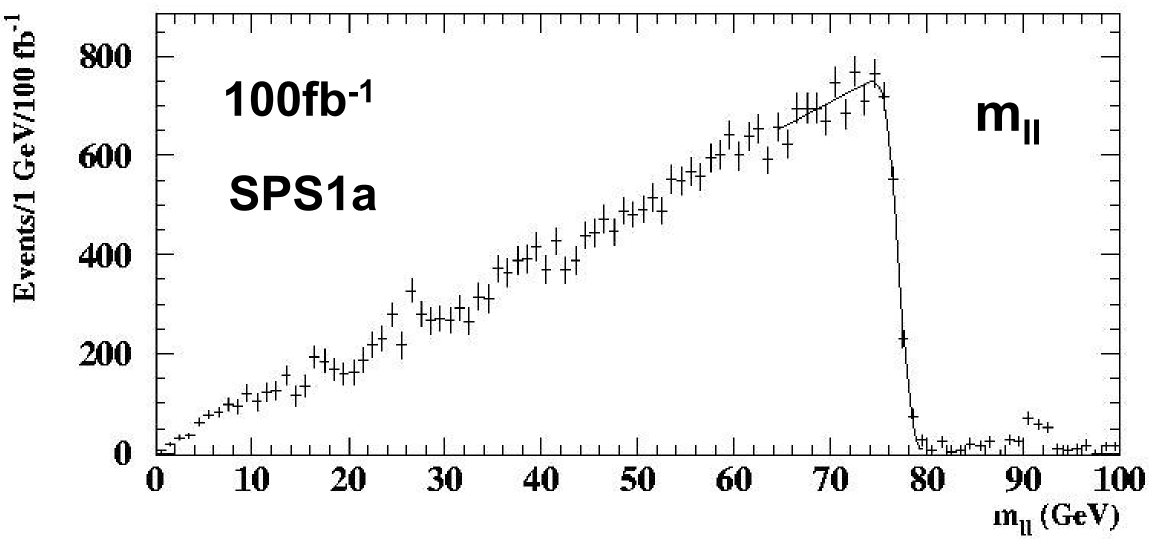


Invariant Mass Techniques

Phys.Rev.D55 (1997) 5520;
JHEP 0009 (2000) 004

- End-points in invariant mass distributions of aggregated decay products from single chain.
- Works for two sequential 2-body decays, also for 3-body decays
- Does not require symmetric chain ☺
- 'Model-independent' ☺
- Can not obtain individual masses from sequential 2-body or 3-body decays alone ☹

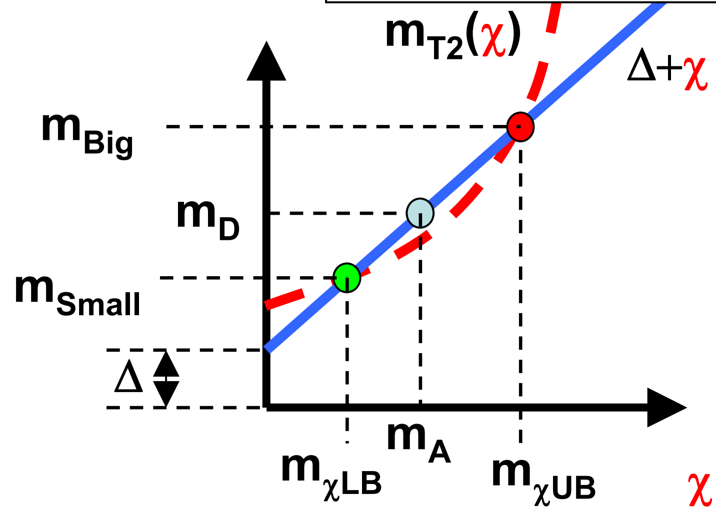


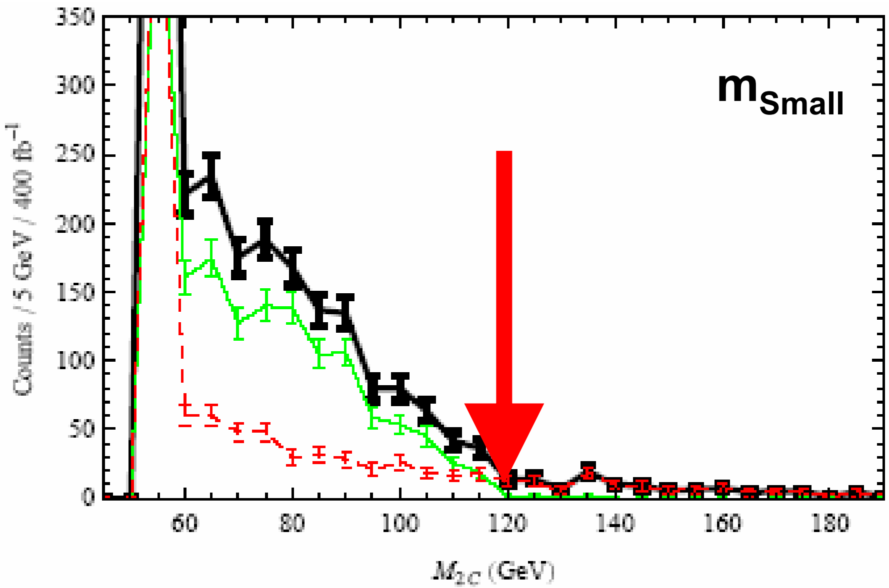


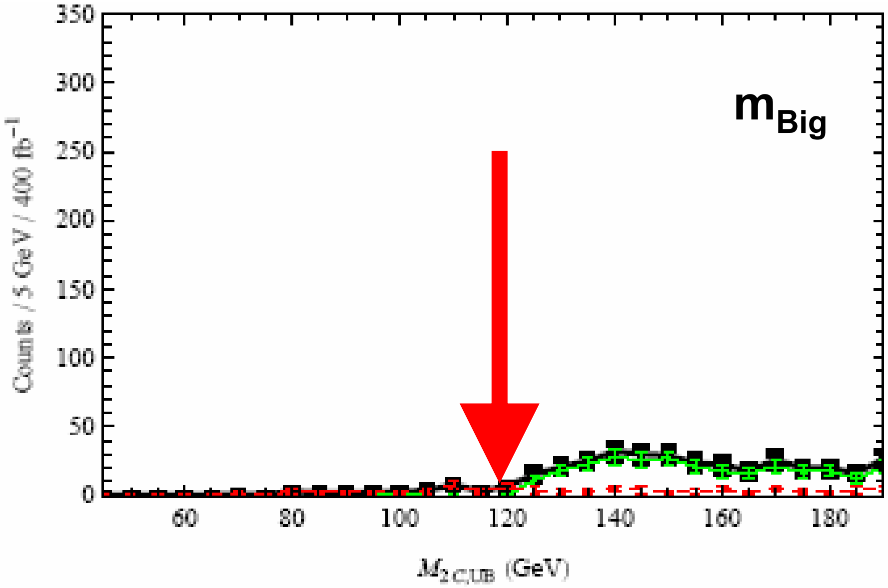
Hybrid Methods

- Combine ‘transverse momentum’ and endpoint mass constraints.
- E.g.1: Solve M_{CT} and invariant mass constraints simultaneously.
- E.g.2: Use invariant mass endpoint measure of m_D-m_A to solve χ ambiguity in m_{T2} event-by-event
  - Requires large transverse CoM boost

Phys.Rev.D78 (2008) 056006



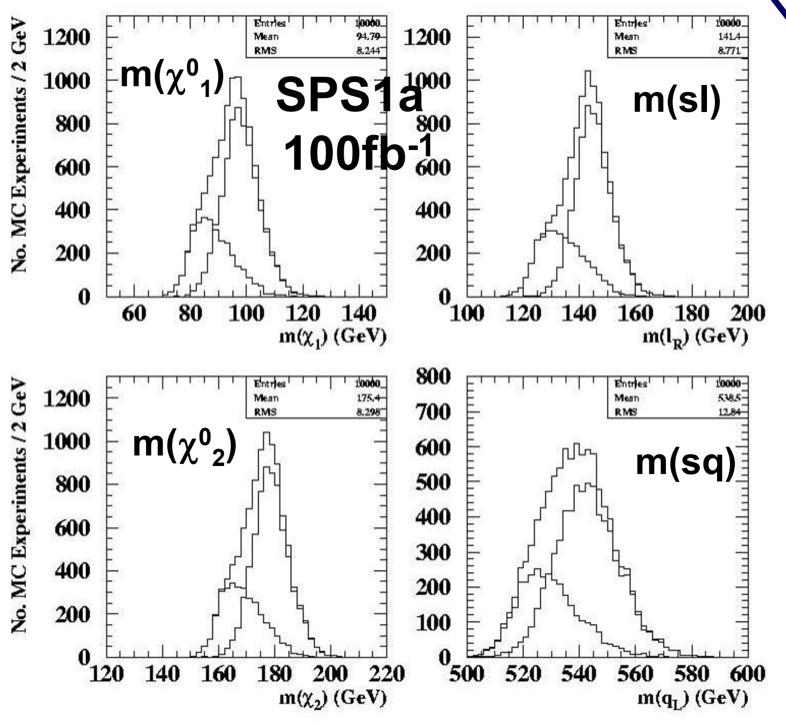


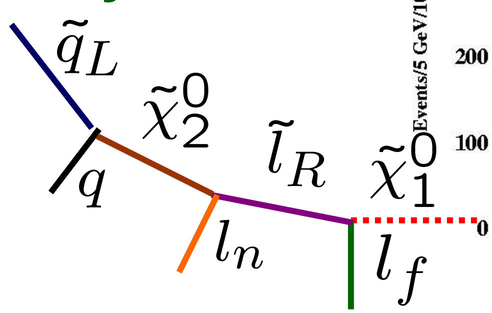


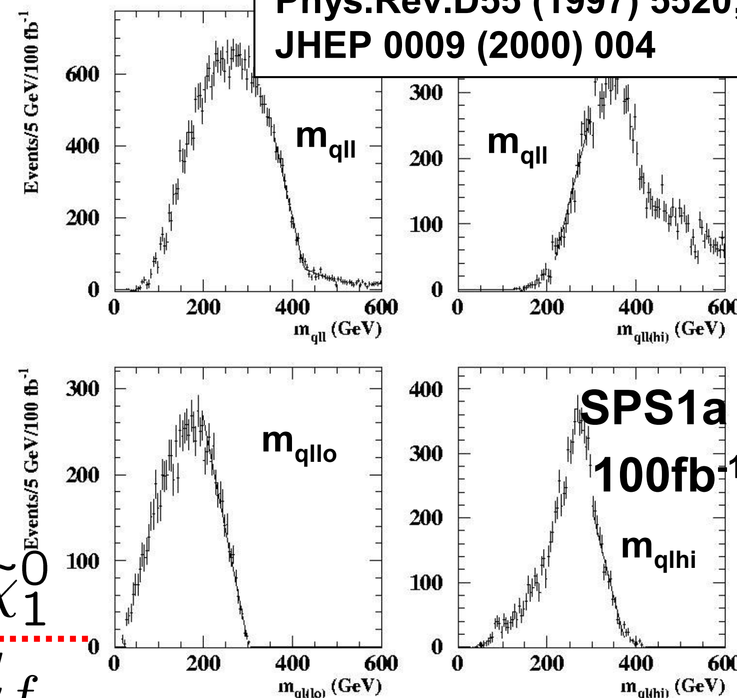
Double 3-Step Sl Chain

- With at least 3-steps in one chain measure invariant mass endpoints from combinations of decay products
- Does not require symmetric chain ☺
- ‘Model-independent’ ☺
- Canonical example: sq_L decay chain.

Phys.Rev.D55 (1997) 5520;
JHEP 0009 (2000) 004





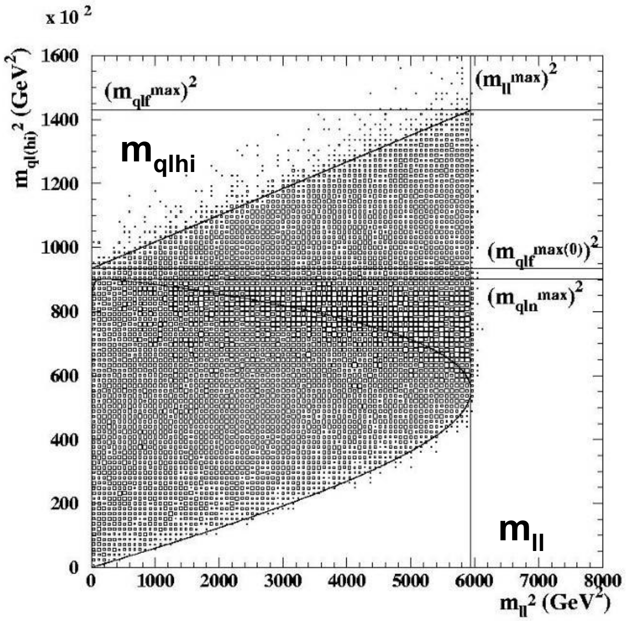


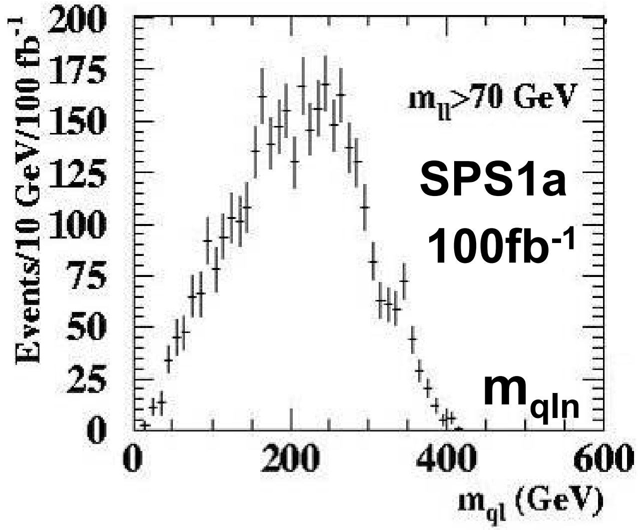
- Very exclusive → lots of lumi ☹
- Requires good knowledge of decay chain ☹
- Solve constraint equations analytically or with numerical fit: potential ambiguities ☹

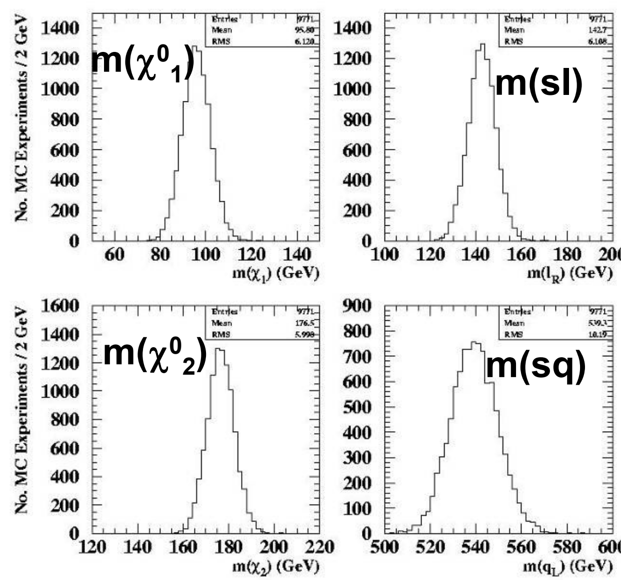
Endpoint Correlations

arXiv:0902.2331 [hep-ph]

- Cannot distinguish l_n from l_f ➔ ambiguity
- Improve statistical precision by studying correlations between mass observables ➔ potentially infinite number of observables!
- $m(ql_f)$ endpoint changes with $m(ll)$, $m(ql_n)$ endpoint does not.
- Potentially removes $m(ql_{hi})$ / $m(ql_{lo})$ endpoint ambiguity ☺
- Also allows discrimination of two-body lepton-producing decays from three-body via $m(ql)$ mass ratio at small $m(ll)$ ☺



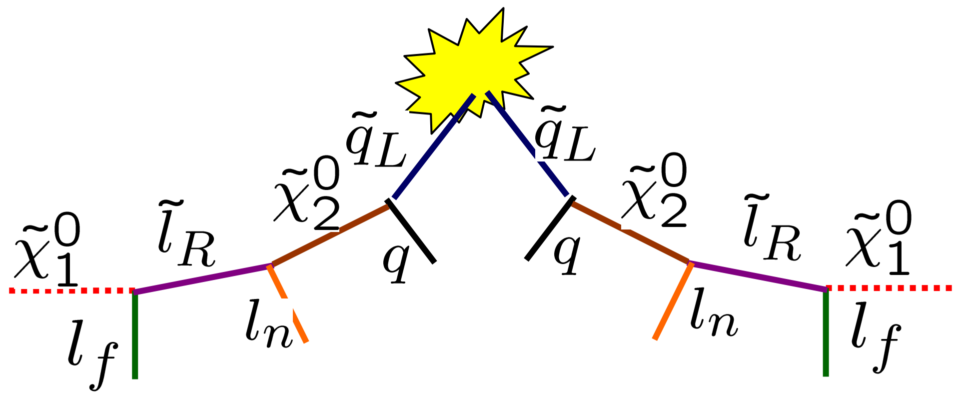


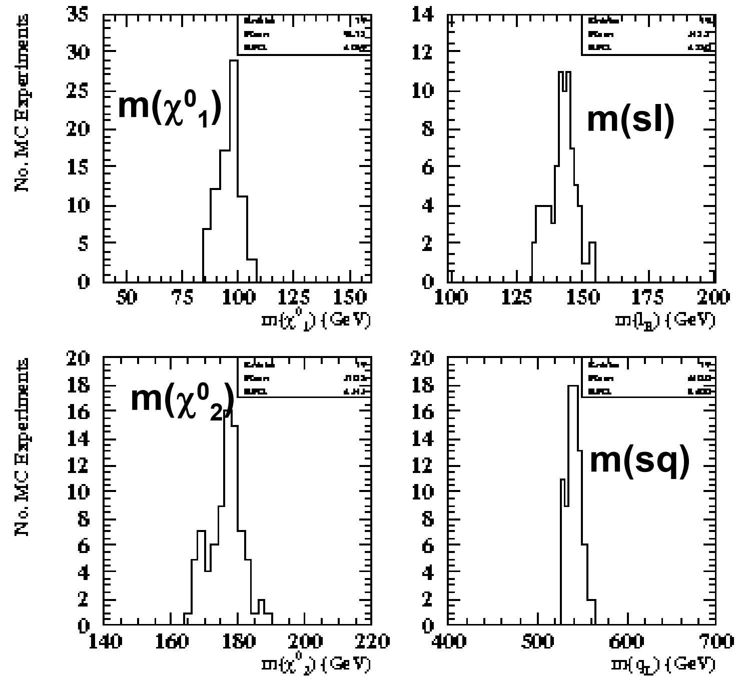


Double Symmetric 3-Step SI

JHEP 0805 (2008) 014

- Endpoint constraints from 3-step chains determine masses
- With masses can reconstruct invisible momentum of A in each observed 3-step chain.
- Can hence determine expected E_T^{miss} in events with two identical 3-step chains ➔ extra constraints
- Improved mass precision with global minimisation wrt constraints. ☺
- ‘Hybrid’ method



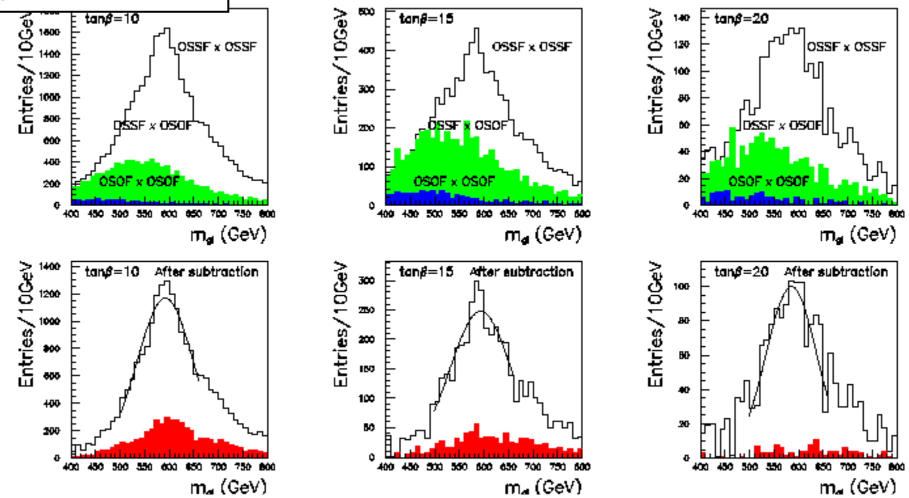


A Different Perspective …

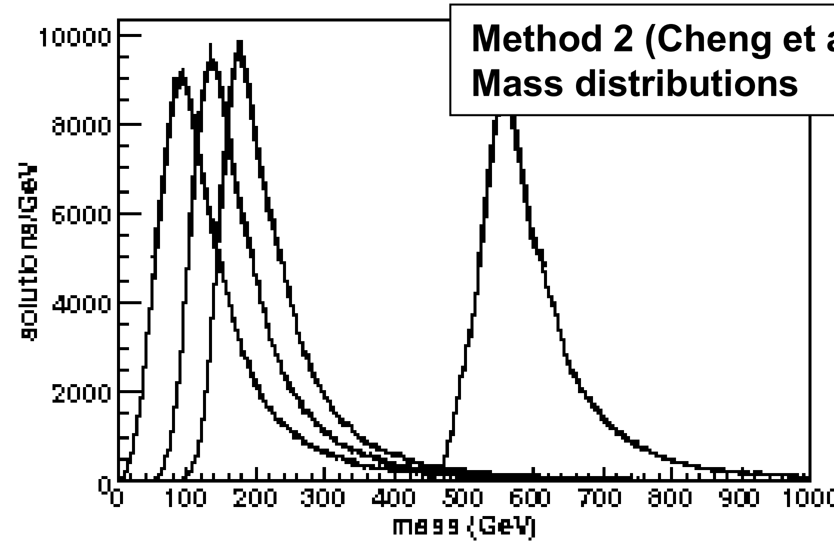
- Can constrain m_Z with just one Z→ll event.
- Not the case for endpoint techniques ➔ any one event may lie anywhere below the endpoint
- Can obtain mass constraints only by using multiple events
- Alternative idea: combine constraints from multiple events explicitly.
- E.g. 1: single 3-step SI chain (‘Mass Relation Method’)
- E.g. 2: double symmetric 3-step SI chain using E_T^{miss} constraint

Method 1 (MRM)
m(gl)

Phys.Rev.D71 (2005) 035008;
Phys.Rev.Lett.100 (2008) 252001



Method 2 (Cheng et al.)
Mass distributions



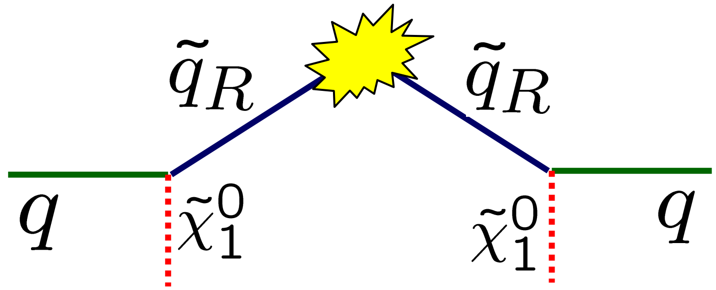
Summary

- Many observables proposed for measuring masses in events with E_T^{miss}.
- Merits (and de-merits) in all ➔ complementarity
- Field progressed significantly over past 12 years
  - Moved from *ad hoc* inclusive measurements to well-motivated exclusive techniques.
- Still missing: simple technique for short decay chains without assumed LSP mass, invariant under transverse CoM boosts.
- Work needed on clusterisation for inclusive methods.
- Presence of ISR jets significant problem for inclusive techniques: can these be rejected (k_T methods etc.)?

Back-Up

Motivation

- **Aim to constrain sparticle masses with short decay chains:**



- **Examples:**

$$\tilde{q}_R \rightarrow q\tilde{\chi}_1^0 \qquad\qquad \tilde{l} \rightarrow l\tilde{\chi}_1^0$$

- **Tool already on the market: 'stransverse mass' m_{T2} (Lester & Summers, Phys.Lett.B463:99-103,1999) used in CSC SUSY-5 and SUSY-6.**
  - **Maximum transverse mass of two decays, minimised over all possible partitions of E_T^{miss} between decays.**
  - **Requires mass of LSP as input**
  - **Some sensitivity in principle to individual masses**
- **New proposal: 'contransverse mass' (JHEP 0804 (2008) 034, arXiv:0802.2879 [hep-ph])**