Quantum particles and fields on the Lattice

Introduction and Motivation

In the course of this lecture we will explore the foundations of both THEORY and TECHNOLOGY instrumental in pushing the limits of our knowledge in high-energy NUCLEAR and PARTICLE physics.

At the end of the lecture: Basis to take up studies in one of many areas highlighted at the annual LATTICE conferences. (2016 edition: conference.ippp.dur.ac.uk/event/470 <u>and</u> inspirehep.net)

A. The high precision frontier

Indirect searches for physics beyond the standard model (BSM) at low energies from discrepancies between theory and experiment.

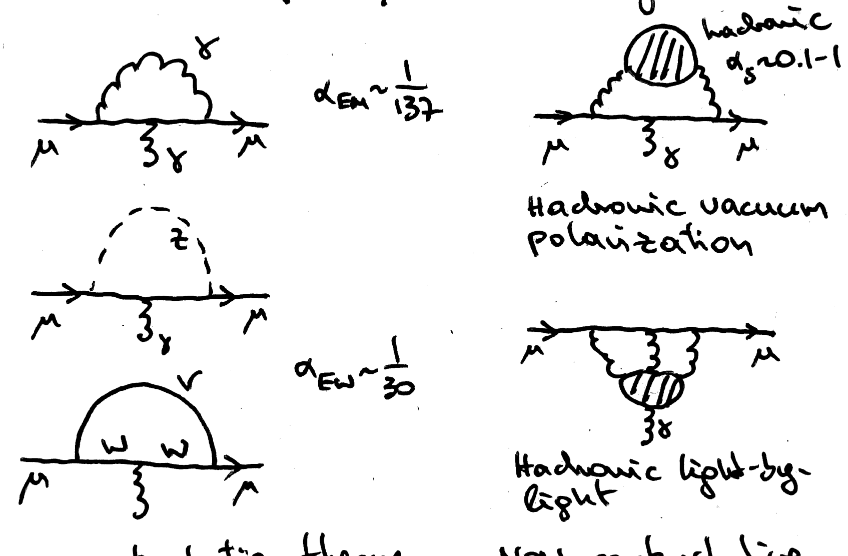
Example: Muon anomalous magnetic moment $(g-2)$

Remember: Energy shift of particle in magnetic field $\Delta E = \vec{\mu}\cdot\vec{B}$ with $\vec{\mu} = g\left(\frac{e}{2m}\right)\vec{s}$. e charge, m mass, $\vec{s}$ spin, g Landé factor

Goal: understand the value of g, different from the "tree-level" result by Dirac $g_D = 2$.

Anomalous magnetic moment $a = \frac{g-2}{2}$ summarizes contributions arising from quantum (field) fluctuations, so called radiative corrections. (see Peskin, Schroeder chap. 7; Jegerlehner Springer Tracts Mod. Phys 274 (2017)) $a_\ell \propto m_{\ell = e,\mu,\tau}$

In terms of Feynman diagrams:



via perturbation theory

NON-perturbative

Status of a_μ

	$\times 10^{-13}$
QED (5loop)	11 658 471 895 (8)
EW	15 400 (100)
HVP LO	692 300 (4200)
NLO	-9.840 (60)
NNLO	1 240 (10)
HLbL	10 500 (2600)
SM (total)	11 659 181 500 (4900)
BNL E821	11 659 209 100 (6300)

$\Rightarrow 3\sigma$ tension

New experiment at Fermilab E989 reduces errors $\sim \times 4$

experimental determination from myons in storage ring

$$\vec{\omega}_{\text{cyclotron}} = \frac{e\vec{B}}{m} \qquad \vec{\omega}_{\text{Larmor}} = g\,\frac{e\vec{B}}{2m} \quad \Rightarrow \quad \vec{\omega}_L - \vec{\omega}_C = a\left(\frac{e}{m}\right)\vec{B}$$

important side note: Talk to experimentalists, what is it that they measure and how can it be connected to what you can compute.

How to improve the situation: Need to understand 

strategy: express needed information in terms of <u>correlation functions</u>, here e.g. of e.m. current $j_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s$

$$a_\mu = \frac{\alpha_{em}}{\pi}\int_0^\infty dq\,\frac{\rho(q)}{q^2}\,G(q) \qquad G(q) = \int_0^1 dx\,\frac{x^2(1-x)}{x^2 + (q^2/m_\mu^2)(1-x)} \quad \text{from QED}$$

spectral function $\rho_{\mu\nu}(k) = \frac{1}{2\pi}\int d^4x\, e^{ikx} \underbrace{\langle 0|[j_\mu(x), j_\nu(0)]|0\rangle}_{\text{compute numerically}}$

Tasks:

① Formulation ② Simulation ③ Data analysis

B. The extreme temperature (and density) frontier

Insight into the properties of nuclear matter and its phases at temperatures relevant for the early universe and relativistic heavy-ion collisions. (review on lattice QCD @ T>0 1504.05274, cosmology 1606.07494)

Starting point $T = 300\text{K} \sim 10^{-4}\,\text{eV} \approx 0$

The world around us is made out of hadrons, while the theory of strong interactions QCD is formulated in terms of quark and gluon fields

$$\mathcal{L} = -\frac{1}{4}F_a^{\mu\nu}F_{a,\mu\nu} + \sum_f \bar{\psi}_f\,(iD^\mu\gamma_\mu - m)\,\psi_f \qquad D_\mu = \partial_\mu - igA_\mu \ \text{covariant derivative}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\underbrace{f^{abc}}_{\text{structure constants}}A^b_\mu A^c_\nu \qquad A^\mu = \underset{\in\mathbb{R}}{A^\mu_a}\,T^a \qquad T^a = \frac{\lambda^a}{2} \leftarrow \text{Gell-Mann matrix}$$

Classic goal of lattice QCD simulations is to pre/post dict properties of hadrons. strategy: compute correlation functions of currents and extract e.g. mass of particles. (Dürr et.al. Science 322, 1224 (2008))

Symmetries play an important role in hadron properties

<u>local gauge invariance</u>: $A^\mu(x) \mapsto G(x)A^\mu(x)G^{-1}(x) - \frac{i}{g}G(x)\partial_\mu G^{-1}(x) \qquad G(x) \in SU(3)$

$\psi \mapsto G(x)\psi \quad \bar{\psi} \mapsto \bar{\psi}G^{-1}(x)$ leads to Asymptotic freedom and confinement

$$\beta(g) = \frac{\partial g}{\partial \log(\mu)} = -\left(11 - \frac{2n_f}{3}\right)\frac{g^3}{16\pi^2}$$

μ: Energy scale at which we resolve the system chiral symmetry ($m_q=0$): $P_{L/R} = \frac{1 \pm \gamma^5}{2}$ $\quad \psi_{L/R} = P_{L/R}\,\psi$ $\quad$ consider $\psi = (u,d)$ $\quad$ III

$\Rightarrow \mathcal{L}_q = \bar{\psi}_L i\not{D}\psi_L + \bar{\psi}_R i\not{D}\psi_R$ $\quad$ can rotate ψ_L or ψ_R by $SU(2)$ matrices

$\Rightarrow \underbrace{SU(2)_L \times SU(2)_R}_{} \times U(1)_V \times U(1)_A \longrightarrow$ broken by axial anomaly $\Gamma(\pi \to 2\gamma) \neq 0$ quantum fluctuations violate symmetry

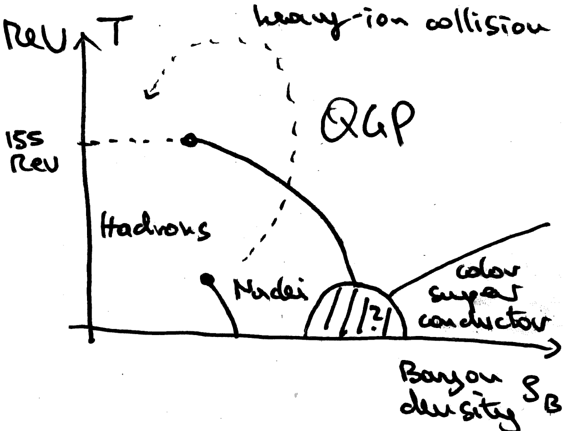
At $T=0$ spontaneously broken to $SU(2)_V$ related to Isospin.

$U(1)_V$: preserved at $T=0$, related to Baryon # conservation.

χ-sym breaking $\rightarrow$ lightness of π particles (Goldstone)
$\rightarrow$ heaviness of n, p via χ-condensate $\langle\bar{q}q\rangle$

If temperature is increased up to $T \sim m_\pi = 140\,\text{MeV}$ can we find phases of QCD where
- χ symmetry is restored?
- quarks are not confined in hadrons?

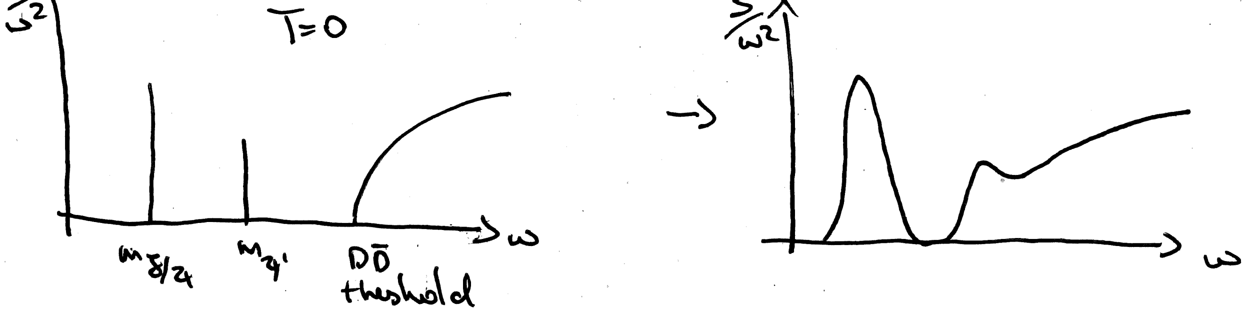
$\Rightarrow$ Current knowledge: YES!
Quark-Gluon Plasma



Active research programs ongoing, related to mapping phase structure @ finite T and ρ_B.

From theory side: How to identify a phase transition? Order parameter!
χ-symmetry (violated by m_q): chiral condensate
confinement (violated by q): Polyakov loop

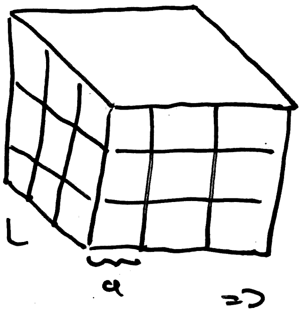
Phenomenological approach: In-medium modification of hadrons e.g. heavy quark-antiquark states. strategy: compute correlation functions of heavy quark currents, compute spectral functions from it.



How do these modifications affect measured particle yields in heavy-ion collisions?

$\Rightarrow$ Diverse topics, active collaboration with experiments

<u>The Lattice:</u> Discretize field theory on a space-time hypercube with periodic boundary conditions. We have lattice spacing "a" and box size $L = aN$ $N \in \mathbb{N}$. Approximate



$$\int dx\, f(x) \approx \sum_k a\, f(ak) \qquad \frac{d}{dx} f(x) \approx \frac{f(x+a) - f(x)}{a}$$

$\phi(x) \to \phi(x_k)$ finite number of field degrees of freedom

$\Rightarrow$ Solve the quantum theory numerically, then perform the limits $a \to 0$ (continuum limit) $L \to \infty$ (thermodynamic lim.)

Pros:
- genuine first principles, non-perturbative approach
- exact, if limits can be taken
- provides a well defined regularization of QFT (compare to Pauli-Villars, dimensional regularization)
- practical realization of the renormalization group program

Cons:
- discretization breaks symmetries of the continuum theory, need to recover in continuum limit.
- simulations carried out in unphysical Euclidean time need to connect back to Minkowski domain.
- exploration of finite Baryon density challenging (sign-problem)
- Realistic simulations in QCD require supercomputers.

$\Rightarrow$ Algorithmic development is an important aspect of lattice theory, to make best use of todays computing resources:

Moore's law: # of transistors doubles every 18 months 1965-2013 CPU
2013-... GPU
change of paradigm: parallel architectures, e.g. GPU, multicore.

<u>Outline of the lecture:</u>

1. Spin-models (Ising) thermodynamics, phase-transitions.
2. Monte-Carlo methods
3. Solving PDE's (Schrödinger)
4. Path Integrals
5. Scalar fields transfer matrix, cont. limit, symmetry breaking
6. Data analysis
7. Gauge theory Wilson formulation, improved actions scale setting, algorithms
8. Fermions doubling, Z-sym
9. Lattice QCD @ $T>0$
10. The real-time challenge
11. The sign-problem