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Renormalization group study of the Hubbard model at the van Hove singular point

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Abstract

We study a two-dimensional Hubbard model with a Fermi surface containing the saddle points (π, 0) and (0, π). Including Cooper and Peierls channel contributions leads to a one-loop renormalization group flow to strong coupling. Various fixed points are found by varying hopping energies as well as Coulomb repulsion. Natures of these fixed points are investigated through response functions.

Keywords: Hubbard Model; RIB state; Van Hove singularity

1. Introduction

Fermi surface patching [1] is a method to study renormalization group (RG) flows of Fermionic systems in dimension larger than one. This has been used to study the effect of the Umklapp scattering in two-dimensional Hubbard model near half-filling [2,3], as well as effects of the van Hove singularity [4–8]. In this paper, we discuss the RG flows of the two-dimensional Hubbard model with both van Hove singularity and Umklapp scattering. The basis of attraction toward various fixed points is investigated within t'/t and U/t parameter space, where t (t') is the n.n. (n.n.n.) hoppings and U is the Coulomb repulsion. Nature of the fixed points are also discussed.

2. One-loop RG flow

We study the case with a Fermi surface touching the saddle points (π, 0) and (0, π). We restrict ourselves to a small t'/t region. Due to the van Hove singularity, the leading singularity arises at the saddle points. The susceptibility for the Cooper channel at $q = 0$ shows a log-square divergence $\chi_0^{\rm pp}(\omega) \propto \ln^2(\omega)$. For the Peierls channel at $Q = (\pi, \pi)$, there exists a crossover: $\chi_Q^{\rm ph}(\omega) \propto \ln^2(\omega)$ at $\omega \gg |t'|$ while $\chi_Q^{\rm ph}(\omega) \propto \ln(\omega)$ at $\omega \ll |t'|$. The Peierls channel at $q = 0$ and the Cooper channel at $q = Q$ also diverges log-linearly but have smaller coefficients at $t'/t \ll 1$, and therefore are neglected in this paper. Possible ferromagnetism arising from these terms at increased values of t'/t has been discussed in Refs. [8,9].

Taking into account the most singular parts of the Fermi surface, we study the two-patch model where patches are taken at the saddle points (π, 0) and (0, π). There are four species of interaction vertices at these patches as defined in Ref. [3]. The RG flow equations is given by [3,6]

$$\dot g_1 = 2d_1 g_1(g_2 - g_1), \quad (1)$$

$$\dot g_2 = d_1(g_2^2 + g_3^2), \quad (2)$$

$$\dot g_3 = -2g_3 g_4 + 2d_1 g_3(2g_2 - g_1), \quad (3)$$

$$\dot g_4 = -(g_3^2 + g_4^2). \quad (4)$$

Here, $\dot g_i \equiv ({\rm d}g_i)/({\rm d}y)$ where $y \equiv \ln^2(\omega/E_0) \propto \chi_0^{\rm pp}(\omega)$. We define d_1 as $d_1(y) = {\rm d}\chi_Q^{\rm ph}/{\rm d}y$. The asymptotic forms are $d_1(y) \to 1$ at $y \approx 1$ and $d_1(y) \sim \ln|t/t'|/\sqrt{y}$ as $y \to \infty$. We study the repulsive Hubbard-model by taking an equal initial value for all couplings $g_i = U > 0$ (i = 1–4).

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The case $d_1 \equiv 1$ arises in the limit $t' = 0$ as well as in the strong U region where t' is irrelevant in the Peierls channel divergence. This case, where both Cooper and Peierls channels have log-square singularity, was studied by Schulz [4] and also by Dzyaloshinskii [5]. The most divergent term appears at the antiferromagnetic (AF) susceptibility, which has the same exponent as d-wave superconductivity (d-SC) but is dominant due to the next leading divergent terms. The uniform charge susceptibility is suppressed to zero. Thus we have a Mott insulator with an AF order.

Dzyaloshinskii treated the limit $d_1 \equiv 0$ [7] where Eqs. (3) and (4) combine to give $\dot{g}_\pm = -g_\pm^2$ with $g_\pm = g_4 \pm g_3$. From the initial condition $g_3 = g_4 = U$, these RG equations lead to a weak-coupling fixed point $g_\pm \to 0$ which he discusses as a Tomonaga–Luttinger fixed point. However, with nonzero d_1 in Eq. (3), g_- flows into negative region and finally to a strong coupling fixed point $g_- \to -\infty$. For the two-patch Hubbard model with generic values of $U > 0$, RG flows to strong coupling fixed points.

3. Nature of the fixed points

Difference in ω dependences of the two channels creates an additional energy scale which causes susceptibility exponents to vary as a function of U. Within the RG scheme, these exponents are determined by d_1 at the critical point which monotonically increases as a function of U.

In the small U limit, we have $d_1 \ll 1$ and the RG flows to $g_+ \to 0$ and $g_- \to -\infty$. Hence we have $g_3 \to +\infty$ and $g_4 \to -\infty$. The most dominant susceptibility of this fixed point is d-SC with spin gap. As U is increased, the strength of AF correlation increases and finally becomes identically divergent as d-SC correlation at $d_1 = 1$, and a transition to Mott-AF state occurs here.

What is speculated from the charge compressibility calculation is that the transition to an insulating state seems to occur at a smaller value of U. Namely, as U is increased, phase transitions from the d-SC to a new insulating phase with dominant d-wave pairing occurs, and then transition to the AF insulating phase follows later. This new insulating phase in the intermediate region also has a spin gap, and can be interpreted as an RVB state defined by Gutzwiller projected d-SC state. The origin of such insulating behaviour should be due to the Umklapp process [2] at van Hove singular points. RVB state seems to emerge at the region of competition between d-SC and AF correlations.

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