Ultracold gases and Functional renormalization I

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Work in collaboration with

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C. Wetterich

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Very nice model system to test methods of quantum and statistical field theory!

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$$\begin{aligned}\mathcal{L} &= \psi^\dagger(\partial_\tau - \vec{\nabla}^2 - \mu)\psi + \varphi^*(\partial_\tau - \frac{1}{2}\vec{\nabla}^2 - 2\mu + \nu)\varphi \\ &\quad - h(\varphi^* \psi_1 \psi_2 + h.c.).\end{aligned}$$

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These are effective theories on the length scale of the Bohr radius $a_0 \approx 0.5 \times 10^{-10} m$.

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- Galilean invariance is broken explicitely by a thermal bath for $T > 0$.

The grand canonical ensemble

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- $\Gamma_k[\phi]$ is the *average action* or *flowing action*.
- Grand canonical potential is obtained from $\beta\Omega_G = \Gamma_k[\phi]$ for $k = 0$ and $J = 0$.

How the flowing action flows

Simple and exact flow equation (Wetterich 1993)

$$\partial_k \Gamma_k[\phi] = \frac{1}{2} \mathsf{STr} \left(\Gamma_k^{(2)}[\phi] + R_k \right)^{-1} \partial_k R_k .$$

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  - Solve these equations numerically.

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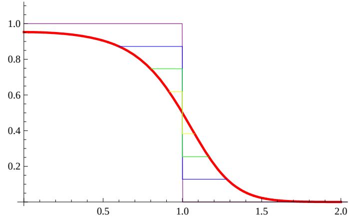
- In our case it is usefull to have R_k independent from frequency. Matsubara summation can then be done analytically.
- For fermions we choose a cutoff that regularizes the fermi surface.



Truncations

For many purposes *derivative expansions* are suitable approximations. For example we use for the BCS-BEC Crossover

$$\Gamma_k = \int_{\tau,\vec{x}} \Big\{ \psi^\dagger(\partial_\tau - \vec{\nabla}^2 - \mu)\psi + \varphi^*(Z_\varphi \partial_\tau - A_\varphi \frac{1}{2}\vec{\nabla}^2)\varphi \\ -h(\varphi^*\psi_1\psi_2 + h.c.) + \frac{1}{2}\lambda_\psi(\psi^\dagger\psi)^2 + U_k(\varphi^*\varphi, \mu) \Big\}$$

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- The effective potential U_k contains no derivatives - describes homogeneous fields.
- Wave-function renormalization and self-energy corrections for fermions can be included as well.

The effective potential

- We use a Taylor expansion around the minimum ρ_0

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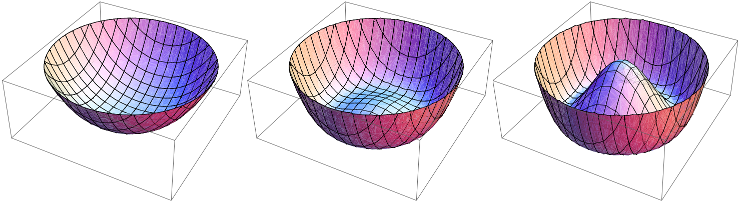
- Symmetry breaking:



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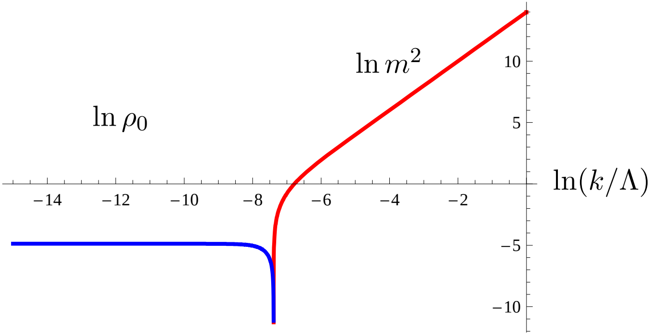
- Typical flow:



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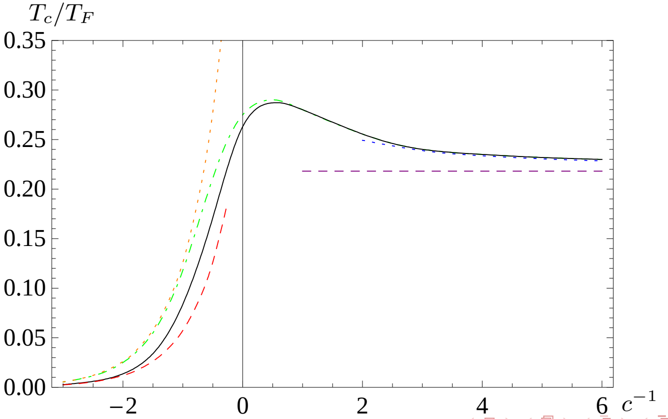

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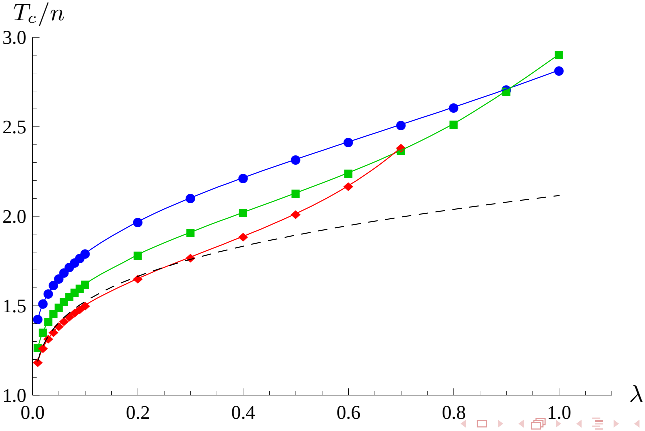

Solving the flow equation - Thermodynamic observables

We calculate the grand canonical potential and can therefore access many thermodynamic observables!

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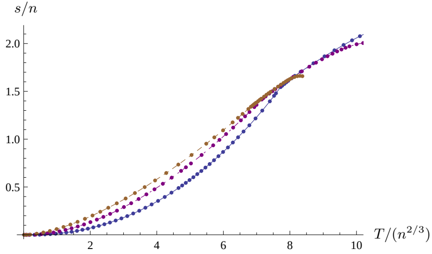

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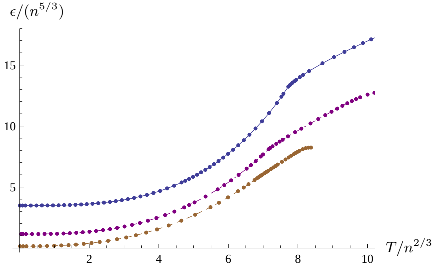

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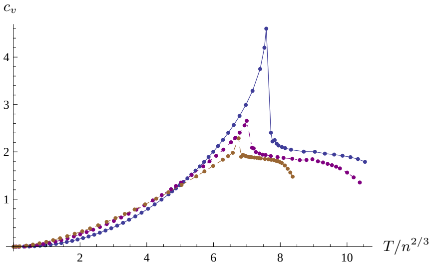

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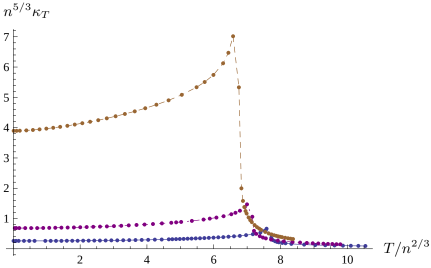

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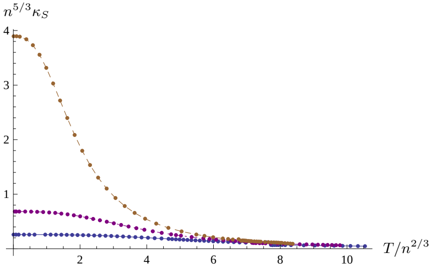

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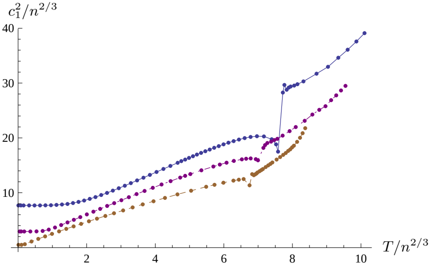

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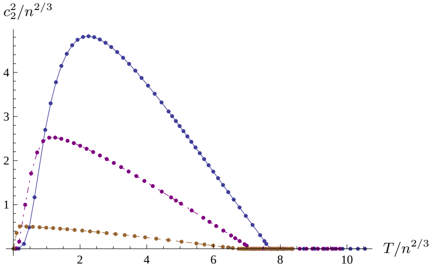

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Solving the flow equation - Occupation numbers

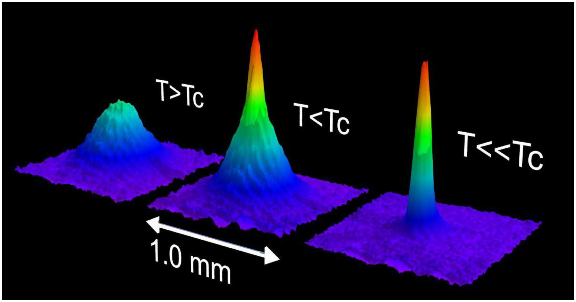
Usually density can be written as

$$n = \int_{\vec{p}} n(\vec{p})$$

with Occupation number $n(\vec{p})$. Example: Homogeneous Bose gas

$$n(\vec{p}) = n_c\,\delta^{(d)}(\vec{p}) + n_T(\vec{p}).$$

Occupation numbers are measured in time-of-flight experiments.



Picture from W. Ketterle, MIT.

Flow equations for occupation numbers

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- Use momentum-dependent chemical potential $\mu = \mu(\vec{p})$

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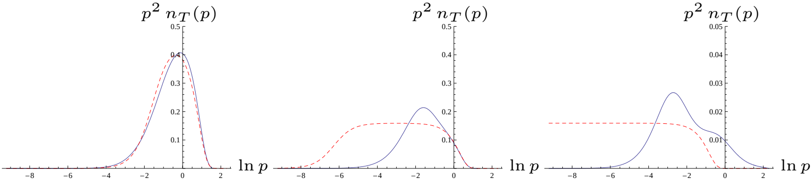
- Example: Bose gas in $d = 2$ with finite size.



$T > T_c$, $n_c = 0$ $\quad$ $T < T_c$, $n_c/n = 0.4$ $\quad$ $T \ll T_c$, $n_c/n = 0.9$

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Consider model with global SU(3) symmetry in truncation

$$\begin{aligned}\Gamma_k &= \int_x \psi^\dagger(\partial_\tau - \vec{\nabla}^2 - \mu)\psi + \varphi^\dagger(\partial_\tau - \vec{\nabla}^2/2 + m_\varphi^2)\varphi \\ &\quad + \chi^*(\partial_\tau - \vec{\nabla}^2/3 + m_\chi^2)\chi \\ &\quad + h\epsilon_{ijk}(\varphi_i^*\psi_j\psi_k + h.c.) \quad + g(\varphi_i\psi_i^*\chi + h.c.)\end{aligned}$$

Flow equations for new physics

- 1 component Fermi gas - no s-wave interaction
- 2 component Fermi gas - BCS-BEC crossover
- 3 component Fermi gas - ??
  On the lattice: Trion formation (RAPP ET AL. 2007).

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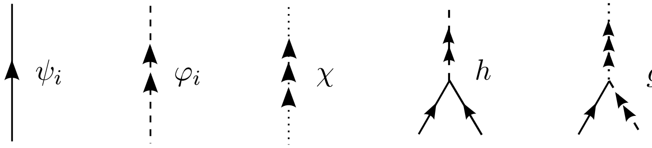
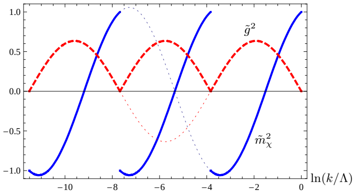
atoms: $\psi = (\psi_1, \psi_2, \psi_3)$, bosons: $\varphi = (\varphi_1, \varphi_2, \varphi_3)$ trion: χ.



Flow equations for new physics

- At $n = T = 0$ limit-cycle scaling for g^2 and m_χ^2



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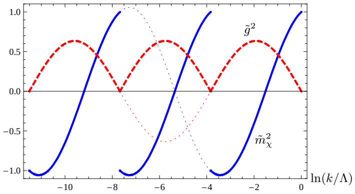

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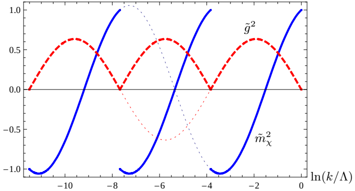

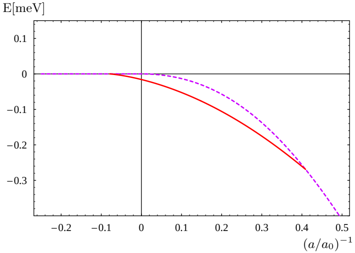

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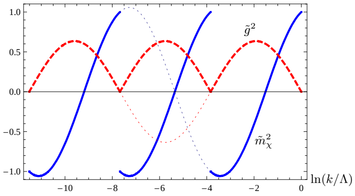

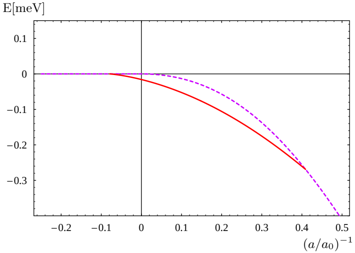

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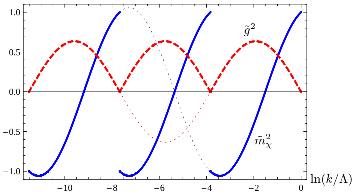

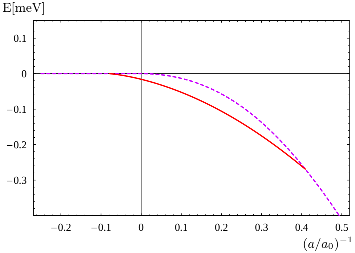

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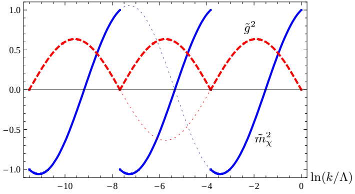

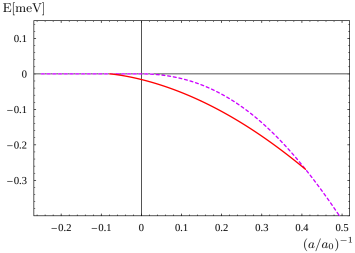

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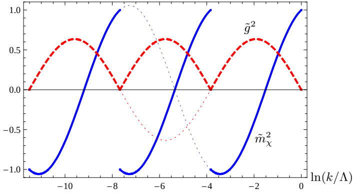

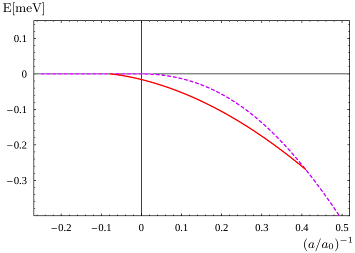

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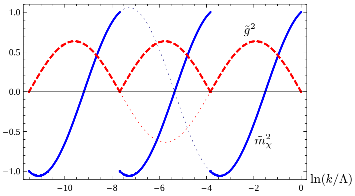

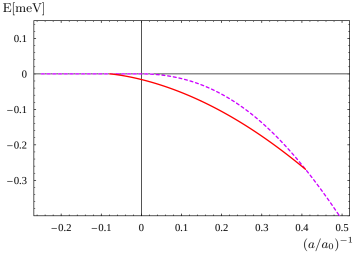

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- Thank you for your attention!