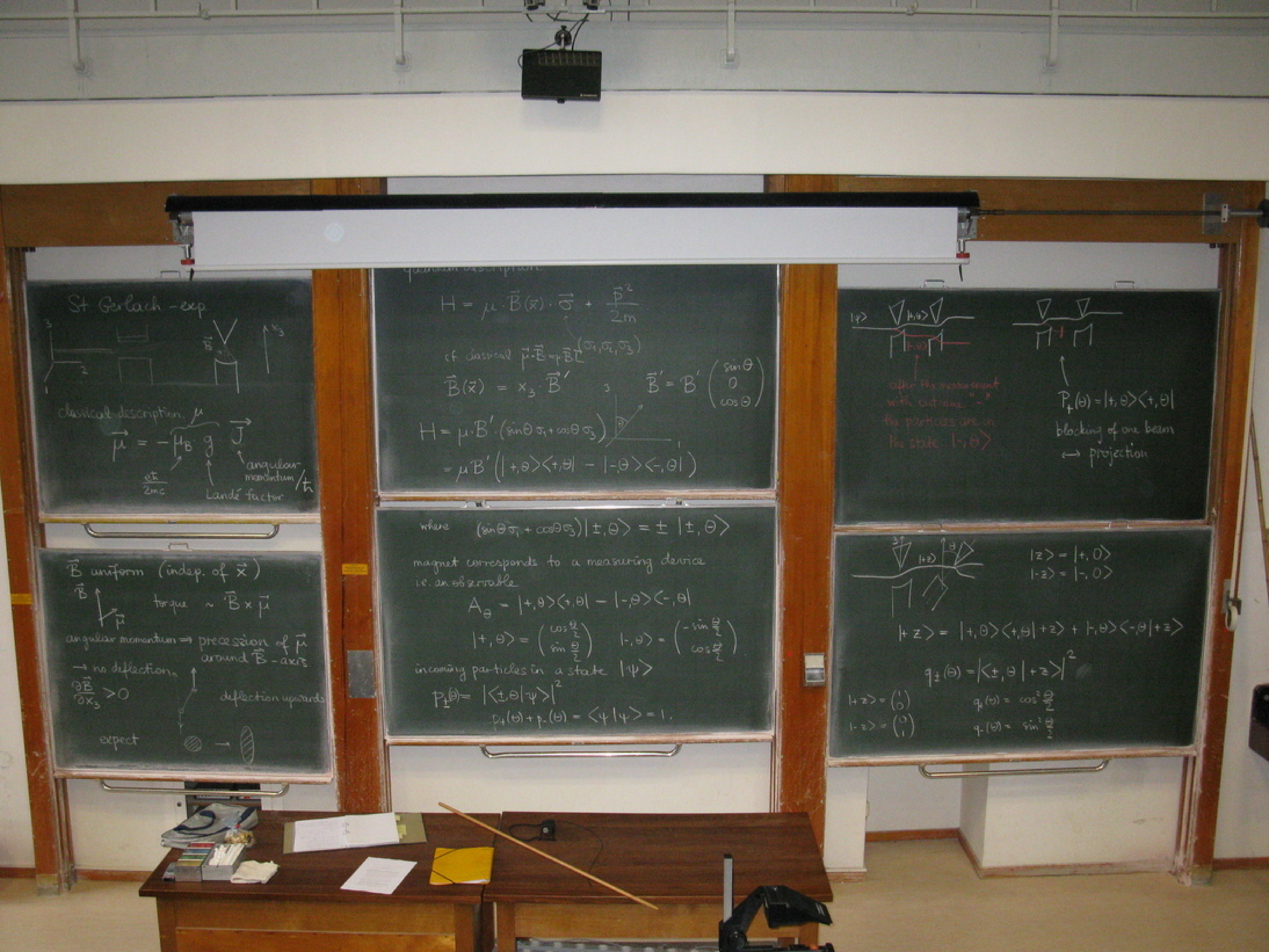
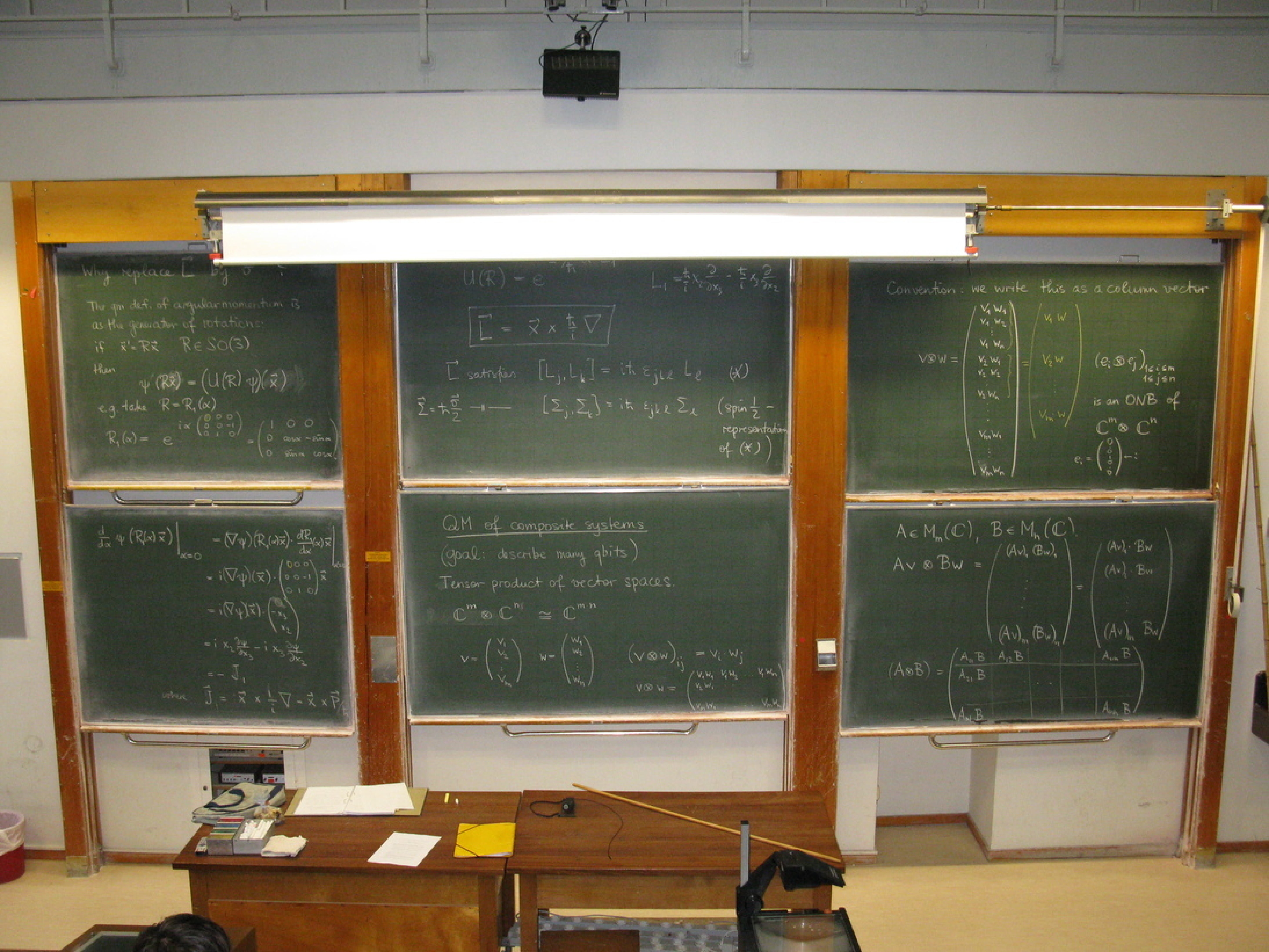
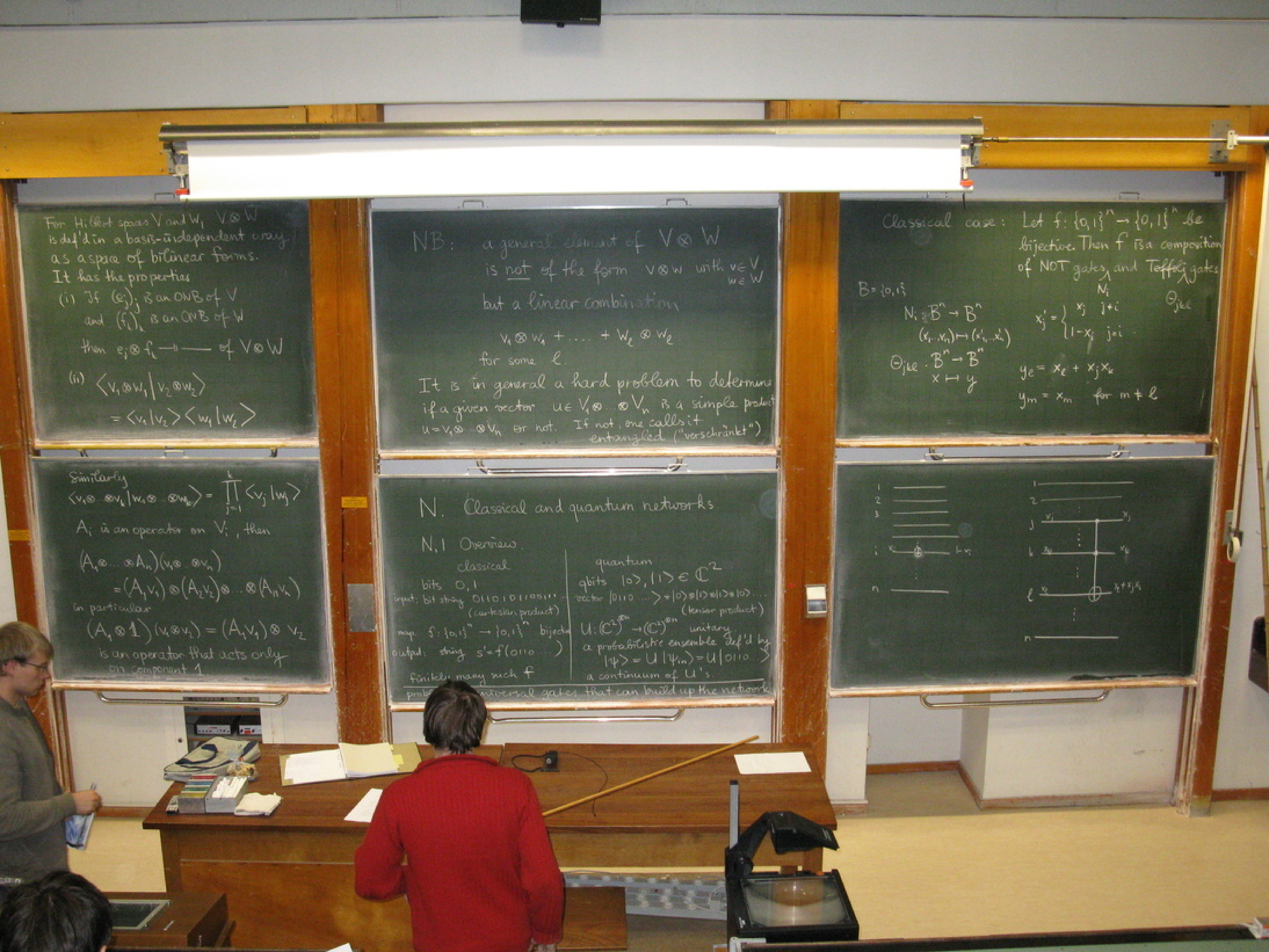

St Gerlach-exp.
classical description
$\vec{\mu} = -\underbrace{\mu_B}_{\frac{e\hbar}{2mc}}\, g\, \vec{J}$
μ
angular momentum $/\hbar$
Landé factor
$\vec{B}$ uniform (indep. of $\vec{x}$)
torque $\sim \vec{B} \times \vec{\mu}$
angular momentum $\Rightarrow$ precession of $\vec{\mu}$ around $\vec{B}$-axis
$\rightarrow$ no deflection.
$\frac{\partial \vec{B}}{\partial x_3} > 0$
deflection upwards
expect
$H = \mu \cdot \vec{B}(x) \cdot \vec{\sigma} + \frac{\vec{p}^2}{2m}$
$(\sigma_1, \sigma_2, \sigma_3)$
cf classical $\vec{\mu} \cdot \vec{B} = \mu \vec{B} \vec{L}$
$\vec{B}(\vec{x}) = x_3 \cdot \vec{B}'$
$\vec{B}' = B' \begin{pmatrix} \sin\theta \\ 0 \\ \cos\theta \end{pmatrix}$
$H = \mu \cdot B' \cdot (\sin\theta\, \sigma_1 + \cos\theta\, \sigma_3)$
$= \mu B' \left(|+,\theta\rangle\langle +,\theta| - |-,\theta\rangle\langle -,\theta| \right)$
where $(\sin\theta\, \sigma_1 + \cos\theta\, \sigma_3)\, |\pm,\theta\rangle = \pm\, |\pm,\theta\rangle$
magnet corresponds to a measuring device
i.e. an observable
$A_\theta = |+,\theta\rangle\langle +,\theta| - |-,\theta\rangle\langle -,\theta|$
$|+,\theta\rangle = \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} \end{pmatrix}$ $|-,\theta\rangle = \begin{pmatrix} -\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \end{pmatrix}$
incoming particles in a state $|\psi\rangle$
$p_\pm(\theta) = |\langle \pm,\theta | \psi \rangle|^2$
$p_+(\theta) + p_-(\theta) = \langle \psi | \psi \rangle = 1.$
$|\psi\rangle$
$|+,\theta\rangle$
$|-,\theta\rangle$
after the measurement with outcome "−" the particles are in the state $|-,\theta\rangle$
$P_+(\theta) = |+,\theta\rangle\langle +,\theta|$
blocking of one beam
$\longleftrightarrow$ projection
$|+z\rangle$
$|z\rangle = |+,0\rangle$
$|-z\rangle = |-,0\rangle$
$|+z\rangle = |+,\theta\rangle\langle +,\theta | +z\rangle + |-,\theta\rangle\langle -,\theta | +z\rangle$
$q_\pm(\theta) = |\langle \pm,\theta | +z \rangle|^2$
$|+z\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$|-z\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
$q_+(\theta) = \cos^2\frac{\theta}{2}$
$q_-(\theta) = \sin^2\frac{\theta}{2}$


Why replace L by σ
The qm def. of angular momentum is
as the generator of rotations:
if x' = Rx R ∈ SO(3)
then
ψ'(Rx) = (U(R) ψ)(x)
e.g. take R = R1(α)
R1(α) = e
U(R) = e
L = x × ħ/i ∇
L satisfies [Lj, Lk] = iħ εjkl Ll (*)
Σ = ħ σ/2 ↦ [Σj, Σk] = iħ εjkl Σl (spin 1/2 -
representation
of (*))
Convention: we write this as a column vector
V⊗W =
is an ONB of
ℂ^m ⊗ ℂ^n
QM of composite systems
(goal: describe many qbits)
Tensor product of vector spaces.
ℂ^m ⊗ ℂ^n ≅ ℂ^(m·n)
A ∈ Mm(ℂ), B ∈ Mn(ℂ)
Av ⊗ Bw =
(A⊗B) =



For Hilbert spaces V and W, $V \otimes W$
is def'd in a basis-independent way
as a space of bilinear forms.
It has the properties
(i) If $(e_j)_j$ is an ONB of V
and $(f_l)_k$ is an ONB of W
then $e_j \otimes f_k$ —— of $V \otimes W$
(ii) $\langle v_1 \otimes w_1 | v_2 \otimes w_2 \rangle$
$= \langle v_1 | v_2 \rangle \langle w_1 | w_2 \rangle$
Similarly
$\langle v_1 \otimes \dots \otimes v_k | w_1 \otimes \dots \otimes w_k \rangle = \prod_{j=1}^{k} \langle v_j | w_j \rangle$
A_i is an operator on V_i, then
$(A_1 \otimes \dots \otimes A_n)(v_1 \otimes \dots \otimes v_n)$
$= (A_1 v_1) \otimes (A_2 v_2) \otimes \dots \otimes (A_n v_n)$
in particular
$(A_1 \otimes \mathbb{1})(v_1 \otimes v_2) = (A_1 v_1) \otimes v_2$
is an operator that acts only
on component 1
NB: a general element of $V \otimes W$
is not of the form $v \otimes w$ with $v \in V$, $w \in W$
but a linear combination
$v_1 \otimes w_1 + \dots + w_\ell \otimes w_\ell$
for some ℓ.
It is in general a hard problem to determine
if a given vector $u \in V_1 \otimes \dots \otimes V_n$ is a simple product
$u = v_1 \otimes \dots \otimes v_n$ or not. If not, one calls it
entangled ("verschränkt")
N. Classical and quantum networks.
N.1 Overview.
classical
bits 0, 1
input: bit string 0110101100111...
(cartesian product)
map: $f: \{0,1\}^n \to \{0,1\}^n$ bijective
output: string $s' = f(0110\dots)$
finitely many such f
quantum
qbits $|0\rangle, |1\rangle \in \mathbb{C}^2$
vector $|0110\dots\rangle = |0\rangle \otimes |1\rangle \otimes |1\rangle \otimes |0\rangle \dots$
(tensor product)
$U: (\mathbb{C}^2)^{\otimes n} \to (\mathbb{C}^2)^{\otimes n}$ unitary
a probabilistic ensemble def'd by
$|\psi\rangle = U|\psi_{in}\rangle = U|0110\dots\rangle$
a continuum of U's.
problem: universal gates that can build up the network
Classical case: Let $f: \{0,1\}^n \to \{0,1\}^n$ be
bijective. Then f is a composition
of NOT gates N_i and Toffoli gates $\Theta_{jk\ell}$
$B = \{0,1\}$
$N_i: B^n \to B^n$
$(x_1 \dots x_n) \mapsto (x_1' \dots x_n')$
$x_j' = \begin{cases} x_j & j \neq i \\ 1 - x_j & j = i \end{cases}$
$\Theta_{jk\ell}: B^n \to B^n$
$x \mapsto y$
$y_\ell = x_\ell + x_j x_k$
$y_m = x_m$ for $m \neq \ell$
1
2
3
i
x_i
$1-x_i$
n
1
2
j
x_j
x_j
k
x_k
x_k
ℓ
x_ℓ
$x_\ell + x_j x_k$
n