Exercise Sheet 8, Theoretical Physics III (Quantum Mechanics)

Class exercises discussed in the tutorials of Week 9 (14/12/07)

Class exercise P8: More linear algebra (3 points)

Let A and B be Hermitian operators on a Hilbert space, and $|\,a\rangle$ be an eigenstate of A with eigenvalue a.

a) Give a condition for AB to be Hermitian.

b) What is the adjoint of the commutator $[A, B]$? Find $c \in \mathbb{C}$ such that $c[A, B]$ is Hermitian.

c) Compute the expectation value of $[A, B]$ in the state $|\,a\rangle$.

d) Assume A is invertible. Show that $|\,a\rangle$ is an eigenstate of A^{-1}, and compute the eigenvalue.

e) Is the projector $|\,a\rangle\,\langle a\,|$ invertible?

Class exercise P9: Three-dimensional Hilbert space (2 points)

Let $|\,\rho\rangle = (1, 1, 0)^T$ and $|\,\psi\rangle = (1, 0, 1)^T$ be vectors in the Hilbert space $\mathbb{C}^3$, for which an orthonormal basis is given by $|\,e_1\rangle = (1, 0, 0)^T, |\,e_2\rangle = (0, 1, 0)^T, |\,e_3\rangle = (0, 0, 1)^T$.

a) Find the matrix elements of $A \equiv |\,\rho\rangle\,\langle\psi\,|$ in this basis.

b) Compute $A^\dagger$. Is A Hermitian?

c) Determine the eigenvalues of A.