10th Exercise Sheet: Electrodynamics, Summer Term '06

Prof. M. G. Schmidt, J. Braun

June 30, 2006

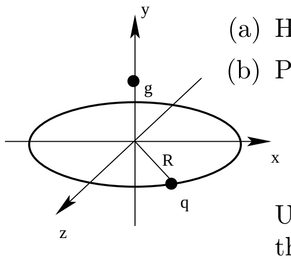
Submission on **July 7, 2006** during the lecture

10. 1. **('Präsenzübung': Dirac's charge quantization and magnetic monopole, 1+1 marks)** If we allow for magnetic monopoles, then Maxwell's equations would read:

$$\vec{\nabla} \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{j}_e\,, \qquad \vec{\nabla}\cdot\vec{B} = 4\pi\rho_m\,, \qquad \vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} - \frac{4\pi}{c}\vec{j}_m\,, \qquad \vec{\nabla}\cdot\vec{E} = 4\pi\rho_e\,.$$

Here, $\rho_{e/m}$ and $\vec{j}_{e/m}$ denote the electric and magnetic charge and current density, respectively. Consider the following problem (see sketch):

A magnetic monopole with magnetic charge g moves along the y-axis 'on a rail' with velocity $\vec{v} = v\,\vec{e}_y$. A ring with radius R is located in the x-z-plane with an electric charge q is sitting on it. The electric charge can move freely on the ring.



(a) How does Faraday's induction law read if magnetic monopoles would exist?

(b) Prove that the total momentum of the electric charge q is given by

$$P_\phi = \int_{-\infty}^{\infty} dt\; qE_\phi = -\frac{2\,qg}{Rc}\,.$$

Use Faraday's induction law and assume that $\Phi_m(t \to -\infty) = \Phi_m(t \to \infty)$ (Φ_m denotes the magnetic flux through the ring).

(c) What do the results of exercise 10.1.(b) imply, if you assume that the total angular momentum of the electric charge is quantized, i. e. $|\vec{L}| = n\hbar$.

10. 2. **(electromagnetic waves in a medium, 6 marks)** Consider an electromagnetic wave in a medium consisting of free electrons with electron density n_e. The electric and magnetic components of the wave are given by

$$\vec{E} = \vec{E}_0(\vec{r})\,\mathrm{e}^{-i\omega t} \quad \text{and} \quad \vec{B} = \vec{B}_0(\vec{r})\,\mathrm{e}^{-i\omega t}\,.$$

(a) **(1.5 marks)** Calculate the current density in the medium which is generated by the electric field. Neglect the interaction between the electrons and the effect of the magnetic field! For which velocities of the electrons can the effect of the magnetic field be neglected, if you assume $|\vec{E}| \sim |\vec{B}|$?

(b) **(3.5 marks)** Using Maxwell's equations, write the differential equations for the spatial dependence of the electromagnetic wave in such a medium. **Hint:** Assume that the Maxwell equation I (nomenclature as in the lecture) is homogeneous, i. e. $\vec{\nabla}\cdot\vec{E} = 0$, but take the current density calculated in exercise (a) into account!

(c) **(1 mark)** From the equations of exercise (b), find the necessary and sufficient condition on the electron density in which the electromagnetic waves propagate in this medium indefinitely.

10. 3. **(spherical waves, 6 marks)**

(a) **(2 marks)** Prove that any function of the form

$$F(\vec{r}, t) = f(\vec{e}\vec{r} - ct)$$

fulfills the homogenous wave equation, where $\vec{e}$ denotes an unit vector. Assume that the function f is two times continuously differentiable.

(b) **(2 marks)** Check (explicitly) if the function $\mathrm{e}^{i(kr\pm\omega t)}/r$ is a solution of the homogenous wave equation. What is the physical interpretation of this ansatz? ($k = |\vec{k}|$ and $r = \vec{r}$)

(c) **(2 marks)** Make ansätze for the potentials $\vec{A}$ and ϕ along the lines of exercise (b). Give the solutions for $\vec{A}$ and ϕ in Lorenz-gauge!

See reverse!

10. 4. **(accelerated charges, 6 marks)**

(a) **(2.5 marks)** Calculate the Poynting vector of the electromagnetic field radiated by an accelerated charge e by means of the results for the electric and magnetic field derived in the lecture (non-relativistic approximation):

$$\vec{E}_{\text{str.}} = \frac{e}{c^2 R^3} \left[\left(\vec{R}\dot{\vec{v}} \right) \vec{R} - R^2 \dot{\vec{v}} \right] \qquad \text{and} \qquad \vec{B}_{\text{str.}} = \frac{e}{c^2 R^2} \left[\dot{\vec{v}} \times \vec{R} \right] .$$

Assume that $\dot{\vec{v}} \parallel \vec{e}_z$!

(b) **(1 mark)** From the result of exercise (a), find the radiated power!

(c) **(1 mark)** Give the radiated power for a particle (mass m and charge e) with

$$\dot{\vec{v}} = \frac{e}{m\,c} \vec{v} \times \vec{B}_{\text{ext.}} \quad \text{and} \quad \vec{v} \perp \vec{B}_{\text{ext.}} ,$$

where $\vec{B}_{ext}$ is an external magnetic field in which the particle propagates.

(d) **(1.5 marks)** Calculate classically the radiated power of an electron that orbits an hydrogen atom. The radius of the orbit is given by a_B (Bohr's constant). Assume that the electron has a constant angular velocity!