# 11th Exercise Sheet: Electrodynamics, Summer Term '06

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**Submission** on **Friday 14, 2006** during the lecture

11. 1. **(Präsenzübung: collider-energies, 1+1 marks)** Consider the reaction $p + p \rightarrow p + p + p + \bar{p}$ , where an antiproton $\bar{p}$ and a proton $p$ with equal masses $m_p = m_{\bar{p}} = 938\,\mathrm{MeV}/c^2$ are produced.

   (a) fixed target experiment: What is the minimal energy to make this reaction possible if a proton collides with another proton at rest?

   (b) collider experiment: What is the minimal energy for the protons in a proton-proton-collider in order to make this reaction possible if the momenta of the both protons are given by $\vec{p_1} = -\vec{p_2}$?

   **Notes:**

   - four-momenta: $p_\mu = \left(\frac{E}{c}, \vec{p}\right)$
   - relativistic energy-momentum relation: $E = \sqrt{c^2\vec{p}^2 + m^2c^4}$
   - lorentz-invariant energy quantity $s$ (square of the sum of the four-momenta of each particle of the system):

$$s = c^2 \left( \sum_i p_\mu^{(i)} \right)^2 .$$

11. 2. **(oscillating electric dipole, 7 marks)** An electric dipole $\vec{P}$ is oscillating with a frequency $\omega$ and amplitude $P_0$, i. e. $\vec{P} = \vec{P}_0 \mathrm{e}^{i\omega t}$. The dipole is located at a distance $a/2$ from an infinitely extended conducting plate. The dipole vector is parallel to the surface of the plate. Calculate the electromagnetic field and the time-averaged angular distribution of the emitted radiation $\frac{dP}{d\Omega}$ of the dipole, for distances $r \gg \lambda = \frac{2\pi c}{\omega}$. How does the result reduce if you assume $\lambda \gg a$?
**Hint:** Assume that the conducting plate is represented by the $y$-$z$-plane. What is the effect of the conducting plane on the $x > 0$ space? Think of the method of mirror charges! Neglect terms of higher order in $\frac{1}{r}$ in the calculation of the electric and magnetic fields.

11. 3. **(special Lorentz transformations and Galilei transformations, 4 marks)** A reference-frame $I'$ is moving with velocity $v$ in $x_1$-direction relative to a reference-frame $I$. Give the (special) Lorentz transformations for the variables $t = \frac{x^0}{c}$, $x^1$, $x^2$, $x^3$ and $t' = \frac{x'^0}{c}$, $x'^1$, $x'^2$, $x'^3$. Expand the transformation formulas for the coordinates in powers of $\frac{v}{c}$ and prove that, in leading order, the (special) Lorentz-transformations reduce to (special) Galilei transformations between the two reference frames.

11. 4. **('stick paradox', 7 marks)** A stick of length 10 cm is moving along the $x$-axis with velocity $v_s = \frac{\sqrt{3}}{2}c$ parallel to a plate with a hole of length 10 cm. The plate is moving along the $y$-axis with velocity $u$, in such a way that the center of the stick and the center of the hole are lying on top of each other at time $t = 0$, in the reference-frame $K$ of the plate.

   (a) **(1 mark)** How long is the stick in the reference-frame $K$ of the plate? Can the stick pass the hole?

   (b) **(6 marks)** How big is the hole in the reference-frame $K'$ of the stick? Can the stick pass the hole in this reference-frame? If yes, do both ends of the stick pass the hole at the same time?