6th Exercise Sheet: Electrodynamics, Summer Term '06

Prof. M. G. Schmidt, J. Braun

June 2, 2006

Submission on **June 9, 2006** during the lecture

6. 1. **(Präsenzübung: infinite long cylinder and magnetic fields, 1+1 marks)** Consider an infinite long conducting cylinder of radius R with a homogenous current flow $\vec{j}$. (homogenous current flow means that $\vec{j}$ is constant in the entire cross section of the conductor)

   (a) Calculate the vector potential $\vec{A}(\vec{x})$ inside and outside of the conductor.

   (b) Calculate the magnetic induction $\vec{B}(\vec{x})$ inside and outside of the conductor.

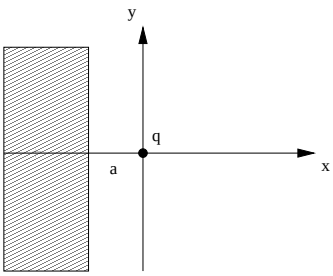
6. 2. **(point charge and dielectric matter, 5 marks)** Consider a point charge q that is located in the vacuum at the origin $r = 0$ with distance a from a dielectric half-space (dielectric constant ϵ).

   (a) **(3 marks)** Prove that the potential in the vacuum can be written as

   $$\phi_1 = \frac{q}{\sqrt{x^2+y^2+z^2}} + \frac{q'}{\sqrt{(x+b)^2+y^2+z^2}}$$

   and in the dielektric half-space as

   $$\phi_2 = \frac{q''}{\sqrt{x^2+y^2+z^2}}$$

   Calculate the parameter b and the charges q', q''.

   (b) **(2 marks)** Calculate and sketch the electric field of the arrangement for $\epsilon > 1$ and $\epsilon < 1$.

6. 3. **(vector potentials and gauge transformations, 4 marks)** Consider a homogenous magnetic field in z-direction. Construct this magnetic field by two vector potentials, one of them pointing in the x-direction and the other one pointing in the y-direction. Find a gauge transformation that connects both potentials.

6. 4. **(rotating ball and Coulomb-gauge, 9 marks)** Consider a rotating charged ball of radius R. The charge Q of the ball is distributed homogenously on the surface of the ball. The ball rotates with a constant angular velocity ω and the rotation axis is passing through the centre of the ball. (**Hints:** It is convenient to use spherical coordinates!)

   (a) **(2 marks)** Write the charge density ρ and the current density vector $\vec{j}$ by means of Delta-distributions and verify that $\vec{\nabla} \cdot \vec{j} = 0$.

   (b) **(4 marks)** Calculate the vector potential $\vec{A}(\vec{x})$ such that $\vec{\nabla} \cdot \vec{A} = 0$ (Coulomb-gauge).

   (c) **(3 marks)** Calculate the magnetic induction $\vec{B}(\vec{x})$ and prove by an explicit calculation that the field equations

   $$\vec{\nabla} \cdot \vec{B} = 0 \qquad \text{and} \qquad \vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{j}$$

   are fulfilled.

Hendrik Antoon Lorentz

Born: 18 July 1853 in Arnhem, Netherlands
Died: 4 Feb 1928 in Haarlem, Netherlands

Click the picture above
to see eight larger pictures

Show birthplace location

Previous (Chronologically) Next Biographies Index
Previous (Alphabetically) Next Main index

Hendrik Lorentz attended Mr Timmer's Primary School in Arnhem until he was 13 years of age when he entered the new High School there. He entered the University of Leiden in 1870 but, in 1872, he returned to Arnhem to take up teaching evening classes. He worked for his doctorate while holding the teaching post.

Lorentz refined Maxwell's electromagnetic theory in his doctoral thesis *The theory of the reflection and refraction of light* presented in 1875. He was appointed professor of mathematical physics at Leiden University in 1878. He remained in this post until he retired in 1912 when Ehrenfest was appointed to his chair. After retiring from this chair, Lorentz was appointed director of research at the Teyler Institute, Haarlem. However, he retained an honorary position at Leiden, where he continued to lecture.

Before the existence of electrons was proved, Lorentz proposed that light waves were due to oscillations of an electric charge in the atom. Lorentz developed his mathematical theory of the electron for which he received the Nobel Prize in 1902. The Nobel prize was awarded jointly to Lorentz and Pieter Zeeman, a student of Lorentz. Zeeman had verified experimentally Lorentz's theoretical work on atomic structure, demonstrating the effect of a strong magnetic field on the oscillations by measuring the change in the wavelength of the light produced.

Lorentz is also famed for his work on the FitzGerald-Lorentz contraction, which is a contraction in the length of an object at relativistic speeds. Lorentz transformations, which he introduced in 1904, form the basis of Einstein's special theory of relativity. They describe the increase of mass, the shortening of length, and the time dilation of a body moving at speeds close to the velocity of light.

Lorentz was chairman of the first Solvay Conference held in Brussels in the autumn of 1911. This conference looked at the problems of having two approaches, namely that of classical physics and of quantum theory. However Lorentz never fully accepted quantum theory and always hoped that it would be possible to incorporate it back into the classical approach. He said in his presidential address at the opening ceremony of the conference:-

> *In this stage of affairs there appeared to us like a wonderful ray of light the beautiful hypothesis of energy elements which was first expounded by* Planck *and then extended by* Einstein *and Nernst, and others to many phenomena. It has opened for us unexpected vistas, even those, who consider it with a certain suspicion, must admit its importance and fruitfulness.*

Some of Lorentz's numerous publications are highlighted in [3]:-

> *In an early memoir, which became famous, Lorentz applied for the first time considerations relating to discrete molecules to electric propagation in material bodies, and incidentally arrived at a rational reflection-equivalent for each substance independent of its density. In* 1884 *he began to study the effect which magnetization exerts on the polarization of reflected light. His "Théorie Electromagnétique de Maxwell et son application aux Corps Mouvants" and his "Versuch einer Theorie der Elektrischen und Optischen Erscheinungen in bewegten Körpern" were published in* 1892 *and* 1895 *respectively. They embodied the first systematic appearance of the electrodynamic principle of relativity, and in* 1920 *he brought out "The Einstein Theory of Relativity: A Concise Statement". In* 1909 *he published his "Theory of Electrons", based on a series of lectures at Columbia University, and in* 1916 *he published in French at Leipzig an account of statistical thermodynamic theories, based on lectures delivered at the Collège de France in* 1912. *An edition of his University lectures, entitled "Lessons on Theoretical Physics", began to appear, under his supervision, in* 1919. *He was also the author of a textbook of the differential and integral calculus; "Visible and Invisible Movements",* 1901; *and "Clerk Maxwell's Electromagnetic Theory",* 1924.

In [7] O W Richardson describes Lorentz as:-

> *... a man of remarkable intellectual powers ... . Although steeped in his own investigation of the moment, he always seemed to have in his immediate grasp its ramifications into every corner of the universe. ... The singular clearness of his writings provides a striking reflection of his wonderful powers in this respect. .... He possessed and successfully employed the mental vivacity which is necessary to follow the interplay of discussion, the insight which is required to extract those statements which illuminate the real difficulties, and the wisdom to lead the discussion among fruitful channels, and he did this so skilfully that the process was hardly perceptible.*

Lorentz received a great many honours for his outstanding work. He was elected a Fellow of the Royal Society in 1905. The Society awarded him their Rumford Medal in 1908 and their Copley Medal in 1918.

The respect in which Lorentz was held in The Netherlands is seen in Richardson's description of his funeral [7]:-

> *The funeral took place at Haarlem at noon on Friday,* 10 *February. At the stroke of twelve the State telegraph and telephone services of Holland were suspended for three minutes as a revered tribute to the greatest man Holland has produced in our time. It was attended by many colleagues and distinguished physicists from foreign countries. The President, Sir Ernest Rutherford, represented the Royal Society and made an appreciative oration by the graveside.*