8. Übungsblatt zur Elektrodynamik, Sommersemester ’06

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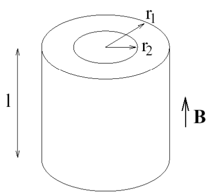
Abgabe am **Freitag, den 23.06.2006** in der Vorlesung

8. 1. **(Präsenzübung: Induktion und mechanischer Drehimpuls, 1+1 Punkte)** Betrachten Sie einen Zylinderkondensator der Länge l mit Außenradius r_1 und Innenradius r_2 mit einer Ladung q außen und $-q$ innen. Zwischen den beiden Zylindern sei Vakuum. Parallel zur Symmetrieachse sei ein Magnetfeld B_0. Berechnen Sie das **mechanische Drehmoment** und den **mechanischen Drehimpuls**, den das System erhält, ...



(a) ... wenn der Kondensator entladen wird.

(b) ... wenn das Magnetfeld abgeschaltet wird.

Hinweis: Berechnen Sie zunächst die Strahlungsimpulsdichte $\vec{S} = \frac{1}{4\pi c}\vec{E}\times\vec{B}$. Damit läßt sich dann ein Drehimpuls in z-Richtung errechnen. Überlegen Sie sich dann, was passiert, wenn Sie den Kondensator entladen bzw. das Magnetfeld abschalten.

8. 2. **(Indukion in einer Leiterschleife, 4 Punkte)** Eine leitende Kreisschleife mit Radius r_0 rotiert in einem homogenen magnetischen Feld der Stärke B_0 mit einer Winkelgeschwindigkeit $\vec{w}$ um einen Durchmesser, welcher senkrecht auf der Feldrichtung von $\vec{B}$ steht.

(a) **(2 Punkte)** Berechnen Sie die in der Schleife induzierte Spannung U.

(b) **(2 Punkte)** Beziehen Sie im Folgenden in Ihre Rechnung die Tatsache ein, daß die Schleife einen elektrischen Widerstand R besitzt und damit eine elektrische Leistung $P = UI$ erbringt. Berechnen Sie das Drehmoment, das auf die Schleife ausgeübt werden muss, damit sie sich mit konstanter Winkelgeschwindigkeit ω_0 dreht!

8. 3. **(Maxwell’scher Spannungstensor I, 2 Punkte)** Berechnen Sie den Strahlungsdruck $P_{\text{str.}}$, den ein elektromagnetisches Feld auf eine Fläche in der y-z-Ebene ausübt. Unterscheiden Sie dabei zwei Fälle:

(a) **(1 Punkte)** isotrope Strahlung

(b) **(1 Punkte)** senkrecht auf die Fläche auftreffende Strahlung

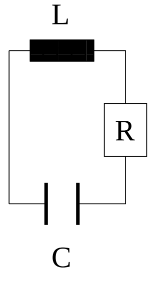
8. 4. **(Maxwell’scher Spannungstensor II, 4 Punkte)** Betrachten Sie eine Punktladung q im Abstand r' vor einer in der x_2-x_3-Ebene liegenden unendlich ausgedehnten Metallplatte, die geerdet ist.

(a) **(2 Punkte)** Berechnen Sie den Maxwell’schen Spannungstensor T_{ij} für Punkte in der x_2-x_3-Ebene.

(b) **(2 Punkte)** Berechnen Sie mit Hilfe des Ergebnisses von (a) die Kraft der Punktladung auf die leitende Platte.

Hinweis: Verwenden Sie das Ergebnis für das Potential bzw. die Green’sche Funktion für das Problem ’Punktladung vor einer leitenden Platte’ (siehe Vorlesungsskript S. 41; das Skript ist auf der Homepage zur Vorlesung zum Download bereitgestellt). Um den Maxwell’schen Spannungstensor berechnen zu können, müssen Sie zunächst das elektrische Feld für die Anordnung bestimmen.

8. 5. **(Schwingkreis, 8 Punkte)** Betrachten Sie die in der Skizze angegebene Anordnung aus einem ohmschen Widerstand R, einer Spule mit dem Selbstinduktionskoeffizienten L und einem Kondensator C. Zum Zeitpunkt $t = 0$ sei die zwischen den Kondensatorplatten anliegende Spannung gegeben durch $U_C(t = 0) = U_0$. Weiterhin sei $I(t = 0) = 0$.



(a) **(1 Punkt)** Wie lautet die Differentialgleichung für die zwischen den Kondensatorplatten anliegende Spannung $U_C(t)$?

(b) **(4 Punkte)** Geben Sie die allgemeine Lösung $U_C(t)$ dieser Differentialgleichung an. Berücksichtigen Sie dabei die vorgegebenen Anfangsbedingungen.

(c) **(3 Punkte)** Für welche Werte von L, R und C erhält man den sogenannten Kriechfall, den aperiodischen Grenzfall sowie den Schwingfall? Geben Sie für alle drei Fälle die Lösung $U_C(t)$ an und skizzieren Sie diese.

André Marie Ampère

Born: 20 Jan 1775 in Lyon, France
Died: 10 June 1836 in Marseilles, France

Click the picture above
to see five larger pictures

Show birthplace location

Previous (Chronologically) Next Biographies Index
Previous (Alphabetically) Next Main index

André-Marie Ampère's father, Jean-Jacques Ampère, was a prosperous man who owned a home in Lyon and a country house in Poleymieux, which is only 10 km from Lyon. Up till André-Marie was seven years old the family spent most of the year in Lyon except the summer months which were spent at Poleymieux. However, in 1782, the home at Poleymieux became their main residence since André-Marie's father wished to spend more time on his son's education. Only a short time in winter was spent at Lyon where André-Marie's father saw to his business interests.

Despite not attending school, André-Marie was to be given an excellent education. He describes this education in autobiographical writings (rather strangely referring to himself in the third person):-

> *His father, who had never ceased to cultivate Latin and French literature, as well as several branches of science, raised him himself in the country near the city where he was born. He never required him to study anything, but he knew how to inspire in him a desire to know. Before being able to read, the young Ampère's greatest pleasure was to listen to passages from* Buffon*'s natural history.*

Ampère read articles from *L'Encyclopédie* many of which, Arago remarked many years later, he could recite in full in later life. Arago also claims that Ampère read the *Encyclopédie* starting at volume 1 and reading the articles in alphabetical order. Whether Ampère's later desire for classification in all subjects arose from this education, or whether he enjoyed Buffon and the *Encyclopédie* because of a natural liking for classifying, is hard to say.

It has been claimed that Ampère had mastered all known mathematics by the age of twelve years but this seems somewhat of an exaggeration since, by Ampère's own account, he did not start to read elementary mathematics books until he was 13 years old. However Ampère was always one to feel very confident in his own abilities and he certainly began to develop his own mathematical ideas very quickly and he began to write a treatise on conic sections. Ampère had no contacts with anyone with any depth of mathematical knowledge so it is not surprising that he felt that his ideas were original.

While still only 13 years old Ampère submitted his first paper to the Académie de Lyon. This work attempted to solve the problem of constructing a line of the same length as an arc of a circle. His method involves the use of infinitesimals but since Ampère had not studied the calculus the paper was not found worthy of publication. Shortly after writing the article Ampère began to read d'Alembert's article on the differential calculus in the *Encyclopédie* and realised that he must learn more mathematics.

After taking a few lessons in the differential and integral calculus from a monk in Lyon, Ampère began to study works by Euler and Bernoulli. He then acquired a copy of the 1788 edition of Lagrange's *Mécanique analytique* and began serious study of the work. Ampère writes (again writing about himself in the third person):-

> *... the reading of [Mécanique analytique] had animated him with a new ardour. He repeated all the calculations in it ...*

However his life was soon to be shattered. The French Revolution began with the storming of the Bastille on 14 July 1789 but the effect on the Poleymieux region was not very great at first. Ampère's father kept out of trouble until late in 1791 when he accepted the position of Justice of the Peace in Lyon. This post made it virtually impossible for him to avoid trouble but the first tragedy to hit the family was in 1792 when André-Marie's sister died. The city of Lyon refused to carry out instructions from Paris and the city was besieged for two months. On the fall of the city Ampère's father was arrested for issuing an arrest warrant for the Jacobin Chevalier who had then been put to death. Ampère's father went to the guillotine with remarkable composure writing to Ampère's mother from his cell:-

> *I desire my death to be the seal of a general reconciliation between all our brothers; I pardon those who rejoice in it, those who provoked it, and those who ordered it....*

The effect on Ampère of his father's death was devastating. He gave up his studies of *Mécanique analytique* and did not return to the study of mathematics for 18 months. He only returned to something like his old self when he met a girl, Julie, who he fell deeply in love with. Julie seemed less attracted to Ampère:-

> *He has no manners; he is awkward, shy and presents himself poorly.*

Despite this coolness they were engaged to be married in 1797 and Ampère decided he better show that he could earn a living so began tutoring mathematics in Lyon. He married Julie in 1799 and their son Jean-Jacques was born in 1800. Ampère continued tutoring mathematics until 1802 when he was appointed professor of physics and chemistry at Bourg Ecole Centrale. This was a difficult time for Ampère since Julie became ill before he made the move to Bourg leaving her at Poleymieux.

While Ampère was in Bourg he spent much time teaching physics and chemistry but his research was in mathematics. This research resulted in him composing a treatise on probability, *The Mathematical Theory of Games*, which he submitted to the Paris Academy in 1803. Laplace noticed an error, explaining the error to Ampère in a letter, which Ampère was able to correct and the treatise was reprinted. In fact the treatise was modified a number of times and Ampère was reluctant to call it completed for fear that further changes might be required. This work was followed by one on the calculus of variations in 1803.

After a year in Bourg, Ampère moved closer to Poleymieux being appointed to a mathematics position at the Lycée in Lyon on Delambre's recommendation. His time spent in Lyon had been made difficult due to the continuing decline in his wife's health. Mathematically he continued to produce good work, this time an interesting treatise on analytic geometry. Like a number of other mathematicians, Ampère seemed able to concentrate on his theorems despite the personal tragedy around him and, sadly, this would be required of him throughout his unhappy life. After his wife died in July 1803, Ampère was left with feelings of guilt for he had lived apart from his wife during much of their short marriage. He decided to leave Lyon for Paris. Hofman writes in [4] regarding his feelings following his wife's death:-

> *His subsequent depression contributed to his decision to take the earliest opportunity to leave Lyon for new surroundings in Paris. Later he would regret this decision. The Lyon friends who attempted to fill the emotional void left by Julie's death were missed painfully. Although Ampère gradually adjusted to the priority disputes and infighting of the Parisian scientific community, he always longed for a return to the intellectual life he experienced in Lyon.*

By this time Ampère had a fair reputation as both a teacher of mathematics and as a research mathematician and on the strength of this reputation he was appointed répétiteur (basically a tutor) in analysis at the Ecole Polytechnique in 1804. Without a formal education and formal qualifications his appointment is surprising but shows that his potential was recognised at this stage. His life, already containing many tragedies, did not improve and he embarked on a disastrous marriage. Lagrange and Delambre attended his wedding to Jenny on 1 August 1806 but, before the birth of their daughter on 6 July 1807, the couple were living apart and were not on speaking terms. They were legally separated in 1808 and Ampère was given custody of their daughter Albine.

Appointed professor of mathematics at the Ecole Polytechnique in 1809 he held posts there until 1828. Ampère and Cauchy shared the teaching of analysis and mechanics and there was a great contrast between the two with Cauchy's rigorous analysis teaching leading to great mathematical progress but found extremely difficult by students who greatly preferred Ampère's more conventional approach to analysis and mechanics. Ampère was appointed to a chair at Université de France in 1826 which he held until his death.

In Paris Ampère worked on a wide variety of topics. Although a mathematics professor, his interests included, in addition to mathematics, metaphysics, physics and chemistry. In mathematics he worked on partial differential equations, producing a classification which he presented to the Institut in 1814. This seems to have been a crucial step in his election to the Institut National des Sciences in November 1814 when he defeated Cauchy, receiving 28 of the 56 votes cast.

Ampère was also making significant contributions to chemistry. In 1811 he suggested that an anhydrous acid prepared two years earlier was a compound of hydrogen with an unknown element, analogous to chlorine, for which he suggested the name fluorine. After concentrating on mathematics as he sought admission to the Institut, Ampère returned to chemistry after his election in 1814 and produced a classification of elements in 1816.

Ampère also worked on the theory of light, publishing on refraction of light in 1815. By 1816 he was a strong advocate of a wave theory of light, agreeing with Fresnel and opposed to Biot and Laplace who advocated a corpuscular theory. Fresnel became a good friend of Ampère's and lodged at Ampère's home from 1822 until his death in 1827.

In the early 1820s, Ampère attempted to give a combined theory of electricity and magnetism after hearing about experimental results by the Danish physicist Hans Christian Orsted. Ampère formulated a circuit force law and treated magnetism by postulating small closed circuits inside the magnetised substance.

It is worth commenting on how quickly Ampère produced this theory, the inspiration striking him immediately he heard of Orsted's experimental results. Orsted's work was reported the Academy in Paris on 4 September 1820 by Arago and a week later Arago repeated Orsted's experiment at an Academy meeting. Ampère demonstrated various magnetic / electrical effects to the Academy over the next weeks and he had discovered electrodynamical forces between linear wires before the end of September. He spoke on his law of addition of electrodynamical forces at the Academy on 6 November 1820 and on the symmetry principle in the following month. Ampère wrote up the work he had described to the Academy with remarkable speed and it was published in the *Annales de Chimie et de Physique*.

Ampère was assisted over the next few years in his work by Felix Savary whose help in getting Ampère to write up his results was invaluable [4]:-

> *... beginning with the memoir he completed early in* 1823, Savary *now made much more creative contributions. But more than his creativity, it was* Savary*'s discipline and ability to concentrate at length on specific problems that proved especially valuable to Ampère. There is room to speculate that, without* Savary*'s aid, Ampère might never have found time to complete the detailed calculations required to apply his force law to magnetic phenomena.*

However Ampère was not the only one to react quickly to Arago's report of Orsted's experiment. Biot, with his assistant Savart, also quickly conducted experiments and reported to the Academy in October 1820. This led to the Biot-Savart Law. Another who worked on magnetism at this time was Poisson who insisted on treating magnetism without any reference to electricity. Poisson had already written two important memoirs on electricity and he published two on magnetism in 1826.

Ampère's most important publication on electricity and magnetism was also published in 1826. It is called *Memoir on the Mathematical Theory of Electrodynamic Phenomena, Uniquely Deduced from Experience* and contained a mathematical derivation of the electrodynamic force law and describes four experiments. Maxwell, writing about this Memoir in 1879, says:-

> *We can scarcely believe that Ampère really discovered the law of action by means of the experiments which he describes. We are led to suspect, what, indeed, he tells us himself, that he discovered the law by some process which he has not shown us, and that when he had afterwards built up a perfect demonstration he removed all traces of the scaffolding by which he had raised it.*

Ampère's theory became fundamental for 19th century developments in electricity and magnetism. Faraday discovered electromagnetic induction in 1831 and, after initially believing that he had himself

Fortsetzung folgt ...