

I. Exponential fitting of correlation functions

Based on real-world Monte-Carlo correlation functions along $N_\tau=32$ steps in Euclidean time

$$D(\tau_k) = \frac{1}{N_{meas}} \sum_j^{N_{meas}} D^j(\tau_k)$$

from a study of heavy quark-antiquark bound states, carry out the determination of the ground state energy and a reliable error estimation. Averages are found in CorrelatorAvgAlongTau.dat, individual measurements in the ./AnalysisData subfolder.

- a. To check the quality of the Monte-Carlo estimates, determine whether the correlation functions remain correlated over Monte Carlo time t . Since this statement is specific for an individual observable, check it for $D(1)$, $D(16)$ and $D(32)$. (Data is prepared in CorrelatorTauXXMC.dat)

Since the formula for $\tilde{A}(t)$ via FFT shown in the lecture operates on observables, which had its mean subtracted we have:

$$G(\omega) = \text{FFT}[f(t) - \langle f \rangle]$$

$$\tilde{A}(t) = \text{IFFT}[|G(\omega)|^2]$$

(Use the fftw library to carry out the required Fourier transform.)

- b. Using the Lapack and GSL library, write a program that carries out a Jackknifed multi exponential fit on the correlator data provided. To this end write down the derivative of the likelihood function with respect to the fit parameters.

Since we are dealing with a symmetric real matrix we can use the appropriate optimized Eigenvalue/Eigenvector routines of the Lapack library.
<http://www.netlib.org/lapack/double/dsyev.f>

The minimization for the likelihood can be implemented using a numerical minimizer of the GSL library that only needs the function value and its derivative: `gsl_multimin_fdfminimizer_vector_bfgs2`

As an example (using a minimizer which also requires the second derivative, unnecessary in our case) you can have a look at
http://www.gnu.org/software/gsl/manual/html_node/Multimin-Examples.html#Multimin-Examples

- c. How many exponentials do you need to reliably determine the ground state mass? Can you make a statement about the first excited state?