I. Simulating SU(3) gauge fields on the lattice

Since there does not exist an equivalently simple parametrization of SU(3) as we had for SU(2) we need to operate on full 3x3 complex matrices.

a. Write a new data structure that can act as SU(3) matrix. Implement matrix multiplication, the conjugate transpose and the trace for these matrices. (You can simply use for loops in this exercise, in production code, one would hard-code these operations)

b. The Metropolis proposal matrix can now be written as product of three individual proposal matrices, each of which affects only one of the 2x2 subblocks of the original link

$$U' = TSRU,\quad R = \begin{pmatrix} r_{11} & r_{12} & 0 \\ r_{21} & r_{22} & 0 \\ 0 & 0 & 1 \end{pmatrix}, S = \begin{pmatrix} s_{11} & 0 & s_{12} \\ 0 & 1 & 0 \\ s_{21} & 0 & s_{22} \end{pmatrix}, T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & t_{11} & t_{12} \\ 0 & t_{21} & t_{22} \end{pmatrix}$$

Use the proposal function from last week to generate R,S and T. For the Markov Chain to fulfill detailed balance we need to add an additional random choice on whether to act with TSR from the left or from the right, i.e. whether to use TSR or $(TSR)^\dagger$

c. Check if the links are affected by rounding errors to deviate from SU(3)

d. (Extra credit) Write a subroutine that calculates the Wilson Loop and plot its dependence on Euclidean time for different separation distances.