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Quantum fields on the lattice: Excercise sheet Nr. 9 — Sommersemester 2015

Complex Langevin

We consider a toy model of lattice QCD with nonzero chemical potential given by the partition function

$$Z = \int_0^{2\pi} \frac{dx}{2\pi} e^{\beta \cos x} (1 + \kappa \cos(x - i\mu)) \tag{1}$$

Consider the observables e^{ix}, e^{-ix} and the density $\partial_\mu \ln Z$. Calculate the exact averages using numerical integration in the interval $[0, 2\pi]$.
Set up the complex Langevin dynamics for the complexified variable $x + iy$ using a drift term calculated from the effective action $S_{eff} = -\beta \cos x - \ln(1 + \kappa \cos(x - i\mu))$.
The discretised Langevin equation reads:

$$z(\tau + \Delta\tau) = - \left. \partial_x S(x) \right|_{x \to z} \Delta\tau + \eta(\tau)\sqrt{2\Delta\tau}, \tag{2}$$

where η is a Gaussian random variable with unit mean square. To avoid runaways, one tipycally uses $\Delta\tau = 10^{-4}..10^{-5}$. Allow some thermalization until $\tau = 10$ or so before taking averages.
Use the parameters $\beta = 1$ and $\kappa = 1/2$, and look at the observables for several different μ values, and compare with the exact values.
Look at the scatter plot of $x + iy$ on the complex plane at $\mu = 0.1, 0.5$, and 1.0.