On the role of the quark-gluon vertex in confinement and chiral symmetry breaking

Or: How to climb Olympus Mons?

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Outline

1. How are confinement, DχSB, and the U$_A$(1) anomaly related?

2. Coupling quarks to gluons
   - Quark propagator and quark-gluon vertex
   - Dynamically generated quark confinement
   - η' mass from infrared divergent Green functions
   - The effect of an IR divergent quark-antiquark interaction kernel

3. Conclusions and Outlook

How are confinement, DχSB, and U$_A$(1) related?

"Traditional" conjecture:

- Confinement induces DχSB:
  - Heuristic argument –> momentum but not spin flip at "bag wall".
  - Infrared slavery –> supercritical coupling.

However:

- DχSB without confinement
  - Explicit examples.
  - Structure of Dirac operator.

Talk by Szabolcz Borsanyi:

- Evidence for $T_\chi = T_{\rm dec}$.

Vafa & Witten, NPB234 (1984):

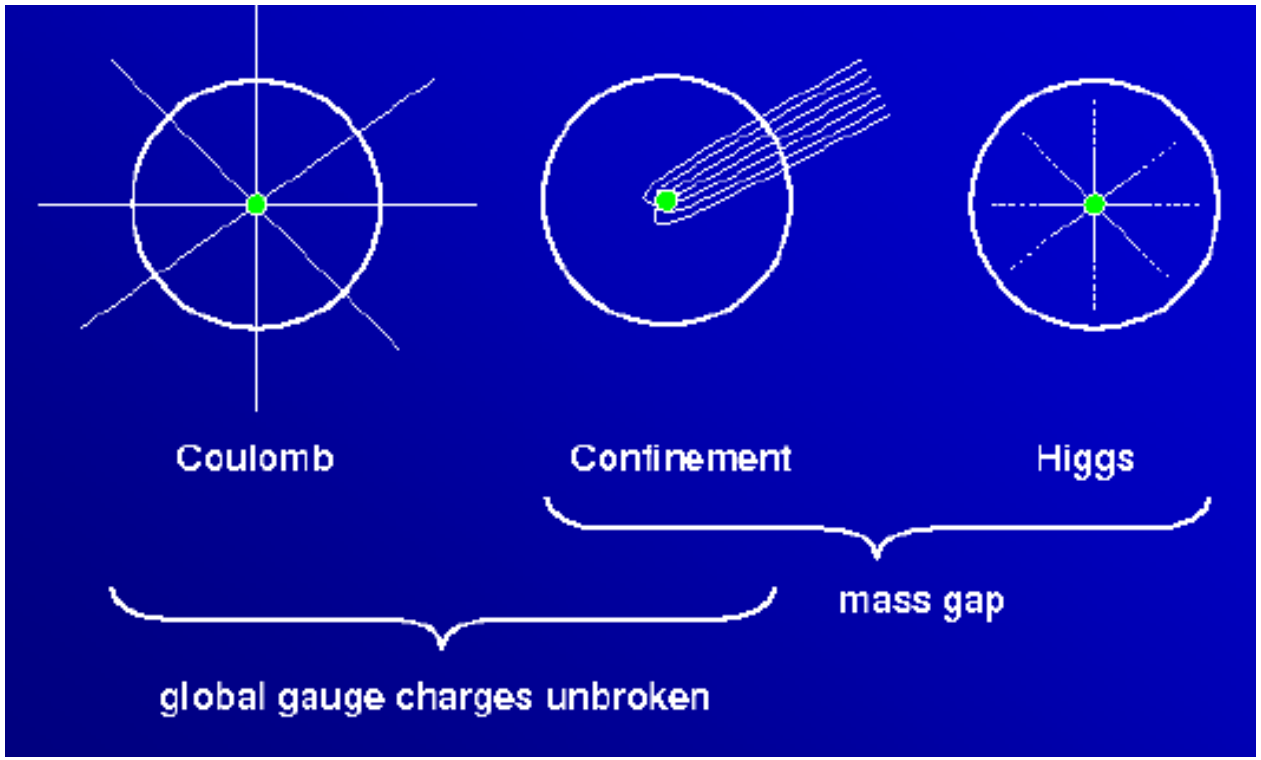
- U$_A$(1) always anomalous in vector-like gauge theories with $\Theta = 0$

These arguments apply at zero T and μ!

How are confinement, DχSB, and U$_A$(1) related?

Wilson loop gives only a clear criterion in the absence of quarks!

A possible definition of *confinement* in the presence of fundamental charges:



How are confinement, DχSB, and U$_A$(1) related?

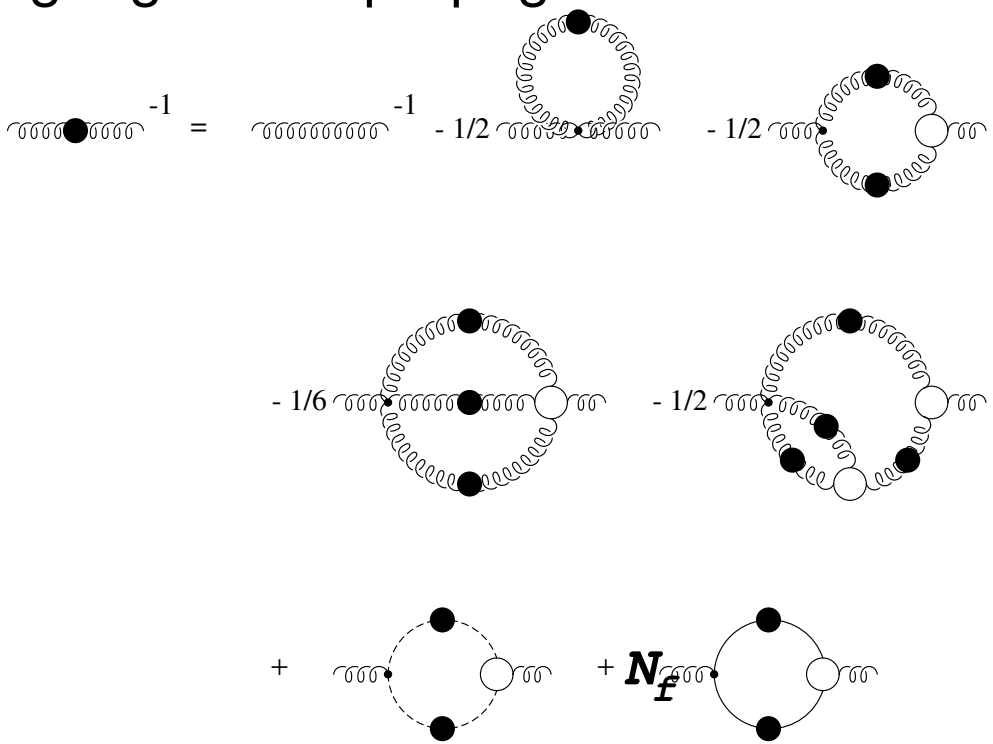
DχSB and U$_A$(1) anomaly

imply

- the existence of topologically non-trivial gluon field configurations and quark would-be zero modes,
  i.e., properties of Dirac operator wanted.
- the existence of a supercritical coupling and infrared divergencies,
  i.e., properties of quark-gluon coupling wanted.
- the dynamical generation of Lorentz-scalar couplings and "constituent" quark masses
  i.e., properties of quark propagator wanted.

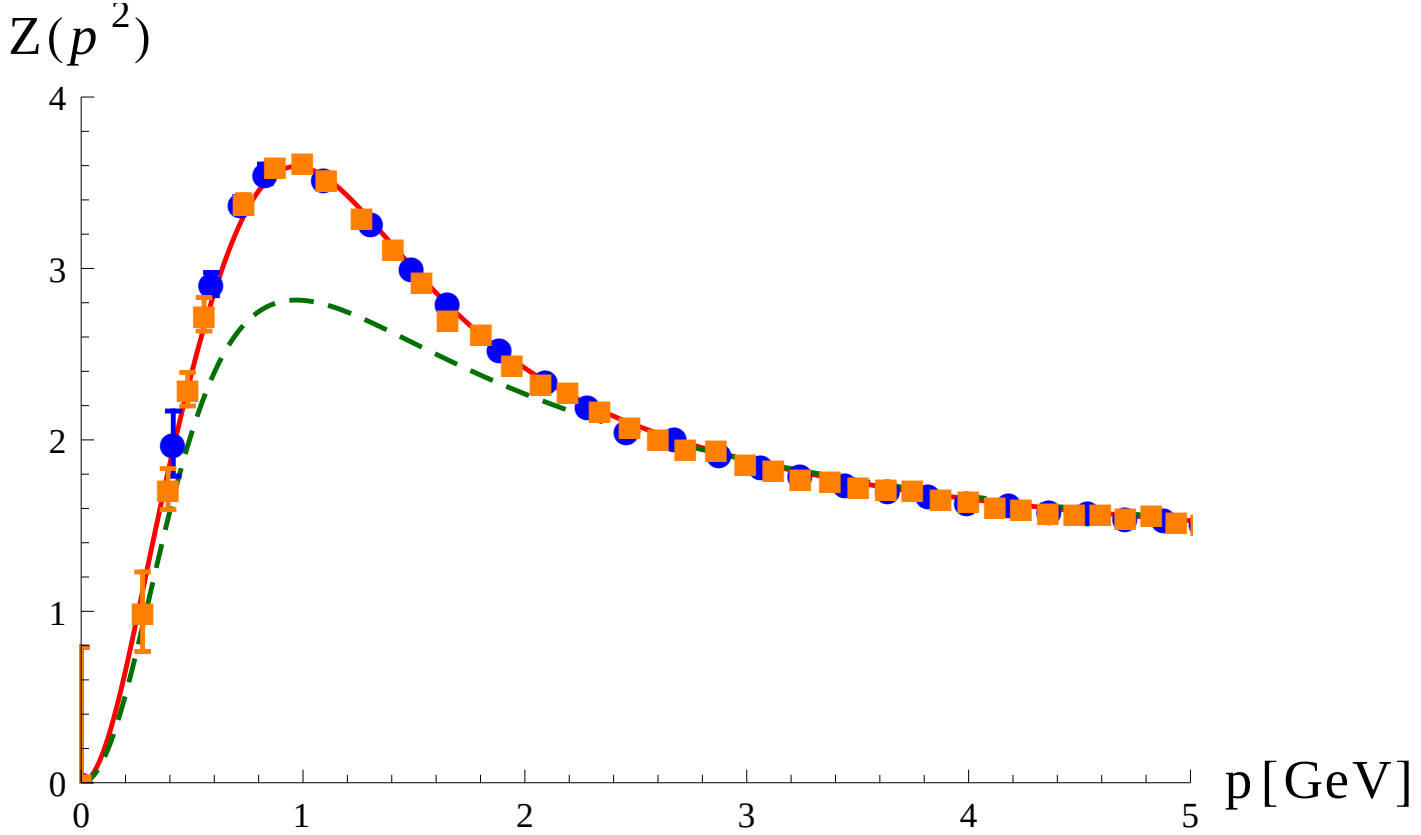
Coupling quarks to gluons

DSEs for Landau gauge QCD propagators:



Coupling quarks to gluons

Gluon propagator dressing function $Z(p^2)$ with a dynamic ghost-gluon and a model three-gluon vertex (*cf.* lattice [A. Maas], contains a zero!):



red curve: M.Q. Huber and L. von Smekal, arXiv:1211.6092
green dashed curve: C.S. Fischer and R.A., Phys. Lett. **B536** (2002) 177
lattice: A. Sternbeck, PhD thesis 2006

Coupling quarks to gluons

R.A., C.S. Fischer, F. Llanes-Estrada, K. Schwenzer, Annals Phys. **324** (2009) 106;
R.A., M. Hopfer, A. Windisch, in preparation.

Chiral symmetry dynamically or explicitely broken:

- quark propagator infrared finite:

$$S(p) = \frac{\not{p} - M(p^2)}{p^2 + M^2(p^2)} Z_f(p^2) \to \frac{Z_f \not{p}}{M^2} + \frac{Z_f}{M}$$

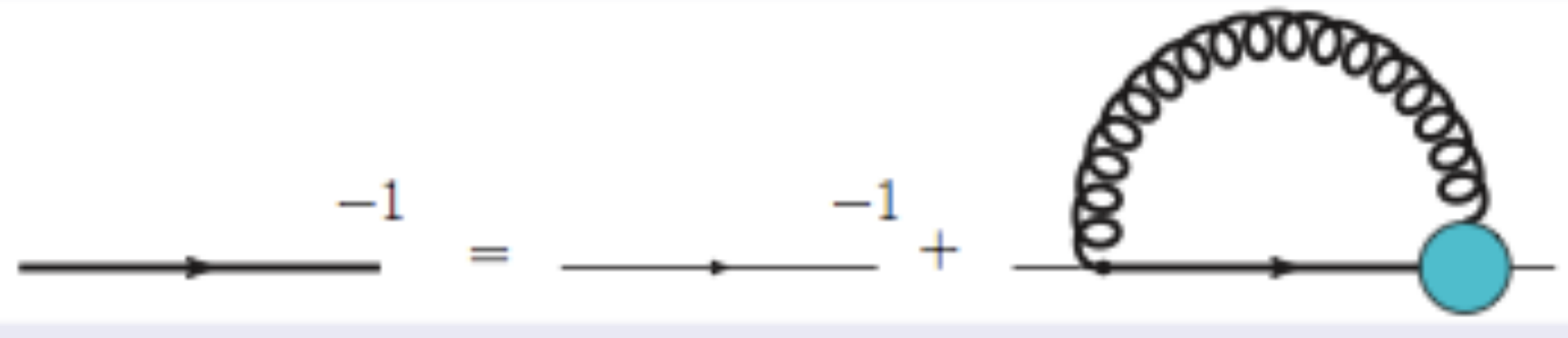
AND

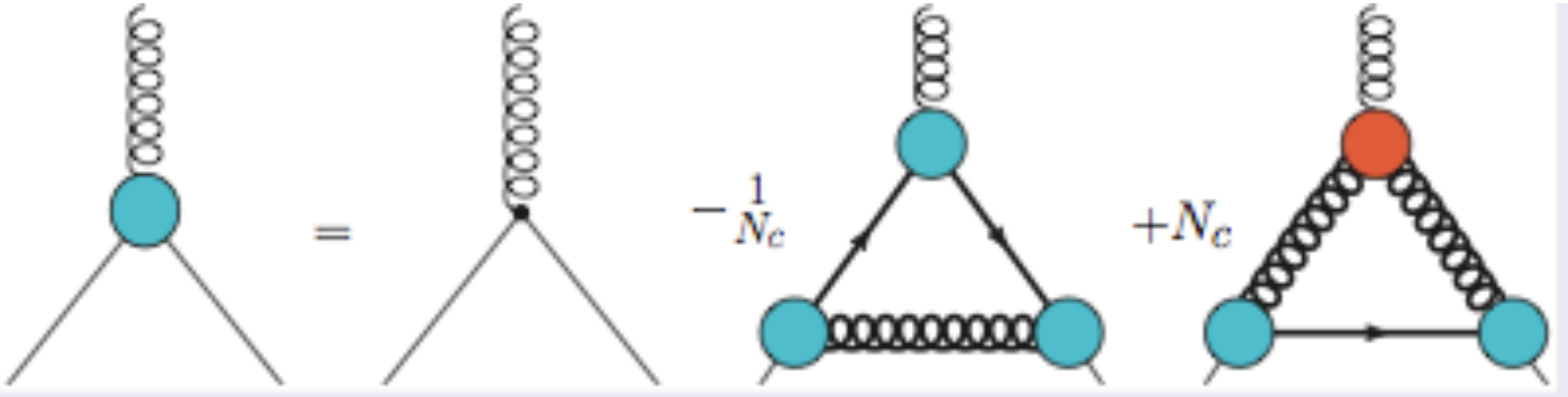
- quark-gluon vertex
  incl. dynamically generated χSB tensors structures,
  infrared enhanced:

$$\Gamma_\mu = ig \sum_{i=1}^{12} \lambda_i G_\mu^i \,, \quad G_\mu^1 = \gamma_\mu \,, \quad G_\mu^2 = \hat{p}_\mu \,, \quad G_\mu^3 = \ldots$$

Coupling quarks to gluons

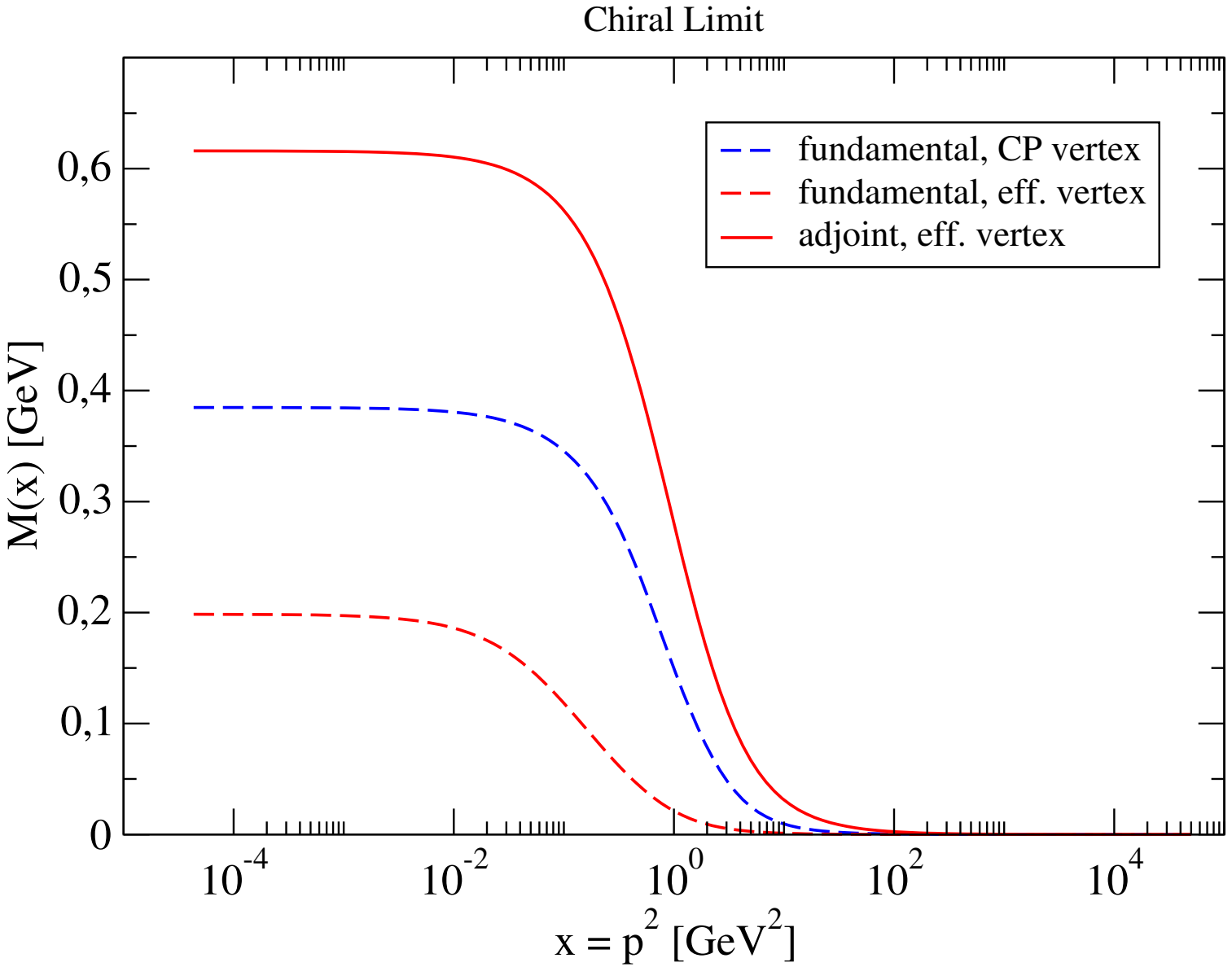
DSEs for quark propagator and quark-gluon vertex:





Coupling quarks to gluons

Quark mass function with models for QGV:



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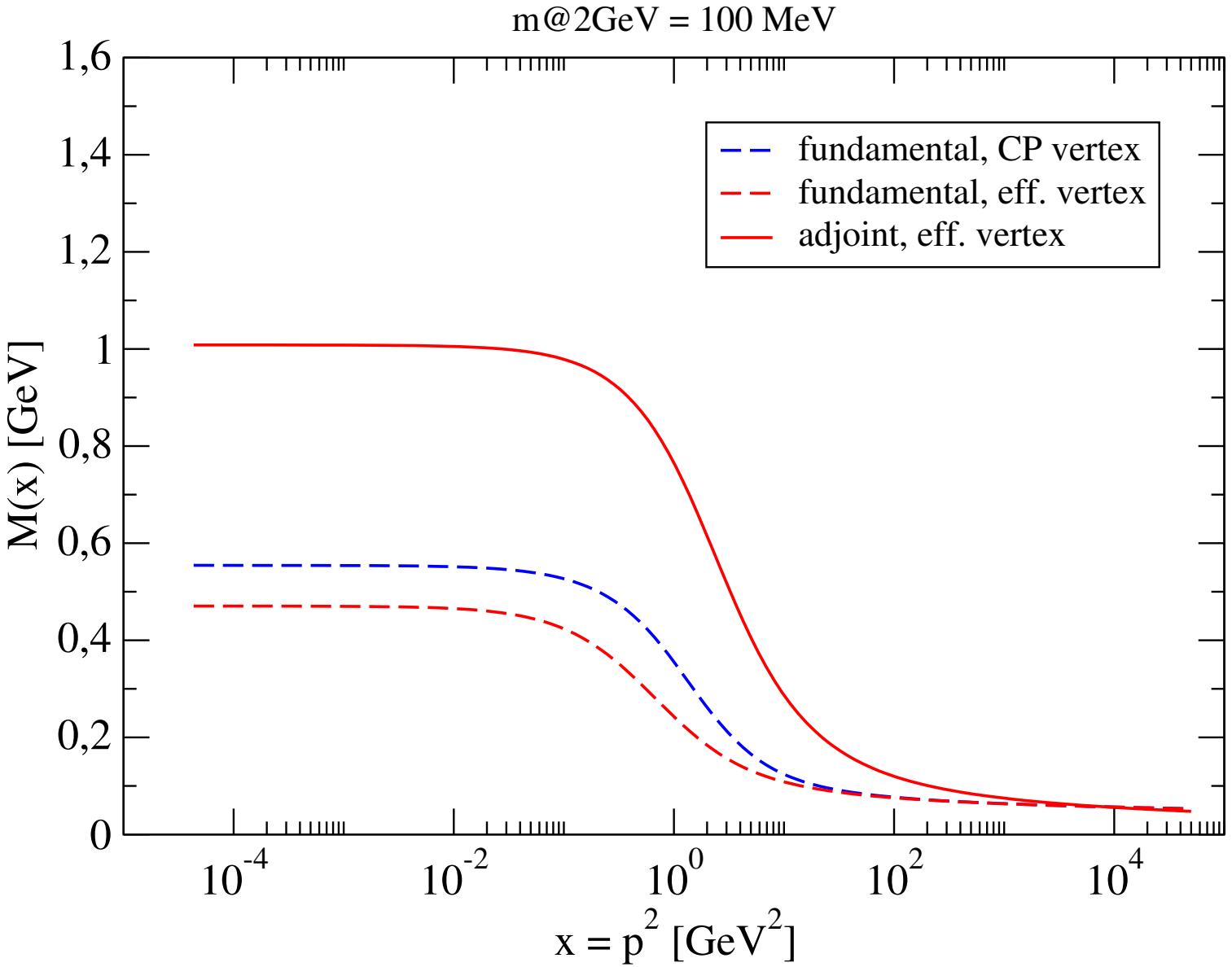
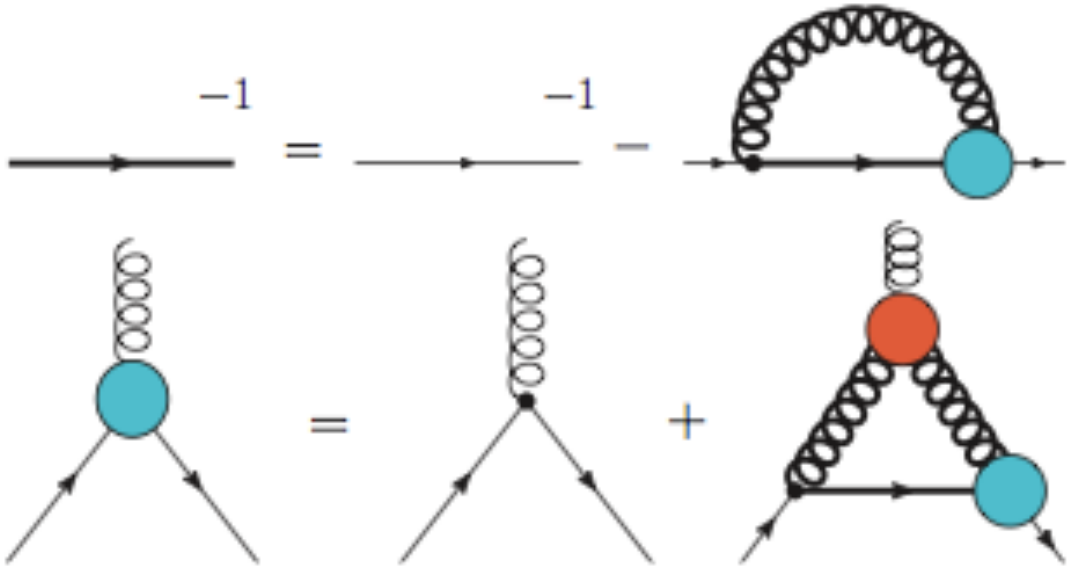


Coupling quarks to gluons

Solving for the quark-gluon vertex:
Preliminary results for a simplified system

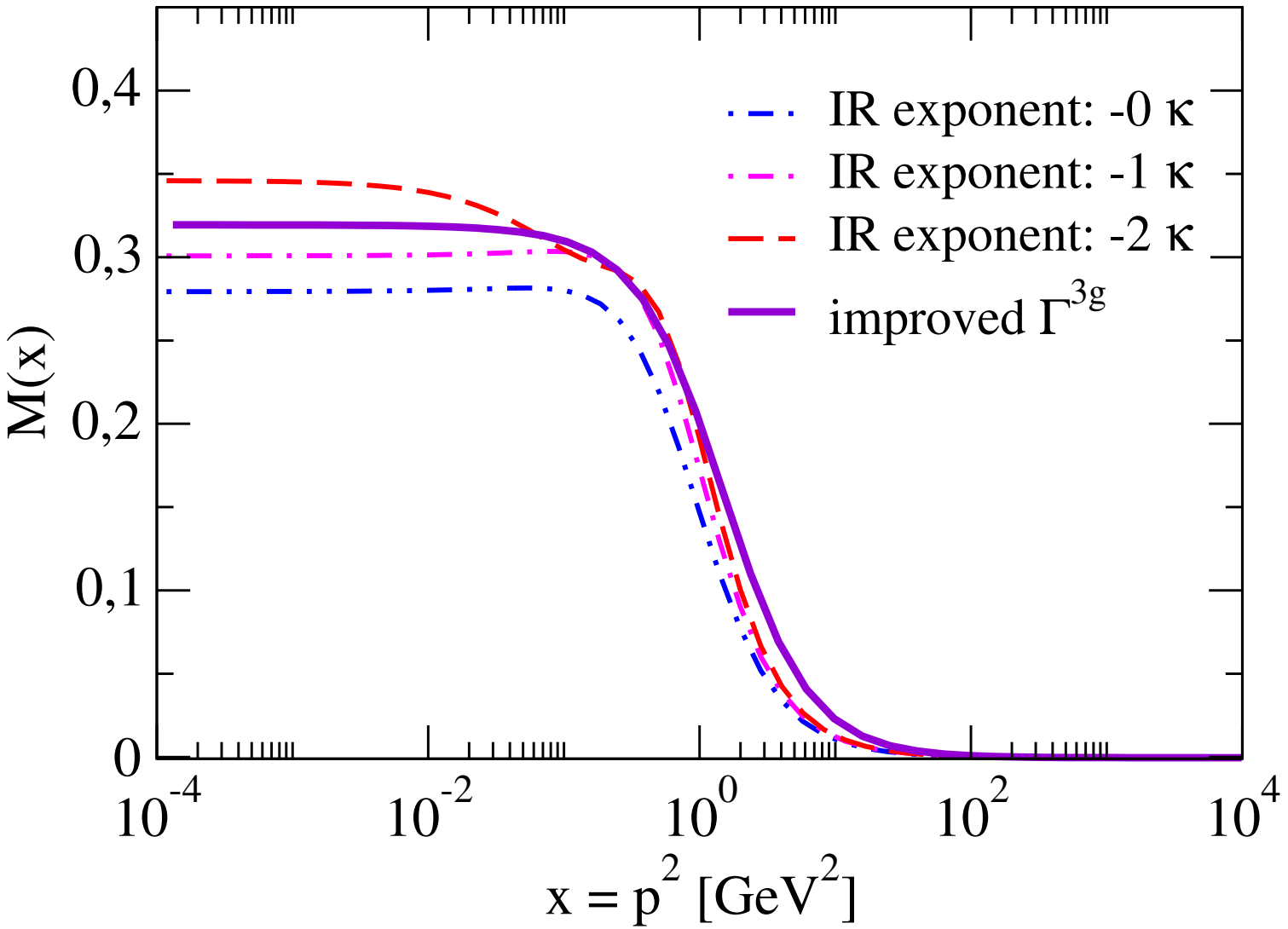
- self-consistent solution of the quark-gluon vertex DSE in a truncation including all 12 tensor structures



  - scaling-type gluon propagator
  - improved model for three-gluon vertex (M.Q. Huber and L. von Smekal)

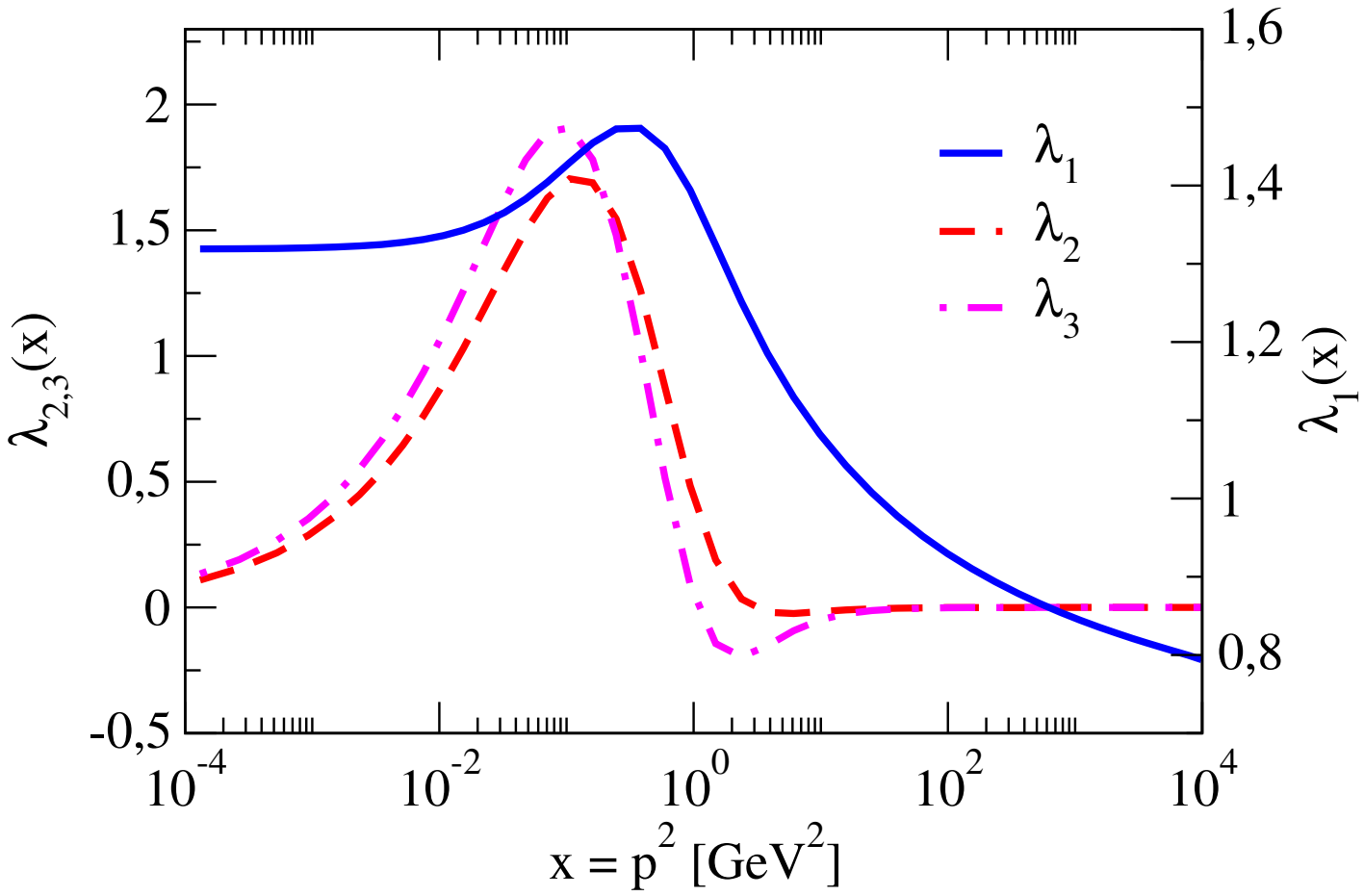
Coupling quarks to gluons

Quark mass function with calculated QGV:



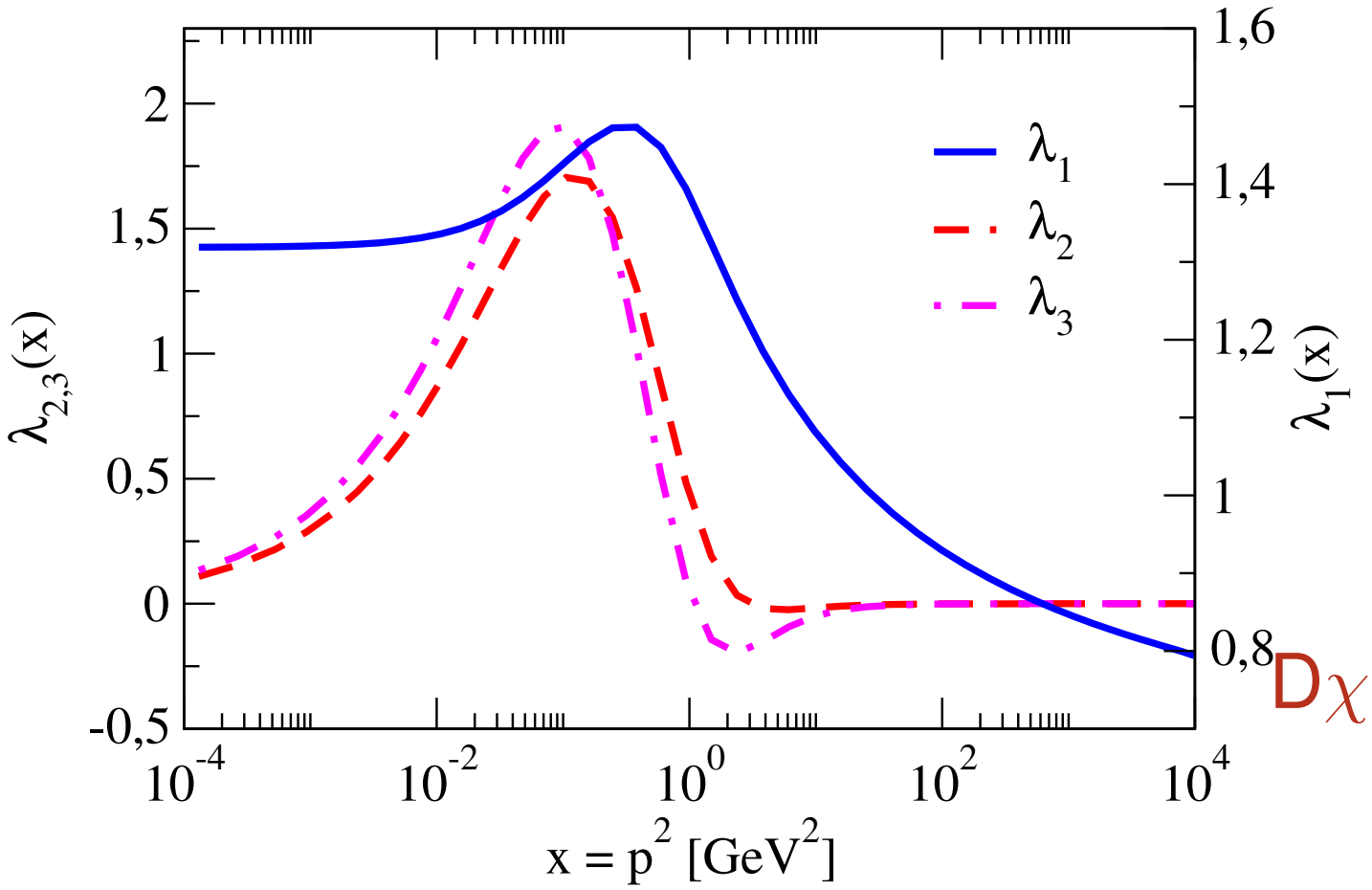
Coupling quarks to gluons

Leading tensor structure λ_1, calculated QGV,
symm. momenta $x = p_1^2 = p_2^2 = p_3^2$: Significant IR enhancement!



Coupling quarks to gluons

Subleading DχSB tensor structures $\lambda_2, \ldots,$ calculated QGV:



DχSB in QGV!!!

Coupling quarks to gluons

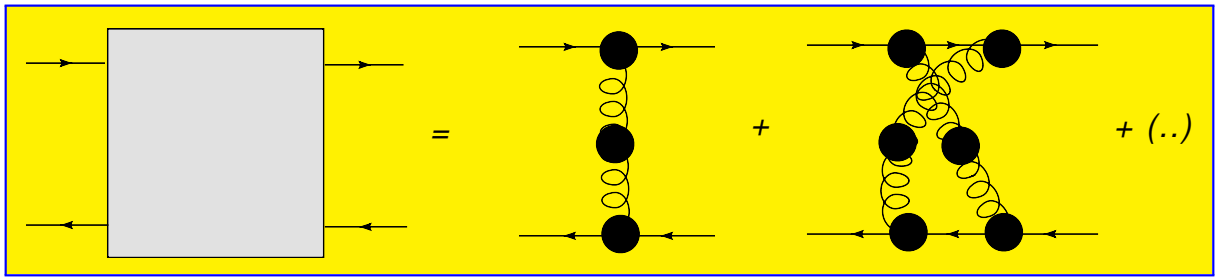
- shown preliminary calculation:
  few hours on GPU cluster
- to be done:
  reformulate with transverse parts only and solve full system

Expected from infrared analysis within scaling solution:

- infrared divergence $\lambda_{1,2,\ldots} \sim (p^2)^{-1/2-\kappa}$
  R.A., C. Fischer, F. Llanes-Estrada, K. Schwenzer, Ann. Phys. **324** (2009) 106.
- quark confinement due to infrared slavery
- self-consistently generated scalar conining quark-gluon interaction

Dynamically induced scalar quark confinement

Scaling solution:
"Quenched´´ quark-antiquark potential



infrared divergent such that

$$V(\mathbf{r}) = \int \frac{d^3p}{(2\pi)^3} H(p^0 = 0, \mathbf{p}) e^{i\mathbf{pr}} \quad \sim \quad |\mathbf{r}|$$

i.e. linear, vector+scalar, quark confinement!

η' mass from IR divergent Green functions

R.A., C. S. Fischer, R. Williams, Eur. Phys. J. A **38** (2008) 53.

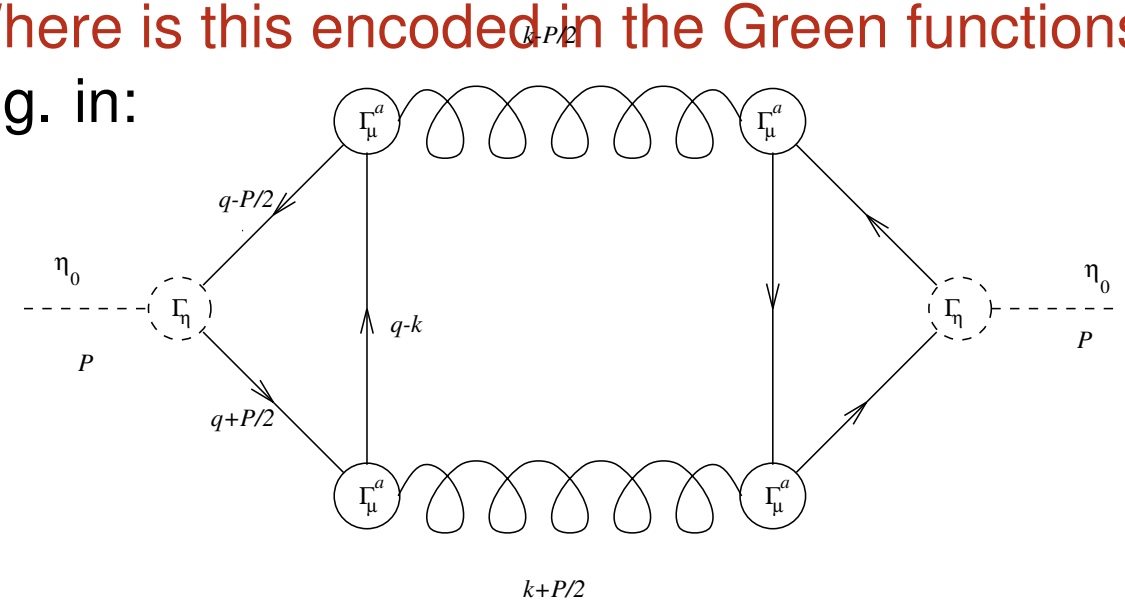
$U_A(1)$ symmetry anomalous $\Rightarrow$ η' mass $\gg$ π mass

QCD vacuum: winding number spots as, *e.g.*, instantons, couple

to chiral quark zero modes $\Rightarrow$ $U_A(1)$ symmetry broken!

Where is this encoded in the Green functions?

E.g. in:



$$\Gamma_\mu D^{\mu\nu} \Gamma_\nu \propto 1/k^4$$

J. B. Kogut and L. Susskind, Phys. Rev. D **10** (1974) 3468.

η' mass from IR divergent Green functions

However: Infinitely many diagrams (*n*-gluon exchange) contribute!

diamond diagram only
with DSE results for gluon & quark propagators & quark-gluon vertex
(provides correct pseudoscalar and vector meson masses):

$$\chi^2 \approx (160\mathrm{MeV})^4 \text{ vs. phenomenological value } (180\mathrm{MeV})^4$$

results in: $m_\eta = 479\mathrm{MeV}$, $m_{\eta'} = 906\mathrm{MeV}$, $\theta = -23^0$.

Conclusion:
(Fluct.) topologically non-trivial fields $\Leftrightarrow$ IR singularities of GF!
. . . another view to generate the Witten-Venezanio mechanism . . .

Confinement, DχSB, and U$_A$(1)

Enforce chiral symmetry in Wigner-Weyl mode:

$$S(p) = \left(\frac{\not{p} + M(p^2)}{p^2 + M^2(p^2)} Z_f(p^2) \right)_{M \to 0} \to \frac{Z_f \not{p}}{p^2}$$

AND

$$\Gamma_\mu = ig \sum_{i=1}^{12} \lambda_i G^i_\mu \,, \quad G^1_\mu = \gamma_\mu \,, \quad G^2_\mu = \hat{p}_\mu \,, \quad \ldots$$

WITH $\lambda_{1,3,\ldots} \sim (p^2)^{-\kappa}$ **and** $\lambda_{2,4,\ldots} = 0$**.**

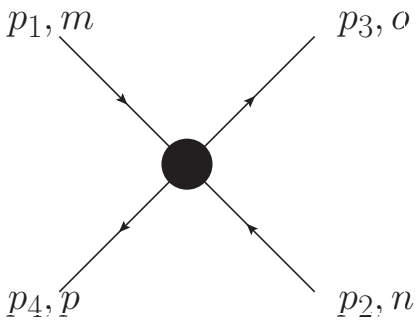
Quark-antiquark potential: No confinement

At vanishing T and μ: **Confinement $\Leftrightarrow$ DχSB**
Confinement $\Leftrightarrow$ U$_A$(1)

Assuming an IR divergent 4-point function

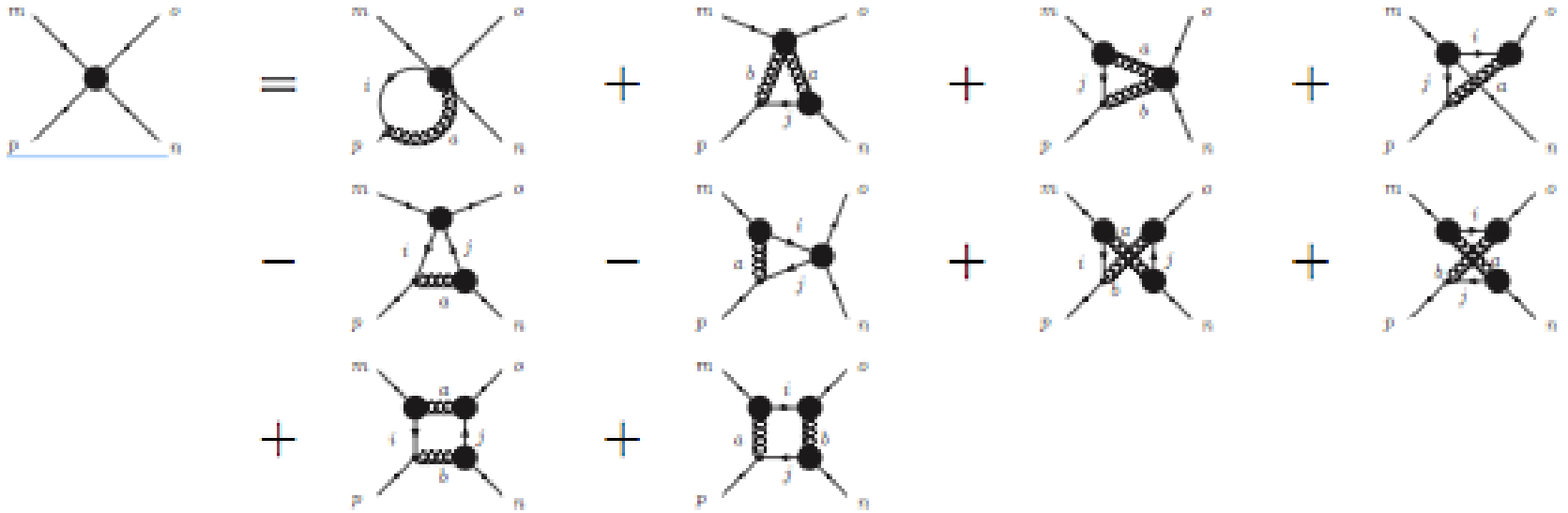
M. Mitter, RA, in preparation

- Assume quark 4-point function to be maximally IR singular, *i.e.*, $\propto \mathbf{1}/\mathbf{k}^4$:



$$\underset{\propto}{p_1 \to p_3} \left. \frac{1}{(p_1 - p_3)^4} \right|_{reg.}$$

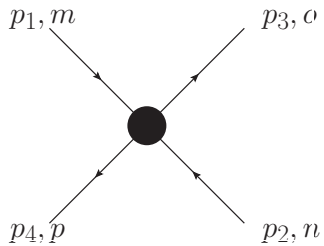
- Put *e.g.* DSE for 4-quark function:



Consequences an IR divergent 4-point function

- For simplicity: Analysis first for fundamentally charged scalar!
- Consistency requirements:

  ☺ Boundedness of higher n-point functions to $1/k^4 \Longrightarrow$ matter-gluon vertex less singular $\Longrightarrow$ **colour structure**



$$\overset{p_1 \to p_3}{\propto} \left. \frac{\delta_{mo}\delta_{np}}{(p_1 - p_3)^4} \right|_{reg.}$$

  ☺ One-gluon exchange fails to reproduce this colour structure!

  ☺ All 4-point functions (4-gluon, ghost-gluon, matter-gluon, matter-ghost) inherit the $1/k^4$ singularity in specific colour channels.

  ☺ Higher n-point functions contain contributions $\propto 1/k^4$ with k being the momentum transfer between two coloured clusters.

  ☺ Propagators and 3-point functions protected by cancellations.

- Decoupling theorem circumvented by IR singularities:
  One heavy fundamental charge induce changes in the IR behaviour of YM Green's functions!?!

Consequences an IR divergent 4-point function

- Assumption of confining IR singularity in matter-matter scattering kernel leads to several wanted features.
- Especially Casimir scaling!
- No decoupling of infinitely heavy charges?
- Further to be clarified:
  - Absence of van-der-Waals forces?
  - *N*-ality?
  - Relation to dynamical chiral symmetry breaking / restoration?
  - ...

Conclusions and Outlook

Landau gauge QCD Green functions:

- ☺ Coupled system of quark propagator and quark-gluon vertex DSEs under investigation.
- ☺ Chiral symmetry dynamically broken! In 2- and **3-**point function!
- ☺ Quark/matter confinement: Analysis of IR divergencies!
  Color structure, Casimir scaling, no decoupling of heavy d.o.f., ...

- ☺ Quark/matter-gluon vertex:
  - quark/matter confinement, DχSB, $U_A(1)$

- ☺ Find simplifying features of quark-gluon vertex at $T = 0$
- ☺ Calculate / model at $T \neq 0$ and $\mu \neq 0$
  - phase transition at $T \neq 0$
  - finite density, color-superconducting phase(s)

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