Supersymmetry on the lattice and simulations of supersymmetric Yang-Mills theory

Georg Bergner
ITP GU Frankfurt

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

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In collaboration with I. Montvay, G. Münster, U. D. Özugurel, S. Piemonte, D. Sandbrink

Supersymmetry

$$\left\{ Q_i \, , \, \bar{Q}_j \right\} = 2\delta_{ij}\gamma^{\mu} P_{\mu}$$

- fermions $\overset{\text{SUSY}}{\rightleftharpoons}$ bosons
- degeneracy of bosonic and fermonic states; mass degeneracy
- only non-trivial interplay between internal symmetries and space-time symmetry
- spontaneous SUSY breking only if Witten index $\Delta = n_B^{E=0} - n_F^{E=0}$ is zero
- Lebniz rule essential: $\partial(fg) = (\partial f)g + f(\partial g)$

The lattice

- discretize, cutoff in momentum space
- (controlled) breaking of space-time symmetries
  ⇒ uncontrolled SUSY breaking
- derivative operators replaced by difference operators with no Leibniz rule
  ⇒ breaks SUSY
- fermonic doubling problem, Wilson mass term
  ⇒ breaks SUSY
- gauge fields represented as link variables
  ⇒ different for fermions and bosons
- Nielsen-Ninomiya theorem:
  No-Go for (naive) lattice chiral symmetry

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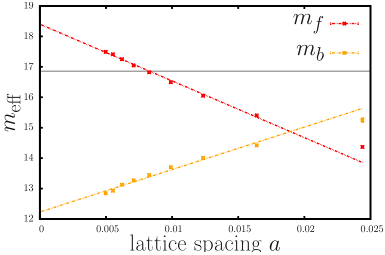
No-Go theorem for lattice supersymmetry

- No way to realize (naive) supersymmetry on the lattice!
- like Nielsen-Ninomiya theorem[1]:
  locality contradicts with SUSY
- general No-Go even without Wilson mass or gauge fields !

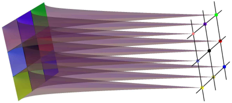


[1] [Kato, Sakamoto, So, JHEP 0805 (2008)], [GB, JHEP 1001 (2010)]

Ginsparg-Wilson relation

- solution for chiral symmetry: Ginsparg-Wilson relation
- modified symmetry relation: $\bar{\gamma}_{5,\mathrm{def}}\mathcal{D} + \mathcal{D}\gamma_{5,\mathrm{def}} = 0$
  replaces naive symmetry: $\{\gamma_5, \mathcal{D}\} = 0$
- resembles relevant properties of the symmetry on the lattice
- derivation based on a renormalization group step
  (integrating out the continuum degrees of freedom)

$$e^{-S_L[\phi]} = \int d\varphi\, e^{-R[\varphi,\phi]-S[\varphi]}$$



Generalized Ginsparg-Wilson relation[1]

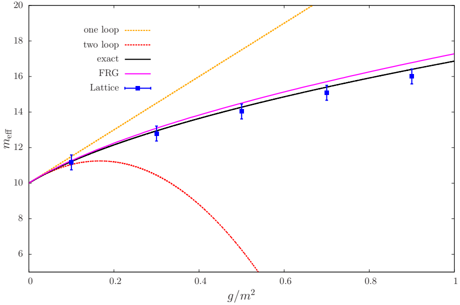
$$M^{ij}_{nm}\phi^j_m \frac{\delta S_L}{\delta \phi^i_n} = (M\alpha^{-1})^{ij}_{nm}\left(\frac{\delta S}{\delta \phi^j_m}\frac{\delta S_L}{\delta \phi^i_n} - \frac{\delta^2 S_L}{\delta \phi^j_m \delta \phi^i_n}\right)$$

- naive lattice symmetry generator M
- deformed regulator dependent rhs.
  $R[\varphi,\phi] = \frac{1}{2}(\phi - \Phi[\varphi])\alpha(\phi - \Phi[\varphi])$
- general relation for any symmetry,
  but space-time symmetry and SUSY introduces non-locality
- non-polynomial solutions

[1] [GB, Bruckmann, Pawlowski, Phys. Rev. D 79 (2009)], [GB, Bruckmann, Echigo, Igarashi, Pawlowski, Schierenberg, arXiv:1212.0219]

Lattice supersymmetry

- no general solution for Lattice SUSY; only model dependent solutions
- problem solved[1] in low dimensions
- FRG alternative non-perturbative method[2]



[1] [Golterman,Petcher Nucl. Phys. B319 (1989)], [Catterall, Gregory, Phys. Lett. B 487 (2000)], [Giedt, Koniuk, Poppitz, Yavin, JHEP 0412 (2004)], [G.B, Kästner, Uhlmann, Wipf, Annals Phys. 323 (2008)], [Baumgartner, Wenger, PoS LATTICE 2011],. . .

[2] [Synatschke, GB, Gies, Wipf, JHEP03 (2009)]

Super Yang-Mills theory

Supersymmetric Yang-Mills theory:

$$\mathcal{L} = \mathrm{Tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\bar{\psi}\not{D}\psi - \frac{m_g}{2}\bar{\psi}\psi\right]$$

- supersymmetric counterpart of Yang-Mills theory; but in several respects similar to QCD
- ψ Majorana fermion in the adjoint representation

Supersymmetric Yang-Mills theory: Symmetries

SUSY

- gluino mass term m_g $\Rightarrow$ soft SUSY breaking

$U_R(1)$ symmetry, "chiral symmetry": $\psi \rightarrow e^{-i\theta\gamma_5}\psi$

- $U_R(1)$ anomaly: $\theta = \frac{k\pi}{N_c}$, $U_R(1) \rightarrow \mathbb{Z}_{2N_c}$
- $U_R(1)$ spontaneous breaking: $\mathbb{Z}_{2N_c} \stackrel{\langle\bar{\psi}\psi\rangle\neq 0}{\rightarrow} \mathbb{Z}_2$

Supersymmetric Yang-Mills theory: effective actions

symmetries + confinement → low energy effective theory

- exact value of $\langle\bar{\psi}\psi\rangle$
- exact beta function
- low energy effective actions:
  1. multiplet[1]:
  mesons : $a - f_0$ and $a - \eta'$
  fermionic gluino-glue
- 2. multiplet[2]:
  glueballs: 0^{++} and 0^{-+}
  fermionic gluino-glue

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[1] [Veneziano, Yankielowicz, Phys.Lett.B113 (1982)]
[2] [Farrar, Gabadadze, Schwetz, Phys.Rev. D58 (1998)]

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Supersymmetry

All particles of a multiplet must have the same mass.

[1] [Veneziano, Yankielowicz, Phys.Lett.B113 (1982)]

[2] [Farrar, Gabadadze, Schwetz, Phys.Rev. D58 (1998)]

Why study supersymmetric Yang-Mills theory on the lattice ?

1. extension of the standard model
   - gauge part of SUSY models
   - non-perturbative sector important: check effective actions etc.
2. extension of the standard model
   - SUSY: adjoint - $1/2$ - flavor - QCD
   - technicolor: adjoint - 1 - flavor - QCD
3. Connection to QCD
   - orientifold planar equivalence: SYM $\leftrightarrow$ QCD
   - Remnants of SYM in QCD ?
   - comparison with one flavor QCD

Supersymmetric Yang-Mills theory on the lattice

Lattice action:

$$\mathcal{S}_L = \beta \sum_P \left(1 - \frac{1}{N_c} \Re U_P\right) + \frac{1}{2} \sum_{xy} \bar{\psi}_x \left(\mathrm{D}_w(m_g)\right)_{xy} \psi_y$$

- Wilson fermions:

$$\mathrm{D}_w = 1 - \kappa \sum_{\mu=1}^{4} \left[(1-\gamma_\mu)_{\alpha,\beta} T_\mu + (1+\gamma_\mu)_{\alpha,\beta} T_\mu^\dagger\right]$$

gauge invariant transport: $T_\mu \psi(x) = V_\mu \psi(x+\hat{\mu})$;

$$\kappa = \frac{1}{2(m_g+4)}$$

- links in adjoint representation: $(V_\mu)_{ab} = 2\mathrm{Tr}[U_\mu^\dagger T^a U_\mu T^b]$

Lattice SYM: Symmetries

Wilson fermions:

- explicit breaking of symmetries: ~~chiral Sym. ($U_R(1)$)~~, ~~SUSY~~

fine tuning:

- add counterterms to action
- tune coefficients to obtain signal of restored symmetry

special case of SYM:

- tuning of m_g enough to recover chiral symmetry [1]
- same tuning enough to recover supersymmetry [2]

[1] [Bochicchio et al., Nucl.Phys.B262 (1985)]

[2] [Veneziano, Curci, Nucl.Phys.B292 (1987)]

Recovering symmetry

Fine-tuning:

chiral limit = SUSY limit $+O(a)$, obtained at critical $\kappa(m_g)$

- no fine tuning with Ginsparg-Wilson fermions (overlap/domainwall) fermions[3]; but too expensive

practical determination of critical κ:

- limit of zero mass of adjoint pion $(a-\pi)$

$\Rightarrow$ definition of gluino mass: $\propto (m_{a-\pi})^2$

[3] [Fleming, Kogut, Vranas, Phys. Rev. D 64 (2001)], [Endres, Phys. Rev. D 79 (2009)], [JLQCD, PoS Lattice 2011]

The sign problem in supersymmetric theories

- $Z \propto \Delta$ (periodic boundary conditions)
- $\Delta = 0$ from fluctuating sign of fermion path integral
- Majorana fermions:

$$\int \mathcal{D}\psi e^{-\frac{1}{2}\int \bar{\psi} D\psi} = \mathrm{Pf}(CD) = \mathrm{sign}(\mathrm{Pf}(CD))\sqrt{\det D}$$

$\Rightarrow$ severe sign problem if spontaneous SUSY breaking possible[1]

[1] [Wozar, Wipf, Annals Phys. 327 (2012)], [Wenger]

The sign problem in SYM on the lattice

- continuum $SU(N_c)$ SYM theory: $\Delta = N_c$

$\Rightarrow$ no sign problem in the continuum
or with Ginsparg-Wilson fermions

- Wilson fermions: sign problem even in SYM
- reweighting: $\mathrm{sign}(\mathrm{Pf}(CD))$
- vanishes in continuum limit, not severe;
but technical problem: Pf computation

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Status of the simulations

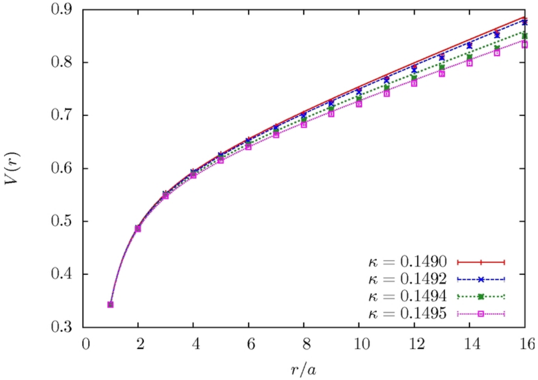
- main focus: mass-spectrum of SYM
- gauge group SU(2), adjoint: SO(3)
- simulations similar to $N_f = 1$ QCD (SU(3))
- PHMC: approximate $|\,\mathrm{Pf}(CD)|$
- improvements to reduce lattice artifacts:
  tree level Symanzik improved gauge action; stout smearing

Non-perturbative investigations on the lattice

- confinement
- same mass for particles of multiplet

	multiplet 1	**multiplet 2**
scalar	meson a $-f_0$	glueball 0^{++}
pseudoscalar	meson a $-\eta'$	glueball 0^{-+}
fermion	gluino-glue	gluino-glue

Confinement and the static quark-antiquark potential



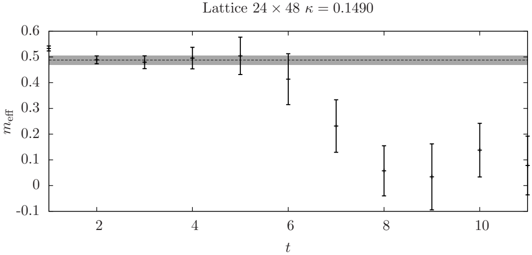
- good agreement with $V(r) = v_0 + c/r + \sigma r$ (confining)

$\Rightarrow$ sets the scale to compare with QCD/YM simulations

The spectrum of SYM on the lattice: bosonic operators

- glueball operators

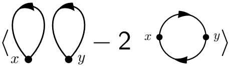
0^{++}



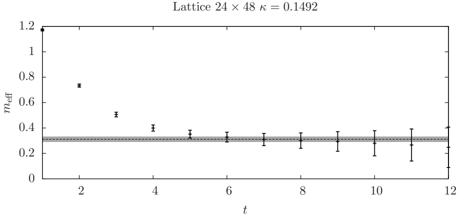
(0^{-+} currently insufficient statistic to obtain masses)

$\Rightarrow$ variational smearing methods (APE, HYP)

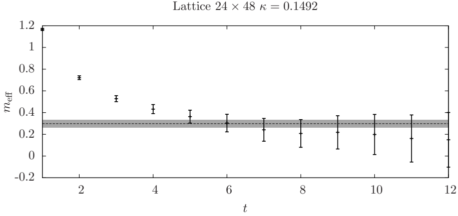
The spectrum of SYM on the lattice: bosonic operators

- mesons $\langle \bar{\psi}(x)\gamma_5\psi(x)\,\bar{\psi}(y)\gamma_5\psi(y)\rangle = \langle$  $\rangle$

pseudoscalar $a-\eta'$



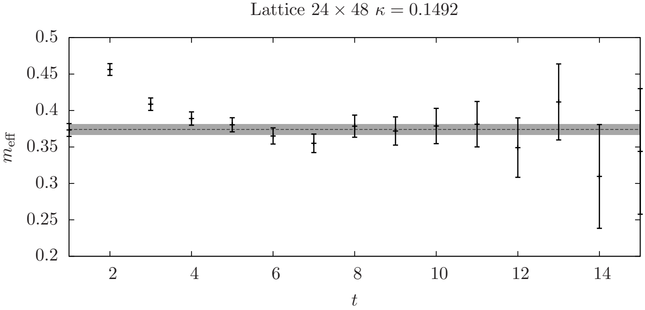
scalar $a-f_0$



$\Rightarrow$ disconnected contributions: SET + TEA/precond. TEA + TS

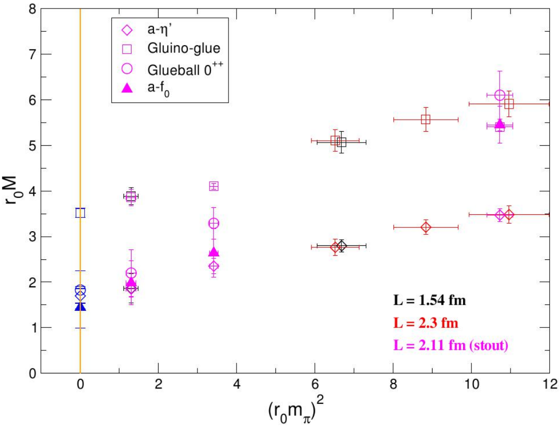
The gluino-glue particle

- gluino-glue fermionic operator $\sigma^{\mu\nu}\mathrm{Tr}[F_{\mu\nu}\psi]$
- $F_{\mu\nu}$ represented by clover plaquette



$\Rightarrow$ APE smearing on gauge fields + Jacobi smearing on ψ

Mass gap at $\beta = 1.6$ [1]



⇒ unexpected mass gap

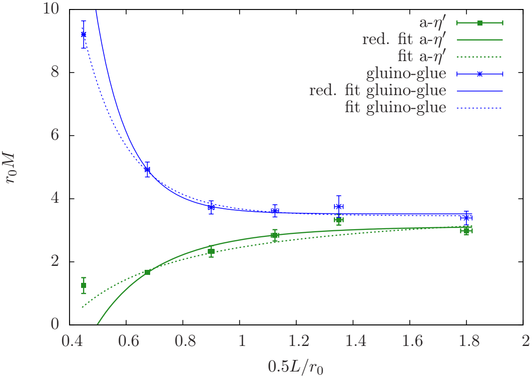
[1] [Demmouche et al., Eur.Phys.J.C69 (2010)]

Estimating finite size effects

- asymptotic behavior[1] for large L:
  $m(L) \approx m_0 + CL^{-1} \exp(-\alpha m_0 L)$
- best signal: gluino-glue
- reasonable signal: $a - \eta'$, but large deviation from asymptotic behavior due to systematic errors (excited states, disconnected contributions)
- simulations at lattice sizes:
  $8^3 \times 16$, $12^3 \times 24$, $16^3 \times 36$, $20^3 \times 40$, $24^3 \times 48$, $32^3 \times 64$
- chiral extrapolation of infinite volume limit at different $m_{a-\pi}$

[1] [Lüscher, Commun. Math. Phys. 104 (1986)], [Münster, Nucl. Phys. B 249 (1985)]

Dependence of the mass gap on the finite volume[1]

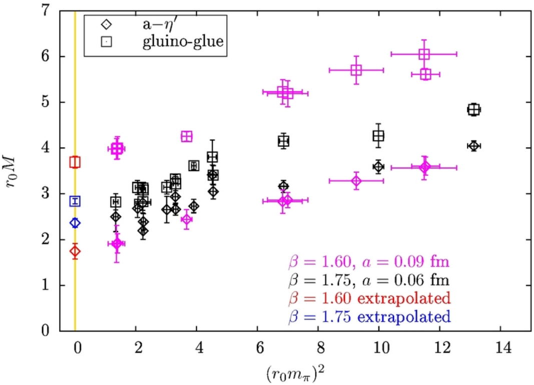


⇒ finite volume effects increase mass gap

⇒ influence of finite size effects small at moderate lattice sizes

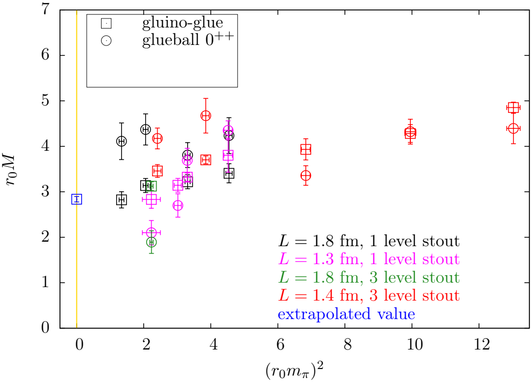
[1] [GB, Berheide, Montvay, Münster, Özugurel, Sandbrink, arXiv:1206.2341]

The influence of the finite lattice spacing



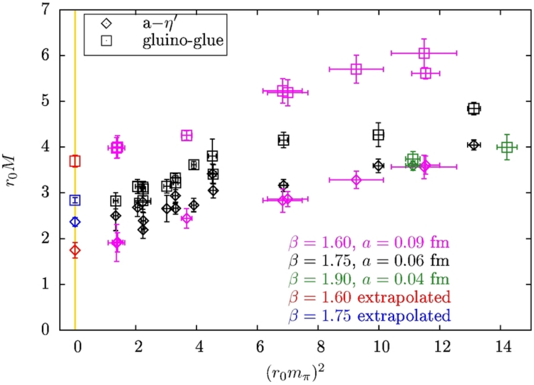
⇒ smaller lattice spacing considerably reduces the mass gap

The results of the mass spectrum: $L = 1.35\text{fm}$



- still difficult to determine glueballs and $a - f_0$
- masses of the multiplet close to each other

Final explanation for the mass gap (?)



$\Rightarrow$ very preliminary results for the finest lattice (need further investigations)

Conclusions and outlook

- effects that increase the mass gap (gluino-glue gets heavier than bosonic $a - \eta'$)
  1. finite size effects: negligible at sizes above 1.2 fm
  2. finite lattice spacing: most relevant influence
- first preliminary indications that the gap might be due to lattice artifacts
- most important current concern is large noise, especially for the scalar particles (0^{++}, $a - f_0$)

$\Rightarrow$ large statistic at moderate lattice volume

$\Rightarrow$ clover improvement under investigations